El Soussi *et al. EURASIP Journal on Wireless Communications and Networking* (2015) 2015:133
DOI 10.1186/s13638-015-0361-y

EURASIP Journal on Wireless Communications and Networking
a SpringerOpen Journal

RESEARCH Open Access

DF-based sum-rate optimization for multicarrier multiple access relay channel

Mohieddine El Soussi[1*], Abdellatif Zaidi[2,3] and Luc Vandendorpe[1]

Abstract

We consider a system that consists of two sources, a half-duplex relay and a destination. The sources want to transmit their messages reliably to the destination with the help of the relay. We study and analyze the performance of a transmission scheme in which the relay implements a decode-and-forward strategy. We assume that all the channels are frequency selective, and in order to cope with that, we incorporate Orthogonal Frequency-Division Multiplexing (OFDM) transmission into the system. In contrast to previous works, both sources can transmit their messages using all subcarriers and the relay can decide to help none, only one, or both sources. For this scheme, we discuss the design criteria and evaluate the achievable sum-rate. Next, we study and solve the problem of resource allocation aiming at maximizing the achievable sum-rate. We propose an iterative coordinate-descent algorithm that finds a solution that is at least a local optimum. We show through numerical examples the effectiveness of the algorithms and illustrate the benefits of allowing both sources to transmit on all subcarriers.

Keywords: Relay channel; Decode-and-forward; OFDM; Optimization

1 Introduction

Relaying has been introduced to extend system coverage, enhance spectrum efficiency, and improve the performance of wireless systems. Cooperative relay networks have been studied extensively for many wireless systems [1-3]. In a typical relay system, the relay helps the transmitters by forwarding the transmitted messages to the destination. Different efficient relaying protocols have been proposed in the literature, including amplify-and-forward (AF), decode-and-forward (DF), and compress-and-forward (CF) [2,4]. Each protocol has its advantages and its disadvantages; and which scheme outperforms the others depends on the network topology and channel conditions. Capacity bounds and rate regions have been established in [5] for the standard three-terminal Gaussian relay channel and in [4,6] for the Gaussian multiple access relay channel (MARC). The reader may also refer to [7-9] for some related works.

In the context of cooperative communication, multicarrier transmission techniques, such as the popular Orthogonal Frequency-Division Multiplexing (OFDM) and its multiuser version Orthogonal Frequency-Division Multiple Access (OFDMA), constitute promising tools that can offer high data rate. In particular, this is due to the fact that these techniques permit to handle frequency selectivity and harness multiuser diversity. Essentially for these reasons, these techniques have been adopted in most next-generation wireless standards and are generally considered in the context of relay-aided communications in frequency selective channels.

In this paper, we consider communication over a multicarrier two-source multiaccess channel in which the transmission is aided by a relay node, i.e., a multicarrier two-source MARC. The communication takes place over two transmission periods or time slots. The sources transmit only during the first transmission period. The relay is half-duplex, implements the decode-and-forward protocol, and transmits only during the second transmission period. We propose a multicarrier transmission scheme based on OFDM where, in contrast to the OFDMA scheme [10], each subcarrier can be used by both sources simultaneously. In this paper, we refer to this scheme as OFDM for convenience. For this scheme, we derive the achievable sum-rate. Also, we study the problem of allocating the resources and selecting the relay operation mode (i.e., active or idle) optimally in order to maximize

*Correspondence: mohieddine.elsoussi@uclouvain.be
[1]ICTEAM, Université catholique de Louvain, Place du Levant, 2, 1348 Louvain-la-Neuve, Belgium
Full list of author information is available at the end of the article

the obtained sum-rate. Some of the key issues that we consider are related to the selection of appropriate relay operation mode for every subcarrier and the allocation of power at the two sources and at the relay.

1.1 Literature overview

For a point-to-point OFDM transmission aided by a DF relay node, some resource allocation algorithms have been proposed and studied in the literature. For example, in [11], the authors investigate the problem of maximizing the sum-rate. Depending on the fading coefficients, on each subcarrier, the relay node can be either idle or active. If the relay is idle, the source transmits a new independent symbol in the second time slot. This transmission protocol is extended for the scenarios in which the transmission involves multiple relays, and the related resource allocation problems are solved in [12-14]. The problem of resource allocation over a two-way DF relayed channel has been investigated as well in [15,16].

For OFDMA systems that involve relays, some related contributions have been proposed in the literature. These include [17] and [18], in which the authors consider respectively the maximization of the achievable sum-rate and the maximization of a weighted sum goodput. In [19], the authors jointly optimize the relay strategies and physical-layer resources in a multiuser network, where each user can act as a relay. In [20] and [21], the authors study capacity regions of OFDMA multiple access networks that comprise AF and DF relays. They also investigate a problem of subcarrier assignment for given powers at the sources and the relay. The reader may also refer to [22-24] for some related works.

For multiaccess relay networks, in [7], the authors investigate a problem of power allocation for ergodic fading orthogonal MARC in which two sources communicate with a destination and with the help of a half-duplex relay. The authors show that the sum-rate belongs to one of the different cases and they optimize the power allocation in order to maximize the sum-rate for all the cases. In comparison with our setting, their relay uses non-regenerative DF [22] where it transmits, during the second transmission period, to the destination a codeword independent from the ones transmitted by the sources. However, in our setting, the relay estimates the symbols sent by the sources and then forwards them to the destination, i.e., the relay implements regenerative DF [22] in which the relay uses the same codebook as that used by the sources, and thus, it transmits the same codewords as those sent by the sources. For this reason, the decoding method and the achievable sum-rate are different. Hence, the problem formulation and the corresponding resource allocation are different as well.

The setting that we consider is connected to [10] where the authors study the problem of resource allocation for a multiuser relay network with orthogonal channel access that uses OFDMA. They consider different relay strategies and maximize the sum-rate under individual power constraint. However, the setup in [10] does not consider the case in which the sources are allowed to transmit their messages using the same subcarrier. In the current work, we show the advantage of allowing both sources to transmit on all subcarriers.

1.2 Contributions

The main contributions of this paper can be summarized as follows. For the multicarrier multiaccess relay network that we consider, we propose a transmission scheme that uses OFDM where both sources are allowed to simultaneously transmit their codewords on every subcarrier. The relay can decide to help none, only one, or both sources. Whenever it is active, the relay transmits on the *same* subcarrier as that utilized by the source(s). Also, if, for a given subcarrier, the relay helps both sources simultaneously, it re-encodes the decoded sources' codewords via superposition coding. The decoding procedure at the relay is based on successive decoding, and at the destination, it is based on successive decoding and maximum-ratio combining (MRC). In this work, we adopt successive decoding at the decoders since it has lower complexity than joint decoding. At this level, we should mention that, in contrast to a standard multiple access channel in which the achievable sum-rate does not depend on which decoding order is considered (assuming perfect decoding), in the presence of relay nodes, i.e., for multiple access relay networks, different decoding orders at the relay and at the destination generally yield different achievable sum-rates. Taking this aspect into consideration, we consider all possible decoding orders combinations and select the appropriate combination that offers the largest sum-rate. In addition to the decoding orders, the relay operation modes (i.e, helping none, only one, or both sources simultaneously) obviously also influence the sum-rate that is achievable per subcarrier, and, so, thereby the total offered sum-rate. We show that the greatest advantage of the proposed method over the OFDMA one lies in the cases where the relay helps only one of the sources. In these cases, one of the sources is clustered with the relay, i.e., it is close to the relay and far from the destination; and the other one is clustered with the destination, i.e., it is close to the destination and far from the relay. Therefore, one of the sources will be helped by the relay to transmit its message while the other source will communicate with the destination through the direct link. This provides a larger degree of freedom and significantly improves the rate since each transmitted message is decoded at the destination at the same subcarrier in a different transmission period.

For the multicarrier transmission scheme that we consider, we study and solve the problem of maximizing the offered sum-rate under individual power constraints. The optimization problem consists of i) selecting the appropriate relay operation mode (i.e., helping none, only one, or both sources simultaneously) for every subcarrier, ii) choosing the best decoding orders at the relay (if active) and the destination for every subcarrier, and iii) allocating the powers on each subcarrier and transmitting terminal. The resulting optimization problem is mixed-integer program since some of the variables are constrained to be integers, while other variables are allowed to be non-integers, and so, it is not easy to solve it optimally. We propose an iterative algorithm that is based on a coordinate descent approach and that, for every subcarrier, finds the best relay operation mode and decoding orders at the relay (if active) and the destination, and appropriate powers for the terminals transmitting on that subcarrier, alternately. The iterations stop when convergence to a stationary point is obtained. For given relay operation mode and decoding orders combination, the problem of allocating the powers appropriately is non-convex and non-linear. Since optimally solving this problem is difficult, we propose an algorithm that is based on geometric programming approach and a successive convex approximation method [25] and that provides a solution that is at least a local optimum.

Our analysis shows that by allowing the sources to possibly transmit on the same subcarrier simultaneously, one can afford a larger sum-rate, i.e., the OFDM-based transmission scheme offers a substantial sum-rate gain over the one that is based on OFDMA. The analysis also shows the convergence of the proposed algorithm with a reasonable complexity. We illustrate our results through some numerical examples.

1.3 Outline and notation

An outline of the remainder of this paper is as follows. Section 2 describes in more details the system model that we consider in this work. Section 3 contains some known results from the literature for the setup under consideration where the sources transmit on orthogonal channels, i.e., using OFDMA transmission scheme. In Section 4, we analyze the sum-rate that is achievable using the OFDM scheme. Section 5 contains the optimization problem as well as the algorithms that we propose. In Section 6, we consider an improvement to the transmission schemes described in this work. Section 7 contains some numerical examples, and Section 8 concludes the paper.

The following notations are used throughout the paper. Lowercase boldface letters are used to denote vectors, e.g., $\mathbf{x}$. Calligraphic letters designate alphabets, i.e., $\mathcal{X}$. The cardinality of a set $\mathcal{X}$ is denoted by $|\mathcal{X}|$. For vectors, we write $\mathbf{x} \in \mathbb{A}^n$, e.g., $\mathbb{A} = \mathbb{R}$ or $\mathbb{A} = \mathbb{C}$, to mean that $\mathbf{x}$ is a column vector of size n, with its elements taken from the set $\mathbb{A}$. For a vector $\mathbf{x} \in \mathbb{R}^n$, $\|\mathbf{x}\|$ designates the norm of $\mathbf{x}$ in terms of Euclidean distance. We use $[x]^+$ to denote $\max\{0, x\}$. Finally, for a complex-valued number $z = x + jy \in \mathbb{C}$, the notations $\mathrm{Re}\{z\}$ and $\mathrm{Im}\{z\}$ refer respectively to the real part and imaginary part of $z \in \mathbb{C}$, i.e., $\mathrm{Re}\{z\} = x$ and $\mathrm{Im}\{z\} = y$ and the notation z^* refer to the complex conjugate of z, i.e., $z^* = x - jy$.

2 System model

We consider a multiaccess relay network that comprises two sources (**A** and **B**), a relay node (**R**) and a destination (**D**), as shown in Figure 1. The sources **A** and **B** want to transmit their messages, $W_a \in \mathcal{W}_a$ and $W_b \in \mathcal{W}_b$, to the

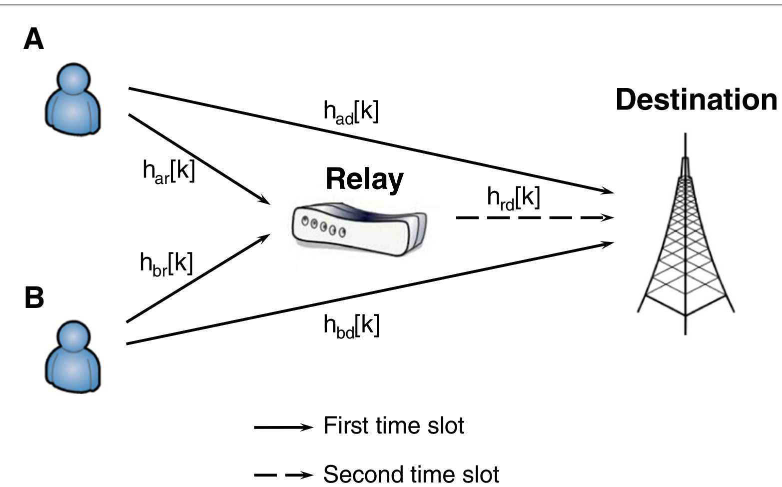

Figure 1 Multiple access relay channel with a half-duplex relay.

destination with the help of the relay. The relay is half-duplex and implements a regenerative DF strategy. The communication takes place in $2n$ channel uses (n is the number of channel uses required to transmit a codeword) and is divided into two periods or time slots with equal durations. All the channels are assumed to be frequency selective, and in order to cope with that, we incorporate OFDM transmission into the system. As usually assumed in similar settings, we assume that appropriate cyclic prefix is employed, turning the channel into a number of parallel subchannels.

There are in total K subcarriers that can be used by the sources for the transmission. In the OFDM-based transmission, both sources transmit simultaneously on the same subcarrier k. The encoding and transmission scheme on subcarrier k, $1 \leq k \leq K$ is as follows. During the first transmission period, source **A** transmits the codeword $\mathbf{x}_a[k]$ over the channel. Similarly, source **B** transmits the codeword $\mathbf{x}_b[k]$ over the channel. During this period, the outputs at the relay and the destination on subcarrier k are given by

$$\begin{aligned} \mathbf{y}_r[k] &= h_{\mathrm{ar}}[k]\,\mathbf{x}_a[k] + h_{\mathrm{br}}[k]\,\mathbf{x}_b[k] + \mathbf{z}_r[k] \\ \mathbf{y}_d[k] &= h_{\mathrm{ad}}[k]\,\mathbf{x}_a[k] + h_{\mathrm{bd}}[k]\,\mathbf{x}_b[k] + \mathbf{z}_d[k], \end{aligned} \tag{1}$$

where $h_{\mathrm{ar}}[k]$ and $h_{\mathrm{br}}[k]$ are the channel gains on the links to the relay; $h_{\mathrm{ad}}[k]$ and $h_{\mathrm{bd}}[k]$ are the channel gains on the links to the destination; the vector $\mathbf{z}_r[k]$ is the additive noise at the relay, and the vector $\mathbf{z}_d[k]$ is the additive noise at the destination. These noise vectors, on subcarrier k, are mutually independent and are independently and identically distributed (i.i.d) with components drawn according to the circular complex Gaussian distribution with zero mean and variance N.

Assuming that it decodes correctly the codewords transmitted by the sources, during the second transmission period the relay re-encodes the codewords using the same codebook employed by the sources. Thus, during this period, the output at the destination on subcarrier k is given by

$$\tilde{\mathbf{y}}_d[k] = h_{\mathrm{rd}}[k]\,\tilde{\mathbf{x}}_r[k] + \tilde{\mathbf{z}}_d[k], \tag{2}$$

where $h_{\mathrm{rd}}[k]$ is the channel gain on the link to the destination; and the vector $\tilde{\mathbf{z}}_d[k]$, on subcarrier k, is the additive noise at the destination during this period, assumed to be independent from all other noise vectors and i.i.d. with components drawn according to a circular complex Gaussian distribution with zero mean and variance N. We should note that the relay signals the destination if one or two codewords are forwarded through the control information.

Throughout the paper, we assume that the carrier frequency and symbol timing of the sources are perfectly synchronized at the relay and the destination. Also, we assume that the states of the channel are known perfectly to all terminals in which they can be estimated at the receivers and fed back to the transmitters. Thus, we assume perfect channel state information at the receivers (CSIR) and perfect channel state information at the transmitters (CSIT), and that these CSIs remain constant over a transmission period. We also assume that we have perfect decoding at the relay and the destination.

Furthermore, the noise signals at the relay and the destination are independent from each other and i.i.d circular complex Gaussian, with zero mean and variance N. Also, we consider the following individual constraints on the transmitted power,

$$\begin{aligned} \sum_{k=1}^{K} \frac{1}{n}\mathbb{E}[\,\|\mathbf{x}_a[k]\|^2] &= \sum_{k=1}^{K} \beta_a^2[k]\,P \leq P_a, \\ \sum_{k=1}^{K} \frac{1}{n}\mathbb{E}[\,\|\mathbf{x}_b[k]\|^2] &= \sum_{k=1}^{K} \beta_b^2[k]\,P \leq P_b, \\ \sum_{k=1}^{K} \frac{1}{n}\mathbb{E}[\,\|\tilde{\mathbf{x}}_r[k]\|^2] &= \sum_{k=1}^{K} \beta_r^2[k]\,P \leq P_r \end{aligned} \tag{3}$$

where $P_a \geq 0$, $P_b \geq 0$, and $P_r \geq 0$ are power constraints imposed on the system; $P \geq 0$ is given. The constraints in (3) are the total power used by source **A**, source **B**, and relay **R**, respectively, during the whole transmission.

For convenience, let $\beta_a[k] \geq 0$ and $\beta_b[k] \geq 0$ be non-negative scalars such that $\beta_a^2[k]\,P$ and $\beta_b^2[k]\,P$ be the powers used at source **A** and source **B** on subcarrier k, respectively. Similarly, let $\beta_r[k] \geq 0$ be a non-negative scalar such that $\beta_r^2[k]\,P$ be the power used by relay **R** on subcarrier k. Also, let $\beta_{\mathrm{ar}}^2[k]\,P$ be the fraction of the power that the relay uses to help source **A**, and $\beta_{\mathrm{br}}^2[k]\,P$ be the fraction of the power that the relay uses to help source **B**, with $\beta_{\mathrm{ar}}^2[k] + \beta_{\mathrm{br}}^2[k] = \beta_r^2[k]$. Finally, we will sometimes use the shorthand vector notation $\boldsymbol{\beta}[k] = [\beta_a[k], \beta_b[k], \beta_{\mathrm{ar}}[k], \beta_{\mathrm{br}}[k]]^T \in \mathbb{R}^4$.

3 Achievable sum-rate using OFDMA transmission

In this section, we present the achievable sum-rate for the MARC model that we study using the OFDMA transmission scheme [10].

In the OFDMA transmission scheme, each source transmits its messages using its allocated subcarriers. Let $\mathbb{K}_A$ and $\mathbb{K}_B$ be the sets of subcarriers assigned to source **A** and source **B**, respectively. Each source, using its allocated subcarriers, can transmit its messages either with the help of the relay, i.e., through the relay link or without the help of the relay, i.e., through the direct link. Hence, the achievable sum-rate for the OFDMA transmission scheme is given by [10],

$$R_{\text{sum}}^{\text{OFDMA}} = \frac{1}{2} \sum_{k \in \mathbb{K}_A} \max \left\{ \log_2 \left(1 + \frac{\beta_a^2[k] \, |h_{\text{ad}}[k]|^2 P}{N} \right), \min \left\{ \log_2 \left(1 + \frac{\beta_a^2[k] \, |h_{\text{ar}}[k]|^2 P}{N} \right), \log_2 \left(1 + \frac{\beta_a^2[k] \, |h_{\text{ad}}[k]|^2 P}{N} + \frac{\beta_{\text{ar}}^2[k] \, |h_{\text{rd}}[k]|^2 P}{N} \right) \right\} \right\} + \frac{1}{2} \sum_{k \in \mathbb{K}_B} \max \left\{ \log_2 \left(1 + \frac{\beta_b^2[k] \, |h_{\text{bd}}[k]|^2 P}{N} \right), \min \left\{ \log_2 \left(1 + \frac{\beta_b^2[k] \, |h_{\text{br}}[k]|^2 P}{N} \right), \log_2 \left(1 + \frac{\beta_b^2[k] \, |h_{\text{bd}}[k]|^2 P}{N} + \frac{\beta_{\text{br}}^2[k] \, |h_{\text{rd}}[k]|^2 P}{N} \right) \right\} \right\}. \tag{4}$$

We should note that to maximize $R_{\text{sum}}^{\text{OFDMA}}$, we need to properly allocate the subcarriers among the two sources and allocate the powers per subcarrier at the sources and the relay. For that, we use (Algorithm 2, [10]) to allocate the subcarriers and (Algorithm 4, [10]) to allocate the powers. These algorithms will be used for comparisons in Section 7.

4 Sum-rate analysis for the OFDM-based transmission

In this section, we describe and analyze the OFDM multicarrier transmission scheme from the achievable sum-rate viewpoint.

The following proposition provides an achievable sum-rate for the multiaccess relay model of Figure 1, using OFDM multicarrier transmission.

Proposition 1. *For given channel states* $\{h_{\text{ar}}[k], h_{\text{br}}[k], h_{\text{ad}}[k], h_{\text{bd}}[k], h_{\text{rd}}[k]\}_{k=1}^{K}$*, the following sum-rate is achievable for the multiaccess relay channel of Figure 1:*

$$\text{(P1)}: \quad R_{\text{sum}}^{\text{OFDM}} = \max \sum_{k=1}^{K} \max_{1 \le l \le 7} R_l[k], \tag{5}$$

for $1 \le k \le K$*,* $1 \le l \le 7$*, and* $R_l[k]$ *are defined as in Definition 1 in Appendix 1; the outer maximization is over* $\{\boldsymbol{\beta}[k]\}_{k=1}^{K}$*, with* $\boldsymbol{\beta}[k] = [\beta_a[k], \beta_b[k], \beta_{\text{ar}}[k], \beta_{\text{br}}[k]]^T$*, such that* $\sum_{k=1}^{K} \beta_a^2[k] P \le P_a$*,* $\sum_{k=1}^{K} \beta_b^2[k] P \le P_b$*, and* $\sum_{k=1}^{K} (\beta_{\text{ar}}^2[k] + \beta_{\text{br}}^2[k]) P \le P_r$*.*

The proof of (P1) can be found in Appendix 2. The following remark reveals certain aspects related to the coding scheme and is useful for a better understanding of the proof and its structure.

Remark 1. In this scheme, in contrast to the OFDMA scheme, both sources are allowed to simultaneously transmit on every subcarrier. The relay is half-duplex and implements regenerative decode-and-forward strategy on the symbols transmitted on each subcarrier. It can decide to help none, only one, or both sources simultaneously. If, for a given subcarrier, the relay helps both sources, it decodes the sources' codewords successively. Then, on the same subcarrier, it shares its power among the two codewords and superimposes the information that is intended to help source **A** and the one that is intended to help source **B**. The destination, using what it receives during the two transmission periods, decodes the sources' codewords successively, and the decoding operations are based on maximum-ratio combining. Different decoding orders combinations (at the relay, if applicable, and at the destination) generally result in different achievable sum-rates, and the selection of the appropriate decoding order depends on the fading coefficients and the allocated power. In addition to the decoding orders at the relay and the destination, the relay operation mode (i.e, helping none, only one, or both sources) influences the achievable sum-rate. This leads to 13 different cases if all possible combinations are considered using the decoding orders and the relay operation modes. However, it can be shown that whenever the relay helps only one of the sources (by decoding and forwarding the codeword transmitted by that source), this codeword should be decoded first at the destination. When the relay helps the two sources simultaneously, a total of four possible decoding orders need to be investigated and compared (two possible decoding orders at the destination for each possible decoding order

Table 1 Different useful cases for the OFDM multicarrier transmission

	Decoding order at the relay	Decoding order at the destination	Case
Direct transmission	N.A.	No decoding order	1
The relay forwards $\mathbf{x}_b[k]$	$\mathbf{x}_b[k]$	$\mathbf{x}_b[k] \rightarrow \mathbf{x}_a[k]$	2
The relay forwards $\mathbf{x}_a[k]$ and $\mathbf{x}_b[k]$	$\mathbf{x}_b[k] \rightarrow \mathbf{x}_a[k]$	$\mathbf{x}_b[k] \rightarrow \mathbf{x}_a[k]$	3
The relay forwards $\mathbf{x}_a[k]$ and $\mathbf{x}_b[k]$	$\mathbf{x}_a[k] \rightarrow \mathbf{x}_b[k]$	$\mathbf{x}_b[k] \rightarrow \mathbf{x}_a[k]$	4
The relay forwards $\mathbf{x}_a[k]$	$\mathbf{x}_a[k]$	$\mathbf{x}_a[k] \rightarrow \mathbf{x}_b[k]$	5
The relay forwards $\mathbf{x}_a[k]$ and $\mathbf{x}_b[k]$	$\mathbf{x}_a[k] \rightarrow \mathbf{x}_b[k]$	$\mathbf{x}_a[k] \rightarrow \mathbf{x}_b[k]$	6
The relay forwards $\mathbf{x}_a[k]$ and $\mathbf{x}_b[k]$	$\mathbf{x}_b[k] \rightarrow \mathbf{x}_a[k]$	$\mathbf{x}_a[k] \rightarrow \mathbf{x}_b[k]$	7

N.A., not applicable.

at the relay). Hence, out of the 13 *a priori* possible cases only 7 actually are of interest. These cases are summarized in Table 1, and their corresponding sum-rates, $R_l[k]$, $1 \leq l \leq 7$, are given in Definition 1.

Remark 2. The greatest advantage of the OFDM scheme of (P1) over the OFDMA scheme lies in the cases where the relay helps only one of the sources, i.e., case 2 and case 5. In these cases, one of the sources is clustered with the relay, i.e., it is close to the relay and far from the destination; the other one is clustered with the destination, i.e., it is close to the destination and far from the relay. To illustrate this point, we consider a scenario where each source has a strong link and a weak link. Let us suppose that, without loss of generality, source **A** has a strong relay link, i.e., it is close to the relay, and a weak direct link, i.e., it is far from the destination, and source **B** has a weak relay link and a strong direct link. This means that, during the two transmission periods and by allowing the sources to simultaneously transmit on the same subcarrier, source **A** communicates with the destination through the relay link and source **B** communicates with the destination through the direct link. Note that the interference generated by source **A** on the direct link is small since source **A** has a weak direct link. Similarly, the interference generated by source **B** on the relay link is small since source **B** has a weak relay link. Thus, using the proposed scheme, the optimal policy for source **A** and source **B** can be approximated to be a water-filling solution over all subcarriers to the relay and to the destination, respectively, and the optimal policy for the relay can be approximated to be a water-filling solution over all subcarriers to the destination. On the contrary, using the OFDMA scheme, the optimal policy for each source is a water-filling solution over the allocated subcarriers. Therefore, the proposed scheme uses the two transmission periods to serve both sources, and each source has access to a larger container (subcarriers) during the water-filling solution which yields a higher sum-rate compared with the OFDMA scheme. As a result, the proposed scheme has a larger degree of freedom compared with the OFDMA scheme. This is illustrated through some numerical examples as shown in Section 7.

Remark 3. We should note that the OFDM scheme of (P1) always outperforms the OFDMA scheme, and in worst case scenario, it has the same performance. This can be verified by investigating the achievable sum-rate of both schemes. It can easily be seen that the optimum power policy $\{\boldsymbol{\beta}^{\star}[k]\}_{k=1}^{K}$ obtained by maximizing the sum-rate of the OFDMA scheme yields the same sum-rate if it is used with the OFDM scheme. Thus, the OFDMA transmission scheme acts as a lower bound for the OFDM transmission scheme.

Remark 4. As described in Remark 1, there exist seven cases for the two-source MARC. However, in order to decrease the computational complexity, we can consider only three cases (1, 2, and 5) and still benefit from the larger degree of freedom (e.g., see Remark 2). We should note that by considering cases 1, 2, and 5, the complexity is dramatically reduced with the expense of a lower transmission sum-rate in some regimes as we will see in the numerical examples in Section 7.

Remark 5. The system model that we study can be extended to the case of multiple sources. This can be done by allocating a subcarrier k to only two sources, the first source is close to the relay and the second source is close to the destination. This means that only cases 2 and 5 are considered. In this way, we can benefit from a larger degree of freedom, decrease the complexity, and achieve a larger sum-rate as explained in Remark 2.

5 Sum-rate optimization

In this section, we study the problem of maximizing the offered sum-rate given in (5) under individual power constraints. The optimization problem comprises i) selecting the appropriate relay operation mode (i.e., helping none, only one, or both sources simultaneously) for every subcarrier, ii) choosing the best decoding orders at the relay (if active) and at the destination for every subcarrier, and iii) allocating the powers on each subcarrier at the transmitting terminals. In what follows, we study the optimization problem in its general form, i.e., considering the seven cases; however, this can be modified to the situation where less cases are considered.

5.1 Problem formulation

Consider the sum-rate $R_{\text{sum}}^{\text{OFDM}}$ as given by (5) in (P1). The optimization problem can be equivalently stated as

$$\text{(A)} \ : \ \max \qquad \sum_{k=1}^{K}\sum_{l=1}^{7} a_l[k]\, R_l[k]\,, \tag{6}$$

for $1 \leq k \leq K$, $1 \leq l \leq 7$, and $a_l[k]$ is an indicator whose value should be 0 or 1, and $R_l[k]$ is defined as in Definition 1; the maximization is over $\{\boldsymbol{\beta}[k]\}_{k=1}^{K}$, satisfying

$$\sum_{k=1}^{K}\beta_a^2[k]\,P \leq P_a,\ \sum_{k=1}^{K}\beta_b^2[k]\,P \leq P_b,\ \sum_{k=1}^{K}\left(\beta_{\text{ar}}^2[k]+\beta_{\text{br}}^2[k]\right)P \leq P_r, \tag{7}$$

and over $\{\mathbf{a}[k]\}_{k=1}^{K}$, with $\mathbf{a}[k] = [a_1[k], a_2[k], \ldots, a_7[k]]^T$, such that

$$\|\mathbf{a}[k]\|^2 \leq 1, \ \text{ for } 1 \leq k \leq K. \tag{8}$$

The optimization problem (A) is a mixed-integer program, and, so, it is not easy to solve it optimally. We propose an iterative optimization where we find the appropriate powers $\{\boldsymbol{\beta}[k]\}_{k=1}^{K}$ and indicators $\{\mathbf{a}[k]\}_{k=1}^{K}$, alternately. We should note that the selection of $\{\mathbf{a}[k]\}_{k=1}^{K}$ determines the decoding orders at the relay and the destination, and the relay operation mode (i.e., helping none, only one, or both sources simultaneously).

Let us, with a slight abuse of notation, denote by $R_{\text{sum}}^{\text{OFDM}}[\iota]$ the value of the sum-rate at some iteration $\iota \geq 0$. We develop the following iterative algorithm 'Algorithm IP' to allocate the indicators and the powers alternately in such a way that $R_{\text{sum}}^{\text{OFDM}}$ is maximized.

Algorithm IP Iterative algorithm for computing $R_{\text{sum}}^{\text{OFDM}}$ as given by (5)

1: Initialization: set $\iota = 1$
2: Set $\{\boldsymbol{\beta}[k] = \boldsymbol{\beta}^{(\iota-1)}[k]\}_{k=1}^{K}$ in (6), and solve the obtained problem as we will describe in Section 5.2. Denote by $\{\mathbf{a}^{(\iota)}[k]\}_{k=1}^{K}$ the found $\{\mathbf{a}[k]\}_{k=1}^{K}$
3: Set $\{\mathbf{a}[k] = \mathbf{a}^{(\iota)}[k]\}_{k=1}^{K}$ in (6), and solve the obtained problem using 'Algorithm P.' Denote by $\{\boldsymbol{\beta}^{(\iota)}[k]\}_{k=1}^{K}$ the found $\{\boldsymbol{\beta}[k]\}_{k=1}^{K}$
4: Increment the iteration index as $\iota = \iota + 1$, and go back to step 2
5: Terminate if $|R_{\text{sum}}^{\text{OFDM}}[\iota] - R_{\text{sum}}^{\text{OFDM}}[\iota-1]| \leq \epsilon_1$

In 'Algorithm IP', we compute the power values given by $\{\boldsymbol{\beta}[k]\}_{k=1}^{K}$ and the indicator values given by $\{\mathbf{a}[k]\}_{k=1}^{K}$, alternately. More specifically, at iteration $\iota \geq 1$, the algorithm computes appropriate indicator values $\{\mathbf{a}^{(\iota)}[k]\}_{k=1}^{K}$ that maximize (6) with the choice of the power values $\{\boldsymbol{\beta}[k]\}_{k=1}^{K}$ set to their values obtained from the previous iteration, i.e., $\{\boldsymbol{\beta}[k] = \boldsymbol{\beta}^{(\iota-1)}[k]\}_{k=1}^{K}$ (for the initialization, we set $\{\boldsymbol{\beta}^{(0)}[k]\}_{k=1}^{K}$ according to a uniform power allocation). This subproblem is an integer linear program (ILP) problem [26] and can be solved by selecting the case that yields the largest sum-rate $R_l[k]$, $1 \leq l \leq 7$, on each subcarrier k. Next, the power values $\{\boldsymbol{\beta}^{(\iota)}[k]\}_{k=1}^{K}$ can be computed in order to maximize (6) with the choice of $\{\mathbf{a}[k] = \mathbf{a}^{(\iota)}[k]\}_{k=1}^{K}$. This subproblem can be formulated as a complementary geometric programming problem that is an intractable nondeterministic polynomial-time (NP)-hard problem. To obtain a solution for the power values $\{\boldsymbol{\beta}^{(\iota)}[k]\}_{k=1}^{K}$, we use a successive convex optimization approach and a geometric programming (see 'Algorithm P' below). The iterative algorithm ('Algorithm IP') terminates if $|R_{\text{sum}}^{\text{OFDM}}[\iota] - R_{\text{sum}}^{\text{OFDM}}[\iota-1]|$ is smaller than a prescribed small strictly positive constant ϵ_1 - in this case, the maximized sum-rate is $R_{\text{sum}}^{\text{OFDM}}[\iota]$ and is attained using the power values $\{\boldsymbol{\beta}^{\star}[k] = \boldsymbol{\beta}^{(\iota)}[k]\}_{k=1}^{K}$ and indicator values $\{\mathbf{a}^{\star}[k] = \mathbf{a}^{(\iota)}[k]\}_{k=1}^{K}$.

In the following two sections, we study the aforementioned two subproblems of problem (A) and describe the proposed algorithms.

5.2 Indicator allocation

In this section, we aim at finding the indicator values $\{\mathbf{a}[k]\}_{k=1}^{K}$ for a given choice of power values $\{\boldsymbol{\beta}[k]\}_{k=1}^{K}$. The objective function in (6) can be stated as

$$\max \quad \sum_{k=1}^{K}\sum_{l=1}^{7} a_l[k]\, R_l[k]\,, \tag{9a}$$

$$\text{s. t.} \quad \|\mathbf{a}[k]\|^2 \leq 1, \text{ for } 1 \leq k \leq K \tag{9b}$$

$$a_l[k] \in \{0,1\},\ l \in \{1,2,\ldots,7\}, \text{ for } 1 \leq k \leq K. \tag{9c}$$

We can see, from (9a), that the optimum value of $\mathbf{a}[k]$, at a subcarrier k, can be obtained by investigating the sum-rate $R_l[k]$ for $1 \leq l \leq 7$. The indicator $\mathbf{a}[k]$ is calculated in such a way that the largest sum-rate $R_l[k]$ is selected, and it is given by

$$a_l[k] = \begin{cases} 1, & l = \arg\max_{1\leq l\leq 7}\ R_l[k] \\ 0, & \text{otherwise.} \end{cases}$$

Hence, the largest sum-rate at subcarrrier k is

$$\tilde{R}[k] = \max_{1\leq l\leq 7}\ R_l[k]\,. \tag{10}$$

5.3 Power allocation

In this section, we aim at calculating $\{\boldsymbol{\beta}[k]\}_{k=1}^{K}$ for a given choice of $\{\mathbf{a}[k]\}_{k=1}^{K}$. The objective function in (6) can be stated as

$$\max \quad \sum_{k=1}^{K} \tilde{R}[k]\,, \tag{11a}$$

$$\text{s. t.} \quad \sum_{k=1}^{K} \beta_a^2[k]\,P \leq P_a, \quad \sum_{k=1}^{K} \beta_b^2[k]\,P \leq P_b,$$

$$\sum_{k=1}^{K} \left(\beta_{\text{ar}}^2[k] + \beta_{\text{br}}^2[k]\right) P \leq P_r, \tag{11b}$$

$$\beta_i[k] \geq 0,\ i \in \{a, b, \text{ar}, \text{br}\}, \text{ for } 1 \leq \text{k} \leq \text{K} \tag{11c}$$

where $\tilde{R}[k]$ is given in (10). The maximization of $\sum_{k=1}^{K}\tilde{R}[k]$ can be equivalently stated as the minimization of $2^{-2\sum_{k=1}^{K}\tilde{R}[k]}$ which is the minimization of $\prod_{k=1}^{K}\max\{f_1(\boldsymbol{\beta}[k]), f_2(\boldsymbol{\beta}[k])\}\max\{f_3(\boldsymbol{\beta}[k]), f_4(\boldsymbol{\beta}[k])\}$. The functions $f_j(\boldsymbol{\beta}[k])$, $1 \leq j \leq 4$, are given in Table 2 for the cases described in Section 4. For brevity, we did not include in Table 2 the functions of the remaining three cases (case 5, case 6, and case 7), since these functions can be obtained from the functions of case 2, case 3, and case 4, respectively, by swapping the indices a and b. Thus,the optimization problem to maximize $\sum_{k=1}^{K}\tilde{R}[k]$ can be equivalently written as

Table 2 Useful functions for the analysis of the cases described in Section 4

	$\left(f_1(\beta[k])\right)^{-1}$	$\left(f_2(\beta[k])\right)^{-1}$	$\left(f_3(\beta[k])\right)^{-1}$	$\left(f_4(\beta[k])\right)^{-1}$
Case 1	$1+\frac{\beta_a^2[k]\lvert h_{\mathrm{ad}}[k]\rvert^2 P}{N}$	$1+\frac{\beta_a^2[k]\lvert h_{\mathrm{ad}}[k]\rvert^2 P}{N}$	$1+\frac{\beta_b^2[k]\lvert h_{\mathrm{bd}}[k]\rvert^2 P}{N+\beta_a^2[k]\lvert h_{\mathrm{ad}}[k]\rvert^2 P}$	$1+\frac{\beta_b^2[k]\lvert h_{\mathrm{bd}}[k]\rvert^2 P}{N+\beta_a^2[k]\lvert h_{\mathrm{ad}}[k]\rvert^2 P}$
Case 2	$1+\frac{\beta_a^2[k]\lvert h_{\mathrm{ad}}[k]\rvert^2 P}{N}$	$1+\frac{\beta_a^2[k]\lvert h_{\mathrm{ad}}[k]\rvert^2 P}{N}$	$1+\frac{\beta_b^2[k]\lvert h_{\mathrm{br}}[k]\rvert^2 P}{N+\beta_a^2[k]\lvert h_{\mathrm{ar}}[k]\rvert^2 P}$	$1+\frac{\beta_b^2[k]\lvert h_{\mathrm{bd}}[k]\rvert^2 P}{N+\beta_a^2[k]\lvert h_{\mathrm{ad}}[k]\rvert^2 P}+\frac{\beta_{\mathrm{br}}^2[k]\lvert h_{\mathrm{rd}}[k]\rvert^2 P}{N}$
Case 3	$1+\frac{\beta_a^2[k]\lvert h_{\mathrm{ar}}[k]\rvert^2 P}{N}$	$1+\frac{\beta_a^2[k]\lvert h_{\mathrm{ad}}[k]\rvert^2 P}{N}+\frac{\beta_{\mathrm{ar}}^2[k]\lvert h_{\mathrm{rd}}[k]\rvert^2 P}{N}$	$1+\frac{\beta_b^2[k]\lvert h_{\mathrm{br}}[k]\rvert^2 P}{N+\beta_a^2[k]\lvert h_{\mathrm{ar}}[k]\rvert^2 P}$	$1+\mathrm{snr}_b[k]$
Case 4	$1+\frac{\beta_a^2[k]\lvert h_{\mathrm{ar}}[k]\rvert^2 P}{N+\beta_b^2[k]\lvert h_{\mathrm{br}}[k]\rvert^2 P}$	$1+\frac{\beta_a^2[k]\lvert h_{\mathrm{ad}}[k]\rvert^2 P}{N}+\frac{\beta_{\mathrm{ar}}^2[k]\lvert h_{\mathrm{rd}}[k]\rvert^2 P}{N}$	$1+\frac{\beta_b^2[k]\lvert h_{\mathrm{br}}[k]\rvert^2 P}{N}$	$1+\mathrm{snr}_b[k]$

$$
\begin{aligned}
\min \quad & \prod_{k=1}^{K} \Delta_a[k]\,\Delta_b[k], && \text{(12a)}\\
\text{s. t.} \quad & \Delta_a[k] \geq f_1(\boldsymbol{\beta}[k]),\ \ \Delta_a[k] \geq f_2(\boldsymbol{\beta}[k]) && \text{(12b)}\\
& \Delta_b[k] \geq f_3(\boldsymbol{\beta}[k]),\ \ \Delta_b[k] \geq f_4(\boldsymbol{\beta}[k]) && \text{(12c)}\\
& \sum_{k=1}^{K} \beta_a^2[k]\,P \leq P_a,\ \ \sum_{k=1}^{K} \beta_b^2[k]\,P \leq P_b, && \\
& \sum_{k=1}^{K} \left(\beta_{\mathrm{ar}}^2[k] + \beta_{\mathrm{br}}^2[k]\right) P \leq P_r, && \text{(12d)}\\
& \beta_i[k] \geq 0,\ i \in \{a, b, \mathrm{ar}, \mathrm{br}\}, && \text{(12e)}\\
& \Delta_a[k] \geq 0,\ \Delta_b[k] \geq 0 \in \mathbb{R},\ \text{for } 1 \leq k \leq K && \text{(12f)}
\end{aligned}
$$

where $\Delta_a[k]$ and $\Delta_b[k]$ are simultaneously extra optimization variables and the objective function. Also, it is easy to see that the power values $\{\boldsymbol{\beta}[k]\}_{k=1}^{K}$ that achieve the minimum value of $\prod_{k=1}^{K} \Delta_a[k]\,\Delta_b[k]$ also achieve the maximum value of the objective function in (6).

The optimization problem (12) is non-linear and non-convex. Thus, it is not easy to obtain the optimum solution with reasonable complexity. We consider geometric programming (GP) to obtain a solution for $\{\boldsymbol{\beta}[k]\}_{k=1}^{K}$. GP is a special form of convex optimization for which efficient algorithms have been developed [25,27].

We can rewrite the optimization problem (12) as

$$
\begin{aligned}
\min \quad & \prod_{k=1}^{K} \Delta_a[k]\,\Delta_b[k], && \text{(13a)}\\
\text{s. t.} \quad & \frac{p_1(\boldsymbol{\beta}[k], \Delta_a[k])}{g_1(\boldsymbol{\beta}[k], \Delta_a[k])} \leq 1,\ \ \frac{p_2(\boldsymbol{\beta}[k], \Delta_a[k])}{g_2(\boldsymbol{\beta}[k], \Delta_a[k])} \leq 1 && \text{(13b)}\\
& \frac{p_3(\boldsymbol{\beta}[k], \Delta_b[k])}{g_3(\boldsymbol{\beta}[k], \Delta_b[k])} \leq 1,\ \ \frac{p_4(\boldsymbol{\beta}[k], \Delta_b[k])}{g_4(\boldsymbol{\beta}[k], \Delta_b[k])} \leq 1 && \text{(13c)}\\
& \sum_{k=1}^{K} \beta_a^2[k]\,P \leq P_a,\ \ \sum_{k=1}^{K} \beta_b^2[k]\,P \leq P_b, && \\
& \sum_{k=1}^{K} \left(\beta_{\mathrm{ar}}^2[k] + \beta_{\mathrm{br}}^2[k]\right) P \leq P_r, && \text{(13d)}\\
& \beta_i^2[k] \geq 0,\ \ i \in \{a, b, \mathrm{ar}, \mathrm{br}\}, && \text{(13e)}\\
& \Delta_a[k] \geq 0,\ \Delta_b[k] \geq 0 \in \mathbb{R},\ \text{for } 1 \leq k \leq K, && \text{(13f)}
\end{aligned}
$$

where $p_j()$ and $g_j()$, $j = 1, \ldots, 4$, are posynomial functions [25] that can be obtained from the ratios $f_j()/\Delta_a$, $j = 1, 2$, and $f_j()/\Delta_b$, $j = 3, 4$. The calculation of $p_j()$ and $g_j()$ is omitted for brevity. Note that the constraints in (13b) and (13c) correspond to the constraints in (12b) and (12c), respectively.

The constraints in (13b) and (13c) contain functions that are non-posynomial since a ratio of two posynomials is in general not a posynomial [27]. Minimizing or upper bounding a ratio between two posynomials belongs to a class of non-convex problems known as complementary GP (CGP) [25]. Complementary GP is an intractable NP-hard problem. Since optimally solving this problem is difficult, we propose a method that provides a solution that is at least a local optimum. In this method, we find the power vectors $\{\boldsymbol{\beta}[k]\}_{k=1}^{K}$ by series of approximations each of which can be solved in an efficient way. This means that the power vectors $\{\boldsymbol{\beta}[k]\}_{k=1}^{K}$ are obtained first by turning CGP into GP by approximating the denominator of the ratio of posynomials, $g_j(\boldsymbol{\beta}[k], \Delta_i[k])$, with a monomial $\tilde{g}_j(\boldsymbol{\beta}[k], \Delta_i[k])$, then solving the resulting GP problem using interior point approach. To improve the accuracy of the approximation, the found solution of the GP problem is used as initial value to approximate again the posynomial function $g_j(\boldsymbol{\beta}[k], \Delta_i[k])$ with a monomial. Then the resulting GP problem is solved using an interior point approach. This process is repeated until convergence as described in 'Algorithm P' which is provably convergent

[27] since all the conditions for convergence (section IV.A, [27]) are satisfied. Thus, we find the power vectors $\{\boldsymbol{\beta}[k]\}_{k=1}^{K}$ by solving a series of GPs. Each GP in the iteration loop tries to improve the accuracy of the approximation to a particular minimum in the original feasible region. We should note that we use (Lemma 1, [27]) to approximate the posynomial function $g_j(\boldsymbol{\beta}[k], \Delta_i[k])$ with a monomial function $\tilde{g}_j(\boldsymbol{\beta}[k], \Delta_i[k])$ around some initial value.

The solution of the problem obtained using the convex approximations is also solution for the original problem (13), i.e., satisfies the Karush-Kuhn-Tucker (KKT) conditions of the original problem [27].

Algorithm P Power allocation

1: Set $\left\{\boldsymbol{\beta}^{(0)}[k]\right\}_{k=1}^{K}$ to an initial value. Compute $\left\{\Delta_a^{(0)}[k]\right\}_{k=1}^{K}$ and $\left\{\Delta_b^{(0)}[k]\right\}_{k=1}^{K}$ using $\left\{\boldsymbol{\beta}^{(0)}[k]\right\}_{k=1}^{K}$ and set $\iota_1 = 1$ and $k = 1$
2: **While** $k \leq K$ **do**
3: Approximate $g_1\left(\boldsymbol{\beta}^{(\iota_1)}[k], \Delta_a^{(\iota_1)}[k]\right)$ with $\tilde{g}_1\left(\boldsymbol{\beta}^{(\iota_1)}[k], \Delta_a^{(\iota_1)}[k]\right)$ and $g_2\left(\boldsymbol{\beta}^{(\iota_1)}[k], \Delta_a^{(\iota_1)}[k]\right)$ with $\tilde{g}_2\left(\boldsymbol{\beta}^{(\iota_1)}[k], \Delta_a^{(\iota_1)}[k]\right)$ around $\boldsymbol{\beta}^{(\iota_1-1)}[k]$ and $\Delta_a^{(\iota_1-1)}[k]$ using (Lemma 1, [27])
4: Approximate $g_3\left(\boldsymbol{\beta}^{(\iota_1)}[k], \Delta_b^{(\iota_1)}[k]\right)$ with $\tilde{g}_3\left(\boldsymbol{\beta}^{(\iota_1)}[k], \Delta_b^{(\iota_1)}[k]\right)$ and $g_4\left(\boldsymbol{\beta}^{(\iota_1)}[k], \Delta_b^{(\iota_1)}[k]\right)$ with $\tilde{g}_4\left(\boldsymbol{\beta}^{(\iota_1)}[k], \Delta_b^{(\iota_1)}[k]\right)$ around $\boldsymbol{\beta}^{(\iota_1-1)}[k]$ and $\Delta_b^{(\iota_1-1)}[k]$ using (Lemma 1, [27])
5: Increment k as $k = k + 1$
6: **end while**
7: Solve the resulting approximated GP problem using an interior point approach. Denote the found solutions as $\left\{\boldsymbol{\beta}^{(\iota_1)}[k]\right\}_{k=1}^{K}$, $\left\{\Delta_a^{(\iota_1)}[k]\right\}_{k=1}^{K}$, and $\left\{\Delta_b^{(\iota_1)}[k]\right\}_{k=1}^{K}$
8: Increment the iteration index as $\iota_1 = \iota_1 + 1$ and go back to Step 2 using $\{\boldsymbol{\beta}[k]\}_{k=1}^{K}$, $\{\Delta_a[k]\}_{k=1}^{K}$ and $\{\Delta_b[k]\}_{k=1}^{K}$ of step 7
9: Terminate if $|R_{\text{sum}}^{\text{OFDM}}[\iota_1] - R_{\text{sum}}^{\text{OFDM}}[\iota_1 - 1]| \leq \epsilon_2$

6 Improved transmission schemes

In this section, we consider an improvement to the two transmission schemes described in Sections 3 and 4. In the improved transmission scheme, the sources, in addition to the transmitted messages during the first transmission period, transmit new independent messages during the second transmission period whenever the relay is idle (not active). This means that, in the OFDM transmission scheme, source **A**, on subcarrier k, transmits a codeword $\mathbf{x}_a[k]$ during the first transmission period and then transmits a new independent codeword $\tilde{\mathbf{x}}_a[k]$ during the second transmission period. Similarly, source **B** transmits a codeword $\mathbf{x}_b[k]$ during the first transmission period and then transmits a new independent codeword $\tilde{\mathbf{x}}_b[k]$ during the second transmission period. Hence, the achievable sum-rate for case 1 defined in Definition 1 can be rewritten as

$$R_1[k] = \log_2\left(1 + \frac{\beta_a^2[k]\,|h_{\text{ad}}[k]|^2 P}{2N} + \frac{\beta_b^2[k]\,|h_{\text{bd}}[k]|^2 P}{2N}\right). \tag{14}$$

Similarly, for the OFDMA transmission scheme, whenever the relay is not active on subcarrier k, the source transmits a new independent message during the second transmission period. The achievable sum-rate for the OFDMA transmission scheme can be rewritten as

$$\begin{aligned} R_{\text{sum}}^{\text{OFDMA}} = \frac{1}{2} \sum_{k \in \mathbb{K}_A} \max \Bigg\{ & 2\log_2\left(1 + \frac{\beta_a^2[k]\,|h_{\text{ad}}[k]|^2 P}{2N}\right), \\ & \min\Bigg\{\log_2\left(1 + \frac{\beta_a^2[k]\,|h_{\text{ar}}[k]|^2 P}{N}\right), \\ & \log_2\left(1 + \frac{\beta_a^2[k]\,|h_{\text{ad}}[k]|^2 P}{N} + \frac{\beta_{\text{ar}}^2[k]\,|h_{\text{rd}}[k]|^2 P}{N}\right)\Bigg\}\Bigg\} \\ + \frac{1}{2} \sum_{k \in \mathbb{K}_B} \max \Bigg\{ & 2\log_2\left(1 + \frac{\beta_b^2[k]\,|h_{\text{bd}}[k]|^2 P}{2N}\right), \\ & \min\Bigg\{\log_2\left(1 + \frac{\beta_b^2[k]\,|h_{\text{br}}[k]|^2 P}{N}\right), \\ & \log_2\left(1 + \frac{\beta_b^2[k]\,|h_{\text{bd}}[k]|^2 P}{N} + \frac{\beta_{\text{br}}^2[k]\,|h_{\text{rd}}[k]|^2 P}{N}\right)\Bigg\}\Bigg\}. \end{aligned} \tag{15}$$

The optimization problem to maximize the sum-rate with the improved OFDM transmission scheme can be formulated and solved using the algorithm described in Section 5. This can be done by replacing the sum-rate of case 1 ($R_1[k]$) given in (16) by the one given in (14). Similarly, the optimization problem to maximize the sum-rate of the improved OFDMA transmission scheme can be solved using the 'Algorithm 2' and 'Algorithm 4' as described in [10].

7 Numerical examples

Throughout this section, we set the number of subcarriers to $K = 128$. The channel impulse response (CIR) between node i and node j is modeled as a delay line with length $L = 32$ taps. The taps are assumed to be i.i.d circular complex Gaussian distributed with zero mean and variance σ_{ij}^2. More specifically, the taps have a variance σ_{ar}^2 for the link from source **A** to the relay, σ_{br}^2 for the link from source **B** to the relay, and σ_{rd}^2 for the link from the relay to the destination. Similar assumptions and notations are used for the direct links from the sources to the destination. The frequency response $\{h_{\rm ar}, h_{\rm br}\}$, $\{h_{\rm ad}, h_{\rm bd}\}$, and $\{h_{\rm rd}\}$ are computed by taking K-points fast Fourier transform of the CIRs. Furthermore, we assume that, at every time instant, all the nodes know, or can estimate with high accuracy, the values taken by the channel coefficients, $\{h_{\rm ar}, h_{\rm br}, h_{\rm ad}, h_{\rm bd}, h_{\rm rd}\}$, at that time, i.e., CSIs are assumed to be perfectly known. Also, we set $P_a = P_b = P_r = P = 20$ dBW.

In order to illustrate the theoretical analysis and the effectiveness of the OFDM transmission scheme of (P1), we compare it with the OFDMA transmission scheme. To maximize the sum-rate of the OFDMA scheme, we use (Algorithm 2, [10]) to allocate the subcarriers and (Algorithm 4, [10]) to allocate the powers. Recall that OFDMA-based scheme allows only one source to transmit on each subcarrier. On the contrary, OFDM-based scheme does not have such a restriction and it allows the sources to transmit on all subcarriers.

Figure 2 depicts the sum-rate obtained using the OFDMA transmission scheme of [10], i.e., $R_{\rm sum}^{\rm OFDMA}$ ([10]), and the sum-rate obtained using the OFDM transmission scheme of (P1) under different levels of complexity: i) all the decoding orders and all the relay operation modes are considered, i.e., $R_{\rm sum}^{\rm OFDM}$ (All cases), ii) only the cases where the decoding orders are the same at the relay and the destination, and the relay helps none or one source are considered, i.e., $R_{\rm sum}^{\rm OFDM}$ (Cases 1,2,5). The sum-rates are taken as functions of $10\log(P/N)$ (in decibels). We should note that the curves correspond to numerical values of channel coefficients chosen such that $\sigma_{\rm ar}^2 = \sigma_{\rm br}^2 = 20$ dB, $\sigma_{\rm rd}^2 = 20$ dB, and $\sigma_{\rm ad}^2 = \sigma_{\rm bd}^2 = 20$ dB.

For the example shown in Figure 2, we observe that the transmission scheme of (P1) outperforms the transmission scheme of [10] in terms of sum-rate. Also, we observe that the transmission scheme of (P1) considering all cases and the transmission scheme of (P1) considering cases '1, 2, 5' give almost the same performance in terms of sum-rate. However, it can easily be seen that the complexity of the latter is less than that of the former since in the latter, we only consider three cases instead of seven cases.

Figures 3 and 4 depict the same curves for other combinations of channel coefficients. We observe that the gap between the transmission scheme of (P1) and the transmission scheme of [10] in Figure 3 is larger compared to the gap shown in the other figures (related to this aspect, recall the discussion in Remark 2). For the example shown in Figure 3, we can observe that since the source-relay link of source **A** is 26 dB stronger than the source-relay link of source **B** and the source-destination link of source **B** is 26 dB stronger than the source-destination link of source **A**, both sources can reliably transmit their messages using all the subcarriers during the two transmission periods with a small interference. This explains the reason why the sum-rate of (P1) is much larger than the sum-rate of [10] and shows the advantage of the OFDM scheme given in (P1) over the OFDMA scheme. Also, we notice that the OFDM scheme has higher degree of freedom compared with the OFDMA scheme.

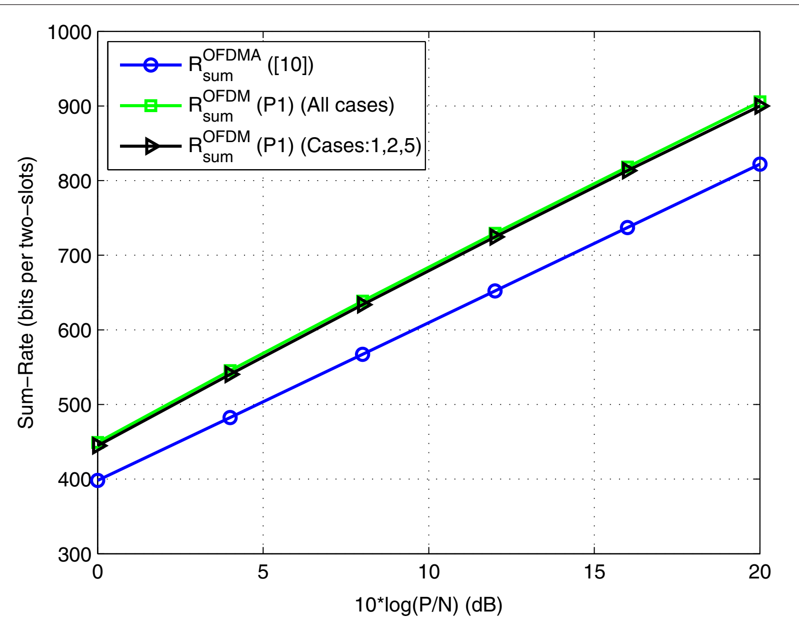

Figure 2 Sum-rate comparison. Numerical values are $K = 128$, $\sigma_{\rm ar}^2 = 20$ dB, $\sigma_{\rm br}^2 = 20$ dB, $\sigma_{\rm rd}^2 = 20$ dB, and $\sigma_{\rm ad}^2 = \sigma_{\rm bd}^2 = 20$ dB.

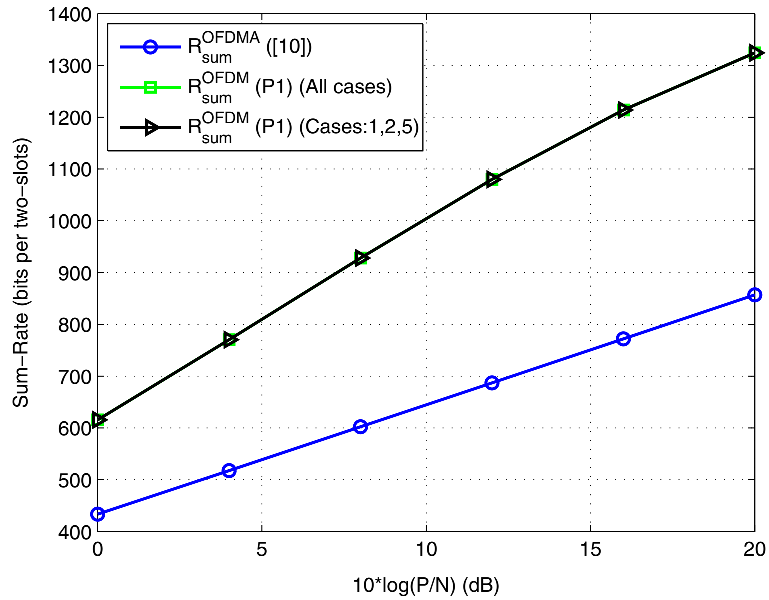

Figure 3 Sum-rate comparison. Numerical values are $K = 128$, $\sigma_{ar}^2 = 26$ dB, $\sigma_{br}^2 = 0$ dB, $\sigma_{rd}^2 = 20$ dB, $\sigma_{ad}^2 = 0$ dB, and $\sigma_{bd}^2 = 26$ dB.

In Figures 2 and 3, we notice that the sum-rate of (P1) considering all cases and the sum-rate of (P1) considering cases '1, 2, 5' yield the same sum-rate. However, we can see that, in Figure 4, the sum-rate of (P1) considering cases '1, 2, 5' is smaller than the sum-rate of (P1) considering all cases and slightly better than the sum-rate of [10]. This can be explained by investigating the channel strength of the different links. We can see that both sources have a strong source-relay link and a weak source-destination link. Hence, using case 2, the transmitted symbol of source **B** encounters high interference at the relay due to the transmitted symbol of source **A**. Therefore, source **A** must transmit its symbol with a power much lower than the one of source **B**. Since source **A** has a weak source-destination link, the destination will decode the message of source **A** at a small rate. Similarly, we can

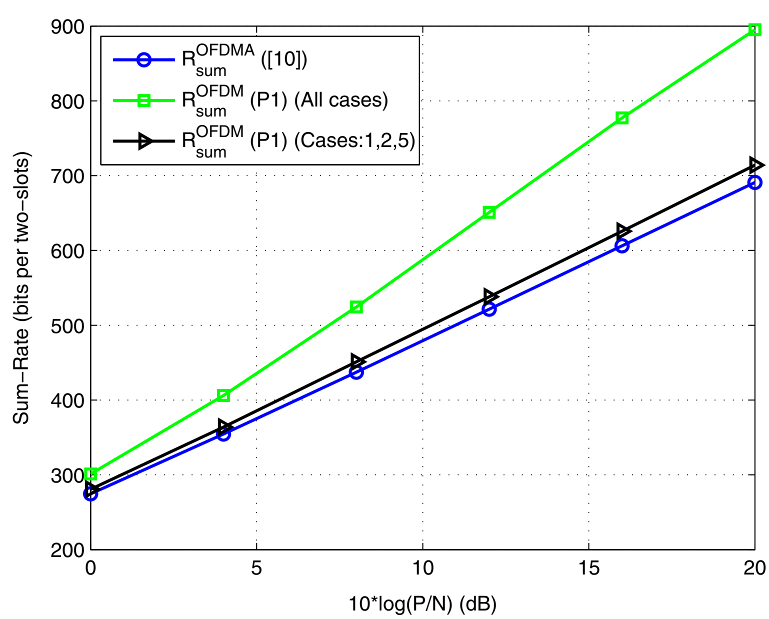

Figure 4 Sum-rate comparison. Numerical values are $K = 128$, $\sigma_{ar}^2 = 26$ dB, $\sigma_{br}^2 = 26$ dB, $\sigma_{rd}^2 = 20$ dB, and $\sigma_{ad}^2 = \sigma_{bd}^2 = 0$ dB.

see that using case 5, source **B** must transmit its symbol with a power much lower than the one of source **A** and the destination will decode the message of source **B** at a small rate. Hence, the OFDM transmission scheme of (P1) using only cases '1, 2, 5' has a performance similar to that of the OFDMA scheme of [10]. Nevertheless, by considering different decoding orders at the relay and the destination, we can observe that the OFDM transmission scheme of (P1) considering all cases outperforms the OFDMA scheme of [10] as shown in Figure 4. Also, in Figure 4, we notice that different decoding orders yield different sum-rates.

Figures 5, 6, and 7 depict the sum-rate obtained using the OFDMA transmission of [10], i.e., $R_{\text{sum}}^{\text{OFDMA}}$ ([10]), the sum-rate obtained using the improved version of OFDMA transmission of [10] given in (15), i.e., $R_{\text{sum}}^{\text{OFDMA}}$ ([10] Improved), the sum-rate obtained using the OFDM transmission scheme of (P1) considering all the cases, i.e., $R_{\text{sum}}^{\text{OFDM}}$ (P1), and the sum-rate obtained using the improved version of OFDM transmission scheme of (P1) considering all the cases, i.e., $R_{\text{sum}}^{\text{OFDM}}$ ((P1) Improved).

In Figure 5, we can observe that both improved transmission schemes have the same performance and that both outperform the other transmission schemes. Since all links have same channel variances (σ^2), it is more beneficial to allow the sources to transmit during the two transmission periods through the direct link than to transmit during only the first transmission period through the relay link [11]. Therefore, both improved schemes yield the same performance.

In Figures 6 and 7, we observe that the improved OFDM scheme outperforms the other schemes. Also, we observe that, in Figure 6, the OFDM scheme of (P1) outperforms the improved OFDMA scheme of [10] between 0 and 16 dB. This means that, in this interval, it is better to use the relay link in parallel with the direct link (i.e., case 2 or case 5) than to use the direct link during the two transmission periods (related to this aspect, recall the discussion in Remark 2). We also observe how much improvement the improved OFDMA scheme brought compared with the OFDMA scheme.

In Figure 7, we observe that the improved OFDM scheme of (P1) and the OFDM scheme of (P1) have the same performance. Since the source-relay link and the relay-destination link are stronger than the source-destination link, it is more beneficial to use the relay link in parallel with the direct link (i.e., cases 2 to 7) than to use the direct link during the two transmission periods. However, for the OFDMA scheme, it is still more beneficial to use the direct link during the two transmission periods than to use the relay link, especially at high values of P/N.

Moreover, we consider the model shown in Figure 8, and we study the performance of the two transmission schemes as functions of the relay position d. For this model, we set the number of subcarriers to $K = 128$. The CIR between node i and node j is modeled as a delay line with length $L = 32$ taps. The taps are assumed to be i.i.d circular complex Gaussian distributed with zero mean and variance d_{ij}^{-3}, where d_{ij} is the distance between nodes i and j. Also, we set $10\log(P/N) = 40$ dB. In Figure 9, we can observe the performance of the two transmission schemes with respect to the relay position.

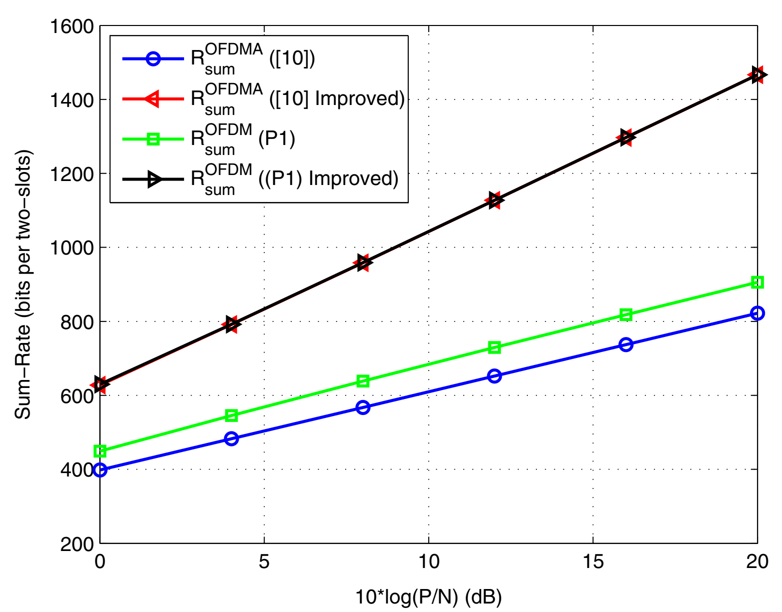

Figure 5 Sum-rate comparison. Numerical values are $K = 128$, $\sigma_{\text{ar}}^2 = 20$ dB, $\sigma_{\text{br}}^2 = 20$ dB, $\sigma_{\text{rd}}^2 = 20$ dB, and $\sigma_{\text{ad}}^2 = \sigma_{\text{bd}}^2 = 20$ dB.

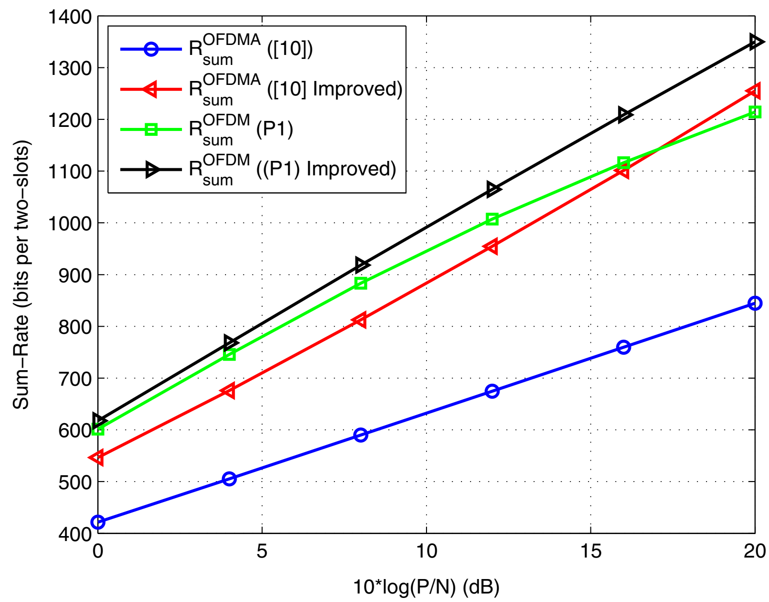

Figure 6 Sum-rate comparison. Numerical values are $K = 128$, $\sigma_{ar}^2 = 26$ dB, $\sigma_{br}^2 = 0$ dB, $\sigma_{rd}^2 = 26$ dB, $\sigma_{ad}^2 = 0$ dB, and $\sigma_{bd}^2 = 20$ dB.

In the previous comparisons, we have focused on the performance of the OFDM scheme over the OFDMA one in terms of sum-rate. It is worth pointing out the performance-complexity trade-offs. In what follows, we discuss the convergence and complexity of 'Algorithm IP'. Recall that 'Algorithm IP' involves allocating the powers and selecting the best relay operation mode and decoding orders alternately in an iterative manner. Also, we compare the computational complexity of the proposed algorithms with the one described in [10] for the OFDMA scheme.

The complexity to obtain the relay operation mode and the decoding orders is $7K$, since the algorithm needs to calculate the sum-rate for the seven cases presented in Table 1 per subcarrier. We should note that the complexity can be decreased to $3K$ if we only consider the three cases (1, 2, and 5) and still achieving a sum-rate that is larger than what is obtained using the OFDMA scheme.

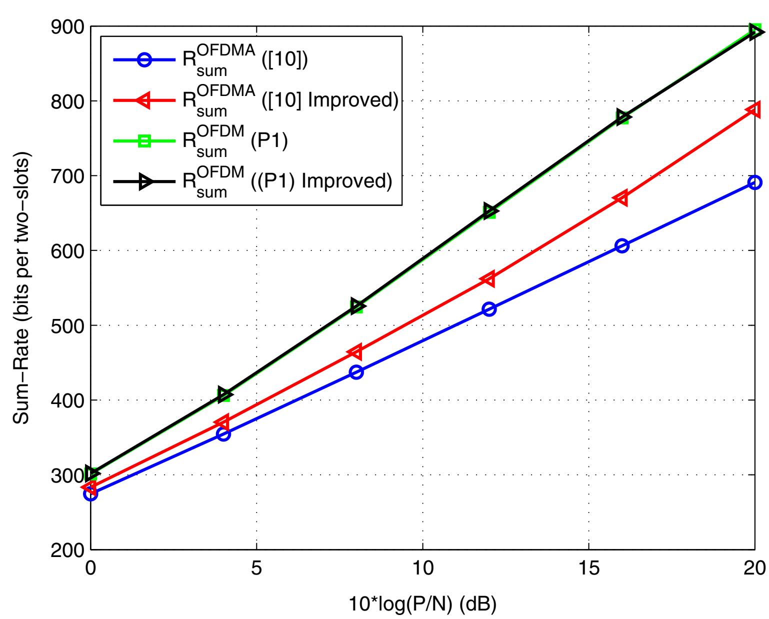

Figure 7 Sum-rate comparison. Numerical values are $K = 128$, $\sigma_{ar}^2 = 26$ dB, $\sigma_{br}^2 = 26$ dB, $\sigma_{rd}^2 = 20$ dB, and $\sigma_{ad}^2 = \sigma_{bd}^2 = 0$ dB.

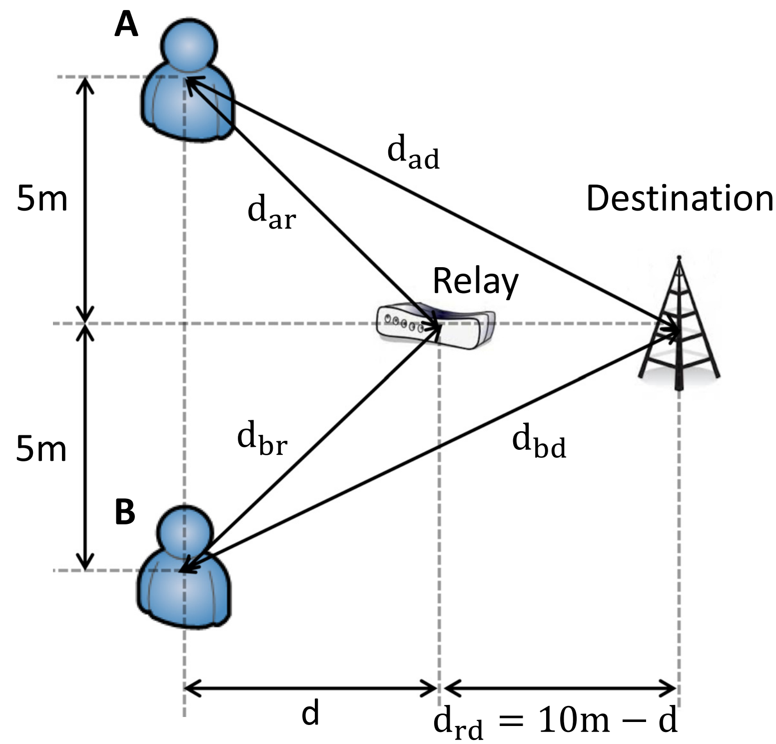

Figure 8 Two-sources MARC model.

Therefore, a smaller amount of cases can be selected in order to reduce the complexity, at the expense of lower sum-rate (in some situations). We should also note that there is no case allocation for the OFDMA scheme. However, there is subcarrier allocation. The complexity to allocate the subcarriers using (Algorithm 2, [10]) is $\mathcal{O}(4K^2)$.

The complexity of 'Algorithm P' can be calculated numerically as follows. Table 3 indicates the average number of iterations and the average run time needed to

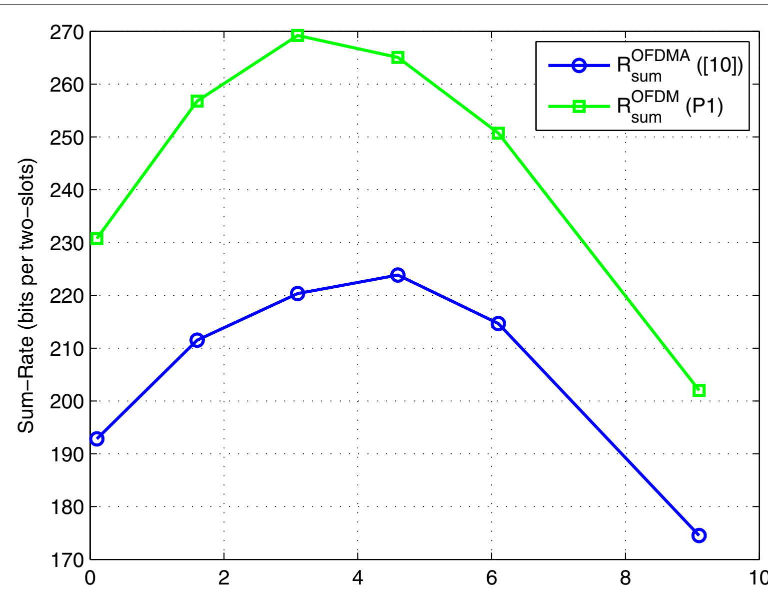

Figure 9 Sum-rate comparison. Numerical values are $K = 128$ and $10\log(P/N) = 40$ dB.

Table 3 Complexity analysis for the power allocation algorithms

	S1: OFDMA (Algorithms 2 and 4, [10])	S2: OFDM Algorithm IP
Average number of iterations needed to converge	13.64	3.24
Average run time needed to converge (seconds)	0.47	175.6
Average sum-rate (bits per two-slots)	861.5	1,335.4
Power allocation optimization	Separate	Joint

converge for two different setups. The first one (S1) is based on OFDMA scheme, and the algorithms used to allocate the subcarriers and the powers are (Algorithm 2, [10]) and (Algorithm 4, [10]), respectively. The second one (S2) is based on OFDM scheme, and the algorithm used to allocate the cases and the powers is 'Algorithm IP'. Table 3 also indicates the average sum-rate and the type of optimization that has been used for the power allocation, i.e., if the powers at the different nodes are allocated jointly or separately. We should note that these results are taken over 100 channel realizations and the numerical values of channel coefficients are chosen such that $\sigma_{\mathrm{ar}}^2 = 26$ dB, $\sigma_{\mathrm{br}}^2 = 0$ dB, $\sigma_{\mathrm{rd}}^2 = 20$ dB, $\sigma_{\mathrm{ad}}^2 = 0$ dB, and $\sigma_{\mathrm{bd}}^2 = 26$ dB which corresponds to the situation given in Figure 3. We also set the exit condition $\epsilon_2 = 10^{-2}$ and $10\log(P/N) = 20$ dB. We can see that (S2) has better performance than (S1) in terms of sum-rate. However, the run time needed for (S2) to converge is higher than the one for (S1). This can be explained as follows: since the power allocation for (S2) is done in a joint manner and since more efforts are needed to provide a better solution, more time is needed to converge. Thus, (S2) yields higher sum-rate than (S1) at the cost of higher complexity. We can clearly see the trade-off between the complexity and the performance.

Since the optimization problem (6) is a mixed-integer program problem, we investigate the convergence of 'Algorithm IP' by comparing it with one in which the relay operation mode and the decoding order search on each subcarrier is performed in an exhaustive manner, i.e., all possible combinations are considered, and the power allocation is kept as in Section 5.3. Note that, using this exhaustive search algorithm, for the relay operation mode and the decoding orders to be chosen optimally, the search should consider 7^K combinations. Let $R_{\mathrm{sum}}^{\mathrm{Ex}}$ denote the sum-rate obtained by using the described exhaustive search-based algorithm.

In Figure 10, we compare the maximized sum-rate $R_{\mathrm{sum}}^{\mathrm{OFDM}}$ achieved by 'Algorithm IP' with $R_{\mathrm{sum}}^{\mathrm{Ex}}$ achieved by the exhaustive search-based algorithm for $K = 4$ and ten given channel realizations. Figure 10 shows that 'Algorithm IP' has the same performance as the exhaustive search-based algorithm. Note that the exhaustive search-based algorithm is more largely time and computational resources consuming, especially at a large value of K. Also, note that 'Algorithm IP' is based on coordinate descent method, and it yields increasing sum-rate as the iterations continue until the power vectors $\{\boldsymbol{\beta}[k]\}_{k=1}^{K}$ are saturated,

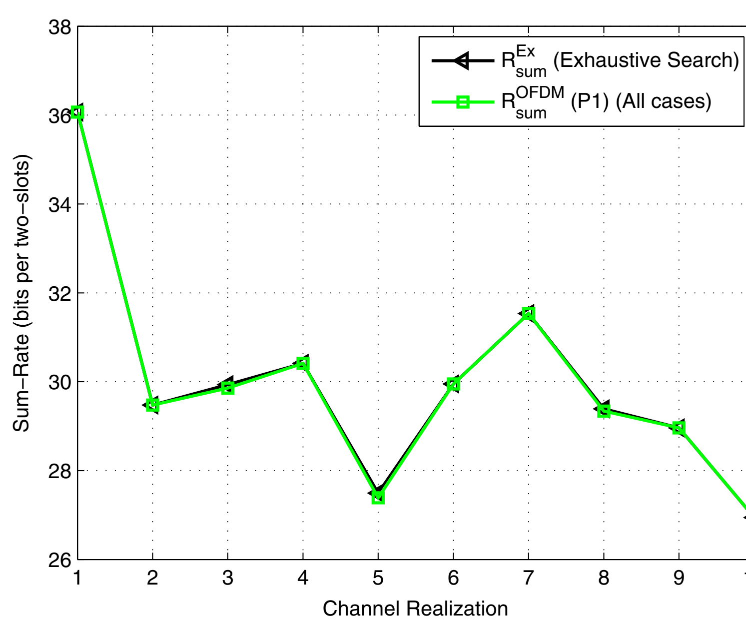

Figure 10 'Algorithm IP' vs Exhaustive search. Numerical values are $K = 4$, $10\log(P/N) = 20$ dB, $\sigma_{\mathrm{ar}}^2 = \sigma_{\mathrm{br}}^2 = \sigma_{\mathrm{rd}}^2 = 20$ dB, and $\sigma_{\mathrm{ad}}^2 = \sigma_{\mathrm{bd}}^2 = 20$ dB.

and thus, convergence is guaranteed. Finally, we discuss the complexity of 'Algorithm IP'. The average number of iterations required for 'Algorithm IP' to converge is no more than three iterations. We should note that this result is taken over 100 channel realizations and the numerical values of channel coefficients are chosen such that $\sigma^2_{\rm ar} = \sigma^2_{\rm br} = 26$ dB, $\sigma^2_{\rm rd} = 20$ dB, and $\sigma^2_{\rm ad} = \sigma^2_{\rm bd} = 0$ dB. Also, we use $K = 128$, $\epsilon_1 = \epsilon_2 = 10^{-2}$, and $10\log(P/N) = 20$ dB.

We should note that the OFDM scheme has some additional complexities that depend on various parameters and that are beyond the scope of this paper, e.g., the complexity at the receivers due to the operation of successive decoding. We should also note that if we only consider cases 2 and 5, there is no need of successive decoding at the destination since each transmitted codeword is decoded at the destination in a different time slot. Thus, in general, the complexity of OFDM scheme can be decreased at the expense of lower sum-rate which is still larger than the one obtained using OFDMA scheme.

8 Conclusions

We consider communication over a two-source multiaccess relay channel in which all the channels are assumed to be frequency selective. In order to handle the frequency selectivity of the channels, we incorporate OFDM transmission into the system. We study and analyze the performance of a transmission scheme in which the relay is half-duplex and implements a decode-and-forward strategy. In contrast to previous works, both sources can transmit their messages using all subcarriers and the relay can decide to help none, only one, or both sources. For this scheme, we derive the achievable sum-rate and study the problem of allocating the powers, selecting the relay operation modes and the decoding orders at the relay and the destination optimally in a way to maximize the obtained sum-rate. We propose an iterative coordinate-descent algorithm that finds a solution that is at least local optimum. We illustrate our results through some numerical examples. In particular, our analysis shows that by allowing the sources to possibly transmit on the same subcarrier simultaneously, one can afford a larger sum-rate, i.e., the OFDM-based transmission scheme offers a substantial sum-rate gain over the one that is based on OFDMA.

Appendix 1: Some useful definitions

Definition 1. For given channel states $\{h_{\rm ar}[k], h_{\rm br}[k], h_{\rm ad}[k], h_{\rm bd}[k], h_{\rm rd}[k]\}_{k=1}^K$, and power policy $\{\boldsymbol{\beta}[k]\}_{k=1}^K$, $1 \le k \le K$, we define

$$R_1[k] = \frac{1}{2}\log_2\left(1 + \frac{\beta_a^2[k]\,|h_{\rm ad}[k]|^2P}{N} + \frac{\beta_b^2[k]\,|h_{\rm bd}[k]|^2P}{N}\right) \tag{16}$$

$$\begin{aligned}R_2[k] = \frac{1}{2}\min\Bigg\{&\log_2\left(1 + \frac{\beta_b^2[k]\,|h_{\rm br}[k]|^2P}{N + \beta_a^2[k]\,|h_{\rm ar}[k]|^2P}\right),\\ &\log_2\left(1 + \frac{\beta_b^2[k]\,|h_{\rm bd}[k]|^2P}{N + \beta_a^2[k]\,|h_{\rm ad}[k]|^2P} + \frac{\beta_{\rm br}^2[k]\,|h_{\rm rd}[k]|^2P}{N}\right)\Bigg\}\\ &+ \frac{1}{2}\log_2\left(1 + \frac{\beta_a^2[k]\,|h_{\rm ad}[k]|^2P}{N}\right)\end{aligned} \tag{17}$$

$$\begin{aligned}R_3[k] = \frac{1}{2}\min\Bigg\{&\log_2\left(1 + \frac{\beta_a^2[k]\,|h_{\rm ar}[k]|^2P}{N}\right),\\ &\log_2\left(1 + \frac{\beta_a^2[k]\,|h_{\rm ad}[k]|^2P}{N} + \frac{\beta_{\rm ar}^2[k]\,|h_{\rm rd}[k]|^2P}{N}\right)\Bigg\}\\ +\frac{1}{2}\min\Bigg\{&\log_2\left(1 + \frac{\beta_b^2[k]\,|h_{\rm br}[k]|^2P}{N + \beta_a^2[k]\,|h_{\rm ar}[k]|^2P}\right),\\ &\log_2\left(1 + \mathrm{snr_b}[k]\right)\Bigg\}\end{aligned} \tag{18}$$

$$\begin{aligned}R_4[k] = \frac{1}{2}\min\Bigg\{&\log_2\left(1 + \frac{\beta_a^2[k]\,|h_{\rm ar}[k]|^2P}{N + \beta_b^2[k]\,|h_{\rm br}[k]|^2P}\right),\\ &\log_2\left(1 + \frac{\beta_a^2[k]\,|h_{\rm ad}[k]|^2P}{N} + \frac{\beta_{\rm ar}^2[k]\,|h_{\rm rd}[k]|^2P}{N}\right)\Bigg\}\\ + \frac{1}{2}\min\Bigg\{&\log_2\left(1+\frac{\beta_b^2[k]\,|h_{\rm br}[k]|^2P}{N}\right), \log_2\left(1 + \mathrm{snr_b}[k]\right)\Bigg\}.\end{aligned} \tag{19}$$

where $\mathrm{snr}_b[k]$ is defined as in Definition 2. Also, let $R_5[k]$, $R_6[k]$, and $R_7[k]$ be obtained by swapping the indices a and b in $R_2[k]$, $R_3[k]$, and $R_4[k]$, respectively.

Definition 2. For given channel states $\{h_{\rm ar}[k], h_{\rm br}[k], h_{\rm ad}[k], h_{\rm bd}[k], h_{\rm rd}[k]\}_{k=1}^K$, and power policy $\{\boldsymbol{\beta}[k]\}_{k=1}^K$, $1 \le k \le K$, we define

$$\Theta_b^{(1)}[k] = \frac{N\left(\beta_b^2[k]\,|h_{\rm bd}[k]|^2 + \beta_{\rm br}^2[k]\,|h_{\rm rd}[k]|^2\right)P + \beta_b^2[k]\,|h_{\rm bd}[k]|^2\beta_{\rm ar}^2[k]\,|h_{\rm rd}[k]|^2P^2}{N^2 + \beta_a^2[k]\,|h_{\rm ad}[k]|^2PN + \beta_{\rm ar}^2[k]\,|h_{\rm rd}[k]|^2PN} \tag{20}$$

$$\Theta_b^{(2)}[k] = \frac{\beta_{\rm br}^2[k]\,|h_{\rm rd}[k]|^2\beta_a^2[k]\,|h_{\rm ad}[k]|^2P^2}{N^2 + \beta_a^2[k]\,|h_{\rm ad}[k]|^2PN + \beta_{\rm ar}^2[k]\,|h_{\rm rd}[k]|^2PN} - \frac{2\beta_a[k]\,\beta_b[k]\,\beta_{\rm ar}[k]\,\beta_{\rm br}[k]\,Re\left\{h_{\rm bd}^*[k]\,h_{\rm ad}[k]\right\}|h_{\rm rd}[k]|^2P^2}{N^2 + \beta_a^2[k]\,|h_{\rm ad}[k]|^2PN + \beta_{\rm ar}^2[k]\,|h_{\rm rd}[k]|^2PN} \tag{21}$$

$$\mathrm{snr}_b[k] = \Theta_b^{(1)}[k] + \Theta_b^{(2)}[k]. \tag{22}$$

Also, let $\Theta_{\rm a}^{(1)}[k]$, $\Theta_{\rm a}^{(2)}[k]$, and $\mathrm{snr}_a[k]$ be obtained by swapping the indices a and b in $\Theta_b^{(1)}[k]$, $\Theta_b^{(2)}[k]$, and $\mathrm{snr}_b[k]$, respectively.

Appendix 2: Proof of Proposition 1

Recall the seven possible cases that we mentioned in Remark 1, summarized in Table 1. In what follows, because of symmetry, we only analyze the following four cases for the transmission on subcarrier k, $1 \leq k \leq K$: *Case 1)* transmission to the destination on subcarrier k utilizes only the direct links, i.e., the relay remains idle on subcarrier k, *Case 2)* the relay helps only one source on subcarrier k, e.g., source **B** by decoding and forwarding the transmitted symbol $\mathbf{x}_b[k]$, *Case 3)* the relay helps both sources simultaneously on subcarrier k, and the codeword $\mathbf{x}_b[k]$ of source **B** is decoded first at both relay and destination, and *Case 4)* the relay helps both sources simultaneously on subcarrier k, with the codeword $\mathbf{x}_a[k]$ of source **A** decoded first at the relay and the codeword $\mathbf{x}_b[k]$ of source **B** decoded first at the destination. The analysis of the remaining three cases (obtained respectively from case 2, case 3, and case 4 by swapping the roles of the sources) can be obtained straightforwardly by symmetry. For each of the four cases that will be analyzed, we first describe the decoding procedures at the relay and the destination and then analyze the achievable sum-rate.

Case 1 *Transmission using only direct links:* This scenario corresponds to a regular MAC, and the sum-rate that is achievable on subcarrier k, $1 \leq k \leq K$, can easily be shown (Theorem 4.4, [28]) to be $R_1[k]$ as given by (16) in Definition 1.

Case 2 *The relay helps only source **B**:* At the end of the first transmission period, the relay gets the output vector $\mathbf{y}_r[k]$ given by (1). The relay utilizes joint typicality decoding to decode the codeword $\mathbf{x}_b[k]$ transmitted by source **B** on subcarrier k, $1 \leq k \leq K$. In doing so, the relay treats the codeword $\mathbf{x}_a[k]$ transmitted by source **A** as unknown noise or interference. We assume that the codeword $\mathbf{x}_a[k]$ is considered as normal distributed interference. For large n, the decoding can be done reliably at rate

$$R_{\mathrm{br}}^{(2)}[k] = \frac{1}{2}\log_2\left(1 + \frac{\beta_b^2[k]\,|h_{\mathrm{br}}[k]|^2 P}{N + \beta_a^2[k]\,|h_{\mathrm{ar}}[k]|^2 P}\right), \quad (23)$$

where the upperscript refers to the case in hand and the lowerscript refers to the channel link. The relay then forwards the decoded codeword on the same subcarrier k to the destination, during the second transmission period. To this end, the relay sends

$$\tilde{\mathbf{x}}_r[k] = \sqrt{\frac{\beta_{\mathrm{br}}^2[k]}{\beta_b^2[k]}}\mathbf{x}_b[k]\,. \quad (24)$$

The destination, using its output components $(\mathbf{y}_d[k], \tilde{\mathbf{y}}_d[k])$, decodes the codewords transmitted by both sources successively. Given that the relay helps only source **B**, it can be shown relatively straightforwardly that, in this case, decoding the relayed codeword $\mathbf{x}_b[k]$ first, i.e., before canceling out its contribution and decoding the non-relayed codeword $\mathbf{x}_a[k]$, results in a sum-rate that is larger than the one that would be achievable if the decoding of the codewords at the destination is performed in the reverse order. Thus, the destination first decodes codeword $\mathbf{x}_b[k]$, cancels its contribution out, and then decodes codeword $\mathbf{x}_a[k]$. In order to decode codeword $\mathbf{x}_b[k]$, the destination combines the output components $\mathbf{y}_d[k]$ and $\tilde{\mathbf{y}}_d[k]$ to their maximum ratio, i.e., using standard MRC. It can be shown that, for large n, the decoding of codeword $\mathbf{x}_b[k]$ can be decoded reliably at rate

$$R_{\mathrm{bd}}^{(2)}[k] = \frac{1}{2}\log_2\left(1 + \frac{\beta_b^2[k]\,|h_{\mathrm{bd}}[k]|^2 P}{N + \beta_a^2[k]\,|h_{\mathrm{ad}}[k]|^2 P} + \frac{\beta_{\mathrm{br}}^2[k]\,|h_{\mathrm{rd}}[k]|^2 P}{N}\right). \quad (25)$$

Next, the destination subtracts out the contribution of $\mathbf{x}_b[k]$ from $\mathbf{y}_d[k]$ and, so, decodes the codeword $\mathbf{x}_a[k]$ free of interference. It can be shown that, for large n, this can be done reliably at rate

$$R_{\mathrm{ad}}^{(2)}[k] = \frac{1}{2}\log_2\left(1 + \frac{\beta_a^2[k]\,|h_{\mathrm{ad}}[k]|^2}{N}\right). \quad (26)$$

From the above, it follows that, in this case, the destination can decode reliably the sources' codewords that are transmitted on subcarrier k, $1 \leq k \leq K$, as long as n is large and these codewords are sent at a sum-rate that is no larger than the sum of $R_{\mathrm{ad}}^{(2)}[k]$ and the minimum among $R_{\mathrm{br}}^{(2)}[k]$ and $R_{\mathrm{bd}}^{(2)}[k]$, i.e., $R_2[k]$ as given by (17) in Definition 1.

Case 3 *The relay helps both sources, and the decoding orders at the relay and the destination are identical:* In this case, we assume that the relay helps both sources and that the relay and the destination first decode codeword $\mathbf{x}_b[k]$, cancel out its contribution, and then decode codeword $\mathbf{x}_a[k]$.

Consider first the decoding operations at the relay. At the end of the first transmission period, the relay gets the output vector $\mathbf{y}_r[k]$ and decodes the codeword $\mathbf{x}_b[k]$ exactly as in case 2. Thus, for large n, the relay can get the correct $\mathbf{x}_b[k]$ at rate $R_{\mathrm{br}}^{(3)}[k] = R_{\mathrm{br}}^{(2)}[k]$ as given by (23). The relay then subtracts out the contribution of $\mathbf{x}_b[k]$ from $\mathbf{y}_r[k]$ and then decodes codeword $\mathbf{x}_a[k]$, again using a joint typicality decoding. Similarly, for large n, this can be done reliably at rate

$$R_{\mathrm{ar}}^{(3)}[k] = \frac{1}{2}\log_2\left(1 + \frac{\beta_a^2[k]\,|h_{\mathrm{ar}}[k]|^2 P}{N}\right). \quad (27)$$

During the second transmission period, the relay helps both sources and transmits their codewords simultaneously on subcarrier k. To this end, the relay shares its power among re-transmitting codeword $\mathbf{x}_a[k]$ and re-transmitting codeword $\mathbf{x}_b[k]$, on the same subcarrier k, using superposition coding. That is, the relay sends

$$\tilde{\mathbf{x}}_r[k] = \sqrt{\frac{\beta_{\mathrm{ar}}^2[k]}{\beta_a^2[k]}}\mathbf{x}_a[k] + \sqrt{\frac{\beta_{\mathrm{br}}^2[k]}{\beta_b^2[k]}}\mathbf{x}_b[k]. \tag{28}$$

The destination, using its output components $(\mathbf{y}_d[k], \tilde{\mathbf{y}}_d[k])$, decodes the codewords transmitted by both sources successively, in the same order this is performed at the relay. More precisely, the destination first decodes codeword $\mathbf{x}_b[k]$, cancels its contribution out, and then decodes codeword $\mathbf{x}_a[k]$. In order to decode codeword $\mathbf{x}_b[k]$, the destination combines the output components $\mathbf{y}_d[k]$ and $\tilde{\mathbf{y}}_d[k]$ to their maximum ratio. Through straightforward algebra, which we omit for brevity, it can be shown that, for large n, the destination can get the correct $\mathbf{x}_b[k]$ at rate

$$R_{\mathrm{bd}}^{(3)}[k] = \frac{1}{2}\log_2\left(1 + \mathrm{snr}_b[k]\right), \tag{29}$$

where $\mathrm{snr}_b[k]$ is given in Definition 2. Next, the destination subtracts out the contribution of codeword $\mathbf{x}_b[k]$ from $(\mathbf{y}_d[k], \tilde{\mathbf{y}}_d[k])$ and combines the resulting equivalent output components using MRC to decode codeword $\mathbf{x}_a[k]$. Again, through straightforward algebra, which we omit for brevity, it can be shown that, for large n, the destination can get the correct $\mathbf{x}_a[k]$ at rate

$$R_{\mathrm{ad}}^{(3)}[k] = \frac{1}{2}\log_2\left(1 + \frac{\beta_a^2[k]\,|h_{\mathrm{ad}}[k]|^2 P}{N} + \frac{\beta_{\mathrm{ar}}^2[k]\,|h_{\mathrm{rd}}[k]|^2 P}{N}\right). \tag{30}$$

From the above, it follows that, in this case, the destination can decode reliably the sources' codewords that are transmitted on subcarrier k, $1 \le k \le K$, as long as n is large and these codewords are sent at a sum-rate that is no larger than the sum of the minimum among $R_{\mathrm{ar}}^{(3)}[k]$ and $R_{\mathrm{ad}}^{(3)}[k]$ and the minimum among $R_{\mathrm{br}}^{(3)}[k]$ and $R_{\mathrm{bd}}^{(3)}[k]$, i.e., $R_3[k]$ as given by (18) in Definition 1.

Case 4 The relay helps both sources, and the decoding orders at the relay and the destination are different: In this case, we assume that the relay helps both sources and that the relay and the destination decode the sources' codewords in different orders. In particular, in what follows, we analyze the case in which the decoding order at the relay is such that codeword $\mathbf{x}_a[k]$ is decoded first, and the decoding at the destination is maintained as in case 3 above.

Consider first the decoding operations at the relay. At the end of the first transmission period, the relay gets the output vector $\mathbf{y}_r[k]$ given by (1). Proceeding along the lines in the analysis of case 3 above, but the roles of codewords $\mathbf{x}_a[k]$ and $\mathbf{x}_b[k]$ swapped, it can be shown that, for large n, the relay can get the correct $\mathbf{x}_a[k]$ at rate

$$R_{\mathrm{ar}}^{(4)}[k] = \frac{1}{2}\log_2\left(1 + \frac{\beta_a^2[k]\,|h_{\mathrm{ar}}[k]|^2 P}{N + \beta_b^2[k]\,|h_{\mathrm{br}}[k]|^2 P}\right) \tag{31}$$

and the correct $\mathbf{x}_b[k]$ at rate

$$R_{\mathrm{br}}^{(4)}[k] = \frac{1}{2}\log_2\left(1 + \frac{\beta_b^2[k]\,|h_{\mathrm{br}}[k]|^2 P}{N}\right). \tag{32}$$

The decoding at the destination is exactly as in case 3. Thus, for large n, the destination can first get the correct $\mathbf{x}_b[k]$ at rate $R_{\mathrm{bd}}^{(4)}[k] = R_{\mathrm{bd}}^{(3)}[k]$ as given by (29) and then subtract its contribution out and get the correct codeword $\mathbf{x}_a[k]$ at rate $R_{\mathrm{ad}}^{(4)}[k] = R_{\mathrm{ad}}^{(3)}[k]$ as given by (30).

From the above, it follows that, in this case, the destination can decode reliably the sources' codewords that are transmitted on subcarrier k, $1 \le k \le K$, as long as n is large and these codewords are sent at a sum-rate that is no larger than the sum of the minimum among $R_{\mathrm{ar}}^{(4)}[k]$ and $R_{\mathrm{ad}}^{(4)}[k]$ and the minimum among $R_{\mathrm{br}}^{(4)}[k]$ and $R_{\mathrm{bd}}^{(4)}[k]$, i.e., $R_4[k]$ as given by (19) in Definition 1.

This completes the analysis of cases 1 to 4. The analysis of case 5, case 6, and case 7 in Table 1 can be obtained straightforwardly respectively from the analysis of case 2, case 3, and case 4, by swapping the roles of source **A** and source **B**. This leads to the associated sum-rates $R_5[k]$, $R_6[k]$, and $R_7[k]$ as given in Definition 1.

Summary: For given channel states $\{h_{\mathrm{ar}}[k], h_{\mathrm{br}}[k], h_{\mathrm{ad}}[k], h_{\mathrm{bd}}[k], h_{\mathrm{rd}}[k]\}_{k=1}^K$ and power policy $\{\boldsymbol{\beta}[k]\}_{k=1}^K$, the sum-rates of $R_l[k]$ bits per second, $1 \le l \le 7$, are achievable on subcarrier k, $1 \le k \le K$, using the OFDM-based transmission that we described. Thus, the sum-rate $R[k] = \max_{1 \le l \le 7} R_l[k]$ on subcarrier k, i.e., the maximum among the seven sum-rates $\{R_l[k]\}_{l=1}^7$, is obtained by selecting for subcarrier k the coding scheme that offers the larger per-subcarrier sum-rate among those of the aforementioned seven cases. Next, since OFDM transforms the channel into a set of K *parallel* subchannels, the total sum-rate that is offered through the transmission, over all subchannels, is obtained by simply summing over all subchannels the individual achievable per-subcarrier sum-rates [28].

Competing interests
The authors declare that they have no competing interests.

Acknowledgements
This work has been supported in part by the IAP BESTCOM project funded by BELSPO and FP7 project NEWCOM#. The authors would like to thank Prof. Stephen Boyd and his team, Information Systems Laboratory, Stanford University, USA, for providing the software and programs used in the optimization part in this paper and Dr. Tao Wang for some helpful discussions on some aspects of this work. Also, the authors would like to thank the anonymous reviewers for their valuable suggestions to improve the quality of this work.

The material in this paper has been presented in part at the International Symposium on Communications, Control and Signal Processing, Limassol, Cyprus, March 2010.

Author details
[1]ICTEAM, Université catholique de Louvain, Place du Levant, 2, 1348 Louvain-la-Neuve, Belgium. [2]Laboratoire d'Informatique Gaspard Monge, Université Paris-Est, Marne la Vallée Cedex 2, 77454, France. [3]Mathematical and Algorithmic Sciences Lab., France Research Center, Huawei Technologies, Paris, France.

Received: 17 October 2014 Accepted: 15 April 2015
Published online: 09 May 2015

References

1. P Gupta, PR Kumar, The capacity of wireless networks. IEEE Trans. Inf. Theory. **46**, 388–404 (2000)
2. NJ Laneman, DN Tse, GW Wornell, Cooperative diversity in wireless networks: efficient protocols and outage behavior. IEEE Trans. Inf. Theory. **50**, 3062–3080 (2004)
3. A Sendonaris, E Erkip, B Aazhang, in *IEEE International Symposium on Information Theory*. Increasing uplink capacity via user cooperation diversity (IEEE press Piscataway, NJ, 1998), p. 156
4. G Kramer, M Gastpar, P Gupta, Cooperative strategies and capacity theorems for relay networks. IEEE Trans. Inf. Theory. **51**, 3037–3063 (2005)
5. A Host-Madsen, J Zhang, Capacity bounds and pair allocation for wireless relay channels. IEEE Trans. Inf. Theory. **51**, 2020–2040 (2005)
6. L Sankaranarayanan, G Kramer, NB Mandayam, in *Proc. First IEEE Conference on Sensor and Ad Hoc Communications and Networks*. Hierarchical sensor networks: capacity bounds and cooperative strategies using the multiple-access relay channel model (IEEE press Piscataway, NJ, 2004), pp. 191–199
7. L Sankar, Y Liang, VH Poor, NB Mandayam, in *IEEE International Symposium on Information Theory*. Opportunistic communications in an orthogonal multiaccess relay channel (IEEE press Piscataway, NJ, 2007), pp. 1261–1265
8. M El-Soussi, A Zaidi, J Louveaux, L Vandendorpe, in *International Symposium on Communications, Control and Signal Processing (ISCCSP)*. Sum-rate optimized power allocation for the OFDM multiple access relay channel (IEEE press Piscataway, NJ, 2010), pp. 1–6
9. M El-Soussi, A Zaidi, L Vandendorpe, Compute-and-forward on a multiaccess relay channel: coding and symmetric-rate optimization. IEEE Trans. Wireless Commun. **13**, 1932–1947 (2014)
10. S Schedler, V Kuehn, Resource allocation for the multiple-access relay channels and OFDMA. EURASIP J. Adv. Signal Process. **120**, 1–20 (2013)
11. L Vandendorpe, J Louveaux, O Oguz, A Zaidi, Rate-optimized power allocation for DF-relayed OFDM transmission under sum and individual power constraints. EURASIP J. Wireless Commun. Netw. **814278**, 1–11 (2009)
12. T Wang, L Vandendorpe, Sum rate maximized resource allocation in multiple DF relays aided OFDM transmission. IEEE J. Sel. Areas Commun. **29**, 1559–1571 (2011)
13. T Wang, L Vandendorpe, WSR maximized resource allocation in multiple DF relays aided OFDMA downlink transmission. IEEE Trans. Sig. Proc. **59**, 3964–3976 (2011)
14. K Bakanoglu, S Tomasin, E Erkip, Resource allocation for the parallel relay channel with multiple relays. IEEE Trans. Wireless Commun. **10**, 792–802 (2011)
15. K Jitvanichphaibool, R Zhang, YC Liang, Optimal resource allocation for two-way relay-assisted OFDMA. IEEE Trans. Vehicul. Tech. **58**, 3311–3321 (2009)
16. HN Vu, HY Kong, Joint subcarrier matching and power allocation in OFDM two-way relay systems. J Commun Netw. **14**, 257–266 (2012)
17. M Kaneko, P Popovski, in *IEEE Int. Conf. Commun*. Radio resource allocation algorithm for relay-aided cellular OFDMA system (IEEE press Piscataway, NJ, 2007), pp. 4831–4836
18. Y Cui, VKN Lau, R Wang, Distributive subband allocation, power and rate control for relay-assisted OFDMA cellular system with imperfect system state knowledge. IEEE Trans. Wireless Commun. **8**, 5096–5102 (2009)
19. T Ng, W Yu, Joint optimization of relay strategies and resource allocations in cooperative cellular networks. IEEE J. Sel. Areas Commun. **25**, 328–339 (2007)
20. G Li, H Liu, in *38th Annu. Asilomar Conf. Signals, Syst. Comput*. On the capacity of the broadband relay networks (IEEE press Piscataway, NJ, 2004), pp. 1318–1322
21. G Li, H Liu, Resource allocation for OFDMA relay networks with fairness constraints. IEEE J. Sel. Areas on Commun. **24**, 2061–2069 (2006)
22. W Mesbah, TN Davidson, Power and resource allocation for orthogonal multiple access relay systems. EURASIP J. Adv. Signal Process. **476125**, 1–15 (2008)
23. J Joung, S Sun, Power efficient resource allocation for downlink OFDMA relay cellular networks. IEEE Trans. Signal Process. **60**, 2447–2459 (2012)
24. W Dang, M Tao, H Mu, J Huang, Subcarrier-pair based resource allocation for cooperative multi-relay OFDM systems. IEEE Trans. Wireless Commun. **9**, 1640–1649 (2010)
25. M Chiang, *Geometric programming for communication systems*. (Foundation and Trends in Commun. Inf. Theory, Massachusetts, 2005), pp. 1–156
26. LA Wolsey, *Integer Programming*. (Wiley, New York, 1998), pp. 1–264
27. M Chiang, CW Tan, DP Palomar, D O'Neill, D Julian, Power control by geometric programming. IEEE Trans. Wireless Commun. **6**, 2640–2651 (2007)
28. AE Gamal, Y-H Kim, *Network Information Theory*. (Cambridge University Press, New York, 2012), pp. 1–685