Hindawi Publishing Corporation
EURASIP Journal on Wireless Communications and Networking
Volume 2007, Article ID 30548, 7 pages
doi:10.1155/2007/30548

Research Article

Performance Analysis of Space-Time Block Codes in Flat Fading MIMO Channels with Offsets

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Received 19 June 2006; Revised 14 October 2006; Accepted 11 February 2007

Recommended by Yonghui Li

We consider the effect of imperfect carrier offset compensation on the performance of space-time block codes. The symbol error rate (SER) for orthogonal space-time block code (OSTBC) is derived here by taking into account the carrier offset and the resulting imperfect channel state information (CSI) in Rayleigh flat fading MIMO wireless channels with offsets.

1. INTRODUCTION

Use of space-time codes with multiple transmit antennas has generated a lot of interest for increasing spectral efficiency as well as improved performance in wireless communications. Although, the literature on space-time coding is quite rich now, the orthogonal designs of Alamouti [1], Tarokh et al. [2, 3], Naguib and Seshádri [4] remain popular. The strength of orthogonal designs is that these lead to simple, optimal receiver structure due to the possibility of decoupled detection along orthogonal dimensions of space and time.

Presence of a frequency offset between the transmitter and receiver, which could arrive due to oscillator instabilities, or relative motion between the two, however, has the potential to destroy this orthogonality and hence the optimality of the corresponding receiver. Several authors have, therefore, proposed methods based on pilot symbol transmission [5] or even blind methods [6] to estimate and compensate for the frequency offset under different channel conditions. Nevertheless, some residual offset remains, which adversely affects the code orthogonality and leads to increased symbol error rate (SER).

The purpose of this paper is to analyze the effect of such a residual frequency offset on performance of MIMO system. More specifically, we obtain a general result for calculating the SER in the presence of imperfect carrier offset knowledge (COK) and compensation, and the resulting imperfect CSI (due to imperfect COK and noise). The results of [7–12] which deal with the cases of performance analysis of OSTBC systems in the presence of imperfect CSI (due to noise alone) follow as special cases of the analysis presented here.

The outline of the paper is as follows. Formulation of the problem is accomplished in Section 2. Mean square error (MSE) in the channel estimates due to the residual offset error (ROE) is obtained in Section 3. In Section 4, we discuss the decoding of OSTBC data. In Section 5, the probability of error analysis in the presence of imperfect offset compensation is presented and we discuss the analytical and simulation results in Section 6. Section 7, contains some conclusions and in the appendix we derive the total interference power in the estimation of OSTBC data.

NOTATIONS

Throughout the paper we have used the following notations: $\mathbf{B}$ is used for matrix, $\mathbf{b}$ is used for vector, $b \in \mathbf{b}$ or $\mathbf{B}$, B and b are used for variables, $[\cdot]^H$ is used for hermitian of matrix or vector, $[\cdot]^T$ is used for transpose of matrix or vector,$[\mathbf{I}]$ is used for identity matrix, and $[\cdot]^*$ is used for conjugate matrix or vector.

2. PROBLEM FORMULATION

2.1. System modeling

Here for simplicity of analysis we restrict our attention to the simpler case of a MISO system (i.e., one with m transmit and

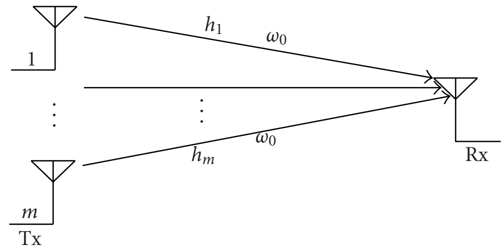

Figure 1: MISO system considered in the problem.

single receive antennas) shown in Figure 1, in which the frequency offset between each of the transmitters and the receiver is the same, as will happen when the source of frequency offset is primarily due to oscillator drift or platform motion.

Here h_k are channel gains between kth transmit antenna and receive antenna and ω_0 is the frequency offset. The received data vector corresponding to one frame of transmitted data in the presence of carrier offset is given by

$$\mathbf{y} = \mathbf{\Omega}(\omega_0)\mathbf{F}\mathbf{h} + \mathbf{e}, \tag{1}$$

where $\mathbf{y} = [y_1 y_2 \cdots y_{N_t+N}]^{\mathbf{T}}$; N_t and N being the number of time intervals over which, respectively, pilot symbols and unknown symbols are transmitted, $\mathbf{F}$ consists of data formatting matrix $[\mathbf{S}^t\mathbf{S}]$ corresponding to one frame; $\mathbf{S}^t$ and $\mathbf{S}$ being the pilot symbol matrix and space-time matrix, respectively, $\mathbf{h} = [h_1\ \ h_2\ \ \cdots\ \ h_m]^T$ denotes the channel gain vector with statistically independent complex circular Gaussian components of variance σ^2, and stationary over a frame duration, $\mathbf{e}$ is the AWGN noise vector $[e_1 e_2 \cdots e_{N_t+N}]^{\mathbf{T}}$ with a power density of $N_0/2$ per dimension, and $\mathbf{\Omega}(\omega_0) = \operatorname{diag}\{\exp(j\omega_0), \exp(j2\omega_0), \ldots, \exp(j(N_t + N)\omega_0)\}$ denotes the carrier offset matrix.

2.2. *Estimation of carrier offset and imperfect compensation of the received data*

Maximum likelihood (ML) estimation of transmitted data requires the perfect knowledge of the carrier offset and the channel. The offset can be estimated through the use of pilot symbols [5], or using blind method [6]. However, a few pilot symbols are almost always necessary, for estimation of the channel gains. Considering all this, therefore, we use in our analysis a generalized frame consisting of an orthogonal pilot symbol matrix (typically proportional to the identity matrix) and the STBC data matrix. In any case, the estimation of the offset cannot be perfect due to the limitations over the data rate, and delay and processing complexity. There could also be additional constraint in the form of time varying nature of the unknown channel. Thus, a residual offset error will always remain in the received data even after its compensation based on its estimated value. This can be explained as follows: if $\hat{\omega}_0$ denotes the estimated value of the offset ω_0, we have

$$\hat{\omega}_0 = \omega_0 - \Delta\omega, \tag{2}$$

where $\Delta\omega$ is a residual offset error (ROE) the amount of which depends upon the efficiency of the estimator. The compensated received data vector will be

$$\mathbf{y}_c = \mathbf{\Omega}(-\hat{\omega}_0)\mathbf{y} = \mathbf{\Omega}(\Delta\omega)\mathbf{F}\mathbf{h} + \mathbf{\Omega}(-\hat{\omega}_0)\mathbf{e}. \tag{3}$$

It is reasonable to consider ROE to be normally distributed with zero mean and variance σ_ω^2. We have also assumed that carrier offset and hence ROE remains constant over a data frame. The problem of interest here is to analytically find the performance of the receiver in the presence of ROE.

3. MEAN SQUARE ERROR IN THE ESTIMATION OF CHANNEL GAINS IN THE PRESENCE OF RESIDUAL OFFSET ERROR

Although it is possible to continue with the general case of m transmit antennas, the treatment and solution becomes cumbersome, especially since the details will also depend on the specific OSTBC used. On the other hand, the principle behind the analysis can be easily illustrated by considering the special case of two transmit antennas, employing the famous Alamouti code [1]. Suppose we transmit K orthogonal pilot symbol blocks of 2×2 size and L Alamouti code blocks over a frame. One such frame is depicted in Figure 2, where x is a pilot symbol of unit power and $s_r(k)$ represents the rth symbol transmitted by the kth antenna and $x, s_r(k) \in M$-QAM.

The compensated received vector corresponding to K training data blocks (denoted here by matrix $\mathbf{P}$) can be expressed as

$$\begin{aligned}\breve{\mathbf{y}}_c &= [y_c^1 y_c^2 \cdots y_c^{2K}]^T \\ &= \mathbf{\Omega}(\Delta\omega)_{2K\times 2K}\mathbf{P}\mathbf{h} + \mathbf{\Omega}(-\hat{\omega}_0)_{2K\times 2K}\mathbf{e}_{2K\times 1},\end{aligned} \tag{4}$$

where $\mathbf{\Omega}(\Delta\omega)_{2K\times 2K}$ is the ROE matrix and $\mathbf{\Omega}(-\hat{\omega}_0)_{2K\times 2K}$ is compensating matrix, respectively, corresponding to K pilot blocks, and $\mathbf{e}_{2K\times 1}$ is the noise in pilot data. It may be noted that the last term in (4) can still be modeled as complex, circular Gaussian and contains independent components. As the receiver already has the information about $\mathbf{P}$, we can find the ML estimate of the channel gains as follows [2, 4]:

$$\hat{\mathbf{h}} = \frac{1}{K|x|^2}\mathbf{P}^H\breve{\mathbf{y}}_c\,. \tag{5}$$

Substituting the value of $\breve{\mathbf{y}}_c$ from (4) into (5), we get

$$\begin{aligned}\hat{\mathbf{h}} &= \frac{1}{K|x|^2}\mathbf{P}^H\mathbf{\Omega}(\Delta\omega)_{2K\times 2K}\mathbf{P}\mathbf{h} \\ &\quad + \frac{1}{K|x|^2}\mathbf{P}^H\mathbf{\Omega}(-\hat{\omega}_0)_{2K\times 2K}\mathbf{e}_{2K\times 1}.\end{aligned} \tag{6}$$

In the low mobility scenario where the carrier offset is mainly because of the oscillator instabilities, its value is very small and if sufficient training data is transmitted or an efficient blind estimator is used, the variance σ_ω^2 of ROE is generally very small ($\sigma_\omega^2 \ll 1$), thus we can comfortably use a

$$\underbrace{\begin{bmatrix} x & 0 & \cdots & x & 0 \\ 0 & x & \cdots & 0 & x \end{bmatrix}}_{\text{Training data } (K \text{ blocks})} \underbrace{\begin{bmatrix} s_1(1) & s_2(1) & s_3(1) & s_4(1)\cdots s_{2L-1}(1) & s_{2L}(1) \\ -s_2^*(2) & s_1^*(2) & -s_4^*(2) & s_3^*(2)\cdots -s_{2L}^*(2) & s_{2L-1}^*(2) \end{bmatrix}}_{\text{STBC data } (L \text{ blocks})}$$

Figure 2: Complete frame for two transmit antennas and single receive antenna case.

first order Taylor series approximation for exponential terms in $\mathbf{\Omega}(\Delta\omega)_{2K\times 2K}$ as

$$\exp(jN\Delta\omega) = 1 + jN\Delta\omega. \tag{7}$$

After a simple manipulation, we can find the estimates of channel gains as

$$\begin{aligned} \widehat{\mathbf{h}} &= \begin{bmatrix} h_1 \\ h_2 \end{bmatrix} + \underbrace{\begin{bmatrix} jK\Delta\omega h_1 \\ j(1+K)\Delta\omega h_2 \end{bmatrix}}_{\text{error due to residual offset}} \\ &\quad + \underbrace{\frac{1}{K|x|^2}\mathbf{P}^H\mathbf{\Omega}(-\widehat{\omega}_0)_{2K\times 2K}\mathbf{e}_{2K\times 1}}_{\text{error due to noise}} = \mathbf{h} + \Delta\mathbf{h}, \end{aligned} \tag{8}$$

where $\Delta\mathbf{h} = \left[\begin{smallmatrix} jK\Delta\omega h_1 \\ j(1+K)\Delta\omega h_2 \end{smallmatrix}\right] + (1/K|x|^2)\mathbf{P}^H\mathbf{\Omega}(-\widehat{\omega}_0)_{2K\times 2K}\mathbf{e}_{2K}$ is the total error in estimates. It is easy to see that there are two distinct interfering terms in (8) due to ROE and AWGN noise. In the previous work [7–12], the interference only due to the AWGN noise is considered. However, here in (8) we are also taking into account the effect of the interference due to ROE. The mean square error (MSE) of channel estimate in (8) can be found as follows:

$$\begin{aligned} \text{MSE} &= \frac{1}{2}\operatorname{Tr}\left\{E_{\mathbf{h},\Delta\omega,\mathbf{e}}\left\{\Delta\mathbf{h}\Delta\mathbf{h}^H\right\}\right\} \\ &= \frac{1}{2}\operatorname{Tr}\Bigg\{E_{\mathbf{h},\Delta\omega,\mathbf{e}}\Bigg\{\Bigg(\begin{bmatrix} jK\Delta\omega h_1 \\ j(1+K)\Delta\omega h_2 \end{bmatrix} \\ &\qquad + \frac{1}{K|x|^2}\mathbf{P}^H\mathbf{\Omega}(-\widehat{\omega}_0)_{2K\times 2K}\mathbf{e}_{2K\times 1}\Bigg) \\ &\qquad \cdot\Bigg(\begin{bmatrix} jK\Delta\omega h_1 \\ j(1+K)\Delta\omega h_2 \end{bmatrix} \\ &\qquad + \frac{1}{K|x|^2}\mathbf{P}^H\mathbf{\Omega}(-\widehat{\omega}_0)_{2K\times 2K}\mathbf{e}_{2K\times 1}\Bigg)^H\Bigg\}\Bigg\}. \end{aligned} \tag{9}$$

Assuming, elements of $\mathbf{h}$, $\Delta\omega$ and elements of $\mathbf{e}$ are statistically independent of each other, the expectation of cross terms will be zero and the MSE would be simplified as follows:

$$\begin{aligned} \text{MSE} &= \frac{1}{2}\operatorname{Tr}\Bigg\{E_{\mathbf{h},\Delta\omega}\Bigg\{\Bigg(\begin{bmatrix} jK\Delta\omega h_1 \\ j(1+K)\Delta\omega h_2 \end{bmatrix}\begin{bmatrix} jK\Delta\omega h_1 \\ j(1+K)\Delta\omega h_2 \end{bmatrix}^H\Bigg)\Bigg\} \\ &\quad + E_{\mathbf{e}}\Bigg\{\Bigg(\frac{1}{K|x|^2}\mathbf{P}^H\mathbf{\Omega}(-\widehat{\omega}_0)_{2K\times 2K}\mathbf{e}_{2K\times 1}\Bigg) \\ &\qquad \cdot\Bigg(\frac{1}{K|x|^2}\mathbf{P}^H\mathbf{\Omega}(-\widehat{\omega}_0)_{2K\times 2K}\mathbf{e}_{2K\times 1}\Bigg)^H\Bigg\}\Bigg\} \\ &= \left(\frac{(K^2+(1+K)^2)}{2}\right)\sigma_\omega^2\sigma^2 + \left(\frac{1}{K}\right)\left(\frac{N_0}{|x|^2}\right). \end{aligned} \tag{10}$$

This generalizes the results of mean square channel estimation error in AWGN noise only [7, 8] to the case where there is also a residual offset present in the data being used for channel estimation. It is clear that the expression reduces to that in [7, 8], when $\sigma_\omega^2 = 0$. Figure 3 depicts the results in a graphical form for two pilot blocks. It is also satisfying to see that the results match closely (except very large ROEs) those based on experimental simulations. The effect of σ_ω^2 is seen to be very prominent as it introduces a floor in MSE value, independent of SNR.

4. ESTIMATION OF OSTBC DATA

Next, we consider the compensated received data vector corresponding to the OSTBC part of the frame. Consider the lth STBC (Alamouti) block, which can be written as [1]

$$\begin{aligned} \mathbf{z} = \left[y_c^{(2\nu-1)}\left(y_c^{2\nu}\right)^*\right]^T &= \begin{bmatrix} e^{j(2\nu-1)\Delta\omega} & 0 \\ 0 & e^{-j2\nu\Delta\omega} \end{bmatrix}\mathbf{Hs} \\ &\quad + \begin{bmatrix} e^{j(2\nu-1)\widehat{\omega}_0} & 0 \\ 0 & e^{-j2\nu\widehat{\omega}_0} \end{bmatrix}\left[e_{2\nu-1} \;\; e_{2\nu}^*\right]^T, \end{aligned} \tag{11}$$

where $\nu = K + l$, $\mathbf{H} = \left[\begin{smallmatrix} h_1 & h_2 \\ h_2^* & -h_1^* \end{smallmatrix}\right]$, and $\mathbf{s} = [s_{2l-1} \;\; s_{2l}]^T$. If the channel is known perfectly, then the ML estimation rule for obtaining estimate of $\mathbf{s}$ is given as

$$\widehat{\mathbf{s}} = \arg\min \|\mathbf{r} - \rho\mathbf{s}\|, \tag{12}$$

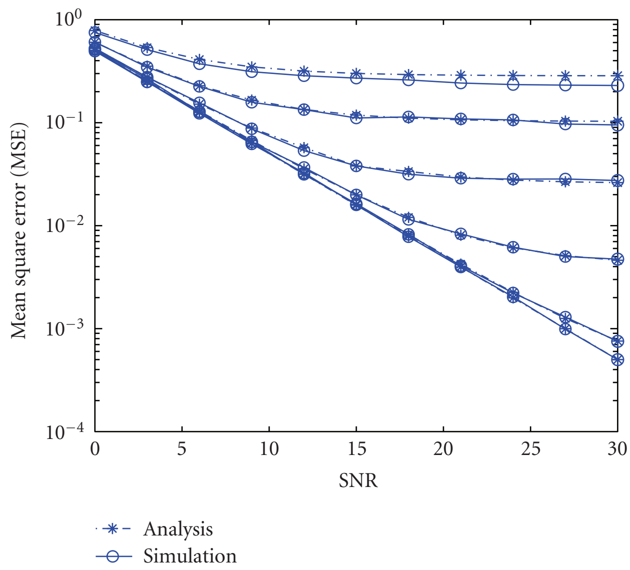

FIGURE 3: Analytical and experimental plots of MSE in the channel estimates for different values of ROE; $\sigma_\omega^2 = (2\pi/30)^2, (2\pi/50)^2, (2\pi/100)^2, (2\pi/250)^2, (2\pi/1000)^2, 0$ from uppermost to downmost, respectively.

where

$$\rho = \mathbf{H}^H\mathbf{H}, \qquad \mathbf{r} = \mathbf{H}^H\mathbf{z}. \tag{13}$$

In the presence of channel estimation errors, as discussed in Section 3, the vector $\mathbf{r}$ will be equal to

$$\mathbf{r} = \tilde{\mathbf{H}}^H\mathbf{z} = (\mathbf{H} + \Delta\mathbf{H})^H\mathbf{z}, \tag{14}$$

where $\Delta\mathbf{H} = \left[\begin{smallmatrix} \Delta h_1 & \Delta h_2 \\ \Delta h_2^* & -\Delta h_1^* \end{smallmatrix}\right]$. Substituting the value of $\mathbf{z}$ from (11) into (14), we get

$$\mathbf{r} = \tilde{\mathbf{H}}^H\Bigg(\underbrace{\begin{bmatrix} e^{j(2\nu-1)\Delta\omega} & 0 \\ 0 & e^{-j2\nu\Delta\omega} \end{bmatrix}}_{I}\mathbf{Hs} + \begin{bmatrix} e^{j(2\nu-1)\hat{\omega}_0} & 0 \\ 0 & e^{-j2\nu\hat{\omega}_0} \end{bmatrix}\begin{bmatrix} e_{(2\nu-1)} & e_{2\nu}^* \end{bmatrix}^T\Bigg). \tag{15}$$

Applying Taylor series approximation for the exponential term in the term (I) in (15), we will get

$$\mathbf{r} = \tilde{\mathbf{H}}^H\mathbf{Hs} + \tilde{\mathbf{H}}^H\begin{bmatrix} j(2\nu-1)\Delta\omega & 0 \\ 0 & -j2\nu\Delta\omega \end{bmatrix}\mathbf{Hs} + \tilde{\mathbf{H}}^H\begin{bmatrix} e^{j(2\nu-1)\hat{\omega}_0} & 0 \\ 0 & e^{-j2\nu\hat{\omega}_0} \end{bmatrix}\begin{bmatrix} e_{2\nu-1} & e_{2\nu}^* \end{bmatrix}^T. \tag{16}$$

From (14) and (16), $\mathbf{r}$ can be expressed as

$$\mathbf{r} = \rho\mathbf{s} + \underbrace{\Delta\mathbf{H}^H\mathbf{Hs}}_{\text{interfering term (1)}} + \underbrace{\tilde{\mathbf{H}}^H\mathbf{H}_\Omega\mathbf{s}}_{\text{interfering term (2)}} + \underbrace{\tilde{\mathbf{H}}^H\begin{bmatrix} e^{j(2\nu-1)\hat{\omega}_0} & 0 \\ 0 & e^{-j2\nu\hat{\omega}_0} \end{bmatrix}\begin{bmatrix} e_{(2\nu-1)} & e_{2\nu}^* \end{bmatrix}^T}_{\text{interfering term (3)}}, \tag{17}$$

where $\mathbf{H}_\Omega = \left[\begin{smallmatrix} j(2\nu-1)\Delta\omega & 0 \\ 0 & -j2\nu\Delta\omega \end{smallmatrix}\right]\mathbf{H}$. Clearly, estimation of $\hat{\mathbf{s}}$ via minimization of (12) would be affected by the interfering terms (1)–(3) shown in (17). In the next section, we carry out an SER analysis by first obtaining expression for the total interference power and its subsequent effect on the signal-to-interference ratio (SIR).

5. ERROR PROBABILITY ANALYSIS

In order to obtain an expression for the SIR, and hence for the probability of error, we need to find the total interference power in (17). To simplify the analysis, we restrict ourself to those cases when ω_0 is typically much smaller than the symbol period and if a sufficiently efficient estimator like [5, 6] is used for carrier offset estimation, $\Delta\omega$ is also very less than the symbol period. Under this restriction and assuming channel, noise, training data and S-T data independent of each other and of zero mean, the correlations between $\Delta\mathbf{h}$ and $\mathbf{h}$, $\Delta\mathbf{h}$ and $\Delta\omega$, and $\tilde{\mathbf{H}}$ and $\mathbf{H}_\Omega$, which mainly depend upon ω_0 and $\Delta\omega$, would be so small that these could be neglected. We make use of this assumption in the following analysis for simplicity, but without any loss of generality. In this case, the total interference power in (17) is obtained in the appendix and the average interfering power will be

$$\begin{aligned} \text{Power}_{\text{avg}} &= 2E_s\sigma^2\,\text{MSE} + 2(2\nu-1)^2E_s\sigma_\omega^2\sigma^2(\sigma^2 + \text{MSE}) \\ &\quad + 2(\sigma^2 + \text{MSE})N_0. \end{aligned} \tag{18}$$

Since the channel is modeled as complex Gaussian random variable with variance σ^2, hence $E\{\sum_{i=1}^m |h_i|^2\} = m\sigma^2$ and the average SIR per channel will be

$$\begin{aligned} \bar{\gamma} &= \frac{E_s}{2\,\text{Power}_{\text{avg}}}\sigma^2 \\ &= \frac{E_s}{2\{E_s\sigma^2\,\text{MSE} + (2\nu-1)^2E_s\sigma_\omega^2\sigma^2(\sigma^2+\text{MSE}) + (\sigma^2+\text{MSE})N_0\}}\sigma^2, \end{aligned} \tag{19}$$

where E_s is signal power. If there is no carrier offset present, that is, σ_ω^2=0, and channel variance is unity, that is, σ^2=1, (19) reduces into the following conventional form [7–12]:

$$\bar{\gamma} = \frac{E_s}{2\{E_s\,\text{MSE} + N_0 + N_0\,\text{MSE}\}}. \tag{20}$$

Hence, (19) is more general form of SIR than (20) and therefore, our analysis presents a comprehensive view of the behavior of STBC data in the presence of carrier offset. Further, the expression of exact probability of error for M-QAM data received over J-independent flat fading Rayleigh channels, in the terms of SIR, is suggested in [13] as

$$\begin{aligned}\mathbf{P}_e = {} & 4\left(1-\frac{1}{\sqrt{M}}\right)\left(\frac{1-\mu_c}{2}\right)^J \sum_{l=0}^{J-1}\binom{J-1+l}{l}\left(\frac{1+\mu_c}{2}\right)^J \\ & -4\left(1-\frac{1}{\sqrt{M}}\right)^2 \\ & \cdot\left(\frac{1}{4}-\frac{\mu_c}{\pi}\left(\left[\frac{\pi}{2}-\arctan\mu_c\right]\sum_{l=0}^{J-1}\frac{\binom{2l}{l}}{4(1+g_{\text{QAM}}\overline{\gamma})^l}\right.\right. \\ & \qquad -\sin(\arctan\mu_c)\sum_{l=1}^{J-1}\sum_{i=1}^{l}\frac{T_{il}}{(1+g_{\text{QAM}}\overline{\gamma})^l} \\ & \qquad \left.\left.\cdot\left[\cos(\arctan\mu_c)\right]^{2(l-i)+1}\right)\right),\end{aligned} \tag{21}$$

where

$$\mu_c = \sqrt{\frac{g_{\text{QAM}}\overline{\gamma}}{1+g_{\text{QAM}}\overline{\gamma}}},\quad g_{\text{QAM}} = \frac{3}{2(M-1)},$$
$$T_{il} = \frac{\binom{2l}{l}}{\left(\binom{2(l-i)}{(l-i)}4^i(2(l-i)+1)\right)}. \tag{22}$$

Probability of error in the frame consisting of L blocks of OSTBC data will be

$$\overline{P}_e = \frac{1}{L}\sum_{i=1}^{L}(P_e)_i, \tag{23}$$

where $(P_e)_i$ denote the error probability of ith OSTBC block. As all the interference terms in (17) consist of Gaussian data and have zero mean and diagonal covariance matrices (see the appendix), we may assume without loss of generality that all the interference terms are Gaussian distributed with zero mean and certain diagonal covariance matrices.

6. ANALYTICAL AND SIMULATION RESULTS

The analytical and simulation results for a frame consisting of two pilot blocks and three OSTBC blocks are shown in Figures 4–6. All the simulations are performed with the 16-QAM data. The average power transmitted in a time interval is kept unity. The MISO system of two transmit antennas and a single receive antenna employs Alamouti code. The channel gains are assumed circular, complex Gaussian with unit variance and stationary over one frame duration (flat fading). The analytical plots of SIR and probability of error are plotted under the same conditions as those of experiments.

Figure 4 shows the effect of ROE on the average SIR per channel with -30 dB MSE, in channel estimates. Here, we have plotted (19) with different values of ROE. It is easy to see that there is not much improvement in SIR with the increase in SNR at large values of ROE, which is quite intuitive. Hence, our analytical formula of SIR presents a feasible view of the behavior of OSTBC imperfect knowledge of carrier offset in MIMO channels.

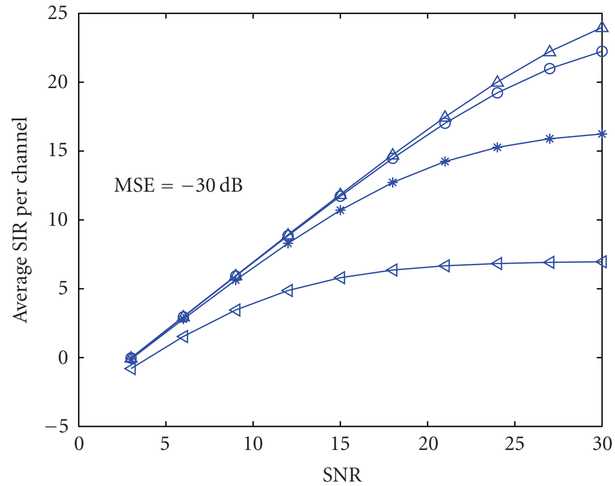

FIGURE 4: Plot of average SIR per channel versus SNR for MSE = -30 dB (graphs are plotted for $\sigma_\omega^2 = 0$, $(2\pi/1000)^2$, $(2\pi)^2/10^5$, $(2\pi/100)^2$ from uppermost to downmost, resp.).

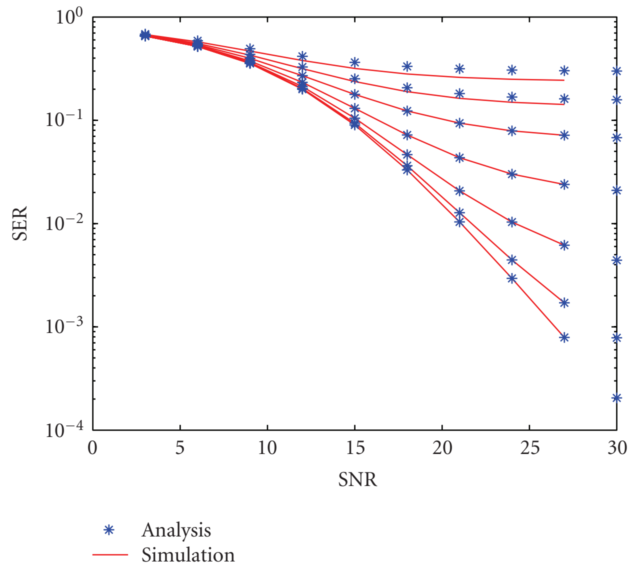

FIGURE 5: SER versus SNR plots for 16 QAM, with no MSE (graphs are plotted for $\sigma_\omega^2 = (2\pi)^2/10000$, $(2\pi)^2/20000$, $(2\pi/200)^2$, $(2\pi/300)^2$, $(2\pi/500)^2$, $(2\pi/1000)^2$, 0 from uppermost to downmost, resp.).

Figures 5 and 6 show the analytical and experimental, probability of error plots with different values of MSE in channel estimates and with different values of ROE. It is very much satisfying to see that the analytical results match closely those based on experimental simulations for small value of residual carrier offsets. However, for the large values of offset error, the analytical results do not follow the simulation results very tightly because our assumption of uncorrelatedness between different quantities (Section 5) gets violated in

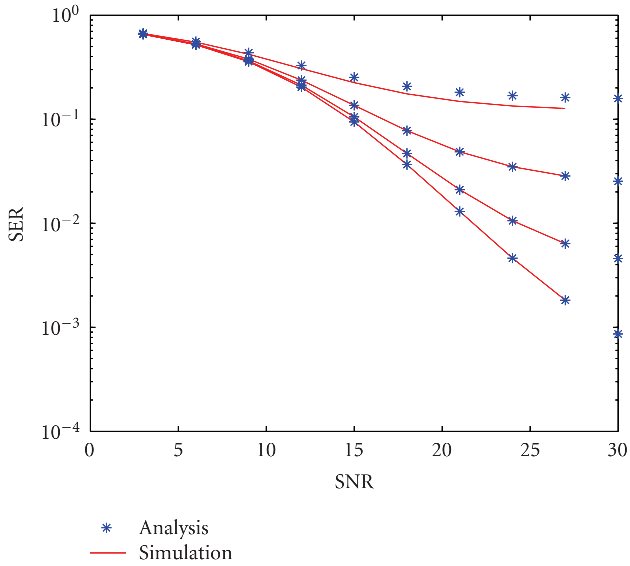

FIGURE 6: SER versus SNR plots for 16 QAM, with MSE = −40 dB (graphs are plotted for $\sigma_\omega^2 = (2\pi)^2/20000$, $(2\pi)^2/80000$, $(2\pi/500)^2$, $(2\pi/1000)^2$ from uppermost to down most, resp.).

such cases. Nevertheless, our analysis is still able to provide an approximate picture of the behavior of the S-T data with large residual offset errors.

7. CONCLUSIONS

We have performed a mathematical analysis of the behavior of orthogonal space-time codes with imperfect carrier offset compensation in MIMO channels. We have considered the effect of imperfect carrier offset knowledge over the estimates of the channel gains and resulting probability of error in the final decoding of OSTBC data. Our analysis also includes the effect of imperfect channel state information due to AWGN noise, over the decoding of OSTBC data. Hence, it presents a comprehensive view of the performance of OSTBC with imperfect knowledge of small carrier offsets (in case of small oscillator drifts or low mobility and an efficient offset estimator) in flat fading MIMO channels with offsets. The proposed analysis can also predict the approximate behavior of S-T data with large carrier offsets (in case of high mobility or highly unstable oscillators and an inefficient offset estimator).

APPENDIX

A. DERIVATION OF TOTAL INTERFERENCE POWER IN THE ESTIMATION OF OSTBC DATA

We will find the expression of the total interference power in (17) here. There are three interfering terms in (17). Initially, we will calculate power of each term separately and finally we will sum the power of all terms to find the total interference power. Before proceeding to the power calculation, we can also assume $\mathbf{s}$ being a vector of statistically independent symbols, which are also independent of channel, carrier offset, channel estimation error, and ROE.

A.1. Power of first interfering term

In view of the discussion of Section 5, we can write

$$E\{\Delta\mathbf{H}^H\mathbf{H}\mathbf{s}\} = E\{\Delta\mathbf{H}^H\}E\{\mathbf{H}\}E\{\mathbf{s}\} = 0, \qquad \text{(A.1)}$$

implying that the first term has zero mean. Further, it can be shown that $E\{\mathbf{s}\mathbf{s}^H\} = (E_s/2)[\mathbf{I}]$, $E\{\mathbf{H}\mathbf{H}^H\} = 2\sigma^2[\mathbf{I}]$ and $E\{\Delta\mathbf{H}^H\Delta\mathbf{H}\} = 2(\text{MSE})[\mathbf{I}]$, where $[\mathbf{I}]$ is identity matrix of 2×2. In view of the uncorrelatedness assumption of $\Delta\mathbf{H}$, $\mathbf{H}$ and $\mathbf{s}$, and using the results of [14], the covariance matrix associated with this term can be found as follows:

$$\begin{aligned} E\{(\Delta\mathbf{H}^H\mathbf{H}\mathbf{s})(\Delta\mathbf{H}^H\mathbf{H}\mathbf{s})^H\} &= E\{\Delta\mathbf{H}^H[E\{\mathbf{H}(E\{\mathbf{s}\mathbf{s}^H\})\mathbf{H}^H\}]\Delta\mathbf{H}\} \\ &= 2E_s\sigma^2(\text{MSE})\begin{bmatrix}1&0\\0&1\end{bmatrix}. \end{aligned} \qquad \text{(A.2)}$$

A.2. Power of second interfering term

The mean of the second interfering term, as per the discussion of Section 5, will be

$$E\{\widetilde{\mathbf{H}}^H\mathbf{H}_\Omega\mathbf{s}\} = E\{\widetilde{\mathbf{H}}^H\}E\{\mathbf{H}_\Omega\}E\{\mathbf{s}\} = 0, \qquad \text{(A.3)}$$

implying that the second term also has zero mean. Further, it can be shown that $E\{\mathbf{H}_\Omega\mathbf{H}_\Omega^H\} \cong 2(2v-1)^2\sigma_\omega^2\sigma^2[\mathbf{I}]$ and $E\{\widetilde{\mathbf{H}}^H\widetilde{\mathbf{H}}\} = 2(\sigma^2+\text{MSE})[\mathbf{I}]$. In view of the uncorrelatedness assumption of $\widetilde{\mathbf{H}}$, $\mathbf{H}_\Omega$ and $\mathbf{s}$, and using the results of [14], the covariance matrix associated with this term can be found as follows:

$$\begin{aligned} &E\{(\widetilde{\mathbf{H}}^H\mathbf{H}_\Omega\mathbf{s})(\widetilde{\mathbf{H}}^H\mathbf{H}_\Omega\mathbf{s})^H\} \\ &\quad = E\{\widetilde{\mathbf{H}}^H[E\{\mathbf{H}_\Omega(E\{\mathbf{s}\mathbf{s}^H\})\mathbf{H}_\Omega^H\}]\widetilde{\mathbf{H}}\} \\ &\quad = 2(2v-1)^2E_s\sigma_\omega^2\sigma^2(\sigma^2+\text{MSE})\begin{bmatrix}1&0\\0&1\end{bmatrix}. \end{aligned} \qquad \text{(A.4)}$$

A.3. Power of third interfering term

Assuming $\mathbf{e}$, $\widetilde{\mathbf{H}}$ and $\hat{\omega}_0$ being statistically independent of each other, the mean of the third interfering term will be

$$\begin{aligned} &E\left\{\widetilde{\mathbf{H}}^H\begin{bmatrix}e^{j(2v-1)\hat{\omega}_0}&0\\0&e^{-j2v\hat{\omega}_0}\end{bmatrix}\begin{bmatrix}e_{(2v-1)}&e_{2v}^*\end{bmatrix}^T\right\} \\ &\quad = E\{\widetilde{\mathbf{H}}^H\}E\left\{\begin{bmatrix}e^{j(2v-1)\hat{\omega}_0}&0\\0&e^{-j2v\hat{\omega}_0}\end{bmatrix}\right\}E\left\{\begin{bmatrix}e_{(2v-1)}&e_{2v}^*\end{bmatrix}^T\right\} = 0, \end{aligned} \qquad \text{(A.5)}$$

implying that the third term also has zero mean. Further, it can be shown that

$$\begin{aligned} &E\left\{\begin{bmatrix}e_{(2v-1)}\\e_{2v}^*\end{bmatrix}\begin{bmatrix}e_{(2v-1)}^*&e_{2v}\end{bmatrix}\right\} = N_0[\mathbf{I}], \\ &E\left\{\begin{bmatrix}e^{j(2v-1)\hat{\omega}_0}&0\\0&e^{-j2v\hat{\omega}_0}\end{bmatrix}\begin{bmatrix}e^{-j(2v-1)\hat{\omega}_0}&0\\0&e^{j2v\hat{\omega}_0}\end{bmatrix}\right\} = [\mathbf{I}]. \end{aligned} \qquad \text{(A.6)}$$

Using the results of [14], the covariance matrix can be found as follows:

$$
\begin{aligned}
&E\left\{\left(\tilde{\mathbf{H}}^H\begin{bmatrix} e^{j(2\nu-1)\hat{\omega}_0} & 0 \\ 0 & e^{-j2\nu\hat{\omega}_0}\end{bmatrix}\begin{bmatrix} e_{(2\nu-1)} & e_{2\nu}^*\end{bmatrix}^T\right)\right.\\
&\quad\left.\times\left(\tilde{\mathbf{H}}^H\begin{bmatrix} e^{j(2\nu-1)\hat{\omega}_0} & 0 \\ 0 & e^{-j2\nu\hat{\omega}_0}\end{bmatrix}\begin{bmatrix} e_{(2\nu-1)} & e_{2\nu}^*\end{bmatrix}^T\right)^H\right\}\\
&\quad= E\left\{\tilde{\mathbf{H}}^H\left[E\left\{\begin{bmatrix} e^{j(2\nu-1)\hat{\omega}_0} & 0 \\ 0 & e^{-j2\nu\hat{\omega}_0}\end{bmatrix}\right.\right.\right.\\
&\qquad\qquad\times\left(E\left\{\begin{bmatrix} e_{(2\nu-1)} \\ e_{2\nu}^*\end{bmatrix}\begin{bmatrix} e_{(2\nu-1)}^* & e_{2\nu}\end{bmatrix}\right\}\right)\\
&\qquad\qquad\left.\left.\left.\times\begin{bmatrix} e^{-j(2\nu-1)\hat{\omega}_0} & 0 \\ 0 & e^{j2\nu\hat{\omega}_0}\end{bmatrix}\right\}\right]\tilde{\mathbf{H}}\right\}\\
&\quad= 2\left(\sigma^2+\mathrm{MSE}\right)N_0\begin{bmatrix} 1 & 0 \\ 0 & 1\end{bmatrix}.
\end{aligned}
\tag{A.7}
$$

Apparently, all interfering terms are distributed identically with zero mean and their covariance matrices are proportional to the identity matrix. Further, we note that the power in the three terms can be simply added, since, these can be shown to be mutually uncorrelated. Hence, the total interfering power will be

$$
\begin{aligned}
\text{Power}_{\text{tot}} &= 2E_s\sigma^2\,\mathrm{MSE}\begin{bmatrix} 1 & 0 \\ 0 & 1\end{bmatrix}\\
&\quad+2(2\nu-1)^2E_s\sigma_\omega^2\sigma^2\left(\sigma^2+\mathrm{MSE}\right)\begin{bmatrix} 1 & 0 \\ 0 & 1\end{bmatrix}\\
&\quad+2\left(\sigma^2+\mathrm{MSE}\right)N_0\begin{bmatrix} 1 & 0 \\ 0 & 1\end{bmatrix}.
\end{aligned}
\tag{A.8}
$$

ACKNOWLEDGMENT

The authors are extremely thankful to Professor Are Hjørungnes, UniK, University of Oslo, for his help provided in the derivation of the expectation in the appendix.

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