Hindawi Publishing Corporation
EURASIP Journal on Wireless Communications and Networking
Volume 2007, Article ID 78156, 8 pages
doi:10.1155/2007/78156

Research Article

Robust Adaptive OFDM with Diversity for Time-Varying Channels

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Received 23 November 2006; Accepted 17 April 2007

Recommended by Sangarapillai Lambotharan

The performance of an orthogonal frequency-division multiplexing (OFDM) system can be significantly increased by using adaptive modulation and transmit diversity. An accurate estimate of the channel, however, is required at the transmitter to realize this benefit. Due to the time-varying nature of the channel, this estimate may be outdated by the time it is used for detection. This results in a mismatch between the actual channel and its estimate as seen by the transmitter. In this paper, we investigate adaptive OFDM with transmit and receive diversities, and evaluate the detrimental effects of this channel mismatch. We also describe a robust scheme based on using past estimates of the channel. We show that the effects of the mismatch can be significantly reduced with a combination of diversity and multiple channel estimates. In addition, to reduce the amount of feedback, the subband approach is introduced where a common channel estimate for a number of subcarriers is fedback to the transmitter, and the effect of this method on the achievable rate is analyzed.

1. INTRODUCTION

The performance of wireless communication systems can be significantly improved by adaptively matching the transmission parameters such as rate, power level, or coding type to the channel frequency response [1–3]. When OFDM is used in a wideband frequency-selective channel, different subchannels usually experience different channel gains. The system capacity can then be maximized if the data rate and power level of each subcarrier are adjusted according to the channel gain of that subcarrier. To take advantage of this property of OFDM, bit-loading algorithms that adaptively distribute the input bits over the available subcarriers have been proposed [4–6].

To achieve the performance gain of adaptive modulation, an accurate estimate of the channel response, as seen at the receiver, is required at the transmitter. One approach to accomplish this is to measure the channel state at the receiver and feedback the estimate to the transmitter. Due to the time-varying nature of the wireless channel, however, if the Doppler is large enough, this information may be outdated by the time it is used for detection resulting in an imperfect channel state information (CSI) at the transmitter. Channel estimation errors or errors introduced in the feedback channel might also cause imperfections in the CSI. The performance of adaptive loading algorithms degrades when imperfect CSI is used to compute the bit distribution over the subcarriers and this degradation has been investigated by several authors [7–9]. Another method to improve the performance is to increase the diversity by using multiple antennas. One of the options is to use an antenna array at the transmitter to form a beam in a specific direction to maximize the signal power at the receiver. This type of transmit diversity is called beamforming and it also requires an accurate estimate of the channel response. The performance of beamforming degrades when there is a mismatch between the actual channel characteristics and the estimate [10].

It is common for most high-speed wireless systems to suffer from imperfections in CSI due to Doppler, constraints on the size of the CSI data that can be fedback to the transmitter, or channel estimation errors. This has led many researchers to investigate the robust optimization of transmission strategies with imperfect CSI for single-carrier and OFDM systems, possibly with multiple antennas, and several solutions have been presented [11–18]. Robust optimization techniques can be classified as stochastic or worst-case approaches [19, 20]. In the stochastic approach, the optimization parameter is modeled as a random variable with a known distribution, and the expectations of the objective and constraint functions with respect to the parameter are used to compute the solution [21]. In the worst-case approach, the error in the optimization parameter is given to lie in a set and the solution is computed by solving the optimization problem with the worst-case objective and constraint functions for any parameter error in the given set [22]. In [13], the stochastic approach is used to optimize a system with multiple transmitter antennas, and in [15] the approach is used to design a general MIMO transmission system. An adaptive MIMO-OFDM system with imperfect CSI is designed with the stochastic approach in [12]. The worst-case approach has been studied widely and, in [23, 24], the theory and applications of this approach are discussed. A worst-case MSE precoder for MIMO channels with imperfect CSI is designed in [25]. This approach is also used in [26, 27] to design robust adaptive beamformers, and in [28] to design a robust minimum variance beamformer.

In this paper, we propose a robust adaptive modulation scheme for OFDM with transmit and/or receive diversity. The scheme is based on the idea of using outdated estimates of the channel, as suggested in [12, 21], to characterize the statistics of the current channel more reliably. This scheme is an example of a stochastically robust design method. The outline of the paper is as follows: in Section 2, we introduce the system model. In Section 3, adaptive OFDM with transmit diversity is studied, the detrimental effects of a time-varying channel is investigated, and the proposed robust scheme is introduced. Then, in Section 4, the scheme is extended to the case where the receiver is also equipped with multiple antennas to provide receive diversity. It is shown with simulations that multiple outdated channel estimates and transmit/receive diversity significantly reduce the degradation due to channel mismatch. To reduce the amount of feedback, the subband approach is introduced in Section 5 where a common channel estimate for a number of subcarriers is fedback to the transmitter, and the effect of this method on the achievable rate is analyzed. Finally, in Section 6, conclusions are presented.

2. SYSTEM MODEL

We assume that the system is free of any intersymbol interference (ISI) or intercarrier interference (ICI). The time index for an OFDM block is denoted as n, k is the frequency index for a given subcarrier, and the information symbol on this subcarrier of the nth OFDM block is denoted as $S[n,k]$. The number of transmit antennas is denoted by N_t, and we initially assume one receive antenna. The channel between each transmit and receive antenna pair is independent and is modeled as a multipath with an exponential power delay profile. The channel frequency response at time n and for subcarrier k between the transmit antenna array and the receiver is denoted by the $N_t \times 1$ vector $\mathbf{H}[n,k]$. Any individual ith component of $\mathbf{H}[n,k]$ represents the frequency response between the ith transmit antenna and the receiver and can be modeled as an independent complex Gaussian random variable with zero mean and unit variance. Each information symbol $S[n,k]$ is sent over all of the transmit antennas after being weighted by a beamforming vector $\mathbf{W}$ which has unit norm.

When there are also N_r receive antennas, the channel between the transmitter and the receiver will be denoted as a $N_r \times N_t$ matrix $\mathbf{H}[n,k]$. Each entry in $\mathbf{H}[n,k]$ is modeled as an independent complex Gaussian random variable with zero mean and unit variance. If the beamforming vector is again denoted as $\mathbf{W}$, then the equivalent channel response between the transmitter and the receiver becomes $\mathbf{H}[n,k]\mathbf{W}$.

3. ADAPTIVE MODULATION WITH TRANSMIT DIVERSITY ONLY

3.1. Adaptation with a single channel estimate

The received signal on subcarrier k in block n is $\mathbf{r} = \mathbf{H}^T\mathbf{W}S + \mathbf{n}$, where $\mathbf{n}$ denotes the additive white Gaussian noise with zero mean and variance N_0, and where we have omitted the OFDM block and subcarrier indices for the sake of simplicity. When there is a single receive antenna, the weight vector $\mathbf{W}$ that maximizes the SNR is $\mathbf{H}^*/\|\mathbf{H}\|$ where $*$ denotes the complex conjugate. This choice of the weight vector is equivalent to performing maximal ratio combining at the transmitter. As described in [29], the probability of error for QAM modulation on each subcarrier can be approximated by

$$P_e = c_1 \exp\left(\frac{-c_2\,\mathrm{SNR}}{2^R - 1}\right), \tag{1}$$

where in this case, $\mathrm{SNR} = |\mathbf{H}^T\mathbf{W}|^2(E_s/N_0)$, $c_1 = 0.2$, $c_2 = 1.6$, R is the number of bits transmitted per symbol on the kth subcarrier, and E_s is the signal energy. Let us assume that there is a time delay D between when the channel is estimated and when it is actually used to compute the beamforming vector. Then, if the channel response at time n is given by $\mathbf{H}(n)$, where we have omitted the subchannel index k, the beamforming vector is derived from the outdated channel estimate $\mathbf{H}(n-D)$ and it becomes $\mathbf{W} = \mathbf{H}(n-D)^*/\|\mathbf{H}(n-D)\|$, that is, a mismatch occurs in the coefficients of the optimal beamforming vector and the actual one due to the delay of D.

The relation between the channel vector at two different time instances can be studied with Jakes' model [30]. Using this model, the relation between the channel vectors at times n and $n-D$ can be written as $\mathbf{H}(n) = \rho\mathbf{H}(n-D) + \boldsymbol{\varepsilon}$, where ρ is the correlation coefficient, $\boldsymbol{\varepsilon}$ is white and Gaussian with covariance matrix $(1-\rho^2)\mathbf{I}$, and $\mathbf{I}$ is the identity matrix with dimension $N_t \times N_t$. The correlation coefficient $\rho = J_0(2\pi f_m D)$, where $J_0(\cdot)$ is the zeroth-order Bessel function and f_m is the maximum Doppler frequency.

For a given error probability, the achievable bit rate is computed from (1) and then the corresponding subcarrier is loaded accordingly. When there is time variation in the channel, however, this approach results in degraded performance due to the outdated channel estimate. One robust approach is then to compute the average probability of error, $E(P_e)$, conditioned on the outdated estimates of the channel, and then find the achievable bit rate R for the given $E(P_e)$ [12]. To this end, let us denote $\mathcal{X} = \mathbf{H}^T\mathbf{W}$. Then,

$$\begin{aligned}\mathcal{X} &= [\rho\mathbf{H}(n-D)+\boldsymbol{\varepsilon}]^T\frac{\mathbf{H}(n-D)^*}{||\mathbf{H}(n-D)||}\\ &= \rho||\mathbf{H}(n-D)|| + \boldsymbol{\varepsilon}^T\frac{\mathbf{H}(n-D)^*}{||\mathbf{H}(n-D)||}.\end{aligned} \tag{2}$$

Conditioned on $\mathbf{H}(n-D)$, $\mathcal{X}$ is a complex Gaussian random variable with mean $m_{\mathcal{X}} = \rho\|\mathbf{H}(n-D)\|$ and variance $\sigma^2_{\mathcal{X}}$ given by

$$\begin{aligned}\sigma^2_{\mathcal{X}} &= E\left\{\left(\boldsymbol{\varepsilon}^T\frac{\mathbf{H}(m-D)^*}{||\mathbf{H}(m-D)||}\right)\left(\boldsymbol{\varepsilon}^T\frac{\mathbf{H}(m-D)^*}{||\mathbf{H}(m-D)||}\right)^*\right\}\\ &= 1-\rho^2.\end{aligned} \tag{3}$$

Then, the average probability of error given by $\mathbf{H}(n-D)$ is calculated as

$$E(P_e) = \int c_1\exp\left(\frac{-c_2|\mathcal{X}|^2(E_s/N_0)}{2^R-1}\right)f_{\mathcal{X}}(x)dx, \tag{4}$$

where $f_{\mathcal{X}}(x)$ is the distribution function of the complex random variable $\mathcal{X}$. Evaluating (4), we get

$$E(P_e) = c_1\frac{2^R-1}{a+(2^R-1)}\exp\left(\frac{-b}{a+(2^R-1)}\right), \tag{5}$$

where the constants are defined as $a = c_2\sigma^2_{\mathcal{X}}(E_s/N_0)$ and $b = c_2|m_{\mathcal{X}}|^2(E_s/N_0)$ with $\mathcal{X} = \mathbf{H}^T\mathbf{W}$, $m_{\mathcal{X}} = \rho\|\mathbf{H}(n-D)\|$ and $\sigma^2_{\mathcal{X}} = 1-\rho^2$. For a target $E(P_e)$, the achievable rate R (b/s/Hz) can be determined from (5) by resorting to numerical methods. In this work, the *fsolve* function from the Matlab Optimization Toolbox [31] was used for this computation.

3.2. Adaptation with multiple channel estimates

One approach for reducing the degradation caused by the channel mismatch problem is to use multiple past estimates of the channel to obtain a more reliable overall statistical characterization of the channel. This approach was effectively used in [12] for adaptive OFDM with a single transmit antenna. To elaborate this idea, let us assume that we have two outdated estimates of the current channel $\mathbf{H}(n)$ with delays D and $2D$ given as $\mathbf{H}(n-D)$, and $\mathbf{H}(n-2D)$. Then, $\mathbf{H}(n)$, $\mathbf{H}(n-D)$, and $\mathbf{H}(n-2D)$ are jointly Gaussian with the mean vector $\mathbf{M}$ and covariance matrix $\boldsymbol{\Sigma}$ given as

$$\mathbf{M} = \begin{bmatrix}\mathbf{0}\\ \mathbf{0}\\ \mathbf{0}\end{bmatrix}, \qquad \boldsymbol{\Sigma} = \begin{bmatrix}\boldsymbol{\Sigma}_{\mathbf{H}_1\mathbf{H}_1} & \boldsymbol{\Sigma}_{\mathbf{H}_1\mathbf{H}_2} & \boldsymbol{\Sigma}_{\mathbf{H}_1\mathbf{H}_3}\\ \boldsymbol{\Sigma}_{\mathbf{H}_2\mathbf{H}_1} & \boldsymbol{\Sigma}_{\mathbf{H}_2\mathbf{H}_2} & \boldsymbol{\Sigma}_{\mathbf{H}_2\mathbf{H}_3}\\ \boldsymbol{\Sigma}_{\mathbf{H}_3\mathbf{H}_1} & \boldsymbol{\Sigma}_{\mathbf{H}_3\mathbf{H}_2} & \boldsymbol{\Sigma}_{\mathbf{H}_3\mathbf{H}_3}\end{bmatrix}, \tag{6}$$

where $\mathbf{0}$ is a vector of zeros with dimension $N_t \times 1$, $\mathbf{H}_1 = \mathbf{H}(n)$, $\mathbf{H}_2 = \mathbf{H}(n-D)$, and $\mathbf{H}_3 = \mathbf{H}(n-2D)$.

If the correlation coefficients are defined as $\rho_1 = J_0(2\pi f_m D)$ and $\rho_2 = J_0(2\pi f_m 2D)$, then $\mathbf{H}(n) = \rho_1\mathbf{H}(n-D) + \boldsymbol{\varepsilon}_1$, $\mathbf{H}(n) = \rho_2\mathbf{H}(n-2D)+\boldsymbol{\varepsilon}_2$, and $\mathbf{H}(n-D) = \rho_1\mathbf{H}(n-2D)+\boldsymbol{\varepsilon}_3$, where $\boldsymbol{\varepsilon}_1$ and $\boldsymbol{\varepsilon}_3$ are complex Gaussian with covariance $(1-\rho_1^2)\mathbf{I}_{N_t\times N_t}$, $\boldsymbol{\varepsilon}_2$ is complex Gaussian with covariance $(1-\rho_2^2)\mathbf{I}_{N_t\times N_t}$, and $\mathbf{I}_{N_t\times N_t}$ is the identity matrix of dimension $N_t \times N_t$. With these equalities, we can easily show that the elements of the covariance matrix are

$$\begin{aligned}\boldsymbol{\Sigma}_{\mathbf{H}_1\mathbf{H}_1} &= \boldsymbol{\Sigma}_{\mathbf{H}_2\mathbf{H}_2} = \boldsymbol{\Sigma}_{\mathbf{H}_3\mathbf{H}_3} = \mathbf{I}_{N_t\times N_t},\\ \boldsymbol{\Sigma}_{\mathbf{H}_1\mathbf{H}_2} &= \boldsymbol{\Sigma}_{\mathbf{H}_2\mathbf{H}_1} = \boldsymbol{\Sigma}_{\mathbf{H}_2\mathbf{H}_3} = \rho_1\mathbf{I}_{N_t\times N_t},\\ \boldsymbol{\Sigma}_{\mathbf{H}_1\mathbf{H}_3} &= \boldsymbol{\Sigma}_{\mathbf{H}_3\mathbf{H}_1} = \rho_2\mathbf{I}_{N_t\times N_t}.\end{aligned} \tag{7}$$

Now from (6),

$$\begin{bmatrix}\mathbf{H}(n)\\ \mathbf{H}(n-D)\\ \mathbf{H}(n-2D)\end{bmatrix} \sim CN\left\{\begin{bmatrix}\mathbf{0}\\ \mathbf{0}\\ \mathbf{0}\end{bmatrix}, \begin{bmatrix}\boldsymbol{\Sigma}_{11} & \boldsymbol{\Sigma}_{12}\\ \boldsymbol{\Sigma}_{21} & \boldsymbol{\Sigma}_{22}\end{bmatrix}\right\}, \tag{8}$$

where we have defined

$$\begin{aligned}\boldsymbol{\Sigma}_{11} &= [\mathbf{I}_{N_t\times N_t}], & \boldsymbol{\Sigma}_{12} &= \begin{bmatrix}\rho_1\mathbf{I}_{N_t\times N_t} & \rho_2\mathbf{I}_{N_t\times N_t}\end{bmatrix},\\ \boldsymbol{\Sigma}_{21} &= \begin{bmatrix}\rho_1\mathbf{I}_{N_t\times N_t}\\ \rho_2\mathbf{I}_{N_t\times N_t}\end{bmatrix}, & \boldsymbol{\Sigma}_{22} &= \begin{bmatrix}\mathbf{I}_{N_t\times N_t} & \rho_1\mathbf{I}_{N_t\times N_t}\\ \rho_1\mathbf{I}_{N_t\times N_t} & \mathbf{I}_{N_t\times N_t}\end{bmatrix}.\end{aligned} \tag{9}$$

Conditioned on $\mathbf{H}(n-D)$ and $\mathbf{H}(n-2D)$, $\mathbf{H}(n)$ is Gaussian with mean $\mathbf{M}_{\mathbf{H}}$ and covariance matrix $\boldsymbol{\Sigma}_{\mathbf{H}}$, given by [32]

$$\begin{aligned}\mathbf{M}_{\mathbf{H}} &= \boldsymbol{\Sigma}_{12}\boldsymbol{\Sigma}_{22}^{-1}\begin{bmatrix}\mathbf{H}(n-D)\\ \mathbf{H}(n-2D)\end{bmatrix}\\ &= \frac{\rho_1(1-\rho_2)}{1-\rho_1^2}\mathbf{H}(n-D) + \frac{-\rho_1^2+\rho_2}{1-\rho_1^2}\mathbf{H}(n-2D),\\ \boldsymbol{\Sigma}_{\mathbf{H}} &= \boldsymbol{\Sigma}_{11} - \boldsymbol{\Sigma}_{12}\boldsymbol{\Sigma}_{22}^{-1}\boldsymbol{\Sigma}_{21} = \frac{1-2\rho_1^2+2\rho_1^2\rho_2-\rho_2^2}{1-\rho_1^2}\mathbf{I}_{N_t\times N_t}.\end{aligned} \tag{10}$$

Therefore, $\mathcal{X} = \mathbf{H}^T\mathbf{W}$ is a complex Gaussian random variable with mean $m_{\mathcal{X}}$ and variance $\sigma^2_{\mathcal{X}}$, where

$$\begin{aligned}m_{\mathcal{X}} &= \frac{\rho_1(1-\rho_2)}{1-\rho_1^2}||\mathbf{H}(n-D)||\\ &\quad + \frac{-\rho_1^2+\rho_2}{1-\rho_1^2}\mathbf{H}(n-2D)^T\frac{\mathbf{H}(n-D)^*}{||\mathbf{H}(n-D)||},\\ \sigma^2_{\mathcal{X}} &= \frac{1-2\rho_1^2+2\rho_1^2\rho_2-\rho_2^2}{1-\rho_1^2}.\end{aligned} \tag{11}$$

The average bit error probability in this case can similarly be computed from (5) by using (11).

3.3. Simulation results

In this section, simulation results are presented to quantify the performance of an adaptive system which has multiple transmit antennas and uses outdated channel estimates as proposed previously. Here, we set the average target error probability to 10^{-3} and calculate the achievable rate for a large number of channel realizations. We assume that there are no errors due to noise in the receiver estimate of the channel. A sample simulation result for the achievable rate as a function of E_s/N_0 is provided in Figure 1 where a single outdated estimate has been used. The number of transmit antennas, N_t, and the Doppler-delay product, $f_m D$, are parameters. When $f_m D = 0$, the actual and estimated channel responses are the same and the best performance is achieved.

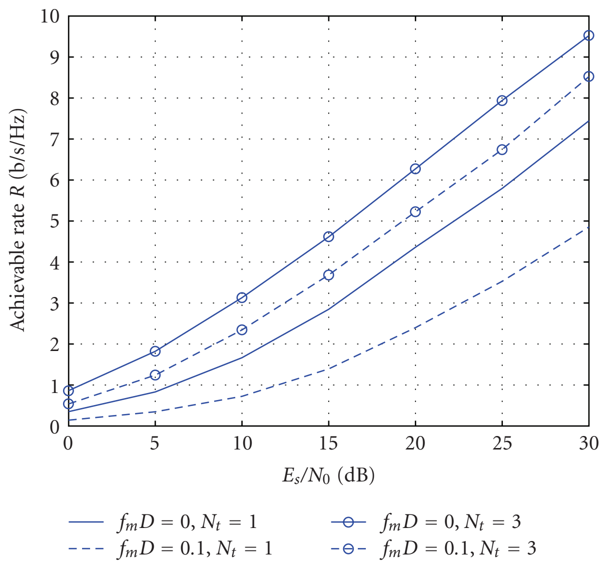

Figure 1: Achievable rate with multiple transmit antennas only and one outdated channel estimate.

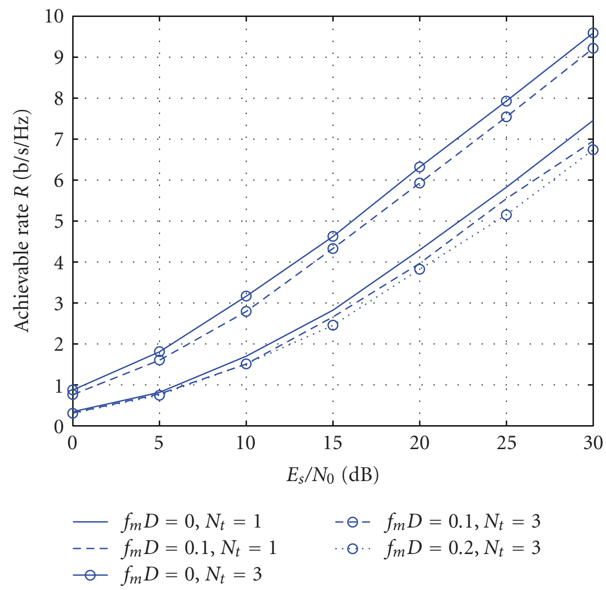

Figure 2: Achievable rate with multiple transmit antennas only and two outdated channel estimates.

When $f_mD > 0$, a mismatch occurs and a degradation results as expected. A sample value of $f_mD = 0.1$ is used in the simulations. This could correspond to a Doppler frequency of 165 Hz (e.g., a carrier frequency of 2 GHz and a vehicle speed of 55 mph), and a delay of about 600 microseconds (e.g., 3 OFDM blocks composed of 1024 subchannels and occupying 5 MHz of bandwidth). From Figure 1, we see that delay causes significant degradation when one antenna is used at the transmitter. When the number of antennas is increased to three, the relative degradation is smaller and the achievable rate with mismatch is even better than the single transmit-antenna case with no mismatch. So, either a power saving can be achieved or a higher Doppler-delay product term can be tolerated.

To investigate the additional benefits of using multiple past estimates, simulation results for a system similar to that of Figure 1 are presented in Figure 2. The results from Figure 2 show that with two estimates, the loss in achievable bit rate due to channel mismatch is minimized even with a single transmit antenna. The system with multiple transmit antennas can, of course, tolerate much higher Doppler rates or longer packets. For example, with three antennas and $f_mD = 0.2$, the performance is close to that of one transmitter antenna with no delay. The results show that the detrimental effects of delay can be significantly reduced with a combination of transmitter diversity and the use of multiple channel estimates.

4. ADAPTIVE MODULATION WITH TRANSMIT AND RECEIVE DIVERSITIES

In this section, we extend the above analysis to the case where multiple antennas are deployed at the receiver as well as at the transmitter. To maximize the SNR, the receiver performs maximal ratio combining (MRC). With MRC, the received SNR becomes $\text{SNR} = (\mathbf{HW})^H(\mathbf{HW})(E_s/N_0) = \mathbf{W}^H\mathbf{H}^H\mathbf{HW}(E_s/N_0) = |\mathbf{HW}|^2(E_s/N_0)$, where the superscript H denotes the Hermitian. The received SNR is maximized if the beamforming vector $\mathbf{W}$ is chosen to be the eigenvector $\mathbf{\Lambda}$, corresponding to the largest eigenvalue $\lambda_{\max}$ of $\mathbf{H}^H\mathbf{H}$ [33].

Similar to the previous analysis, we need to compute the average error probability conditioned on the outdated estimates of the channel. Let us denote $\mathbf{g} = \mathbf{H\Lambda}$ so that the received SNR can be written as $\text{SNR} = (E_s/N_0)\mathbf{g}^H\mathbf{g}$, and assume that we have a single outdated estimate of the channel, $\mathbf{H}(n-D)$. Then, the current channel is $\mathbf{H} = \rho\mathbf{H}(n-D) + \boldsymbol{\varepsilon}$, where each element of $\boldsymbol{\varepsilon}$ is white and Gaussian with variance $1-\rho^2$, and $\mathbf{g} = \mathbf{H\Lambda} = \rho\mathbf{H}(n-D)\mathbf{\Lambda} + \boldsymbol{\varepsilon}\mathbf{\Lambda}$. From this, we see that given the past estimate of the channel, $\mathbf{g}$ is a complex Gaussian random vector with mean and covariance given as $\mathbf{g} \sim CN(\rho\mathbf{H}(n-D)\mathbf{\Lambda}, \sigma_\varepsilon^2\mathbf{I}_{N_r\times N_r})$, where $\sigma_\varepsilon^2 = 1-\rho^2$.

We already know that the bit error probability can be computed as

$$P_e = c_1 \exp\left(\frac{-c_2(E_s/N_0)\mathbf{g}^H\mathbf{g}}{2^R-1}\right). \tag{12}$$

To find the expectation of (12) given the past channel estimate, we use the following identity from [16, 34]: if $\mathbf{z} \sim CN(\boldsymbol{\mu}, \boldsymbol{\Sigma})$, then $E_z(\exp(-\mathbf{z}^H\mathbf{A}\mathbf{z})) = \exp(-\mu^H\mathbf{A}(\mathbf{I}+\boldsymbol{\Sigma}\mathbf{A})^{-1}\mu)/\det(\mathbf{I}+\boldsymbol{\Sigma}\mathbf{A})$.

Substituting $\mathbf{z} = \mathbf{g}$ and $\mathbf{A} = (c_2(E_s/N_0)/(2^R-1))\mathbf{I}$ with matching dimensions, and after making the necessary computations, we find the resulting average error probability as

$$E(P_e) = c_1\frac{1}{(1+\boldsymbol{\sigma}_\varepsilon^2K)^{N_r}}\exp\left(-\frac{K}{1+\boldsymbol{\sigma}_\varepsilon^2K}\rho^2\left|\mathbf{H}(n-D)\mathbf{\Lambda}\right|^2\right), \tag{13}$$

where $|\mathbf{H}(n-D)\mathbf{\Lambda}|^2 = \lambda_{\max}$ and $K = (c_2E_s/N_0)/(2^R-1)$. The achievable bit rate for a target $E(P_e)$ in this case is computed

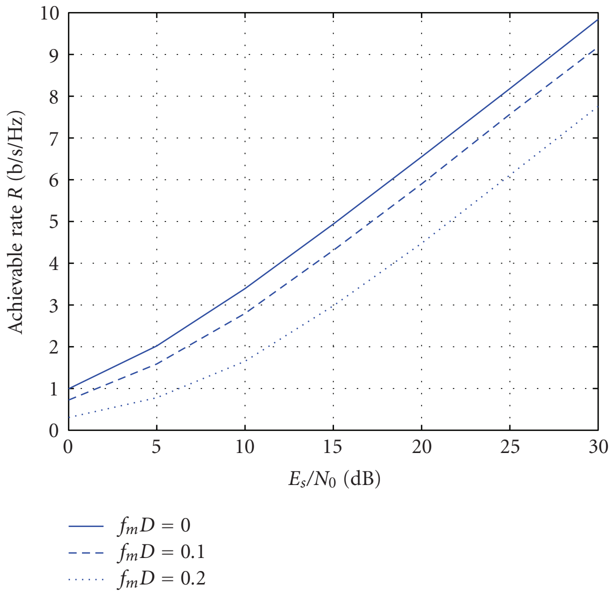

FIGURE 3: Achievable rate with multiple transmit-receive antennas ($N_t = 2$, $N_r = 2$) and one outdated channel estimate.

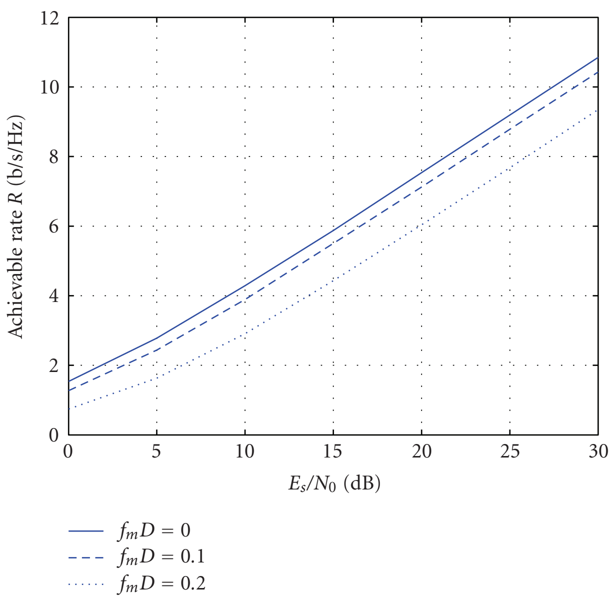

FIGURE 4: Achievable rate with multiple transmit-receive antennas ($N_t = 3$, $N_r = 3$) and one outdated channel estimate.

by averaging (13) over $\lambda_{\max}$ with Monte Carlo simulations and inverting (13) numerically to compute R. If we have multiple channel estimates, computing the achievable bit rate follows a similar route.

The effect of having multiple antennas at the receiver as well as at the transmitter is also studied with simulations and the results are presented in Figures 3 and 4 for the case where a single outdated channel estimate is used. The results show that, as expected, the performance and robustness of the system against channel delays are increased as more antennas are used at the receiver. When Figures 2 and 3 are compared, we see that if $f_mD = 0$ or $f_mD = 0.1$, the performance of the system with two transmit and two receive antennas with a single outdated channel estimate is similar to the performance of the system with three transmit antennas and one receive antenna, but with two outdated channel estimates. However, the increase in performance is more considerable when the delay gets larger. As an example, when $f_mD = 0.2$, the achievable rate is about 6.8 bits/s/Hz in Figure 2. The achievable rate increases to about 7.8 bits/s/Hz in Figure 3, which means a gain of 1 bit/s/Hz. It is interesting to note that the increase in achievable bit rate is larger when the system experiences larger delays. This observation illustrates that the robustness of the system against the time variance of the channel increases as more receive antennas are added.

5. ADAPTIVE MODULATION WITH SUBBAND FEEDBACK

In the previous discussion, we assumed that a channel estimate for each single subcarrier is sent back to the transmitter by the receiver. In practice, this increases the amount of feedback significantly. A possible method to decrease the amount of feedback is to use a single channel estimate for a number of subcarriers in a subband. For example, the average of the channel estimates of a number of highly correlated subcarriers in a subband might be used as an estimate for all these subcarriers. In this case, the channel estimate of the subcarrier k at the nth OFDM block can be written as

$$\widehat{H}(n,k) = \frac{1}{2M+1}\sum_{\Delta k=-M}^{M} H(n+\Delta n, \ell+\Delta k), \qquad (14)$$

where $\mathcal{S} = [\ell - M, \dots, \ell, \dots, \ell + M]$ denotes the subband that consists of $2M+1$ subcarriers, $k \in \mathcal{S}$, and Δn specifies the amount of feedback delay in OFDM blocks.

As we have seen before, the achievable rate depends on the correlation between the actual channel and its estimate. To quantify this correlation, assume that the multipath channel has P paths, where $\tau_p(t)$ and $\gamma_p(t)$ are the delay and attenuation factors of the pth path at time t, respectively [35]. Given this, the frequency response of the kth subcarrier at the nth OFDM block can be written as

$$H(n,k) = \sum_{p=1}^{P} \gamma_p(nT) \exp\left(\frac{-j2\pi k \tau_p}{KT_s}\right), \qquad (15)$$

where T_s denotes the sampling period, T is the duration of an OFDM block, and K is the number of subcarriers. We also have assumed that path gains remain constant over an OFDM block and the path delays do not change with time. Then, the correlation between the actual channel of the kth

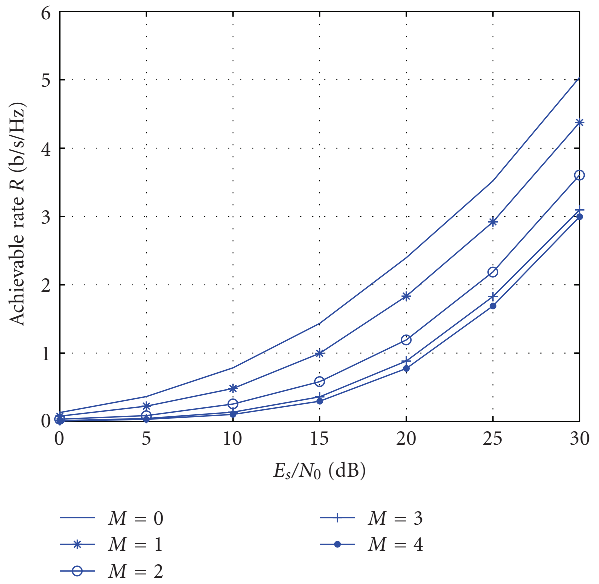

FIGURE 5: Achievable rate with one transmit antennas and one outdated channel estimate.

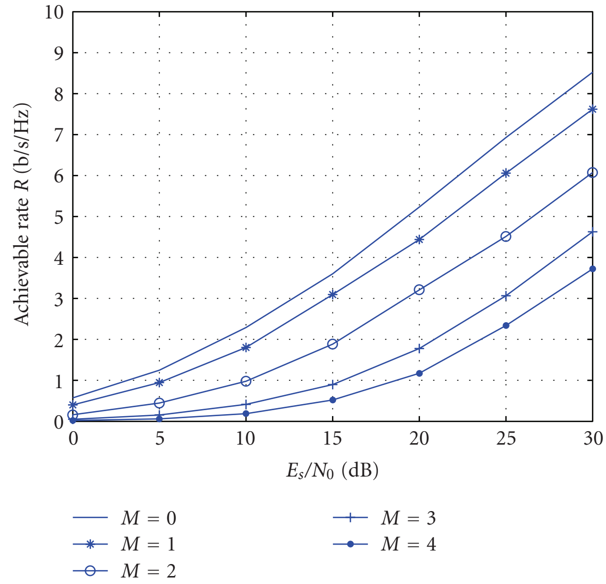

FIGURE 6: Achievable rate with multiple transmit antennas only and one outdated channel estimate.

subcarrier at the nth OFDM block and its estimate can be computed as

$$\begin{aligned}
\rho &= E\{\widehat{H}(n,k)H^*(n,k)\} \\
&= \frac{1}{2M+1}\sum_{\Delta k=-M}^{M}\sum_{p=1}^{P}\gamma_p((n+\Delta n)T) \\
&\quad\times \exp\left(\frac{-j2\pi(\ell+\Delta k)\tau_p}{KT_s}\right)\sum_{p'=1}^{P}\gamma_{p'}^*(nT)\exp\left(\frac{2\pi k\tau_{p'}^*}{KT_s}\right) \\
&= \frac{1}{2M+1}\sum_{\Delta k=-M}^{M}\sum_{p=1}^{P}E\{\gamma_p((n+\Delta n)T)\gamma_{p'}^*(nT)\} \\
&\quad\times \exp\left(\frac{-j2\pi(\ell+\Delta k-k)\tau_p}{KT_s}\right) \\
&= \frac{1}{2M+1}\sum_{\Delta k=-M}^{M}\sum_{p=1}^{P}\sigma_{\gamma_p}^2 J_0(2\pi f_m\Delta nT) \\
&\quad\times \exp\left(\frac{-j2\pi(\ell+\Delta k-k)\tau_p}{KT_s}\right) \\
&= \frac{1}{2M+1}J_0(2\pi f_m\Delta nT)\sum_{p=1}^{P}\sigma_{\gamma_p}^2\sum_{\Delta k=-M}^{M} \\
&\quad\times \exp\left(\frac{-j2\pi(\ell+\Delta k-k)\tau_p}{KT_s}\right).
\end{aligned} \tag{16}$$

Note that when $\Delta k = 0$, by $\sum_{p=1}^{P}\sigma_{\gamma_p}^2 = 1$, the correlation reduces to $\rho = J_0(2\pi f_m D)$ and the feedback delay is $D = \Delta nT$ resulting in $\widehat{H}(n,k) = H(n-D,k)$, as in the previous sections. Equation (16) implies that increasing the subband size decreases the correlation that results in a less reliable channel estimate. This, in turn, is expected to reduce the achievable rate.

The effect of the subband approach on the achievable rate is also studied with simulations. In the simulations, a multipath channel with an exponential power delay profile and an RMS delay spread of 5 microseconds is used resulting in a coherence bandwidth of about 44 kHz. It is also assumed that $T_s = 1$ microsecond and $K = 128$; with these numbers, the subcarrier spacing is about 8 kHz. Figures 5 and 6 illustrate the achievable rate for various M values when a single channel estimate is used with $f_m D = 0.1$. The number of transmit antennas is 1 for Figure 5 and 3 for Figure 6. From the figures, we can see that when $M = 0$, the results are the same as in Figure 1. Increasing M, however, results in a reduction of the achievable rate due to the less reliable channel estimate. For example, for $M = 0, 1, 2, 3, 4$, the correlation values are $\rho = 0.9037, 0.8623, 0.7870, 0.6911, 0.5895$, respectively. We see that although using multiple antennas results in higher achievable rates, the rate of reduction in R with increasing M is faster than when a single antenna is used. This is due to the fact that in this case, the channel estimates for all antennas start to become less reliable.

6. CONCLUSIONS

In this paper, a robust bit-loading algorithm for an adaptive OFDM system with transmit and/or receive diversity that operates in time-varying channels is proposed. The time variation causes the channel estimates to be outdated resulting in a mismatch between the actual and estimated channels and decreasing the performance significantly. The proposed method exploits the correlation between the actual channel and its outdated estimate(s) to increase the robustness of the link adaptation. It is shown that creating diversity with multiple transmit and receive antennas and using more past estimates of the channel is helpful in decreasing the performance degradation due to channel mismatch, even though this mismatch has a detrimental effect on the effectiveness of the transmit diversity. In addition, to reduce the amount of feedback, a method that uses a single channel estimate for a number of subcarriers in a subband is introduced and the tradeoff between the subband size and achievable rate is analyzed.

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