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Research Article

On the Throughput Capacity of Large Wireless Ad Hoc Networks Confined to a Region of Fixed Area

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We study the throughput capacity of large ad hoc networks confined to a square region of fixed area, thus exploring the dependence of the achievable throughput on the spatial node density. We find that there exists the value of the node density (the "critical" density) depending on the ratio of the total noise power to the transmit power such that the throughput increases as $n^{(\alpha-1)/2}$ at first, reaches a maximum, and then decreases as $n^{-1/2}$.

1. INTRODUCTION

Wireless networks consist of a number of nodes which communicate with each other using high-frequency radio waves. Some of these networks have a wired backbone or infrastructure with only the last hop being wireless. Cellular phones and wireless networks using 802.11 (WI-FI) are examples of this. Ad hoc networks are another type of wireless networks. They are formed by a collection of nodes without the aid of any fixed infrastructure. Since there are no base stations to route data through, the data needs to be routed to the destination by using the nodes in a multihop fashion.

The problem of throughput capacity of ad hoc wireless networks has received a lot of attention starting with the article [1] in which it was shown that the throughput of random networks with uniform spatial node distribution and random source-destination pairs location scales asymptotically as $\Theta(1/\sqrt{n\log n})$. This original result was later extended in many directions. Thus, in [2] it was shown that the node mobility can be used to remove this adverse scaling behavior at the expense of the end-to-end delay. The tradeoff between capacity and delay was studied in some detail in [3–7]. The original bounds on the throughput were also tightened using percolation theory in [8]. The problem of the throughput was also studied under somewhat different assumptions (one source-destination pair) in [9] and the throughput was found to scale as $\Theta(\log n)$ even if arbitrary complex network coding is allowed. Finally, very encouraging results were obtained in [10, 11] where it was shown that for a network employing an "idealized" ultra-wideband hardware (see, e.g., [12, 13] for a description of some aspects of the UWB technology) (with infinite bandwidth), the throughput in fact grows with the total node number as $\Theta(n^{(\alpha-1)/2})$, where α is the path loss exponent.

The main goal of this paper is to demonstrate that the decreasing behavior of the throughput first found in [1] and the increasing behavior in the UWB networks analyzed in [10, 11] are essentially two different "branches" of the throughput that can coexist in the same network. To this end, we analyze the uniform per node throughput of large ad hoc networks with uniform spatial node distribution that are confined to a square region of fixed area A. The available bandwidth is assumed to have a fixed (but arbitrary) value W. We find that the behavior of the throughput (up to numerical constants that are left undetermined by the analysis) can have two different regimes. The "switching" between the two regimes happens at around the "critical" value of the spatial node density that is determined by the ratio of the total noise power to the transmit power. For node densities below critical, the throughput is found to increase as $\Theta(n^{(\alpha-1)/2})$ (where α is the path loss exponent), and for node densities above critical, the throughput decreases roughly as $\Theta(1/\sqrt{n})$. The first regime corresponds to the behavior reported in [10, 11] for ultra-wideband systems. The second regime is the behavior found in [1]. The physical reasons for the two regimes can be qualitatively described as follows.

(i) For spatial node densities below critical, noise dominates interference, and two effects are simultaneously at work. The first effect is the increase of the throughput with node density due to the increase of received power to noise ratio. The second effect is the decrease of the throughput due to the increase of the number of relays between sources and destinations. The overall result is the increase mentioned above.

(ii) For spatial node densities above critical, the interference begins dominating the noise, and the first effect goes away since now as the node density increases (and, hence, the typical internode distance decreases), the interference grows at the same rate as the received power. Thus, the second effect (originally reported in [1]) becomes the only one leading to the decrease of throughput for larger networks.

We consider n wireless nodes uniformly distributed over a square area of area A. So the density of the nodes has the value $\rho = n/A$. More precisely, in the following we assume that the node density is fixed, that is, the area A is filled with nodes according to a two-dimensional Poisson process with density ρ. This means that $n = A\rho$ is the expected total number of nodes, with the actual node number possibly being different. Since for large n the difference is relatively negligible and does not affect any of the results, we ignore it in the following. To avoid unnecessary complications with the boundary, we assume that periodic boundary conditions are imposed, that is, the square is really a torus. We assume that each node i has a randomly chosen destination $d(i)$ whose identity does not change. Each node i has an unlimited amount of data to send to $d(i)$. Each node is constrained to a maximum transmit power of P. The available bandwidth is equal to W. The time is assumed to be divided into slots of unit length.

Let $M_j(t)$ be the number of bits received by the node j in time slot t. A uniform throughput of $\mathcal{T}$ is said to be feasible if

$$\liminf_{T\to\infty} \frac{1}{T}\sum_{t=1}^{T} M_{d(i)}(t) \geq \mathcal{T} \tag{1}$$

for all $i \in \mathcal{N}$.

If, in a given time slot t, the node i is transmitting to another node j (which does not have to be $d(i)$ if relaying of data is used), then the rate of transmission (and the number of bits transmitted from i to j during the slot) is given by

$$R_{t,ij} = W \log\left(1 + \mathrm{SINR}_{t,ij}\right), \tag{2}$$

where

$$\mathrm{SINR}_{t,ij} = \frac{P_i/\left|\mathbf{x}_i - \mathbf{x}_j\right|^{\alpha}}{N_0 W + I_{t,ij}} = \frac{P_i/\left|\mathbf{x}_i - \mathbf{x}_j\right|^{\alpha}}{N_0 W + \sum_{k\neq i} P_k/\left|\mathbf{x}_k - \mathbf{x}_j\right|^{\alpha}}, \tag{3}$$

with α being a constant usually between 2 and 4 that describes signal attenuation with distance, N_0 being the noise spectral density.

We will use the notation

$$F \equiv \frac{P}{N_0 W}, \tag{4}$$

where convenient.

We will make use of the useful auxiliary quantity, called information transport capacity. Let us enumerate all information bits originated during the period of time T, by denoting them v_i, $i = 1, 2, \ldots, N$. Let s_i be the distance travelled by the bit v_i from source to destination. We say that the information transport capacity of C_T is feasible if

$$\liminf_{T\to\infty} \frac{1}{T}\sum_{i=1}^{N} s_i \geq C_T. \tag{5}$$

Let γ be a purely numerical constant. We introduce the "critical" node density

$$\rho_{\mathrm{cr}} = \gamma \left(\frac{N_0 W}{P}\right)^{2/\alpha} \tag{6}$$

and let $n_{\mathrm{cr}} = \rho_{\mathrm{cr}} A$. Using this notation, we can state the main result of the paper as follows.

Main result

There exist purely numerical constants b_1, b_2, b_1', and b_2' independent of all network parameters such that the uniform per node throughput of an ad hoc network confined to an square region of (dimensionless) area A satisfies the inequalities (these bounds can actually be tightened by getting rid of powers of $\log n$ in the lower bounds; see Section 5 for more details)

$$\begin{gathered} b_1 \frac{P}{N_0 A^{\alpha/2}} \frac{n^{(\alpha-1)/2}}{\log^{(\alpha+1)/2}} \leq \mathcal{T} \leq b_2 \frac{P}{N_0 A^{\alpha/2}} n^{(\alpha-1)/2} \quad \text{if } n < n_{\mathrm{cr}}, \\ b_1' \frac{W}{\sqrt{n \log^{\alpha+1}}} \leq \mathcal{T} \leq b_2' \frac{W}{\sqrt{n}} \quad \text{if } n > n_{\mathrm{cr}}. \end{gathered} \tag{7}$$

The dependence of the uniform throughput on the total node number n is schematically depicted in Figure 1. We see that there exists an "optimal" node number $n_{\mathrm{cr}} = \rho_{\mathrm{cr}} A$ such that the throughput reaches its highest value for n close to n_{cr}. The physical reason for such behavior of the throughput can be described as follows. For low enough node density, the typical received signal power (and interference) is dominated by the noise power, and, as a result, interference can be neglected. Thus, in this regime, the throughput increases with the node density (and hence with the total node number as the area of the region is fixed). When the node density becomes high enough the increase of the throughput with density stops. The reason is that the interference begins dominating the noise power and it increases roughly proportionally to the signal power resulting in constant SINR and hence, independence of the throughput of the node density. Therefore, the effect found in [1] ($1/\sqrt{n}$ dependence due to the increase of the number of relays between sources and destinations) takes over and we obtain the corresponding decrease of the throughput.

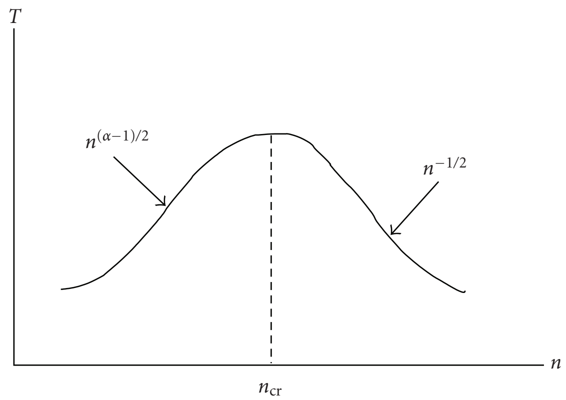

FIGURE 1: Schematic dependence of the throughput on the number of nodes in case of constant network area and starting node density below critical.

Note that since the node density is bounded from above because of a finite physical size of nodes, the critical node density ρ_{cr} may not be reached at all if the total noise power $N_0 W$ is much larger than the transmit power P. In such a case one may never see the downward part of the throughput curve, and the highest uniform throughput will be reached for the largest possible total node number. This can be the case for some ultra-wideband systems.

The rest of the paper is organized as follows. In Section 2, we find upper bounds on the information transport capacity. In Section 3, we use these bounds to obtain upper bounds on throughput. In Section 4, we study achievability of these bounds. In Section 5, we explain how the bounds can be tightened using the percolation theory approach, and, finally, Section 6 presents conclusions.

2. UPPER BOUNDS ON TRANSPORT CAPACITY

In this section, we find several upper bounds on the information transport capacity which will be used to obtain upper bounds on the uniform throughput.

2.1. Upper bound induced by interference

The following theorem was proved in [14], and we state it without proof.

Theorem 1. *The total transport capacity of the network is upper bounded as*

$$C_T \le c_1(\alpha) W \sqrt{An}. \tag{8}$$

2.2. Density independent upper bound induced by noise

Now, let us find a different (node density dependent) upper bound on transport capacity. We begin with a simple auxiliary result.

Lemma 1. *Maximum of the function*

$$f(r) = r \log\left(1 + \frac{F}{r^\alpha}\right) \tag{9}$$

over nonnegative values of r is achieved at

$$r^* = \left(\frac{F}{\exp\left(W\left(-\alpha/e^\alpha\right) + \alpha\right)}\right)^{1/\alpha} \tag{10}$$

and is equal to

$$f(r^*) = \left(\frac{FB(\alpha)}{\alpha - B(\alpha)}\right)^{1/\alpha} \log\left(\frac{\alpha}{B(\alpha)}\right), \tag{11}$$

where $B(\alpha) \equiv -W(-\alpha e^{-\alpha})$.

Proof. It is easy to see that $f(r) \ge 0$ for $r \ge 0$. In addition, $\lim_{r\to 0} f(r) = \lim_{r\to\infty} f(r) = 0$ and $f(r)$ has a single local maximum which is also global. This maximum can be found solving the equation $f'(r) = 0$ which leads to the statement of the lemma. □

We can now state the upper bound itself.

Theorem 2. *The total transport capacity of the network is upper bounded as*

$$C_T \le c_2\left(\frac{P}{N_0 W}\right)^{1/\alpha} \sqrt{\rho} W \sqrt{An}. \tag{12}$$

Proof. Consider a given time slot. A contribution $C_{T,ij}$ to the total transport capacity of a transmission from node i to node j can be upper bounded as follows:

$$\begin{aligned} C_{T,ij} &\le r_{ij} W \log\left(1 + \frac{P/r_{ij}^\alpha}{N_0 W + I_j}\right) \\ &\le r_{ij} W \log\left(1 + \frac{P/r_{ij}^\alpha}{N_0 W}\right) \\ &\le \max_r rW \log\left(1 + \frac{P/r^\alpha}{N_0 W}\right). \end{aligned} \tag{13}$$

Applying Lemma 1 to the last line in the above equation, we obtain

$$\begin{aligned} C_{T,ij} &\le W\left(\frac{(P/N_0 W)B(\alpha)}{\alpha - B(\alpha)}\right)^{1/\alpha} \log\left(\frac{\alpha}{B(\alpha)}\right) \\ &= W c_2(\alpha)\left(\frac{P}{N_0 W}\right)^{1/\alpha}, \end{aligned} \tag{14}$$

where $c_2(\alpha) \equiv (B(\alpha)/(\alpha - B(\alpha)))^{1/\alpha} \log(\alpha/B(\alpha))$.

Since no more than n simultaneous successful transmissions can take place in a time slot, multiplying (14) by n, we obtain

$$\begin{aligned} C_T &\le W c_2(\alpha)\left(\frac{P}{N_0 W}\right)^{1/\alpha} n \\ &= W c_2(\alpha)\left(\frac{P}{N_0 W}\right)^{1/\alpha} \sqrt{\rho}\sqrt{An}, \end{aligned} \tag{15}$$

where we have used the identity $n^2 = \rho A n$. □

The upper bound on transport capacity obtained in Theorem 2 does not take into account the actual internode distance since it optimizes over it. It is easy to see that if the node density is such that the typical internode distance is significantly larger than the optimal distance (10), a tighter bound can be obtained. We investigate this topic next.

2.3. Node density induced upper bound on transport capacity

To derive this upper bound, we need a few preliminary results that we formulate as lemmas. We assume that $\alpha < 3$, for convenience. If $\alpha \geq 3$ a slightly different proof technique can be used to arrive at the same results. For node i, let the node $\hat{i}$ be its nearest neighbor. The next lemma gives the probability density function $p(r)$ for the nearest neighbor distance $r_{i\hat{i}}$.

Lemma 2. *The pdf for the nearest neighbor distance $r_{i\hat{i}}$ is given by*

$$p(r) = 2\pi\rho r e^{-\pi\rho r^2}. \tag{16}$$

Proof. Select an arbitrary node i. Draw a circle $\mathcal{C}$ of radius r around i. We can write

$$\begin{aligned}\Pr\{r_{i\hat{i}} > r\} &= \Pr\{\text{there is no node in } \mathcal{C}\}\\ &= e^{-\pi r^2 \rho}.\end{aligned} \tag{17}$$

Therefore, the cdf of the nearest neighbor distance is

$$P(r) = \Pr\{r_{i\hat{i}} < r\} = 1 - e^{-\pi r^2 \rho}. \tag{18}$$

Taking a derivative of it, we arrive at the statement of the lemma. □

Now, let b be a positive number, and let $I(b)$ be the following integral:

$$I(b) \equiv \int_0^\infty y^2 e^{-by^2} \log\left(1 + \frac{1}{y^\alpha}\right) dy. \tag{19}$$

We can establish the following upper bound on $I(b)$.

Lemma 3. *The following inequality holds:*

$$I(b) \leq c_1(\alpha) b^{(\alpha-3)/2}. \tag{20}$$

Proof. Using the inequality $\log(1 + x) \leq x$ valid for all nonnegative values of x, we obtain

$$I(b) \leq \int_0^\infty y^{2-\alpha} e^{-by^2} dy = c_1(\alpha) b^{(\alpha-3)/2}, \tag{21}$$

where $c_1(\alpha)$ is independent of b. □

We can now use Lemma 3 to put an upper bound on the expected value of the information transport "quantum" $C_{T,i\hat{i}}$ in case every node transmits to its nearest neighbor.

Lemma 4. *The following inequality holds:*

$$E(C_{T,i\hat{i}}) \leq W c_2(\alpha) F \rho^{(\alpha-1)/2}. \tag{22}$$

Proof. Using Lemma 2, we obtain that

$$\begin{aligned}E(C_{T,i\hat{i}}) &\leq W \int_0^\infty r \log\left(1 + \frac{F}{r^\alpha}\right) \cdot 2\pi r \rho e^{-\pi\rho r^2} dr\\ &= 2\pi\rho W \int_0^\infty r^2 e^{-\pi\rho r^2} \log\left(1 + \frac{F}{r^\alpha}\right) dr.\end{aligned} \tag{23}$$

Introducing a new variable $y \equiv r/F^{1/\alpha}$ in the above expression, we obtain

$$E(C_{T,i\hat{i}}) \leq 2\pi\rho F^{3/\alpha} W \int_0^\infty y^2 e^{-\pi\rho F^{2/\alpha} y^2} \log\left(1 + \frac{1}{y^\alpha}\right) dy. \tag{24}$$

Denoting $b \equiv \pi\rho C^{2/\alpha}$, we can apply Lemma 3 and obtain that

$$\begin{aligned}E(C_{T,i\hat{i}}) &\leq 2\pi\rho F^{3/\alpha} W \cdot c_1(\alpha) \left(\rho F^{2/\alpha}\right)^{(\alpha-3)/2}\\ &= W c_2(\alpha) F \rho^{(\alpha-1)/2}.\end{aligned} \tag{25}$$

□

Now, let M be the $n \times n$ covariance matrix of the quantities $C_{T,i\hat{i}}$ for $i = 1, 2, \ldots, n$. Due to uniformity of the nodes distribution, all diagonal elements of M are equal and all off-diagonal elements of M are also equal. The following lemma shows that the latter (off-diagonal) are much smaller than the former (diagonal), or, in other words, the quantities $C_{T,i\hat{i}}$ are almost independent.

Lemma 5. *The following inequality holds:*

$$\operatorname{Cov}(C_{T,i\hat{i}}, C_{T,j\hat{j}}) \leq \frac{3}{n-1} \operatorname{Var}(C_{T,i\hat{i}}). \tag{26}$$

Proof. Let us introduce the following notation. $A_{i,j}$ is the event that the node closest to node i is node j. $A_{ij,s}$ is the event that the nodes i and j share the closest node, that is, the same node is both the closest to i and closest to j. Also denote by A_{ind} the event that $C_{T,i\hat{i}}$ is independent of $C_{T,j\hat{j}}$.

Note that the quantities $C_{T,i\hat{i}}$ and $C_{T,j\hat{j}}$ may be mutually dependent only if either j is the node closest to i, i is the node closest to j, or the nodes i and j share the closest node. In other words, using the notation introduced above, we have

$$\overline{A}_{\text{ind}} = A_{i,j} \cup A_{j,i} \cup A_{ij,s}. \tag{27}$$

Therefore,

$$\Pr(\overline{A}_{\text{ind}}) \leq \Pr(A_{i,j}) + \Pr(A_{j,i}) + \Pr(A_{ij,s}). \tag{28}$$

We also have, due to uniformity of nodes distribution,

$$\Pr(A_{i,j}) = \Pr(A_{j,i}) = \frac{1}{n-1}. \tag{29}$$

The event $A_{ij,s}$ can be written as $A_{ij,s} = \bigcup_{k \neq i,j} (A_{i,k} \cap A_{j,k})$. Therefore,

$$\Pr(A_{ij,s}) \leq \sum_{k \neq i,j} \Pr(A_{i,k} \cap A_{j,k}) = (n-2)\left(\frac{1}{n-1}\right)^2, \tag{30}$$

where in the last step we have used the independence of the events $A_{i,k}$ and $A_{j,k}$. Substituting (29) and (30) into (28) we obtain

$$\Pr(\overline{A}_{\text{ind}}) \leq \frac{2}{n-1} + \frac{n-2}{(n-1)^2} < \frac{3}{n-1}. \tag{31}$$

Using the total probability formula, we can calculate the covariance $\mathrm{Cov}(r_{i\hat{i}}, C_{T,j\hat{j}})$ as

$$\begin{aligned}\mathrm{Cov}(r_{i\hat{i}}, C_{T,j\hat{j}}) &= \mathrm{Cov}(C_{T,i\hat{i}}, C_{T,j\hat{j}} \mid A_{\mathrm{ind}})\Pr(A_{\mathrm{ind}}) \\ &\quad + \mathrm{Cov}(C_{T,i\hat{i}}, C_{T,j\hat{j}} \mid \overline{A}_{\mathrm{ind}})\Pr(\overline{A}_{\mathrm{ind}}) \\ &\leq 0\cdot\Pr(A_{\mathrm{ind}}) + \mathrm{Var}(C_{T,i\hat{i}})\Pr(\overline{A}_{\mathrm{ind}}) \\ &\leq \mathrm{Var}(C_{T,i\hat{i}})\cdot\frac{3}{n-1},\end{aligned} \tag{32}$$

where we have used (31) in the last step. The proof is complete. □

Let $\overline{C}'_T$ be sample mean of the "quantum" of the transport capacity $C_{T,i\hat{i}}$:

$$\overline{C}'_T = \frac{1}{|\mathcal{N}|}\sum_{i=1}^{|\mathcal{N}|} C_{T,i\hat{i}}, \tag{33}$$

where $\mathcal{N}$ is the set of nodes transmitting in the chosen time slot.

The following lemma shows that, for large n, the value of $\overline{C}'_T$ can be upper bounded in much the same way as the expected value $E(C_{T,i\hat{i}})$.

Lemma 6. *The relation*

$$\overline{C}'_T \leq W c_3(\alpha) F \rho^{(\alpha-1)/2} \tag{34}$$

holds with high probability.

Proof. We have the following bound on the variance:

$$\mathrm{Var}\left(\sum_{i=1}^{n} C_{T,i\hat{i}}\right) = n\mathrm{Var}(C_{T,i\hat{i}}) + n(n-1)\mathrm{Cov}(C_{T,i\hat{i}}, C_{T,j\hat{j}}) \tag{35}$$

for arbitrary i and j. Using Lemma 5, we obtain

$$\mathrm{Var}\left(\sum_{i=1}^{n} C_{T,i\hat{i}}\right) \leq 4n\mathrm{Var}(C_{T,i\hat{i}}), \tag{36}$$

and, therefore,

$$\mathrm{Var}\left(\overline{C}'_T\right) \leq \frac{4n}{n^2}\mathrm{Var}(C_{T,i\hat{i}}). \tag{37}$$

Taking the square root we obtain

$$\mathrm{Std}\left(\overline{C}'_T\right) \leq \frac{2}{\sqrt{n}}\mathrm{Std}(C_{T,i\hat{i}}). \tag{38}$$

An application of Chebyshev's inequality gives

$$\Pr\left(\overline{C}'_T \geq E\left(\overline{C}'_T\right) + t\,\mathrm{Std}\left(\overline{C}'_T\right)\right) \leq \frac{1}{t^2}. \tag{39}$$

Setting $t = n^{1/4}$ and recalling that $E(\overline{C}'_T) = E(C'_T)$ we obtain that

$$\Pr\left(\overline{C}'_T \geq E(C_{T,i\hat{i}}) + O(n^{-1/4})\mathrm{Std}(C_{T,i\hat{i}})\right) \leq \frac{1}{\sqrt{n}}, \tag{40}$$

which, taken together with the result of Lemma 4, implies that

$$\Pr\left(\overline{C}'_T \geq W c_{21}(\alpha) F \rho^{(\alpha-1)/2} + O(n^{-1/4})\mathrm{Std}(C_{T,i\hat{i}})\right) \leq \frac{1}{\sqrt{n}}. \tag{41}$$

Since $\mathrm{Std}(C_{T,i\hat{i}})$ is independent (for a given ρ) of n, this completes the proof of the lemma. □

Now, let $n_<$ be the number of nearest neighbor distances $r_{i\hat{i}}$ that do not exceed r^*:

$$n_< = \left|\{r_{i\hat{i}} \mid r_{i\hat{i}} \leq r^*\}\right|. \tag{42}$$

Lemma 7. *The inequality*

$$n_< \leq \hat{c}(\alpha) n \rho F^{2/\alpha} \tag{43}$$

holds with high probability.

Proof. The probability that the nearest neighbor distance $r_{i\hat{i}}$ is less than r^* can be found as

$$\begin{aligned}\Pr(r_{i\hat{i}} < r^*) = P(r^*) &= 1 - \exp\left(-\pi\rho r^{*2}\right) \\ &= 1 - \exp\left(-\hat{c}_1(\alpha)\rho F^{2/\alpha}\right) \leq \hat{c}_1(\alpha)\rho F^{2/\alpha}.\end{aligned} \tag{44}$$

Therefore, the expected value of $n_<$ can be found as

$$E(n_<) = n\Pr(r_{i\hat{i}} < r^*) \leq n\hat{c}_1(\alpha)\rho F^{2/\alpha}. \tag{45}$$

Now, v_i let be an indicator variable such that

$$v_i = \begin{cases} 1 & \text{if } r_{i\hat{i}} < r^*, \\ 0 & \text{otherwise.} \end{cases} \tag{46}$$

Then $n_< = \sum_{i=1}^{n} v_i$, and, therefore, for the variance of $n_<$ (making use of Lemma 5) we have

$$\mathrm{Var}(n_<) \leq 4n\mathrm{Var}(v_i) \leq 4n = \hat{c}_2 n, \tag{47}$$

where $\hat{c}_2 = 4$ is a constant independent of n.

We can now use Chebyshev's inequality to obtain

$$\Pr(n_< \geq E(n_<) + t\,\mathrm{Std}(n_<)) \leq \frac{1}{t^2}, \tag{48}$$

which implies

$$\Pr\left(n_< \geq \hat{c}_1(\alpha) n \rho F^{2/\alpha} + t\hat{c}_2\sqrt{n}\right) \leq \frac{1}{t^2}. \tag{49}$$

Choosing $t = n^{1/4}$, we finally obtain

$$\Pr\left(n_< \geq \hat{c}(\alpha) n \rho F^{2/\alpha}\right) \leq \frac{1}{\sqrt{n}}, \tag{50}$$

which proves the lemma. □

We are now prepared to derive an upper bound that is tighter than the previous one for small node densities.

Theorem 3. *The total transport capacity of the network is upper bounded as*

$$C_T \le c_3(\alpha) W\left(\frac{P}{N_0 W}\right)\rho^{\alpha/2}\sqrt{An} \tag{51}$$

with high probability.

Proof. Since the "quantum" $C_{T,ij}$ of the transport capacity is maximized for $r_{ij} = r^*$ where r^* is given in (10), the following bound on $C_{T,ij}$ holds for all i:

$$C_{T,ij} \le \begin{cases} Wr^* \log\left(1+\dfrac{F}{r^{*\alpha}}\right) & \text{if } r_{i\hat{i}} \le r^*, \\ Wr_{i\hat{i}} \log\left(1+\dfrac{F}{r_{i\hat{i}}^{\alpha}}\right) & \text{if } r_{i\hat{i}} > r^*. \end{cases} \tag{52}$$

We can upper bound the total transport capacity as follows:

$$C_T \le n_< \cdot Wr^* \log\left(1+\frac{F}{r^{*\alpha}}\right) + n\cdot \overline{C}'_T. \tag{53}$$

Using Lemmas 1, 6, and 7, we see that it follows from (53) that, with high probability,

$$C_T \le W\hat{c}(\alpha)n\rho F^{2/\alpha}\cdot\tilde{c}(\alpha)F^{1/\alpha} + Wnc_3(\alpha)F\rho^{(\alpha-1)/2}. \tag{54}$$

Finally, we obtain

$$C_T \le Wc_3(\alpha)F\rho^{(\alpha-1)/2}\left(1+\overline{c}_3(\alpha)\left(F^{1/\alpha}\sqrt{\rho}\right)^{3-\alpha}\right)n. \tag{55}$$

Using the identity $n = \sqrt{\rho A n}$ in the above equation and recalling the definition of F, we arrive at the statement of the theorem. □

3. UPPER BOUND ON THROUGHPUT

In this section, we use the upper bounds on information transport capacity found in the previous section, to find upper bounds on throughput.

Let $g(n,\rho)$ be an arbitrary function of n and ρ, and let b_1 and b_2 be constants (quantities independent of n and ρ). We have the following lemma relating upper bounds on transport capacity and throughput.

Lemma 8. *Suppose that the total transport capacity is upper bounded as*

$$C_T \le b_1 g(n,\rho) \tag{56}$$

with high probability. Then the throughput can be upper bounded as

$$\mathcal{T} \le \frac{b_2 g(\rho,n)}{\sqrt{An}} \tag{57}$$

with high probability.

Proof. Suppose that for any constant b_2, the throughput exceeds the quantity $b_2 g(\rho,n)/\sqrt{An}$ with high probability. We will show that this implies that for any constant c, the transport capacity exceeds $cg(n,\rho)$ also with high probability. The lemma then would be proved by contradiction.

Let us denote by the distance between the node i and its destination by d_i. Let $\overline{d}$ be the sample mean $(1/n)\sum_{i=1}^{n} d_i$. Since the quantities d_i are mutually independent, we have for the standard deviation of $\overline{d}$:

$$\text{Std}(\overline{d}) = \frac{1}{\sqrt{n}}\text{Std}(d_i) = \frac{1}{\sqrt{n}}h_2\sqrt{A}, \tag{58}$$

where h_2 does not depend on n. On the other hand, clearly,

$$E(\overline{d}) = E(d_i) = h_1\sqrt{A}, \tag{59}$$

where the number h_1 depends only on the shape of the region containing the network but not on the number of nodes in it. An application of Chebyshev's inequality then yields

$$\Pr\left(\overline{d} \le h_1\sqrt{A} - t\frac{1}{\sqrt{n}}h_2\sqrt{A}\right) \le \frac{1}{t^2}. \tag{60}$$

Setting $t = n^{1/4}$, we obtain that for large enough n,

$$\Pr\left(\overline{d} \le h_3\sqrt{A}\right) \le \frac{1}{\sqrt{n}}, \tag{61}$$

where h_3 is independent of n. This implies that

$$\sum_{i=1}^{n} d_i \ge nh_3\sqrt{A}, \tag{62}$$

with high probability.

Now assume that for any constant b_2, the throughput satisfies

$$\mathcal{T} > \frac{b_2 g(\rho,n)}{\sqrt{An}} \tag{63}$$

with high probability. Then, for the total transport capacity, we have using (62) that

$$C_T \ge \mathcal{T}\sum_{i=1}^{n} d_i \ge b_2 h_3 g(\rho,n) \tag{64}$$

with high probability, and the lemma is proved. □

We can now combine the results of Theorems 1, 3, and Lemma 8 to obtain upper bounds on throughput.

Theorem 4. *Upper bounds on the uniform throughput $\mathcal{T}$ are given by*

$$\mathcal{T} \le \min\left\{\tilde{c}_1(\alpha)\frac{W}{\sqrt{n}},\ \tilde{c}_2(\alpha)\frac{W^{(\alpha-1)/\alpha}P^{1/\alpha}}{N_0^{1/\alpha}\sqrt{A}},\ \tilde{c}_3(\alpha)\frac{P}{N_0 A^{\alpha/2}}n^{(\alpha-1)/2}\right\}. \tag{65}$$

Proof. To prove the theorem we only need to combine the results of Theorems 1, 2, 3 with that of Lemma 8 and substitute $\rho = n/A$. □

Note that all three bounds are become the same (up to a numerical constant) for $n = n_{\text{cr}} = \rho_{\text{cr}}A$, whereas for $n < n_{\text{cr}}$, the third bound (the node density induced one) becomes the tightest one, and for $n > n_{\text{cr}}$, the first bound (interference induced) is the tightest bound.

4. LOWER BOUNDS ON THROUGHPUT

In this section, we address the achievability of these upper bounds found in the previous sections.

4.1. Tessellation

The tessellation of the square region that turns out to be convenient for our goals is the regular one: we divide it into identical smaller squares with side a each. Anticipating the transmission strategy to be employed below, we choose the parameter a in such a way that every cell can always directly communicate with 4 of its neighbors using the smallest common range of communication that in turn is chosen in a way to ensure connectivity with high probability. Using results from [15], for connectivity, we have to employ the range

$$r_c(\rho) = \sqrt{\frac{c' A \log n}{n}} = \sqrt{\frac{c' A \log A\rho}{A\rho}}, \tag{66}$$

where $c' > 1/\pi$. We chose $c' = 10$ for simplicity. Then, to ensure that each cell can directly communicate with 4 neighbors, one needs to set the cell size to be

$$a(\rho) = \frac{r_c(\rho)}{\sqrt{5}}. \tag{67}$$

So the total number of cells in the system is equal to

$$m_s = \frac{A}{a^2} = \frac{n}{2\log n}. \tag{68}$$

We will denote the cells in the system by $C_i, i = 1, 2, \ldots, m_s$.

4.2. Transmission schedule

We define a transmission policy $\pi(d)$. We organize transmission in the following way. The entire system is tesselated into square cells of area $a(\rho)^2$. The routing of packets between cells proceeds as follows. To route a packet between two cells, we employ at most two straight lines: one vertical and one horizontal. (It is possible that only one straight line is needed.) Each time a packet is transmitted from a cell to an adjacent cell (see Figure 2). If a node is transmitting to another node, and the receiving node is very close to another transmitting node (such a situation is shown in Figure 3), then the receiving node may experience very large interference. To avoid this situation, we are enforcing a square region around each transmitter where no other nodes may transmit. This square has sides of length $2d+1$ cells. Figure 2 shows the case of $d = 2$.

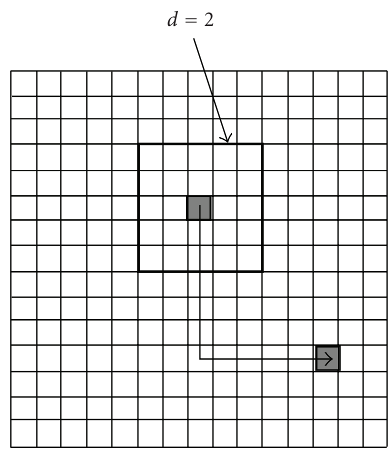

Figure 2: Routes between cells are along at most two straight lines.

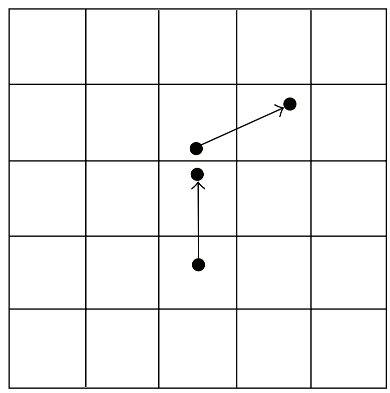

Figure 3: The node in the center cell may experience very large interference in this situation.

4.3. Bound on interference

Lemma 9. *Under the transmission strategy $\pi(d)$,*

$$I_j \le hP_{ij} \tag{69}$$

with P_{ij} as the power received by node j from node i, I_j as the interference at node j, and h is a constant. In other words, the total interference is bounded by a constant multiple of the received power.

Proof. It is easy to see that adding contributions from all possible interferers, the total interference at the location of node j can be upper bounded as

$$\begin{aligned} I_j &\le \frac{P}{(da)^\alpha} 8 + \frac{P}{(da)^\alpha} 16 + \cdots \\ &= \frac{P}{a^\alpha} \sum_{i=1}^{\infty} \frac{4(2i-1)}{(id)^\alpha} \\ &\le \frac{8P}{a^\alpha d^\alpha} \sum_{i=1}^{\infty} \frac{1}{i^{\alpha-1}}. \end{aligned} \tag{70}$$

On the other hand, in policy $\pi(d)$, the power received at node j from node i can be lower bounded as

$$P_{ij} \ge \frac{P}{(\sqrt{5}a)^\alpha}. \tag{71}$$

Substituting (71) into (70), we obtain

$$I_j \le \frac{8 \cdot 5^{\alpha/2} P_{ij}}{d^\alpha} \sum_{i=1}^{\infty} \frac{1}{i^{\alpha-1}}. \tag{72}$$

Finally, since for $\alpha > 2$, $\sum_{i=1}^{\infty} 1/i^{\alpha-1} < \infty$, we can combine all constants in (72) into one and write

$$I_j \leq h(\alpha) P_{ij}, \tag{73}$$

with $h(\alpha)$ being just a constant, which proves the lemma. □

4.4. Number of nodes in a cell

To make the transmission schedule presented below feasible, we need to ensure that every cell contains at least one node with high probability. Given the square geometry we have chosen, this is easy to do. Indeed, let us compute the probability that a given cell does not have any nodes in it. If a single node is placed in the system, the probability that a cell does not contain that node is the ratio of area outside the cell over the total area. For n nodes, this ratio is raised to the n power. Since the area of a cell is $a(\rho)^2$,

$$\begin{aligned} P(\text{no node in a cell}) &= \left(1 - \frac{a(\rho)^2}{A}\right)^n = \left(1 - \frac{2\log n}{n}\right)^n \\ &\leq e^{-2\log n} = (n)^{-2}. \end{aligned} \tag{74}$$

Multiplying (74) by the number of cells (68), we obtain, by the union bound, that the probability that there exists a cell that does not contain a single node is upper bounded by $1/(2n \log n)$, which means that every cell has at least one node with high probability.

4.5. Number of routes through a given cell

Let us consider a given cell C_i and count the number of routes passing through it. Let us denote this number by N_i.

Lemma 10. *The inequality*

$$\max_i \ N_i < \sqrt{32n \log n} \tag{75}$$

holds with high probability.

Proof. Obviously, the number of vertical components of the routes passing through C_i does not exceed the number of cells found in the vertical strip with the width of a (see Figure 2). It is clear that the expected number of "vertical" routes N_i^v in a cell satisfy the inequality

$$\begin{aligned} E(N_i^v) &\leq \rho a(\rho)\sqrt{A} = A\rho\sqrt{\frac{2\log n}{n}} \\ E(N_i^v) &\leq \sqrt{2n\log n}. \end{aligned} \tag{76}$$

Then, using the fact that the node locations are independent, we can apply the Chernoff bound to obtain

$$P(N_i \geq (1+\epsilon)E(N_i)) \leq e^{-\epsilon^2 E(N_i)/4}. \tag{77}$$

Now we can choose $\epsilon = 1$ and rewrite (77) as

$$P\left(N_i^v \geq \sqrt{8n \log n}\right) \leq e^{-\left(\sqrt{2n\log n}\right)/4}, \tag{78}$$

and so, using the union bound, we obtain

$$P\left(\max_i N_i^v \geq \sqrt{8n\log n}\right) \leq \frac{n}{2\log n} e^{-\sqrt{(n\log n)/8}}. \tag{79}$$

Exactly the same argument holds for the number N_i^h of horizontal components of routes passing through C_i. We obtain

$$P\left(\max_i N_i^h \geq \sqrt{8n\log n}\right) \leq \frac{n}{2\log n} e^{-\sqrt{(n\log n)/8}}. \tag{80}$$

Since the total number of routes passing through C_i is $N_i = N_i^v + N_i^h$, we can combine (79) and (80), and use the union bound to obtain

$$P\left(\max_i \ N_i \geq \sqrt{32n\log n}\right) \leq \frac{n}{\log n} e^{-\sqrt{(n\log n)/8}}, \tag{81}$$

which proves the lemma. □

4.6. Interference limited network achievable throughput

Now we are prepared to compute a lower bound on the achievable per node throughput for systems where the interference is the limiting factor.

Theorem 5. *For node densities $\rho \geq \rho_{\text{cr}}$, the throughput*

$$\mathcal{T} = \frac{\widetilde{b}_1 W}{\sqrt{n \log^{\alpha+1} n}} \tag{82}$$

is achievable with high probability.

Proof. We begin with finding a lower bound on the transmission rate in policy $\pi(d)$. The transmission rate from node i to node j has the value

$$R_{t,ij} = W \log\left(1 + \text{SINR}_{t,ij}\right), \tag{83}$$

with

$$\text{SINR}_{t,ij} = \frac{P_{ij}}{N_0 W + I_j}. \tag{84}$$

For transmission policy $\pi(d)$, the power received at node j can be lower bounded as

$$P_{ij} \geq \frac{P}{r_c(\rho)^\alpha} = \frac{P\rho^{\alpha/2}}{(10\log n)^{\alpha/2}}. \tag{85}$$

For $\rho \geq \rho_{\text{cr}}$, we have $P\rho^{\alpha/2} > \gamma^{2/\alpha} N_0 W$ and it follows from (85) that

$$P_{ij} \geq \frac{\gamma^{2/\alpha} N_0 W}{(10\log n)^{\alpha/2}}. \tag{86}$$

Substituting (86) and the result of Lemma 9 into (84), we obtain that, for large enough n, for any time slot t,

$$\mathrm{SINR}_{t,ij} \geq \frac{h_4}{(\log n)^{\alpha/2}}, \tag{87}$$

where h_4 is a constant.

Now, substituting (87) into (83), we obtain that, for large enough n,

$$R_{t,ij} \geq \frac{h_5 W}{(\log n)^{\alpha/2}}, \tag{88}$$

where h_5 is another constant.

On the other hand, in policy $\pi(d)$, each cell can transmit at least once in every $(d+1)^2$ time slots, and, according to Lemma 10, each cell C_i can serve each route passing through it at least once in every $\sqrt{32n \log n}$ time slot in which it transmits. This implies that the throughput of at least

$$\mathcal{T} \geq \frac{\min R_{t,ij}}{(d+1)^2\sqrt{32n \log n}} \tag{89}$$

can be achieved. Substituting (88) into (89) and combining all constants into one, we obtain the statement of the theorem. □

4.7. Power limited network achievable throughput

Theorem 6. *For node densities $\rho < \rho_{\mathrm{cr}}$, the throughput*

$$\begin{aligned}\mathcal{T} &= \frac{\widetilde{b}_3 W}{\sqrt{n \log^{\alpha+1} n}}\left(\frac{P}{N_0 W}\right)\rho^{\alpha/2}\\ &= \widetilde{b}_3 \frac{P}{N_0 A^{\alpha/2}} \cdot \frac{n^{(\alpha-1)/2}}{\log^{(\alpha+1)/2} n}\end{aligned} \tag{90}$$

is achievable with high probability.

Proof. Again, as in Theorem 5, we begin with finding a lower bound on the transmission rate in policy $\pi(d)$. The transmission rate from node i to node j and the signal to noise and interference ratio have the same form (83) and (84) as in Theorem 5. The lower bound (85) on the power received at node j is implied by the policy $\pi(d)$ and holds in this case as well. We can now combine (84), (85), and Lemma 9 to obtain the following lower bound on $\mathrm{SINR}_{t,ij}$:

$$\mathrm{SINR}_{t,ij} \geq \frac{P\rho^{\alpha/2}}{N_0 W (10 \log n)^{\alpha/2} + hP\rho^{\alpha/2}}. \tag{91}$$

Since $\rho < \rho_{\mathrm{cr}}$ which implies that $P\rho^{\alpha/2} < \gamma^{2/\alpha} N_0 W$, it follows from (91) that

$$\mathrm{SINR}_{t,ij} \geq \frac{P\rho^{\alpha/2}}{N_0 W ((10 \log n)^{\alpha/2} + \gamma^{2/\alpha})}. \tag{92}$$

Therefore, for large enough n, we can write

$$\mathrm{SINR}_{t,ij} \geq \frac{h_6 P\rho^{\alpha/2}}{N_0 W (\log n)^{\alpha/2}}, \tag{93}$$

where h_6 is a constant.

Substituting (93) into (83), we obtain that, for large enough n,

$$R_{t,ij} \geq \frac{h_7 W (P/N_0 W)\rho^{\alpha/2}}{(\log n)^{\alpha/2}}, \tag{94}$$

where h_7 is another constant.

In policy $\pi(d)$, each cell can transmit at least once in every $(d+1)^2$ time slots. Also, according to Lemma 10, each cell C_i can serve each route passing through it at least once in every $\sqrt{32n \log n}$ time slots in which it transmits. This implies, just like in Theorem 5, that the throughput of at least

$$\mathcal{T} \geq \frac{\min R_{t,ij}}{(d+1)^2\sqrt{32n \log n}} \tag{95}$$

can be achieved. Substituting (94) into (95) and combining all constants into one, we obtain the statement of the theorem. □

5. TIGHTENING THE BOUNDS

Although the main focus of this paper is to demonstrate the possible switching behavior of achievable throughput at the critical node density, it is possible to tighten the bounds presented in the paper slightly using the percolation theory methods employed in [8, 11]. Namely, in order to tighten the lower bound of Theorems 5 and 6, it is sufficient to observe that the use of percolation theory approach allows to construct a transmission policy $\pi'(d)$ with the following properties.

(i) The transmission range of $r_c'(\rho) = \sqrt{c''A/n} = \sqrt{c''/\rho}$ (for some constant c'') for node-to-node transmissions can be employed. (In policy $\pi'(d)$ described in [8, 11], there are phases which use longer hop lengths. It is shown, however, that these phases are not bottlenecks for the overall throughput.)

(ii) Each node serves as a relay for no more than $c\sqrt{n}$ source-destination pairs, where c is a constant independent of n.

(iii) The presence of a "silence zone" in policy $\pi'(d)$ (just like in $\pi(d)$) makes Lemma 9 still valid.

Then it is easy to see that the lower bounds of Theorems 5 and 6 could be tightened as follows.

Changes in Theorem 5

In Theorem 5, the expression (85) would read (using property 1)

$$P_{ij} \geq \frac{P\rho^{\alpha/2}}{c''^{\alpha/2}}, \tag{96}$$

and (for $\rho \geq \rho_{cr}$) the expression (86) would get replaced with

$$P_{ij} \geq \frac{\gamma^{\alpha/2} N_0 W}{c''^{\alpha/2}}. \tag{97}$$

This, in turn, would imply that, for any time slot t, $\mathrm{SINR}_{t,ij} \geq h_4$ and $R_{t,ij} \geq h_5 W$ for some constants h_4 and h_5. Using property 2 of the policy $\pi'(d)$, we obtain the lower bound of the achievable with high probability throughput

$$\mathcal{T} \geq \frac{\widetilde{b}_1 W}{\sqrt{n}} \tag{98}$$

for some constant $\widetilde{b}_1$.

Changes in Theorem 6

In Theorem 6, the expression (91) would get replaced with

$$\mathrm{SINR}_{t,ij} \geq \frac{P\rho^{\alpha/2}}{N_0 W c''^{\alpha/2} + hP\rho^{\alpha/2}}, \tag{99}$$

and, for $\rho < \rho_{cr}$, the bounds (93) and (94) would become

$$\begin{aligned} \mathrm{SINR}_{t,ij} &\geq \frac{h_6 P\rho^{\alpha/2}}{N_0 W}, \\ R_{t,ij} &\geq h_7 W\left(\frac{P}{N_0 W}\right)\rho^{\alpha/2}, \end{aligned} \tag{100}$$

respectively. Using property 2 of the policy $\pi'(d)$, we see that the throughput satisfying

$$\mathcal{T} \geq \frac{\widetilde{b}_3 W}{\sqrt{n}}\left(\frac{P}{N_0 W}\right)\rho^{\alpha/2} = \frac{\widetilde{b}_3 P n^{(\alpha-1)/2}}{N_0 A^{\alpha/2}} \tag{101}$$

for a constant $\widetilde{b}_3$ can be achieved with high probability.

We can summarize the above changes in the following Theorem.

Theorem 7. *If $\rho < \rho_{cr}$, the throughput of*

$$\mathcal{T} \geq \frac{\widetilde{b}_3 P n^{(\alpha-1)/2}}{N_0 A^{\alpha/2}} \tag{102}$$

is achievable with high probability

If $\rho \geq \rho_{cr}$, the throughput of

$$\mathcal{T} \geq \frac{\widetilde{b}_1 W}{\sqrt{n}} \tag{103}$$

is achievable with high probability.

6. CONCLUSION

This paper examined the uniform throughput of large ad hoc networks confined to a region of fixed area. It was found that, for a large enough total area, as the total number of nodes increases, the achievable throughput can exhibit an up-and-down behavior reaching a a maximum at a critical spatial node density that is proportional to a power of the ratio of the total noise power to the transmitted power $(N_0 W/P)^{2/\alpha}$.

While the spatial node density is below the critical value, the achievable per node throughput increases as $n^{(\alpha-1)/2}$. In this regime, the total noise power dominates the interference power and the effect of the increasing SINR is able to overcome the effect of increasing number of relays thus leading to an overall increase of the achievable throughput. When the spatial node density is above critical, the further increase of the spatial node density (and hence the total node number) does not lead to the further increase of the SINR (since now interference dominates noise and grows at the same rate of the received power). Therefore, the effect of increasing number of relays takes over and leads to a decrease of the throughput as $n^{-1/2}$.

Note that the critical node density ρ_{cr} can be very small for non-ultra-wideband systems, and the increasing branch of the throughput may not be seen in practice. On the other hand, for ultra-wideband systems with the ratio $N_0 W/P$ significantly larger than 1, the maximum node density (limited by the physical size of transceivers) may be reached before the critical node density, thus rendering the decreasing branch of the throughput practically unobservable. The former case corresponds to the situation studied in [1] and the latter to the "ideal" ultra-wideband setup explored in [10, 11]. The result of this paper pertains to the general case which can involve "switching" from the increasing to the decreasing branch.

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