Hindawi Publishing Corporation
EURASIP Journal on Wireless Communications and Networking
Volume 2008, Article ID 130747, 12 pages
doi:10.1155/2008/130747

Research Article

Interference Cancellation Schemes for Single-Carrier Block Transmission with Insufficient Cyclic Prefix

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Received 30 April 2007; Revised 13 August 2007; Accepted 3 October 2007

Recommended by Arne Svensson

This paper proposes intersymbol interference (ISI) and interblock interference (IBI) cancellation schemes at the transmitter and the receiver for the single-carrier block transmission with insufficient cyclic prefix (CP). The proposed scheme at the transmitter can exterminate the interferences by only setting some signals in the transmitted signal block to be the same as those of the previous transmitted signal block. On the other hand, the proposed schemes at the receiver can cancel the interferences without any change in the transmitted signals compared to the conventional method. The IBI components are reduced by using previously detected data signals, while for the ISI cancellation, we firstly change the defective channel matrix into a circulant matrix by using the tentative decisions, which are obtained by our newly derived frequency domain equalization (FDE), and then the conventional FDE is performed to compensate the ISI. Moreover, we propose a pilot signal configuration, which enables us to estimate a channel impulse response whose order is greater than the guard interval (GI). Computer simulations show that the proposed interference cancellation schemes can significantly improve bit error rate (BER) performance, and the validity of the proposed channel estimation scheme is also demonstrated.

1. INTRODUCTION

A block transmission with cyclic prefix (CP), including orthogonal frequency division multiplexing (OFDM) [1, 2] and single-carrier block transmission with the CP (SC-CP) [3, 4], has been drawing much attention as a promising candidate for the 4G (4th generation) mobile communications systems. The insertion of the CP as a guard interval (GI) at the transmitter and the removal of the CP at the receiver eliminate interblock interference (IBI), if all the delayed signals exist within the GI. Moreover, the insertion and the removal of the CP convert the effect of the channel from a linear convolution to a circular convolution. This means that the CP operation converts a Toeplitz channel matrix into a circulant matrix, therefore, the intersymbol interference (ISI) of the received signal can be effectively equalized by a discrete frequency domain equalizer (FDE) using fast Fourier transform (FFT).

The existence of delayed signals beyond the GI deteriorates the performance of the block transmission with the CP. This is because, with the delayed signals, the IBI cannot be eliminated by the CP removal and the channel matrix is no longer the circulant matrix. In order to overcome the performance degradation due to the insufficient GI, a considerable number of studies have been made on the issue, such as impulse response shortening [5], utilization of an adaptive antenna array [6], per tone equalization [7–9], and overlap FDE [10, 11]. All these methods can improve the performance, however, they increase the computational or system complexity, which may spoil the most important feature of the FDE-based system of simplicity.

In this paper, we propose simple ISI and IBI cancellation schemes for the SC-CP system with the insufficient (or even without) GI. The proposed schemes can be separated into two different approaches as follows:

(1) an interference cancellation approach by controlling the transmitted signal (payload) in the transmitter without any increase in the computational complexity in the receiver but with some reduction of transmission rate;

(2) an approach by the signal processing in the receiver without any reduction of transmission rate but with slight increase in the computational complexity at the receiver.

In the SC-CP system, only limited number of symbols in a transmitted block cause the interferences, while all the information data contribute to the interferences in the OFDM system. Taking advantage of this feature of the SC-CP system, the first approach (or the proposed scheme at the transmitter) can exterminate the interference by only setting some signals in the transmitted signal block to be the same as those of the previous transmitted signal block without changing any parameters or configuration of the receiver. Therefore, it can be said that the proposed scheme cancels the interferences at the cost of the transmission rate, and in this sense, the proposed scheme is similar to the SC-CP system with a variable length GI. However, the proposed scheme does not require any frame resynchronization, which is necessary for the variable GI systems. So far, to the best of authors' knowledge, no countermeasure against the insufficient GI based on the data signal (payload) modification has been proposed. On the other hand, the second approach (or the proposed schemes at receiver) can cancel the interference without any reduction of the transmission rate. In the block transmission schemes, the equalization and demodulation processing is commonly conducted in a block-by-block manner, therefore, the IBI could be reduced by using previously detected data signals. For the ISI cancellation, we firstly generate replica signals of the ISI using tentative decisions in order to make the defective channel matrix circulant, and then the conventional FDE is performed to compensate the ISI. As for the replica signals, we propose two tentative decision generation methods, where our newly derived FDE is utilized. We also derive linear equalizers using minimum mean-square-error (MMSE) criterion for the sake of performance benchmark. Moreover, we propose a pilot signal configuration for the SC-CP system, which enables us to estimate channel impulse response even when the channel order is greater than the GI length. Computer simulations show that the proposed interference cancellation schemes can significantly improve bit error rate (BER) performance of the SC-CP system with the insufficient GI, and especially, the proposed interference cancellation scheme at the receiver can outperform the linear MMSE equalizer while it requires much lower computational complexity than the linear equalizer. Also, the validity of the proposed channel estimation scheme is demonstrated via computer simulations.

Note that there is a common point among the proposed method with the second approach and the methods proposed in [15–18] in the sense that all these methods utilize a certain estimate of the interference due to the insufficient GI in order to obtain the same received signal model as the conventional block transmission system with the CP. However, there are differences in the ways of obtaining the *estimate* of interference. The work [15] has been proposed for multicarrier transmission and is applied to the SC-CP system in [16], while the iterative processing is required in order to obtain good performance because of the different nature of the interference between the multicarrier and the single-carrier signals. In [17], instead of the iterative cancellation, more reliable *estimate* of the interference is obtained based on the log-likelihood ratios (LLRs) of the coded bits. The scope of [18] is a bit different from other methods and it devises the configuration or structure of the CP in order to reduce the loss of the CP transmission, whereas it also utilizes the interference canceller. On the other hand, the contributions of our method especially against [16–18] will be as follows:

(1) the derivation of the closed form MMSE FDE weights taking in consideration the interference due to the insufficient GI;

(2) the replica signal of the interference is generated taking advantages of the temporal localization nature of the interference.

As far as the computational complexity is concerned, all the methods (our method with the second approach and methods in [16–18]) require comparably low complexity because of the utilization of the computationally efficient FDE, although the method in [16] could require a bit higher complexity due to the iterative approach in order to obtain the same performance depending on the channel conditions.

The rest of this paper is organized as follows. Section 2 introduces the signal model of the SC-CP system with the insufficient GI. Sections 3, 4, and 5 describe the proposed interference cancellation scheme at the transmitter, the proposed schemes at the receiver, and the proposed pilot configuration for the channel estimation, respectively. Computer simulation results are presented in Section 6, and finally, conclusions are given in Section 7.

2. SIGNAL MODELING

Figure 1 shows a basic configuration of the SC-CP system. Let $\mathbf{s}(n) = [s_0(n), \dots, s_{M-1}(n)]^T$, where the superscript $(\cdot)^T$ stands for the transpose, be the nth information signal block of size $M \times 1$. The transmitted signal block $\mathbf{s}'(n)$ of size $(M+K) \times 1$ is generated from $\mathbf{s}(n)$ by adding the CP of K symbols length as the GI, namely,

$$\mathbf{s}'(n) = \mathbf{T}_{\text{cp}}\mathbf{s}(n), \tag{1}$$

where $\mathbf{T}_{\text{cp}}$ denotes the CP insertion matrix of size $(M+K)\times M$ defined as

$$\mathbf{T}_{\text{cp}} = \begin{bmatrix} \mathbf{0}_{K\times(M-K)} & \mathbf{I}_{K\times K} \\ \multicolumn{2}{c}{\mathbf{I}_{M\times M}} \end{bmatrix}. \tag{2}$$

$\mathbf{0}_{K\times(M-K)}$ is a zero matrix of size $K \times (M-K)$, and $\mathbf{I}_{M\times M}$ is an identity matrix of size $M \times M$.

The received signal block $\mathbf{r}'(n)$ is written as

$$\mathbf{r}'(n) = \mathbf{H}_0\mathbf{s}'(n) + \mathbf{H}_1\mathbf{s}'(n-1) + \mathbf{n}'(n), \tag{3}$$

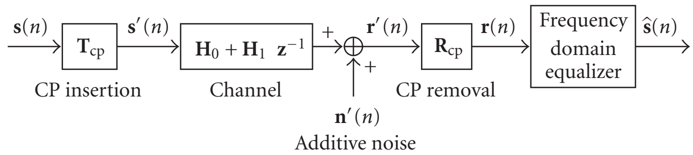

Figure 1: Basic configuration of SC-CP system.

where $\mathbf{n}'(n)$ is a channel noise vector of size $(M+K)\times 1$. $\mathbf{H}_0$ and $\mathbf{H}_1$ denote $(M+K)\times(M+K)$ channel matrices defined as

$$\mathbf{H}_0 = \begin{bmatrix} h_0 & 0 & \cdots & \cdots & \cdots & 0 \\ \vdots & \ddots & \ddots & & & \vdots \\ h_L & & \ddots & \ddots & & \vdots \\ 0 & \ddots & & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & & \ddots & 0 \\ 0 & \cdots & 0 & h_L & \cdots & h_0 \end{bmatrix}, \tag{4}$$

$$\mathbf{H}_1 = \begin{bmatrix} & h_L & \cdots & h_1 \\ & 0 & \ddots & \vdots \\ & \vdots & \ddots & h_L \\ \mathbf{0}_{(M+K)\times(M+K-L)} & \vdots & & 0 \\ & \vdots & & \vdots \\ & 0 & \cdots & 0 \end{bmatrix},$$

where $\{h_0,\ldots,h_L\}$ denotes the channel impulse response.

After discarding the CP portion of the received signal block $\mathbf{r}'(n)$, the received signal block $\mathbf{r}(n)$ of size $M\times 1$ can be written as

$$\begin{aligned} \mathbf{r}(n) &= \mathbf{R}_{\mathrm{cp}}\mathbf{r}'(n) \\ &= \mathbf{R}_{\mathrm{cp}}\mathbf{H}_0\mathbf{T}_{\mathrm{cp}}\mathbf{s}(n) + \mathbf{R}_{\mathrm{cp}}\mathbf{H}_1\mathbf{T}_{\mathrm{cp}}\mathbf{s}(n-1) + \mathbf{n}(n), \end{aligned} \tag{5}$$

where $\mathbf{R}_{\mathrm{cp}}$ denotes the CP discarding matrix of size $M\times(M+K)$ defined as

$$\mathbf{R}_{\mathrm{cp}} = \begin{bmatrix} \mathbf{0}_{M\times K} & \mathbf{I}_{M\times M} \end{bmatrix}, \tag{6}$$

and $\mathbf{n}(n) = \mathbf{R}_{\mathrm{cp}}\mathbf{n}'(n)$.

If the length of the GI is sufficiently long, namely, $K > L-1$, it can be easily verified that $\mathbf{R}_{\mathrm{cp}}\mathbf{H}_1\mathbf{T}_{\mathrm{cp}}$ becomes a zero matrix, and the nth received signal block has no IBI component from the $(n-1)$th transmitted signal block. Moreover, if $K > L-1$, $\mathbf{R}_{\mathrm{cp}}\mathbf{H}_0\mathbf{T}_{\mathrm{cp}}$ becomes a circulant matrix of size $M\times M$, which means that the one-tap FDE can equalize the ISI effectively.

However, if the length of the GI is insufficient ($K \le L-1$), $\mathbf{R}_{\mathrm{cp}}\mathbf{H}_1\mathbf{T}_{\mathrm{cp}}$ is no longer a zero matrix. Instead, $\mathbf{R}_{\mathrm{cp}}\mathbf{H}_1\mathbf{T}_{\mathrm{cp}}$ can be written as

$$\mathbf{R}_{\mathrm{cp}}\mathbf{H}_1\mathbf{T}_{\mathrm{cp}} = \begin{bmatrix} & h_L & \cdots & h_{K+1} \\ & 0 & \ddots & \vdots \\ & \vdots & \ddots & h_L \\ \mathbf{0}_{M\times(M-L+K)} & \vdots & & 0 \\ & \vdots & & \vdots \\ & 0 & \cdots & 0 \end{bmatrix}. \tag{7}$$

This means that the IBI from the $(n-1)$th transmitted signal block remains even after the CP removal operation at the receiver. $\mathbf{R}_{\mathrm{cp}}\mathbf{H}_0\mathbf{T}_{\mathrm{cp}}$ can be written as

$$\mathbf{R}_{\mathrm{cp}}\mathbf{H}_0\mathbf{T}_{\mathrm{cp}} = \begin{bmatrix} h_0 & 0 & \cdots & \cdots & \cdots & 0 & h_K & \cdots & h_1 \\ \vdots & \ddots & \ddots & & & \vdots & \vdots & & \vdots \\ \vdots & & \ddots & \ddots & & 0 & h_L & & \vdots \\ \vdots & & & \ddots & \ddots & & \ddots & \ddots & \vdots \\ h_L & & & & \ddots & \ddots & & \ddots & h_L \\ 0 & \ddots & & & & \ddots & \ddots & & 0 \\ \vdots & \ddots & \ddots & & & & \ddots & \ddots & \vdots \\ \vdots & & \ddots & \ddots & & & & \ddots & 0 \\ 0 & \cdots & \cdots & 0 & h_L & \cdots & \cdots & \cdots & h_0 \end{bmatrix}. \tag{8}$$

Note that $\mathbf{R}_{\mathrm{cp}}\mathbf{H}_0\mathbf{T}_{\mathrm{cp}}$ is no longer a circulant matrix. Therefore, it is difficult for the one-tap FDE to equalize the received signal block distorted by the ISI.

Although $\mathbf{R}_{\mathrm{cp}}\mathbf{H}_0\mathbf{T}_{\mathrm{cp}}$ is no longer a circulant matrix as mentioned above, it is also true that the matrix still has a structure close to circulant. In order to present the proposed interference cancellation schemes, we separate the matrix $\mathbf{R}_{\mathrm{cp}}\mathbf{H}_0\mathbf{T}_{\mathrm{cp}}$ into two matrices, namely, a circulant part and a compensation part, as

$$\mathbf{R}_{\mathrm{cp}}\mathbf{H}_0\mathbf{T}_{\mathrm{cp}} = \mathbf{C} - \mathbf{C}_{\mathrm{ISI}}, \tag{9}$$

where $\mathbf{C}$ is a circulant matrix whose first column is the same as that of the matrix $\mathbf{R}_{\text{cp}}\mathbf{H}_0\mathbf{T}_{\text{cp}}$, namely,

$$\mathbf{C} = \begin{bmatrix} h_0 & 0 & \cdots & 0 & h_L & \cdots & h_1 \\ \vdots & \ddots & \ddots & & \ddots & \ddots & \vdots \\ \vdots & & \ddots & \ddots & & \ddots & h_L \\ h_L & & & \ddots & \ddots & & 0 \\ 0 & \ddots & & & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & & & \ddots & 0 \\ 0 & \cdots & 0 & h_L & \cdots & \cdots & h_0 \end{bmatrix}, \tag{10}$$

and $\mathbf{C}_{\text{ISI}}$ is the compensation term given by

$$\mathbf{C}_{\text{ISI}} = \left[\mathbf{0}_{M\times(M-L)} \quad \begin{matrix} h_L & \cdots & h_{K+1} \\ 0 & \ddots & \vdots \\ \vdots & \ddots & h_L \\ \vdots & & 0 \\ \vdots & & \vdots \\ 0 & \cdots & 0 \end{matrix} \quad \mathbf{0}_{M\times K} \right]. \tag{11}$$

Using $\mathbf{C}$ and $\mathbf{C}_{\text{ISI}}$, the nth received signal block $\mathbf{r}(n)$ after the CP removal can be rewritten as

$$\mathbf{r}(n) = \mathbf{C}\mathbf{s}(n) - \mathbf{C}_{\text{ISI}}\mathbf{s}(n) + \mathbf{C}_{\text{IBI}}\mathbf{s}(n-1) + \mathbf{n}(n), \tag{12}$$

where $\mathbf{C}_{\text{IBI}}$ is defined as

$$\mathbf{C}_{\text{IBI}} = \mathbf{R}_{\text{cp}}\mathbf{H}_1\mathbf{T}_{\text{cp}}. \tag{13}$$

3. PROPOSED INTERFERENCE CANCELLATION SCHEME AT TRANSMITTER

In this section, we propose a simple interference cancellation scheme, which is performed in the transmitter. Although the proposed scheme requires a certain reduction of the transmission rate, conventional receivers can be used without any modification.

From (12), we can see that the first term of the right hand, $\mathbf{C}\mathbf{s}(n)$, can be equalized using the FDE, since $\mathbf{C}$ is a circulant matrix, while the second and the third terms could result in the ISI and the IBI components, respectively, at the FDE output. However, if

$$\mathbf{C}_{\text{ISI}}\mathbf{s}(n) = \mathbf{C}_{\text{IBI}}\mathbf{s}(n-1) \tag{14}$$

holds, the received signal block $\mathbf{r}(n)$ can be written as

$$\mathbf{r}(n) = \mathbf{C}\mathbf{s}(n) + \mathbf{n}(n), \tag{15}$$

which is the same form as the received signal block with the sufficient GI.

Inspection of (7) and (11) reveals that the two matrices, $\mathbf{C}_{\text{ISI}}$ and $\mathbf{C}_{\text{IBI}}$, share the same elements with the same arrangement, although they are not the same matrices. Namely, if we circularly shift all the elements of $\mathbf{C}_{\text{ISI}}$ to the right side by K columns, then we obtain $\mathbf{C}_{\text{IBI}}$. It is easily verified that $\mathbf{C}_{\text{ISI}}$ and $\mathbf{C}_{\text{IBI}}$ can be related as

$$\mathbf{C}_{\text{ISI}}\mathbf{S}^K = \mathbf{C}_{\text{IBI}}, \tag{16}$$

where the $M \times M$ shifting matrix $\mathbf{S}$ is defined as

$$\mathbf{S} = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ \vdots & & \ddots & \ddots & 0 \\ 0 & & & \ddots & 1 \\ 1 & 0 & \cdots & \cdots & 0 \end{bmatrix}. \tag{17}$$

Also, $\mathbf{S}^K$ stands for the K times multiplications of $\mathbf{S}$.

Using (16), (14) can be modified as

$$\mathbf{C}_{\text{ISI}}\mathbf{s}(n) = \mathbf{C}_{\text{ISI}}\mathbf{S}^K\mathbf{s}(n-1). \tag{18}$$

Therefore, taking advantage of the fact that only the limited number of columns of $\mathbf{C}_{\text{ISI}}$ has nonzero elements, the condition imposed on the transmitted signals for the equality in (14) to be true is given by

$$s_m(n) = s_{m+K}(n-1), \quad m = M-L,\ldots,M-K-1. \tag{19}$$

From the condition, we can see that the interferences can be eliminated by just setting the transmitted signal, while the transmission rate of the proposed scheme is $(M-L+K)/M$ times the transmission rate of the original SC-CP system. Therefore, it can be said that the proposed scheme eliminates the interferences due to the insufficient GI at the cost of reduction of the transmission rate. Figure 2 shows the proposed transmitted signal configuration for the interference elimination.

Note that the proposed transmission scheme does not require the transmitter to know the detailed channel state information (CSI), such as an instantaneous channel impulse response. The transmitter only has to know the channel order L, which is not difficult to feed back from the receiver and could be estimated by using the received signal of the reverse link in the case of time division duplex (TDD) systems.

With the proposed transmission scheme (19), the received signal block $\mathbf{r}(n)$ can be written as (15). Since $\mathbf{C}$ is a circulant matrix, it can be diagonalized by the discrete Fourier transform (DFT) matrix $\mathbf{D}$ of size $M \times M$ as [13]

$$\mathbf{C} = \mathbf{D}^H\mathbf{\Lambda}\mathbf{D}, \tag{20}$$

where the superscript H denotes the Hermitian transpose, and $\mathbf{\Lambda}$ is a diagonal matrix, whose diagonal elements are $\{\lambda_0,\ldots,\lambda_{M-1}\}$. Also, $\mathbf{\Lambda}$ can be calculated as

$$\mathbf{\Lambda} = \text{diag}\left\{\mathbf{D}\begin{bmatrix} h_0 \\ \vdots \\ h_L \\ \mathbf{0}_{(M-L-1)\times 1} \end{bmatrix}\right\}, \tag{21}$$

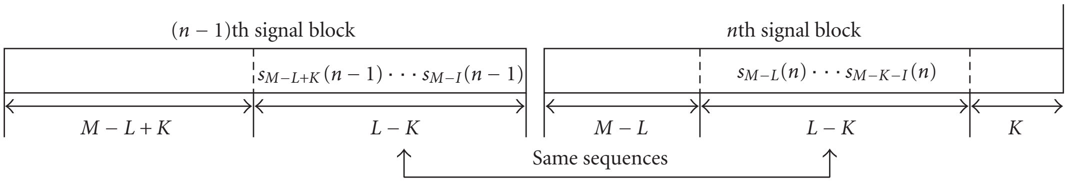

FIGURE 2: Transmitted signal format for interference cancellation.

where diag$\{\mathbf{v}\}$ denotes a diagonal matrix, whose diagonal elements are the same as the elements of vector $\mathbf{v}$. The one-tap FDE can be formulated as $\mathbf{D}^H\mathbf{\Gamma}_{\text{cnv}}\mathbf{D}$, where $\mathbf{\Gamma}_{\text{cnv}}$ is a diagonal matrix with the diagonal elements of $\{\gamma_0^{\text{cnv}},\ldots,\gamma_{M-1}^{\text{cnv}}\}$. For MMSE equalization, the equalizer weights γ_m^{cnv} are given by

$$\gamma_m^{\text{cnv}} = \frac{\lambda_m^*}{|\lambda_m|^2 + \sigma_n^2/\sigma_s^2}, \quad m = 0,\ldots,M-1, \tag{22}$$

where the superscript $*$ denotes the complex conjugate, σ_n^2 is the variance of the additive channel noise, and σ_s^2 is the variance of the transmitted data symbols. In this way, the conventional equalization methods for the SC-CP system can be applied to the proposed scheme. The fundamental difference between the equalization in the proposed transmission scheme and in the conventional SC-CP system is that the channel order can be greater than the length of the GI in the proposed scheme.

4. PROPOSED INTERFERENCE CANCELLATION SCHEME AT RECEIVER

In this section, we propose interference cancellation schemes at the receiver. Unlike the proposed method in Section 3, the proposed schemes in this section can cancel the interferences without any reduction of the transmission rate, while they somewhat increase the complexity of the receiver.

4.1. Interblock interference cancellation

In the block transmission schemes, the equalization and the detection are commonly conducted in a block-by-block manner, therefore, the IBI component $\mathbf{C}_{\text{IBI}}\mathbf{s}(n-1)$ could be cancelled by using the previously detected data vector $\tilde{\mathbf{s}}(n-1)$. In the proposed method, we cancel the IBI by subtracting $\mathbf{C}_{\text{IBI}}\tilde{\mathbf{s}}(n-1)$ from $\mathbf{r}(n)$. After the IBI cancellation, the received signal vector $\bar{\mathbf{r}}(n)$ can be written as

$$\begin{aligned}\bar{\mathbf{r}}(n) &= \mathbf{r}(n) - \mathbf{C}_{\text{IBI}}\tilde{\mathbf{s}}(n-1),\\ &\approx (\mathbf{C}-\mathbf{C}_{\text{ISI}})\mathbf{s}(n) + \mathbf{n}(n),\end{aligned} \tag{23}$$

where $\approx$ becomes an equality when $\tilde{\mathbf{s}}(n-1) = \mathbf{s}(n-1)$. Figure 3 shows the configuration of the proposed IBI canceller. In this figure, the feedback path stands for the processing of the IBI cancellation using the previously detected data vector $\tilde{\mathbf{s}}(n-1)$. The block of ISI canceller (equalizer) will be discussed in detail in the next section.

4.2. Intersymbol interference cancellation

In this section, we show ISI cancellation (or equalization) methods assuming that the IBI components are completely cancelled, namely,

$$\begin{aligned}\bar{\mathbf{r}}(n) &= (\mathbf{C}-\mathbf{C}_{\text{ISI}})\mathbf{s}(n) + \mathbf{n}(n),\\ &= \mathbf{R}_{\text{cp}}\mathbf{H}_0\mathbf{T}_{\text{cp}}\mathbf{s}(n) + \mathbf{n}(n).\end{aligned} \tag{24}$$

In the following, we firstly derive a linear equalizer, which will be a benchmark of the proposed method, although it requires high computational complexity compared to the FDE approach. Then, we derive the FDE weight for the SC-CP system with insufficient GI based on MMSE criterion. Finally, we describe the details of the proposed ISI cancellation method, which utilizes the FDE and the replica signal generator. Note that all these methods correspond to the ISI canceller (equalizer) in Figure 3.

(1) Linear equalization

As shown in Figure 4, where a linear equalizer matrix of $\mathbf{\Omega}$ is employed as the ISI canceller, the output of the linear equalizer can be written as

$$\hat{s}_{lnr}(n) = \mathbf{\Omega}\bar{\mathbf{r}}(n) = \mathbf{\Omega}\mathbf{R}_{\text{cp}}\mathbf{H}_0\mathbf{T}_{\text{cp}} + \mathbf{\Omega}\mathbf{r}(n). \tag{25}$$

In order to determine the equalizer weights, we have employed MMSE criterion. The MMSE equalizer can be obtained by minimizing $E\{\text{tr}[(\hat{\mathbf{s}}(n)-\mathbf{s}(n))(\hat{\mathbf{s}}(n)-\mathbf{s}(n))^H]\}$, where $E\{\cdot\}$ and tr$[\cdot]$ denote ensemble average and trace of the matrix, respectively. By solving the minimization problem, the MMSE equalizer weight can be given by

$$\mathbf{\Omega} = (\mathbf{R}_{\text{cp}}\mathbf{H}_0\mathbf{T}_{\text{cp}})^H \cdot \left\{\mathbf{R}_{\text{cp}}\mathbf{H}_0\mathbf{T}_{\text{cp}}(\mathbf{R}_{\text{cp}}\mathbf{H}_0\mathbf{T}_{\text{cp}})^H + \frac{\sigma_n^2}{\sigma_s^2}\mathbf{I}_M\right\}^{-1}. \tag{26}$$

(2) One-tap frequency domain equalization

The channel matrix $\mathbf{R}_{\text{cp}}\mathbf{H}_0\mathbf{T}_{\text{cp}}$ is no longer a circulant, therefore, the one-tap FDE cannot perfectly equalize the distorted received signal even when the IBIs are completely cancelled. However, the FDE is still attractive because of the simplicity of the implementation using FFT. As shown in Figure 5, where the one-tap frequency domain equalizer of $\mathbf{D^H\Gamma D}$ is

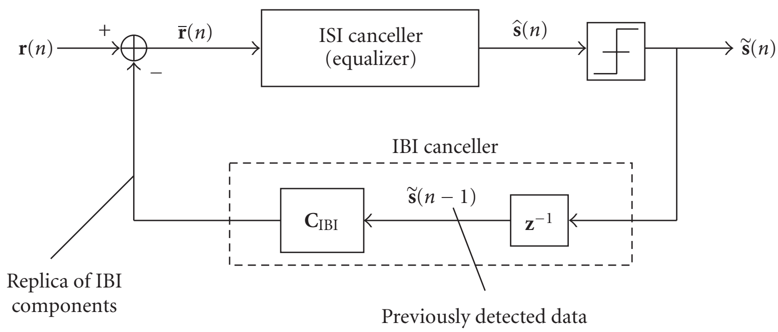

Figure 3: IBI canceller.

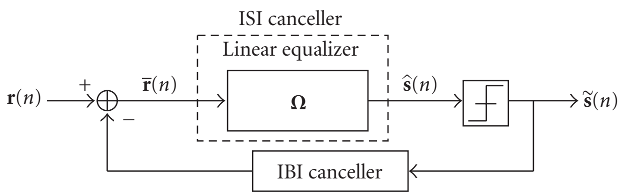

Figure 4: ISI canceller: linear equalizer.

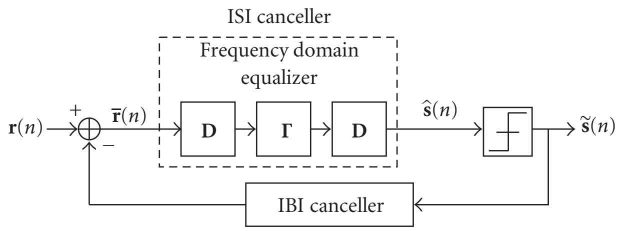

Figure 5: ISI canceller: FDE.

employed as the ISI canceller, the output of the FDE for the SC-CP system with the insufficient GI is given by

$$\begin{aligned}\hat{\mathbf{s}}_{\text{fde}}(n) &= \mathbf{D}^H\boldsymbol{\Gamma}\mathbf{D}\bar{\mathbf{r}}(n)\\ &= \mathbf{D}^H\boldsymbol{\Gamma}\mathbf{D}(\mathbf{C}-\mathbf{C}_{\text{ISI}})\mathbf{s}(n)+\mathbf{D}^H\boldsymbol{\Gamma}\mathbf{D}\mathbf{n}(n).\end{aligned} \tag{27}$$

$\boldsymbol{\Gamma}$ is a diagonal matrix, whose diagonal elements are $\gamma_0,\ldots,\gamma_{M-1}$, and the mth element γ_m is given by (see the appendix)

$$\begin{aligned}\gamma_m &= \frac{\lambda_m^* - g_{m,m}^*}{\left|\lambda_m - g_{m,m}\right|^2+\sum_{i=0,\,i\neq m}^{M-1}\left|g_{m,i}\right|^2+\left(\sigma_n^2/\sigma_s^2\right)},\\ g_{m,n} &= \frac{1}{M}\sum_{l=0}^{L-K-1}\sum_{i=0}^{l}h_{L-i}e^{j(2\pi/M)\{n(M-L+l)-mi\}},\\ g_{m,m} &= \frac{1}{M}\sum_{l=0}^{L-K-1}\sum_{i=0}^{l}h_{L-i}e^{j(2\pi/M)m(M-L+l-i)},\\ \sum_{m=0}^{M-1}\left|g_{m,n}\right|^2 &= \frac{1}{M}\sum_{l=0}^{L-K-1}\sum_{i=0}^{l}\sum_{l'=0}^{L-K-1}\left|h_{L-i}\right|^2 e^{j(2\pi/M)n(l-l')}.\end{aligned} \tag{28}$$

(3) FDE with replica signal generator

The proposed FDE (27) requires low computational complexity and can achieve better performance than the conventional FDE, however, it still suffers from performance degradation due to the defective channel matrix $\mathbf{R}_{\text{cp}}\mathbf{H}_0\mathbf{T}_{\text{cp}}(=\ \mathbf{C}-\mathbf{C}_{\text{ISI}})$. In order to further improve the performance of the FDE, we propose to utilize a replica signal of $\mathbf{C}_{\text{ISI}}\mathbf{s}(n)$, which is generated from a tentative decision $\tilde{\tilde{\mathbf{s}}}(n) = [\tilde{\tilde{s}}_0(n),\ldots,\tilde{\tilde{s}}_{M-1}(n)]^T$. The main idea of the proposed method is that, by adding the replica signal $\mathbf{C}_{\text{ISI}}\tilde{\tilde{\mathbf{s}}}(n)$ to $\bar{\mathbf{r}}(n)$, we can obtain a received signal vector $\bar{\bar{\mathbf{r}}}(n)$, which is distorted only by the circulant matrix $\mathbf{C}$ in the ideal case, as

$$\begin{aligned}\bar{\bar{\mathbf{r}}}(n) &= \bar{\mathbf{r}}(n)+\mathbf{C}_{\text{ISI}}\tilde{\tilde{\mathbf{s}}}(n),\\ &\approx \mathbf{C}\mathbf{s}(n)+\mathbf{n}(n).\end{aligned} \tag{29}$$

Then, the conventional FDE can efficiently equalize $\bar{\bar{\mathbf{r}}}(n)$ as

$$\hat{\mathbf{s}}_{\text{cancel}}(n) = \mathbf{D}^H\boldsymbol{\Gamma}_{\text{cnv}}\mathbf{D}\bar{\bar{\mathbf{r}}}(n), \tag{30}$$

where $\boldsymbol{\Gamma}_{\text{cnv}}$ is the diagonal matrix, whose diagonal elements are defined by (22).

As for the tentative decision used for the replica signal generation, we consider two schemes as follows.

(1) *Tentative Decision 1*: in this scheme, we directly utilize the output of the proposed FDE (27) for the tentative decision, namely,

$$\tilde{\tilde{\mathbf{s}}}(n) = \tilde{\mathbf{s}}_{\text{fde}}(n) = \langle\hat{\mathbf{s}}_{\text{fde}}(n)\rangle, \tag{31}$$

where $\langle\cdot\rangle$ stands for the detection operation. Figure 6 shows the configuration of the proposed receiver using the *tentative decision 1* for the replica signal generation. In this figure, the combined parts of the replica signal generation and the conventional FDE correspond to the ISI canceller in Figure 3.

(2) *Tentative Decision 2*: although the idea of the *tentative decision 1* is simple and easily understood, we cannot have sufficient performance gain with the decision. The reason for the poor performance gain can be

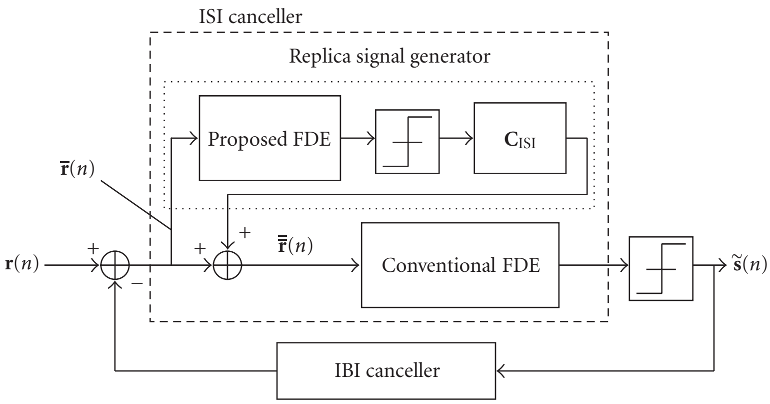

Figure 6: ISI canceller: FDE with replica signal generator using *Tentative Decision 1*.

explained as follows. Since $\mathbf{C}_{\text{ISI}}$ has nonzero elements only in $L-K$ columns, we have

$$\begin{aligned}\mathbf{C}_{\text{ISI}}\mathbf{s}(n) &= \mathbf{C}_{\text{ISI}}\begin{bmatrix}\mathbf{0}_{(M-L)\times 1}\\ \mathbf{s}^{\text{sub}}(n)\\ \mathbf{0}_{K\times 1}\end{bmatrix}\\ &= \mathbf{C}_{\text{ISI}}\mathbf{F}_s\mathbf{F}_s^T\mathbf{s}(n),\end{aligned} \tag{32}$$

where $\mathbf{s}^{\text{sub}}(n) = [s_{M-L}(n),\ldots,s_{M-K-1}(n)]^T = \mathbf{F}_s^T\mathbf{s}(n)$, and

$$\mathbf{F}_s = \begin{bmatrix}\mathbf{0}_{(M-L)\times(L-K)}\\ \mathbf{I}_{L-K}\\ \mathbf{0}_{K\times(L-K)}\end{bmatrix}. \tag{33}$$

This means that only the corresponding tentative decision $\tilde{\tilde{\mathbf{s}}}^{\text{sub}}(n)$, which is defined in the same way as $\mathbf{s}^{\text{sub}}(n)$, is required for the replica signal generation. However, if we recall the received signal model (24), we can see that the power of $\mathbf{s}^{\text{sub}}(n)$ included in $\bar{\mathbf{r}}$ is smaller than the other transmitted signals due to the defectiveness of the channel matrix $\mathbf{R}_{\text{cp}}\mathbf{H}_0\mathbf{T}_{\text{cp}}$. Therefore, the reliability of the corresponding FDE output $\tilde{\mathbf{s}}_{\text{fde}}^{\text{sub}}(n) = \mathbf{F}_s^T\tilde{\mathbf{s}}_{\text{fde}}(n)$ is lower than the other signals, which results in the poor performance gain of the *tentative decision 1*.

Note that the utilization of the *tentative decision 1* combined with the IBI canceller is similar to the method proposed in [15] for the multicarrier systems, although the conventional FDE weights are used also for the replica signal generation. In the case of multicarrier transmission, the interference due to the insufficient GI is spread in the discrete frequency domain, which makes such a simple approach applicable to the multicarrier case. Although the same approach as [15] is applied to the SC-CP system in [16], the iterative interference cancellation is also employed in order to improve the performance.

Based on the discussion above, we propose to utilize not $\tilde{\mathbf{s}}_{\text{fde}}^{\text{sub}}(n)$ but the rest of $\tilde{\mathbf{s}}_{\text{fde}}(n)$ to generate the replica signal of $\mathbf{s}^{\text{sub}}(n)$. This can be achieved by using the key relation of

$$\begin{aligned}&\mathbf{R}_{\text{cp}}\mathbf{H}_0\mathbf{T}_{\text{cp}}\mathbf{s}(n) - \mathbf{C}(\mathbf{I}_M - \mathbf{F}_s\mathbf{F}_s^T)\mathbf{s}(n)\\ &\quad= \mathbf{R}_{\text{cp}}\mathbf{H}_0\mathbf{T}_{\text{cp}}\mathbf{s}(n) - \mathbf{C}\left(\mathbf{s}(n) - \begin{bmatrix}\mathbf{0}_{(M-L)\times 1}\\ \mathbf{s}^{\text{sub}}(n)\\ \mathbf{0}_{K\times 1}\end{bmatrix}\right)\\ &\quad= \mathbf{R}_{\text{cp}}\mathbf{H}_0\mathbf{T}_{\text{cp}}\begin{bmatrix}\mathbf{0}_{(M-L)\times 1}\\ \mathbf{s}^{\text{sub}}(n)\\ \mathbf{0}_{K\times 1}\end{bmatrix}.\end{aligned} \tag{34}$$

By substituting $\bar{\mathbf{r}}(n)$ for $\mathbf{R}_{\text{cp}}\mathbf{H}_0\mathbf{T}_{\text{cp}}\mathbf{s}(n)$, $\tilde{\mathbf{s}}_{\text{fde}}(n)$ for $\mathbf{s}(n)$, and $\tilde{\mathbf{s}}_{\text{fde}}^{\text{sub}}(n)$ for $\mathbf{s}^{\text{sub}}(n)$ in (34), we have

$$\begin{aligned}\bar{\mathbf{r}}'(n) &\overset{\text{def}}{=} \bar{\mathbf{r}}(n) - \mathbf{C}\left(\tilde{\mathbf{s}}_{\text{fde}}(n) - \begin{bmatrix}\mathbf{0}_{(M-L)\times 1}\\ \tilde{\mathbf{s}}_{\text{fde}}^{\text{sub}}(n)\\ \mathbf{0}_{K\times 1}\end{bmatrix}\right)\\ &\approx \mathbf{R}_{\text{cp}}\mathbf{H}_0\mathbf{T}_{\text{cp}}\begin{bmatrix}\mathbf{0}_{(M-L)\times 1}\\ \mathbf{s}^{\text{sub}}(n)\\ \mathbf{0}_{K\times 1}\end{bmatrix}.\end{aligned} \tag{35}$$

Furthermore, defining

$$\mathbf{F}_r = \begin{bmatrix}\mathbf{0}_{(M-L)\times L}\\ \mathbf{I}_L\end{bmatrix}, \tag{36}$$

and $\bar{\mathbf{r}}'^{\,\text{sub}}(n) = \mathbf{F}_r^T\bar{\mathbf{r}}'(n)$, we finally have

$$\bar{\mathbf{r}}'^{\,\text{sub}}(n) \approx \mathbf{E}\mathbf{s}^{\text{sub}}(n), \tag{37}$$

where

$$\mathbf{E} = \mathbf{F}_r^T\mathbf{R}_{\text{cp}}\mathbf{H}_0\mathbf{T}_{\text{cp}}\mathbf{F}_s = \begin{bmatrix}h_0 & & \mathbf{0}\\ \vdots & \ddots & \\ \vdots & & h_0\\ \vdots & & \vdots\\ h_{L-1} & \cdots & h_k\end{bmatrix}. \tag{38}$$

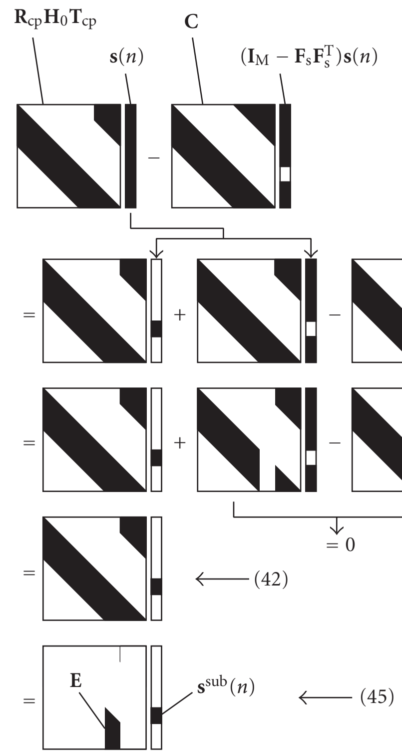

Figure 7: Derivation of key relations.

Figure 7 explains how to obtain the relations of (34) and (37), where the colored parts stand for nonzero elements of the matrices or vectors. The transmitted signal vector $\mathbf{s}(n)$ is separated into two vectors of $\left[\mathbf{0}_{1\times(M-L)}\ \ \mathbf{s}^{\mathrm{sub}\,T}(n)\ \ \mathbf{0}_{1\times K}\right]^T$ and $(\mathbf{I}_M - \mathbf{F}_s\mathbf{F}_s^T)\mathbf{s}(n)$ in the development from the fist line to the second. In the third line, we have set all the elements of the columns, which correspond to zero entries of the vector, to be zero in the second and the third terms. Then, we obtain the relation of (34). Moreover, in the same way as the third line, by setting all the columns, which correspond to the zero entries of the vector, to be zero, we finally obtain the relation of (37), although the effects of noise or detection errors are ignored in this derivation.

By solving the overdetermined system of (37), the tentative decision for the replica generation can be given by

$$\tilde{\tilde{\mathbf{s}}}^{\mathrm{sub}}(n) = \left\langle \left(\mathbf{E}^H\mathbf{E}\right)^{-1}\mathbf{E}\overline{\mathbf{r}}'^{\,\mathrm{sub}}(n)\right\rangle. \tag{39}$$

The schematic diagram of the proposed receiver with the *tentative decision 2* is shown in Figure 8. In this figure, the uppermost path is used to obtain the second term of left-hand side of (34). After the multiplication by the matrix $\mathbf{F}_r^T$, we obtain the vector $\overline{\mathbf{r}}'^{\mathrm{sub}}(n)$ of (37), therefore, the estimate of $\mathbf{s}^{\mathrm{sub}T}(n)$, which is required for the replica signal generation, is obtained by multiplying the pseudoinverse matrix $\mathbf{E}$.

5. PROPOSED CHANNEL ESTIMATION SCHEME

The proposed schemes can effectively eliminate or cancel the ISI and the IBI components, however, they require the receiver to know the channel impulse response, whose order may be greater than the length of the GI. In this section, we propose a pilot signal configuration for the computationally efficient channel estimation for the proposed interference cancellation schemes.

Let $\mathbf{p}(n) = [p_0(n), \ldots, p_{M-1}(n)]^T$ denote the nth pilot signal block of length M. After the CP removal, the corresponding received pilot signal block, $\mathbf{r}_p(n)$, can be written as

$$\mathbf{r}_p(n) = \mathbf{C}\mathbf{p}(n) - \mathbf{C}_{\mathrm{ISI}}\mathbf{p}(n) + \mathbf{C}_{\mathrm{IBI}}\mathbf{p}(n-1) + \mathbf{n}(n), \tag{40}$$

where $\mathbf{C}$, $\mathbf{C}_{\mathrm{ISI}}$, and $\mathbf{C}_{\mathrm{IBI}}$ are the same as the matrices defined in (10), (11), and (13), respectively. Therefore, if we have

$$\begin{aligned}\mathbf{C}_{\mathrm{ISI}}\mathbf{p}(n) &= \mathbf{C}_{\mathrm{IBI}}\mathbf{p}(n-1)\\ &= \mathbf{C}_{\mathrm{ISI}}\mathbf{S}^K\mathbf{p}(n-1),\end{aligned} \tag{41}$$

the received pilot signal block $\mathbf{r}_p(n)$ can be written as

$$\mathbf{r}_p(n) = \mathbf{C}\mathbf{p}(n) + \mathbf{n}(n). \tag{42}$$

From (41), it can be said that if the two consecutive pilot signal blocks, $\mathbf{p}(n-1)$ and $\mathbf{p}(n)$, have the relation of

$$\mathbf{p}(n) = \mathbf{S}^K\mathbf{p}(n-1), \tag{43}$$

the equality of (41) is always true regardless of $\mathbf{C}_{\mathrm{ISI}}$. Although we can also employ the same condition as (19), the condition of (43) will be more suited for the pilot signals. This is because the second pilot signal can generate only the cyclic shift operation from a predetermined pilot signal. Figure 9

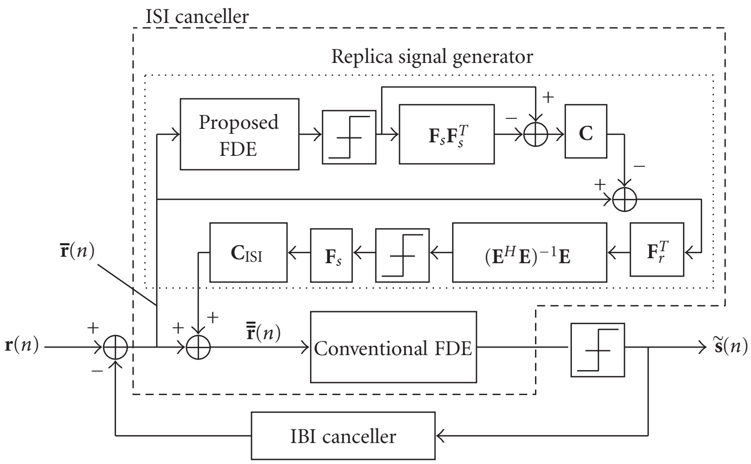

Figure 8: ISI canceller: FDE with replica signal generator using *tentative decision 2*.

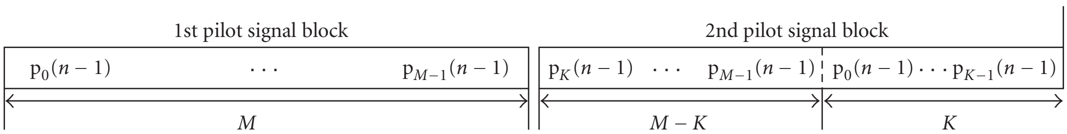

Figure 9: Pilot signal configuration for insufficient GI.

shows the proposed pilot signal configuration for the channel estimation.

Now that we have the received pilot signal block given by (42), we can estimate the channel impulse response, whose order is possibly greater than the length of the CP, by using conventional channel estimation schemes. For example, since the received pilot signal block can be modified as

$$\begin{aligned}\mathbf{r}_p(n) &= \mathbf{C}\mathbf{p}(n) + \mathbf{n}(n)\\ &= \mathbf{Q}(n)\mathbf{h} + \mathbf{n}(n),\end{aligned} \tag{44}$$

where $\mathbf{Q}(n)$ is an $M \times (L+1)$ circulant matrix defined as

$$\mathbf{Q}(n) = \begin{bmatrix} p_0(n) & p_{M-1}(n) & \cdots & p_{M-L+1}(n) \\ p_1(n) & p_0(n) & \ddots & \vdots \\ \vdots & & \ddots & p_{M-1}(n) \\ \vdots & \vdots & & p_0(n) \\ \vdots & & & \vdots \\ p_{M-1}(n) & p_{M-2}(n) & \cdots & p_{M-L}(n) \end{bmatrix}, \tag{45}$$

and $\mathbf{h}$ is the channel impulse response vector defined as $\mathbf{h} = [h_0, \ldots, h_L]^T$, the channel impulse response is estimated as [14]

$$\hat{\mathbf{h}} = \left(\mathbf{Q}(n)^H\mathbf{Q}(n)\right)^{-1}\mathbf{Q}(n)^H\mathbf{r}_p. \tag{46}$$

Also, more computationally efficient channel estimation can be achieved in the DFT domain. The DFT of the received pilot signal $\mathbf{r}_p(n)$ is given by

$$\begin{aligned}\mathbf{D}\mathbf{r}_p(n) &= \mathbf{D}\mathbf{C}\mathbf{p}(n) + \mathbf{D}\mathbf{n}(n) = \mathbf{\Lambda}\mathbf{P}(n) + \mathbf{N}(n)\\ &= \operatorname{diag}\{\mathbf{P}(n)\}\mathbf{H} + \mathbf{N}(n),\end{aligned} \tag{47}$$

Table 1: System parameters.

Mod./demod. scheme	QPSK/coherent detection
FFT length	$M = 64$
Guard interval	$K = 16$
Channel order	$L = 20$
Channel model	9-path Rayleigh fading channel
Channel noise	Additive white Gaussian noise

where $\mathbf{P}(n)$=$\mathbf{D}\mathbf{p}(n)$, $\mathbf{N}(n)$=$\mathbf{D}\mathbf{n}(n)$, and $\mathbf{H}$=$\mathbf{D}[\mathbf{h}^T\ \mathbf{0}_{1\times(M-L-1)}]^T$, therefore, the frequency response of the channel $\mathbf{H}$ can be estimated as

$$\hat{\mathbf{H}} = \left(\operatorname{diag}\{\mathbf{P}(n)\}\right)^{-1}\mathbf{D}\mathbf{r}_p(n). \tag{48}$$

Note that, since $\mathbf{P}(n)$ is known to the receiver a priori, the calculation of $\hat{\mathbf{H}}$ is efficiently conducted using the FFT.

6. COMPUTER SIMULATION

In order to confirm the validity of the proposed interference cancellation and the channel estimation schemes, we have conducted computer simulations. System parameters used in the computer simulations are summarized in Table 1.

As a modulation/demodulation scheme, QPSK modulation with a coherent detection is employed. The FFT length (or the information block size), the length of GI, and the channel order are set to be $M = 64$, $K = 16$, and $L = 20$, respectively. 9-path frequency selective Rayleigh fading channel with uniform delay power profile is used for the channel model. In order to evaluate the performance against solely the frequency selectivity of the channel, no time variation of

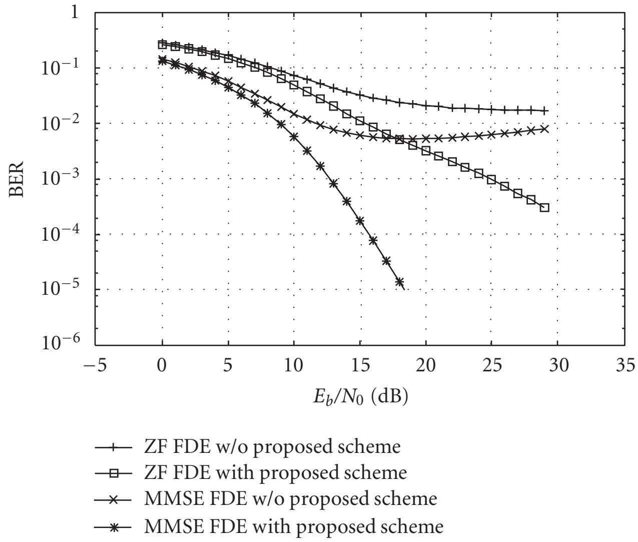

FIGURE 10: BER performance of interference canceller at transmitter.

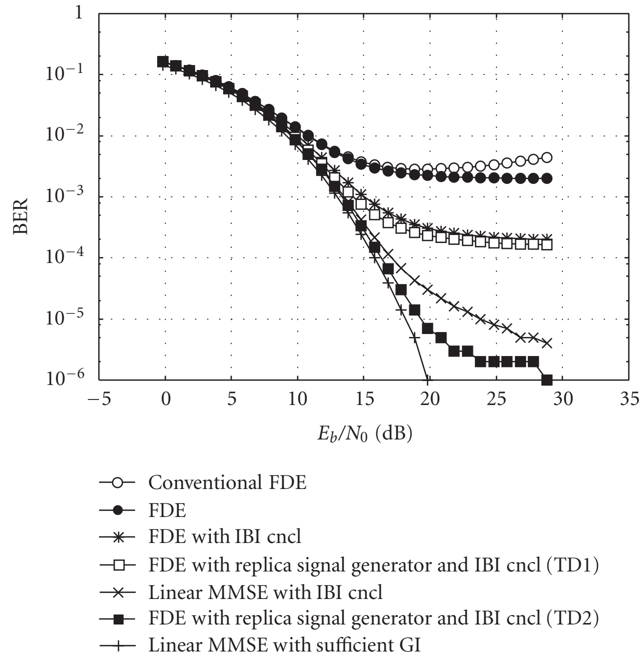

FIGURE 11: BER performance of interference canceller at receiver.

the channel has been assumed. Also, additive white Gaussian noise (AWGN) is assumed as the channel noise. In the computer simulation of the proposed interference cancellation schemes, perfect channel estimation is assumed in order to evaluate the attainable BER performance by the employment of the proposed schemes.

Figure 10 shows the BER performance versus the ratio of the energy per bit to the noise power density (E_b/N_0) of the proposed scheme in Section 2 with the MMSE-based FDE. The BER performances of the SC-CP system without the proposed transmission scheme are also plotted in the same figure. Note that the transmission rate of the proposed method in this figure is $(M - L + K)/M = 0.9375$ times that of the conventional SC-CP system. From this figure, we can see that the proposed scheme can improve the BER performance significantly at the cost of transmission rate, while the performance of the SC-CP system without the proposed scheme is degraded due to the ISI and the IBI caused by the insufficient GI.

Figure 11 shows the BER performances versus the E_b/N_0 of the following 8 schemes as follows:

(1) *conventional FDE*: the conventional FDE (22) without the IBI canceller;
(2) *FDE*: the proposed FDE (27) without the IBI canceller;
(3) *FDE with IBI cncl*: the proposed FDE (27) with the IBI canceller;
(4) *FDE with replica signal generator and IBI cncl (TD1)*: the conventional FDE (22) with the replica signal generator using *tentative decision 1* and the IBI canceller;
(5) *FDE with replica signal generator and IBI cncl (TD2)*: the conventional FDE (22) with the replica signal generator using *tentative decision 2* and the IBI canceller;
(6) *Linear MMSE with IBI cncl*: the linear MMSE equalizer (26) with the IBI canceller;
(7) *Linear MMSE with Sufficient GI*: the linear MMSE equalizer (26) (or equivalently the conventional MMSE FDE (22)) with sufficient length of the GI ($K = 20$).

From this figure, we can see that the proposed FDE with the replica signal generation using the *tentative decision 2* and the IBI canceller can achieve the best performance among the systems with the insufficient GI, and the performance is close to the linear MMSE equalizer with the sufficient GI. Amazingly enough, FDE with replica signal generator and IBI cncl (TD2) can outperform even the linear MMSE equalizer with the IBI canceller, while the proposed FDE requires much lower computational complexity than the linear equalizer—thanks to the implementation using the FFT. Also, it should be noted that even only the proposed IBI cancellation can significantly improve the BER performance.

Figure 12 shows the mean-square errors (MSEs) of the channel estimation schemes (46) and (48) versus the E_b/N_0 with and without the proposed pilot signal configuration. The MSE is defined as

$$\mathrm{MSE} = \frac{1}{N_{\mathrm{trial}}} \sum_{i=1}^{N_{\mathrm{trial}}} \frac{\|\mathbf{h} - \overline{\mathbf{h}}\|^2}{\|\mathbf{h}\|^2}, \tag{49}$$

where $\|\cdot\|$ denotes the Euclidean norm, and N_{trial} denotes the number of channel realizations and is set to be 1 000 in the simulations. From this figure, we can see that the proposed pilot signal configuration can achieve accurate channel estimation even when the channel order is greater than the length of the GI.

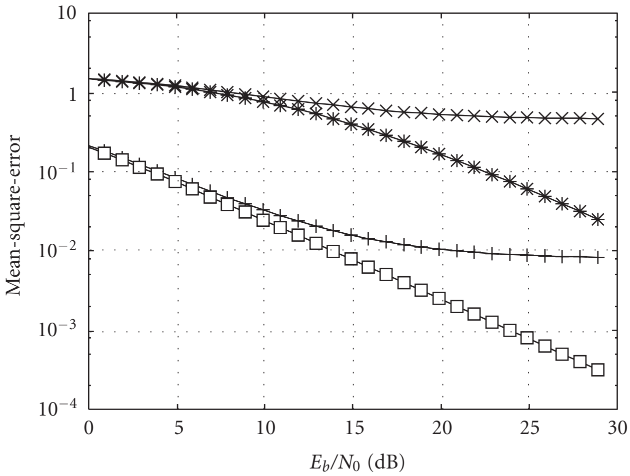

FIGURE 12: Channel estimation error.

7. CONCLUSION

We have proposed ISI and IBI cancellation schemes for the SC-CP system with the insufficient GI. Moreover, we have proposed a pilot signal configuration for the channel estimation, where the channel order is possibly greater than the length of the GI. The proposed interference cancellation scheme at the transmitter can exterminate the interferences without changing any configuration of the receiver at the cost of some reduction of the transmission rate. On the other hand, the proposed interference cancellation schemes at the receiver increase the complexity of the receiver to some extent, however, they can efficiently cancel the interferences without any reduction of the transmission rate. The performances of the proposed interference cancellation schemes and the channel estimation schemes are evaluated via computer simulations. From all the results, it can be concluded that the proposed schemes could be a simple but powerful solution for the SC-CP system with the insufficient GI.

APPENDIX

Here, we derive MMSE FDE weights of the SC-CP scheme with the insufficient GI. Since the received signal vector can be rewritten as

$$\begin{aligned}\bar{\mathbf{r}}(n) &= \mathbf{R}_{\text{cp}}\mathbf{H}_0\mathbf{T}_{\text{cp}}\mathbf{s}(n) + \mathbf{n}(n)\\ &= \mathbf{D}^H\mathbf{\Lambda}\mathbf{D}\mathbf{s}(n) - \mathbf{C}_{\text{ISI}}\mathbf{s}(n) + \mathbf{n}(n),\end{aligned} \tag{A.1}$$

denoting the FDE weights by a diagonal matrix $\mathbf{\Gamma}$, whose diagonal components are $\gamma_0, \dots, \gamma_{M-1}$, the FDE output can be given by

$$\begin{aligned}\widehat{\mathbf{s}}_{\text{fde}}(n) &= \mathbf{D}^H\mathbf{\Gamma}\mathbf{D}\bar{\mathbf{r}}(n)\\ &= \mathbf{D}^H\mathbf{\Gamma}\mathbf{\Lambda}\mathbf{D}\mathbf{s}(n) - \mathbf{D}^H\mathbf{\Gamma}\mathbf{D}\mathbf{C}_{\text{ISI}}\mathbf{s}(n) + \mathbf{D}^H\mathbf{\Gamma}\mathbf{D}\mathbf{n}(n).\end{aligned} \tag{A.2}$$

In order to derive the MMSE weights, we define a cost function J to be minimized as

$$\begin{aligned} J &= E\{\text{tr}[(\widehat{\mathbf{s}}(n) - \mathbf{s}(n))(\widehat{\mathbf{s}}^H(n) - \mathbf{s}^H(n))]\}\\ &= \text{tr}\big[\sigma_s^2\{\mathbf{D}^H\mathbf{\Gamma}\mathbf{\Lambda}\mathbf{\Lambda}^H\mathbf{\Gamma}^H\mathbf{D} - \mathbf{D}^H\mathbf{\Gamma}\mathbf{\Lambda}\mathbf{D}\mathbf{C}_{\text{ISI}}^H\mathbf{D}^H\mathbf{\Gamma}^H\mathbf{D}\\ &\qquad - \mathbf{D}^H\mathbf{\Gamma}\mathbf{\Lambda}\mathbf{D} - \mathbf{D}^H\mathbf{\Gamma}\mathbf{D}\mathbf{C}_{\text{ISI}}\mathbf{D}^H\mathbf{\Lambda}^H\mathbf{\Gamma}^H\mathbf{D}\\ &\qquad + \mathbf{D}^H\mathbf{\Gamma}\mathbf{D}\mathbf{C}_{\text{ISI}}\mathbf{C}_{\text{ISI}}^H\mathbf{D}^H\mathbf{\Gamma}^H\mathbf{D} + \mathbf{D}^H\mathbf{\Gamma}\mathbf{D}\mathbf{C}_{\text{ISI}}\\ &\qquad - \mathbf{D}^H\mathbf{\Lambda}^H\mathbf{\Gamma}^H\mathbf{D} + \mathbf{C}_{\text{ISI}}^H\mathbf{D}^H\mathbf{\Gamma}^H\mathbf{D} + \mathbf{I}_M\}\\ &\quad + \sigma_n^2\mathbf{D}^H\mathbf{\Gamma}\mathbf{\Gamma}^H\mathbf{D}\big].\end{aligned} \tag{A.3}$$

Ignoring the term $\text{tr}[\sigma_s^2\mathbf{I}_M]$, which has no elements of $\mathbf{\Gamma}$, the cost function J can be redefined as

$$\begin{aligned} J &= \sigma_s^2\{\text{tr}[\mathbf{\Gamma}\mathbf{\Lambda}\mathbf{\Lambda}^H\mathbf{\Gamma}^H] - \text{tr}[\mathbf{\Gamma}\mathbf{\Lambda}\mathbf{D}\mathbf{C}_{\text{ISI}}^H\mathbf{D}^H\mathbf{\Gamma}^H] - \text{tr}[\mathbf{\Gamma}\mathbf{\Lambda}]\\ &\qquad - \text{tr}[\mathbf{\Gamma}\mathbf{D}\mathbf{C}_{\text{ISI}}\mathbf{D}^H\mathbf{\Lambda}^H\mathbf{\Gamma}^H] + \text{tr}[\mathbf{\Gamma}\mathbf{D}\mathbf{C}_{\text{ISI}}\mathbf{C}_{\text{ISI}}^H\mathbf{D}^H\mathbf{\Gamma}^H]\\ &\qquad + \text{tr}[\mathbf{\Gamma}\mathbf{D}\mathbf{C}_{\text{ISI}}\mathbf{D}^H] - \text{tr}[\mathbf{\Lambda}^H\mathbf{\Gamma}^H] + \text{tr}[\mathbf{D}\mathbf{C}_{\text{ISI}}^H\mathbf{D}^H\mathbf{\Gamma}^H]\}\\ &\quad + \sigma_n^2\text{tr}[\mathbf{\Gamma}\mathbf{\Gamma}^H].\end{aligned} \tag{A.4}$$

Moreover, defining a matrix as

$$\mathbf{G} = \mathbf{D}\mathbf{C}_{\text{ISI}}\mathbf{D}^H, \tag{A.5}$$

and the (m, n) element of $\mathbf{G}$ as $g_{m,n}(m, n = 0, \dots, M-1)$, we have

$$\begin{aligned} J &= \sigma_s^2\sum_{m=0}^{M-1}\Bigg(|\lambda_m|^2|\gamma_m|^2 - |\gamma_m|^2\lambda_m g_{m,m}^* - \gamma_m\lambda_m\\ &\qquad - \lambda_m^*|\gamma_m|^2 g_{m,m} + |\gamma_m|^2\sum_{i=0}^{M-1}|g_{m,i}|^2 + \gamma_m g_{m,m}\\ &\qquad - \lambda_m^*\gamma_m^* + g_{m,m}^*\gamma_m^*\Bigg) + \sigma_n^2|\gamma_m|^2.\end{aligned} \tag{A.6}$$

The differentiation of J with respect to γ_m^* is given by

$$\begin{aligned}\frac{\partial J'}{\partial\gamma_m^*} &= \sigma_s^2\Bigg\{|\lambda_m|^2\,|\,\gamma_m - \lambda_m g_{m,m}^*\gamma_m - \lambda_m^* g_{m,m}\gamma_m\\ &\qquad + \gamma_m\sum_{i=0}^{M-1}|g_{m,i}|^2 - \lambda_m^* + g_{m,m}^*\Bigg\} + \sigma_n^2\gamma_m.\end{aligned} \tag{A.7}$$

By solving $\partial J'/\partial\gamma_m^* = 0$, we have the MMSE weight of (28) as

$$\begin{aligned}\gamma_m &= \frac{\lambda_m^* - g_{m,m}^*}{|\lambda_m|^2 - \lambda_m g_{m,m}^* - \lambda_m^* g_{m,m} + \sum_{i=0}^{M-1}|g_{m,i}|^2 + \sigma_n^2/\sigma_s^2}\\ &= \frac{\lambda_m^* - g_{m,m}^*}{|\lambda_m - g_{m,m}|^2 + \sum_{i=0, i\neq m}^{M-1}|g_{m,i}|^2 + \sigma_n^2/\sigma_s^2},\end{aligned} \tag{A.8}$$

where

$$g_{m,n} = \frac{1}{M} \sum_{l=0}^{L-K-1} \sum_{i=0}^{l} h_{L-i} e^{j(2\pi/M)\{n(M-L+l)-mi\}},$$
$$g_{m,m} = \frac{1}{M} \sum_{l=0}^{L-K-1} \sum_{i=0}^{l} h_{L-i} e^{j(2\pi/M)m(M-L+l-i)},$$
$$\sum_{m=0}^{M-1} |g_{m,n}|^2 = \frac{1}{M} \sum_{l=0}^{L-K-1} \sum_{i=0}^{l} \sum_{l'=0}^{L-K-1} |h_{L-i}|^2 e^{j(2\pi/M)n(l-l')}. \tag{A.9}$$

ACKNOWLEDGMENTS

This work was supported in part by the International Communications Foundation (ICF), Tokyo, and by the Grantin-Aid for Scientific Research, Grant no. 17760305, from the Ministry of Education, Science, Sports, and Culture of Japan.

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