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Research Article

Guaranteed Performance Region in Fading Orthogonal Space-Time Coded Broadcast Channels

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Recently, the capacity region of the MIMO broadcast channel (BC) was completely characterized and duality between MIMO multiple access channel (MAC) and MIMO BC with perfect channel state information (CSI) at transmitter and receiver was established. In this work, we propose a MIMO BC approach in which only information about the channel norm is available at the base and hence no joint beamforming and dirty paper precoding (DPC) can be applied. However, a certain set of individual performances in terms of MSE or zero-outage rates can be guaranteed at any time by applying an orthogonal space-time block code (OSTBC). The guaranteed MSE region without superposition coding is characterized in closed form and the impact of diversity, fading statistics, and number of transmit antennas and receive antennas is analyzed. Finally, six CSI and precoding scenarios with different levels of CSI and precoding are compared numerically in terms of their guaranteed MSE region. Depending on the long-term SNR, superposition coding as well as successive interference cancellation (SIC) with norm feedback performs better than linear precoding with perfect CSI.

1. INTRODUCTION

Wireless multiuser systems are characterized by different performance measures. The choice of the performance measure depends either on the fading characteristics (e.g., fast or slow fading correspond to ergodic and outage capacity [1]) or on the type of service (e.g., elastic or nonelastic traffic correspond to average and outage or zero-outage capacity).

Consider the downlink broadcast channel (BC). In [2], the ergodic BC region was analyzed. Further on, in [3] the zero-outage BC region was studied and time-division (TD) was investigated. Whereas the latter is the interference avoiding case, the code division (CD) with successive interference cancellation (SIC) and without SIC (CDWO) are the full interference cases. The delay-limited capacity (DLC) region of CD contains the region of TD which contains the CDWO region, that is, CD is superior to TD, which in turn is superior to CDWO.

With respect to the uplink multiple access channel (MAC), in [4] the ergodic MAC region, and in [5] the delay-limited capacity (DLC) region were characterized. A useful property for the analysis and optimal power allocation is the polymatroid structure of the capacity region [4, 5]. The optimality of allowing for full interference (CD) is also shown in [6] by studying the ergodic capacity region of the MIMO MAC with different amount of user collision.

In [7], the capacity region with minimal rate requirements of the fading BC is studied. A certain part of the long-term transmit power is used to fulfill the minimum rate requirements, while the remaining part of the long-term transmit power is used to maximize the ergodic capacity region. Recently, in [8], the capacity of fading broadcast channels with rate constraints is analyzed. A general framework is provided to represent ergodic, zero-outage, minimum-rate, outage, and limited-jitter capacity regions. More recently, the performance under these hard fairness constraints was compared to the performance of the proportional scheduler in [9]. All these results were derived under the assumption of perfect channel state information (CSI) at the base as well as at the mobiles.

We consider the downlink and assume that information about the average channel power instead of perfect CSI is available at the base as well as perfect CSI at the receivers. This is a form of partial CSI which can be achieved by norm feedback. The combination of norm feedback and covariance information has been analyzed for single-user systems in [10]. The BC setting is studied in [11]. Then the base applies an orthogonal space-time code (OSTBC) and *can* apply superposition coding or dirty paper precoding (DPC) on the effective OSTBC channels.

The disadvantage of the notion of delay-limited capacity or zero-outage capacity is that capacity in general can only be approached with long codes. In contrast, the mean-squared error (*MSE*), $0 \leq MSE \leq 1$, for the linear multiuser MMSE receiver can be computed for each transmitted symbol. When studying the *MSE* region, the polymatroidal structure of the capacity region cannot directly be exploited. in this work, we study the guaranteed *MSE* region in a fading BC under long-term sum power constraints. This region could also be called delay-limited or zero-outage *MSE* region. All *MSE* tuples that lie in the guaranteed *MSE* region can be achieved for all joint fading states and for each transmitted symbol vector.

We compare the cases where the mobiles are either assumed to perform successive decoding or treat all other user signals as noise. For single-input single-output Rayleigh fading channels, it turns out that the guaranteed *MSE* point is the tuple $(1, 1, \ldots, 1)$. Thus in order to achieve nontrivial *MSE* points, for example, spatial diversity has to be exploited. Since full CSI feedback seems impractical, only the channel norm is feedback from the mobiles to base and an OSTBC is applied at the transmitter. One advantage of OSTBC is the simple receive processing at the mobiles. One disadvantage of OSTBC is that the higher the number of transmit antennas, the lower the code rate which can be supported [12, 13]. Recently, this rate loss or rate reduction was characterized completely for OSTBC without linear processing of information symbols [14]. Note that the rate reduction derived in [14] has been conjectured in [13] to hold for OSTBC with linear processing of information symbols as well. The optimality of a full-rate OSTBC has been shown for the MIMO BC without CSI at the base in [15].

The contributions of the paper are summarized below as follows.

(1) A system concept for how to achieve nonunity guaranteed *MSE* region by utilizing OSTBC, and limited channel norm feedback is presented in Section 2.2.

(2) Optimal resource allocation with and without successive decoding to guarantee *MSE* requirements in all fading states with minimum long-term sum transmit power is performed. We derive a closed form characterization of the full interference guaranteed *MSE* region (Theorem 1) (and the corresponding DLC region—Corollary 1).

(3) Feasibility analysis of QoS requirements as a function of the number of users K, number of transmit n_T and receive n_R antennas using the performance measure effective bandwidth from [16] is performed (14).

(4) The impact of the fading statistic is analyzed: the guaranteed *MSE* region shrinks with increased spatial correlation (Theorem 2). The guaranteed common *MSE* decreases with asymmetric user distribution (Theorem 3). We optimize the user placement for long-term power reduction under QoS requirements in Section 3.4.

(5) In Section 5, we compare guaranteed *MSE* regions for the following six cases:

  (i) norm feedback and linear precoding without SIC (CDWO);
  (ii) norm feedback and linear precoding with time sharing (TD);
  (iii) norm feedback and superposition coding with SIC (CD);
  (iv) perfect CSI and beamforming (BFWO);
  (v) perfect CSI and time-sharing (BFTD);
  (vi) perfect CSI and DPC (BF).

(6) Depending on the SNR working point (CD) outperforms (BFWO).

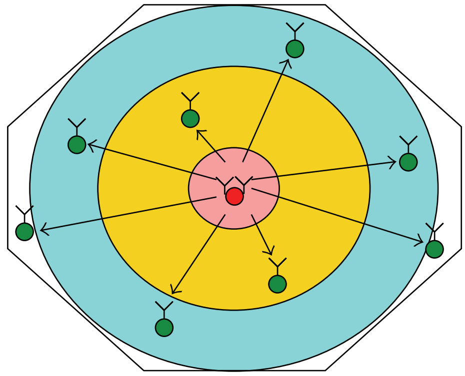

Figure 1: Cellular downlink transmission.

2. SYSTEM MODEL, CHANNEL MODEL, AND PRELIMINARIES

2.1. System model

The system model in Figure 1 consists of K mobile users and one base station. Each user k requests a certain QoS level that has to be fulfilled throughout the transmission in *every* fading realization. For complexity reasons, we assume that the mobile users apply a linear MMSE receiver. The QoS requirements are formulated in terms of *MSE* requirements $m_1, \ldots, m_K$, since the *MSE* is closely related to other practical performance measures, for example, the SINR and the BER.

2.2. Transmitter structure

The base station has multiple antennas (n_T), the mobiles have $n_{R,1} = \cdots = n_{R,K} = n_R$ antennas. Denote the channels to the users as $\mathbf{H}_1, \ldots, \mathbf{H}_K$. The base applies an OSTBC [12, 17] as shown in Figure 2. The data stream vectors $\mathbf{d}_1, \ldots, \mathbf{d}_K$ of dimension $1 \times M$ of the K users are weighted by a power allocation $p_1, \ldots, p_K$ and added before they come into the

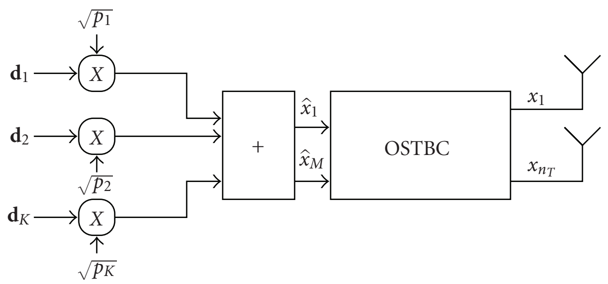

Figure 2: Transmitter structure.

OSTBC as $\hat{x}_1, \ldots, \hat{x}_M$. The output of the OSTBC is a vector $\mathbf{x} = [x_1, \ldots, x_{n_T}]$ of dimension $1 \times n_T$. The code-rate is given by $r_c = M/n_T$.

Each mobile first performs channel-matched filtering according to the effective OSTBC channel. Afterwards the received signal at user k of stream n is given by

$$y_{k,n} = a_k \sum_{l=1}^{K} \overline{x}_{l,n} + n_{k,l}, \quad 1 \le n \le M \tag{1}$$

with fading coefficients $\alpha_k = a_k^2 = \|\mathbf{H}_k\|^2/n_T = (1/n_T)\operatorname{tr}(\mathbf{H}_k\mathbf{H}_k^H)$, transmit stream n intended for user l as $\overline{x}_{l,n}$ and noise for stream n as $n_{k,l}$. There are M parallel streams for each mobile. However, all streams have the same properties in terms of a_k and noise statistics and the same interference. Therefore, we restrict our attention without loss of generality to the first stream $n = 1$ and omit the index in the following. Let p_k be the power allocated to user k, that is, $p_k = \mathbb{E}[|x_k|^2]$. Denote the long-term sum transmit power constraint at the base station as P, that is,

$$\mathbb{E}_{a_1,\ldots,a_k}\left[\sum_{k=1}^{K} p_k(a_1, \ldots, a_k)\right] \le P. \tag{2}$$

The noise power at the receivers is $\sigma_k^2 = 1/\rho$. The transmit power is distributed uniformly over the n_T transmit antennas and each data stream has an effective power p_k/n_T. We incorporate this weighting into the statistics of $\alpha_k = \|\mathbf{H}_k\|^2/n_T$. The transmit power to noise power is given by $\text{SNR} = P\rho$, which is called long-term transmit SNR. Later, we will use the name short-term SINR s_k of a user k to denote the instantaneous SINR achieved for a given channel realization.

The mobiles feedback their fading coefficient $a_1, \ldots, a_K$ to the base and we assume these numbers are perfectly known at the base station. The base has perfect information about the channel norm, but not about the complete fading vectors. Further on, in the case with SIC at the mobiles, we assume that the signals $\overline{x}_1, \ldots, \overline{x}_K$ are encoded by, for example, superposition coding and the mobiles perform ideal SIC.

2.3. *Channel model and measure of spatial correlation and user distribution*

The following assumptions are made regarding the channel matrices $\mathbf{H}_1, \ldots, \mathbf{H}_K$. The fading processes of users k and l for $k \neq l$ are independently distributed. The channels of the users are spatially correlated according to the Kronecker model, that is, $\mathbf{H}_k = \sqrt{c_k}\mathbf{T}_k^{1/2}\mathbf{W}_k\mathbf{R}_k^{1/2}$ with random matrix $\mathbf{W}_k$ with zero-mean unit-variance complex Gaussian distributed entries, transmit correlation matrix $\mathbf{T}_k$, receive correlation matrix $\mathbf{R}_k$, and long-term fading coefficient c_k for user $1 \le k \le K$.

Denote the eigenvalue decomposition of the channel correlation matrices as $\mathbf{T}_k = \mathbf{U}_k\mathbf{\Lambda}_k\mathbf{U}_k^H$ and the vector with eigenvalues of user $1 \le k \le K$ as $\boldsymbol{\lambda}_k = [\lambda_{1,k}, \ldots, \lambda_{n_T,k}]$ and $\mathbf{R}_k = \mathbf{V}_k\mathbf{\Gamma}_k\mathbf{V}^H$ with eigenvalues of user $1 \le k \le K$ as $\boldsymbol{\gamma}_k = [\gamma_{1,k}, \ldots, \gamma_{n_R,k}]$. In order to compare different spatial correlation scenarios, we use majorization theory [18]. The measure of correlation is defined and explained in [19, Section 4.1.2]. A correlation matrix $\mathbf{R}_1$ is "more correlated" than $\mathbf{R}_2$ if the vector of eigenvalues of the correlation matrix one majorizes the vector of eigenvalues of the correlation matrix two, that is, $\boldsymbol{\lambda}_1 \succeq \boldsymbol{\lambda}_2$. This means that the sum of the ℓ largest eigenvalues of the correlation matrix one is larger than or equal to the sum of the ℓ largest eigenvalues of the correlation matrix two for all $1 \le \ell < n_T$ and the traces of $\mathbf{R}_1$ and $\mathbf{R}_2$ are equal, that is,

$$\begin{gathered}\sum_{k=1}^{\ell}\lambda_{1,k} \ge \sum_{k=1}^{\ell}\lambda_{2,k}, \quad \forall 1 \le \ell \le n_T,\\ \sum_{k=1}^{n_T}\lambda_{1,k} = \sum_{k=1}^{n_T}\lambda_{2,k}.\end{gathered} \tag{3}$$

The long-term fading coefficient c_k depends mainly on the distance of the user from the base station. The measure of user distribution based on majorization theory is defined in [19, Section 4.2.1]. Collect the fading variances of all users in a vector $\mathbf{c} = [c_1, \ldots, c_K]$. Then a user distribution $\mathbf{c}$ is "more spread out" (less symmetrically distributed users) than $\mathbf{d}$ if $\mathbf{c}$ majorizes $\mathbf{d}$, that is, $\mathbf{c} \succeq \mathbf{d}$.

A function $\phi : \mathbb{R}^{n_T} \to \mathbb{R}_0^+$ which maps from the set of vectors of dimension n_T to the set of nonnegative numbers is called Schur-convex if for $\mathbf{c} \succeq \mathbf{d}$, it follows that $\phi(\mathbf{c}) \le \phi(\mathbf{d})$. In words, this means that the function is monotonic increasing with respect to the partial order induced by majorization. A function is called Schur-concave if it is monotonic decreasing with respect to the majorization order. For more properties and examples, the interested reader is referred to [19].

3. GUARANTEED PERFORMANCE REGION WITHOUT SIC

For nonelastic traffic, like video stream or gaming services, a certain performance measure has to be guaranteed for all channel states. The *MSE* is a measure which works on a symbol by symbol basis. Therefore, hard delay constraints can be nicely expressed in terms of guaranteed *MSE* requirements. Since also many other performance measures can be mapped to the *MSE*, we study the guaranteed *MSE* region in this paper.

3.1. *Characterization of guaranteed MSE region*

Suppose that the users do not perform successive interference cancellation and the base station does only power allocation.

This case is called "code division without interference cancellation" (CDWO) in the terminology of [3].

The individual instantaneous *MSE* of user k without precoding is given by

$$m_k = 1 - p_k \frac{\rho\alpha_k}{1 + \rho\alpha_k P_s} \tag{4}$$

with the instantaneous sum power $P_s = \sum_{k=1}^{K} p_k$. Denote the guaranteed *MSE* region as $\mathcal{M}$. The following result describes the guaranteed *MSE* region without SIC and full collisions.

Theorem 1. *The MSE tuple* $(m_1, \ldots, m_K)$ *with* $0 \le m_k \le 1$ *is in the guaranteed MSE region* $\mathcal{M}$, *that is,* $(m_1, \ldots, m_K) \in \mathcal{M}$, *with* CDWO *if and only if*

$$\sum_{k=1}^{K} \mathbb{E}\left[\frac{1}{\alpha_k}\right](1 - m_k) \le \text{SNR}\left(1 - \sum_{k=1}^{K}(1 - m_k)\right). \tag{5}$$

Proof. First, we prove that the *MSE*s can be guaranteed if (5) is fulfilled. Solve (4) for p_k to obtain

$$p_k = (1 - m_k)\frac{1 + \rho\alpha_k P_s}{\rho\alpha_k}. \tag{6}$$

The sum power P_s is

$$P_s = \sum_{k=1}^{K} p_k = \sum_{k=1}^{K}(1 - m_k)\frac{1 + \rho\alpha_k P_s}{\rho\alpha_k}. \tag{7}$$

Solve (7) for P_s to obtain

$$P_s = \frac{\sum_{k=1}^{K}(1 - m_k)(1/\rho\alpha_k)}{1 - \sum_{k=1}^{K}(1 - m_k)}. \tag{8}$$

The instantaneous power allocation P_s and the long-term power constraint are related by $\mathbb{E}[P_s] \le P$. Taking the average with respect to the fading realizations, α_k yields the inequality in (5).

For the converse direction choose the set of *MSE*s $\mathbf{m} = [m_1, \ldots, m_K]$ such that the condition in (5) is fulfilled with equality. Choose a vector $\boldsymbol{\epsilon} = [\epsilon_1, \ldots, \epsilon_K]$ with small real numbers $\epsilon_k \ge 0$ for $1 \le k \le K$ with at least one entry greater than zero. Next, we show that it is not possible to support the *MSE* requirements $\widetilde{\mathbf{m}} = \mathbf{m} - \boldsymbol{\epsilon}$. Consider user k for which $\widetilde{m}_k < m_k$. Define $u_k = (1 + \rho\alpha_k P_s)/\rho\alpha_k$ and note that $u_k > 0$. The minimum instantaneous power $\widetilde{p}_k$ that is needed to support $\widetilde{m}_k$ is

$$\begin{aligned} \widetilde{p}_k &= (1 - \widetilde{m}_k)u_k \\ &= (1 - m_k + \epsilon_k)u_k \\ &= (1 - m_k)u_k + \epsilon_k u_k \\ &= p_k + \epsilon_k u_k > p_k. \end{aligned} \tag{9}$$

Since every instantaneous power $\widetilde{p}_k$ of user k with decreased *MSE* requirement $\widetilde{m}_k$ is strictly larger than the instantaneous power p_k of user k for the original *MSE* requirement m_k, the instantaneous sum power P_s as well as its average $\mathbb{E}[P_s]$ is strictly increased. Therefore, any *MSE* vector $\widetilde{\mathbf{m}}$ outside the region defined in (5) cannot be guaranteed under the same long-term power constraint SNR. □

Remark 1. The *MSE* tuple $(m_1, \ldots, m_K)$ is not feasible if

$$\sum_{k=1}^{K} m_k < K - 1, \tag{10}$$

since then the RHS of (5) is not positive. The condition for feasibility in (10) can be interpreted in terms of the effective bandwidth defined in [16]. The effective bandwidth of user $1 \le k \le K$ is defined in terms of SINR s_k of user $1 \le k \le K$ as

$$\frac{s_k}{1 + s_k} = 1 - \frac{1}{1 + s_k} = 1 - m_k. \tag{11}$$

Therefore, condition (10) yields

$$\sum_{k=1}^{K} \frac{s_k}{1 + s_k} < 1, \tag{12}$$

which corresponds to the result in [16] with processing gain $N = 1$. Note that in [16] the nonfading Gaussian MAC and BC are studied with synchronous CDMA and linear MMSE multiuser receivers. Therefore, they provide a lower bound on the guaranteed *MSE* region in (5) in which fading is present.

Remark 2. If all *MSE* requirements are equal $m_1 = \cdots = m_K = m$, the condition in (5) simplifies to

$$\sum_{k=1}^{K} \mathbb{E}\left[\frac{1}{\alpha_k}\right] < \text{SNR}\left(\frac{1}{1 - m} - K\right). \tag{13}$$

The condition in (13) can be rewritten with SINR requirement $s = 1/m - 1$ as (the interpretation is that K users are admissible in the system if the condition is fulfilled)

$$K < \frac{1}{s} + 1 - \frac{\sum_{k=1}^{K}\mathbb{E}[1/\alpha_k]}{\text{SNR}} \tag{14}$$

in order to compare the results to [16]. The last term in the RHS of (14) arises due to the fading channels and long-term transmit power constraint.

Remark 3. The *MSE* region is empty, that is, consists only of the point $(1, 1, \ldots, 1)$, if the channels are Rayleigh fading because then $\mathbb{E}[1/\alpha_k] = \infty$.

Since the *MSE* m_k and the SINR s_k as well as the transmission rate r_k are closely connected by

$$r_k = -\log_2(m_k) = \log_2(1 + s_k), \tag{15}$$

the result regarding the guaranteed *MSE* region can be transformed to give the delay-limited or zero-outage capacity region. The detour over the guaranteed *MSE* region yields a simple and novel characterization of the DLC-region in the following corollary.

Corollary 1. *The zero-outage capacity region consists of all rates* $r_1, \ldots, r_K$ *for which*

$$\frac{\sum_{k=1}^{K}\mathbb{E}[1/\alpha_k](1 - 2^{-r_k})}{1 - \sum_{k=1}^{K}(1 - 2^{-r_k})} \le \text{SNR}. \tag{16}$$

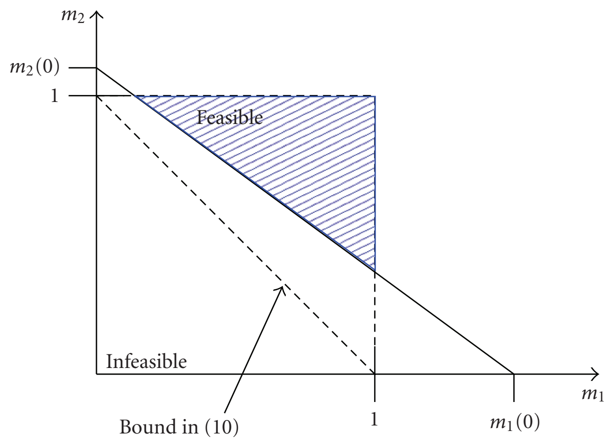

FIGURE 3: Guaranteed *MSE* region with linear precoding and full collision.

Remark 4. For the DLC-region, the feasibility condition in (10) reads

$$\sum_{k=1}^{K} 2^{-r_k} \geq K - 1. \quad (17)$$

In contrast to [3, Section III.B], we obtain in (16) a simple-closed form expression for the delay-limited capacity region of CDWO that will be further analyzed with respect to the tradeoff between diversity and code rate of the OSTBC loss below.

3.2. Two-user special case

Consider the two-user special case and denote $\mu_1 = \mathbb{E}[1/\alpha_1]$ and $\mu_2 = \mathbb{E}[1/\alpha_2]$. Then the *MSE* of user one can be expressed by the *MSE* of user two and vice versa, that is,

$$m_2(m_1) \geq \frac{\mu_1 + \mu_2 + \text{SNR} - m_1(\mu_1 + \text{SNR})}{\mu_2 + \text{SNR}}, \quad m_1(m_2) \geq \frac{\mu_1 + \mu_2 + \text{SNR} - m_2(\mu_2 + \text{SNR})}{\mu_1 + \text{SNR}}. \quad (18)$$

The guaranteed *MSE* region is then characterized by the two *MSE* points on the axes, that is, $m_1(0) = \mu_2/(\mu_1 + \text{SNR}) + 1$ and $m_2(0) = \mu_1/(\mu_2 + \text{SNR}) + 1$. This is illustrated in Figure 3. The hatched area is the guaranteed *MSE* region. It is lower bounded by the line through $m_1(0)$ and $m_2(0)$ in (18). The dashed line in Figure 3 corresponds to the feasibility condition in (10). Note that *MSE* tuples, in which one or more components are greater than one, are not achievable. Therefore, the guaranteed *MSE* region is inside the unit box.

3.3. Impact of fading statistics and user distribution

The guaranteed *MSE* region depends on the expectations $\mathbb{E}[1/\alpha_k]$ for $1 \leq k \leq K$. The expectation has been analyzed in [20] with respect to spatial correlation. The results apply also to the multiuser setting. Write the guaranteed *MSE* region as a function of the spatial correlations $\mathcal{M}(\boldsymbol{\lambda}_1, \ldots, \boldsymbol{\lambda}_K)$.

Theorem 2. *The guaranteed MSE region without* SIC *shrinks with increasing spatial correlation at the base station, that is,*

$$\boldsymbol{\lambda}_k \succeq \boldsymbol{\gamma}_k \quad \text{for } 1 \leq k \leq K \Longrightarrow \mathcal{M}(\boldsymbol{\lambda}_1, \ldots, \boldsymbol{\lambda}_K) \subseteq \mathcal{M}(\boldsymbol{\gamma}_1, \ldots, \boldsymbol{\gamma}_K). \quad (19)$$

Proof. The required SNR in (5) depends on the spatial statistics of the channels via $\mathbb{E}[1/\alpha_k]$. Since the expression in (5) decouples in terms of the users $1 \leq k \leq K$, we focus on one user k. Fix the receive correlation $\mathbf{R}_k$. The statistics of $\alpha_k = 1/n_T \operatorname{tr}(c_k \mathbf{R}_k \mathbf{W}_k \mathbf{T}_k \mathbf{W}_k^H)$ does not change if we multiply $\mathbf{W}$ from left with unitary $\mathbf{V}_k^H$ and from right with unitary $\mathbf{U}_k$. The resulting expectation can be rewritten as

$$h(\boldsymbol{\lambda}) = \mathbb{E}\left[\frac{1}{\alpha_k}\right] = \mathbb{E}\left[\frac{n_T}{c_k \operatorname{tr}(\boldsymbol{\Gamma}_k \mathbf{W}_k \boldsymbol{\Lambda}_k \mathbf{W}_k^H)}\right] = n_T \mathbb{E}\left[\left(c_k \sum_{l=1}^{n_T} \lambda_{k,l} \operatorname{tr}\left(\widetilde{\mathbf{W}}_{k,l} \widetilde{\mathbf{W}}_{k,l}^H\right)\right)^{-1}\right] \quad (20)$$

with $\widetilde{\mathbf{W}} = \boldsymbol{\Gamma}^{1/2}\mathbf{W}$. From [19, Theorem 2.15], it follows that $h(\boldsymbol{\lambda})$ is Schur-convex because $1/x$ is a convex function, that is, the value of $\mathbb{E}[1/\alpha_k]$ decreases for less correlation and the region gets larger. □

Define the guaranteed *MSE* region as a function of the receive correlation eigenvalue vectors $\mathcal{M}(\boldsymbol{\gamma}_1, \ldots, \boldsymbol{\gamma}_K)$.

Corollary 2. *The guaranteed MSE region without* SIC *shrinks with increasing spatial correlation at the mobile terminals, that is,*

$$\boldsymbol{\zeta}_k \succeq \boldsymbol{\gamma}_k \quad \text{for } 1 \leq k \leq K \Longrightarrow \mathcal{M}(\boldsymbol{\zeta}_1, \ldots, \boldsymbol{\zeta}_K) \subseteq \mathcal{M}(\boldsymbol{\gamma}_1, \ldots, \boldsymbol{\gamma}_K). \quad (21)$$

This result follows from Theorem 2 by keeping the transmit correlation fixed and analyzing the *MSE* region as a function of the receive correlation.

Next, for the case in which the users have equal *MSE* requirements and spatially uncorrelated channels, the impact of the user distribution is characterized. Write the guaranteed *MSE* region as a function of the user distribution $\mathcal{M}(\mathbf{c})$.

Theorem 3. *Assume that all users have the same MSE requirement $m_1 = \cdots = m_K = m = 1/(1+s)$ and spatially uncorrelated channels $\mathbf{R}_k = \mathbf{I}$, $\mathbf{T}_k = \mathbf{I}$ for all $1 \leq k \leq K$. Then the common MSE as a function of the user distribution $m(\mathbf{c})$ is Schur-convex with respect to $\mathbf{c}$, that is,*

$$\mathbf{c} \succeq \mathbf{d} \Longrightarrow \mathcal{M}(\mathbf{c}) \geq \mathcal{M}(\mathbf{d}). \quad (22)$$

Proof. We note from (13) that the necessary and sufficient condition for the overall *MSE* requirement m and for spatially uncorrelated channels $\boldsymbol{\lambda}_k = \mathbf{1}$ for $1 \leq k \leq K$ is

$$m(\mathbf{c}) \geq 1 - \frac{\text{SNR}}{K\text{SNR} + (n_T/n_T n_R - 1)\sum_{k=1}^{K}(1/c_k)}. \quad (23)$$

The function $\sum_{k=1}^{K}(1/c_k)$ if symmetric with respect to $\mathbf{c}$ and convex. The argument vector of a symmetric function can be permuted without changing the value of the function. This implies Schur-convexity [19, Proposition 2.8]. The inverse term is Schur-concave and the negative inverse term is Schur-convex again. Hence the function minimum *MSE* requirement $m(\mathbf{c})$ is Schur-convex with respect to $\mathbf{c}$. □

Remark 5. The smallest (best) guaranteed *MSE* is obtained for spatially uncorrelated channels and symmetrically distributed users. That means for OSTBC using n_T transmit and n_R receive antennas, the expectation in (5) of the effective channel for this upper bound incorporating the power $1/n_T$ per antenna is given by $\mathbb{E}[1/\alpha_k] = n_T/n_T n_R - 1$.

Remark 6. For scenarios in which the users have different spatial correlations or different QoS requirements, the impact of the user distribution is not as clear as in (23). Imagine a scenario in which one user has a much larger QoS requirement than all other users. Obviously, it is beneficial in terms of long-term transmit power if this user is closer to the base. Section 3.4. studies unequal QoS requirements and optimal user placements.

3.4. Optimal user placement with QoS requirements

Consider the case in which the *MSE* requirements $m_1, \ldots, m_K$ are fixed and known, but the user distribution $c_1, \ldots, c_K$ can be influenced under a total average path-loss constraint $\sum_{k=1}^{K} c_k = K$. Otherwise the optimal user placement is to place all users as close as possible to the BS. The objective is to minimize the total average transmit power at the base station. For convenience, define

$$\delta_k = \mathbb{E}\left[\frac{1}{\operatorname{tr}\left(\mathbf{T}_k \mathbf{W}_k^H \mathbf{R}_k \mathbf{W}_k\right)}\right](1 - m_k). \tag{24}$$

The programming problem that finds the optimal user placement which minimizes the average transmit power under *MSE* requirements is

$$\min_{c_1,\ldots,c_K} \sum_{k=1}^{K} \frac{\delta_k}{c_k} \quad \text{s.t.} \sum_{k=1}^{K} c_k = K, \; c_k \geq 0, \; 1 \leq k \leq K. \tag{25}$$

Lemma 1. *The optimal user placement solving* (25) *is given by*

$$c_k^* = K \frac{\sqrt{\delta_k}}{\sum_{l=1}^{K} \sqrt{\delta_l}} \tag{26}$$

and the corresponding condition for the guaranteed MSE region MSE reads*

$$\frac{\left(\sum_{k=1}^{K} \sqrt{\delta_k}\right)^2}{K} \leq \text{SNR}\left(1 - \sum_{k=1}^{K} (1 - m_k)\right). \tag{27}$$

Proof. The optimal user placement is found by the necessary Karush-Kuhn-Tucker optimality conditions [21]. The Lagrangian function with Lagrangian multiplier for sum constraint μ is given by

$$L(\mathbf{c}, \mu) = \sum_{k=1}^{K} \frac{\delta_k}{c_k} + \mu\left(\sum_{k=1}^{K} c_k - K\right). \tag{28}$$

Note that we do not need Lagrangian multipliers for the nonnegativeness constraint since the objective function itself acts as a barrier function. The first optimality condition gives

$$\frac{\partial L(\mathbf{c}, \mu)}{\partial c_l} = -\frac{\delta_l}{c_l^2} + \mu = 0 \Longrightarrow c_l^2 = \frac{\delta_l}{\mu} \Longrightarrow c_l^* = \sqrt{\frac{\delta_l}{\mu}} \tag{29}$$

which corresponds to (26). Note that μ is chosen such that $\sum_{k=1}^{K} c_k = K$. Insert the solution from (26) into (5) to obtain (27). □

Remark 7. Note that the region in (27) still shrinks with spatial correlation.

3.5. Effect of number of transmit and receive antennas on required SNR

Fix an *MSE* tuple $m_1, \ldots, m_K$ and assume the users have independent and identically distributed channels according to complex Gaussian, zero-mean with symmetrically distributed users $\mathbf{c} = \mathbf{1}$ and spatially uncorrelated channels $\mathbf{R}_k = \mathbf{I}$, $\mathbf{T}_k = \mathbf{I}$ for all $1 \leq k \leq K$. Then the required SNR reads

$$\text{SNR} \geq \left(\frac{n_T}{n_T n_R - 1}\right)\left(\frac{1}{1 - \sum_{k=1}^{K} (1 - m_k)} - 1\right). \tag{30}$$

For n_R approaching infinity, the first term on the RHS goes to zero. The impact of n_T in (30) is more complicated, since the code rate of the OSTBC depends on n_T, which tends to one half for n_T approaching infinity [14]. Note that the rate loss is characterized by [13] as

$$r_c(n_T) = \frac{m}{n_T} = \frac{\lfloor (n_T + 1)/2 \rfloor + 1}{2\lfloor (n_T + 1)/2 \rfloor}. \tag{31}$$

Note that the code rate in (31) is lower and upper bounded by

$$\frac{1}{2} + \frac{1}{n_T + 1} \leq r_c(n_T) \leq \frac{1}{2} + \frac{1}{n_T}. \tag{32}$$

On the one hand, increasing diversity has the positive effect on improving the first term on the RHS of (30), but also the negative effect by decreasing the code rate. This tradeoff is analyzed for single-user systems in [22]. Assume $r_1 = r_2 = \cdots = r_K = R$. From (16) it follows:

$$\text{SNR} \geq \frac{n_T}{n_T n_R - 1}\left(\frac{1}{1 - K\left(1 - 2^{-R/r_c(n_T)}\right)} - 1\right). \tag{33}$$

In (33), the first term on the RHS decreases with increasing n_T. The second term increases with increasing n_T.

For small rates R, the RHS in (33) can be approximated by the first term of the Taylor series expansion at $R = 0$ as

$$f(n_T) \approx \frac{n_T}{n_T - 1} \frac{K \log(2)}{r_c(n_T)}. \tag{34}$$

The first derivative of $f(n_T)$, with respect to, n_T is negative for the lower bound in (32) for $n_T \leq 6$ and for the

Table 1: Evaluation of (30) for $K = 2$ and rate $R = 0.1$.

n_T	2	3	4	5	6	7	8	9	10
SNR [dB]	− 5.10	− 4.93	− 5.438	− 5.12	− 5.29	− 5.09	− 5.18	− 5.03	− 5.08

upper bound in (32) for $n_T < 4$, respectively, and positive otherwise. This means that for small rates it does not help to increase the number of transmit antennas from two to four (or three to five). However, increasing the number of transmit antennas from six to eight (or seven to nine) improves performance. This is illustrated in Table 1.

3.6. Moment constraints

Additional moment constraints P_ℓ that limit the ℓth moment of the transmit power probability distribution specialize to the usual long-term power constraint with $\ell = 1$ and to peak power constraints with $\ell = \infty$. The moment constraint

$$\mathbb{E}[P_s^\ell] \leq P_\ell \tag{35}$$

lead to the following guaranteed *MSE* region:

$$\left(\frac{1}{1-K+\sum_{k=1}^K m_k}\right)^\ell \mathbb{E}\left[\left(\sum_{k=1}^K (1-m_k)\frac{1}{\rho\alpha_k}\right)^\ell\right] \leq P_\ell. \tag{36}$$

Note that for diversity systems, the expectation in (36) is finite only if $\ell + 1$ diversity branches, for example, transmit antennas are available [20].

3.7. Guaranteed MSE region with time-sharing

For the case in which time-sharing is used to satisfy the QoS requirements, we divide one fading block into K small subblocks of duration $\tau_k \geq 0$ such that $\sum_{k=1}^K \tau_k = 1$ [3, Section 3.3]. Time-sharing influences the achievable rates r_k to a fraction $\tau_k r_k$. However, it can be also applied if the performance is measured in terms of *MSE*. The longer the block, the smaller the resulting *MSE*. The connection between rate and *MSE* from (15) yields

$$\tau_k r_k = \tau_k \log\left(\frac{1}{m_k}\right) = \log\left(\frac{1}{m_k^{\tau_k}}\right). \tag{37}$$

The power allocated to user k in subblock k is p_k. Thus the sum power is given by $\sum_{k=1}^K \tau_k p_k$. In each subblock, only one user k is active. Therefore, (4) changes using (37) to

$$m_k = \left(\frac{1}{1+\rho\alpha_k p_k}\right)^{\tau_k}. \tag{38}$$

In order to satisfy the *MSE* constraints m_k, the instantaneous transmit power

$$p_k = \frac{m_k^{-1/\tau_k} - 1}{\rho\alpha_k} \tag{39}$$

is needed. The instantaneous sum power is given by

$$P_s = \sum_{k=1}^K \tau_k p_k = \sum_{k=1}^K \tau_k \left(\frac{m_k^{-1/\tau_k} - 1}{\rho\alpha_k}\right). \tag{40}$$

The optimal time-sharing parameters $\tau_1, \ldots, \tau_K$ are found by solving the programming problem

$$\min_{\tau_1,\ldots,\tau_K \geq 0} \sum_{k=1}^K \tau_k \left(\frac{m_k^{-1/\tau_k} - 1}{\rho\alpha_k}\right) \quad \text{s.t.} \sum_{k=1}^K \tau_k = 1. \tag{41}$$

The optimization problem in (41) is a convex optimization problem because the constraint set if a convex set and the objective function to be minimized is convex, that is, the second derivative with respect to τ_l is nonnegative,

$$\frac{\partial^2 \sum_{k=1}^K \tau_k \left((m_k^{-1/\tau_k} - 1)/\rho\alpha_k\right)}{\partial \tau_l^2} = \frac{m_l^{-1/\tau_l} \log(m_l)^2}{\tau_l^3 \rho\alpha_l} \geq 0. \tag{42}$$

Hence the programming problem in (41) can be solved efficiently by any convex optimization tool [21]. However, it can be simplified from a vector optimization problem to a simple scalar problem exploiting the Karush-Kuhn-Tucker (KKT) optimality conditions.

Theorem 4. *The optimal time-sharing parameter $\tau_1, \ldots, \tau_K$ can be found by solving first the scalar problem*

$$\sum_{k=1}^K \frac{\log(m_k)}{L_w((-1+\nu\alpha_k\rho)/e)+1} = -1 \tag{43}$$

with respect to ν and then compute for $1 \leq k \leq K$ the time-sharing parameter

$$\tau_k = -\frac{\log(m_k)}{L_w((-1+\nu\alpha_k\rho)/e)+1}, \tag{44}$$

where L_w is the Lambert-W function. The Lambert-W function, also called the omega function, is the inverse function of $f(W) = W \exp(W)$ [23].

Proof. Since the problem is convex and it has at least one feasible solution, we can use the necessary and sufficient KKT conditions in order to characterize the solution. Introduce the Lagrangian as follows:

$$L(\tau_1, \ldots, \tau_K, \nu) = \sum_{k=1}^K \tau_k \left(\frac{m_k^{-1/\tau_k} - 1}{\rho\alpha_k}\right) - \nu\left(1 - \sum_{k=1}^K \tau_k\right). \tag{45}$$

The set of KKT conditions is given for all $1 \le l \le K$ by

$$\frac{\tau_l m_l^{-1/\tau_l} - \tau_l + m_l^{-1/\tau_l} \log(m_l)}{\tau_l \rho \alpha_l} = -\nu,$$
$$\tau_l \ge 0,\ \nu > 0,\ \nu\left(1 - \sum_{k=1}^{K} \tau_k\right) = 0. \tag{46}$$

Solving the first KKT condition in (46) with respect to τ_l gives

$$\tau_l = -\frac{\log(m_l)}{L_w(-(\nu\rho\alpha_l + 1)/e) + 1}. \tag{47}$$

In order to fulfill the constraint that the sum of the time-sharing parameter is equal to one, ν has to solve (43) and (47) corresponds to (44). □

4. GUARANTEED *MSE* REGION WITH DIFFERENT TYPES OF CSI AND NONLINEAR PRECODING

In this section, we discuss three further scenarios. In the first case, the base station has still only knowledge about the channel norm, but can apply nonlinear precoding. In the second and third scenarios, we assume that the base station has perfect CSI and study the linear and nonlinear precoding case.

4.1. Guaranteed MSE region with superposition coding and SIC

If the users apply successive decoding without error propagation, the *MSE* of user k is given by

$$m_k = 1 - \frac{p_k \rho \alpha_k}{1 + \alpha_k \rho p_k + \alpha_k \rho \sum_{l \in \mathcal{S}_k} p_l} \tag{48}$$

with the interference set $\mathcal{S}_k$ containing all users not yet subtracted, that is,

$$\mathcal{S}_k(\alpha_1, \ldots, \alpha_K) = \{1 \le l \le K : \alpha_l > \alpha_k\}. \tag{49}$$

Sort the fading channel realizations by $\alpha_{\pi_1} > \alpha_{\pi_2} > \cdots > \alpha_{\pi_K}$. Denote the probability that a certain order π of all possible $K!$ orders occur by $p(\pi)$. The set of the $K!$ orders is denoted by $\mathcal{P}$. The function $\mathbf{1}(x)$ is the indicator function, that is, $\mathbf{1}(x) = 1$ if event x is true or $\mathbf{1}(x) = 0$ if event x is false.

Theorem 5. *For code division (CD) with successive decoding, the MSE tuple $m_1, \ldots, m_K$ can be guaranteed if*

$$\sum_{\pi \in \mathcal{P}} \mathbb{E}\Bigg[\mathbf{1}(\alpha_{\pi_1} > \alpha_{\pi_2} > \cdots > \alpha_{\pi_K}) \cdot \left[\frac{1}{\alpha_{\pi_K}}\left(\frac{1}{m_{\pi_K}} - 1\right) + \sum_{k=1}^{K-1} \frac{1}{\alpha_{\pi_k}}\left(\frac{1}{m_{\pi_k}} - 1\right) \prod_{l=k+1}^{K} \frac{1}{m_{\pi_l}}\right]\Bigg] \le \text{SNR}. \tag{50}$$

Proof. Assume that the channel realization to be ordered according to $\alpha_1 > \alpha_2 > \cdots > \alpha_K$. The cases that two or more realizations have equal power have zero probability. According to (48), the achievable *MSE* with power allocation p_k are given by

$$m_k = 1 - \frac{p_k \rho \alpha_k}{1 + \rho \alpha_k \sum_{l=1}^{k} p_l} \quad \text{for } 1 \le k \le K. \tag{51}$$

To support a certain *MSE* tuple $m_1, \ldots, m_K$, the transmit powers are

$$p_k = \left(\frac{1}{m_k} - 1\right)\left(\frac{1}{\rho \alpha_k} + \sum_{l=1}^{k-1} p_l\right) \quad \text{for } 1 \le k \le K. \tag{52}$$

The SNR is given by $\text{SNR} = \rho \mathbb{E} \sum_{k=1}^{K} p_k$, where the expectation is with respect to $\alpha_1, \ldots, \alpha_K$. Using (52) to compute the sum power and taking the average yields (50). Note that we compute the minimal transmit powers only for one decoding ordering when averaging. For all fading realizations, the indicator function chooses the optimal decoding order. □

4.1.1. Two-user scenario

Consider the two-user scenario and denote $s_1 = 1/m_1 - 1$ and $s_2 = 1/m_2 - 1$ and $w_1 = s_1(1/\alpha_1 + s_2/\alpha_2) + s_2/\alpha_2$ and $w_2 = s_2(1/\alpha_2 + s_1/\alpha_1) + s_1/\alpha_1$. Then the following *MSE* tuple m_1, m_2 can be supported (If α_1 and α_2 are independently distributed, the expression in (53) is further analyzed in [3]):

$$\text{SNR} \ge \int_{\alpha_2=0}^{\infty} \int_{\alpha_1=0}^{\alpha_2} w_1 p(\alpha_1, \alpha_2)\, d\alpha_1 d\alpha_2 + \int_{\alpha_1=0}^{\infty} \int_{\alpha_2=0}^{\alpha_1} w_2 p(\alpha_2, \alpha_1)\, d\alpha_2 d\alpha_1. \tag{53}$$

4.2. Perfect CSI and linear precoding without SIC

In Sections 4.2 and 4.3, we focus on the case in which the users have only single antennas because otherwise multistream transmission and optimization of a full rank transmit covariance matrix is required.

For the case in which the base station has perfect CSI and performs linear precoding for two users with single antennas, the optimal beamformers and power allocation is found according to [24, Section 4.3.2]. Define $a_1 = \|\mathbf{h}_1\|^2$, $a_2 = \|\mathbf{h}_2\|^2$, and $\chi = |\mathbf{h}_1^H \mathbf{h}_2|^2$. The average transmit power needed to support SINR requirements s_1, s_2 is given by

$$\mathbb{E}\left[\frac{-d_1 + \sqrt{d_1^2 + 4b_1c_1}}{2c_1}\right] + \mathbb{E}\left[\frac{-d_2 + \sqrt{d_2^2 + 4b_2c_2}}{2c_2}\right] \tag{54}$$

with $d_1 = a_1a_2(1+s_2-s_1-s_1s_2) + (s_1-s_2)\chi$, $b_1 = s_1a_2(1+s_2)$, $c_1 = a_1^2a_2(1+s_2) - (1+s_2)a_1\chi$, $d_2 = a_1a_2(1+s_1-s_2-s_1s_2) + (s_2-s_1)\chi$, $b_2 = s_2a_1(1+s_1)$, and $c_2 = a_2^2a_1(1+s_1) - (1+s_1)a_2\chi$.

Input: channel realizations $\mathbf{h}_1, \dots, \mathbf{h}_K$, feasible rate r_2.
For DPC order 2→1: required power to satisfy QoS-contraint (r_1, r_2) is given by $p_1 = (2^{r_1} - 1)/\rho\|\mathbf{h}_1\|^2$ and p_2 solves

$$r_1 + r_2 = \log\det\left(\mathbf{I} + \frac{2^{r_1} - 1}{\|\mathbf{h}_1\|^2}\mathbf{h}_1\mathbf{h}_1^H + \rho p_2 \mathbf{h}_2\mathbf{h}_2^H\right).$$

For DPC order 1→2: required power to satisfy QoS-contraint (r_1, r_2) is given by $q_2 = (2^{r_2} - 1)/\rho\|\mathbf{h}_2\|^2$ and q_1 solves

$$r_1 + r_2 = \log\det\left(\mathbf{I} + \frac{2^{r_2} - 1}{\|\mathbf{h}_2\|^2}\mathbf{h}_2\mathbf{h}_2^H + \rho q_1 \mathbf{h}_1\mathbf{h}_1^H\right).$$

Find r_1 such that $\mathbb{E}\,\min(p_1 + p_2, q_1 + q_2) = P$.
Output: rate tuple (r_1, r_2) which lies in the DLC region.

Algorithm 1: Compute the DLC region for 2-user MISO BC with perfect CSI and DPC.

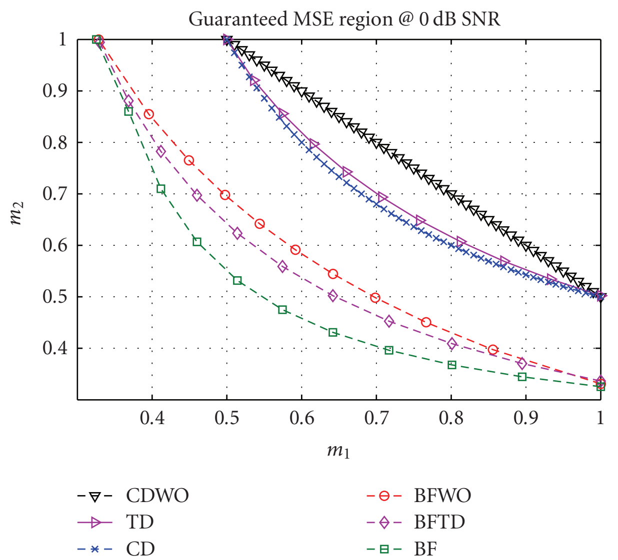

Figure 4: Guaranteed *MSE* region with and without superposition coding and with full collisions compared to perfect CSI and nonlinear and linear precoding with and without time-sharing for SNR 0 dB.

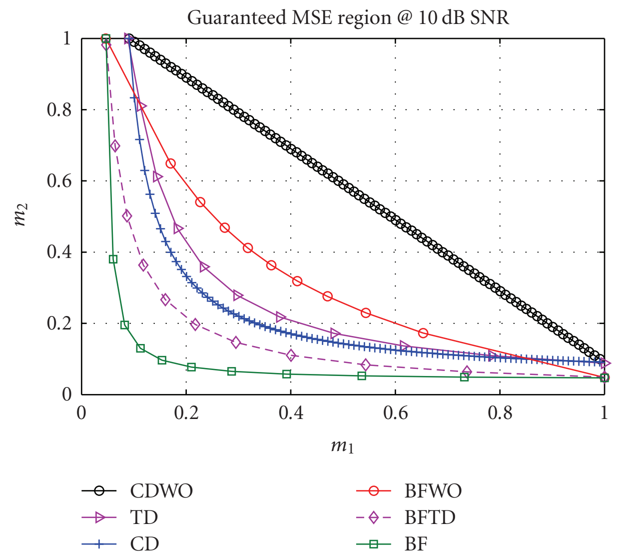

Figure 5: Guaranteed *MSE* region with and without superposition coding and with full collisions compared to perfect CSI and nonlinear and linear precoding with and without time-sharing for SNR 10 dB.

The expectation in (54) is with respect to a_1, a_2, and χ with the statistics of $\mathbf{h}_1$ and $\mathbf{h}_2$.

Based on the close relation of the SINR and *MSE* given in (15), the *MSE* requirements can be obtained immediately from the SINR requirements.

4.3. Perfect CSI and nonlinear precoding with SIC

For the case in which the base station has perfect CSI and performs nonlinear precoding for two users with single antennas, the DLC region is computed according to Algorithm 1.

Once the rate tuple is obtained by Algorithm 1, the *MSE* tuple can be computed using (15).

4.4. Perfect CSI and TD

For time-sharing, the only difference between the guaranteed QoS-region with norm feedback and perfect CSI is the beamforming gain of n_T. Therefore, the same approach as outlined in Section 3.7 can be used to compute the performance region.

5. ILLUSTRATIONS

5.1. Symmetric and spatially uncorrelated scenario

In Figure 4, the guaranteed *MSE* region using superposition coding with SIC (SC-SIC) and without SC-SIC is compared for the symmetric fading scenario and two transmit antennas $n_T = 2$ and long-term SNR 0 dB. The channels of the two users are spatially uncorrelated and both users have unit average channel power. In Figure 4, it can be observed that the largest guaranteed *MSE* region is achieved with perfect CSI and DPC (BF) closely followed by beamforming and time-sharing (BFTD). The beamforming without precoding and SIC (BFWO) is third best. Note that in low-SNR regime the beamforming gain is dominant and all three regions achieved by beamforming (perfect CSI) are larger than the regions achieved by norm feedback and OSTBC. For norm

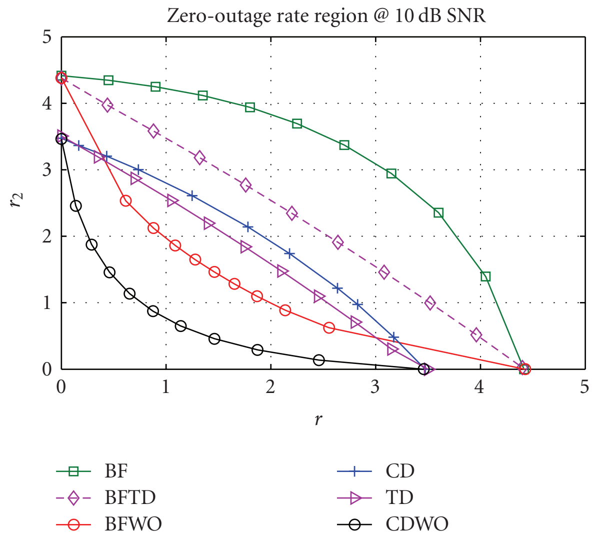

FIGURE 6: Zero-outage capacity region with and without superposition coding and with full collisions compared to perfect CSI and nonlinear and linear precoding with and without time-sharing for SNR 10 dB.

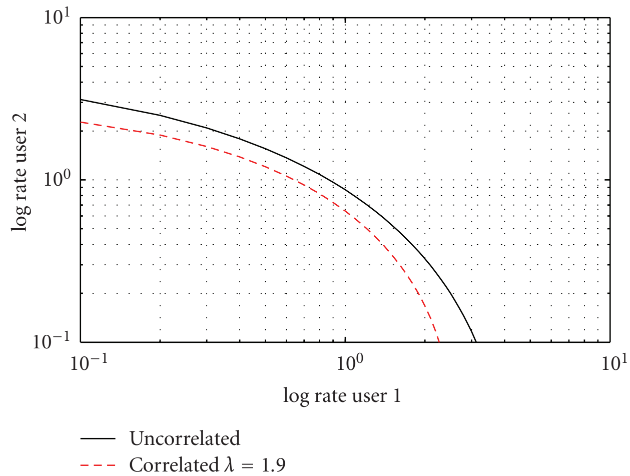

FIGURE 7: Zero-outage capacity region (CDWO) for MISO BC with two transmit antennas and two users for different correlation scenarios $\lambda = 1$ and $\lambda = 1.9$.

feedback, the largest region is obtained for superposition coding and SIC (CD) at the mobiles closely followed by time-sharing (TD) and finally without SIC (CDWO).

In Figure 5, the guaranteed *MSE* region using superposition coding and SIC and without SIC are compared for the symmetric fading scenario and two transmit antennas $n_T = 2$ and long-term SNR 10 dB. In Figure 5, it can be observed that the largest guaranteed *MSE* region is still achieved by BF closely followed by BFTD. Next, the order depends on the *MSE* requirements: for very asymmetrical *MSE* requirements, the beamforming gain dominates and BFWO is better than the norm feedback schemes (CD, TD, and CDWO). For more symmetrical *MSE* requirements, CD and TD outperform BFWO. CDWO has the smallest guaranteed *MSE* region.

The gain by superposition coding and SIC is visible especially for medium (and high) SNR in Figure 5. The corresponding zero-outage capacity region is convex for superposition coding and SIC, whereas it is concave without [3]. It can be observed that for small SNR, the beamforming gain weights more than the nonlinear precoding and BFWO as well as BF outperform CD and CDWO. However, for SNR of 10 dB, there is an intersection between the BFWO and the CD curve. The reason for this behavior is that the system gets interference limited rather than power limited for higher SNR.

In Figure 6, the delay-limited or zero-outage capacity region for the same scenario as in Figure 5 is shown. An interesting observation is that BFTD seems like standard time-sharing between the single-user rates, whereas CDTD is convex region. This is in agreement with the results from [3, Figure 3]. The reason for this behavior is that for larger rates (or small *MSE*s) the TD region approaches a straight line, whereas for small rates (or large *MSE*s) the TD region is more convex.

5.2. Impact of spatial correlation on CDWO

In Figure 7, the zero-outage rate region for two users and two transmit antennas with symmetric correlation for different scenarios is shown. Note that completely correlated transmit antennas lead to zero-outage capacity. The uncorrelated scenario leads to $\mathbb{E}[1/\alpha_1] = \mathbb{E}[1/\alpha_2] = 1$, whereas correlation λ increases this value to

$$\mathbb{E}\left[\frac{1}{\alpha_1}\right] = \mathbb{E}\left[\frac{1}{\alpha_2}\right] = \frac{\log(\lambda) - \log(2-\lambda)}{2\lambda - 2}. \quad (55)$$

In Figure 7, the impact of spatial correlation on the zero-outage rate region with CDWO can be observed. As predicted by Theorem 2, the region shrinks with increased correlation.

5.3. Optimal user placement in CDWO

In Figure 8, the guaranteed *MSE* region with CDWO is shown for SNR 0 dB and 10 dB with symmetric and optimal user placement from Section 3.4. Furthermore, the optimal user placement for the two user scenario as a function of m_1 with $m_2 = 1 - m_1$ is shown in the lower-left corner. It can be seen that only for very unequal *MSE* requirements, the user location is very different from the symmetrical state $c_1 = c_2 = 1$. This explains the improvement of the *MSE* at large m_1 and m_2 and the neglecting gain at medium *MSE*s.

6. CONCLUSION

The guaranteed *MSE* region of an orthogonal space-time block coded MIMO BC with normfeedback was characterized in closed form and the impact of fading statistics, user distribution, and number of transmit and receive antennas was analyzed. As a byproduct the DLC region was also completely characterized. Finally, a comparison to the perfect

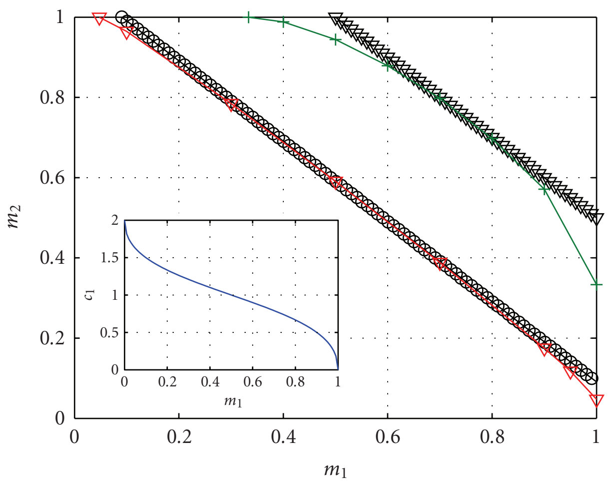

FIGURE 8: Guaranteed *MSE* region for symmetric and optimal user placement for SNR 0 dB and 10 dB in CDWO mode.

CSI and beamforming case with and without time-sharing was performed. The results indicate that in some scenarios SIC does matter more than perfect CSI.

Note that the results can be applied also to multibeam opportunistic beamforming systems with power allocation [25].

One open problem is about duality, that is, is the guaranteed *MSE* region of the BC with norm feedback under long-term sum power constraint equal to the guaranteed *MSE* region of the MAC with norm feedback under long-term sum power constraint. Recent results in [26] indicate that duality does not hold for all types of CSI.

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