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RESEARCH

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Robust resource allocation for cognitive relay networks with multiple primary users

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Abstract

A robust resource allocation (RA) algorithm for cognitive relay networks with multiple primary users considering joint channel uncertainty and interference uncertainty is proposed to maximize the capacity of the networks subject to the interference threshold limitations of primary users' receivers (PU-RXs) and the total power constraint of secondary user's transmitter and relays. Ellipsoid set and interval set are adopted to describe the uncertainty parameters. The robust relay selection and power allocation problems are separately formulated as semi-infinite programming (SIP) problems. With the worst-case approach, the SIP problems are transformed into equivalent convex optimization problems and solved by Lagrange dual decomposition method. Numerical results show the impact of channel uncertainties and validation of the proposed robust algorithm for strict guarantee the interference threshold requirements at different PU-RXs.

Keywords: Cognitive relay networks, Robust resource allocation, Channel and interference uncertainty, Worst-case approach, Lagrange dual decomposition method

1 Introduction

Cognitive radio (CR) is a promising technology to alleviate the looming spectrum scarcity crisis by opportunistic accessing the licensed spectrum [1]. In underlay mode of CR networks (CRNs), secondary users (SUs) can share the same radio spectrums allocated to primary users (PUs), provided that the interference to PUs' receivers (PU-RXs) generated by SUs' transmitters (SU-TXs) are below the specified interference thresholds [2]. At the same time, orthogonal frequency division multiplexing (OFDM) is widely accepted as a potential air interface physical layer technology for CRNs owing to its high flexibility for allocating radio resources [3]. In OFDM-based CRNs, SUs adopt OFDM modulation and cannot cause mutual interference. However, there exists the scenario of severe channel attenuation between the pair of SUs, and large power requirement for reliable data transmission can bring harmful interference to PU-RXs and degrade system performance in both CRNs and the licensed networks. Therefore, end to end transmission for conventional CRNs cannot satisfy the quality of service (QoS) requirements of PUs.

In order to increase spectrum efficiency, extend coverage area, and achieve spatial diversity gain, cooperative relay technique is introduced into CRNs [4–6]. Decode-and-forward (DF) [7] and amplify-and-forward (AF) [8] are most widely known relay transmission protocols. In DF case, relay decodes the signal received from cognitive source, encodes it, and forwards the regenerated signal to cognitive destination. As for AF protocol, relay amplifies the signal received from cognitive source with amplify factor and then forwards it to cognitive destination. With the help of cooperative relays, the transmit power of cognitive source is reduced and the interference to PU-RXs is mitigated, which can prohibit the performance degradation of primary network.

Resource allocation (RA) is a very important issue for CRNs with cooperative relays, which can further improve the system performance. Most existing literatures on RA for OFDM-based relay CRNs assume that perfect channel state information (CSI) is available [9, 10]. In [9], a joint subcarrier selection and power allocation scheme using variational inequality in OFDM-based cognitive relay networks is proposed to maximize the system throughput. In [10], an asymptotically optimal subcarrier pairing, relay

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selection, and power allocation scheme is obtained in DF relayed OFDM-based cognitive system under the individual power constraints in source and relays. These optimization problems with perfect CSI are referred as the nominal problems [11]. Due to time varying and random nature of wireless system, this assumption is invalid. Therefore, the solutions to nominal problems are not robust.

Robust optimization theory has been applied in CRNs to overcome uncertainty parameters in nominal problems in recent years [12, 13]. Worst-case approach [14] and Bayesian approach [15] are widely used in robust optimization problems. For worst-case approach, uncertainty parameters belong to a predefined uncertainty region, so that this approach can strictly guarantee QoS of PU-RXs for all realizations of the uncertainty in that region. Currently, most works on worst-case approach focus on how to find a proper set to describe the characteristics of the uncertainty parameters. However, there is no unified way to define an uncertainty set, and the main principle is to preserve the convexity of the original nominal problem and obtain a solvable optimization problem. In [16], the authors study robust beamforming and power allocation problem in cognitive relay networks with imperfect CSI modeled by Gaussian random variables. In [17], the robust ergodic uplink resource allocation problem for secure communication with imperfect CSI in relay-assisted CRNs is investigated. The channel uncertainty set is represented by an ellipsoid uncertainty region. In [18], the authors study robust worst-case interference control and use general norm and polyhedron model to describe channel uncertainty set in underlay CRNs. Bayesian approach handles uncertainty parameters with certain statistical properties and satisfies QoS of PU-RXs in a probabilistic manner. Generally, worst-case approach is more appealing since the interference to PU-RXs should be below the interference threshold under any circumstances, even in the presence of the worst-case scenario.

Recently, some research works have been conducted on robust RA in cognitive relay networks considering the impacts of uncertainty parameters [19–21]. In [19], the authors propose a joint relay pre-coder and power allocation design to minimize the sum mean-square error (MSE) of the transceiver node with channel uncertainties between SU and SU. In [20], the outage performance of cognitive relay networks is analyzed with the channel uncertainties between SU and PU. In [21], robust resource allocation problem for cooperative CRNs is optimized with the channel uncertainties between SU and SU, SU and PU. However, the interference uncertainty is not considered in these works.

Different from most existing literatures, this paper investigates the robust RA problem for OFDM-based cognitive relay network with multiple PUs, where channel uncertainty and interference uncertainty are all considered. Ellipsoid set and interval set are jointly used to characterize the uncertainty region sets. Under the total power budget constraint of SU-TX and relays, the optimization objective is to maximize capacity of the cognitive relay network while maintaining the interference to PU-RXs under their prescribed thresholds. The major contributions of this work are as follows.

1. Robust RA problem is solved through robust relay selection and power allocation separately under the consideration of both channel uncertainty and interference uncertainty characterized by ellipsoid set and interval set.
2. Robust relay selection and power allocation problems are formulated as semi-infinite programming (SIP) problems. And they are converted into equivalent convex optimization by worst-case approach and solved by Lagrange dual decomposition method.
3. Closed form analytical solutions to robust relay selection and power allocation have been derived. Numerical results show that the proposed robust algorithm outperforms the non-robust ones for ensuring QoS of multiple PU-RXs under the influence of the uncertainties.

The rest of this paper is organized as follows. The system model and nominal RA problem formulation are described in Section 2. The robust RA problem is proposed and solved in Section 3. The performance of the proposed robust algorithm is illustrated with simulation results in Section 4. Finally, conclusions are drawn and future works are given in Section 5.

2 System model and nominal RA problem formulation

2.1 System model

We consider an OFDM-based cognitive relay network with a primary network as shown in Fig. 1. The related symbol explanations are given in Table 1. It is assumed that there is no direct link between the cognitive source (S) and the cognitive destination (D), so that S and D communicate with each other through multiple relays. We also assume that there are K relays and N subcarriers. The relays operate in time division half-duplex mode with DF protocol. Signal transmissions are conducted in two time slots. In the first time slot, S transmits signals to relay node $R_k, k \in [1:K]$. While in the second time slot, R_k forwards the regenerated signals to D through the subcarrier which has the same sequence number as the first time slot. The capacity of the i^{th}, $i \in [1:N]$ subcarrier in the two time slots can be expressed as

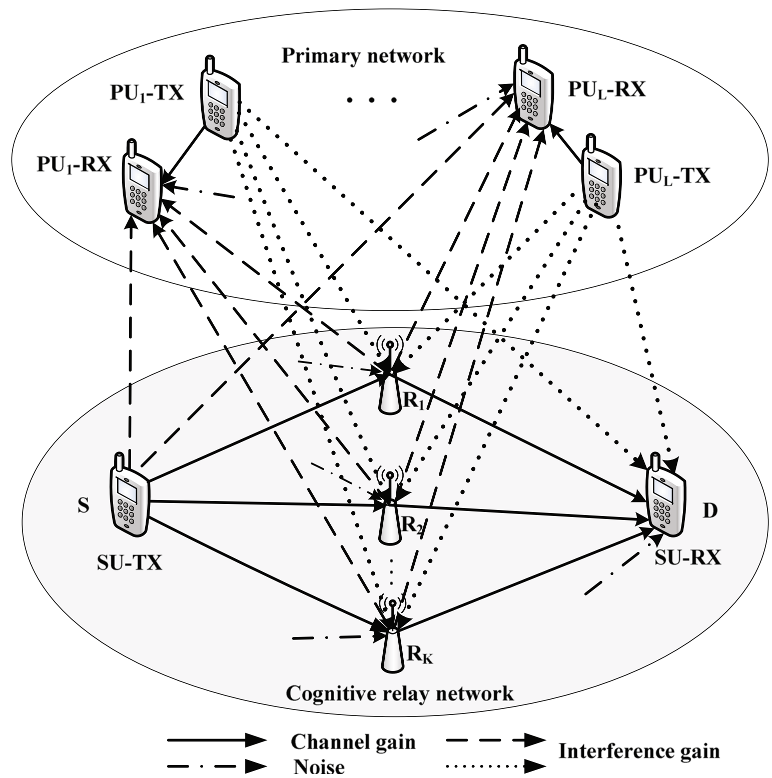

Fig. 1 System model

$$C_{SR_k}^i = \frac{1}{2}\log_2\left(1+\frac{\left|h_{SR_k}^i\right|^2 P_{SR_k}^i}{\sigma_{ik}^2+\sum_{l=1}^{L} J_{ik}^l}\right) \quad (1)$$

$$C_{R_kD}^i = \frac{1}{2}\log_2\left(1+\frac{\left|h_{R_kD}^i\right|^2 P_{R_kD}^i}{\sigma_{iD}^2+\sum_{l=1}^{L} J_{iD}^l}\right) \quad (2)$$

Table 1 Symbol introduction

Symbol	Specification
$h_{SR_k}^i$	Channel gain of the i^{th} subcarrier transmitted on SR_k link
$h_{R_kD}^i$	Channel gain of the i^{th} subcarrier transmitted on R_kD link
$P_{SR_k}^i$	Transmit power of the i^{th} subcarrier transmitted on SR_k link
$P_{R_kD}^i$	Transmit power of the i^{th} subcarrier transmitted on R_kD link
$h_{SP_l}^i$	Channel gain of the i^{th} subcarrier transmitted from S to the l^{th} PU-RX (SP_l link)
$h_{R_kP_l}^i$	Channel gain of the i^{th} subcarrier transmitted from R_k to the l^{th} PU-RX (R_kP_l link)
σ_{ik}^2	Noise power of the i^{th} subcarrier transmitted on SR_k link
σ_{iD}^2	Noise power of the i^{th} subcarrier transmitted on R_kD link
J_{ik}^l	Interference power to the i^{th} subcarrier transmitted on SR_k link caused by the l^{th} PU-TX
J_{iD}^l	Interference power to the i^{th} subcarrier transmitted on R_kD link caused by the l^{th} PU-TX

where $C_{SR_k}^i$ and $C_{R_kD}^i$ represent the capacity of the i^{th} subcarrier transmitted from S to R_k (SR_k link) and from R_k to D (R_kD link), respectively. L is the number of primary users. J_{ik}^l and J_{iD}^l can be modeled as the additive white Gaussian noise (AWGN) described in [22]. In order to simplify mathematical analysis, we assume that the interference power from PU-TXs plus the noise power to the i^{th} subcarrier in the first time slot and the second time slot are equal, which can be denoted by $\sigma_i^2 = \sigma_{ik}^2 + \sum_{l=1}^{L} J_{ik}^l = \sigma_{iD}^2 + \sum_{l=1}^{L} J_{iD}^l$ as described in [23]. In fact, the achievable rate of each subcarrier is the minimum rate of the two time slots. Therefore, the capacity of the i^{th} subcarrier for the cognitive relay network can be given as

$$C_k^i = \min\left\{C_{SR_k}^i, C_{R_kD}^i\right\} \quad (3)$$

As for DF protocol, C_k^i can achieve maximum capacity when the signal to interference plus noise ratio (SINR) at R_k equals to that at D, then we can have the following relation for the i^{th} subcarrier as [23]

$$a_k^i P_{SR_k}^i = b_k^i P_{R_kD}^i \quad (4)$$

where $a_k^i = \frac{\left|h_{SR_k}^i\right|^2}{\sigma_i^2}, b_k^i = \frac{\left|h_{R_kD}^i\right|^2}{\sigma_i^2}$. We define $H_k^i = \frac{\left|h_{SR_k}^i\right|^2\left|h_{R_kD}^i\right|^2}{\left|h_{SR_k}^i\right|^2+\left|h_{R_kD}^i\right|^2}$ and $\alpha_k^i = \frac{H_k^i}{\sigma_i^2}$ to represent the normalized channel gain and the equivalent channel gain of the i^{th} subcarrier in the two time slots, respectively. The total transmit power on the i^{th} subcarrier in the two time slots is

$$P_k^i = P_{SR_k}^i + P_{R_kD}^i \tag{5}$$

According to (4) and (5), the maximum capacity of the i^{th} subcarrier can be rewritten as

$$C_k^i = \frac{1}{2}\log_2\left(1+\alpha_k^i P_k^i\right) \tag{6}$$

Note that in cognitive relay network, both SU-TX and relays have to take the interference thresholds of different PU-RXs into account. Therefore, the interferences introduced to the l^{th} PU-RX in the two time slots are

$$I_{SP_l} = \sum_{k=1}^{K}\sum_{i=1}^{N} \rho_k^i P_{SR_k}^i \left|h_{SP_l}^i\right|^2 \le I_{th}^l, \forall l \tag{7}$$

$$I_{RP_l} = \sum_{k=1}^{K}\sum_{i=1}^{N} \rho_k^i P_{R_kD}^i \left|h_{R_kP_l}^i\right|^2 \le I_{th}^l, \forall l \tag{8}$$

where I_{SP_l} and I_{RP_l} are the interference to the l^{th} PU-RX from the total subcarriers transmitted between S and all relays (SR link) and between all relays and D (RD link), respectively. ρ_k^i is a binary factor indicating whether the i^{th} subcarrier is allocated to R_k. And I_{th}^l is the interference threshold of the l^{th} PU-RX.

According to the definition of H_k^i and P_k^i, (7) and (8) can be rewritten as

$$I_{SP_l} = \sum_{k=1}^{K}\sum_{i=1}^{N} \rho_k^i P_k^i H_k^i G_{S,k,l}^i \le I_{th}^l, \forall l \tag{9}$$

$$I_{RP_l} = \sum_{k=1}^{K}\sum_{i=1}^{N} \rho_k^i P_k^i H_k^i G_{k,l,D}^i \le I_{th}^l, \forall l \tag{10}$$

where $G_{S,k,l}^i = \frac{\left|h_{SP_l}^i\right|^2}{\left|h_{SR_k}^i\right|^2}$ is defined to represent the normalized channel gain of the i^{th} subcarrier for SP_l link and SR_k link, and $G_{k,l,D}^i = \frac{\left|h_{R_kP_l}^i\right|^2}{\left|h_{R_kD}^i\right|^2}$ is defined to describe the normalized channel gain of the i^{th} subcarrier for R_kP_l link and R_kD link.

2.2 Nominal RA problem formulation

The optimization objective is to maximize the capacity of the cognitive relay network while maintaining the interference to PU-RXs below their interference thresholds under the total power constraint. Mathematically, this optimization can be formulated as

Nominal RA problem (P0):

$$\max_{\rho_k^i,\, P_k^i \geqslant 0} \sum_{k=1}^{K}\sum_{i=1}^{N} \rho_k^i \frac{1}{2}\log_2\left(1+\alpha_k^i P_k^i\right) \tag{11}$$

subject to

$$\begin{aligned}
&C1: I_{SP_l} \le I_{th}^l, \forall l \\
&C2: I_{RP_l} \le I_{th}^l, \forall l \\
&C3: \textstyle\sum_{k=1}^{K}\sum_{i=1}^{N} \rho_k^i P_k^i \le P_{\text{total}} \\
&C4: \textstyle\sum_{k=1}^{K} \rho_k^i = 1, \forall i \\
&C5: \rho_k^i \in \{0,1\}, \forall k,\, i
\end{aligned} \tag{12}$$

where $C1$ and $C2$ are the interference constraints of the l^{th} PU-RX in the first time slot and the second time slot, respectively. $C3$ is the transmit power constraint, and P_{total} is the maximum total transmit power. $C4$ and $C5$ are the relay selection constraints to indicate that only one relay is selected by each subcarrier to guarantee the exclusiveness of subcarrier. If $\rho_k^i = 1$, R_k is allocated to the i^{th} subcarrier, otherwise not.

Apparently, **P0** is a mixed binary integer programming problem, and it is difficult to achieve a joint relay selection and power allocation solution. Therefore, we will solve this problem by dividing **P0** into two sub-problems, i.e., relay selection and power allocation separately. Firstly, we allocate subcarriers to relays through supposing uniform power distribution over subcarriers (i.e., $P_k^i = \frac{P_{\text{total}}}{N}$). Secondly, power allocation is carried out under the given relay selection scheme. Note that relay selection scheme needs to solve a discrete optimization problem, which is NP-hard. Thus, we relax the integrality constraint on ρ_k^i from $\rho_k^i \in \{0,1\}$ to $\rho_k^i \in (0,1]$ and relax $\sum_{k=1}^{K} \rho_k^i = 1$ to $\sum_{k=1}^{K} \rho_k^i \le 1$, which can transform the discrete optimization problem into a continuous linear optimization problem [24].

Nominal relay selection problem (P1):

$$\max_{\rho_k^i} \sum_{k=1}^{K}\sum_{i=1}^{N} \frac{1}{2}\log_2\left(1+\frac{\rho_k^i \alpha_k^i P_{\text{total}}}{N}\right) \tag{13}$$

subject to

$$\begin{aligned}
&C1: \textstyle\sum_{k=1}^{K}\sum_{i=1}^{N} \rho_k^i H_k^i G_{S,k,l}^i P_{\text{total}} \le N I_{th}^l, \forall l \\
&C2: \textstyle\sum_{k=1}^{K}\sum_{i=1}^{N} \rho_k^i H_k^i G_{k,l,D}^i P_{\text{total}} \le N I_{th}^l, \forall l \\
&C3: \textstyle\sum_{k=1}^{K} \rho_k^i \le 1,\ \forall i \\
&C4: \rho_k^i \in (0,1], \forall k,\, i
\end{aligned} \tag{14}$$

This time **P1** is a convex problem, and we can obtain an optimal solution to ρ_k^i after applying Karush-Kuhn-Tucker (KKT) conditions [24]. Therefore, R_k with maximum ρ_k^i is allocated to the i^{th} subcarrier, which can be expressed as

$$K(i) = arg\max_{k} \rho_k^i, \forall i \tag{15}$$

where $K(i) \in [1:K]$ presents the relay selected by the i^{th} subcarrier. Under this relay selection scheme, **P0** can be converted into

Nominal power allocation problem (P2):

$$\max_{P^i_{K(i)}\geqslant 0} \sum_{i=1}^{N} \frac{1}{2}\log_2\left(1+\alpha^i_{K(i)}P^i_{K(i)}\right) \tag{16}$$

subject to

$$\begin{aligned} &C1: \textstyle\sum_{i=1}^{N} H^i_{K(i)}G^i_{S,K(i),l}P^i_{K(i)} \le I^l_{th}, \forall l\\ &C2: \textstyle\sum_{i=1}^{N} H^i_{K(i)}G^i_{K(i),l,D}P^i_{K(i)} \le I^l_{th}, \forall l\\ &C3: \textstyle\sum_{i=1}^{N} P^i_{K(i)} \le P_{\text{total}} \end{aligned} \tag{17}$$

P2 is also a convex optimization problem with linear constraints so that an optimal power allocation analytical solution can also be obtained by using KKT conditions. We define this nominal relay selection and power allocation algorithm under the assumption of the perfect CSI as non-robust algorithm.

3 Robust RA algorithm

In cognitive relay networks, channel gain is very important information for RA. However, perfect channel gain is extremely difficult to obtain due to estimation errors, quantization errors, and feedback delays. The interference power at relay and SU-RX are also inaccuracy due to the measurement errors. The assumption of perfect CSI is so idealistic that it can lead to system performance degradation. In this paper, we study robust RA problem with joint channel uncertainty and interference uncertainty. Based on the robust optimization theory, the uncertainty parameter is modeled by the sum of the estimated value (i.e., nominal value) and additive error (i.e., perturbation part). Mathematically, the uncertainty parameter can be expressed as $g = \bar{g} + \Delta g$, where g is the actual value, $\bar{g}$ is the estimated value, and Δg is the estimated error. We adopt ellipsoid set to describe the channel uncertainty and interval set to describe interference uncertainty respectively. According to the definition of the ellipsoid uncertainty, H^i_k, $G^i_{S,k,l}$, and $G^i_{k,l,D}$ can be formulated as

$$\mathcal{H} = \left\{ H^i_k \middle| \bar{H}^i_k + \Delta H^i_k : \sum_{i=1}^{N} \left|\Delta H^i_k\right|^2 \le \varepsilon^2_l, \forall k, \forall l \right\} \tag{18}$$

$$\mathcal{G} = \left\{ G^i_{S,k,l} \middle| \bar{G}^i_{S,k,l} + \Delta G^i_{S,k,l} : \sum_{i=1}^{N} \left|\Delta G^i_{S,k,l}\right|^2 \le \eta^2_l, \forall k, \forall l \right\} \tag{19}$$

$$\mathcal{F} = \left\{ G^i_{k,l,D} \middle| \bar{G}^i_{k,l,D} + \Delta G^i_{k,l,D} : \sum_{i=1}^{N} \left|\Delta G^i_{k,l,D}\right|^2 \le \delta^2_l, \forall k, \forall l \right\} \tag{20}$$

where $\mathcal{H}$, $\mathcal{G}$, and $\mathcal{F}$ denote the channel uncertainty sets. $\varepsilon_l \in [0,1)$, $\eta_l \in [0,1)$, and $\delta_l \in [0,1)$ are the maximum deviations of H^i_k, $G^i_{S,k,l}$, and $G^i_{k,l,D}$, respectively. We define ψ as the uncertainty set of α^i_k. Based on the definition of the interval uncertainty, α^i_k can be formulated as

$$\psi = \left\{ \alpha^i_k \middle| \bar{\alpha}^i_k + \Delta\alpha^i_k : \left|\Delta\alpha^i_k\right| \le \xi^i_k \bar{\alpha}^i_k \right\} \tag{21}$$

where $\xi^i_k \in [0,1)$ is the upper bound of the uncertainty that determines the size of the uncertainty region. Afterwards, we study robust RA problem with these uncertainty parameters. According to (18)–(21), **P1** can be transformed as

Robust relay selection problem (P3):

$$\max_{\rho^i_k} \sum_{k=1}^{K}\sum_{i=1}^{N} \frac{1}{2}\log_2\left(1+\frac{\rho^i_k P_{\text{total}}\left(\bar{\alpha}^i_k + \Delta\alpha^i_k\right)}{N}\right) \tag{22}$$

subject to

$$\begin{aligned} &C1: \textstyle\sum_{k=1}^{K}\sum_{i=1}^{N} \rho^i_k P_{\text{total}}\left(\bar{H}^i_k + \Delta H^i_k\right)\left(\bar{G}^i_{S,k,l} + \Delta G^i_{S,k,l}\right) \le NI^l_{th}, \forall l\\ &C2: \textstyle\sum_{k=1}^{K}\sum_{i=1}^{N} \rho^i_k P_{\text{total}}\left(\bar{H}^i_k + \Delta H^i_k\right)\left(\bar{G}^i_{k,l,D} + \Delta G^i_{k,l,D}\right) \le NI^l_{th}, \forall l\\ &C3: \textstyle\sum_{k=1}^{K} \rho^i_k \le 1, \forall i\\ &C4: \rho^i_k \in (0,1], \forall k, i\\ &C5: H^i_k \in \mathcal{H},\ G^i_{S,k,l} \in \mathcal{G},\ G^i_{k,l,D} \in \mathcal{F},\ \alpha^i_k \in \psi \end{aligned} \tag{23}$$

P3 is a SIP problem [25] subject to an infinite number of constraints with respect to the sets of $\mathcal{H}$, $\mathcal{G}$, $\mathcal{F}$, and ψ. Through worst-case approach, we can make all the constraints be in the worst-case scenario, and **P3** is transformed into an equivalent problem with finite constraints. We first transform (22) by $\Delta\alpha^i_k = -\xi^i_k\bar{\alpha}^i_k$, which denotes the worst-case equivalent channel gain. Then, according to the Cauchy-Schwartz inequality [26], we have

$$\begin{aligned} \max_{H^i_k\in\mathcal{H}} \sum_{i=1}^{N}\left(\bar{H}^i_k + \Delta H^i_k\right)\rho^i_k &= \sum_{i=1}^{N}\bar{H}^i_k\rho^i_k + \max_{\sum_{i=1}^{N}\left|\Delta H^i_k\right|^2\le\varepsilon^2_l} \sum_{i=1}^{N}\Delta H^i_k\rho^i_k\\ &\le \sum_{i=1}^{N}\bar{H}^i_k\rho^i_k + \sqrt{\sum_{i=1}^{N}\left(\rho^i_k\right)^2}\sqrt{\sum_{i=1}^{N}|\Delta H^i_k|^2}\\ &= \sum_{i=1}^{N}\bar{H}^i_k\rho^i_k + \varepsilon_l\sum_{i=1}^{N}\rho^i_k,\ \forall k, \forall l \end{aligned} \tag{24}$$

Similarity, $C1$ and $C2$ in **P3** can be further formulated as

$$C1: \sum_{k=1}^{K}\sum_{i=1}^{N} \rho^i_k P_{\text{total}}\left(\bar{H}^i_k + \varepsilon_l\right)\left(\bar{G}^i_{S,k,l} + \eta_l\right) \le NI^l_{th}, \forall l \tag{25}$$

$$C2: \sum_{k=1}^{K}\sum_{i=1}^{N} \rho^i_k P_{\text{total}}\left(\bar{H}^i_k + \varepsilon_l\right)\left(\bar{G}^i_{k,l,D} + \delta_l\right) \le NI^l_{th}, \forall l \tag{26}$$

We can see that the transformed **P3** is convex and it can be solved by the Lagrange dual decomposition method [27]. The optimal robust relay selection factor ρ_k^i can be derived in the form of

$$\rho_k^{i*} = \left[\frac{1}{2\ln 2\left(\mathcal{A}_1 + \mathcal{A}_2 + \lambda_i\right)} - \frac{N}{\bar{\alpha}_k^i\left(1-\xi_k^i\right)P_{\text{total}}} \right]^+, \forall k \tag{27}$$

where

$$\mathcal{A}_1 = \sum_{l=1}^{L} \varphi_l P_{\text{total}}\left(\bar{H}_k^i + \varepsilon_l\right)\left(\bar{G}_{S,k,l}^i + \eta_l\right)$$

$$\mathcal{A}_2 = \sum_{l=1}^{L} \beta_l P_{\text{total}}\left(\bar{H}_k^i + \varepsilon_l\right)\left(\bar{G}_{k,l,D}^i + \delta_l\right)$$

$[x]^+ \triangleq \max(0,x)$. $\varphi_l \geqslant 0$, $\beta_l \geqslant 0$ and $\lambda_i \geqslant 0$ are the Lagrange multipliers updated by the sub-gradient method with recursive forms

$$\begin{aligned}\varphi_l(t+1) = &\left[\varphi_l(t) - s_1^l\left(NI_{th}^l - \textstyle\sum_{k=1}^{K}\sum_{i=1}^{N}\rho_k^i P_{\text{total}}\left(\bar{H}_k^i + \varepsilon_l\right)\right.\right.\\ &\left.\left.\left(\bar{G}_{S,k,l}^i + \eta_l\right)\right)\right]^+, \forall l\end{aligned} \tag{28}$$

$$\begin{aligned}\beta_l(t+1) = &\left[\beta_l(t) - s_2^l\left(NI_{th}^l - \textstyle\sum_{k=1}^{K}\sum_{i=1}^{N}\rho_k^i P_{\text{total}}\right.\right.\\ &\left.\left.\left(\bar{H}_k^i + \varepsilon_l\right)\left(\bar{G}_{k,l,D}^i + \delta_l\right)\right)\right]^+, \forall l\end{aligned} \tag{29}$$

$$\lambda_i(t+1) = \left[\lambda_i(t) - s_3^i\left(1 - \sum_{k=1}^{K}\rho_k^i\right)\right]^+, \forall i \tag{30}$$

where t is the iteration number, s_1^l ,s_2^l, and s_3^i are the small positive step sizes. In the sequel, we optimize the robust power allocation problem after obtaining the robust relay selection factor.

Robust power allocation problem (P4):

$$\max_{P_{K(i)}^i \geqslant 0} \sum_{i=1}^{N} \frac{1}{2}\log_2\left(1 + P_{K(i)}^i\left(\bar{\alpha}_{K(i)}^i + \Delta\alpha_{K(i)}^i\right)\right) \tag{31}$$

subject to

$$\begin{aligned}&C1: \textstyle\sum_{i=1}^{N}\left(\bar{H}_{K(i)}^i + \Delta H_{K(i)}^i\right)\left(\bar{G}_{S,K(i),l}^i + \Delta G_{S,K(i),l}^i\right)P_{K(i)}^i \le I_{th}^l, \forall l\\ &C2: \textstyle\sum_{i=1}^{N}\left(\bar{H}_{K(i)}^i + \Delta H_{K(i)}^i\right)\left(\bar{G}_{K(i),l,D}^i + \Delta G_{K(i),l,D}^i\right)P_{K(i)}^i \le I_{th}^l, \forall l\\ &C3: \textstyle\sum_{i=1}^{N} P_{K(i)}^i \le P_{\text{total}}\\ &C4: H_{K(i)}^i \in \mathcal{H},\ G_{S,K(i),l}^i \in \mathcal{G},\ G_{K(i),l,D}^i \in \mathcal{F},\ \alpha_{K(i)}^i \in \psi\end{aligned} \tag{32}$$

Obviously, **P4** is still a SIP problem, so we need to transform it into an equivalent optimization problem limited by the finite constraints with the worst-case approach. First, (31) can be reformulated as $\max_{P_{K(i)}^i \geqslant 0} \sum_{i=1}^{N}\frac{1}{2}\log_2\left(1 + P_{K(i)}^i\bar{\alpha}_{K(i)}^i\left(1-\xi_{K(i)}^i\right)\right)$. Then $C1$ and $C2$ in **P4** can be formulated as

$$C1: \sum_{i=1}^{N}\left(\bar{H}_{K(i)}^i + \varepsilon_l\right)\left(\bar{G}_{S,K(i),l}^i + \eta_l\right)P_{K(i)}^i \le I_{th}^l, \forall l \tag{33}$$

$$C2: \sum_{i=1}^{N}\left(\bar{H}_{K(i)}^i + \varepsilon_l\right)\left(\bar{G}_{K(i),l,D}^i + \delta_l\right)P_{K(i)}^i \le I_{th}^l, \forall l \tag{34}$$

We can see that the transformed **P4** is also convex and its Lagrange function is

$$\begin{aligned}\mathcal{L}\left(\left\{P_{K(i)}^i\right\},\{\mu_l\},\{\nu_l\},\omega\right) = &\sum_{i=1}^{N}\frac{1}{2}\log_2\left(1 + P_{K(i)}^i\bar{\alpha}_{K(i)}^i\left(1-\xi_{K(i)}^i\right)\right)\\ &+\sum_{l=1}^{L}\mu_l\left(I_{th}^l - \sum_{i=1}^{N}\left(\bar{H}_{K(i)}^i + \varepsilon_l\right)\left(\bar{G}_{S,K(i),l}^i + \eta_l\right)P_{K(i)}^i\right)\\ &+\sum_{l=1}^{L}\nu_l\left(I_{th}^l - \sum_{i=1}^{N}\left(\bar{H}_{K(i)}^i + \varepsilon_l\right)\left(\bar{G}_{K(i),l,D}^i + \delta_l\right)P_{K(i)}^i\right)\\ &+\omega\left(P_{\text{total}} - \sum_{i=1}^{N}P_{K(i)}^i\right)\end{aligned} \tag{35}$$

where $\mu_l \geqslant 0$, $\nu_l \geqslant 0$, and $\omega \geqslant 0$ are the Lagrange multipliers. (35) can be solved by the dual decomposition method, i.e.,

$$\min_{\mu_l,\nu_l,\omega} \tilde{g}\left(\{\mu_l\},\{\nu_l\},\omega\right) \tag{36}$$

The dual function of (35) is defined as

$$\tilde{g}(\{\mu_l\},\{\nu_l\},\omega) \triangleq \max_{P_{K(i)}^i \geqslant 0} \mathcal{L}(\{P_{K(i)}^i\},\{\mu_l\},\{\nu_l\},\omega) \tag{37}$$

Substituting (35) to (37), we can get

$$\tilde{g}\left(\{\mu_l\},\{\nu_l\},\omega\right) = \max_{P_{K(i)}^i \geqslant 0}\left[D_0 + \sum_{i=1}^{N} D\left(P_{K(i)}^i\right)\right] \tag{38}$$

where

$$D_0 = \sum_{l=1}^{L}\left(\mu_l + \nu_l\right)I_{th}^l + \omega P_{\text{total}} \tag{39}$$

$$\begin{aligned}D\left(P_{K(i)}^i\right) = &\frac{1}{2}\log_2\left(1 + P_{K(i)}^i\bar{\alpha}_{K(i)}^i\left(1-\xi_{K(i)}^i\right)\right)\\ &-\sum_{l=1}^{L}\mu_l\left(\bar{H}_{K(i)}^i + \varepsilon_l\right)\left(\bar{G}_{S,K(i),l}^i + \eta_l\right)P_{K(i)}^i\\ &-\sum_{l=1}^{L}\nu_l\left(\bar{H}_{K(i)}^i + \varepsilon_l\right)\left(\bar{G}_{K(i),l,D}^i + \delta_l\right)P_{K(i)}^i - \omega P_{K(i)}^i\end{aligned} \tag{40}$$

We can get the optimal solution of (38) though optimizing (40) by solving $\frac{\partial D(P^i_{K(i)})}{\partial P^i_{K(i)}} = 0$, the optimal robust power allocation is derived in the form of

$$P^i_{K(i)}{}^* = \left[\frac{1}{2\ln 2\left(\mathcal{B}_1 + \mathcal{B}_2 + \omega\right)} - \frac{1}{\bar{\alpha}^i_{K(i)}\left(1 - \xi^i_{K(i)}\right)} \right]^+ \tag{41}$$

where

$$\mathcal{B}_1 = \sum_{l=1}^{L} \mu_l \left(\bar{H}^i_{K(i)} + \varepsilon_l\right)\left(\bar{G}^i_{S,K(i),l} + \eta_l\right)$$

$$\mathcal{B}_2 = \sum_{l=1}^{L} \nu_l \left(\bar{H}^i_{K(i)} + \varepsilon_l\right)\left(\bar{G}^i_{K(i),l,D} + \delta_l\right)$$

The Lagrange multipliers can be updated by the sub-gradient method as

$$\mu_l(t+1) = \left[\mu_l(t) - s^l_4\left(I^l_{th} - \sum_{i=1}^{N}\left(\bar{H}^i_{K(i)} + \varepsilon_l\right)\left(\bar{G}^i_{S,K(i),l} + \eta_l\right)P^i_{K(i)}\right)\right]^+, \forall l \tag{42}$$

$$\nu_l(t+1) = \left[\nu_l(t) - s^l_5\left(I^l_{th} - \sum_{i=1}^{N}\left(\bar{H}^i_{K(i)} + \varepsilon_l\right)\left(\bar{G}^i_{K(i),l,D} + \delta_l\right)P^i_{K(i)}\right)\right]^+, \forall l \tag{43}$$

$$\omega(t+1) = \left[\omega(t) - s_6\left(P_{\text{total}} - \sum_{i=1}^{N} P^i_{K(i)}\right)\right]^+ \tag{44}$$

where s^l_4, s^l_5, and s_6 are the small positive step sizes. The iterations are repeated until the robust power allocation process converges. We define this robust relay selection and power allocation algorithm under the influence of different uncertainty parameters as robust algorithm.

The computational complexity of the proposed robust algorithm can be counted roughly as follows. We address the robust resource allocation problem with two steps, i.e., relay selection and power allocation separately. First, we allocate subcarriers to relays assuming uniform power distribution over subcarriers. Each subcarrier selects the best relay (i.e., the relay with highest $\rho^{i\,*}_k$) for itself with computational complexity K. We assume T_1 is the number of iterations with sub-gradient method to get $\rho^{i\,*}_k$. After N subcarriers all finish the relay selection, the computational complexity of this relay selection procedure is $\mathcal{O}(NKT_1)$. Then, power allocation is carried out under the given relay selection scheme. The power allocation problem is decomposed into N parallel power allocation sub-problems. In every sub-problem, we assume T_2 is the number of iterations to obtain $P^i_{K(i)}{}^*$ with sub-gradient method. After N evaluations, the computational complexity of the power allocation is $\mathcal{O}(NT_2)$. To sum up, the overall computational complexity is $\mathcal{O}(N(KT_1 + T_2))$, which is linear to N.

4 Simulation results

In this section, we use Monte Carlo simulation with MATLAB software to show the effectiveness and the outperformance of our proposed robust algorithm. Setting of simulation parameters are described in Table 2. Channel gains $h^i_{SR_k}$, $h^i_{R_kD}$, $h^i_{SP_l}$, and $h^i_{R_kP_l}$ are assumed to be frequency flat Rayleigh fading channels. They are independent and identically distributed (i.i.d.) circularly symmetric complex Gaussian (CSCG) random variables (RVs) and distributed as $h \sim \mathcal{CN}\left(0, \frac{1}{(1+d)^\tau}\right)$, where τ is the path loss coefficient and d is the distance among different nodes in the system. Without lose of generality and to make the realization of the proposed algorithm simple and better understood, we assume $\sigma^2 = \sigma^2_i$, $\xi = \xi^i_k$, $\varepsilon_l = \varepsilon\bar{H}^i_k$, $\eta_l = \eta\bar{G}^i_{S,k,l}$, and $\delta_l = \delta\bar{G}^i_{k,l,D}$, respectively. For notational brevity, let $\zeta = \xi = \varepsilon = \delta = \eta$ and $\zeta \in [0, 1)$ represent the normalized upper bound for the all uncertainty regions. The simulations have been conducted for 20,000 independent channel realizations.

Figure 2 shows the relay selection factors of all subcarriers in both robust algorithm and non-robust algorithm. The relays in these two algorithms are distinguished by different colors. Based on the relay selection scheme in these two algorithms, R_k with maximum ρ^i_k is allocated to the i^{th} subcarrier. In other words, each subcarrier selects the best relay for itself. As for non-robust algorithm, we can observe that the relays selected by subcarriers are R_1, R_2, R_2, R_1, R_1, and R_1 in sequence. When $\zeta = 0.1$, the robust relay selection results are the same as the non-robust algorithm which validates the robustness of the proposed robust relay selection scheme.

Under the aforementioned relay selection scheme, Fig. 3 illustrates the power allocation schemes to subcarriers for the two algorithms. We can see that the power allocated to subcarriers in the non-robust algorithm is higher than that

Table 2 Setting of simulation parameters

Parameters	Value
K	2
L	2
N	6
τ	4
P_{total}	$0.05W$
σ^2	$1 \times 10^{-13}W$
I^1_{th}	$1 \times 10^{-12}W$
I^2_{th}	$0.5 \times 10^{-12}W$

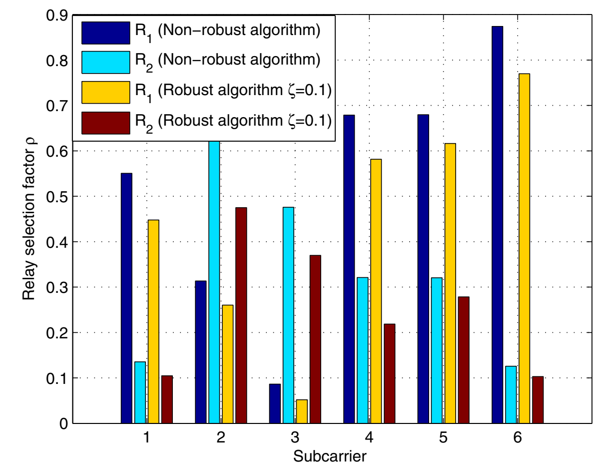

Fig. 2 Relay selection factor of subcarrier

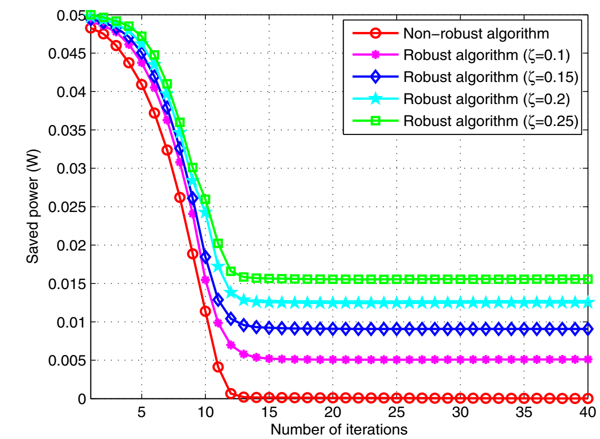

Fig. 4 Saved power

in robust algorithm. According to $C1$ and $C2$ in **P4**, we can find that the transmit power in the robust algorithm has to be reduced to keep the interference thresholds of PU-RXs below their preset limit values under the influence of the uncertainties.

Figure 4 shows the saved power versus the increase number of iterations. We define the saved power as the difference value of the total transmit power P_{total} and the actual transmit power $\sum_{i=1}^{N} P_{K(i)}^{i}$. We find that the saved power in non-robust algorithm converges to zero, which indicates that the non-robust algorithm makes full use of the total transmit power. However, due to the existing uncertainties, the robust algorithm cannot take advantage of the total transmit power. Moreover, the saved power increases with the increase of uncertainties, since the robust algorithm has to reduce more transmit power to cope with the increasing uncertainties.

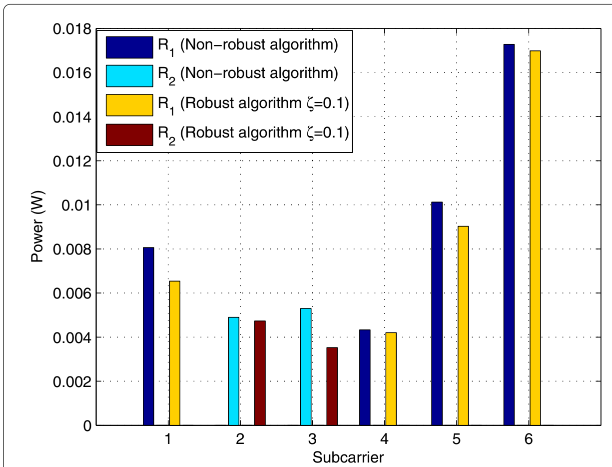

Fig. 3 Power allocation to subcarrier

Figure 5 demonstrates the interference power to each PU-RX from *SR* link and *RD* link in the proposed robust algorithm, non-robust algorithm, method in [9], and method in [10]. In this paper, we assume different PU-RX has different interference threshold. It is also more suitable to practical systems than the assumption that the interference constraints of different PU-RX under the same interference level. Figure 5a, b shows that the proposed robust algorithm can strictly guarantee the interference threshold of each PU-RX, and the interference to each PU-RX in non-robust algorithm, method in [9]and method in [10] are all exceed their interference thresholds, which means that the non-robust algorithm cannot satisfy the QoS of PU-RXs with uncertainty parameters.

Figure 6 shows the capacity convergence results of the two algorithms with the increase number of iterations. We can see that the two algorithms can quickly converge to the equilibrium points. It can also be observed that, although we have maximized the capacity of the cognitive relay network through the robust algorithm with uncertainty parameters, the capacity of the robust algorithm is still lower than that of the non-robust algorithm with perfect CSI. The reason is that the non-robust algorithm can make full use of the total transmit power regardless of the influence of the uncertainty parameters on the interference power to PU-RXs, but the robust algorithm reduces the transmit power to overcome the uncertainties for the QoS of PU-RXs. In addition, the capacity of the robust algorithm decreases with the increase of the uncertainty parameter. Figures 5 and 6 indicate that the robust algorithm outperforms the non-robust algorithm for the perspective of ensuring the QoS of PU-RXs at the expense of system capacity loss.

Figure 7 illustrates the capacity of the non-robust algorithm and the robust algorithm versus the interference threshold under the influence of uncertainties. In order to make simulations more simplified, we assume

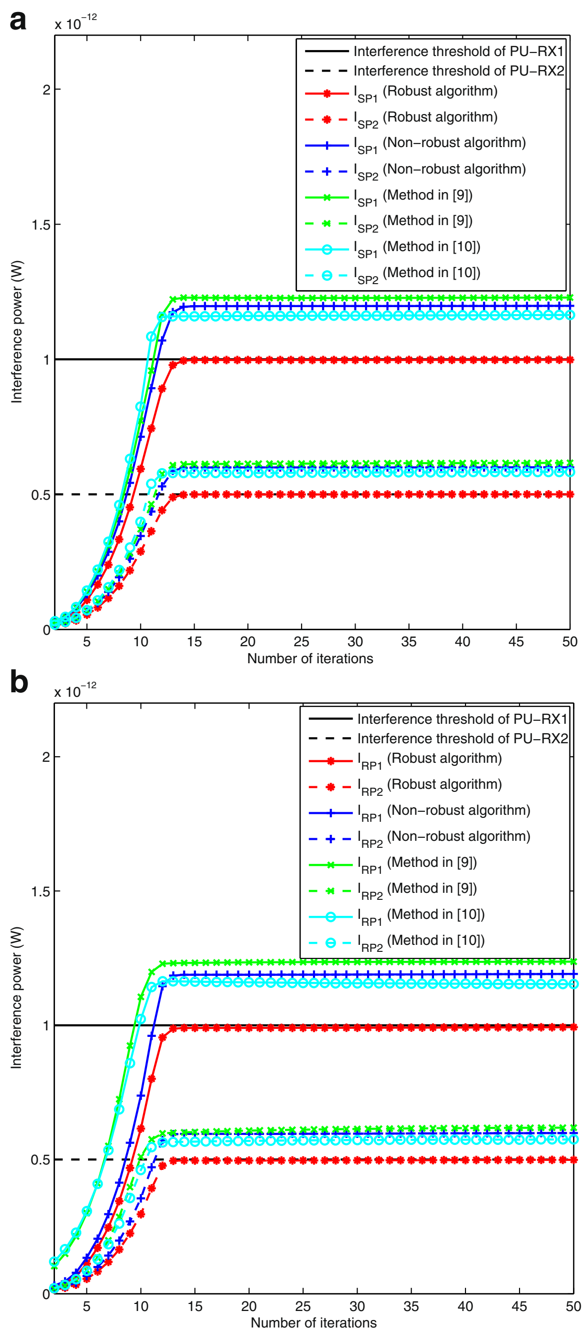

Fig. 5 a, b Interference power to each PU-RX from *SR* link and *RD* link ($\zeta = 0.2$)

that the interference thresholds of different PU-RXs are the same. From Fig. 7, we can see that the capacity of the robust algorithm is lower than that of the non-robust one and decreases with the increase of the uncertainties in accordance with Fig. 6. Moreover, we can find that the capacity of these two algorithms increases with the increase of the interference threshold at first. When power allocation reaches its maximum in the two algorithms, the capacity does not change and becomes constant regardless of the increase of interference threshold.

Figure 8 shows the capacity of the non-robust algorithm and the robust algorithm versus the power budget with the assumption of $I_{th} = I_{th}^1 = I_{th}^2 = 0.8 \times 10^{-12} W$. As expected, the capacity of the robust algorithm is lower than that of the non-robust algorithm and decreases with the increase of the uncertainties. We can also observe

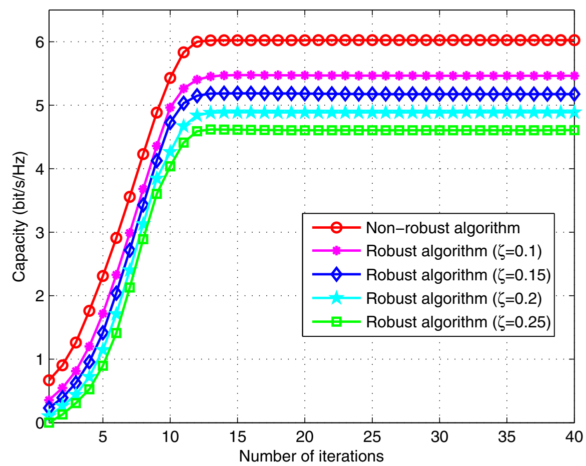

Fig. 6 Convergence of capacity

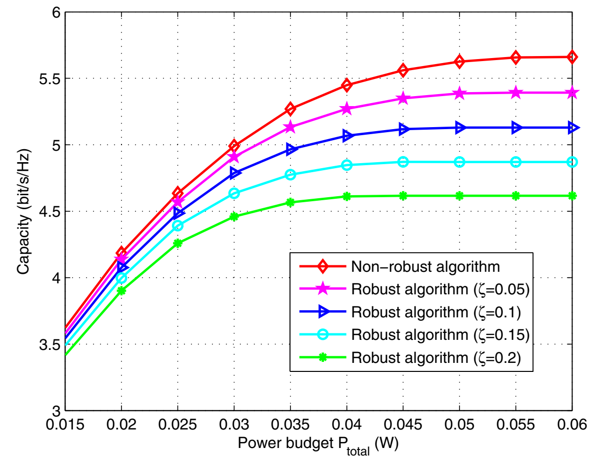

Fig. 8 Capacity versus power budget

that the capacity of the two algorithms increases with the increasing power budget at the beginning. However, when the interference to PU-RXs reaches the preset thresholds, the capacity stops increasing and remains constant despite of the increase of power budget.

Figure 9 gives the effect of the interferencethreshold and the power budget on the capacity under the assumption of $\zeta = 0.2$ .With the increase of the interference threshold, the capacity increases in both robust and non-robust algorithm initially. However, when interference threshold becomes larger to some extent, interference constraint becomes inactive. Power budget becomes the limiting constraint and plays a dominant role. We can also observe that the higher power budget is, the larger the capacity is.

5 Conclusions

In this paper, we have studied robust RA problem in the underlay OFDM-based cognitive relay network with joint channel uncertainty and interference uncertainty. Based on worst-case approach and Lagrange dual decomposition method, closed form analytical solutions to robust relay selection and power allocation have been derived. Numerical results demonstrate the robustness of the proposed robust relay selection scheme. The robust algorithm is superior to the non-robust algorithms in terms of guaranteeing the interference thresholds of different PU-RXs, but the capacity of the robust algorithm is little lower than that of non-robust algorithm for overcoming the uncertainties and also decreases with the increase of the uncertainties. In our future

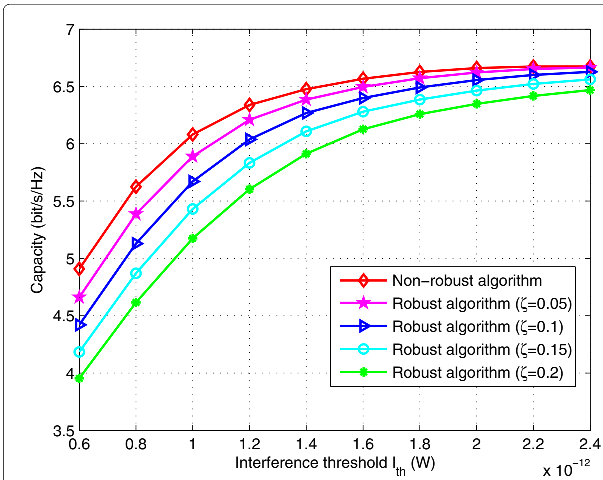

Fig. 7 Capacity versus interference threshold

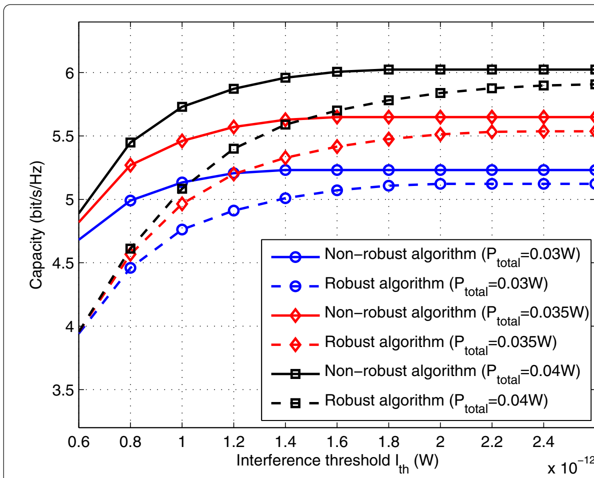

Fig. 9 Effect of interference threshold and power budget on capacity

works, we will extend this frame work to two-way cognitive radio networks with multiple SUs or multiple antennas.

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Authors' contributions
WY contributed in the conception of the study and design of the study and wrote the manuscript. Furthermore, WY carried out the simulation and revised the manuscript. XH helped to perform the analysis with constructive discussions and helped to draft the manuscript. All authors read and approved the final manuscript.

Competing interests
The authors declare that they have no competing interests.

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