Hindawi Publishing Corporation
EURASIP Journal on Wireless Communications and Networking
Volume 2007, Article ID 19249, 16 pages
doi:10.1155/2007/19249

Research Article

An Analysis Framework for Mobility Metrics in Mobile Ad Hoc Networks

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Received 31 January 2006; Revised 9 October 2006; Accepted 9 October 2006

Recommended by Hamid Sadjadpour

Mobile ad hoc networks (MANETs) have inherently dynamic topologies. Under these difficult circumstances, it is essential to have some dependable way of determining the reliability of communication paths. *Mobility metrics* are well suited to this purpose. Several mobility metrics have been proposed in the literature, including link persistence, link duration, link availability, link residual time, and their path equivalents. However, no method has been provided for their exact calculation. Instead, only statistical approximations have been given. In this paper, exact expressions are derived for each of the aforementioned metrics, applicable to both links and paths. We further show relationships between the different metrics, where they exist. Such exact expressions constitute precise mathematical relationships between network connectivity and node mobility. These expressions can, therefore, be employed in a number of ways to improve performance of MANETs such as in the development of efficient algorithms for routing, in route caching, proactive routing, and clustering schemes.

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1. INTRODUCTION

Mobile ad hoc networks (MANETs) are comprised of mobile nodes communicating via (potentially multihop) wireless links. Mobility of the nodes causes communication links to be dynamic, affecting path reliability. Frequent path breakage, requiring discovery of new routes, leads to excessive end-to-end delay and affects the quality of service for delay-sensitive applications.

Understanding node mobility is one of the keys to determine the potential capacity of an ad hoc network. Various mobility metrics have been proposed as measures of topological change in networks. Metrics describing the link or path stability allow adaptive routing in MANETs based on predicted link behavior. A range of routing protocols based on predictive mobility metrics has been shown to increase the packet delivery ratio and to reduce routing overhead [1–6].

We consider a range of mobility metrics: link (path) availability, link (path) persistence, link (path) residual time, and link (path) duration. Many of these metrics have been considered previously, (see [1–3, 7–13]), although the naming has not been consistent. We seek to identify the relationships between the various metrics and provide a consistent nomenclature. In particular, there is considerable confusion in the literature about the term "link availability." The term is generally used to describe the probability that a currently active link will be active at a particular time in the future. However, some authors require that the link should exist for the whole of the intervening period, while others do not. The probability of existence will be considerably increased in the latter case.

To alleviate this confusion, we introduce the new terms *link persistence* and *path persistence* to describe the continuous link and path availabilities, and reserve the term link (path) availability to describe the noncontinuous case [14]. That is, the link (path) persistence is the probability that a link (path) continuously lasts until a future time k given that it existed at time 0. In the perspective of link persistence, once the link is broken, it no longer exists.

We present a theoretical analysis framework for calculating the eight mobility metrics presented, for nodes moving according to a given synthetic mobility model. Our framework can be applied to any mobility model that admits a Markov process describing node separation. This theoretical approach is in contrast to most research to date which has been based on simulation results and empirical analysis of mobility metrics.

Many random mobility models have been proposed [15], however, as yet, statistical analysis of the induced network connectivity is generally unavailable. One of the few which can be described by simple probability distribution functions is the random-walk mobility model (RWMM), which we use to illustrate the use of our framework. (Future work will involve the statistical description of more realistic models, similar to [16], and application of our framework to them.)

The calculated metrics can be useful as an aid to predicting link reliability for routing purposes [5, 17]. Moreover, random mobility models are regularly used for protocol evaluation, so our work is important to facilitate comparison of the evaluation environment with practical implementation environments.

The main contributions of this paper are (1) introduction of notion of link (path) persistence and its calculation method, (2) expressions for the expected link (path) duration and its PDF, (3) expressions for the expected link (path) residual time and its PDF which are derived using a random mobility model rather than a nonrandom travelling pattern (straight-line mobility model), (4) an exact expression for link (path) availability which matches the simulation data well for any given time interval.

We begin with definitions in Section 2 for the mobility metrics we investigate, with a discussion of related work in the literature. In Section 3 we develop two Markov chain models of the evolution of the separation distance between two nodes. In Section 4 the Markov chain models are used to develop exact expressions for the aforementioned mobility metrics. In Section 5 we apply the framework developed in the previous two sections to the random walk mobility model. In Section 6 we compare our theoretical results for the RWMM with simulation results. Finally, we present conclusions and further work in Section 7.

2. MOBILITY METRIC TAXONOMY

We define a series of mobility measures for links and for paths. As explained in the introduction, most of these have appeared in the literature, sometimes under different names, but they have not previously been gathered together as we have done here.

The following definitions do not make any assumptions about what it means for a link to exist, but do assume that it is possible to determine at any point in time whether or not a link does exist. Links are understood to be "on" or "off" at any point in time, as it is common in the existing literature on mobility in MANETs. In reality, fading links are the norm in wireless communication networks at the scales relevant for ad hoc networks [9]. In such cases, link availability is an appropriate metric to employ. However, schemes which use network topology information are sensitive to the length of time for which a link is consistently "on." Therefore, our remaining metrics—persistence, residual time, and duration—assume that the link is "on," and consider how long it will continue to be "on."

An h hop path between two nodes consists of a chain of $h-1$ intermediate nodes connecting them. Each node in the chain has an active link with the nodes either side of it in the chain, effectively forming a transmission path between the two nodes of interest. A *link* could be described as a 1-hop path. We define each of the metrics for paths, and define the corresponding link metrics as special cases for which $h = 1$.

The first two metrics, path (link) availability and persistence, are probabilities—they correspond to the probability that a path (link) exists at a certain time in the future given that it exists now. One can see intuitively that in most situations, this probability decreases as the wait time increases. The difference between availability and persistence lies in the requirement that the path (link) may disappear and reappear during the wait time in the case of availability, but may not do so in the case of persistence.

The remaining metrics are measured in units of time—referring to the length of time that a path (link) exists. Residual time can be measured from any point in the life of the path (link), whereas path (link) duration is measured from the time the path (link) is first "on" until the time the path (link) is next "off." In the case where nodes move according to a synthetic mobility model, the residual time and duration are random variables. We calculate their probability mass functions (PMFs) and expected values in Section 4.

(i) Path availability $\mathcal{A}(t, h)$

Given an active path with h hops between two nodes at time 0, the path availability [5] at time t is defined as the probability that the path exists at time t, given that it existed at time,

$$\mathcal{A}(t,h) \triangleq \Pr\{\text{available at time } t \mid \text{available at time } 0\}. \tag{1}$$

The path may have been broken, possibly several times, between time 0 and time t. The *link availability* is denoted by $\mathcal{A}(t) \triangleq \mathcal{A}(t,1)$.

Path and link availability were proposed by McDonald and Znati [5].

(ii) Path persistence $\mathcal{P}(t, h)$

Given an active path with h hops between two nodes at time 0, the path persistence, as a function of time, is defined as the probability that the path will continuously last until at least time t, given that it existed at time 0,

$$\mathcal{P}(t,h) \triangleq \Pr\{\text{last until at leas time } t \mid \text{available at time } 0\}. \tag{2}$$

That is, $\mathcal{P}(t,h)$ is the probability that the path is continuously in existence from time 0 until at least time t. The *link persistence* is denoted by $\mathcal{P}(t) \triangleq \mathcal{P}(t,1)$.

Link persistence is called "link availability" in [18, 19].

(iii) Path residual time $\mathcal{R}(h)$

Given an active path with h hops between two nodes at time 0 (which may also have been active for some time immediately prior to time 0), the path residual time, $\mathcal{R}(h)$, is the length of time for which the path will continue to exist until it is broken. The *link residual time* is denoted by $\mathcal{R} \triangleq \mathcal{R}(1)$.

Link residual time has been referred to as the "link's residual lifetime" [8], "link available time" [13], "link expiration time" [2], and "expected link lifetime" [3]. Path residual time has been referred to as "path's residual lifetime" [8], "available time in multihop" [13], and "route expiration time" [2].

(iv) Path duration $\mathcal{D}(h)$

Given that a path becomes active at time 0, the path duration [12] $\mathcal{D}(h)$ is the length of time for which the path will continue to exist until it is broken. That is, the path duration is the path residual time from the instant the path first becomes available, and it is a measure of stability of the path between a pair of nodes. It could be understood as a maximal value of the path residual time. The *link duration* [1] is denoted by $\mathcal{D} \triangleq \mathcal{D}(1)$.

We can divide these metrics into two groups based on whether a persistent connection is required (persistence, residual time, and duration) or an intermittent connection is acceptable (availability).

2.1. Related work

Each of the metrics have been studied in various ways by various authors. Here we give a brief overview.

In [5, 11], path availability is used to divide mobile nodes into clusters. The link availability and path availability were theoretically analyzed, for nodes moving according to a variant of the random-walk mobility model. However they employ a Rayleigh approximation for relative movement between a pair of mobile nodes (MNs), which does not work well when taken over short time intervals, particularly for the path availability calculation. By contrast, the calculation method presented in this paper is accurate for any time interval.

Link persistence is calculated approximately by Qin [19] for nodes moving according to the random-walk mobility model (though they call it link availability). In [13] an expression for link persistence is derived for a simple *straight-line* mobility model. A mobility metric that is similar to link persistence is determined in [6, 10] using a combination of calculation and experimental evaluation, for modified random-walk and random waypoint mobility models.

Link (path) residual time is widely used in proactive routing schemes. The mechanism is that when a communicating path is active between two MNs, the destination node can estimate the link (path) residual time by means of a prediction algorithm. New route discovery is initiated early by detecting that an active link is likely to be broken and an alternative route is built before link failure. In many cases, this is achieved by assuming that the MNs do not change movement direction when communicating with each other [2, 3, 13] (a straight-line mobility model), which is clearly quite a restrictive assumption. Link residual time is evaluated by simulation in [8], for nodes moving according to a variety of synthetic mobility models.

The concept of link duration was introduced by Boleng et al. [1] as a mobility metric to enable adaptive routing. Link duration is a good indicator of protocol performance measures such as data packet delivery ratio and end-to-end delay. Furthermore, it is computable in real network implementations without global network knowledge. Bai et al. [7] and Sadagopan et al. [12], investigate link duration and path duration experimentally, for four different mobility models corresponding to routing protocols such as AODV and DSR, based on simulations. Han et al. [20] give an approximate calculation for link duration and path duration for a random waypoint mobility model. In this paper, we determine an exact expression for the PMF of node separation distance when a link is set up and conclude that link (path) duration is a special case of link (path) residual time.

2.2. Metric calculation

In general, each of the above mobility metrics will differ between particular links (paths). If the objective is to predict future connectivity of a particular link (path), specific information about the link (path) must be known—whether measured [18] or assumed [5]. If, on the other hand, the objective is to characterize the degree of mobility of the network as a whole, it is necessary to average over all possible links (paths) [1].

Our framework allows calculation of the mobility metrics under some random mobility model. In this case, link residual time and link duration are random variables. Consequently, the network average link residual time and link duration are also random variables. Thus, we consider the expected value of the network average for these entities.

Mobility models employed in simulation-based performance evaluation usually assume that all nodes move in an i.i.d. random manner. In this case, the expected value of the mobility metric associated with individual links (or paths) will be identical, and equal to the network average. Such assumptions may also provide useful predictions of future connectivity when no *a priori* knowledge of individual node characteristics exists.

We will employ the notation $\overline{\mathcal{A}}(k,h)$, $\overline{\mathcal{P}}(k,h)$, $\overline{\mathcal{R}}(h)$ to denote the *network average* values of availability, persistence, and residual time (omitting the argument $h = 1$ when links, rather than paths, are of interest). Under our assumptions, the link duration $\mathcal{D}$ and path duration $\mathcal{D}(h)$ do not need to be augmented in this manner as the expected value of the network average is identical to the expected value for an individual link (or path).

In our calculations, the link-based mobility metrics, except link duration, depend (only) on the initial separation of nodes. The path-based mobility metrics, except path duration, depend (only) on the initial separation along all hops in the path. Therefore, we augment the notation for availability, persistence, and residual time to include L_0, the separation distance at time 0. The link-based mobility metrics become $\mathcal{A}(k;L_0)$, $\mathcal{P}(k;L_0)$, and $\mathcal{R}(L_0)$. The path-based mobility metrics become $\mathcal{A}(k,h;L_0(1),\dots,L_0(h))$, $\mathcal{P}(k,h;L_0(1),\dots,L_0(h))$, and $\mathcal{R}(h;L_0(1),\dots,L_0(h))$, where

$L_0(i)$ is the initial separation of the nodes constituting the ith hop in a particular path.

Having established definitions for each of the mobility metrics of interest, we next develop generic expressions for each of the mobility metrics, using a Markov chain model. (Using a Markov chain model allows for random mobility models for which no closed-form expression may be found for the PDF of the mobility, which is most often the case.) These expressions may then be applied to any particular random mobility model by substituting in the appropriate PDF. The random-walk mobility model is used as an example in Section 5.

3. MARKOV CHAIN DESCRIPTION OF NODE SEPARATION DISTANCE

A Markov chain model (MCM) gives a model for the evolution of the random process it is describing. We use an MCM to describe the evolution of the separation distance between nodes in an ad hoc network, moving according to a memoryless random mobility model. We will use the MCM to derive mathematical expressions for each of the mobility metrics introduced in Section 2.

In order to apply Markov chain methods, we examine node separation after periods of fixed time length, termed epochs. We assume that the duration of the epochs and the speed of the nodes are such that the path persistence after one epoch, $\mathcal{P}(1, h)$, is approximately one, and the path residual time, $\mathcal{R}(h)$, is considerably more than one epoch. In this case, there is no significant error introduced by discretizing the time via epochs.

3.1. *Notation for model development*

The status of a wireless link depends on numerous system and environmental factors that affect transmitter and receiver's transmission range. A widely applied, albeit optimistic, model is used in this paper, whereby transmission range is approximated by a circle of radius r corresponding to a signal strength threshold. Thus, if the separation distance between a pair of nodes of interest is less than r, it is assumed that the link between them is active.

All of the mobility metrics are based on the probability of a pair of nodes going out of range. That is, we are interested in the behavior of the separation distance between a pair of nodes. An MCM can be employed to calculate the mobility metrics in Section 2 if the separation distance between two nodes is a Markov process. Assume that the movement of nodes in the network can be described by i.i.d. random processes. Let the random variable representing the separation distance between two nodes at epoch m be L_m, and let l_m denote an instance of L_m.[1] We assume that the PDF of the L_{m+1} is dependent only on L_m. Then separation distance is a Markov process and the transition probabilities for the MCM are derived from $f_{L_{m+1}|L_m}(l_{m+1} \mid l_m)$. This PDF is determined by the mobility model being used.

[1] Throughout this paper, we use the convention of capital letters for random variables and the corresponding lowercased letters for instances of random variables.

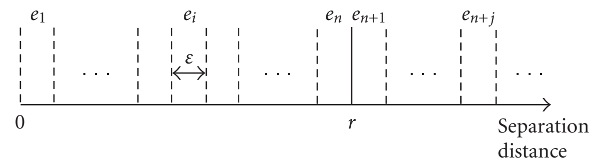

Figure 1: Depiction of state space for distance between a pair of nodes in the intermittent metric group, where communication links for nodes which move outside the transmission range, and back in again, are considered to be the "same" link.

3.2. *State-space derivation*

We divide the node separation distance from 0 to r into n bins of width ε. If a link exists, the node separation at epoch m, L_m, falls into one of these bins. If we label state i, e_i, then the *state space* of the distance between the two nodes is $E = \{e_1, \ldots, e_i, \ldots\}$. The state space for distances greater than r differs for the two mobility metric groups. We examine each group separately below.

3.2.1. *State space for intermittent metric group*

In this case the state space for distances greater than r consists of an infinite number of states, each corresponding to a bin of width ε, as illustrated in Figure 1. The node separation L_m is in e_i if $L_m = l_m$, where

$$(i-1)\varepsilon \le l_m < i\varepsilon, \quad i \in \mathbb{Z}^+. \tag{3}$$

3.2.2. *State space for persistent metric group*

The state space for metrics in the persistent group requires an *absorbing state* which, once reached, cannot be escaped. The absorbing state represents any distance greater than the communication range r. If the distance between the two nodes reaches the absorbing state, the communication link is considered to be broken. If the nodes move back within communication range, a new link is considered to have been formed.

In this model, the state of the node separation distance, $L_m = l_m$, is governed by

$$\begin{aligned} (i-1)\varepsilon \le l_m < i\varepsilon, &\quad i \in [1, \ldots, n], \\ l_m > r, &\quad i = n+1. \end{aligned} \tag{4}$$

3.3. *Initial probability vector*

The Markov chain process is an evolving process. The probability of being in any particular state changes with time. Thus, we begin with an *initial probability* vector which denotes the probability of the initial node separation distance, $L_0 = l_0$, being in each of the states at epoch 0. The initial probability vector $\mathbf{P}(0)$ can be written as

$$\mathbf{P}(0) = \begin{bmatrix} p_1(0) & p_2(0) & \cdots & p_n(0) & \cdots \end{bmatrix}, \tag{5}$$

where

$$p_i(0) = \Pr(l_0 \in e_i) \begin{cases} 1 \le i \le n+1 & \text{for persistent links,} \\ i \in \mathbb{Z}^+ & \text{for intermittent links.} \end{cases} \tag{6}$$

Further, as the links are assumed to be active at epoch 0, that is, in a state with index at most n, $\sum_{i=1}^{n} p_i(0) = 1$.

The choice of $\mathbf{P}(0)$ differs according to whether the objective is to determine the mobility metric for a particular link, or the network average for the metric. In the first case, the initial separation distance, $l_0 < r$, for the link is known, and the initial state, e_i, is determined according to (3) or (4), where $m = 0$ and $i \in [1, \dots, n]$. Then, the initial probability vector, denoted by $\mathbf{P}_{L_0}(0)$, has only one nonzero element:

$$p_i(0) = \begin{cases} 1 & \text{if } l_0 \in e_i, \\ 0 & \text{otherwise.} \end{cases} \tag{7}$$

For network average mobility metrics, it is necessary to determine how the mobile node positions distributed in a two-dimensional space. If the nodes are uniformly distributed over the network area (as it is the case for nodes moving according to a random walk in a bounded region), the distribution of all separation distances is approximately Rayleigh (it is not exact if the network area is bounded). If, in addition, the transmission range is much smaller than the network area, then we can approximate the distribution of node separation distances in the range 0 to r as being linear, as follows:

$$f_{L_0}(l_0) = \begin{cases} \dfrac{2l_0}{r^2}, & 0 \le l_0 \le r, \\ 0, & l_0 > r. \end{cases} \tag{8}$$

Thus, for network average metrics, when nodes are uniformly distributed, the initial condition vector, denoted $\mathbf{P}_{\text{net}}(0)$, has elements

$$p_i(0) = \begin{cases} (2i-1)\dfrac{\varepsilon^2}{r^2}, & 0 \le i \le n, \\ 0, & i > n. \end{cases} \tag{9}$$

To reiterate, this value of $\mathbf{P}_{\text{net}}(0)$ is only appropriate for networks with uniformly distributed nodes. For many interesting mobility models, nodes are not uniformly distributed [21].

A third initial condition vector, $\mathbf{P}_{\text{new}}(0)$, will be introduced in Section 4.1.4 to describe the PDF of node separation for links when they first become active.

3.4. Probability transition matrix

Having established the form of the initial condition vector for the different contexts, we now introduce the probability transmission matrices for the two metric groups.

3.4.1. Intermittent metric group transition matrix

Let the separation distance l_m between two nodes be in state e_i. After one epoch, the separation distance l_{m+1} must be in the range

$$[\max(0, l_m - 2v_{\max}), l_m + 2v_{\max}], \tag{10}$$

where $v_{\max}$ is the maximum speed that can be attained by the nodes. This corresponds to l_{m+1} being in e_j such that

$$j \in [\max(1, i-\gamma), i+\gamma], \quad \gamma := \left\lceil \frac{2v_{\max}}{\varepsilon} \right\rceil, \tag{11}$$

where γ is the maximum number of states that can be crossed in a single epoch. When there is no absorbing state, as depicted in Figure 1, the transition matrix is denoted by the infinite-size matrix $\mathbf{A}_{\text{int}}$, where

$$\mathbf{A}_{\text{int}} = \begin{bmatrix} a_{1,1} & \cdots & a_{1,n} & \cdots \\ \vdots & \ddots & \vdots & \\ a_{n,1} & \cdots & a_{n,n} & \cdots \\ \vdots & & \vdots & \ddots \end{bmatrix}, \tag{12}$$

and $a_{i,j}$ is the probability of transition from e_i to e_j in a given epoch. We note that for all $i, j, a_{i,j} \ge 0$ and $\sum_j a_{i,j} = 1$ (i.e., node i must move somewhere).

To calculate the transition probabilities between any two states in the nonabsorbing state model, as illustrated in Figure 2, consider the state space for the nonabsorbing state model at epoch m. The transition probabilities are given by

$$\begin{aligned} a_{i,j} &= \Pr(e_i \longrightarrow e_j) = \Pr(l_{m+1} \in e_j \mid l_m \in e_i) \\ &= \int_{(j-1)\varepsilon}^{j\varepsilon} \int_{(i-1)\varepsilon}^{i\varepsilon} f_{L_{m+1}|L_m}(l_{m+1} \mid l_m) f_{L_m}(l_m) dl_m \, dl_{m+1}, \end{aligned} \tag{13}$$

where the conditional PDF $f_{L_{m+1}|L_m}(l_{m+1} \mid l_m)$ is dependent upon the particular mobility model. Now, the PDF $f_{L_m}(l_m)$ varies with time m. However, if ε is *sufficiently small*, we can assume that independently of m, L_m is approximately uniformly distributed within the ith bin. In this case,

$$f_{L_m}(l_m) \approx \frac{1}{\varepsilon}. \tag{14}$$

Moreover, we can approximate the PDF of the conditioned separation distance from any point in e_i to any point in e_j by the value of the PDF at the midpoint of the two states, such that

$$f_{L_{m+1}|L_m}(l_{m+1} \in e_j \mid l_m \in e_i) \approx f_{L_{m+1}|L_m}\left(\left(j - \frac{1}{2}\right)\varepsilon \mid \left(i - \frac{1}{2}\right)\varepsilon\right). \tag{15}$$

Thus, we have

$$a_{i,j} \approx \varepsilon f_{L_{m+1}|L_m}\left(\left(j - \frac{1}{2}\right)\varepsilon \mid \left(i - \frac{1}{2}\right)\varepsilon\right), \tag{16}$$

giving us an expression which closely approximates the transition probabilities, as long as we choose the state widths small enough.

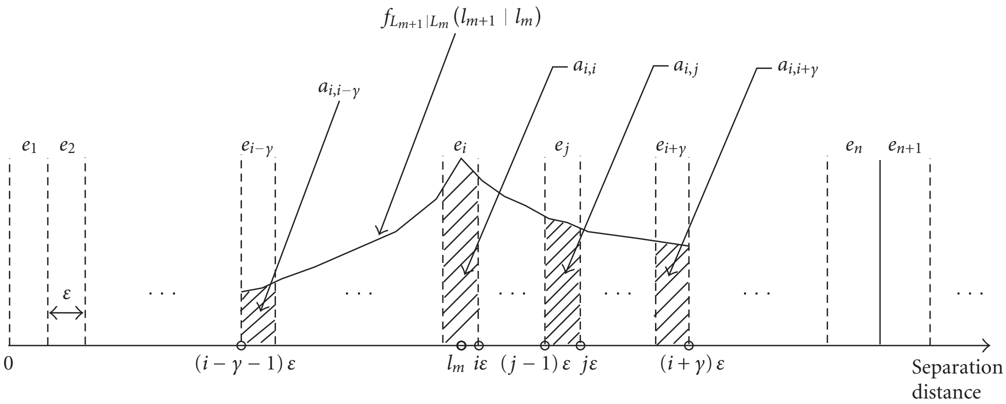

FIGURE 2: Depiction of state space for the nonabsorbing state model, showing the state transition probabilities, $a_{i,j}$, the probability of transferring from e_i to e_j after one epoch, for a given state i and various states j.

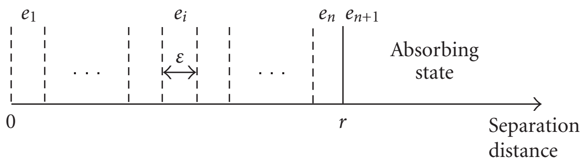

FIGURE 3: State space for distance between a pair of nodes in the persistent metric group, where separations greater than the transmission range (absorbing state) result in a link being discarded.

3.4.2. *Persistent metric group transition matrix*

Recalling that for the persistent metric group, there are $n+1$ possible states, as shown in Figure 3, we let the $(n+1)\times(n+1)$ state transition matrix, with absorbing state, be denoted by $\mathbf{A}_{\text{pst}}$, where

$$\mathbf{A}_{\text{pst}} = \begin{bmatrix} a_{1,1} & \cdots & a_{1,n} & a_{1,n+1} \\ \vdots & \ddots & \vdots & \vdots \\ a_{n,1} & \cdots & a_{n,n} & a_{n,n+1} \\ 0 & \cdots & 0 & 1 \end{bmatrix}. \tag{17}$$

The entries indicating the probabilities of entering the absorbing state, that is, the rightmost column of $\mathbf{A}_{\text{pst}}$, are given by

$$a_{i,n+1} = 1 - \sum_{j=1}^{n} a_{i,j}. \tag{18}$$

The last row of $\mathbf{A}_{\text{pst}}$ indicates the probability of transition from the absorbing state.

The probabilities of moving between each pair of nonabsorbing states are given by the upper left block of $\mathbf{A}_{\text{pst}}$:

$$Q = \begin{bmatrix} a_{1,1} & \cdots & a_{1,n} \\ \vdots & \ddots & \vdots \\ a_{n,1} & \ldots & a_{n,n} \end{bmatrix}, \tag{19}$$

where entry $a_{i,j}$ is given by (16).

3.5. *Separation probability vector after k epochs*

Using the transition matrices defined in Section 3.4, and the initial probability vectors defined in Section 3.3, we can calculate the probability vector of the separation distance after k epochs $\mathbf{P}(k)$. For the intermittent metrics, where there is no absorbing state,

$$\mathbf{P}(k) = \mathbf{P}(0)\mathbf{A}_{\text{int}}^{k}, \tag{20}$$

where $\mathbf{P}(k)$ is an infinite-length vector with elements $p_i(k)$, describing the probability that the separation distance L_k is in e_i at the end of epoch k and $\mathbf{A}_{\text{int}}$ is from (12).

Similarly, for the persistent metrics, where the separation distance state space does include an absorbing state, $\mathbf{P}(k)$ is an $(n+1)$-vector

$$\mathbf{P}(k) = \mathbf{P}(0)\mathbf{A}_{\text{pst}}^{k}, \tag{21}$$

where $\mathbf{A}_{\text{pst}}$ is from (17).

In either case, necessarily,

$$\sum_{i} p_i(k) = 1, \tag{22}$$

where i ranges from 1 to $n+1$ if there is an absorbing state, and from 1 to ∞ if there is no absorbing state.

In summary, $\mathbf{P}(k)$ gives the discrete probability distribution of the separation distance between a pair of nodes after k epochs. It is discrete, but may be made as incremental as desired by appropriately choosing ε, the width of each state.

4. MOBILITY METRIC CALCULATIONS

We have presented expressions for the discrete probability distribution of the separation distance between a pair of nodes at any time in (20) and (21). We now use these to derive expressions for each of the mobility metrics defined in Section 2. Because the Markov chain development requires discrete-time intervals, in our mobility metric calculations, we consider discrete-time versions of the metrics, replacing time t with epoch k.

4.1. Expressions for link-based metrics

Calculation of the link-based metrics is achieved via direct application of Markov chain methods, using the initial probability vectors and transition matrices introduced in Section 3.

4.1.1. Link availability $\mathcal{A}(k)$

Link availability is an intermittent mobility metric—the link may be broken at some time before epoch k, but must be reestablished by epoch k. Thus we use the probability transition matrix with no absorbing state $\mathbf{A}_{\text{int}}$. The probability of the link being in existence after k epochs is the sum of the probabilities of L_k being in one of e_1 to e_n at epoch k. Thus, the link availability is the sum of the first n elements of $\mathbf{P}(k)$ in (20). The general equation for link availability is, therefore,

$$\mathcal{A}(k) = \sum_{i=1}^{n} p_i(k), \tag{23}$$

where $p_i(k)$ are the elements of $\mathbf{P}(k) = \mathbf{P}(0)\mathbf{A}_{\text{int}}^k$.

The link availability for a particular initial separation $\mathcal{A}(k; L_0)$ uses the initial condition vector $\mathbf{P}_{L_0}(0)$ with elements defined in (7). The network average link availability $\overline{\mathcal{A}}(k)$ uses the initial probability vector $\mathbf{P}_{\text{net}}(0)$ from (9).

4.1.2. Link persistence $\mathcal{P}(k)$

Link persistence is determined in the same way as link availability, with the exception that the transition matrix with absorbing state $\mathbf{A}_{\text{pst}}$ is used. Thus, the general equation for link persistence is

$$\mathcal{P}(k) = \sum_{i=1}^{n} p_i(k) = 1 - p_{n+1}(k), \tag{24}$$

where $p_{n+1}(k)$ is the final element of the vector $\mathbf{P}(k) = \mathbf{P}(0)\mathbf{A}_{\text{pst}}^k$.

The link persistence for a particular initial separation, $\mathcal{P}(k; L_0)$ uses the initial condition vector $\mathbf{P}(0) = \mathbf{P}_{L_0}(0)$ with elements defined in (7). The network average link persistence $\overline{\mathcal{P}}(k)$ uses the initial condition vector $\mathbf{P}_{\text{net}}(0)$ from (9).

4.1.3. Link residual time $\mathcal{R}$

The probability that the link residual time is, at most, k is equal to the probability that after epoch k, the separation distance is in the absorbing state e_{n+1}. We can write the (discrete) cumulative density function (CDF), $F_{\mathcal{R}}(k)$, of the link residual time, as

$$F_{\mathcal{R}}(k) = \Pr\{\mathcal{R} \le k\} = p_{n+1}(k), \tag{25}$$

where $p_{n+1}(k)$ is defined in Section 4.1.2. Therefore, the probability mass function (PMF), $f_{\mathcal{R}}(k)$, of the link residual time is

$$f_{\mathcal{R}}(k) = \Pr\{\mathcal{R} = k\} = p_{n+1}(k) - p_{n+1}(k-1). \tag{26}$$

In Section 6 we illustrate that this PMF is approximately exponential.

The expected value of the link residual time can then be written as

$$E\{\mathcal{R}\} = \sum_{k=1}^{\infty} k f_{\mathcal{R}}(k) = \sum_{k=1}^{\infty} k\left[p_{n+1}(k) - p_{n+1}(k-1)\right]. \tag{27}$$

This holds for both link-specific residual time $\mathcal{R}(L_0)$ and network average residual time $\overline{\mathcal{R}}$ by again using the appropriate initial condition vector. Due to the exponential decay of the PMF, terms in this sum are negligible for large k, meaning that truncation at an appropriate point will result in negligible error, allowing feasibility of calculation.

Alternatively, the link residual time can be determined directly from the fundamental matrix, $\mathcal{F}$ [22],

$$\mathcal{F} = \left(I_n - Q\right)^{-1}, \tag{28}$$

where I_n is the $n \times n$ identity matrix, and Q is defined in (19). The sum of the elements of the ith row of $\mathcal{F}$ is the expected link residual time for links starting in e_i,

$$E\{\mathcal{R}(L_0)\} = \sum_{j=1}^{n} \mathcal{F}_{i,j}, \quad L_0 = l_0 \in e_i. \tag{29}$$

The expected value of the network average link residual time is

$$E\{\overline{\mathcal{R}}\} = \sum_{i=1}^{n} p_i(0) \sum_{j=1}^{n} \mathcal{F}_{i,j}, \tag{30}$$

where $p_i(0)$ are elements of $\mathbf{P}_{\text{net}}(0)$ from (9).

4.1.4. Link duration $\mathcal{D}$

Link duration is effectively a special case of the link residual time, with the requirement that $L_0 = r$. That is, the link duration is the link residual time at the time of formation of the link—how long the link lasts from beginning to end. In fact, as the mobility model is discrete in time, $L_0 \in [r - 2v_{\max}, r)$, since we only examine the connectivity at the end of each epoch. Therefore, the link duration can be determined identically to the link residual time, above, with initial condition vector $\mathbf{P}_{\text{new}}(0)$ determined below for the case where nodes are uniformly distributed.

In order to obtain the PDF of the initial separation distance L_0, we consider the conditional PDF of L_{-1}, the node separation distance just prior to the link being established. A pair of nodes with separation distance $L_{-1} \in [r, r+2v_{\max})$ has the potential to form a link in epoch 0. If the nodes are uniformly distributed over the network area, the distribution of separation distances is approximately Rayleigh (it is not exact if the network area is bounded). If the transmission distance $r \ll A$, where A is the network area, then we can approximate the distribution of node separation distances just prior to link establishment as being linear in the range r to

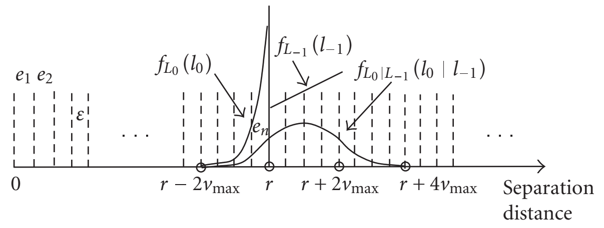

Figure 4: Depiction of PDFs of node separation, with respect to separation distance state space, at epochs −1 and 0, taking into account moves that do and do not result in a link being established. Nodes are assumed to be uniformly distributed.

$r+2v_{\max}$. This is equivalent to saying that the node separation distances are uniformly distributed on a ring with inner radius r and outer radius $r+2v_{\max}$. The PDF of L_{-1} is then

$$f_{L_{-1}}(l_{-1}) = \begin{cases} \dfrac{l_{-1}}{2v_{\max}(r+v_{\max})}, & r \le l_{-1} < r+2v_{\max}, \\ 0 & \text{otherwise.} \end{cases} \tag{31}$$

The marginal PDF of the initial separation distance for new links, $f_{L_0|\text{new}}(l_0 \mid \text{new link})$ is equal to the portion of $f_{L_0|L_{-1}}(l_0 \mid l_{-1})$ that intersects the region $[r-2v_{\max}, r)$, normalized accordingly. Figure 4 illustrates the relationship between $f_{L_{-1}}(l_{-1})$, $f_{L_0|L_{-1}}(l_0 \mid l_{-1})$ and $f_{L_0|\text{new}}(l_0 \mid \text{new link})$ showing approximate shapes for the random-walk mobility model, described in Section 5. Obtaining the PDF $f_{L_0|L_{-1}}(l_0 \mid l_{-1})$ is the same as obtaining the PDF $f_{L_{m+1}|L_m}(l_{m+1} \mid l_m)$ with $m = -1$. Thus, we obtain a discretized version of $f_{L_0}(l_0)$ which is our initial condition vector for new links, $\mathbf{P}_{\text{new}}(0)$, valid when nodes are uniformly distributed.

The new initial condition vector $\mathbf{P}_{\text{new}}(0)$ can be employed to determine the persistence of a newly established link, $\mathcal{P}_{\text{new}}(k)$, in the same way as the link persistence for a particular initial separation and the network average link persistence are determined.

Now, the PMF, $f_{\mathcal{D}}(k)$, of the link duration is given by

$$f_{\mathcal{D}}(k) = p_{n+1}(k) - p_{n+1}(k-1), \tag{32}$$

where $p_{n+1}(k)$ is the final element of the vector $\mathbf{P}(k) = \mathbf{P}_{\text{new}}(0)\mathbf{A}_{\text{pst}}^k$. The expected value of the link duration can be determined either from this PMF, or similar to link residual time, from the fundamental matrix

$$E\{\mathcal{D}\} = \sum_{i=1}^{n} p_i(0) \sum_{j=1}^{n} \mathcal{F}_{i,j}, \tag{33}$$

where $p_i(0)$ are the elements of $\mathbf{P}_{\text{new}}(0)$. (Note that there is no concept of link duration for a given initial separation and that the link duration calculated here is effectively the network average.)

4.2. Path-based metrics

Path-based metrics are determined from link metrics using the assumption that links exist independently of each other. This is true for a randomly chosen path when nodes move according to an i.i.d. random process, even though consecutive links in a path share a common node. (It may not be true when attention is restricted to a particular subset of all possible paths, such as the shortest-distance path between two nodes.)

4.2.1. Path availability $\mathcal{A}(k,h)$

For a path with h hops, path availability is the product of the individual link availabilities of the h hops. If the initial separation distances for each hop in a particular path are $L_0(1), \ldots, L_0(h)$, respectively, the path availability can be calculated using

$$\mathcal{A}(k,h;L_0(1),\ldots,L_0(h)) = \prod_{i=1}^{h} \mathcal{A}(k, L_0(i)), \tag{34}$$

where $\mathcal{A}(k, L_0(i))$ is given by (23). The network average path availability for h-hop paths is given by

$$\overline{\mathcal{A}}(k,h) = \left[\overline{\mathcal{A}}(k)\right]^h, \tag{35}$$

where $\mathcal{A}(k)$ is the network average link availability, as defined in Section 4.1.1.

4.2.2. Path persistence $\mathcal{P}(k,h)$

By using the product of the link persistences for each of the constituent links, the path persistence is given by

$$\mathcal{P}(k,h;L_0(1),\ldots,L_0(h)) = \prod_{i=1}^{h} \mathcal{P}(k, L_0(i)), \tag{36}$$

where $\mathcal{P}(k, L_0(i))$ is given by (24). The network average path persistence for an h-hop path is given by

$$\overline{\mathcal{P}}(k,h) = \left[\overline{\mathcal{P}}(k)\right]^h, \tag{37}$$

where $\overline{\mathcal{P}}(k)$ is the network average link persistence, as defined in Section 4.1.2.

4.2.3. Path residual time $\mathcal{R}(h)$

For a particular path, the path residual time is the length of time that the path continuously lasts without breaking. We can write the CMF, $F_{\mathcal{R}}(k,h)$, of the path residual time, as

$$\begin{aligned} F_{\mathcal{R}}(k,h) &= 1 - \mathcal{P}(k,h) = 1 - \mathbf{P}\,(\text{path lasts} \ge k) \\ &= \mathbf{P}\,(\text{path lasts} \le k). \end{aligned} \tag{38}$$

Therefore the PMF of the path residual time can be written as

$$f_{\mathcal{R}}(k,h) = \mathcal{P}(k-1,h) - \mathcal{P}(k,h). \tag{39}$$

The expected value of the path residual time can be expressed by

$$E\{\mathcal{R}(h)\} = \sum_{k=1}^{\infty} k f_{\mathcal{R}}(k,h) = \sum_{k=1}^{\infty} k\left[\mathcal{P}(k-1,h) - \mathcal{P}(k,h)\right]. \tag{40}$$

There is no equivalent of the fundamental matrix method that was available for link residual time.

4.2.4. Path duration $\mathscr{D}(h)$

To determine the path duration, we need to be precise about the time that the path commences. We will assume that one link in the path has just become active, and all other links are active links with unspecified node separation. That is, the initial condition vector for one of the links is $\mathbf{P}_{\text{new}}(0)$, and the initial condition for the remaining links is $\mathbf{P}_{\text{net}}(0)$. The persistence and all links in the path are considered from the same point in time. Then, the *new path* persistence $\mathscr{P}_{\text{new}}(k,h)$ is given by

$$\mathscr{P}_{\text{new}}(k,h) = \left[\overline{\mathscr{P}}(k)\right]^{h-1}\mathscr{P}_{\text{new}}(k), \tag{41}$$

where $\mathscr{P}_{\text{new}}(k)$ is defined in Section 4.1.4. The PMF of the path duration $f_D(k,h)$ is then

$$f_D(k,h) = \mathscr{P}_{\text{new}}(k-1,h) - \mathscr{P}_{\text{new}}(k,h). \tag{42}$$

In this section, we have derived exact expressions for the mobility metrics using a probability transition matrix derived from the PDF of the node separation after one epoch. In Section 6, we use our calculations to illustrate the values of these mobility metrics for the random-walk mobility model.

5. APPLICATION USING RANDOM-WALK MOBILITY MODEL

The random-walk mobility model (RWMM) is probably the most mathematically tractable mobility model in use. It describes the basic node mobility parameters, velocity, and direction of travel, in terms of known probability distributions. We therefore use the RWMM to illustrate the use of the MCM-derived expressions for the mobility metrics, from Section 3.

We assume that each mobile node moves with a velocity uniformly distributed in both speed $V \sim U[v_{\min}, v_{\max}]$ and direction $\Phi \sim U[0, 2\pi]$. Both the speed and direction change in each epoch but are constant for the duration of an epoch, and are independent of each other. The speed has mean $\overline{v} = (1/2)(v_{\min}+v_{\max})$, and variance, $\sigma_v^2 = (1/12)(v_{\max}-v_{\min})^2$. This random mobility model is widely used to analyze route stability in multihop mobile environments [3, 23].

We saw in Section 3 that the movement-related PDF required for the MCM is $f_{L_{m+1}|L_m}(l_{m+1} \mid l_m)$, where l_m is the separation distance between a pair of nodes at epoch m. To obtain this PDF, we must formulate a description of the behavior of the relative movement.

5.1. Relative movement between two nodes

To determine the PDF $f_{L_{m+1}|L_m}(l_{m+1} \mid l_m)$, we begin with the PDF of the *relative movement* between a given pair of nodes, labelled i and j, whose movements are i.i.d. The relationship between the relative movement vector $\vec{X}$ in any given epoch, and the node velocity vectors $\vec{V}_i$ and $\vec{V}_j$ is $\vec{X} = \vec{V}_j - \vec{V}_i$, as depicted in Figure 5. Let X be the random variable representing the magnitude of $\vec{X}$, similarly for V_i and V_j. The acute angle Ψ between $\vec{V}_i$ and $\vec{V}_j$ is uniformly distributed in $[0,\pi)$, and Ψ, V_i and V_j are independent, so we have the joint PDF

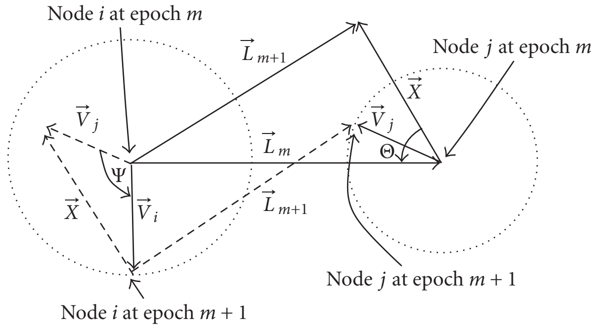

FIGURE 5: Relationship between the node movement vectors $\vec{V}_i$ and $\vec{V}_j$ of nodes i and j, respectively, relative movement vector, $\vec{X}$, separation vector at epoch m, $\vec{L}_m$, and separation vector after one epoch, $\vec{L}_{m+1}$. Solid lines indicate actual vector positions and dashed lines indicate vectors shifted for illustration purposes. The dotted circles indicate the loci of possible positions for nodes i and j at epoch $m+1$.

$$f_{\Psi,V_i,V_j}(\psi,v_i,v_j) = \frac{1}{12\pi\sigma_v^2}. \tag{43}$$

Using the cosine rule, it can be seen that the relative movement X is related to the random variables V_i, V_j, and Ψ by

$$X = \sqrt{V_i^2+V_j^2-2V_iV_j\cos\Psi}. \tag{44}$$

We use the Jacobian transform [24] to obtain the joint PDF:

$$\begin{aligned} f_{X,V_i,V_j}&(x,v_i,v_j) \\ &= \frac{\partial\psi}{\partial x} f_{\Psi,V_i,V_j}(\psi,v_i,v_j) \\ &= \frac{x}{6\pi\sigma_v^2\sqrt{2v_i^2v_j^2+2v_i^2x^2+2v_j^2x^2-v_i^4-v_j^4-x^4}}. \end{aligned} \tag{45}$$

Then the marginal PDF of the magnitude of the relative movement can be found via

$$f_X(x) = \iint_{v_{\min}}^{v_{\max}} f_{X,V_i,V_j}(x,v_i,v_j)\,dv_i\,dv_j, \tag{46}$$

however, there is apparently no closed-form solution to (46). So, (45) and (46) describe the behavior of the relative distance X between a given pair of nodes i and j in any one epoch, given uniform distributions for V_i, V_j, Φ_i, and Φ_j, as previously described.

5.2. Conditional PDF of separation distance

The separation vector at epoch $m+1$ is the sum of the separation vector at epoch m and the relative movement vector, $\vec{L}_{m+1} = \vec{L}_m + \vec{X}$, as shown in Figure 5. The acute angle between $\vec{X}$ and $\vec{L}_m$ is denoted by Θ, as shown in Figure 5. Again we use the Jacobian transform, this time to replace the random variables (X, Θ) with the new pair (L_{m+1}, Θ). The value of new variable L_{m+1} depends on the given value of L_m, so we include the condition in the notation for the new PDF, to obtain

$$f_{L_{m+1},\Theta|L_m}(l_{m+1}, \theta \mid l_m) = \frac{\partial x}{\partial l_{m+1}} f_{X,\Theta}(x, \theta) = \frac{\partial x}{\partial l_{m+1}} f_X(x) f_\Theta(\theta), \tag{47}$$

since the magnitude X and the angle Θ are independent. Θ is uniformly distributed in the interval $[0, \pi]$. The PDF $f_X(x)$ is given in (46) and can be reexpressed in terms of the new variables using

$$X = L_m \cos\Theta \pm \sqrt{L_{m+1}^2 - L_m^2 \sin^2\Theta}. \tag{48}$$

So the new joint PDF is

$$f_{L_{m+1},\Theta|L_m}(l_{m+1}, \theta \mid l_m) = \frac{l_{m+1} f_X\left(l_m \cos\theta \pm \sqrt{l_{m+1}^2 - l_m^2 \sin^2\theta}\right)}{\pi\sqrt{l_{m+1}^2 - l_m^2 \sin^2\theta}}. \tag{49}$$

We then take the marginal PDF with respect to Θ to find the PDF of L_m conditioned on L_{m+1}:

$$f_{L_{m+1}|L_m}(l_{m+1} \mid l_m) = \int_a^b f_{L_{m+1},\Theta|L_m}(l_{m+1}, \theta \mid l_m)\, d\theta. \tag{50}$$

There are several different cases for the relative values of L_m and L_{m+1} which decide the expressions for a and b [25]. Again, there is apparently no closed-form solution to this expression.

Thus, we have the conditional PDF of node separation distance after one epoch. Note that the assumption of identical uniform distributions of V_i and V_j is not necessary to this result, so a similar method could be used to determine the PDF for arbitrarily distributed, independent V_i and V_j.

The PDF (50) can be evaluated at discrete points as indicated in (16), to generate expressions for the mobility metrics for the RWMM.

5.3. Approximation of link residual time and link duration

While, for the RWMM, it is difficult to determine an exact expression for the expected value of the node separation after a given time, it is actually simple to determine the expected value of its square. Let the initial separation distance between a pair of nodes be l_0. Then, after k epochs, from [26] and [27, equation (4.2-11)], the mean square of the separation distance l_k^2 is given by

$$E\{l_k^2\} = l_0^2 + 2k(\overline{v}^2 + \sigma_v^2), \tag{51}$$

where $\overline{v}$ is the mean node speed, and σ_v^2 is the node speed variance.

5.3.1. Link residual time approximation

The mean-square value of the separation distance monotonically increases with k. When k is sufficiently large, $E\{l_k^2\}$ will be greater than r^2. Assuming that the nodes start within range of each other, as required for link residual time calculations to be meaningful, we can expect that the first epoch at which the mean-square value of the separation distance exceeds r^2 will be approximately equal to the link residual time. We denote the separation distance at the end of the epoch when the link is first broken as $r + \delta$, where $0 < \delta < 2v_{\max}$, replace k in (51) with $E\{\mathcal{R}(l_0)\}$, and rearrange to give

$$E\{\mathcal{R}(l_0)\} \approx \frac{(r+\delta)^2 - l_0^2}{2(\overline{v}^2 + \sigma_v^2)}. \tag{52}$$

In [28], we show, via simulation, that $\delta \approx (2/3)\overline{v}$, and δ is negligible when $l_0 \le r/2$.

To determine the expected value of the network average link residual time, we use

$$E\{\overline{\mathcal{R}}\} = \int_0^r E\{\mathcal{R}(l_0)\} f_{L_0}(l_0)\, dl_0, \tag{53}$$

where $f_{L_0}(l_0)$ is given in (8). Thus, the expected value of the network average link residual time $E\{\overline{\mathcal{R}}\}$ is given by

$$E\{\overline{\mathcal{R}}\} = \frac{r^2 + 4r\delta + 2\delta^2}{4(\overline{v}^2 + \sigma_v^2)}. \tag{54}$$

5.3.2. Link duration approximation

To derive an approximate expression for the link duration, we combine the approximate expression for the link residual time in (52) with a linear approximation for the PDF of the initial link separation illustrated in Figure 4. The probability that the initial link separation falls in the region $[r - 2v_{\max}, r - 2\overline{v}]$ is nonzero but negligible. In fact it can be shown that $f_{L_0|\text{new}}(l_0 \mid \text{new link})$ is well approximated by

$$f_{L_0|\text{new}}(l_0 \mid \text{new link}) \approx \begin{cases} \dfrac{l_0 - r + 2\overline{v}}{2\overline{v}^2}, & r - 2\overline{v} \le l_0 < r, \\ 0 & \text{otherwise}. \end{cases} \tag{55}$$

The expected link duration is then

$$\begin{aligned} E\{\mathcal{D}\} &= \int_{r-2\overline{v}}^r E\{\mathcal{R}(l_0)\} f_{L_0|\text{new}}(l_0 \mid \text{new link})\, dl_0 \\ &\approx \frac{\overline{v}(12r - \overline{v})}{9(\overline{v}^2 + \sigma_v^2)}. \end{aligned} \tag{56}$$

Here we have assumed that $2\overline{v} < r$. (If $\overline{v} \ge r$, the mobility model can be considered as a nonrandom travelling model [2, 29].) In Section 6, we compare these approximations to the exact values obtained from (30) and (33).

5.4. Application to other mobility models

Our framework can be applied to any statistical mobility model where nodes move in an i.i.d. manner and node separation evolution relies only on the previous relative position. In particular it can be applied to the random waypoint mobility model, since it is shown in [30] that random waypoint is asymptotic mean stationary.

Our framework does not directly apply to deterministic mobility models such as trace-based models. Indeed, the mobility metrics examined do not make sense with respect to such models. However, for deterministic models, equivalent average mobility metrics may be of interest. These can be calculated using our framework by replacing the PDF $f_{L_{m+1}|L_m}(l_{m+1} \mid l_m)$ with the network average movement between epochs.

6. VERIFICATION AND ANALYSIS OF CALCULATIONS FOR THE RWMM

Simulations were conducted to verify the theoretical calculations in Sections 4 and 5.3. Our calculations assume an unbounded simulation area. However, it is typical for routing protocols to be tested using a simulation package such as NS-2, which confines mobile nodes to a bounded area. Therefore, we include metric performance results for both bounded and unbounded areas in our simulations.

6.1. Simulation environment

We ran simulations with each MN moving at a randomly chosen velocity during each epoch. The speed was uniformly distributed with $v \in [0, v_{\max}]$, such that $\sigma_v^2 = v_{\max}^2/3$, for various values of $v_{\max}$. The direction was uniformly distributed in the range $[0, 2\pi)$. Each MN was equipped with an omnidirectional antenna with maximum transmission range of $r = 100$ units.[2] 100 MNs start in a square plane of side 1000 distance units. For the bounded scenario, MNs which reached the boundary were reflected back into the allowed region for a bounded simulation area. The experiments were repeated for 2000 trials.

For the path residual time and path duration metrics, we compared randomly chosen paths with paths found using the *breadth-first search* (BFS) algorithm [31] to find the minimum hop path between any given pair of nodes. The difference between choosing a minimum hop path and a randomly chosen path is that links in a minimum hop path are likely to be longer than links in a randomly chosen path. Longer links are likely to break sooner, so minimum hop paths turn out to have a shorter residual time and duration than predicted by our calculations for randomly chosen paths.

6.2. Observations

6.2.1. Availability and persistence

Both the *link* availability and the *link* persistence, shown in Figures 6(a) and 6(b), decrease with increasing simulation time, and at a greater rate with increasing ratio of mean node speed to transmission range $\overline{v}/r$. Given the same simulation time and $\overline{v}/r$, the link persistence is much smaller than the corresponding link availability, as would be expected because the link availability allows breakage and reestablishing of links. Further, the *path* availability and the *path* persistence, shown in Figures 6(c) and 6(d), drop off at a greater rate than the link availability and the link persistence, respectively, for the same mean node speed, as would be expected. The path availability and the path persistence also drop off more quickly with an increased number of hops, as there is more chance of an individual link breaking.

6.2.2. Residual time and duration

In Figures 7 and 8, the expected link (path) residual time and the expected link (path) duration have been plotted against $(r/v_{\max})^2$ and $r/v_{\max}$, respectively, each showing a linear relationship. That is,

$$E\{\mathcal{D}\} \propto \frac{r}{v_{\max}}, \qquad E\{\mathcal{D}(h)\} \propto \frac{r}{v_{\max}}, \tag{57}$$

$$E\{\overline{\mathcal{R}}\} \propto \left(\frac{r}{v_{\max}}\right)^2, \qquad E\{\overline{\mathcal{R}}(h)\} \propto \left(\frac{r}{v_{\max}}\right)^2. \tag{58}$$

The relationship in (57) agrees with the experimentally derived relationship as stated in [7]. As expected, $E\{\overline{\mathcal{R}}(h)\}$ and $E\{\mathcal{D}(h)\}$ are much lower than $E\{\overline{\mathcal{R}}\}$ and $E\{\mathcal{D}\}$ for the same communication range to speed ratio. The probability distributions show that $\mathcal{R}(h)$ and $\mathcal{D}(h)$ are more concentrated near the origin than $\mathcal{R}$ and $\mathcal{D}$. Moreover, the link (path) residual time and the link (path) duration are exponentially distributed, which can be seen in Figures 7(c), 7(d), 8(c), and 8(d). This was experimentally determined in [12], and theoretically justified in [20].

It can be observed that the expected link (path) duration is much less than the average expected link (path) residual time as shown in Figures 7(b), 8(b), 7(a), and 8(a), respectively. This is initially a surprise as one would expect the link (path) duration to be effectively a maximal value of link (path) residual time. This is because the distribution of the initial separation of the links differs in the two cases. Measurement of link duration commences when links first form, that is when nodes first move within transmission range of each other. The initial separation L_0 is distributed on the annulus, in the range $[r - 2v_{\max}, r)$, with greater likelihood of being close to r than further within the transmission range. If the nodes continue to move towards each other, the link duration may be long, however, there is a significant probability that the newly formed link may break immediately, reducing the average value of link duration. In contrast, the distribution of the link separation for residual time is nonzero in the range $[0, r)$. The calculation of the average value of the residual time includes nodes which are extremely close to each other, a situation which never arises for many active links. So, the residual time values are somewhat artificially increased. One possible way to remedy this anomaly

[2] We use the generic term "units" rather than, say, m or km because it is the relative and not the absolute distances that are important.

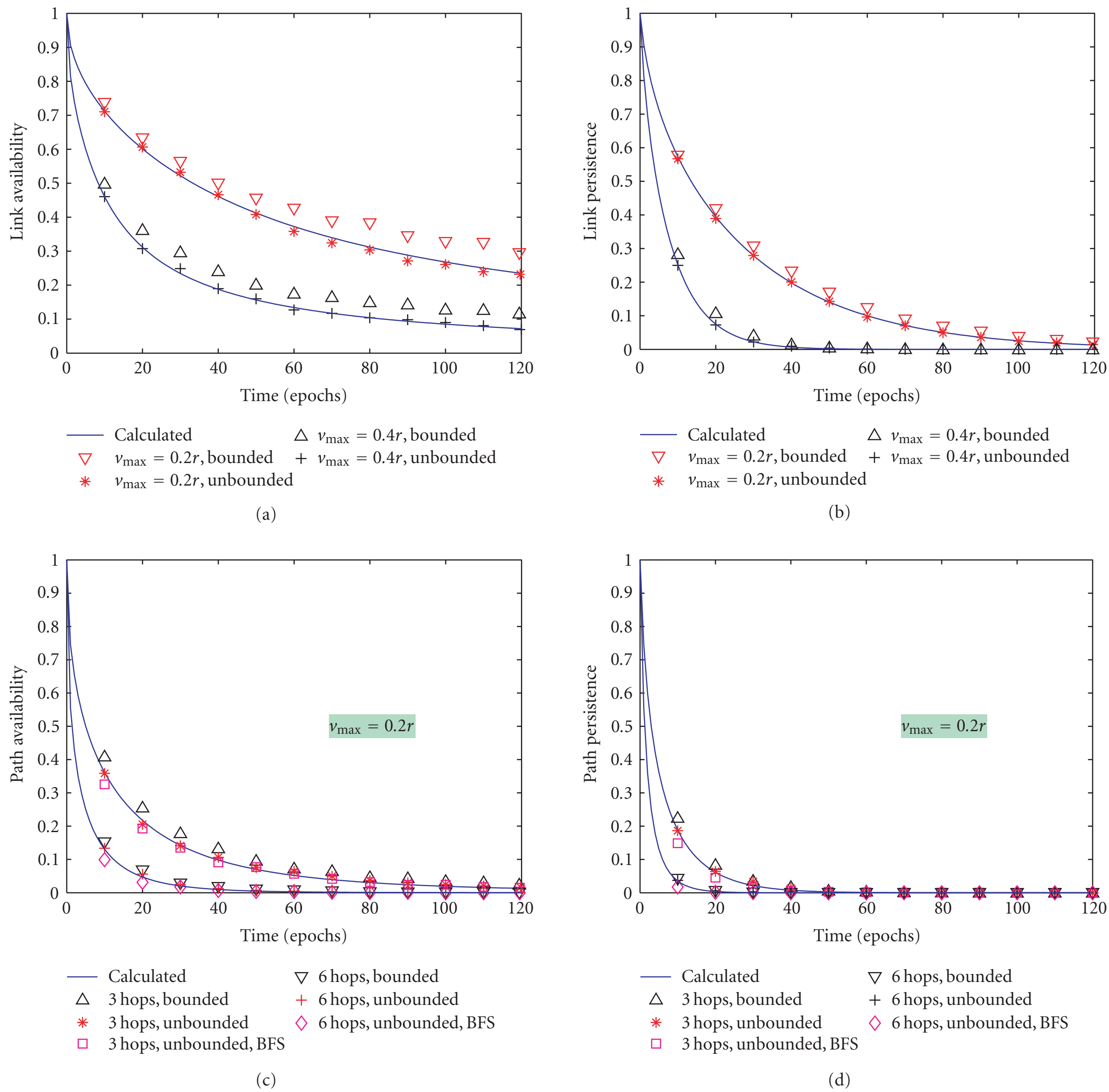

Figure 6: Comparison of metric calculations and simulated results for link (path) availability and link (path) persistence. Each MN moves at a randomly chosen velocity during each epoch, which has uniformly distributed speed in the range $[0, v_{max}]$, and uniformly distributed direction in the range $[0, 2\pi)$. (a) Comparison of calculated and experimental link availability values for both bounded and unbounded simulation areas. Calculated values are from (23). (b) Comparison of calculated and experimental link persistence values for both bounded and unbounded simulation areas. Calculated values are from (24). (c) Comparison of calculated and experimental path availability values for bounded and unbounded simulation areas and the BFS algorithm. Calculated values are from (35). (d) Comparison of calculated and experimental path persistence values for bounded and unbounded simulation areas and the BFS algorithm. Calculated values are from (37).

would be to exclude separation distances below an appropriately chosen threshold.

6.2.3. Effect of bounded simulation area

In the bounded simulation environment, MNs were "reflected" back into the simulation area, if their movement would otherwise take them outside. In this case, node pairs near the edge were more likely to remain in transmission range, and the link (path) availability and link (path) persistence were artificially increased, compared to those for the unbounded simulation area. Consequently, the link (path) residual time and the link (path) duration were increased as well. The experimental results for the bounded area are still close to the calculated results but, as expected, not as well matched.

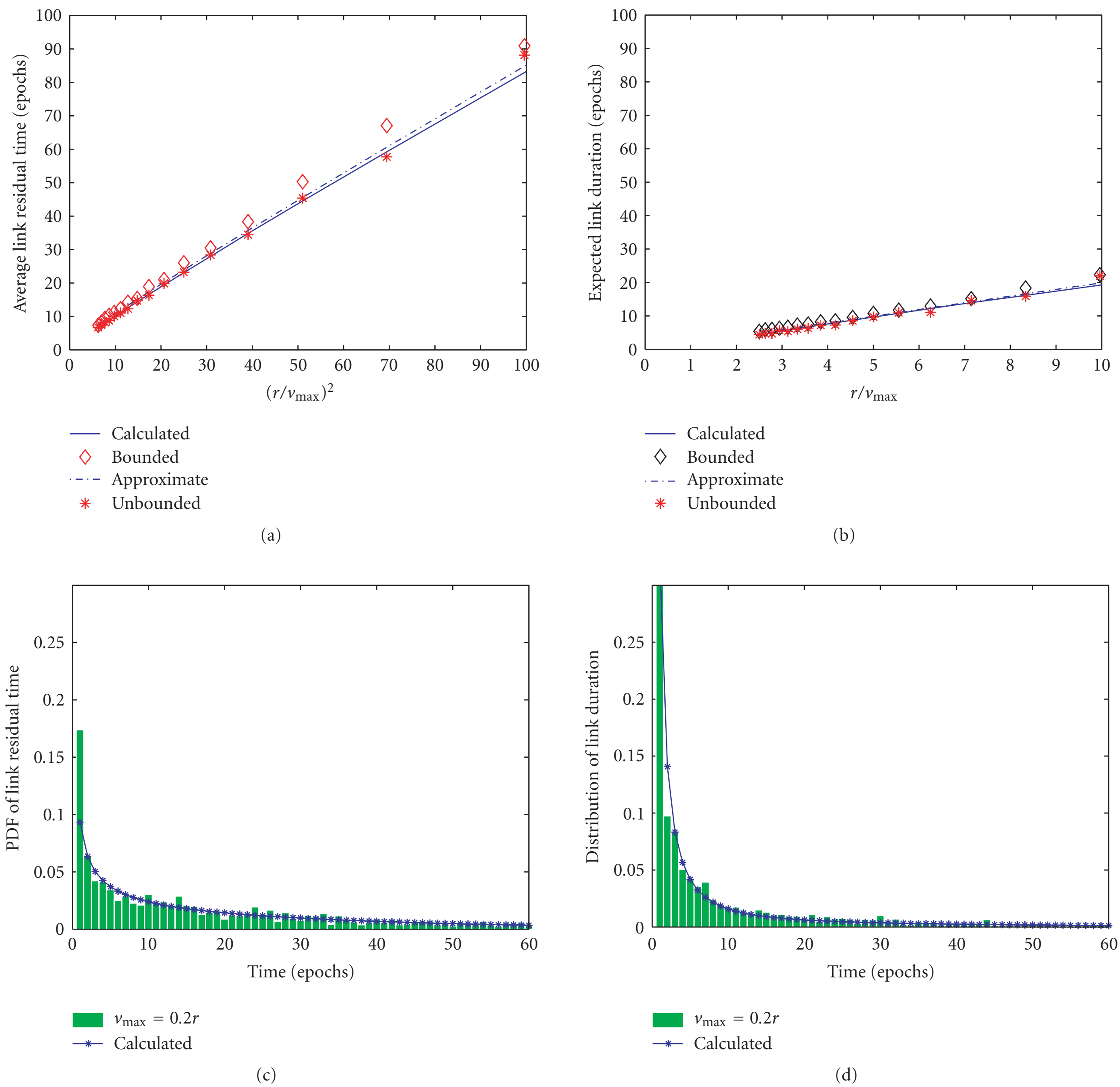

FIGURE 7: Comparison of metric calculations and simulated results for link residual time and link duration. Each MN moves at a randomly chosen velocity during each epoch, which has uniformly distributed speed in the range $[0, v_{\max}]$ and uniformly distributed direction in the range $[0, 2\pi)$ (a) Comparison of calculated, approximate, and experimental average link residual time values for both bounded and unbounded simulation areas. Calculated values are from (30). The approximation is from (54). (b) Comparison of calculated, approximate, and experimental average link duration values for both bounded and unbounded simulation areas. Calculated values are from (33). The approximation is from (56). (c) Comparison of calculated and experimental distributions of the link residual time for an unbounded simulation area. Calculated values are from (26). (d) Comparison of calculated and experimental distributions of the link duration for an unbounded simulation area. Calculated values are from (26).

6.2.4. *Effect of selecting the shortest path*

We used the BFS algorithm to select the shortest path in the path-based mobility metric simulations. Recall from Section 4 that independent link failures are assumed for path-based mobility metrics. For the shortest path, as chosen by the BFS algorithm, however, the hops are correlated. In this case, the probability of a path failing quickly is higher due to the commensurate greater hop lengths, on average, than for a randomly chosen path. The randomly chosen path will likely have more hops with shorter lengths. Therefore, using shortest hop paths, the values of mobility metrics decrease, as shown in Figures 6 and 8. This demonstrates that care must be used in applying our calculations to determine

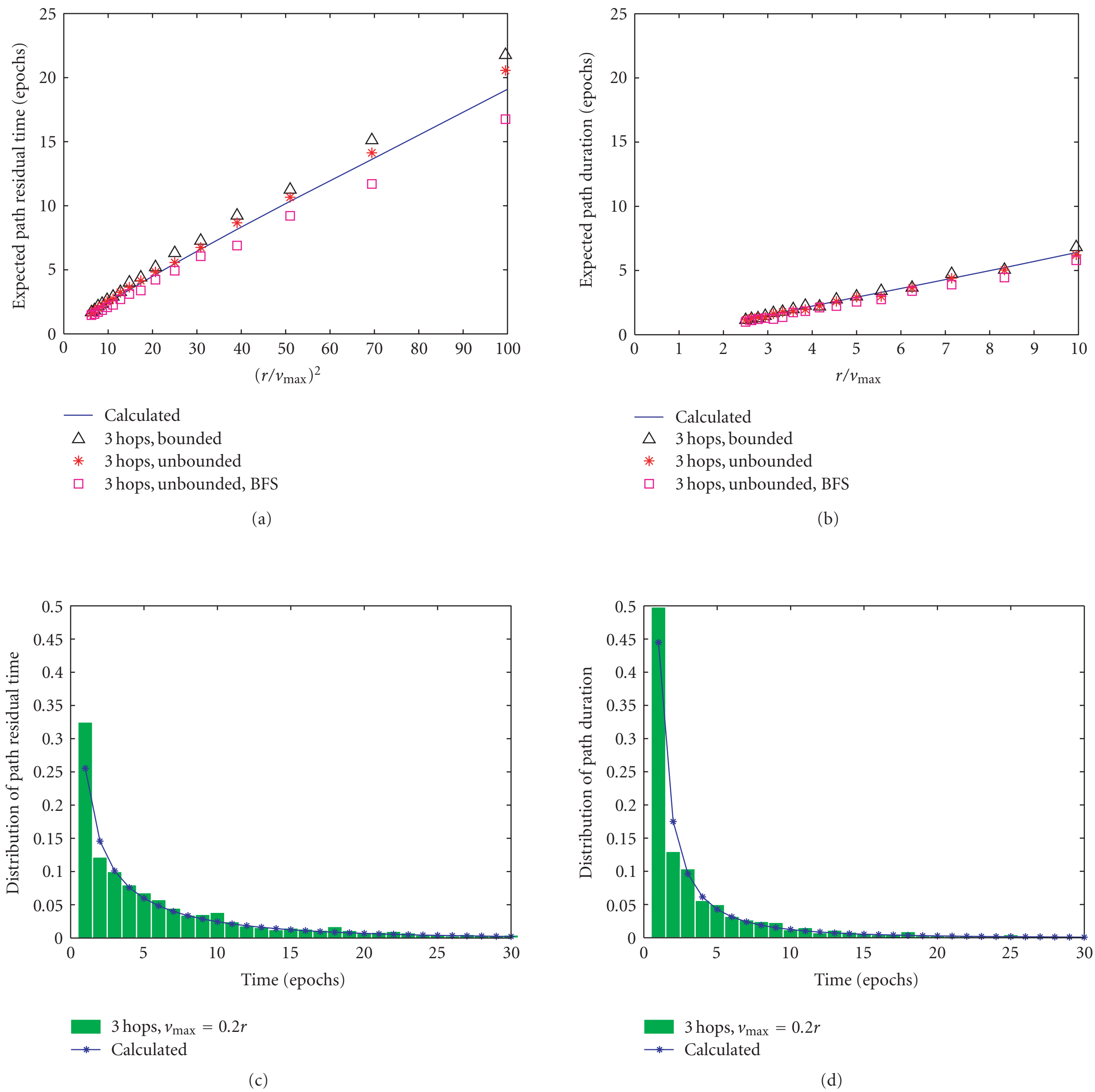

Figure 8: Comparison of metric calculations and simulated results for path residual time and path duration. Each MN moves at a randomly chosen velocity during each epoch, which has uniformly distributed speed in the range $[0, v_{\max}]$ and uniformly distributed direction in the range $[0, 2\pi)$. (a) Comparison of calculated and experimental average path residual time values for bounded and unbounded simulation areas and BFS algorithm. Calculated values are from (40). (b) Comparison of calculated and experimental average path duration values for bounded and unbounded simulation areas and BFS algorithm. Calculated values are from (42). (c) Comparison of calculated and experimental distributions of the path residual time for an unbounded simulation area. Calculated values are from (39). (d) Comparison of calculated and experimental distributions of the path duration for an unbounded simulation area. Calculated values are from (39).

path duration in a particular network routing environment where the method of choosing the paths may adversely affect the path duration. It also suggests that selecting the shortest path for routing is likely to have a detrimental effect on routing performance in MANETs, due to the route discovery necessitated by premature route breakage.

7. CONCLUSIONS

Frequent changes in network topology caused by mobility in mobile ad hoc networks impose great challenges for developing efficient routing algorithms. The theoretical analysis framework presented in this paper provides a better understanding of network behavior under mobility and some fundamental work on the issue of path stability. Apart from the link availability and path availability in previous literature, we propose the link and path persistences for evaluating link and path stabilities. The Markov chain model used in this paper has enabled us to accurately determine a series of mobility metrics. Further, we have presented intuitive and simple expressions (52)–(56) for the link residual time and link duration, for the RWMM, which relate them directly to the ratio between transmission range and node speed. These calculations are useful for comparison of artificial mobility behaviors with actual network implementation scenarios. The analytical results can be readily applied to various adaptive routing protocols that use corresponding mobility metrics. Our next step is to develop statistical descriptions of other, more realistic, mobility models, and to apply this framework to them.

In related work [17, 32], we have utilized our analytical framework to develop adaptive caching strategies that can be used to optimize existing on-demand routing protocols, such as DSR and AODV. We employ the path (link) residual time and path (link) duration as adaptive parameters for route and link caching schemes in on-demand routing protocols, to reduce traffic control overhead and routing delay. We have also begun investigating clustering schemes in MANETs using the mobility metrics calculated in this paper.

ACKNOWLEDGMENTS

The authors wish to thank Dr. Leif Hanlen of the National ICT Australia for his helpful discussions during the development of this work, Dr. J. Boleng from Colorado School of Mines for providing NS-2 source code used for the link duration simulations, and the helpful comments of the various reviewers.

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