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Research Article

Practical Quantize-and-Forward Schemes for the Frequency Division Relay Channel

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We consider relay channels in which the source-destination and relay-destination signals are assumed to be orthogonal and thus have to be recombined at the destination. Assuming memoryless signals at the destination and relay, we propose a low-complexity quantize-and-forward (QF) relaying scheme, which exploits the knowledge of the SNRs of the source-relay and relay-destination channels. Both in static and quasistatic channels, the quantization noise introduced by the relay is shown to be significant in certain scenarios. We therefore propose a maximum likelihood (ML) combiner at the destination, which is shown to compensate for these degradations and to provide significant performance gains. The proposed association, which comprises the QF protocol and ML detector, can be seen, in particular, as a solution for implementing a simple relaying protocol in a digital relay in contrast with the amplify-and-forward protocol which is an analog solution.

1. INTRODUCTION

The channels under investigation in this paper are quasistatic orthogonal relay channels for which orthogonality is defined accordingly to [1]. Since the source-destination channel is assumed to be orthogonal to the relay-destination channel (i.e., the forward channel), the destination receives two distinct signals. For the channels under consideration, there are at least two important technical issues: the relaying protocol and the recombination scheme at the destination. So far, three main types of relaying protocols have been considered in the literature: amplify-and-forward (AF), decode-and-forward (DF), and estimate-and-forward (EF). From the corresponding works, several observations can be made: (a) from information-theoretic studies like [1, 2], it appears that the best choice of the relaying scheme depends on the source-relay channel (i.e., the backward channel) signal-to-noise ratio (SNR) and that of the relay-destination channel; (b) there are not many works dedicated to the design of practical EF schemes although the EF protocol has the potential to perform well for a wide range of relay receive SNRs (in contrast with DF which is generally more suited to relatively high SNRs); (c) the AF protocol is generally chosen for its simplicity but implementation-related issues are often ignored. In particular, while the DF protocol is clearly suited to digital implementations, most of the existing research on the AF protocol makes the questionable assumption that relays are perfect *analog* devices which forward a scaled copy of the received signal.

One of the motivations for the work presented in the paper is precisely to propose low-complexity relaying schemes (comparable to the AF protocol complexity) that can be implemented in a digital relay transceiver (in contrast with the AF protocol) and use the knowledge of the SNRs of the forward and backward channels in order for the relay to optimally adapt to the forward and backward channel conditions. To achieve these goals, the main solution proposed is a quantize-and-forward (QF) protocol for which forwarding is done on a symbol-by-symbol basis and aims to minimize the mean square error (MSE) between the source signal and its reconstructed version at the output of the dequantizer at the destination. Some researchers have also referred to the classic Wyner-Ziv source coding scheme in [3] as QF [4, 5]. Our practical approach, which ultimately aims to minimize the raw bit-error rate (BER) at the destination for a fixed transmit spectral efficiency and does not exploit error correcting coding, differs from these information-theoretic works. It also differs from other practical studies on EF protocols, such as [6–8] in the sense that the corresponding relaying schemes are not analytically optimized by taking the

SNRs of the backward and forward channels into account. Rather, our work is based on the joint source-channel coding approach originally introduced in [9] for the Gaussian point-to-point channel where the authors extended the original iterative Lloyd algorithm by designing a scalar quantizer that takes into account the channel through which the quantized Gaussian source is to be transmitted. The authors of [10] applied this approach in the context of the binary symmetric channel (BSC) and proved that the corresponding distortion is a nonincreasing function of the number of iterations of the optimization algorithm. In this paper, we further extend the iterative algorithm of [9] in the context of quasistatic orthogonal relay channels by taking into account both the forward and backward channels and providing a nonrestrictive sufficient condition for convergence of the derived algorithm, similarly to [10].

This paper is organized as follows: in Section 2, the signal model for the orthogonal relay channel, main assumptions, and notation are given; in Section 3, the proposed QF scheme and a modified AF scheme are provided; in Section 4, we propose an ML detector (MLD) in order to account for the quantization noise introduced by the relay; in Section 5, the proposed schemes are evaluated in terms of raw BER and compared with AF, which serves as a reference strategy; concluding remarks are provided in Section 6.

2. SYSTEM MODEL

The source is assumed to be represented by a discrete-time signal x, which takes its value in the finite set of equiprobable symbols $\mathcal{X} = \{x_1, \ldots, x_{M_s}\}$ and is subject to a unit average power constraint: $E[|x^2|] = 1$. For sake of simplicity, square M_s-QAM symbols with independent real and imaginary parts are assumed. More importantly, the samples of the source, denoted by $x(n)$ where n is the time index, are assumed to be independent and identically distributed (i.i.d.) as in [9, 10]. In the context of digital communications, this assumption is generally valid because of interleaving, dithering, or equivalent operations. In order to limit the relay and receiver complexity, we will not exploit the interactions between the quantizer and the error correcting coders, possibly present at the source and relay. Therefore the assumption made on the source samples and channel model (described just below) implies that there is loss of optimality by assuming *scalar* quantizers, that is, symbol-by-symbol forwarding at the relay, instead of vector quantizers [11]. At each time instant n the source broadcasts the signal $x(n)$, which is received by the destination and relay nodes. The received baseband signals can be written:

$$\begin{aligned} y_{\rm sd}(n) &= h_{\rm sd} \times x(n) + w_{\rm sd}(n), \\ x_{\rm sr}(n) &= h_{\rm sr} \times x(n) + w_{\rm sr}(n), \end{aligned} \tag{1}$$

where $w_{\rm sd}$ and $w_{\rm sr}$ are zero-mean circularly symmetric complex Gaussian noises with variances $\sigma^2_{\rm sd}$ and $\sigma^2_{\rm sr}$, respectively. The complex coefficients $h_{\rm sd}$ and $h_{\rm sr}$ are, respectively, the gains of the source-destination and source-relay channels. In this paper, for simplicity of presentation, most of the derivations are conducted for static channels, so $h_{\rm sd}$ and $h_{\rm sr}$ are constant over the whole transmission. Therefore, the presence of these gains makes only sense in the quasi-static case whereas in the context of static channels they could be removed. In this case, these quantities are assumed to be constant over a block duration and vary from block to block. In the simulation part both cases will be analyzed and Rayleigh block-fading will be assumed for modeling the channel gains in the case of quasistatic channels. In this case, for each block, $h_{\rm sd}$ and $h_{\rm sr}$ are the realizations of two independent Gaussian complex random variables. Note that, thanks to the independency assumption between all the fading gains, the presence of the relay will provide more degrees of freedom in the channel, which will be exploited at the receiver through signal combiners that will provide a diversity order of two instead of one (this assertion can be proven for the two combiners provided in this paper, for more information see [12]). Therefore, one has to keep in mind that in quasistatic channels the performance gain due to the presence of the relay can also come from the qualities of the source-relay and relay-destination channels but is, in general, essentially due to the higher diversity order. In static channels (namely, Gaussian channels or fading channels with a strong Rician component) only a gain in terms of SNR can be expected.

The relay forwards the cooperation signal $x_r(n)$ to the destination. We assume memoryless and zero-delay relaying. The memoryless assumption is a consequence of the previously mentioned independence assumptions while the zero-delay assumption can be satisfied by resynchronizing the direct and cooperation signals at the destination. Under these assumptions $x_r(n) = f(x_{\rm sr}(n))$ for some memoryless function which is chosen to satisfy a unit average power constraint $E[|x_r|^2] = 1$. Since the relay function and channels are memoryless, in the sequel we will at times omit the time index n from the signals. For the QF protocol the relaying function comprises a zero-memory quantization operation (denoted by $\mathcal{Q}$) followed by an M_r-QAM modulation (denoted by $\mathcal{M}$). In the case of the clipped AF protocol, there is no modulation since the relay is assumed to generate a continuous signal. The cooperation signal received at the destination is written:

$$\begin{aligned} y_{\rm rd}(n) &= h_{\rm rd} \times x_r(n) + w_{\rm rd}(n) \\ &= h_{\rm rd} \times f\left[h_{\rm sr}x(n) + w_{\rm sr}(n)\right] + w_{\rm rd}(n), \end{aligned} \tag{2}$$

where the notation is defined above. Orthogonality between the received cooperation signal $y_{\rm rd}$ and direct signal $y_{\rm sd}$ can be implemented by frequency division (FD). The optimal bandwidth allocation issue is beyond the scope of this paper, thus we assume that $y_{\rm sd}$ and $y_{\rm rd}$ have the same bandwidth.

At the destination, two types of combiners can be assumed. We will use either a conventional maximum ratio combiner (MRC) or a more sophisticated detector, namely the MLD, which will be derived in Section 4. The reason for introducing the latter combiner will be clearly explained in Section 4. Figure 1 summarizes the system model when QF is assumed. The notation $\mathcal{D}$ stands for decoder, which jointly incorporates the demodulation and de-quantization operations. On the other hand, when the relay amplifies-and-forward, $\mathcal{D}$ is the identity operator and $\mathcal{Q}$ and $\mathcal{M}$ are

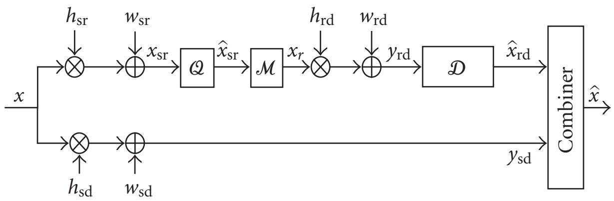

Figure 1: System model for the quantize-and-forward protocol.

replaced with a linear function in the AF case and a nonlinear function in the clipped AF case (Section 3.2).

3. RELAYING SCHEMES

3.1. Optimum quantize-and-forward

The most natural way to quantize and forward the signal received by the relay is to quantize x_{sr} in order to minimize the distortion $D_{00} \triangleq E[|\hat{x}_{sr} - x_{sr}|^2]$, map the quantizer output onto a QAM modulation and send it to the destination. As $\sigma_{rd}^2 \to 0$ and the number or quantization bits increases, this quantization strategy becomes optimum since it achieves the performance of a 1×2 single-input multiple-output (SIMO) system. On the other hand, if x_{sr} is quantized with a high number of bits and sent through a bad cooperation channel, minimizing D_{00} is no longer optimal. This is why minimizing $D_{01} \triangleq E[|\hat{x}_{rd} - x_{sr}|^2]$ can be more efficient as shown by [9, 10, 13, 14] in the context of the point-to-point Gaussian channel. In the context of the relay channel we know that the source-relay channel quality also plays a role in the receiver performance. Therefore, we propose minimizing the MSE between the reconstructed signal $\hat{x}_{rd}$ and the original source signal x, that is, $D_{11} \triangleq E[|\hat{x}_{rd} - x|^2]$, by assuming the SNRs of the forward and backward channels are known to the relay. The disadvantage of minimizing D_{11} is that in the high cooperation regime the SIMO performance is not reached. We will comment on this point further (Section 5.2).

Let us turn our attention to the quantizer itself. Since the signal to be quantized is complex, the quantizer is composed of two "subquantizers," one for the real part of x_{sr} and one for its imaginary part. Quantizing consists in mapping the signal x_{sr} into a pair of rational numbers belonging to $\mathcal{V}^R \times \mathcal{V}^I = \{v_1^R, v_2^R, \ldots, v_L^R\} \times \{v_1^I, v_2^I, \ldots, v_L^I\}$, where $L = 2^{b/2}$ and b is the total number of quantization bits. As the real and imaginary parts of the signal received by the relay are assumed to be independent, the two subquantizers can be designed independently and in the same manner. This is why, from now on, we restrict our attention to the subquantizer of the real part of x_{sr}. The subquantizer maps $x_{sr}^R \triangleq \mathrm{Re}(x_{sr})$ onto the finite set of representatives $\{v_1^R, v_2^R, \ldots, v_L^R\}$. Let $\mathcal{U}^R = \{u_1^R, u_2^R, \ldots, u_{L+1}^R\}$ be the set of the transition levels of the subquantizer. The aforementioned mapping is done as follows: if $x_{sr}^R \in S_j^R = [u_j^R, u_{j+1}^R)$ then its representative is v_j^R, where $j \in \{1, 2, \ldots, L\}$. The quantizer output is then mapped onto the constellation following the idea of [15]. The mapping is done in such a manner that close representatives in the signal space are assigned to close symbols in the modulation space. Therefore, the most likely decision errors which appear in the neighborhood of the symbol associated with the input representative will result in a slight increase in distortion. We now describe the quantizer optimization procedure. To find the optimal pair of subquantizers at the relay, we minimize the MSE D_{11} as follows. The distortion can be written as

$$\begin{aligned} D_{11} &\triangleq E\left[\,|\hat{x}_{rd} - x|^2\right] \\ &= \underbrace{E\left[(\hat{x}_{rd}^R)^2\right] - 2E[\hat{x}_{rd}^R x^R] + E\left[(x^R)^2\right]}_{D_{11}^R} \\ &\quad + \underbrace{E\left[(\hat{x}_{rd}^I)^2\right] - 2E[\hat{x}_{rd}^I x^I] + E\left[(x^I)^2\right]}_{D_{11}^I}. \end{aligned} \tag{3}$$

As D_{11}^R and D_{11}^I can be optimized independently and identically, we focus, hence forth, on minimizing D_{11}^R. The latter quantity can be shown to expand as

$$\begin{aligned} D_{11}^R &= \sum_j (x_j^R)^2 p_j - 2\sum_j x_j^R p_j \sum_{\ell=1}^{L} v_\ell^R \sum_{k=1}^{L} P_{k,\ell}^R \int_{u_k^R}^{u_{k+1}^R} \phi(t - x_j^R)\,dt \\ &\quad + \sum_j p_j \sum_{\ell=1}^{L} (v_\ell^R)^2 \sum_{k=1}^{L} P_{k,\ell}^R \int_{u_k^R}^{u_{k+1}^R} \phi(t - x_j^R)\,dt, \end{aligned} \tag{4}$$

where $\forall j \in \{1, \ldots, \sqrt{M_s}\}$, $p_j = \Pr[X^R = x_j^R]$ (i.e., the channel input statistics), $\forall (k, \ell) \in \{1, \ldots, L\}^2$, $P_{k,\ell}^R = \Pr[\hat{x}_{rd}^R = v_\ell^R \mid \hat{x}_{sr}^R = v_k^R]$ (i.e., the forward channel statistics) and $\phi(t) = (|h_{sr}|/\sqrt{\pi}\sigma_{sr}) \exp(-|h_{sr}|^2 t^2/\sigma_{sr}^2)$ is the probability density function (pdf) of the real noise component $\mathrm{Re}(w_{sr})$ of the signal received by the relay (i.e., the backward channel statistics). Given a number of quantization bits, we now optimize the subquantizer Q^R by minimizing D_{11}^R with respect to the transition levels $\{u_\ell^R\}_{\ell \in \{1,\ldots,L\}}$ and the representatives $\{v_\ell^R\}_{\ell \in \{1,\ldots,L\}}$. For fixed transition levels, the optimum representatives are the centroids of the corresponding quantization cells which are obtained by setting the partial derivatives of D_{11}^R to zero:

$$v_\ell^R = \frac{\sum_{k=1}^{\sqrt{M_s}} x_k^R p_k \sum_{j=1}^{L} P_{j,\ell}^R \int_{u_j^R}^{u_{j+1}^R} \phi(t - x_k^R)\,dt}{\sum_{k=1}^{\sqrt{M_s}} p_k \sum_{j=1}^{L} P_{j,\ell}^R \int_{u_j^R}^{u_{j+1}^R} \phi(t - x_k^R)\,dt}. \tag{5}$$

When the representatives are fixed, it is not trivial, in general, to determine the transition levels explicitly as is the case for conventional channel optimized quantizers such as [10] for which the backward channel is not present. The difficulty is due to the presence of the function $\phi(\cdot)$ in the MSE expression (for more information see Appendix A). Determining the transition levels then requires the use of an exhaustive search algorithm. However, note that there are simple cases such as a 4-QAM source, which is used in the simulations (Section 5), where both the optimum representatives for fixed transition levels and optimum transition levels for fixed representatives can be found. For a 4-QAM constellation, we have $(x^R, x^I) \in \{-A, +A\}^2$. For fixed transition levels, we have for all $\ell \in \{1, \ldots, L\}$ that

$$v_\ell^{R,*} = A \times \frac{\sum_{k=1}^{L} P_{k,\ell}^R \int_{u_k^R}^{u_{k+1}^R} [\phi(t-A) - \phi(t+A)]dt}{\sum_{k=1}^{L} P_{k,\ell}^R \int_{u_k^R}^{u_{k+1}^R} [\phi(t-A) + \phi(t+A)]dt}, \quad (6)$$

and for fixed representatives we have

$$u_\ell^{R,*} = \frac{\sigma_{\rm sr}^2}{2A} \ln \left[\frac{\sum_{k=1}^{L} (P_{\ell,k}^R - P_{\ell-1,k}^R)(A + (1/2)v_k^R)v_k^R}{\sum_{k=1}^{L} (P_{\ell,k}^R - P_{\ell-1,k}^R)(A - (1/2)v_k^R)v_k^R} \right]. \quad (7)$$

Note that in (7) the strict positiveness of the argument of the logarithm ensures the existence of the optimum transition levels. We are now in position to provide the complete iterative optimization procedure. Let i and ϵ be the iteration index and the current value of the estimation error criterion of the iterative algorithm. The algorithm is said to have converged when ϵ reaches $\epsilon_{\max}$.

(i) *Step 1.* Set $i = 0$. Set $\epsilon = 1$. Initialize $\mathcal{V}^R$ and $\mathcal{U}^R$ with the sets (say $\mathcal{V}_{(0)}^R$ and $\mathcal{U}_{(0)}^R$) obtained from [10], which correspond to a local optimum since the backward channel is not taken into account.

(ii) *Step 2.* Set $i \to i+1$. For the fixed partition $\mathcal{U}_{(i-1)}^R$ use (6) to find the optimal codebook $\mathcal{V}_{(i)}^R$. For the fixed codebook $\mathcal{V}_{(i)}^R$ use (7) to obtain the optimal partition $\mathcal{U}_{(i)}^R$. If the realizability condition $u_1^R \leq u_2^R \cdots \leq u_L^R$ is not met, stop the procedure and keep the transition levels provided by the previous iteration.

(iii) *Step 3.* Update ϵ as follows:

$$\epsilon = \frac{\sum_{k=1}^{L} | v_{k(i)}^R - v_{k(i-1)}^R |}{\sum_{k=1}^{L} | v_{k(i)}^R |}. \quad (8)$$

If $\epsilon \geq \epsilon_{\max}$, then go to Step 2, stop otherwise.

As with other iterative algorithms (e.g., the EM algorithm), one cannot easily prove or ensure, in general, convergence to the global optimum. When the backward channel was not present, the authors of [10] proved that the distortion obtained by applying the generalized Lloyd algorithm is a nonincreasing function of the number of iterations. The authors provided a sufficient condition under which the procedure is guaranteed to converge towards a local optimum. The corresponding condition is not restrictive since it can be imposed through the realizability constraint of the transition levels [10] to the iterative procedure without loss of optimality. Recall that this constraint consists in imposing u_ℓ^R to be an increasing function of ℓ. It turns out a similar result can be derived in our context (see Appendix A) if one assumes a zero-mean channel input (i.e., $E[X^R] = 0$) and the backward channel noise to be Gaussian. The obtained condition is as follows: at each iteration step, $\forall \ell \in \{1, \ldots, L-1\}$, $E[\widehat{X}_{\rm rd}^R \mid \widehat{X}_{\rm sr}^R = v_{\ell+1}^R] > E[\widehat{X}_{\rm rd}^R \mid \widehat{X}_{\rm sr}^R = v_\ell^R]$. If this condition is met, the MSE will be a nonincreasing function of the iteration index.

To conclude this section we will make a few comments on the complexity of the proposed protocol. Compared to vector quantizers [16], the proposed solution is much simpler since the creation, storage and computation complexities (for more information see, e.g., [17]) both grow exponentially with the cell dimension (which is 1 for scalar quantizers). If one wants to further decrease the complexity of the quantizer, it is possible to simplify the proposed algorithm by imposing the quantizer to be uniform (equispaced transition levels and representatives). Since the uniform quantizer is entirely specified by its quantization step there is only one parameter to be determined. We will not conduct a complexity analysis here but it can be checked (Appendix C) that the ratio of the optimum QF protocol complexity to that of the uniform version is of the order of the number of iterations of the proposed algorithm, which is typically between 5 and 10 in simulations. The uniform QF protocol can be obtained by using [9] and by specializing the results presented here. The performance of the corresponding scheme will be presented in the simulation part.

3.2. *Clipped amplify-and-forward*

In this section, we propose a modified version of the AF protocol. Our motivation for proposing this new version of AF is threefold. First, it optimizes the same performance criterion as for the QF schemes, that is, the end-to-end distortion. Second, it allows us to fairly compare the scalar QF schemes with the scalar AF scheme given the fact that the conventional AF does not exploit the knowledge of the SNRs of the source-relay and relay-destination channels. Third, the clipped AF bridges the gap between the QF and AF protocols since it allows us to isolate the clipping effect naturally introduced by the QF schemes. So, we now replace the quantizer with a piecewise linear saturation function, which simply clips samples with magnitude above a chosen threshold $\beta > 0$. The linear threshold function which operates independently on the real and imaginary parts of the signal is defined as

$$f_\beta^R(x^R) = \begin{cases} x^R, & |x^R| \leq \beta, \\ \beta \cdot \operatorname{sgn}(x^R), & |x^R| > \beta, \end{cases} \quad (9)$$

where we considered the case of the real part. We see that the relay acts like a perfect AF relay in the region $[-\beta, \beta]$ and limits values outside this region. Our motivation for using this function is to assess the benefits from clipping $x_{\rm sr}$ but in some context better relaying functions can be used. For example the authors of [18] derived the best relaying function in the sense of the raw BER when no direct link is assumed and a BPSK modulation is used both at the source and relay. In our context, the goal is different and the extension of [18] to the case of QAM modulations does not seem to be trivial. In the same spirit, [19] proposed an optimized relaying function in the sense of the mutual information when no direct link is assumed. The rationale for the proposed function (9) is that it preserves the important soft information but does not needlessly expend power relaying large noise samples. Furthermore, it only requires the optimization of a single parameter, that is, the clipping level β. In spite of the seeming simplicity of the relaying function, however, calculating the p.d.f. of saturated Gaussian signals is known to be intractable [20]. After passing the received signal through the saturation function, the signal is scaled by some real parameter α which is chosen to satisfy the average unit power constraint. Of course, in order to ensure coherent reception at the relay node, the incoming signal also has to be equalized. Here, our choice is the MMSE (minimum MSE) equalizer: $x_{\text{sr}}^R(n) = \text{Re}[x_{\text{sr}}(n) \times (h_{\text{sr}}^*/|h_{\text{sr}}|^2)]$. Thus, the cooperation signal is $x_r(n) = \alpha[f_\beta^R(x_{\text{sr}}^R(n)) + j f_\beta^I(x_{\text{sr}}^I(n))]$, where α is such that $E[|x_r|^2] = 1$, and $f_\beta^R(\cdot)$ and $f_\beta^I(\cdot)$ are defined identically since the real and imaginary parts are assumed to be i.i.d. Due to the fact that the data and noise are independent and by calculating the first and the second-order moments (Appendix B) for the random clipped gaussian variable, we find that

$$
\begin{aligned}
&E|x_r|^2 \\
&= \alpha^2 E[f_\beta^2(x_{\text{sr}}^R) + f_\beta^2(x_{\text{sr}}^I)] = 2\alpha^2 E[f_\beta^2(x_{\text{sr}}^R)] \\
&= \frac{2\alpha^2}{\sqrt{M_s}} \sum_{x^R} \Bigg\{ \frac{\sigma_{\text{sr}}^2}{2|h_{\text{sr}}|^2} + (x^R)^2 + \left[\beta^2 - \frac{\sigma_{\text{sr}}^2}{2|h_{\text{sr}}|^2} - (x^R)^2\right] \\
&\qquad \times \left[Q\left(\frac{\beta + x^R}{\sigma_{\text{sr}}/\sqrt{2}|h_{\text{sr}}|}\right) + Q\left(\frac{\beta - x^R}{\sigma_{\text{sr}}/\sqrt{2}|h_{\text{sr}}|}\right)\right] \\
&\qquad - \frac{\sigma_{\text{sr}}}{2\sqrt{\pi}|h_{\text{sr}}|}\Big[(\beta + x^R) e^{-(\beta - x^R)^2/\sigma_{\text{sr}}^2/|h_{\text{sr}}|^2} \\
&\qquad\qquad + (\beta - x^R) e^{-(\beta + x^R)^2/(\sigma_{\text{sr}}^2/|h_{\text{sr}}|^2)}\Big]\Bigg\}
\end{aligned} \tag{10}
$$

which can be set equal to 1 to find the scaling factor α_β that satisfies the power constraint. Note that Q is the classical error function: $Q(x) = (1/\sqrt{2\pi})\int_x^\infty e^{-t^2} dt$. To find the clipping level β, we minimize the MSE between the source signal andthe signal received by the destination on the cooperation channel:

$$
J(\beta) = E\left[\left|\frac{1}{\alpha_\beta}\frac{h_{\text{rd}}^*}{|h_{\text{rd}}|^2} y_{\text{rd}} - x\right|^2\right] \tag{11}
$$

$$
= E\left[\left|f_\beta(x_{\text{sr}}^R) + j\cdot f_\beta(x_{\text{sr}}^I) + \frac{1}{\alpha_\beta}\frac{h_{\text{rd}}^*}{|h_{\text{rd}}|^2} w_{\text{rd}} - x\right|^2\right] \tag{12}
$$

$$
= \frac{2}{\sqrt{M_s}} \sum_{x^R} \left\{ E\left[f_\beta^2\left(x^R + \frac{w_{\text{sr}}^R}{h_{\text{sr}}}\right) - 2x^R f_\beta\left(x^R + \frac{w_{\text{sr}}^R}{h_{\text{sr}}}\right)\right] + \frac{\sigma_{\text{rd}}^2}{\alpha_\beta^2 |h_{\text{rd}}|^2} + 1 \right\}, \tag{13}
$$

where we note that α_β is effectively a function of β since any change in the clipping level affects the scaling required to satisfy the power constraint. The β that minimizes this function cannot be written in closed form. However, it is purely a function of the source-relay and relay-destination SNRs, so it can be computed numerically offline using (13) and the calculation of the first and second-order moments for the clipped Gaussian (Appendix B) and stored in a lookup table. Note that this implies that the relay needs to know the SNRs on its channel both to the transmitter as well as to the receiver, which was also the case in the QF protocol.

4. COMBINING SCHEMES

When the AF protocol is assumed at the relay, the optimum combiner in terms of raw BER is the MRC. When using the clipped version of the AF protocol this is no longer true since the equivalent additive noise in the relay-destination channel is not Gaussian. As already mentioned, calculating the pdf of saturated Gaussian signals is known to be intractable. Therefore, we will still use the MRC at the destination when the clipped AF is used. We will see through the simulation analysis that this issue does not seem to be critical but deriving a better combiner might be seen as an *extension* of this work. On the other hand, when QF is assumed, using the MRC at the destination can lead to a significant performance loss. In this respect the authors have shown in [21] that using the DF protocol with a conventional MRC when the relay is in bad reception conditions can severely degrade the BER performance at the destination with respect to the case without cooperation. This is in part because the relay generates non-Gaussian residual decoding noise that is correlated with the useful signal. For the QF protocol the combiner choice might look less critical since the relay does not make a decision on the transmitted symbols. However, for a low number of quantization bits and relay receive SNR, the answer is not clear. This is why we not only consider the MRC but also propose a more sophisticated detector (namely, the MLD) adapted to the QF protocol, which is derived as follows.

Assume the symbol transmitted by the source is x and the quantizer output $\mathcal{Q}(x_{\text{sr}}) = v_i$. The likelihood $p_{\text{ML}} = p(y_{\text{sd}}, \hat{x}_{\text{rd}} \mid x)$ can be factorized as

$$
\begin{aligned}
p_{\text{ML}} &= p(y_{\text{sd}} \mid x) p(\hat{x}_{\text{rd}} \mid x, y_{\text{sd}}) \\
&= p(y_{\text{sd}} \mid x) \frac{p(y_{\text{sd}} \mid \hat{x}_{\text{rd}}, x) p(\hat{x}_{\text{rd}}, x)}{p(y_{\text{sd}} \mid x) p(x)} \\
&= p(y_{\text{sd}} \mid x) \frac{p(y_{\text{sd}} \mid x) p(\hat{x}_{\text{rd}} \mid x) p(x)}{p(y_{\text{sd}} \mid x) p(x)} \\
&= p(y_{\text{sd}} \mid x) p(\hat{x}_{\text{rd}} \mid x),
\end{aligned} \tag{14}
$$

where $p(y_{\text{sd}} \mid x) = (1/\pi\sigma_{\text{sd}}^2)\exp(-|y_{\text{sd}} - h_{\text{sd}}x|^2/\sigma_{\text{sd}}^2)$. To expand the second term $p(\hat{x}_{\text{rd}} \mid x)$, we recall that $\hat{X}_{\text{rd}} \in \mathcal{V}^R \times \mathcal{V}^I = \{v_1, v_2, \ldots, v_{M_r}\}$, and we make use of the channel transitions probabilities $P_{k,\ell}$ between complex representatives

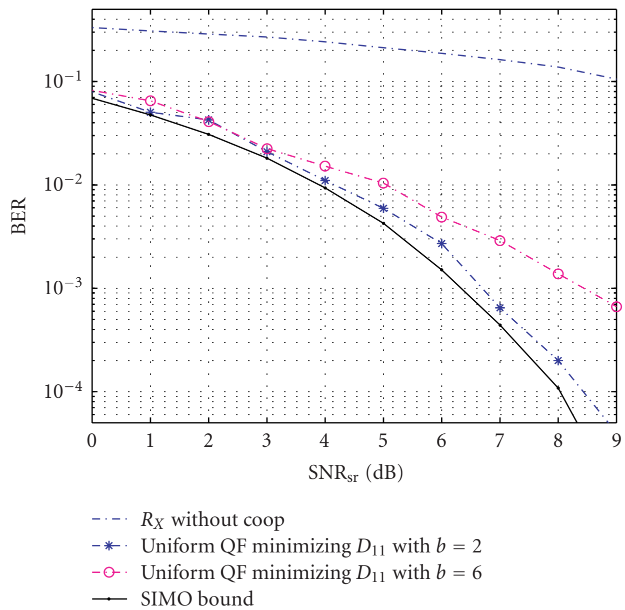

Figure 2: Influence of b on the performance when the uniform QF protocol is used over static channels: raw BER versus SNR_{sr} at the output of the MRC for $\text{SNR}_{\text{rd}} = 10$ dB with $\text{SNR}_{\text{sr}} = \text{SNR}_{\text{sd}} + 10$ dB.

(see Section 3.1) where we have defined $P^R_{k,\ell}$ for the real part of complex representatives. We have

$$\begin{aligned} p(\hat{x}_{\text{rd}} = v_i \mid x) &= \int_{x_{\text{sr}}} p(x_{\text{sr}}, \hat{x}_{\text{rd}} = v_i \mid x)\, dx_{\text{sr}} \\ &= \int_{x_{\text{sr}}} p(x_{\text{sr}} \mid x)\, p(\hat{x}_{\text{rd}} = v_i \mid x_{\text{sr}})\, dx_{\text{sr}} \\ &= \sum_{j=1}^{M_r} \left[\int_{x_{\text{sr}} \in S_j} p(x_{\text{sr}} \mid x) p(\hat{x}_{\text{rd}} = v_i \mid x_{\text{sr}})\, dx_{\text{sr}} \right] \\ &= \sum_{j=1}^{M_r} P_{j,i} \left[\int_{x_{\text{sr}} \in S_j} p(x_{\text{sr}} \mid x)\, dx_{\text{sr}} \right] \\ &= \sum_{\ell=1}^{\sqrt{(M_r)}} \sum_{m=1}^{\sqrt{(M_r)}} P_{j,i} \\ &\quad \times \left[\int_{u^R_\ell}^{u^R_{\ell+1}} \phi(t - x^R)\, dt \int_{u^I_m}^{u^I_{m+1}} \phi(t' - x^I)\, dt' \right], \end{aligned} \tag{15}$$

where the index j corresponds the symbol of the relay alphabet (i.e., $\{1, \dots, M_r\}$) associated with the pair of representatives (v^R_ℓ, v^I_m). Now, by denoting $\underline{s} = (s_1, \dots, s_N)$, the vector of bits associated with the source symbol x allows us to express the log-likelihood ratio for the nth bit:

$$\lambda(s_n) = \log \left[\frac{\sum_{\underline{s} \in S_1^{(n)}} p(y_{\text{sd}} \mid x) p(\hat{x}_{\text{rd}} \mid x)}{\sum_{\underline{s} \in S_0^{(n)}} p(y_{\text{sd}} \mid x) p(\hat{x}_{\text{rd}} \mid x)} \right], \tag{16}$$

where the sets $S_1^{(i)}$ and $S_0^{(i)}$ are defined by $S_1^{(n)} = \{(s_1, \dots, s_N) \in \{0,1\}^N \mid s_n = 1\}$ and $S_0^{(n)} = \{(s_1, \dots, s_N) \in \{0,1\}^N \mid s_n = 0\}$. If $\lambda(s_n) > 0$, then $\hat{s}_n = 1$ and $\hat{s}_n = 0$ otherwise.

5. SIMULATION ANALYSIS

We assume a 4-QAM source and consider different simulation scenarios with the following parameters:

(i) the channels can be either static (Gaussian or purely Rician) or quasistatic (Rayleigh block-fading model); in the latter case the channels are constant over a block duration; each block comprises 100 symbols; we note that the case of static channels can correspond to real situations in wireless communications, for example, fixed users using laptops connected to a hot-spot;

(ii) the relative quality of the relay: $\text{SNR}_{\text{sr}}[\text{dB}] = \text{SNR}_{\text{sd}}[\text{dB}] + \rho$, where $\rho \in \{-5\text{ dB}, 0\text{ dB}, +10\text{ dB}\}$;

(iii) the number of quantization bits used by the QF protocol: $b \in \{2, 6\}$ (i.e., $b/2$ bits per subquantizer);

(iv) the relay-destination channel quality: $\text{SNR}_{\text{rd}}[\text{dB}] \in \{0\text{ dB}, 10\text{ dB}, 30\text{ dB}\}$ with $\text{SNR}_{\text{rd}} = 1/\sigma^2_{\text{rd}}$;

(v) the relaying scheme: AF, optimally clipped AF, uniform QF, and optimum QF; for reference, we will consider the case where no relay is available (a BPSK is then used at the transmitter in order to make a fair comparison in terms of spectral efficiency) and also the full cooperation case; the latter is defined as follows: $\sigma_{\text{rd}} \to 0$ and the AF protocol is used; we will refer to this case as the SIMO bound;

(vi) the combining scheme at the receiver: MRC or MLD.

5.1. *Optimum QF versus uniform QF*

All the simulations we performed showed one significant drawback of the uniform QF relaying protocol. Both in static and quasistatic channels, the receiver performance, when using the *uniform* QF protocol with MRC or MLD, is sensitive to the choice of the number of quantization bits. This tendency is clearly more marked for static channels. For example, see Figures 2 and 3. Figure 2 shows that using the uniform QF with $b = 6$ bits can lead to a significant performance loss. This appears when the source-relay SNR is sufficiently large and the cooperation channel has medium quality. In this situation it is better to decode and forward than quantize and forward a signal that is not robust to cooperation channel noise. When $b/2 = 1$ the uniform QF roughly behaves like DF while it behaves more like AF for $b = 6$, which explains why the performance is better for $b = 6$ in Figure 3. Our interpretation is that the uniform QF has only one degree of freedom (namely, its quantization step) to adapt to SNR_{sr} and SNR_{rd}. For a fixed number of bits, there will always be scenarios where the performance of the uniform QF can be much less than the optimum relaying scheme (AF, DF, or optimum QF) used in the considered setup. On the other hand, the number of quantization bits has much less influence on the performance of the *optimum* QF when the MLD is employed at the receiver. By analyzing many simulations, which are not provided here due to lack of space, we have observed

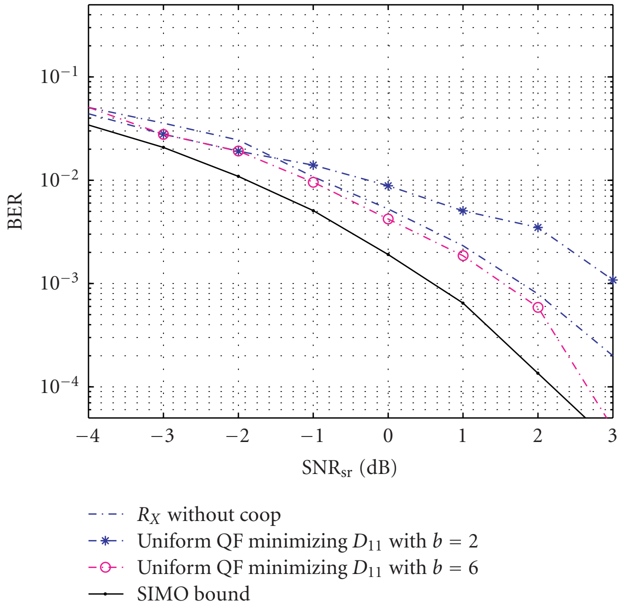

FIGURE 3: Influence of b on the performance when the uniform QF protocol is used over static channels: raw BER versus $\mathrm{SNR_{sr}}$ at the output of the MRC for $\mathrm{SNR_{rd}} = 10$ dB with $\mathrm{SNR_{sr}} = \mathrm{SNR_{sd}} - 5$ dB.

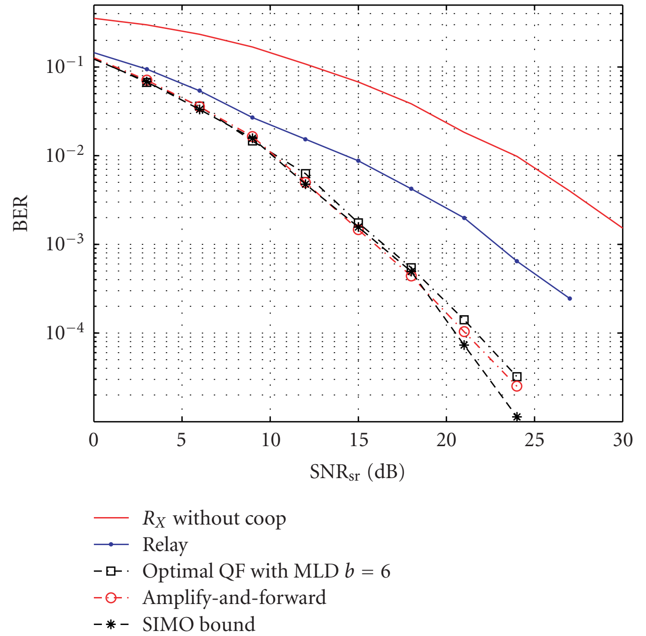

FIGURE 4: Comparison between the optimum QF ($b = 6$) and the AF schemes in quasistatic channels for $\mathrm{SNR_{rd}} = 40$ dB, $\mathrm{SNR_{sr}} = \mathrm{SNR_{sd}} + 10$ dB.

that it is generally better to choose a sufficiently high number of bits (typically 3 bits per dimension) regardless of the SNRs of the different channels. Our explanation is that the optimum quantizer produces a grid of centroids that looks like the source constellation. The constellation in the output of the quantizer looks like a constellation with 2 resolution levels: there are 4 clouds (for a 4-QAM) of centroids, with each cloud comprising 2^{b-2} centroids that are typically concentrated around the cloud center. Depending on $\mathrm{SNR_{sd}}$ and $\mathrm{SNR_{sr}}$, the optimum QF can adapt both the location of the cloud centers and the points around each center.

5.2. *Comparison between the different relaying protocols*

Many simulations showed us the following trend: in quasistatic channels, the receiver performs quite similarly no matter which relaying protocol (AF, clipped AF, or optimum QF) is used, provided that the preferred combining scheme is employed (i.e., the MRC is used for AF and clipped AF, and MLD is used for optimum QF). This is essentially due to the averaging effect of the channel conditions. Figure 4 compares the receiver performance of AF + MRC with optimum QF + MLD. Figure 5 shows that the conventional and clipped AF protocols perform similarly. However, the relaying strategy is more influential in static channels. Figure 6 shows a typical example. Other simulations with different numbers of quantization bits and SNR values can be roughly summarized as follows: for low and medium transmit or cooperation powers, the optimum QF provides the best performance whereas the performance loss in the high cooperation regime is always small, which means that the SIMO bound is almost achieved by optimum QF in the latter regime. Also the AF tends to perform better than the QF protocol in situations where the source-relay channel is bad. Now let us comment on the effect of clipping the signal received by a relay using the AF protocol in static channels. The obtained performance gain obtained by clipping depends on $\mathrm{SNR_{sr}}$ and $\mathrm{SNR_{rd}}$. For low and medium cooperation channel qualities, this

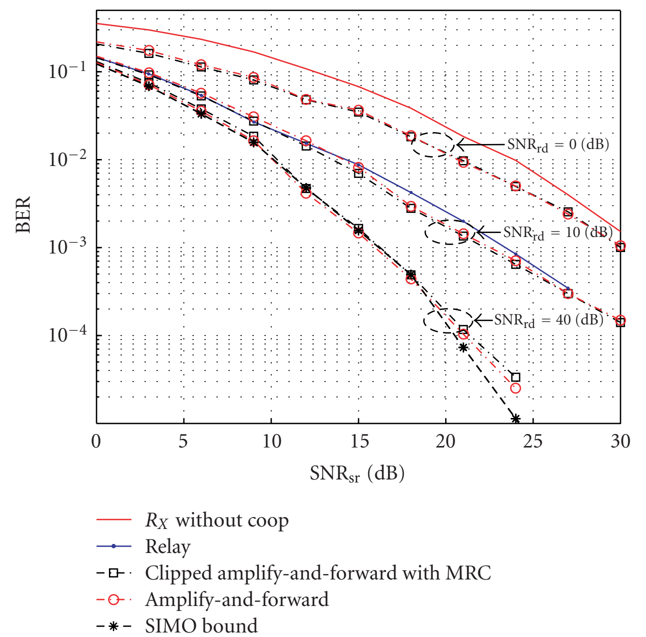

FIGURE 5: Comparison between the AF and clipped AF protocols in quasistatic channels for $\mathrm{SNR_{sr}} = \mathrm{SNR_{sd}} + 10$ dB.

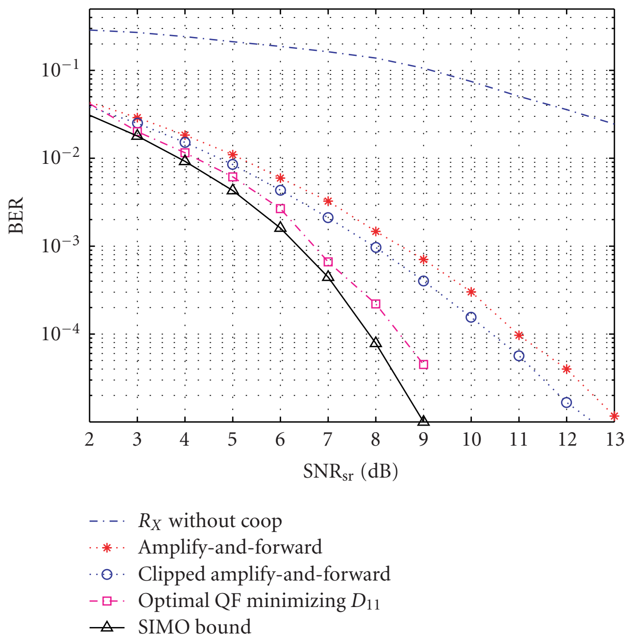

Figure 6: Comparison of the different relaying schemes (AF, clipped AF, optimum QF with $b = 6$) in static channels for $\text{SNR}_{\text{sr}} = \text{SNR}_{\text{sd}} + 10\,\text{dB}$ with $\text{SNR}_{\text{rd}} = 10\,\text{dB}$.

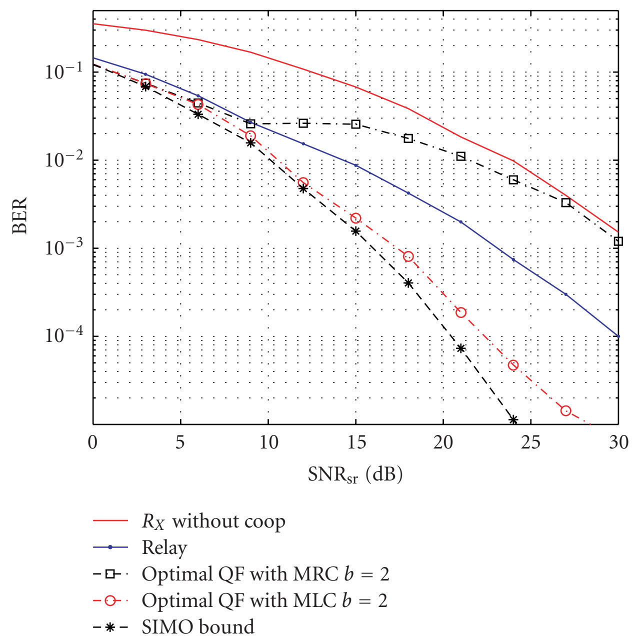

Figure 8: Influence of of the combining scheme for the optimal QF scheme ($b = 2$) in quasistatic channels with $\{\text{SNR}_{\text{rd}} = 10\,\text{dB}, \text{SNR}_{\text{sr}} = \text{SNR}_{\text{sd}} - 10\,\text{dB}\}$.

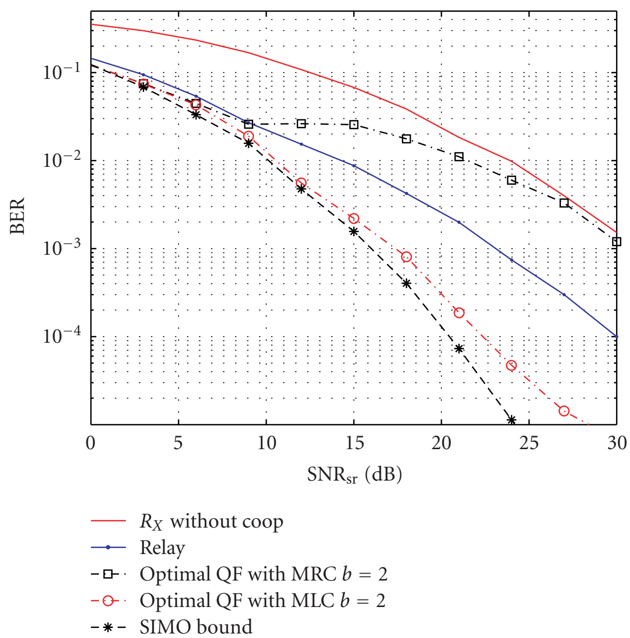

Figure 7: Influence of of the combining scheme for the optimal QF scheme ($b = 2$) in quasistatic channels with $\{\text{SNR}_{\text{rd}} = 40\,\text{dB}, \text{SNR}_{\text{sr}} = \text{SNR}_{\text{sd}} + 10\,\text{dB}\}$.

gain typically ranges from 0.5 dB to 1.5 dB, depending on SNR_{sr}. In the high cooperation regime, it is small and can even be slightly negative since the clipped AF minimizes the distortion while the AF reaches the SIMO bound when $\text{SNR}_{\text{rd}} \to \infty$.

5.3. Importance of the combining scheme for the QF protocol

As already mentioned, when optimum QF is assumed, the facts that the receiver performance is not very sensitive to the number of quantization bits and is close to that obtained by the AF protocol is in part due to the use of the MLD instead of MRC. This can be shown both in static and quasistatic channels. In this subsection, we want to illustrate this point by an explicit comparison. Figures 7 and 8, respectively, represent the receiver performance over quasistatic channels in two markedly different scenarios: (a) a good relay, a good cooperation channel, and $b = 6$; (b) a bad relay, a medium quality cooperation channel, and $b = 2$. In both cases the MLD brings a significant performance gain, which shows the importance of using a receiver structure adapted to the assumed relaying scheme.

6. CONCLUSION

We have proposed a low-complexity quantize-and-forward scheme, which exploits the knowledge of the SNRs of the source-relay and relay-destination channels. In static channels it generally performs close to or better than the conventional or clipped AF protocols. Also, based on knowledge of the SNRs, clipping can provide a nonnegligible (and almost free in terms of complexity) gain with respect to the conventional AF, whose value depends on the different SNRs. Over Rayleigh block-fading channels, we have seen that the optimum QF protocol, provided it is associated with an ML detector, has generally similar performance to the conventional or clipped AF protocols, whatever the simulation scenario. Although the clipped AF and QF protocols can be shown not to be strictly equivalent for a high number of quantization bits (because of the presence of the dequantizer at the end of relay-destination channel), the following comment can be made: since the optimum QF protocol is both scalar and simple and generally performs closely to the AF protocol, this shows that the proposed solution can be seen as a way of implementing a channel-optimized AF-type protocol in a digital relay transceiver. Now, if the relay and receiver complexity can be relaxed, the proposed approach can be improved by exploiting the structure inherent to channel coding, which can be seen as an *extension* of this work.

APPENDICES

A. A SUFFICIENT CONDITION FOR CONVERGENCE OF THE MSE IN THE OPTIMUM QUANTIZER DESIGN

First, we derive the MSE expression in our context:

$$\begin{aligned} D_{11}^R &\triangleq E\big(\widehat{X}_{\mathrm{rd}}^R - X^R\big)^2 \\ &= \sum_{x^R\in\mathcal{X}^R}\sum_{\hat{x}_{\mathrm{rd}}^R\in\mathcal{V}^R}\int_{w_{\mathrm{sr}}^R}\big(\hat{x}_{\mathrm{rd}}^R - x^R\big)^2 p\big(\hat{x}_{\mathrm{rd}}^R, x^R, w_{\mathrm{sr}}^R\big)\,dw_{\mathrm{sr}}^R \\ &= \sum_j\sum_\ell\sum_k \int_{u_k^R - x_j^R}^{u_{k+1}^R - x_j^R}\big(x_j^R - v_\ell^R\big) \\ &\quad\times p\big(v_\ell^R \mid x_j^R, w_{\mathrm{sr}}^R\big)\,p\big(x_j^R\big)\,p\big(w_{\mathrm{sr}}^R\big)\,dw_{\mathrm{sr}}^R \\ &= \sum_{j,k,\ell}\big(x_j^R - v_\ell^R\big)^2 \Pr\big[\hat{x}_{\mathrm{rd}}^R = v_\ell^R \mid \hat{x}_{\mathrm{sr}}^R = v_k^R\big] \\ &\quad\times p\big(x_j^R\big)\int_{u_k^R - x_j^R}^{u_{k+1}^R - x_j^R}\phi\big(w_{\mathrm{sr}}^R\big)\,dw_{\mathrm{sr}}^R \\ &= \sum_{j,k,\ell} p_j P_{k,\ell}\big(x_j^R - v_\ell^R\big)^2 \\ &= \int_{u_k^R}^{u_{k+1}^R}\phi\big(t - x_j^R\big)\,dt. \end{aligned} \tag{A.1}$$

Assume the transition levels to be fixed. Then the MSE is a strictly convex function of v_ℓ^R over $\mathbb{R}$. Indeed, the second partial derivative of the MSE with respect to v_ℓ is given by the following expression: $\partial^2 D_{11}^R/\partial(v_\ell^R)^2 = 2\sum_{j,k} p_j P_{k,\ell}\int_{u_k^R}^{u_{k+1}^R}\phi(t - x_j^R)dt$. For all $\ell \in \{1,\ldots,L\}$, the strict positiveness of this second derivative implies that updating the representatives v_ℓ according to (5) cannot increase the overall MSE. Now, assume the representatives are fixed. The second partial derivative of the MSE with respect to u_ℓ can be expanded as follows:

$$\begin{aligned} \frac{\partial^2 D_{11}^R}{\partial(u_\ell^R)^2} &\overset{(a)}{=} \sum_{j,k} p_j\big[P_{k,\ell} - P_{k,\ell+1}\big]\big(x_j^R - v_k^R\big)^2\phi'\big(u_\ell^R - x_j^R\big) \\ &\overset{(b)}{=} -\frac{2|h_{\mathrm{sr}}|^2}{\sigma_{\mathrm{sr}}^2}\sum_{j,k} p_j\big[P_{k,\ell} - P_{k,\ell+1}\big] \\ &\quad\times\big(x_j^R - v_k^R\big)^2\big(u_\ell^R - x_j^R\big)\,\phi\big(u_\ell^R - x_j^R\big) \\ &= -\frac{2|h_{\mathrm{sr}}|^2}{\sigma_{\mathrm{sr}}^2}\bigg\{u_\ell^R\frac{\partial D_{11}^R}{\partial u_\ell^R} + \sum_{j,k} p_j x_j^R\big[P_{k,\ell} - P_{k,\ell+1}\big] \\ &\quad\times\big(x_j^R - v_k^R\big)^2\phi\big(u_\ell^R - x_j^R\big)\bigg\} \\ &\overset{(c)}{=} \frac{2|h_{\mathrm{sr}}|^2}{\sigma_{\mathrm{sr}}^2}\sum_j p_j x_j^R \\ &\quad\times\Big\{E\big[\big(\widehat{X}_{\mathrm{rd}}^R\big)^2 \mid \widehat{X}_{\mathrm{sr}}^R = v_\ell^R\big] - E\big[\big(\widehat{X}_{\mathrm{rd}}^R\big)^2 \mid \widehat{X}_{\mathrm{sr}}^R = v_{\ell+1}^R\big]\Big\} \\ &\quad\times\phi\big(u_\ell^R - x_j^R\big) + \frac{2|h_{\mathrm{sr}}|^2}{\sigma_{\mathrm{sr}}^2}\sum_j 2p_j\big(x_j^R\big)^2 \\ &\quad\times\Big\{E\big[\widehat{X}_{\mathrm{rd}}^R \mid \widehat{X}_{\mathrm{sr}}^R = v_{\ell+1}^R\big] - E\big[\widehat{X}_{\mathrm{rd}}^R \mid \widehat{X}_{\mathrm{sr}}^R = v_\ell^R\big]\Big\}\phi\big(u_\ell^R - x_j^R\big) \\ &= \frac{2|h_{\mathrm{sr}}|^2}{\sigma_{\mathrm{sr}}^2}E\big[X^R\big] \\ &\quad\times\Big\{E\big[\big(\widehat{X}_{\mathrm{rd}}^R\big)^2 \mid \widehat{X}_{\mathrm{sr}}^R = v_\ell^R\big] - E\big[\big(\widehat{X}_{\mathrm{rd}}^R\big)^2 \mid \widehat{X}_{\mathrm{sr}}^R = v_{\ell+1}^R\big]\Big\} \\ &\quad\times\phi\big(u_\ell^R - x_j^R\big) + \frac{4|h_{\mathrm{sr}}|^2}{\sigma_{\mathrm{sr}}^2}E\big[\big(X^R\big)^2\big] \\ &\quad\times\Big\{E\big[\widehat{X}_{\mathrm{rd}}^R \mid \widehat{X}_{\mathrm{sr}}^R = v_{\ell+1}^R\big] - E\big[\widehat{X}_{\mathrm{rd}}^R \mid \widehat{X}_{\mathrm{sr}}^R = v_\ell^R\big]\Big\}\phi\big(u_\ell^R - x_j^R\big) \\ &\overset{(d)}{=} \frac{4|h_{\mathrm{sr}}|^2}{\sigma_{\mathrm{sr}}^2}E\big[\big(X^R\big)^2\big] \\ &\quad\times\Big\{E\big[\widehat{X}_{\mathrm{rd}}^R \mid \widehat{X}_{\mathrm{sr}}^R = v_{\ell+1}^R\big] - E\big[\widehat{X}_{\mathrm{rd}}^R \mid \widehat{X}_{\mathrm{sr}}^R = v_\ell^R\big]\Big\}\phi\big(u_\ell^R - x_j^R\big), \end{aligned} \tag{A.2}$$

where (a) $\phi'(t) \triangleq (d\phi/dt)(t)$; (b) $\phi'(t) = -(2|h_{\mathrm{sr}}|^2 t/\sigma_{\mathrm{sr}}^2)\,\phi(t)$; (c) the optimum transition levels verify $(\partial D_{11}^R/\partial u_\ell^R)(u_\ell^{R,*}) = 0$ for all ℓ; (d) the channel input X^R is assumed to be a zero-mean random variable. As a consequence, if, for all ℓ, $E[\widehat{X}_{\mathrm{rd}}^R \mid \widehat{X}_{\mathrm{sr}}^R = v_{\ell+1}^R] > E[\widehat{X}_{\mathrm{rd}}^R \mid \widehat{X}_{\mathrm{sr}}^R = v_\ell^R]$, then updating the transition levels in the MSE cannot increase the MSE. This gives a sufficient condition for the convergence of the iterative algorithm under investigation.

B. FIRST- AND SECOND-ORDER MOMENTS OF CLIPPED GAUSSIAN

Let $z\sim\mathcal{N}(\mu,\sigma^2)$, and let $f_\beta(\cdot)$ be the clipping function defined in (9). For $\beta = 1$, the first- and second-order moments of a clipped Gaussian signal are then given by

$$\begin{aligned} E\big[f_1(z)\big] &= \frac{1}{\sqrt{2\pi\sigma^2}}\int_{-\infty}^{\infty} f_1(z)e^{-(z-\mu)^2/2\sigma^2}dz \\ &= \frac{1}{\sqrt{2\pi\sigma^2}}\int_{-1}^{1} z e^{-(z-\mu)^2/2\sigma^2}dz \\ &\quad - \frac{1}{\sqrt{2\pi\sigma^2}}\int_{-\infty}^{-1} e^{-(z-\mu)^2/2\sigma^2}dz \\ &\quad + \frac{1}{\sqrt{2\pi\sigma^2}}\int_{1}^{\infty} e^{-(z-\mu)^2/2\sigma^2}dz \end{aligned}$$

Table 1

	Optimal quantizer	Uniform quantizer
Creation*	$\max\{\mathcal{O}(cL^2A^{2/3}), \mathcal{O}(cS\sqrt{M_s}LA^{2/3}), \mathcal{O}(cS\sqrt{M_s}L^2)\}$	$\max\{\mathcal{O}(SL^2\sqrt{M_s}), \mathcal{O}(S\sqrt{M_s}LA^{2/3})\}$
Storage*	$\mathcal{O}(L)$	$\mathcal{O}(L)$
Computation**	$\mathcal{O}(L)$	$\mathcal{O}(L)$

* Per SNR value.
** Per symbol to quantize.

$$
\begin{aligned}
&= \mu + \frac{1}{\sqrt{2\pi}}\sigma e^{-(1+\mu)^2/2\sigma^2} - e^{-(1-\mu)^2/2\sigma^2} \\
&\quad - \mu\left[Q\left(\frac{1+\mu}{\sigma}\right) + Q\left(\frac{1-\mu}{\sigma}\right)\right] \\
&\quad - Q\left(\frac{1+\mu}{\sigma}\right) + Q\left(\frac{1-\mu}{\sigma}\right), \\
E[f_1^2(z)] &= \frac{1}{\sqrt{2\pi\sigma^2}}\int_{-\infty}^{\infty} f_1^2(z)e^{-(z-\mu)^2/2\sigma^2}dz \\
&= \frac{1}{\sqrt{2\pi\sigma^2}}\int_{-1}^{1} z^2 e^{-(z-\mu)^2/2\sigma^2}dz \\
&\quad + \frac{1}{\sqrt{2\pi\sigma^2}}\int_{-\infty}^{-1} e^{-(z-\mu)^2/2\sigma^2}dz \\
&\quad + \frac{1}{\sqrt{2\pi\sigma^2}}\int_{1}^{\infty} e^{-(z-\mu)^2/2\sigma^2}dz \\
&= (\sigma^2+\mu^2) - \frac{1}{\sqrt{2\pi}}\sigma\Big[(1+\mu)e^{-(1-\mu)^2/2\sigma^2} \\
&\qquad + (1-\mu)e^{-(1+\mu)^2/2\sigma^2}\Big] \\
&\quad + (1-\sigma^2-\mu^2)\left[Q\left(\frac{1+\mu}{\sigma}\right) + Q\left(\frac{1-\mu}{\sigma}\right)\right].
\end{aligned}
\tag{B.1}
$$

C. COMPLEXITY ANALYSIS FOR THE UNIFORM AND OPTIMUM QF PROTOCOLS

See Table 1 *c: number of iterations; A: accuracy in number of used digits; S: number of tested points in the exhaustive search.*

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REFERENCES

[1] A. El Gammal, M. Mohseni, and S. Zahedi, "Bounds on capacity and minimum energy-per-bit for AWGN relay channels," *IEEE Transactions on Information Theory*, vol. 52, no. 4, pp. 1545–1561, 2006.

[2] J. N. Laneman, D. N. C. Tse, and G. W. Wornell, "Cooperative diversity in wireless networks: efficient protocols and outage behavior," *IEEE Transactions on Information Theory*, vol. 50, no. 12, pp. 3062–3080, 2004.

[3] T. M. Cover and A. A. El Gamal, "Capacity theorems for the relay channel," *IEEE Transactions on Information Theory*, vol. 25, no. 5, pp. 572–584, 1979.

[4] M. A. Khojastepour, A. Sabharwal, and B. Aazhang, "Lower bounds on the capacity of gaussian relay channel," in *Proceedings of the 38th Conference on Information Sciences and Systems*, pp. 597–602, Princeton, NJ, USA, March 2004.

[5] M. Katz and S. Shamai, "Relaying protocols for two colocated users," *IEEE Transactions on Information Theory*, vol. 52, no. 6, pp. 2329–2344, 2006.

[6] Z. Liu, V. Stanković, and Z. Xiong, "Wyner-ziv coding for the half-duplex relay channel," in *Proceedings of the IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP '05)*, vol. 5, pp. 1113–1116, 2005.

[7] R. Hu and J. Li, "Exploiting slepian-wolf codes in wireless user cooperation," in *IEEE Workshop on Signal Processing Advances in Wireless Communications, (SPAWC '05)*, vol. 2005, pp. 275–279, 2005.

[8] A. Chakrabarti, A. De Baynast, A. Sabharwal, and B. Aazhang, "Half-duplex estimate-and-forward relaying: bounds and code design," in *IEEE International Symposium on Information Theory (ISIT '06)*, pp. 1239–1243, Seattle, Wash, USA, July 2006.

[9] A. Kurtenbach and P. Wintz, "Quantizing for noisy channels," *IEEE Transactions on Communications*, vol. 17, pp. 291–302, 1969.

[10] N. Farvardin and V. Vaishampayan, "Optimal quantizer design for noisy channels: an approach to combine source-channel coding," *IEEE Transactions on Information Theory*, vol. 33, no. 6, pp. 827–837, 1987.

[11] N. Farvardin, "Study of vector quantization for noisy channels," *IEEE Transactions on Information Theory*, vol. 36, no. 4, pp. 799–809, 1990.

[12] T. Wang, A. Cano, G. B. Giannakis, and J. N. Laneman, "High-performance cooperative demodulation with decode-and-forward relays," *IEEE Transactions on Communications*, vol. 55, no. 7, pp. 1427–1438, 2007.

[13] V. Vaishampayan and N. Farvardin, "Joint design of block source codes and modulation signal sets," *IEEE Transactions on Information Theory*, vol. 38, pp. 1230–1248, 1992.

[14] F. H. Liu, P. Ho, and V. Cuperman, "Joint source and channel coding using a non-linear receiver," in *Proceedings of the IEEE International Conference on Communications*, pp. 1502–1507, 1993.

[15] H. Skinnemoen, "Modulation organized vector quantization, MOR-VQ," in *Proceedings of the IEEE International Symposium on Information Theory*, p. 238, June 1994.

[16] Y. L. Linde, A. Buzo, and R. M. Gray, "An algorithm for vector quantizer design," *IEEE Transactions on Communications*, vol. 28, pp. 84–95, 1980.

[17] J. Hamkins and K. Zeger, "Gaussian source coding with spherical codes," NASA Technical Report, October 2005.

[18] I. Abou-Faycal and M. Médard, "Optimal uncoded regeneration for binary antipodal signaling," in *Proceedings of the IEEE International Conference on Communications*, vol. 2, pp. 742–746, June 2004.

[19] K. S. Gomadam and S. A. Jafar, "On the capacity of memoryless relay networks," in *Proceedings of the IEEE International Conference on Communications*, Istanbul, Turkey, June 2006.

[20] T. C. Huang, "Signaling performance over a piecewise linear limited channel in the presence of interference and gaussian noise," *IEEE Transactions on Communications*, vol. 31, no. 7, pp. 861–870, 1983.

[21] B. Djeumou, S. Lasaulce, and A. G. Klein, "Combining decoded-and-forwarded signals in gaussian cooperative channels," in *Proceedings of the IEEE International Symposium on Signal Processing and Information Technology (ISSPIT '06)*, pp. 622–627, Vancouver, Canada, August 2006.