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Research Article

Employing LSF at Transmitter Eases MMSE Adaptation at Receiver in Asynchronous CDMA Systems

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The Lebesgue spectrum filter (LSF), a finite impulse response (FIR) filter whose coefficients decay exponentially with a negative factor $r := \sqrt{3} - 2$, is shown to be effective preprocessing for spreading code in asynchronous code-division multiple-access (CDMA) systems. The LSF has only been studied independently from the well-known minimum mean-square error (MMSE) filter, an optimal FIR filter in the mean-square error sense. In this paper, we propose an efficient structure, employing the LSF at the transmitter and the MMSE filter at the receiver, for asynchronous CDMA systems. We employ a spreading code preprocessed by the LSF (referred to as LSF-code), and the LSF-code supplies a "best" initial estimate (among the ones obtained without any a priori information) to an adaptive algorithm for the MMSE filter, leading to significant reduction of iterations in adaptation. This is verified by computer simulations. Also we investigate the link between the LSF and the MMSE filter by examining their autocorrelation properties.

1. INTRODUCTION

We study two kinds of finite impulse response (FIR) filters "optimal" in different senses for an asynchronous direct sequence code-division multiple-access (DS/CDMA) system. The first is *the Lebesgue spectrum filter (LSF)* [1], which is a fixed FIR filter given by $\boldsymbol{\ell} := [r, r^2, \dots, r^M]$ (M is the order of LSF), where $r := \sqrt{3} - 2$ is an optimal value for multiple-access interference suppression in asynchronous CDMA systems [2–4]. The second is *the minimum mean-square error (MMSE) filter* [5, 6], which depends on environments and is the optimal linear filter in the sense of minimizing the mean-square error (MSE). It has been reported that the MMSE filter is effective in suppressing multiple access interference (MAI) in the DS/CDMA systems [7–14]. A practical approach to construct the MMSE filter in real time is the adaptive filter [15], and, when the adaptive filter is adopted, the number of iterations in adaptation needs to be significantly small to realize high spectral efficiency.

In this paper, we propose a simple and effective structure, for asynchronous DS/CDMA systems, employing the LSF at the transmitter and the MMSE filter at the receiver. The purpose of employing the LSF is *not* to improve further the MAI suppression capability, but is to reduce the iterations required for adaptation. At the transmitter, we convolve a randomly generated binary sequence with the LSF, and refer to the resulting sequence as LSF-code. Since it provides an adaptive linear receiver with a "best" initial estimate in an average sense, the LSF-code allows the adaptive algorithm to start from a closer point to the MMSE filter than any other codes constructed without any a priori knowledge. As a result, the algorithm provides a reasonable approximation of the MMSE filter in a small number of iterations; in other words, the filter can reduce the adaptation time. It should be mentioned that, although there could exist a preprocessing better than LSF-coding *for each specific situation*, such a preprocessing would require channel state information (CSI) in advance and extra computational costs for encoding/decoding; moreover, the performance will be sensitive to inaccuracy of the CSI. In contrast, LSF-coding requires no a priori knowledge and little extra computational costs. Finally, the autocorrelation properties of the linear receivers are examined, which indicates (i) a connection between the LSF and the MMSE receiver, and (ii) an intrinsic distinction between synchronous and asynchronous systems.

The rest of the paper is organized as follows. In Section 2, the system design, the MMSE receiver, and the LSF-code are described. In Section 3, we compare the performance of the matched-/MMSE-filters for random-/LSF-codes in asynchronous systems under various conditions, and then show that the proposed structure significantly reduces the adaptation-time due to the effect of LSF-code. In Section 4, the autocorrelation properties are studied, followed by the conclusion in Section 5.

2. PRELIMINARIES

In this section, we present the system model, the MMSE receiver, and the design of LSF-code.

2.1. System model

We consider an (asynchronous) uplink CDMA system with K mobile users, described in Figure 1. (In uplink transmission, the users usually transmit their symbols without synchronization, hence the system is *asynchronous* in general.) For simplicity, the carrier modulation/demodulation is not considered in this work (in other words, all the simulations and considerations are carried out with baseband signals). Without any loss of generality, we assume that the 1st user is the desired one. The discrete-time expression of the received baseband signal for the ith transmitted bits is given as follows [5]:

$$\mathbf{r}[i] = A_1 b_1[i]\mathbf{s}_1 + \sum_{j=2}^{K} \left(A_j b_j[i]\mathbf{a}_{0,j} + A_j b_j[i-1]\mathbf{a}_{1,j}\right) + \mathbf{n}[i], \tag{1}$$

where

(i) $A_j \in (0,\infty)$: amplitude of the jth user;

(ii) $\mathbf{s}_j \in \mathbb{R}^N$: spreading code of the jth user ($\|\mathbf{s}_j\| = 1$);

(iii) $N \in \mathbb{N}^*(:= \mathbb{N} \setminus \{0\})$: processing gain;

(iv) $b_j[i] \in \{1,-1\}$: ith transmitted bit of the jth user;

(v) $\mathbf{n}[i] \in \mathbb{R}^N$: noise vector;

(vi) $\mathbf{a}_{0,j} := \phi_{1,j}\left[\begin{smallmatrix}\mathbf{0}_{\tau_j}\\ [\mathbf{s}_j]_{1:N-\tau_j}\end{smallmatrix}\right] + \phi_{2,j}\left[\begin{smallmatrix}\mathbf{0}_{\tau_j+1}\\ [\mathbf{s}_j]_{1:N-\tau_j-1}\end{smallmatrix}\right]$;

(vii) $\mathbf{a}_{1,j} := \phi_{1,j}\left[\begin{smallmatrix}[\mathbf{s}_j]_{N-\tau_j+1:N}\\ \mathbf{0}_{N-\tau_j}\end{smallmatrix}\right] + \phi_{2,j}\left[\begin{smallmatrix}[\mathbf{s}_j]_{N-\tau_j:N}\\ \mathbf{0}_{N-\tau_j-1}\end{smallmatrix}\right]$;

(viii) $\phi_{1,j} := \int_0^{T_c}\psi(t)\psi(t+\delta_j T_c)dt$;

(ix) $\phi_{2,j} := \int_0^{T_c}\psi(t)\psi[t+(1-\delta_j)T_c]dt$;

(x) $\psi(t)$: chip-waveform;

(xi) $\nu_j = (\tau_j + \delta_j)T_c \in [0,T)$: relative delay of the jth user;

(xii) $T \in (0,\infty)$: the bit-duration;

(xiii) $T_c := T/N$: the chip-duration;

(xiv) $\tau_j := \lfloor \nu_j/T_c \rfloor \in \{0,1,\ldots,N-1\} \subset \mathbb{N}$;

(xv) $\delta_j := \nu_j/T_c - \tau_j \in [0,1) \subset \mathbb{R}$.

Here, $[\mathbf{a}]_{b:c}$ designates the subvector of $\mathbf{a}$ corresponding to the bth to cth elements if $b \le c$, otherwise, the null, and $\mathbf{0}_n$, $n \in \mathbb{N}$, denote the zero vector of length n (the simple notation $\mathbf{0}$ will be used to denote the zero vector when its length is clear from the context). In this study, we consider single-path channels and each channel gain $h_j(t)$, $j = 1,2,\ldots,K$, is incorporated into A_j. In the following, we assume the $\psi(t)$ is a rectangular pulse of width T_c in which case $\phi_{1,j} = 1-\delta_j$ and $\phi_{2,j} = \delta_j$. A note on the asynchronous systems is given in the appendix.

2.2. MMSE receiver

In estimation theory, the mean square error (MSE) has been a common criterion. The MSE of a linear filter $\mathbf{h} \in \mathbb{R}^N$ is defined as [5]

$$\mathrm{MSE}(\mathbf{h}) := E\left\{\left(\mathbf{r}[i]^T\mathbf{h} - b_1[i]\right)^2\right\}, \quad \forall \mathbf{h} \in \mathbb{R}^N, \tag{2}$$

where $E\{\cdot\}$ denotes *expectation*. For convenience, the following assumptions regarding the independence of signals and the whiteness of noise are widely used.

Assumption 1. (a) $E\{b_j[i]b_k[i]\} = 0$, $\forall j \neq k \in \{1,2,\ldots,K\}$, $\forall i \in \mathbb{N}$;

(b) $E\{b_j[i]b_j[i-1]\} = 0$, $\forall j \in \{1,2,\ldots,K\}$, $\forall i \in \mathbb{N}$;

(c) $E\{b_j[i]\mathbf{n}[i]\} = E\{b_j[i-1]\mathbf{n}[i]\} = \mathbf{0}$, $\forall j \in \{1,2,\ldots,K\}$, $\forall i \in \mathbb{N}$;

(d) $E\{\mathbf{n}[i]\mathbf{n}[i]^T\} = \sigma_{\mathrm{n}}^2\mathbf{I}$, $\sigma_{\mathrm{n}}^2 > 0$, $\forall i \in \mathbb{N}$.

Under Assumption 1, the MSE in (2) is reduced to

$$\mathrm{MSE}(\mathbf{h}) = \mathbf{h}^T\mathbf{R}\mathbf{h} - 2A_1\mathbf{h}^T\mathbf{s}_1 + 1, \quad \forall \mathbf{h} \in \mathbb{R}^N, \tag{3}$$

where

$$\mathbf{R} := E\{\mathbf{r}[i]\mathbf{r}[i]^T\} = A_1^2\mathbf{s}_1\mathbf{s}_1^T + \sum_{j=2}^{K} A_j^2\left(\mathbf{a}_{0,j}\mathbf{a}_{0,j}^T + \mathbf{a}_{1,j}\mathbf{a}_{1,j}^T\right) + \sigma_{\mathrm{n}}^2\mathbf{I} \tag{4}$$

is the autocorrelation matrix of the received vector $\mathbf{r}[i]$. Defining the $N \times 2(K-1)$ matrix

$$\mathbf{S} := \left[A_2\mathbf{a}_{0,2} \,|\, A_2\mathbf{a}_{1,2} \,|\, \cdots \,|\, A_K\mathbf{a}_{0,K} \,|\, A_K\mathbf{a}_{1,K}\right], \tag{5}$$

$\mathbf{R}$ can be expressed as

$$\mathbf{R} = A_1^2\mathbf{s}_1\mathbf{s}_1^T + \mathbf{S}\mathbf{S}^T + \sigma_{\mathrm{n}}^2\mathbf{I}. \tag{6}$$

A minimizer of (3) is called *the MMSE filter* (or *the MMSE receiver*), which, under Assumption 1(d), is uniquely given by

$$\mathbf{h}_{\mathrm{MMSE}} = A_1\mathbf{R}^{-1}\mathbf{s}_1 \in \mathbb{R}^N. \tag{7}$$

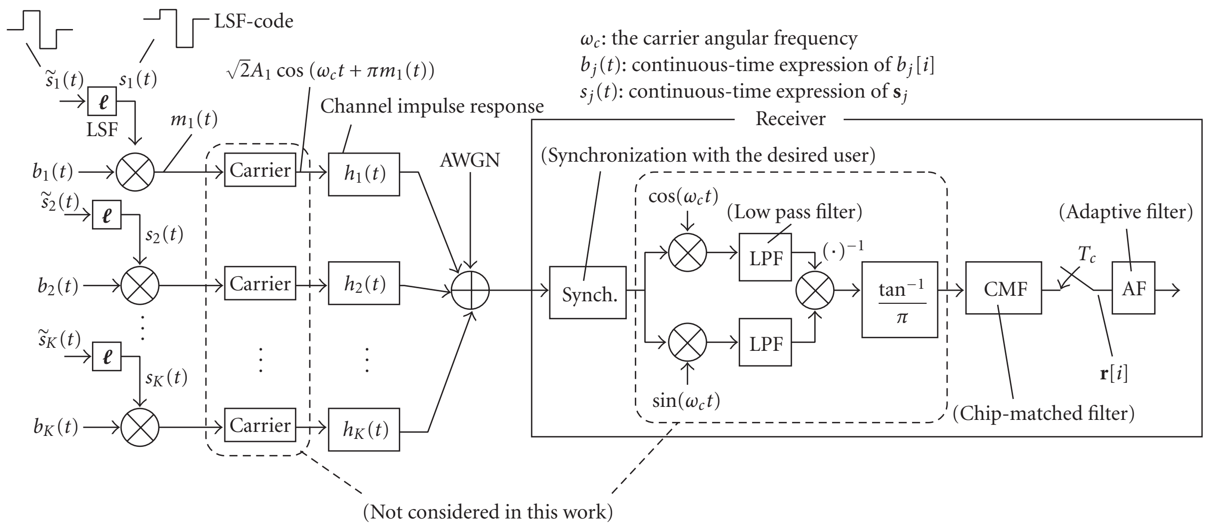


Figure 1: Uplink transmission scheme in a DS-CDMA system with LSF-code and phase shift keying modulation.

It is seen that the MMSE receiver exploits the structure of interference (contained in $\mathbf{R}$), as opposed to the conventional matched filter given simply by

$$\mathbf{h}_{\text{Matched}} := \mathbf{s}_1 \in \mathbb{R}^N. \quad (8)$$

2.3. LSF-code

We present preprocessing for spreading code by means of the LSF [1], which is placed at the transmitter (see Figure 1). The LSF-code for the order M is constructed as follows.

(1) Define the LSF with the order M as follows:

$$\boldsymbol{\ell} := [1, r, r^2, \ldots, r^{M-1}]^T \in \mathbb{R}^M, \quad (9)$$

where $r := \sqrt{3} - 2$.

(2) Given $N \in \mathbb{N}^*$, generate a temporary length-($N + M - 1$) binary random vector $\tilde{\mathbf{s}} \in \{1, -1\}^{N+M-1}$.

(3) Construct a length-N spreading code by normalizing the following vector:

$$\mathbf{s} := \begin{bmatrix} \boldsymbol{\ell}^T [\tilde{\mathbf{s}}]_{1:M} \\ \boldsymbol{\ell}^T [\tilde{\mathbf{s}}]_{2:M+1} \\ \vdots \\ \boldsymbol{\ell}^T [\tilde{\mathbf{s}}]_{N:N+M-1} \end{bmatrix} \in \mathbb{R}^N. \quad (10)$$

In short, the LSF-code is generated by passing a binary random sequence through the LSF $\boldsymbol{\ell}$, hence is no longer binary.

3. PROPOSED STRUCTURE FOR ASYNCHRONOUS CDMA SYSTEMS

In this section, we consider the following four methods (see Table 1):

Table 1: Classification based on modulation and demodulation schemes.

	LSF-code	random-code
MMSE filter	Method 1	Method 2
Matched filter	Method 3	Method 4

(1) modulate with an LSF-code and demodulate with the MMSE filter (which is the proposed structure);

(2) modulate with a random spreading code and demodulate with the MMSE filter;

(3) modulate with an LSF-code and demodulate with the matched filter;

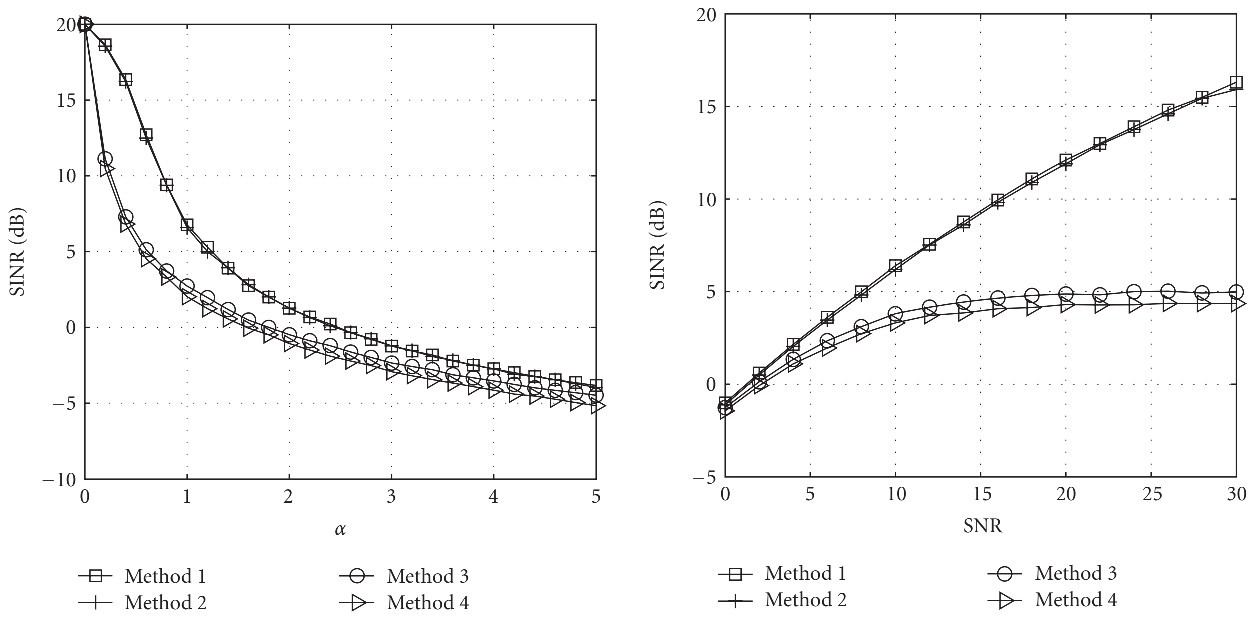
(4) modulate with a random spreading code and demodulate with the matched filter.

Firstly, we show that the MMSE filter (Methods 1 and 2), computed directly with (4) and (7), outperforms the matched filter (Methods 3 and 4). Then, we employ two types of adaptive algorithm to realize Methods 1 and 2, and show that Method 1 (the proposed structure) requires a much smaller number of iterations to converge than Method 2.

3.1. Comparison of four methods

We compare the performance of the four methods for the processing gain $N = 32$ under various conditions. Throughout the section, the order of LSF is set to $M = 3$. We employ the common performance measure called the signal to interference-plus-noise ratio (SINR), which is defined as follows:

$$\text{SINR}(\mathbf{h}) := \frac{E\{(A_1 b_1[i]\langle \mathbf{s}_1, \mathbf{h}\rangle)^2\}}{E\{(\langle \mathbf{r}[i] - A_1 b_1[i]\mathbf{s}_1, \mathbf{h}\rangle)^2\}}, \quad \forall \mathbf{h} \in \mathbb{R}^N. \quad (11)$$



(a) SNR = 20 dB

(b) $K = 20$

FIGURE 2: Comparisons of the four methods for the processing gain $N = 32$. For LSF, we let $M = 3$.

Under Assumption 1, (11) is reduced to

$$\text{SINR}(\mathbf{h}) = \frac{A_1^2 \langle \mathbf{s}_1, \mathbf{h} \rangle^2}{\mathbf{h}^T \mathbf{S}\mathbf{S}^T \mathbf{h} + \sigma_n^2 ||\mathbf{h}^2||}, \quad \forall \mathbf{h} \in \mathbb{R}^N. \tag{12}$$

We firstly fix the signal to noise ratio (SNR) := $10\log_{10}(A_1^2/\sigma_n^2)$ to SNR = 20 dB and change the number of users K (by following the way in [16]) as

$$K := \lfloor \alpha N \rfloor \quad \text{for } \alpha \in [0, 5]. \tag{13}$$

We assume that the amplitudes of all the users are equal. The results are depicted in Figure 2(a). Although omitted for visual clarity, almost identical curves are obtained for $N = 64, 128$; see [17]. This implies that the performance is a function of the ratio between the processing gain N and the number of users K. Thus, the figure is useful when, for example, the designer would like to know how many users can access, for a given N, to the same channel simultaneously with guaranteeing specified SINR performance.

We secondly fix $K = 20$ and change the SNR value (the other conditions are the same as in Figure 2(a)). The results are depicted in Figure 2(b). From Figure 2, for a wide range of situations, the following observation follows.

Observation

(1) Method 1 (or Method 2) performs better than Methods 3 and 4.

(2) The performance of Methods 1 and 2 is almost identical.

(3) The difference between Method 1 and Method 3 (or Method 4) is notable when the number of users is practically small (i.e., $\alpha \leq 1 \Leftrightarrow K \leq N$).

(4) The difference between Method 1 and Method 3 (or Method 4) is notable when SNR is practically large (i.e., SNR $\geq$ 10 dB).

(5) Method 3 performs better than Method 4 (cf. [1]).

The reason for the observation (1) and (2) is given below.

Remark 1. Unlike the MMSE receiver, the LSF does not depend on any specific parameters of the system, and optimizes the average performance in asynchronous systems according to the ergodic theory. This means that LSF is not an optimal FIR filter in each "specific" situation, whereas the MMSE receiver is. This is the reason for the observation (1). In Method 1, the MMSE receiver is constructed so that its convolution with the LSF plays nearly the same role as the MMSE receiver in Method 2. This is the reason for the observation (2).

So, *why should we use the LSF-code?* This will be clarified in the following subsection, but simply stated, the LSF-code allows an adaptive filter to realize (or well approximate) the MMSE receiver in a smaller number of iterations.

We finally examine the resistance of the methods to the *near-far* problem, which occurs when the power controlling systems perform imperfectly. Specifically, we consider the situations in which all interfering users have β times larger amplitudes than the desired one for $\beta = 1, 2, 10$. We set $N = 32$, SNR = 20 dB, and the number of interfering users $K - 1$ ranges between 0 and 40. Figure 3 depicts the results (we omit Method 2, because it is nearly identical to Method 1). It is seen that the performance of Methods 3 and 4 (i.e., matched filter) degrades severely by only a few number of strong interfering users. Meanwhile, Method 1 keeps high SINR performance (above 10 dB) even when the number of strong interfering users is up to $K - 1 = 14$ ($< N/2$). This

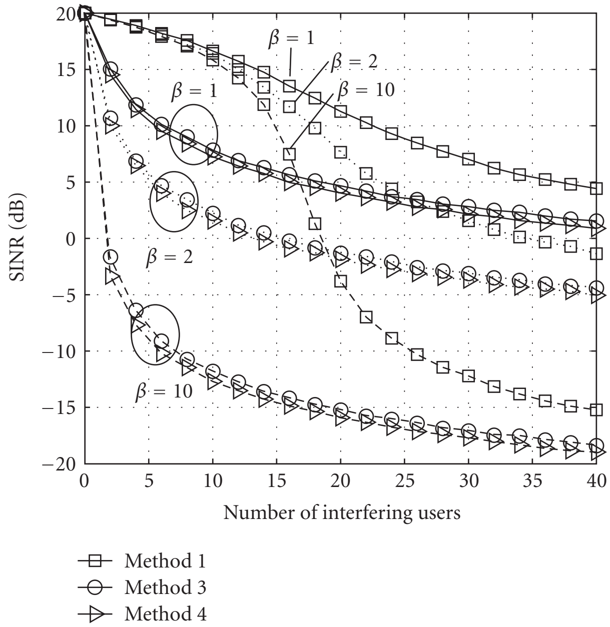


Figure 3: Near-far resistance of the methods for $N = 32$ under SNR = 20 dB. For LSF, we let $M = 3$.

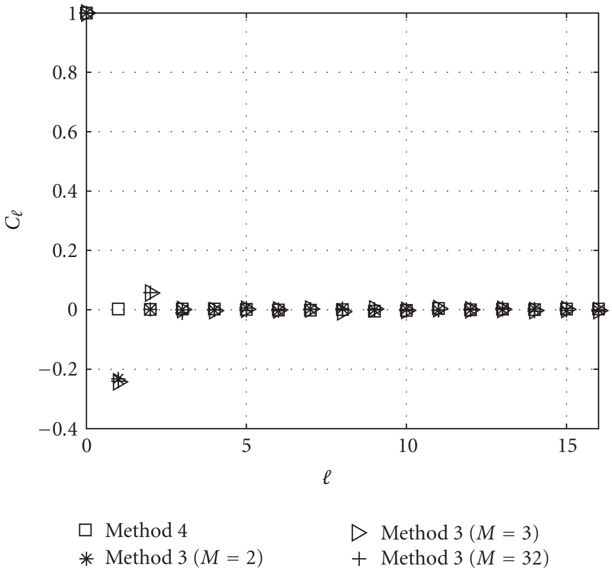


Figure 5: Correlation properties of the matched filter: Method 4 (random code) and Method 3 (LSF-code) for $M = 2, 3, 32$.

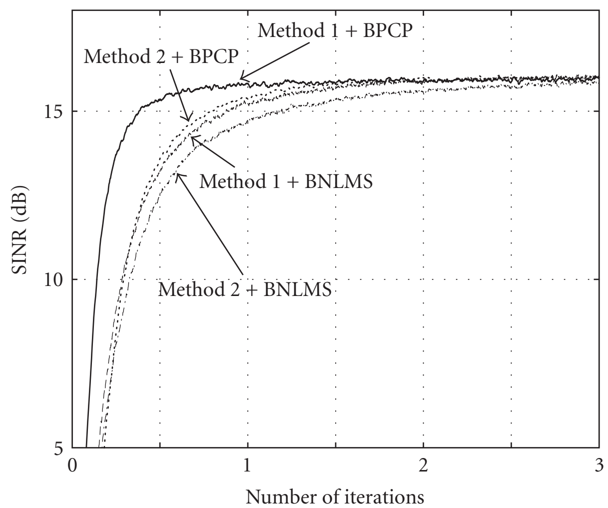


Figure 4: SINR curves for $N = 32$, $K = 8$, $\beta = 10$, $M = 3$ under SNR = 20 dB.

is consistent with one of the results in [18] that, for near-far resistant performance, the system design should satisfy $K - 1 < N/2$ for asynchronous CDMA systems.

3.2. *Method 1 versus Method 2 with adaptive algorithm*

The MMSE receiver (i.e., Method 1 or 2) involves the autocorrelation matrix of the received vector and its inverse. The autocorrelation matrix is in general unavailable and, even if available, the computational costs for its inverse are prohibitively high. Therefore, the adaptive filtering is a practical approach to realize the MMSE receiver in real time. We remark here that the processing in the adaptive filtering algorithm is independent of the choice of spreading codes. (The MMSE-filter coefficients in Methods 1 and 2 are multilevel in general.)

In the adaptive filtering approach, the matched filter, that is, the spreading code of the desired user, is commonly used as an initial estimate. An adaptive algorithm starts from the initial estimate and updates the estimate iteratively according to incoming data for achieving the MMSE receiver as quick as possible. The observation (1) suggests that the use of LSF-code in asynchronous systems provides the adaptive algorithm with a more accurate initial estimate. In other words, the adaptive algorithm can start from a closer point to the optimal MMSE receiver, leading to notable reduction in the number of iterations required to achieve sufficiently high SINR performance.

To verify this, simulations are conducted; Methods 1 and 2 are computed with two blind adaptive filtering algorithms: (i) blind-NLMS (BNLMS) [8] with its step size $\mu = 0.6$, and (ii) BPCP [14] with $q = 16$ parallel projections. The transmitted symbols $b_j[i]$ ($j = 1, 2, \ldots, K$, $i \in \mathbb{N}$) are binary (“+1” or “−1”), and $b_1[i]$ is detected, with an adaptive filter $\mathbf{h}[i] \in \mathbb{R}^N$, by taking the sign of the filter output $\mathbf{h}^T[i]\mathbf{r}[i]$. For a fair comparison, the parameters of each method are adjusted so that the steady-state performance is comparable to each other; in this case, it is meaningful to compare the number of iterations required for achieving a certain level of SINR. For BPCP, we set $\lambda_k = 0.04$ and $\rho = 0.4$ for Method 1, and $\lambda_k = 0.1$ and $\rho = 0.2$ for Method 2. The simulations are performed with $N = 32$ and $K = 8$ under SNR = 20 dB. The order of LSF is set to $M = 3$. We let the interfering users have 10 times larger amplitudes than the desired one (i.e., $\beta = 10$). The results are depicted in Figure 4. We observe that, compared with “Method 2 + BPCP,” “Method 1 +

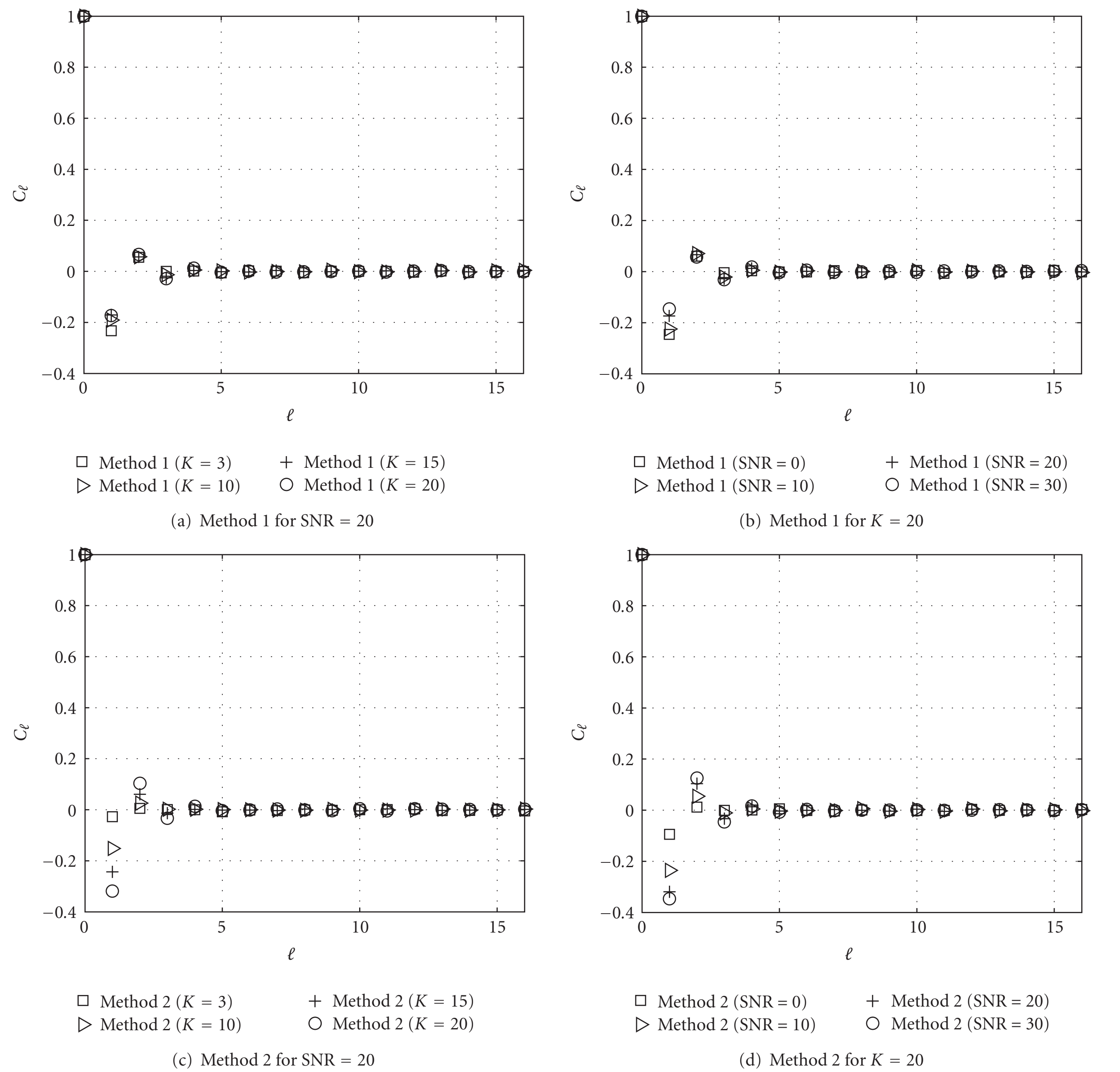


FIGURE 6: Correlation properties, in asynchronous systems, (a) Method 1 for SNR = 20 dB, (b) Method 1 for $K = 20$, (c) Method 2 for SNR = 20 dB, and (d) Method 2 for $K = 20$. We let $M = 3$ for Method 1.

BPCP" reduces the number of iterations required to achieve SINR = 15 dB by half approximately.

4. AUTOCORRELATION PROPERTIES OF FILTERS

We examine the correlation properties of the linear filters studied in the previous section. For a given filter $(\mathbf{0} \neq)\mathbf{h} \in \mathbb{R}^N$, we define its autocorrelation as

$$C_\ell(\mathbf{h}) := \frac{[[\mathbf{h}]_{\ell+1:N}^T \ \ [\mathbf{h}]_{1:\ell}^T]\mathbf{h}}{\|\mathbf{h}\|^2}, \quad \ell = 0, 1, \ldots, N-1. \quad (14)$$

The function C_ℓ has a symmetric property $C_\ell(\mathbf{h}) = C_{N-\ell}(\mathbf{h})$, for $\ell = 1, 2, \ldots, N-1$, $\forall \mathbf{h} \in \mathbb{R}^N$, because

$$\begin{aligned} \|\mathbf{h}\|^2 C_\ell(\mathbf{h}) &= [[\mathbf{h}]_{\ell+1:N}^T \ \ [\mathbf{h}]_{1:\ell}^T] \begin{bmatrix} [\mathbf{h}]_{1:N-\ell} \\ [\mathbf{h}]_{N-\ell+1:N} \end{bmatrix} \\ &= [[\mathbf{h}]_{N-\ell+1:N}^T \ \ [\mathbf{h}]_{1:N-\ell}^T] \begin{bmatrix} [\mathbf{h}]_{1:\ell} \\ [\mathbf{h}]_{\ell+1:N} \end{bmatrix} \\ &= \|\mathbf{h}\|^2 C_{N-\ell}(\mathbf{h}). \end{aligned} \quad (15)$$

Hence, it is sufficient to examine the correlation C_ℓ for $\ell = 0, 1, \ldots, \lceil (N-1)/2 \rceil$. We set $N = 32$ and let all users have the same amplitudes (i.e., $\beta = 1$).

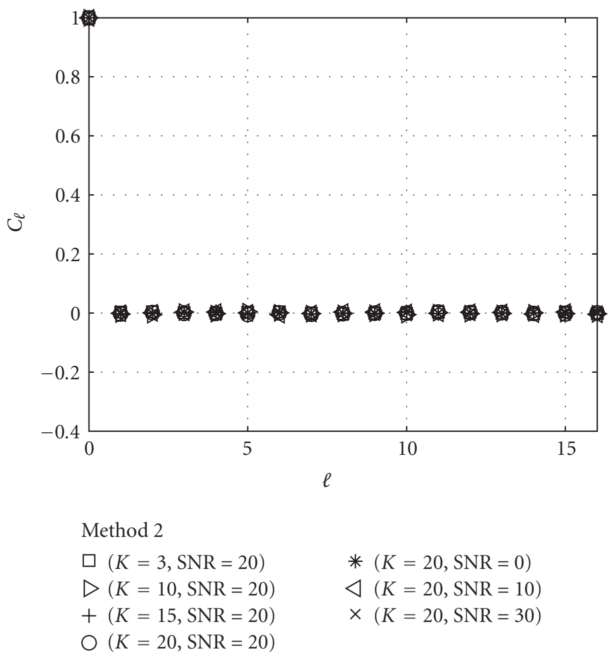


Figure 7: Correlation properties of Method 2 in synchronous systems.

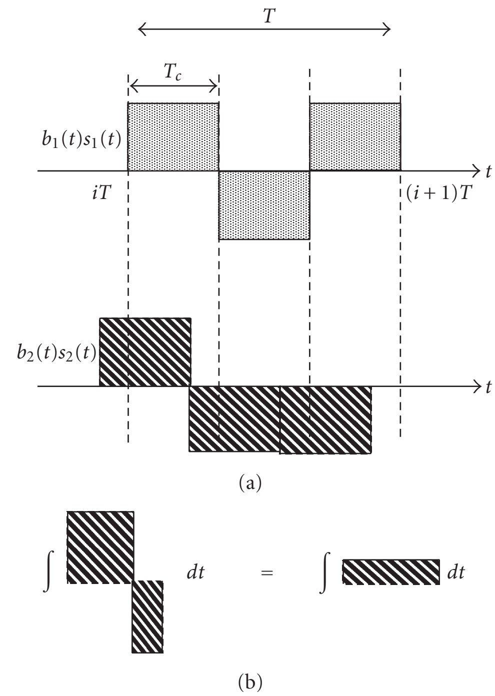


Figure 8: (a) An example of received signals in asynchronous systems with two users, and (b) an illustration of the effective interference reduction that happens in integrating $b_2(t)s_2(t)$ from iT to $iT + T_c$.

Figure 5 depicts the autocorrelation properties of the matched filters used in Method 3 (for the order $M = 2, 3, 32$) and Method 4 [or equivalently the correlation properties of the LSF-codes (for $M = 2, 3, 32$) and the binary random codes]. We compute average correlations of each method over 5000 binary random codes generated independently (recall that the LSF-code is generated with a binary random code). It is seen that the LSF-code has *negative correlation*, that is, $C_\ell \approx (-\delta)^\ell$ for $\delta \in (0, 1)$, which is a desired property *in asynchronous CDMA systems*; see, for example, [19] and the references therein. It is also seen that the use of $M = 3$ and $M = 32$ yields nearly the same correlation, implying that the order $M = 3$ would be reasonable for good performance and low computational costs. The results for $M = 4, 5, \ldots, 31$ are almost identical to the results for $M = 3, 32$.

Next we examine the correlation properties of Methods 1 and 2 (the MMSE-based methods) in asynchronous systems in several situations ($M = 3$ for Method 1). Figure 6 plots the results, where in 6(a) and 6(c) the SNR is fixed to 20 dB and the number of users is changed as $K = 3, 10, 15, 20$, and, in 6(b) and 6(d), $K = 20$ is fixed and SNR is changed as SNR = 0, 10, 20, 30 dB. From Figures 6(a) and 6(b), it is seen that Method 1 has a negative correlation property similar to Method 3 in a wide range of situations. On the other hand, from Figures 6(c) and 6(d), it is seen that Method 2 also has a negative correlation property, but the exponential factor $\delta \in (0, 1)$ depends highly on SNR and K. For instance, δ becomes large when SNR and/or K increases.

To explain this, we show in Figure 7 the correlation properties of Method 2 in synchronous systems under various conditions ($K = 3, 10, 15, 20$, SNR = 0, 10, 20, 30 dB). It is seen that there is no correlation (under any conditions) in case of synchronous systems. Referring to Figure 6(c), we observe that $K = 3$ yields similar results to the case of synchronous systems. This would be because the "degree" of asynchronous is small due to the small number of interfering users. Referring to Figure 6(d), on the other hand, SNR = 0 dB yields closer performance to the case of synchronous systems (i.e., a smaller value of δ) than the cases of SNR = 10, 20, 30 dB. This would be because the noise is dominant over the interfering signals when SNR = 0 dB, making the "degree" of asynchronous small.

Let us clarify here the optimality of the LSF and the MMSE receiver.

(1) The LSF is an optimal FIR filter in asynchronous CDMA systems in an average sense, thus it is situation-independent.

(2) The MMSE receiver is an optimal FIR filter (in general CDMA systems), which is a function of spreading codes and amplitudes of all users and the noise variance (thus it is situation-dependent). Note that such knowledge is *not* required for the adaptive filtering techniques to realize the MMSE receiver (e.g., blind methods such as the one used in Section 3.2 require only the received signal and the signature of the desired user).

Viewing Figure 6 from another side, we could say that, in asynchronous systems, the average correlation property of the MMSE receiver over all situations would roughly be identical to that of the LSF. This is a natural claim from the different senses of optimality, shown above, of the LSF and the MMSE receiver. We finally emphasize that a remarkable distinction is observed between synchronous and asynchronous cases in the correlation property of the MMSE receiver (Method 2).

5. CONCLUSION

In this paper, we have presented an efficient structure employing two kinds of optimal FIR filters, respectively, at the transmitter and the receiver for asynchronous CDMA systems. We have demonstrated that the use of the LSF-code with an adaptive linear receiver yields significant reduction in adaptation-time. The study of autocorrelation properties has shown that (i) the MMSE receiver with the LSF-code has similar correlation to the LSF-code itself in a wide range of scenarios, (ii) the average correlation of the MMSE receiver with a random code in asynchronous systems would roughly be identical to that of the LSF, and (iii) there is a notable difference between synchronous and asynchronous cases for the MMSE receiver.

APPENDIX

In [5], after formulating an asynchronous system as its equivalent synchronous system, it is written that "*we can analyze the asynchronous system considered as a synchronous system with additional interferers*." This is of course true, and the formulation therein is very useful to analyze the convergence properties of adaptive algorithms.

However, if one would like to know precise performance of the algorithm in completely asynchronous systems, then asynchronous systems should be taken into account. The reason can be found in [20], in which Pursley has shown that the asynchronism reduces the "effective" interference. In other words, the performance under the same settings (the length of spreading code, the number of interfering users and their transmitted power, the noise level, etc.) is different in general between synchronous and asynchronous systems.

Nevertheless, most studies on the (adaptive) MMSE receiver have focused solely on a synchronous case [21, 22] (or a "symbol-asynchronous but chip-synchronous" case; i.e., the delays of all users are aligned to the chip timing [16]). Only a few investigations have been done [18, 23] on completely asynchronous cases. This means that the important results in [20] may not widely be known at least in the signal processing community, and this is why we rephrase the fact in this appendix.

To explain the Pursley results intuitively, we give a simple example. We assume that there are only two users with amplitudes equal to one, no noise, no fading, and the spreading code is binary with its length only $N = 3$. Then, the continuous-time expression of the received signal is given by (see Figure 8(a))

$$r(t) = b_1(t)s_1(t) + b_2(t)s_2(t). \tag{A.1}$$

The first element of $\mathbf{r}[i]$, for example, is given as follows:

$$\begin{aligned} r_1[i] &:= \int_{iT}^{iT+T_c} r(t)dt \\ &= \int_{iT}^{iT+T_c} b_1(t)s_1(t)dt + \int_{iT}^{iT+T_c} b_2(t)s_2(t)dt. \end{aligned} \tag{A.2}$$

The second term of (A.2) is illustrated in Figure 8(b), where the equality means that the integral of the positive and negative values (left side) is equal to the integral of the smaller positive values (right side). This is the mechanism of the reduction of effective interference in asynchronous systems, and the same happens in general situations; note that the reduction does *not* happen when the system is chip-synchronous. It would be worth mentioning that it has been shown in [16, 17] that the MMSE receiver (as well as the matched filter) exhibits higher performance in asynchronous systems than in synchronous systems under the fair conditions, although the MMSE receiver is optimal whether the system is asynchronous or not.

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