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Research Article

Performance of Coded Systems with Generalized Selection Diversity in Nakagami Fading

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We investigate the performance of coded diversity systems employing generalized selection combining (GSC) over Nakagami fading channels. In particular, we derive a numerical evaluation method for the cutoff rate of the GSC systems. In addition, we derive a new union bound on the bit-error probability based on the code's transfer function. The proposed bound is general to any coding scheme with a known weight distribution such as convolutional and trellis codes. Results show that the new bound is tight to simulation results for wide ranges of diversity order, Nakagami fading parameter, and signal-to-noise ratio (SNR).

1. INTRODUCTION

Diversity is an effective method to mitigate multipath fading in wireless communication systems. Diversity improves the performance of communication systems by providing a receiver with M independently faded copies of the transmitted signal such that the probability that all these copies are in a deep fade is low. The diversity gain is obtained by combining the received copies at the receiver. The most general diversity combining scheme is the generalized selection combining (GSC), which provides a tradeoff between the high complexity of maximal-ratio combining (MRC) and the poor performance of selection combining (SC). In GSC, the largest M_c branches out of M diversity branches are combined using MRC. The resulting signal-to-noise ratio (SNR) at the output of the combiner is the sum of the SNRs of the largest M_c branches.

A general statistical model for multipath fading is the Nakagami distribution [1]. The error probability and the cutoff rate of GSC over Rayleigh fading channels was analyzed in [2, 3], respectively. In [4], the performance of some special cases of GSC systems over Nakagami fading channels was analyzed. A more general framework to the analysis of GSC systems over Nakagami fading channels was presented in [5] and more recently in [6]. In [7], the cutoff rate and a union bound on the bit-error probability of coded SC systems over Nakagami fading channels were derived. The derivation is based on the transfer function of the code. To the best of our knowledge, no analytical results on the performance of coded GSC systems over Nakagami fading channels exit yet.

In [8], a new approach to analyzing the performance of GSC over Nakagami fading channels was presented. The approach is based on converting the multidimensional integral that appears in the error probability of GSC into a single integral that can be evaluated efficiently. In this paper, we generalize this approach to derive the cutoff rate and a union bound on the bit-error probability of coded GSC over Nakagami fading channels. The bound is based on the transfer function of the code and is simple to evaluate using the Gauss-Leguerre integration (GLI) rule [9]. Results show that the proposed union bound is tight to simulation results for a wide range of Nakagami parameter, SNR values, and diversity orders.

The paper is organized as follows. The coded GSC system is described in Section 2. In Section 3, the cutoff rate of coded GSC systems is derived. In Section 4, the proposed union bound on the bit-error probability is derived, and results are discussed therein. Conclusions are discussed in Section 5.

2. SYSTEM MODEL

The transmitter in a coded system is generally composed of an encoder, interleaver, and a modulator. The encoder might be convolutional, turbo, trellis-coded modulation (TCM), or any other coding scheme. The encoder encodes a block of K information bits into a codeword of L symbols. The code rate is defined as $R_c = K/L$. For the lth symbol in the codeword, the matched filter output of the ith diversity branch is given by

$$y_{l,i} = \sqrt{E_s} a_{l,i} s_l + z_{l,i}, \tag{1}$$

where E_s is the received signal energy per diversity branch and $\mathbf{a}_l = \{a_{l,i}\}_{i=1}^{M}$ are the fading amplitudes affecting the M diversity branches, modeled as independent and identically distributed (i.i.d) Nakagami random variables. Here, we assume ideal interleaving and independent diversity branches. The noise samples $\mathbf{z}_l = \{z_{l,i}\}_{i=1}^{M}$ are i.i.d complex Gaussian random variables with zero-mean and a variance of $N_0/2$ per dimension.

Signals received at different diversity branches are combined such that the performance is improved. In MRC, the received signals at different diversity branch are weighted by the corresponding channel gain. The resulting SNR for symbol l in the codeword is given by $\gamma_l E_s/N_0$, where $\gamma_l = \sum_{i=1}^{M} a_{l,i}^2$. In GSC, the receiver selects the largest M_c diversity branches among the M branches and combines them using MRC. If we arrange the fading amplitudes $a_{l,1}, \ldots, a_{l,M}$ in a descending order $a_{l,(1)} \geq a_{l,(2)} \geq \cdots \geq a_{l,(M)}$, then the SNR at the output of the GSC receiver is given by $\beta_l E_s/N_0$, where $\beta_l = \sum_{i=1}^{M_c} a_{l,(i)}^2$.

3. CUTOFF RATE

The cutoff rate R_0 has been generally referred to as the practical channel capacity. Reliable communication beyond this rate would become very expensive to achieve. Even after the discovery of near-Shannon limit achieving codes such as turbo and LDPC codes [10, 11], the required large block size and inherent delays would make the cutoff rate a valid figure-of-merit to compare different modulation schemes. The cutoff rate for discrete-alphabet modulation schemes [12] is defined as

$$R_0 = 2\log_2|\mathcal{S}| - \log_2\left(\sum_{s_i\in\mathcal{S}}\sum_{s_j\in\mathcal{S}} C(s_i, s_j)\right), \tag{2}$$

where $|\mathcal{S}|$ is the size of the modulation alphabet $\mathcal{S}$ and $C(s_i, s_j)$ is the Chernoff factor defined as

$$C(s_i, s_j) = E_\beta[e^{-\beta d}], \tag{3}$$

where $\beta = \sum_{i=1}^{M_c} a_{(i)}^2$ and $d = E_s|s_i - s_j|^2/4N_0$. Recognizing (3) as the moment generating function (MGF) of the random variable β and using the result of [8], the Chernoff factor can be written as

$$C(s_i, s_j) = M_c \binom{M}{M_c} \int_0^\infty e^{-dx} f_{a^2}(x) [F_{a^2}(x)]^{M-M_c} [\phi_{a^2}(d, x)]^{M_c-1} dx, \tag{4}$$

where $f_{a^2}(x)$ and $F_{a^2}(x)$ are, respectively, the probability density function (pdf) and cumulative distribution function (CDF) of the SNR of each diversity branch, and $\phi_{a^2}(d, x)$ is the marginal MGF [8] defined as

$$\phi_{a^2}(d, x) = \int_x^\infty e^{-dt} f_{a^2}(t) dt. \tag{5}$$

For Nakagami fading channels, the pdf and CDF are given, respectively, by

$$f_{a^2}(x) = \frac{m^m}{\Gamma(m)} x^{m-1} e^{-mx}, \quad x \geq 0,\ m \geq 0.5, \tag{6}$$

$$F_{a^2}(x) = \gamma(m, mx), \quad x \geq 0,\ m \geq 0.5, \tag{7}$$

where $\gamma(a, y) = (1/\Gamma(a)) \int_0^y e^{-t} t^{a-1} dt$ is the incomplete Gamma function and $\Gamma(\cdot)$ is the Gamma function. The marginal MGF for Nakagami fading [8] is given by

$$\phi_{a^2}(d, x) = \frac{1}{\Gamma(m)} \frac{1}{(1 + d/m)^m} [1 - \gamma(m, mx(1 + d/m))]. \tag{8}$$

Substituting (6)–(8) into (4), we obtain

$$\begin{aligned} C(s_i, s_j) = {} & M_c \binom{M}{M_c} \frac{m^m}{\Gamma(m)^{M_c}} \frac{1}{(1 + d/m)^{m(M_c-1)}} \\ & \times \int_0^\infty \exp(-mx(1+d/m)) x^{m-1} [\gamma(m, mx)]^{M-M_c} \\ & \times [1 - \gamma(m, mx(1 + d/m))]^{M_c-1} dx. \end{aligned} \tag{9}$$

Making the change of variable $y = mx(1 + d/m)$ and simplifying, (9) can be written as

$$C(s_i, s_j) = \binom{M}{M_c} \frac{M_c}{[\Gamma(m)(1 + d/m)^m]^{M_c}} \int_0^\infty e^{-y} y^{m-1} g(y) dy, \tag{10}$$

where $g(y)$ is given by

$$g(y) = \left[\gamma\left(m, \frac{y}{1 + d/m}\right)\right]^{M-M_c} [1 - \gamma(m, y)]^{M_c-1}. \tag{11}$$

Using the GLI rule from [9], the integral in (10) can be evaluated efficiently as

$$\int_0^\infty e^{-y} y^{m-1} g(y) dy \approx \sum_{p=1}^{P} w_m(p) g(y_m(p)), \tag{12}$$

where $\{w_m(p)\}$ are the weights of the GLI rule for a specific m and $y_m(p)$ is the pth abscissa. Both $\{w_m(p)\}$ and $\{y_m(p)\}$ are computed according to the GLI rule as in [9]. It was found through our simulations that $P = 20$ is enough to get the required accuracy in the bound.

The cutoff rate of GSC systems with $M = 4$ over Nakagami fading channels with $m = 2$ is shown in Figure 1. In the figure, GSC systems employing 8PSK, QPSK, and BPSK are considered. We observe in the figure that as the

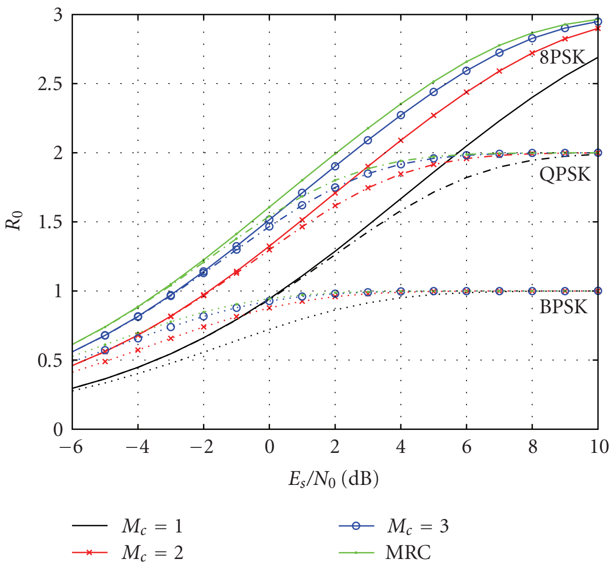

FIGURE 1: Cutoff rate of coded GSC with $M = 4$ and different number of selected diversity branches in Nakagami fading with $m = 2$.

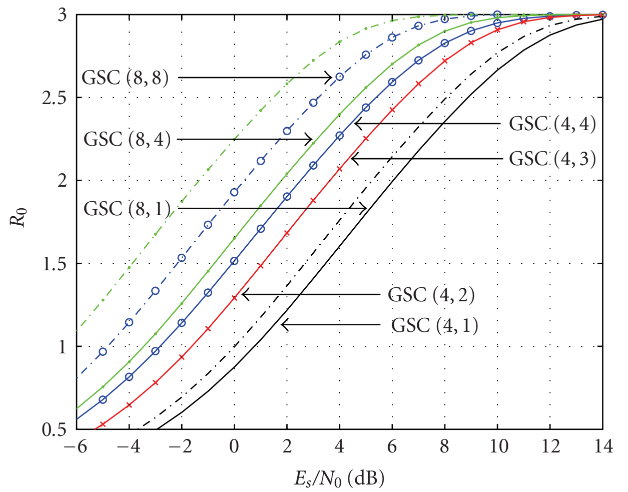

FIGURE 2: Cutoff rate of 8PSK-coded GSC with different number of diversity orders in Nakagami fading with $m = 4$.

number of combined diversity branches increases, the cutoff rate increases. This is expected since combining more diversity branches increases the reliability of the communication system allowing higher transmission rate at the same SNR. Figure 2 shows the cutoff rates of an 8PSK GSC system with different combinations of M and M_c. Note that the proposed evaluation method of the cutoff rate is very simple and efficient as compared with the integral method of [5].

4. BIT-ERROR PROBABILITY

The conditional pairwise error probability (PEP) for coded GSC can be written as

$$P(\mathbf{S} \longrightarrow \hat{\mathbf{S}} \mid \mathbf{A}) = P\left(\sum_{l=1}^{L} \sum_{i=1}^{M_c} \left(\left| y_{l,(i)} - a_{l,(i)} s_l \right|^2 - \left| y_{l,(i)} - a_{l,(i)} \hat{s}_l \right|^2 \right) \geq 0 \mid \mathbf{A} \right), \tag{13}$$

where $y_{l,(i)}$ is the matched filter output corresponding to the diversity branch with fading gain $a_{l,(i)}$, $\mathbf{S}$ and $\hat{\mathbf{S}}$ are the length-L vectors representing the correct and decoded codewords, respectively, and $\mathbf{A}$ is an $L \times M$ matrix containing the fading amplitudes affecting a codeword. The conditional PEP [12] can be simplified as

$$P(\mathbf{S} \longrightarrow \hat{\mathbf{S}} \mid \mathbf{A}) = P\left(\xi \geq \sum_{l=1}^{L} \sum_{i=1}^{M_c} a_{l,(i)}^2 \left| s_l - \hat{s}_l \right|^2 \mid \mathbf{A} \right), \tag{14}$$

where ξ is a zero-mean Gaussian random variable with variance $2LE_s \sum_{l=1}^{L} \sum_{i=1}^{M_c} a_{l,(i)}^2 |s_l - \hat{s}_l|^2$. This probability [12] can be further simplified as

$$P(\mathbf{S} \longrightarrow \hat{\mathbf{S}} \mid \mathbf{A}) = Q\left(\sqrt{2 \sum_{l=1}^{L} \beta_l d_l}\right), \tag{15}$$

where $d_l = E_s |s_l - \hat{s}_l|^2 / 4N_0$ and $\beta_l = \sum_{i=1}^{M_c} a_{l,(i)}^2$ is the normalized SNR at the output of the GSC combiner for symbol l in the codeword. Using the the integral expression of the Q-function, $Q(x) = (1/\pi) \int_0^{\pi/2} e^{(-x^2/2\sin^2\theta)} d\theta$ [13], the unconditional PEP is written as

$$P(\mathbf{S} \longrightarrow \hat{\mathbf{S}}) = \frac{1}{\pi} \int_0^{\pi/2} \prod_{l=1}^{L} E_{\beta_l}\left[e^{-\beta_l d_l \alpha_\theta}\right] d\theta, \tag{16}$$

where $\alpha_\theta = 1/\sin^2\theta$, and the product is due to the independence of the fading variables affecting different symbols. Note that the expectation in (16) is the same as (3). Thus starting from (9), and making the change of variable $y = mx(1 + \beta\alpha_\theta/m)$, the unconditional PEP can be simplified to

$$P(\mathbf{S} \longrightarrow \hat{\mathbf{S}}) = \frac{1}{\pi} \left[\frac{M_c \binom{M}{M_c}}{\Gamma(m)^{M_c}}\right]^{L_\eta} \int_0^{\pi/2} \prod_{l=1}^{L_n} \left\{ \frac{1}{(1 + d_l/m)^{m(M_c - 1)} (1 + d_l \alpha_\theta / m)^m} \times \int_0^{\infty} e^{-y} y^{m-1} h(y) dy \right\} d\theta, \tag{17}$$

where $h(y)$ is given by

$$h(y) = \left[\gamma\left(m, \frac{y}{1 + d/m}\right)\right]^{M - M_c} \left[1 - \gamma\left(m, \frac{y(1 + d_l/m)}{(1 + d_l \alpha_\theta / m)}\right)\right]^{M_c - 1}, \tag{18}$$

and $L_\eta = |\eta|$ represents the minimum time diversity of the code, where $\eta = \{l : s_l \neq \hat{s}_l\}$. Using the transfer function of the code, the union bound on the bit-error probability is finally given by

$$P_b \leq \frac{1}{\pi} \left[\frac{M_c \binom{M}{M_c}}{\Gamma(m)^{M_c}}\right]^{L_\eta} \int_0^{\pi/2} \left\{ \frac{\partial T(\overline{D(\theta)}, I)}{\partial I} \bigg|_{I=1,\, D = e^{-E_s/4N_0}} \right\} d\theta, \tag{19}$$

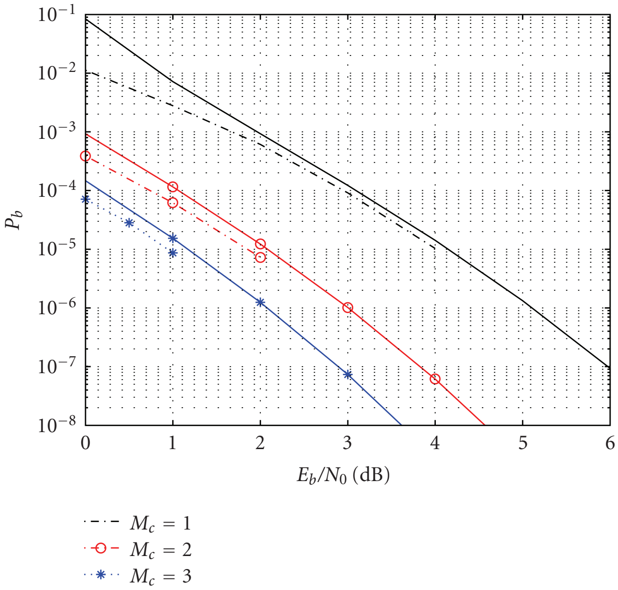

FIGURE 3: Bit-error probability of convolutionally coded GSC with $M = 4$ in Nakagami fading with $m = 2$ (solid: bound, dashed: simulation).

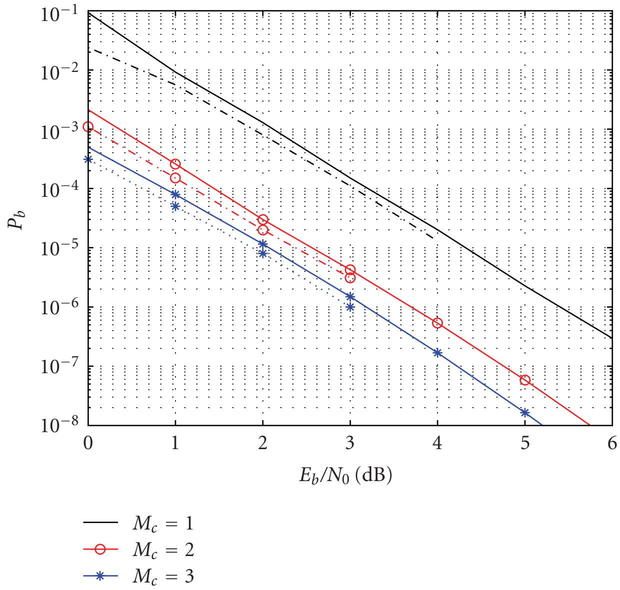

FIGURE 4: Bit-error probability of convolutionally coded GSC with $M = 4$ in Nakagami fading with $m = 0.75$ (solid: bound, dashed: simulation).

where D is a variable whose exponent represents the distance from the all-zero codewords, I is a variable whose exponent represents the number of information bits to the encoder, and $T(\overline{D(\theta)}, I)$ is the transfer function of the code evaluated at $\overline{D(\theta)}$ that is given by

$$\overline{D(\theta)}|_{D=e^{-E_s/4N_0}} = \frac{1}{(1+d_l/m)^{m(M_c-1)}(1+d_l\alpha_\theta/m)^m} \times \int_0^\infty e^{-y}\, y^{m-1} h(y)\, dy, \quad (20)$$

where $h(y)$ is defined in (18). The expression in (20) is evaluated using the GLI rule defined in (12) with $P = 20$, as discussed in Section 3. Once (20) is evaluated for every value of the argument θ, (19) is evaluated using a simple trapezoidal numerical integration [9] since it is a definite integral. It was found that 10 steps are enough to evaluate (19) with a good accuracy.

The proposed bound was evaluated for a rate-1/2 (5, 7) convolutional code and an 8-state 8PSK TCM system presented in [12, Section 5.3]. Nevertheless, the bound is applicable to any coding scheme with a known transfer function such as turbo codes and product codes. Figures 3–5 show the simulation and analytical results for convolutionally and 8PSK TCM-coded systems over different Nakagami fading channels and with different selected diversity branches out of $M = 4$. We observe that the bound is tight to simulation results for a wide range of SNR values, diversity orders, and Nakagami parameters. It is also noted that the bound is appropriate for Nakagami fading channels with noninteger fading parameters. In addition, we note that the bound is simple to evaluate using the GLI rule. Figures 6 and 7 show the performance of convolutional and 8PSK TCM with SC over Nakagami fading channels, respectively. From the figures, we observe that the bound is tight to simulation results for a wide range of Nakagami parameters and diversity orders. It is worth noting that the union bound becomes less tight to simulation results as the SNR decreases, which is a well-known property of the union bounding technique [12].

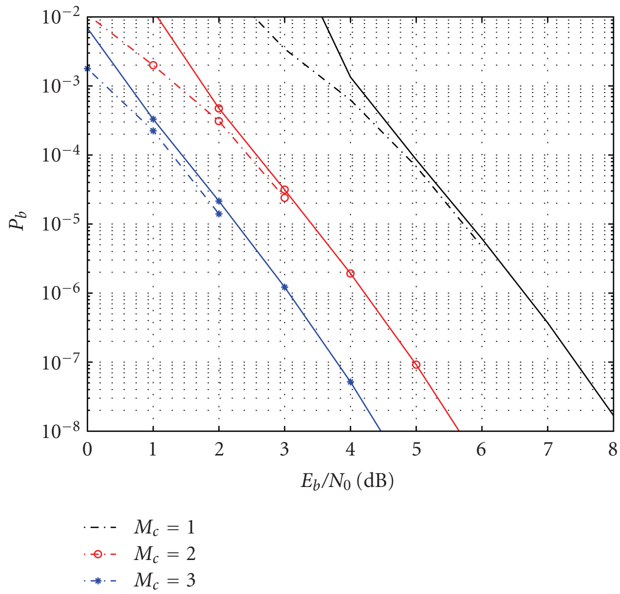

FIGURE 5: Bit-error probability of 8PSK TCM-coded GSC with $M = 4$ in Nakagami fading with $m = 4$ (solid: bound, dashed: simulation).

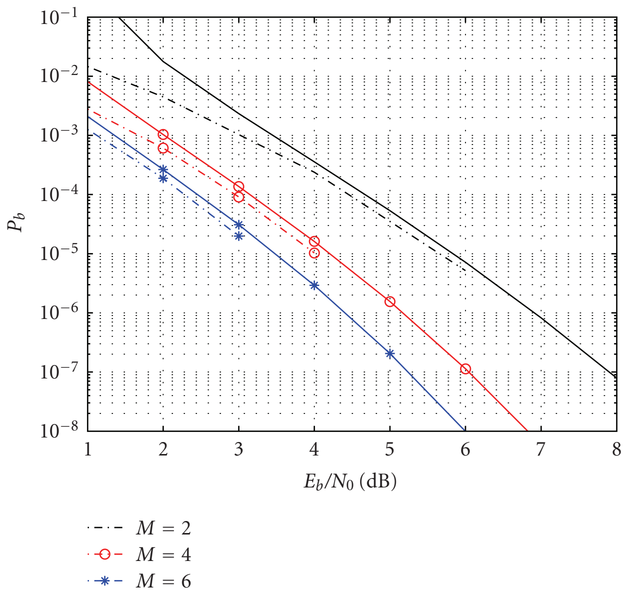

Figure 6: Bit-error probability of convolutionally coded SC with different number of diversity branches in Nakagami fading wih $m = 2$ (solid: bound, dashed: simulation).

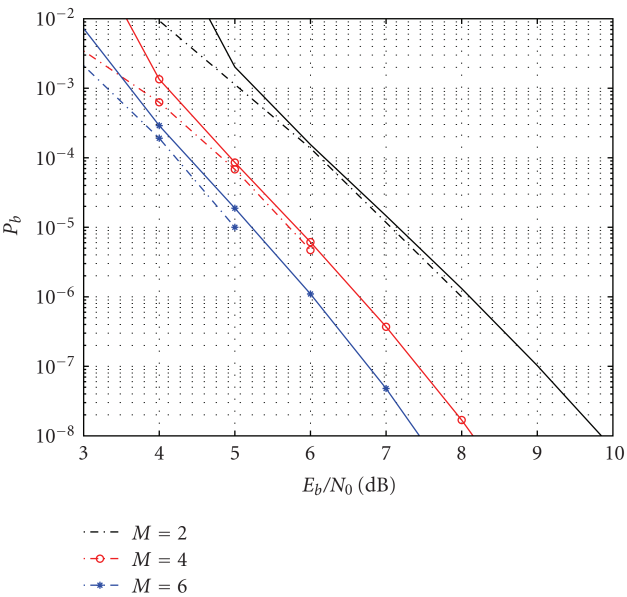

Figure 7: Bit-error probability of 8PSK TCM-coded SC with different number of diversity branches in Nakagami fading wih $m = 4$ (solid: bound, dashed: simulation).

5. CONCLUSIONS

In this paper, we presented a new evaluation method for the cutoff rate of coded GSC systems. In addition, we derived a new union bound on the error probability of coded coherent GSC systems over Nakagami fading channels. Results show that the new bound is tight to simulation results. Furthermore, the bound is general to any coded system with a known transfer function, Nakagami fading with a general Nakagami parameter m and any combinations of diversity order, M, and selected diversity branches, M_c.

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