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Research Article

Interference Aware Routing for Minimum Frame Length Schedules in Wireless Mesh Networks

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The focus of this paper is on routing in wireless mesh networks (WMNs) that results in spatial TDMA (STDMA) schedules with minimum frame length. In particular, the emphasis is on spanning tree construction; and we formulate the joint routing, power control, and scheduling problem as a mixedinteger linear program (MILP). Since this is an $\mathcal{NP}$-complete problem, we propose a low-complexity iterative pruning-based routing scheme that utilizes scheduling information to construct the spanning tree. A randomized version of this scheme is also discussed and numerical investigations reveal that the proposed iterative pruning algorithms outperform previously proposed routing schemes that aim to minimize the transmitted power or interference produced in the network without explicitly taking into account scheduling decisions.

1. INTRODUCTION

Algorithmic aspects of wireless mesh networks (WMNs) are currently a vigorous area of research and have steadily accumulated momentum over the last few years. The leading exponents of this increased interest are the potential multifarious applications of WMNs [1]. Admittedly, the two most important of them are provision of low-cost and rapid-deployable broadband last mile connectivity to the Internet or backhaul support for 3G cells and IEEE 802.11"x" hot spots.

Efficient resource utilization in WMNs calls for scheduling and routing policies that maximize the aggregate throughput of the system. Under this perspective, the central theme of this paper is the design of joint scheduling and shortest-path spanning tree schemes that provide increased system performance. With a preconstructed spanning tree within the mesh network, the cornerstone aim of the scheduling engine is either to maximize the transmission opportunities of active links in a specific time window (frame length) by taking into account the interference caused by simultaneously transmitting nodes or to minimize the time span for all links to transmit, that is, minimize frame length. Concurrent transmissions are of utmost importance since they increase system efficiency but can lead to erroneous reception at the receiver if the level of the received signal is too weak compared to the aggregate interference. Thus, the spatial reuse of timeslots heavily depends on the selected active set of links in the mesh topology; but, the active set of links is constructed by the routing algorithm. Therefore, and as it will become vividly clear in the sequel, there is an interplay between scheduling and routing decisions. The rationale of designing joint routing and scheduling schemes stems exactly from this interplay between the two functionalities.

The medium-access control scheme considered hereafter is based on time division multiple access (TDMA), where time is divided into timeslots and each node can transmit only at predefined timeslots, thus, collisions can be avoided (the same analysis can also be applied for FDMA-based networks). Since nodes are spatially distributed, timeslots can be potentially reused by nodes that are sufficiently far apart. Spatial reusing of timeslots has been defined in the seminal work of Nelson and Kleinrock [2] and is called spatial TDMA (STDMA).

In this paper, we focus on utilizing shortest-path algorithms, which are widely studied and used in practise. The emphasis is on Dijkstra's algorithm which for bounded degree graphs finds the shortest paths from a source node to every other node in $\mathcal{O}(n \log n)$ time, where n is the number of nodes in the network. The cost metric used is the required transmission power for a link (i, j) to be established. We focus on rooted spanning tree construction (for both uplink and downlink) since the mesh mode specifications that have been integrated into the IEEE 802.16-2004 standard are based on tree topology. Our proposed pruning schemes run Dijkstra iteratively, and at each iteration, eliminate (prune) links that produce high interference; the resulting tree's scheduling performance is evaluated by using optimal scheduling or a greedy scheduling heuristic and the tree with the best scheduling performance is kept. Thus, scheduling information is incorporated in making decisions about routing.

We compare the proposed pruning algorithm with other Dijkstra-based schemes. These heuristics use as costs linear combinations of the required power for transmission and the interference produced. We show that proposed pruning algorithm outperforms these heuristics.

The rest of the paper is organized as follows. In Section 2, selected closely related previous works in the area of joint routing and scheduling are outlined. The problem description and the mixed-integer linear program (MILP) formulation are detailed in Section 3. The inherent interplay between routing and scheduling is explained in Section 4. In Section 5, suboptimal joint scheduling and routing schemes are explained; and Section 6 outlines the two flavors of the proposed pruning algorithm. Numerical investigations are reported in Section 7; and finally, Section 8 concludes the paper by outlining the main findings followed by brief remarks on future avenues for research.

2. REVIEW OF SELECTED PRIOR WORKS

After the introduction of the spatial TDMA concept by Nelson and Kleinrock in [2], general timeslot (or channel) assignment scheduling problems have been extensively studied in the literature. The bulk of previous research work focused on graph theoretic solutions by conceiving link scheduling as a graph-coloring problem [3–5]. In the basic setting, graph-coloring approaches aim to tackle the *primary* and *secondary* conflicts between links. More specifically, any pair of directed edges (a, b), (c, d) may be colored with the same color if and only if (i) a, b, c, d are all mutually distinct and (ii) edges (a, d), (c, b) do not belong in the set of edges in the graph. When the first (second) condition fails to hold, then there will be a primary (secondary) conflict between edges (a, b) and (c, d). Scheduling based on graph theoretic tools proved essential for formally defining the problem and for the design of distributed solutions. The limitations on the other hand of these solutions stem from the fact that the aggregate effect of interference of links transmitting in concurrent timeslots (reflected in the signal-to-interference noise ratio (SINR)) is not taken explicitly into account [6]. Hence, a schedule provided by a graph-coloring technique may lead to an infeasible allocation when the SINR thresholds are taken into account. Related to this last point is the observation that an optimal schedule based on graph coloring can be considered as a lower bound of the minimum number of timeslots that can be used in the network.

To fill this void, the authors in [7] have explicitly taken into account the SINR thresholds together with power control for constructing minimum frame length scheduling in STDMA networks with directional antennas. From a computational complexity perspective, even without taking into account the aggregate interference, constructing a transmission schedule of timeslots where all links are scheduled with the minimum number of timeslots (i.e., minimum frame length) has shown to be an $\mathcal{NP}$-complete problem [8].

The work of Tassiulas and Ephremides [9] showed that the capacity region of wireless multihop networks depends on the power allocation vector (which itself depends on channel conditions) as well as the routing and scheduling decisions. This formal characterization of the inherent coupling between power control, scheduling, and routing, sparked a research interest in schemes that attempt to optimize them jointly [10, 11]. These so-called *cross-layer optimization* approaches have recently been extended to take into account end-to-end flow and congestion control decisions (transport layer) [12]. Polynomial complexity algorithms together with necessary and sufficient conditions for optimal scheduling and routing of a predefined set of source-destination rates in mesh networks have been discussed in [13]. In contrast to these previous works, the emphasis in this paper is on how to construct spanning trees that minimize the frame length (in terms of required timeslots) in the mesh network.

Finally, it is worth mentioning that pruning techniques have been mainly used within quality of service (QoS) routing to produce a sparser graph, consisting entirely of feasible links [14, 15]. In these QoS routing schemes, links are deleted from the topology if their available resources do not meet the corresponding constraints. In our case, the incentive for link pruning is a rather different one; pruning is used to delete links that produce high interference to neighbor nodes that can lead to low-spatial reuse of timeslots.

2.1. Contribution of the paper

To the authors best knowledge, this is the first paper that *explicitly* addresses the issue of how to jointly construct a spanning tree while minimizing the required frame length (in terms of the number of timeslots) in a wireless mesh network. In that respect, the contributions of the paper represent measurable progress on the following fronts:

(1) formulation of MILP to perform optimal jointly spanning tree construction and scheduling that minimizes the required frame length in timeslots;

(2) interference aware iterative pruning routing algorithms to construct spanning trees in the WMN with a minimum frame length schedule;

(3) quantification of the gains in terms of scheduling of the pruning schemes compared to previously proposed schemes based on an extensive set of simulations.

It is worth mentioning that even though in this paper we have assumed omnidirectional antennas (0 dB gain) and baseline path loss models, the proposed scheme is independent of the operational characteristics and models used. Thus, results drawn in this paper can be applied for different antenna radiation patterns and/or link gain models.

3. PROBLEM DESCRIPTION

Before embarking our study of suboptimal solutions in later sections, we first formulate the problem of joint routing and scheduling as an MILP. Section 3.1 deals with the mathematical programming formulation for performing STDMA scheduling under the assumption of a predefined route and Section 3.2 augments the scheduling model to incorporate routing decisions.

For performing joint scheduling and routing in WMNs, we consider the graph G, defined by the (V, L) pair, where V is a set of vertices (wireless nodes) and L is the set of transmission links that satisfy the SINR threshold criterion,

$$L = \{(u, v) \mid u, v \in V \text{ s.t. } u \neq v \text{ can transmit to } v \text{ and vice versa}\}. \tag{1}$$

Routing is usually performed using a weighting function $w : L \rightarrow \mathbb{R}$, which assigns a weight to each edge. The weight of an edge is commonly related with the required transmission power, which depends on the Euclidean distance between the nodes and the level of interference. A number of different possible edge weights that implicitly take into account scheduling information for suboptimal routing and scheduling will be discussed in the following sections.

3.1. A mixed-integer linear programming (MILP) formulation for scheduling

We first focus our attention on how to perform optimal scheduling decisions under the assumption that routing paths are preconstructed. Similar formulations appear in [16, 17]. In this case, the routing will create the directed graph $G_S = (V, L_S)$, where $L_S \subseteq L$, and scheduling will be performed on G_S.

We encapsulate power control within the MILP formulation by introducing the variable p_{ijt}, which expresses the transmitted power by node i in link (i, j) at timeslot t, under the constraint that $0 \leq p_{ijt} \leq P_{\max}$ for all t. The variable $P_{\max}$expresses the power ceiling at the transmitting node (without loss of generality $P_{\max}$ is assumed to be equal for all nodes in the WMN). Additionally, we assume that omnidirectional antennas are used by all wireless nodes to transmit and receive signals. Thus, the interference level produced by link (i, j) to all other receiving nodes will be based on their Euclidean distance with node i. With a constant target bit error rate (i.e., $E_b/N_0 = \Gamma$), the transmission can be translated into a signal-to-interference ratio requirement, which will be denoted hereafter as γ. By W we denote the lump sum thermal noise power, and by g_{ij} the link gain between nodes i, j which encapsulates both path loss and slow fading.

To be able to express now the problem in a mathematical programming setting, we introduce the boolean variables x_{ijt} and π_t, which are defined as follows:

$$x_{ijt} = \begin{cases} 1, & \text{if link } (i, j) \text{ active at timeslot } t, \\ 0, & \text{otherwise,} \end{cases} \qquad \pi_t = \begin{cases} 1, & \text{if timeslot } t \text{ is used,} \\ 0, & \text{otherwise.} \end{cases} \tag{2}$$

The mixed-integer linear program for scheduling that minimizes the required frame length in a predefined route on the set of links L_S is denoted as $\mathrm{OS}(L_S)$ and can be written as follows:

$$\min \sum_{t=1}^{M} \pi_t, \tag{3}$$

$$\sum_{(i,j) \in L_S} x_{ijt} \leq \pi_t \cdot |L_S| \quad \forall t, \tag{4}$$

$$\sum_{t=1}^{M} x_{ijt} \geq 1 \quad \forall (i, j) \in L_S, \tag{5}$$

$$\sum_{j:(i,j) \in L_S} x_{ijt} + \sum_{k:(k,i) \in L_S} x_{kit} \leq 1 \quad \forall i \in V, \ \forall t, \tag{6}$$

$$\frac{g_{ij} p_{ijt} + (1 - x_{ijt})\Lambda}{\sum_{(m,n) \in L_S \setminus \{(i,j)\}} g_{mj} p_{mnt} + W} \geq \gamma \quad \forall (i, j) \in L_S, \ \forall t, \tag{7}$$

$$x_{ijt} \leq \frac{p_{ijt} g_{ij}}{W\gamma} \quad \forall (i, j) \in L_S, \ \forall t, \tag{8}$$

$$x_{ijt} \geq p_{ijt}/P_{\max} \quad \forall (i, j) \in L_S, \ \forall t, \tag{9}$$

$$x_{ijt} \in \{0, 1\} \quad \forall (i, j) \in L_S, \ \forall t, \tag{10}$$

$$\pi_t \in \{0, 1\} \quad \forall t, \tag{11}$$

$$0 \leq p_{ijt} \leq P_{\max} \quad \forall (i, j) \in L_S, \ \forall t. \tag{12}$$

In this formulation, an initial frame length M is assumed, where all links can be easily scheduled. For example, an initial frame length value M could be the number of links.

Constraints (4) are the binding constraints for variables π_t and x_{ijt}. The requirement that all links transmit at least once during the frame length is ensured by constraint (5). Constraint (6) is the degree constraint, that is, a node cannot transmit and receive at the same timeslot. Constraint (7) expresses the required SINR threshold that should be satisfied in order to have a successful reception at the receiver. The term $\Lambda(1 - x_{ijt})$ ensures that the inequality is satisfied when link (i, j) does not transmit at timeslot t, for a sufficiently high value of Λ. The binding constraints for variables x_{ijt} and p_{ijt} are shown in (8) and (9). These binding constraints ensure that if link (i, j) is not transmitting at timeslot t, then the transmitted power p_{ijt} is zero and vice versa. Constraint (8) is based on the assumption that all links (i, j) in L_S satisfy the SINR constraint when there are no concurrent transmissions, which is equivalent to $g_{ij} p_{ijt} > \gamma W$.

3.2. Performing joint scheduling and routing

In the previous section, we formulated the scheduling problem given a fixed routing L_S. Allowing flexibility with routing decisions can improve the resulting scheduling. The aim here is to construct a routing such that the number of timeslots in a time frame is minimized. We focus our routing decisions on constructing spanning trees. The direction of the spanning tree depends on whether we are performing uplink or downlink transmission.

We augment the previously defined scheduling model to incorporate both routing (tree construction) and scheduling decisions. Note that the optimal joint routing and scheduling problem operate on the graph $G = (V, L)$. Before describing the new constraints that need to be added, we first introduce the routing variables $y_{i,j}$, which are defined as follows:

$$y_{ij} = \begin{cases} 1, & \text{link } (i, j) \text{ in optimal spanning tree,} \\ 0, & \text{otherwise.} \end{cases} \tag{13}$$

Without loss of generality, we assume that node r is the root node in the constructed spanning tree. Based on the above definitions, the optimal joint scheduling and spanning tree construction problem will be denoted as OSR (L), which is based on the set of all feasible links L. The mathematical formulation of the OSR (L) can be constructed by adding the following routing constraints to the already defined OS (L) formulation:

$$y_{ij} \leq \sum_{t=1}^{M} x_{ijt} \leq y_{ij} \cdot M \quad \forall (i, j) \in L, \tag{14}$$

$$\sum_{(i,j) \in L: i,j \in D} y_{ij} \leq |D| - 1 \quad \forall D \subseteq V, \tag{15}$$

$$\sum_{(i,j) \in L} y_{ij} = |V| - 1 \quad \forall (i, j) \in L, \tag{16}$$

$$\sum_{j \in V:(i,j) \in L} y_{ij} = 1 \quad \forall i \in V \setminus \{r\}, \qquad \sum_{(r,i) \in L} y_{ri} = 0, \tag{17}$$

$$y_{ij} + y_{ji} \leq 1 \quad \forall (i, j) \in L. \tag{18}$$

Constraint (14) binds the boolean variables x_{ijt} and y_{ij} so that a link (i, j) transmits if and only if it belongs to the optimal spanning tree. Constraint (15) ensure that there are no cycles and constraint (16) ensure that there are $|V| - 1$ links. Since an acyclic graph with $|V|$ nodes and $|V| - 1$ edges is a spanning tree, the previous two constraints construct a spanning tree. Constraints (17), (18) ensure that the tree is directed in the uplink direction towards root node r. In the case of downlink, constraint (17) are replaced by the following:

$$\sum_{j \in V:(j,i) \in L} y_{ji} = 1 \quad \forall i \in V \setminus \{r\}, \qquad \sum_{(i,r) \in L} y_{ir} = 0. \tag{19}$$

The OSR (L) formulation constructs a tree that produces schedules with the minimal timeslot frame length. Given that OR (L_S) is an $\mathcal{NP}$-complete problem [8], the $\mathcal{NP}$-completeness of OSR (L) follows.

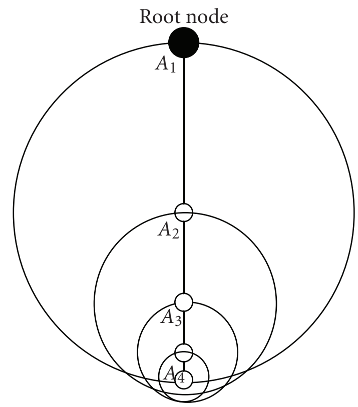


Figure 1: Worst-case scenario for timeslot reuse: the number of required timeslots is equal to the number of edges (i.e., $M = |L|$).

4. THE BINDING NATURE OF SPANNING TREE CONSTRUCTION AND SCHEDULING

The aim of this section is twofold. Firstly, to reveal the closely coupled nature of the spanning tree construction and the scheduling problem by focusing on the uplink transmission problem. Secondly, this discussion will motivate the proposed scheme with polynomial computational complexity for spanning tree construction.

Figure 1 shows the worst-case scenario of a minimum power spanning tree in terms of utilization of the timeslots. As shown in the figure, the transmission areas of the nodes are nested in the sense that each node's transmission area includes all nodes that are further away from the root node. If we define the transmission area of node i as A_i, then nodes $\{i, i+1, i+2 \ldots\}$ fall within the area A_i. This means that each node i cannot transmit at the same timeslot as nodes $i+1, i+2, \ldots$. Thus, no concurrent transmission can occur and the number of timeslots required for all nodes to transmit grows linearly, $\Omega(n)$ with the number of transmitting nodes. On the other hand, Figure 2 depicts a topology where the minimum power spanning tree requires only two timeslots for all the nodes to transmit. Two timeslots is the minimum number required since the degree of the topology is two. As shown in the figure, two timeslots are sufficient since the transmission areas of nodes transmitting at timeslot one (or two) do not overlap. Note that this one-dimensional topology has the minimum interference between nodes that transmit concurrently at timeslots one or two and, in this case, a new timeslot will only be required if the aggregate

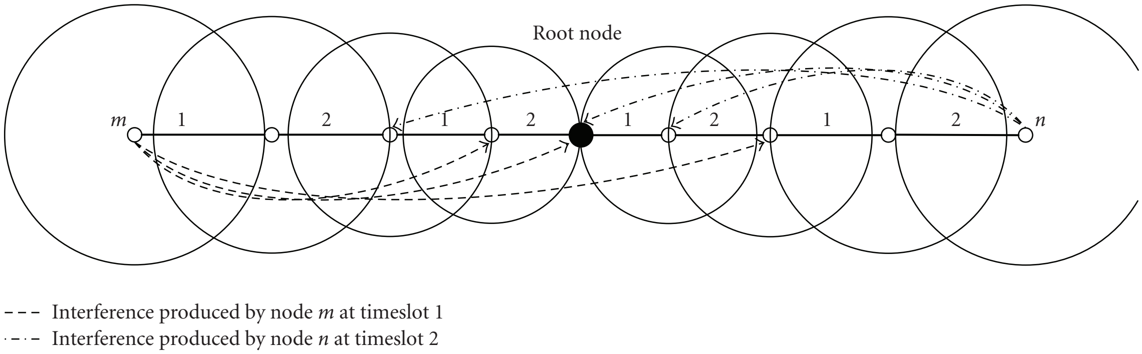


Figure 2: Best-case scenario for timeslot reuse: the number of required timeslots is equal to two.

interference produced by the nodes transmitting at timeslot one or two produce a violation on the SINR threshold.

In reality, we expect that wireless mesh network topologies will lie somewhere in between the worst- and best-case scenarios described above, therefore, efficient algorithms that can provide high spatial reuse of timeslots become crucially important.

5. SHORTEST-PATH TREE CONSTRUCTION SCHEMES IN WMN'S

The joint routing and scheduling problem defined in Section 5 is $\mathcal{NP}$-complete and thus intractable for realistic network sizes. Thus, we turn our attention to existing routing algorithms and try to incorporate scheduling information into routing decisions.

In most widely used routing protocols for constructing trees, the paths are computed based on Dijkstra's algorithm to find shortest-path spanning trees. The weight assigned to each link (i, j), $w(i, j)$ is usually taken to be proportional to the power needed to transmit on link (i, j). In the sequel, we propose Dijkstra-based routing schemes that use different weights with the aim of improving link scheduling. In Section 7, we evaluate the performance of these schemes and compare them to the proposed *interference aware pruning routing scheme*, described in Section 6.

5.1. Minimum power routing (MPR)

This scheme constructs shortest-path spanning trees in $G = (V, L)$ from the root node r to all other nodes $V \setminus \{r\}$ using Dijkstra with transmitted power as a link cost. This cost results in reduction of the overall interference. Given that the transmitted power for link (i, j) relates to the distance between nodes i and j, $d(i, j)$, we define the following cost for MPR:

$$w_P(i, j) = d(i, j)^{\alpha}, \tag{20}$$

where α is the path loss exponent which varies between 2 and 4.

In order to examine the effect of Dijkstra-based routing schemes on scheduling decisions, we assume in this section the following simple interference model: the interference caused during the transmission of link (i, j) only results in unsuccessful reception of nodes that lie within the disc with center i and radius $d(i, j)$. Any receiving nodes that lie outside the disc are unaffected. We call this model as disc-based interference (it is also widely known as the *protocol interference model* [18]).

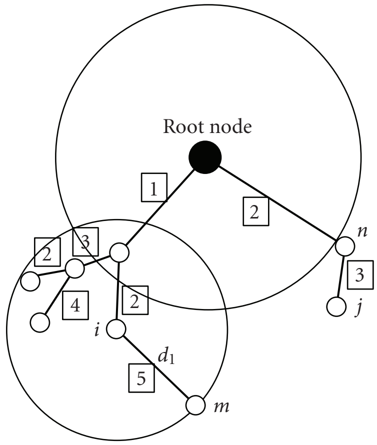
Proposition 1. *Assuming a disc-based interference model, the MPR scheme does not result in a schedule with minimum timeslot frame length.*

Proof. We show this using a counter example. Figure 3(a) shows the shortest-path spanning tree constructed by MPR for the given topology in the case of downlink transmission. In the MPR tree, the transmission of link (i, m) is affecting five links (including the link that has a degree constraint with link (i, m)). These five links require four timeslots (minimum), and since none of them can be reused, the required number of timeslots should be increased by one to accommodate link (i, m). In the tree shown in Figure3(b), node m is connected via node j. In this case, the link (j, m) is affecting two links (the link from root node to node n, and link (n, j)), and, therefore, one of the four timeslots from the other branch of the tree can be reused.

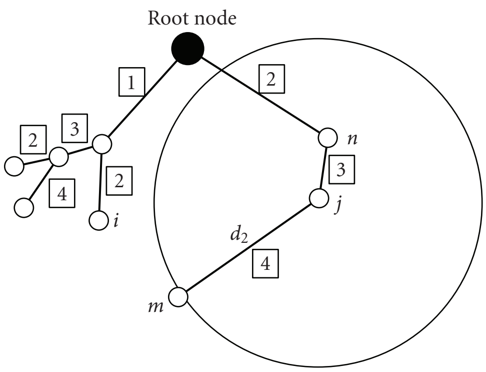
The tree depicted in Figure 3(b) is not a shortest-path spanning tree with respect to w_P since the path from node m to the root node is longer than the equivalent path in Figure 3(a). However, the tree in Figure 3(b) produces a schedule with shorter frame length (in terms of timeslots). □

Shortest-path spanning trees can be computed in polynomial time using the Dijkstra or Bellman-Ford algorithms. A brute force approach to find the tree with the minimum frame length would be to enumerate all possible trees and for each one to perform optimal scheduling. Even without taking into account the embedded scheduling problem, enumerating all trees has exponential computational complexity due to Proposition 2.

Proposition 2 (Cayleys Formula). *The number of labeled trees on n vertices is n^{n-2}.*



(a)



(b)

Figure 3: (a) Minimum power spanning tree (MPST), and (b) a spanning tree that requires less number of timeslots (better spatial reuse). Timeslots are shown within the rectangular boxes.

5.2. Minimum nearest neighborhoods routing—MNR

The MNR algorithm tries to minimize the number of nodes that are within the area of each link in the shortest-path spanning tree. In order to compute such a tree, Dijkstra's algorithm can be deployed where the cost of each link $(i, j) \in L$ is equal to the number of receiving nodes that are within its transmission range (taken to be the disc of center i and radius $d(i, j)$). In this case, the cost can be written as follows:

$$w_N(i, j) = \sum_{n \in V \setminus \{i,j,r\}} \mathcal{I}_{(i,j)}(n), \tag{21}$$

where $\mathcal{I}_{(i,j)}(n)$ is the indicator function which is defined as follows for $n \neq i, j$:

$$\mathcal{I}_{(i,j)}(n) = \begin{cases} 1, & \text{if } d(i,n) \leq d(i,j), \\ 0, & \text{otherwise.} \end{cases} \tag{22}$$

This algorithm can also include a lower bound on the number of nodes that are within the transmission range of each link so that connectivity can be established with high probability [19].

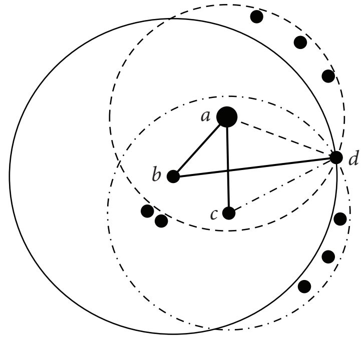


Figure 4: Possible edge crossing in the minimum nearest neighborhoods spanning tree algorithm.

A drawback of this scheme is that it may introduce edge crossings in the constructed tree.

Proposition 3. *The MNR scheme may create shortest-path spanning trees that are nonplanar graphs.*

Proof. Figure 4 shows a possible construction of a spanning tree based on the MNR algorithm. The root node is node a and after the construction of links (a, b), which has cost 0, and (a, c), which has cost 1, the cost of the link (b, d) is 4 (nodes within the circle shown by solid lines) whilst the cost for links (a, d) and (c, d) is 5 and 7, respectively (nodes within the dashed and doted dashed circles, resp.). Thus, the least-cost path to node d is through node b and, therefore, an edge crossing will be introduced. □

Link crossing can be detected and, subsequently, planarity can be restored, but the current proposed techniques need to be adapted before being applied for tree construction (see [20] and references therein).

5.3. Interference based routing—IR

In this case, the actual interference that will be produced to the other receiving nodes in the network is taken into account to produce the cost for every link in the network. More specifically, the cost for link (i, j) is computed as follows:

$$w_I(i, j) = \frac{\sum_{n \in V \setminus \{i,j,r\}} g(i, n)}{g(i, j)}. \tag{23}$$

Therefore, the cost for link (i, j) is inversely proportional to the link gain $g(i, j)$ but weighted with the aggregate link gains of node i to all other receiving nodes in the network. Thus, the actual interference that will be produced by link (i, j) is explicitly taken into account.

5.4. Weighted power and interference routing (WPIR)

In WPIR, the two different metrics (i.e., required power for establishing the link and interference caused by the link) are condensed into a single metric via a linear combination. The cost for link (i, j) can be, therefore, written as follows:

$$w_{PI}(i, j) = \beta w_P(i, j)\Theta + (1 - \beta)w_I(i, j), \tag{24}$$

where β controls the weight of each individual metric in the cost and Θ is a normalizing constant between the average w_P and w_I values. By this linear combination, a single weight is assigned to every link and thus it becomes possible to use a Dijkstra-like algorithm. Since different spanning trees will be constructed with different values of β, a drawback of this scheme is that by linearly combining the two metrics, the optimal weighting value will be different for different topologies. This routing scheme is similar to the one discussed in [21].

6. INTERFERENCE AWARE PRUNING ROUTING ALGORITHM—IAPR

The algorithm presented herein is based on an iterative version of the Dijkstra's shortest-path algorithm. At each iteration of the algorithm, links that produce the highest interference are pruned in later iterations of the algorithm. The idea is that by excluding links that produce severe interference, the spatial timeslot reuse could be enhanced.

At iteration k, a shortest-path spanning tree T_k is constructed using weights w_P based on the set of available links, and scheduling is performed on T_k to find the minimum frame length S_k to schedule all links in the tree. The function $I_k(e)$ is a metric of interference produced by link e at iteration k in shortest-path spanning tree T_k. The spanning tree is updated at each iteration by removing the link with the highest interference and running Dijkstra on the remaining links. We keep the spanning that produced a schedule with the minimum frame length. This continues until the stopping criteria of the algorithm are satisfied. The pseudocode of the proposed IAPR scheme is shown in Algorithm 1.

$\overline{G}(V,\overline{L}) \leftarrow G(V,L)$, pre-processing (see Section 6.2)
$k \leftarrow 0$,
K, maximum number of iterations
T, best spanning tree found so far
S, minimum frame length achieved so far
$T_k \leftarrow \varnothing$, spanning tree at iteration k
$S_k = |\overline{L}|$, frame length achieved at iteration k by tree T_k
repeat
  if $k = 0$ **then**
    $T_{k+1} \leftarrow$ Dijkstra using w_P on $\overline{G}(V,\overline{L})$
  else
    $T_{k+1} \leftarrow$ Modified Dijkstra using w_P on $\overline{G}(V,\overline{L})$
  end if
  $S_{k+1} \leftarrow \text{Schedule}(T_{k+1})$
  if $S_{k+1} < S$ **then**
    $T \leftarrow T_{k+1}$
    $S \leftarrow S_{k+1}$
  end if
  Find link $e \in \overline{L}$ such that $I_{k+1}(e) = \max_{l\in\overline{L}}\{I_{k+1}(l)\}$
  $\overline{L} \leftarrow \overline{L} \setminus \{e\}$
  $k \leftarrow k + 1$
until $(k > K)$
return T, S

ALGORITHM 1: Interference aware pruning routing (IAPR).

6.1. Properties of the IAPR scheme

Since the pruning algorithm eliminates at each iteration links that have been previously used in constructing shortest-path spanning trees, the aggregate shortest-path cost will not decrease with iterations. This characteristic of the IAPR scheme is encapsulated in the following result. Let us denote by $p_k(i)$ the aggregate power for the shortest path in T_k from the root node to node i (downlink case), and let K be the maximum number of iterations that the pruning scheme runs.

Proposition 4. *If by $P_k = \sum_{i\in V\setminus\{r\}} p_k(i)$ we denote the aggregate transmitted power in tree T_k constructed by IAPR algorithm at iterations k, then*

$$P_1 \leq P_2 \leq \cdots \leq P_K. \tag{25}$$

The result below is specific to the downlink case, however, the corresponding result for the uplink holds.

Lemma 1. *Suppose link (i, j) is eliminated at iteration k of the IAPR algorithm in the downlink case. Then, there exists at least one spanning tree found at iteration $k + 1$, T_{k+1} with the following property: all nodes of tree T_k, that node j is not one of their parents, will have the same predecessor in tree T_{k+1}.*

Proof. Since link $(i, j) \in T_k$, (i, j) belongs to the shortest path from root node to node j in T_k. Eliminating (i, j) at iteration k, the shortest-path cost to node j in T_{k+1} will increase (or remain the same). Thus, $p_{k+1}(j) \geq p_k(j)$. Thus, any node that did not have node j as a parent in tree T_k will not have j as a parent in tree T_{k+1}. □

Lemma 1 indicates that trees T_k and T_{k+1} may have a large set of common links. This observation motivates the following modification of the Dijkstra algorithm.

Definition 1. The *modified Dijkstra* algorithm takes as input the graph $G_k = (V, L_k)$, the shortest-path spanning tree T_k of G_k, and a link $(i, j) \in T_k, L_k$, and produces a spanning tree T_{k+1} of $G_{k+1} = (V, L_{k+1})$, where $L_{k+1} = L_k \setminus \{(i, j)\}$.

(1) The set of nodes V is partitioned into two sets: V_1 is the set of nodes whose shortest path from the root node on T_k includes link (i, j), and V_2 is the set of all remaining nodes.

(2) The modified Dijkstra assumes that shortest paths for nodes in V_2 are the same as the ones constructed in tree T_k. Thus, shortest paths for this set of nodes are not recalculated.

(3) The algorithm calculates the shortest paths for the set of nodes in V_1 according to the Dijkstra algorithm.

Proposition 5. *The tree produced by the modified Dijkstra algorithm is a shortest-path spanning tree.*

Proof. The proof follows from Lemma 1. □

The above modification of the Dijkstra algorithm is used to accelerate the updating of trees in the IAPR scheme at iterations $k \geq 1$.

6.2. Preprocessing on the initial graph

In order to accelerate the performance of the algorithm, the following preprocessing step can be implemented. In graph $G(V, L)$ of WMN, the set L of links is reduced by considering only links (i, j) that have $w_N(i, j) \leq N_{\max}$ (i.e., only links with less than $N_{\max}$ neighbors are considered) (see Section 5.2).

6.3. Complexity of IAPR

The computational complexity that pertains one iteration of the algorithm is that of the modified Dijkstra algorithm, the pruning operation, and the scheduling engine. Assuming a greedy packing heuristic for scheduling (see Section 7), the complexity of each aforementioned step in one iteration is $\mathcal{O}(n \log n)$. In the worst-case scenario, the algorithm terminates after K iterations, thus the complexity of the overall computational can be $\mathcal{O}(Kn \log n)$ steps.

6.4. Stopping criterion

In general, a stopping criterion is needed to avoid pruning links that are required to ensure connectivity. A possible stopping criterion could be to hault the algorithm at the iteration at which the remaining links no longer can ensure a connected graph. This would mean that we run the algorithm in the order of $|V|^2$ iterations, in the case of dense networks, that is, complete graphs. However, in practise after the removal of a few high interference links at the beginning, the algorithm will stop improving. Even though the algorithm will not deteriorate after many iterations (since we keep the best schedule), it will be unnecessary to run it until the graph is disconnected. This is intuitive and we have also verified it experimentally as will be shown in later sections. Thus, either a relatively small number of iterations should be chosen or the algorithm should run within some predefined small time limit. An operator, for example, can put a maximum time limit on the computational time for running the routing algorithm. In that case, the number of iterations will be limited by this time limit.

6.5. A randomized version of the IAPR

At each iteration of the IAPR scheme, the link that produces the highest interference is pruned with probability one, irrespectively of whether the framelength is decreased or not. A variation of the scheme could be to check a number of links ordered by the level of interference they produce, and prune the first link whose removal improves the framelength. In this case, a number of pruning options are considered and the scheme proceeds in the direction that improves the framelength. However, in the case that none of the $V - 1$ links of the shortest-path tree (when pruned) improve the framelength, the above scheme will be unable to search further and thus stall. In order to further increase the search space and at the same time avoid stalling, we randomize the above scheme by pruning a link with a small probability p, even though the resulting frame length produced by removing this link is not leading to an improvement. The pseudocode of the randomized version of the IAPR scheme (R-IAPR) is shown in Algorithm 2. In the worst-case scenario, the algorithm in each iteration will test all links in the shortest-path tree. Therefore, the computational complexity of the R-IAPR scheme can be $\mathcal{O}(K(V-1)n \log n)$ steps.

7. NUMERICAL INVESTIGATIONS

In this section, we evaluate the performance of the proposed IAPR scheme (both the deterministic and randomized one) compared to the MPR, MNR, IR, and WPIR schemes that have been detailed in Section 5. Simulations are conducted on different randomly generated WMN topologies. For all different schemes, a simple greedy heuristic for evaluating the scheduling has been used, which is described in Algorithm 3. We denote by S the frame length achieved by either the optimal scheduling or the packing heuristic.

The *packing heuristic* tries to pack as many links as possible in each time slot that have not yet transmitted in previous time slots (list A), giving priority to the ones with the highest transmitted power. This continues until all links have transmitted at least once (list A is empty). This *packing heuristic* is similar to a heuristic used in [16, 22, 23], where it was shown to produce satisfactory solutions.

The IAPR (and R-IAPR) scheme uses the packing heuristic at each iteration to evaluate the scheduling of the current shortest path spanning tree. Further, we use the following function to evaluate the interference caused by each link (i, j) in the shortest-path spanning tree T_k: $I_k((i, j)) = w_N(i, j)$, that is, the number of receiving nodes that are within the disc with center i and radius $d(i, j)$.

For the WPIR scheme, the value of Θ has been selected to normalize the average power weight w_P and the average interference weight w_I. The value of $\beta = 0.5$, which gives equal weight to the two metrics, has been used in the simulations.

For the numerical investigations reported in the following sections the parameterization of the simulation environment is as follows. The path loss model for link (i, j) is $\mathrm{PL}_d(i, j) = \mathrm{PL}(d_o) + 10\,\eta \log_{10}(d(i, j)/d_o)$, where $d(i, j)$ is the distance of link (i, j), $\mathrm{PL}(d_o)$ is the close in distance loss (40 dB) for distance d_o (100 m), and η is the path loss exponent, which is assumed to be equal to 3. The value of the SINR threshold γ is 5 dB. The thermal and background noise at the receiver W is assumed to be 10^{-11} Watt, the carrier frequency 2500 MHz, and the maximum transmission power $P_{\max}$ is equal to 50 Watts for all nodes.

Initialization as in Algorithm 1, p, r (uniformly distributed $[0,1]$ random variable)
$T_0 \leftarrow$ Dijkstra $\overline{G}(V,\overline{L})$; $T_{\text{best}} \leftarrow T_0$
$S_0 \leftarrow$ Schedule(T_0); $S_{\text{best}} \leftarrow S_0$
$k \leftarrow 0$
repeat
  $H_{k+1} \leftarrow$ Links in T_k sorted (descending) by $I_k(\cdot)$
  $S \leftarrow S_k$; flag $\leftarrow 1$; $i \leftarrow 1$
  repeat
    $e \leftarrow H_{k+1}$; $T' \leftarrow$ Dijkstra $\overline{G}(V,\overline{L} \setminus \{e\})$; $S' \leftarrow$ Schedule(T')
    if $S' < S$ **then**
      $\overline{L} \leftarrow \overline{L} \setminus \{e\}$; $S \leftarrow S'$; flag $\leftarrow 0$
    else if $r < p$ **then**
      $\overline{L} \leftarrow \overline{L} \setminus \{e\}$; flag $\leftarrow 0$
    else if $i = V - 1$ **then**
      flag $\leftarrow 0$
    else
      $i \leftarrow i + 1$;
    end if
  until flag = 0
  $T_{k+1} \leftarrow$ Dijkstra $\overline{G}(V,\overline{L})$
  $S_{k+1} \leftarrow$ Schedule(T_{k+1})
  if $S_{k+1} < S_{\text{best}}$ **then**
    $S_{\text{best}} \leftarrow S_{k+1}$; $T_{\text{best}} \leftarrow T_{k+1}$
  end if
  $k \leftarrow k + 1$
until $(k > K)$
return T_{best}, S_{best}

Algorithm 2: Randomized interference aware pruning routing (R-IAPR).

Note: The packing heuristic does not perform power control and assumes that each link transmits with power that is 10% higher than the minimum power needed to transmit on its own, i.e., when there is no interference.

Let A be a list of all links sorted according to transmitted power (highest power first). Let B be an empty list and $t = 1$. At timeslot t schedule the first link in list A for transmission and shift it from list A to list B
repeat
  Proceed down the current list A scheduling links for transmission in timeslot t, if feasible, and shifting them to list B if they transmit
  Let $t \leftarrow t + 1$
until A is empty
return $t - 1$

Algorithm 3: Packing heuristic.

7.1. *Main results*

The performance of the different routing schemes has been tested with varying number of nodes in the WMN. The average frame length (in terms of timeslots) and the standard deviation of the framelength has been measured for 100 random uniformly distributed WMN's with 40, 60, and 80 nodes. The packing heuristic was used to evaluate the scheduling and the results are detailed in Table 1. From Table 1, two interesting conclusions can be drawn. The first one is that in all different scenarios, the IAPR scheme outperforms all the other routing algorithms. The average performance gains in terms of minimum framelength with respect to the second best routing scheme, which is MPR, range between 3.2% and 4.7%. It is interesting to note that the standard deviation of the IAPR averaged across all different scenarios is 13% better than that of the MPR scheme. This is of significant importance because the IARP scheme not only has better average minimum framelength but is also more robust to WMN topologies. Secondly, and as mentioned earlier, the MPR scheme outperforms the other routing schemes, except IAPR. This result reveals also the

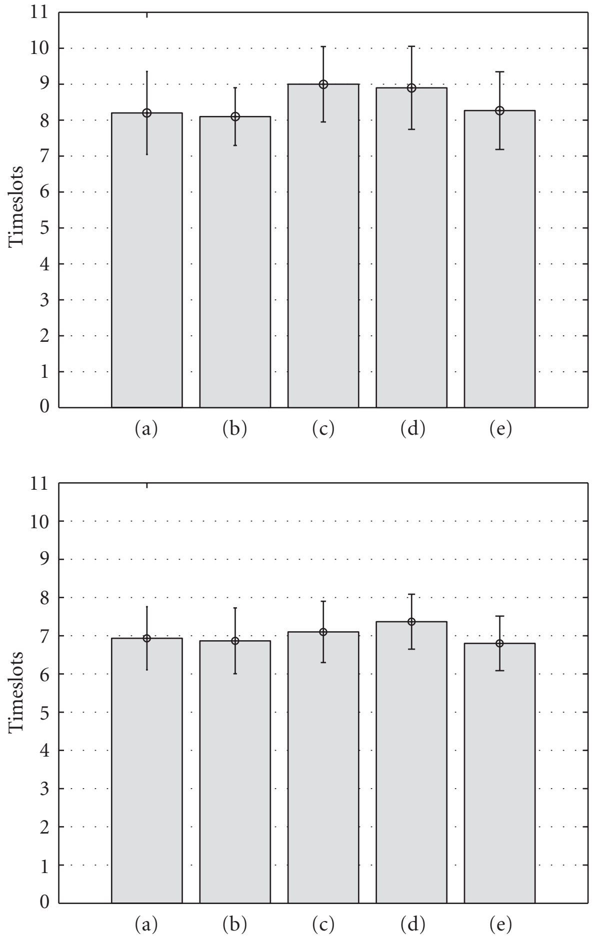


Figure 5: Required number of timeslots for optimal scheduling in the case of top 60 nodes and bottom 40 nodes: (a) minimum power routing (MPR), (b) interference aware pruning routing (IAPR), (c) minimum neighbors routing (MNR), (d) interference routing (IR), and (e) weighted power and interference routing (WPIR).

Table 1: Performance of the routing schemes in WMN topologies with varying number of nodes.

Nodes	40		60		80	
Timeslots	*avg.*	*std*	*avg.*	*std*	*avg.*	*std*
MPR	18.70	1.98	22.33	1.81	24.07	1.71
IAPR	18.10	1.97	21.27	1.57	23.10	1.37
MNR	20.27	1.95	23.8	1.86	26.63	1.63
IR	20.10	1.58	23.6	1.81	26.2	1.84
WPIR	18.93	1.78	22.43	1.55	24.26	1.82

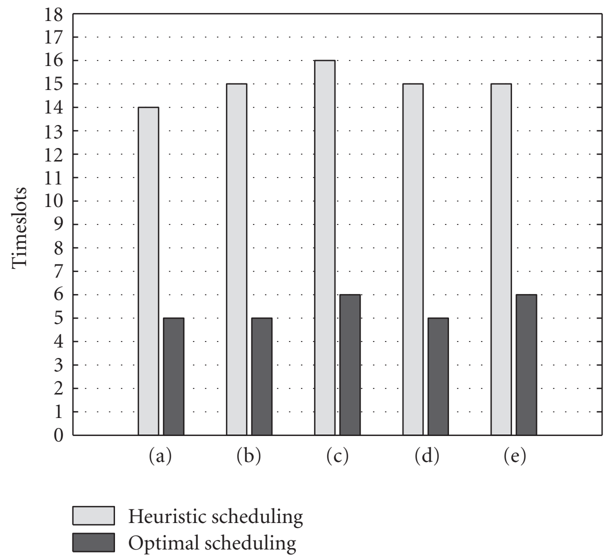


Figure 6: Comparison between packing heuristic and optimal scheduling for the different routing schemes for the case of 18 nodes: (a) interference aware pruning routing (IAPR), (b) minimum power routing (MPR), (c) minimum neighbors-based routing (MNR), (d) interference routing (IR), and (e) weighted power and interference routing (WPIR).

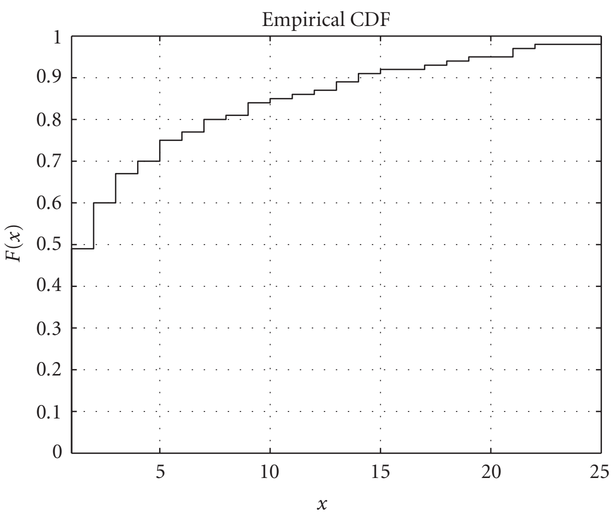


Figure 7: The empirical cumulative distribution function (cdf) of the number of pruning iterations for finding the minimum schedule in terms of timeslots.

inherent difficulties of tuning the β, Θ values for the WPIR so that it can outperform the MPR.

For the same random topologies of 40 and 60 nodes, we test the performance of the different routing schemes using optimal scheduling and the results are shown in Figure 5. When using the optimal scheduling, the average frame length for all different routing schemes is approximately the same (except for MNR and IR schemes which require slightly larger frame lengths). In other words, an optimal scheduling engine can compensate the decisions from the routing engine and thus being able to successfully pack all transmission in almost the same number of timeslots irrespectively of the routing scheme. Despite this fact, the IARP scheme is still very robust to different topologies. As can be seen from the error bar, which expresses the standard deviation, in Figure 5, the std of the frame length for the IARP scheme is approximately 30% less than that of the MPR scheme.

The optimal joint power and routing problem OSR(L) (defined in Section 3.2) has been solved for a small WMN

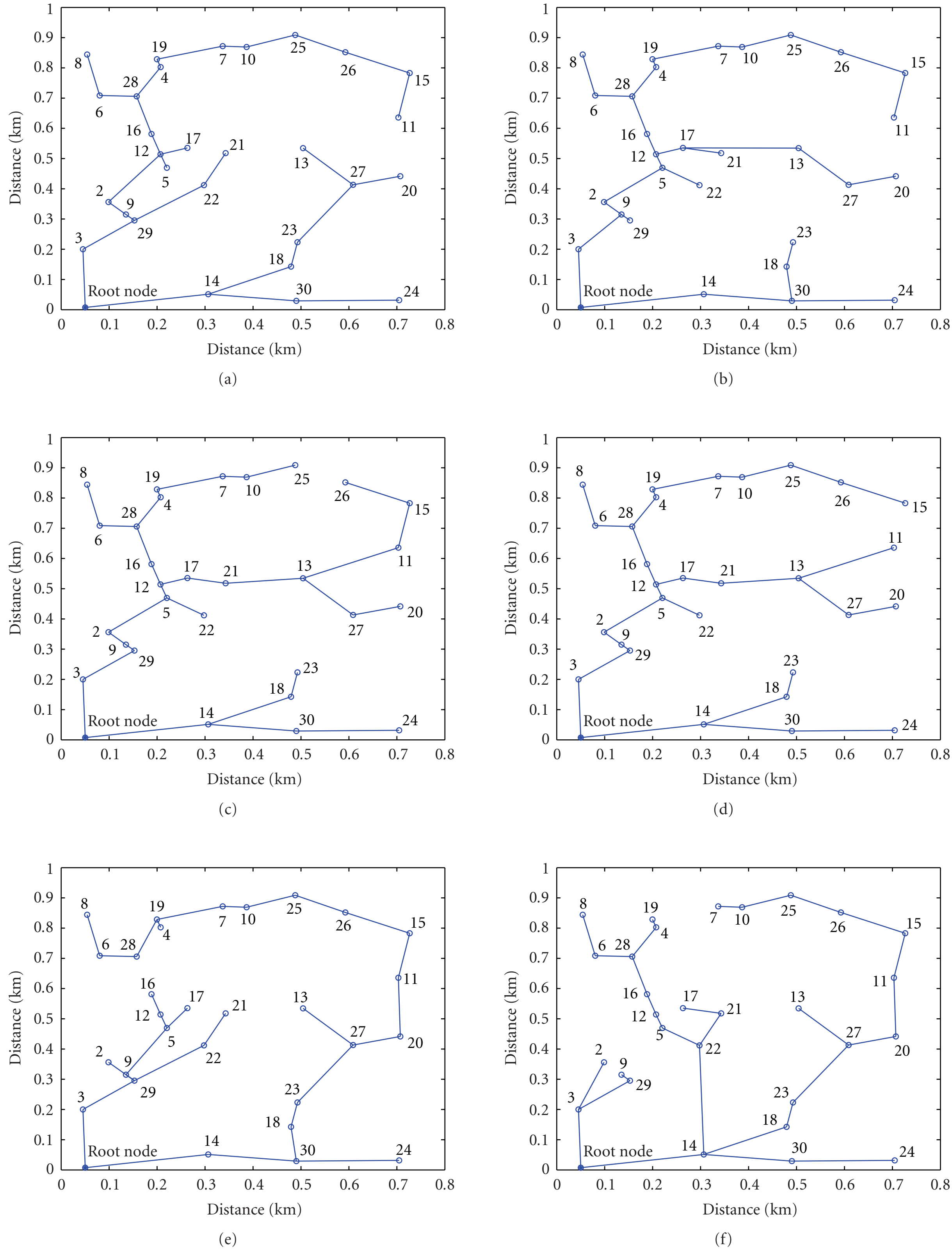


Figure 8: Spanning trees constructed by the different schemes with the assumptions being as follows: 30 nodes (including the root node shown by star) uniformly distributed in a 1 × 1 km plane, path loss exponent is 3, $\gamma = 5$ dB, (a) interference aware pruning routing (IAPR) (15 pruning operations) ($S = 17$), (b) randomized interference aware pruning routing (IAPR) ($P = .01$) ($S = 16$), (c) weighted power and interference routing (WPIR) ($\beta = 0.5$) ($S = 19$), (d) minimum power routing (MPR) ($S = 19$), (e) minimum neighbors routing (MNR) ($S = 17$), and (f) interference routing (IR) ($S = 19$).

with 18 nodes. For this topology, the minimum number of timeslots computed by CPLEX was 5 (this solution was found within 200 seconds). In Figure 6, we compare the number of timeslots computed for the same topology for the different routing schemes with optimal and heuristic scheduling. As can be seen from the figure, when using the heuristic scheduling, the pruning scheme provides a 6.7% improvement compared to the other routing schemes. It is also worth mentioning that when using the optimal scheduling, three out of five routing schemes achieve the same number of timeslots as the optimal joint scheduling and routing.

One interesting question is that if we run the pruning algorithm for a fixed number of iterations, at which iteration will it find the schedule with the minimum possible frame length? To shed some light on that question, we have performed the following experiment. For a specific number of nodes in the network, namely, 40 in this case, 100 uniformly distributed topologies have been generated in a 3×3 km rectangular area. For each topology, we perform 30 pruning operations and store the iteration where the IAPR scheme found the frame length with the minimum number of timeslots. We have repeated this procedure for each topology and the result is shown in Figure 7, which depicts the empirical cumulative distribution function (cdf) that has been obtained by the experiment. The empirical cdf reveals that with 90% probability, the pruning algorithm finds the schedule with the minimum timeslot span in less than 14 iterations.

7.2. *Randomized pruning scheme*

In Section 6.5, we described a variation of the pruning scheme (called R-IAPR) that increases the search space for a better solution by testing more links or by randomizing the search. After demonstrating the improvement performed by the IAPR scheme in Section 7.1, we proceed to demonstrate that the randomized version of the IAPR scheme can offer further improvement.

Table 2 shows the average framelength (in terms of timeslots) and the standard deviation of the frame length, averaged over 100 random uniformly distributed WMN topologies with 40, 60, and 80 nodes, respectively, for the MPR, IAPR, and R-IAPR schemes. For the R-IAPR scheme, we used pruning probability $p = 1/3V$ (see Section 6.5). The average gains of the pruning schemes are also displayed. We observe that the improvement in performance is evident over all three network sizes, with the randomized pruning scheme offering a consistent enhancement in performance over the IAPR scheme.

Table 2: Average timeframes of MPR, IAPR, R-IAPR routing schemes, and average performance gains of the pruning schemes.

Nodes	40		60		80	
Timeslots	*avg.*	*std.*	*avg.*	*std.*	*avg.*	*std.*
MPR	13.17	1.37	14.99	1.59	16.02	1.44
IAPR	12.58	1.32	14.23	1.57	15.45	1.31
R-IAPR	12.23	1.22	13.76	1.31	14.89	1.25
gains over MPR	*avg.*		*avg.*		*avg.*	
IAPR	4.34		4.96		3.42	
R-IAPR	6.94		7.91		6.90	

7.3. *Routing illustrations*

Figure 8 shows the spanning trees constructed by the different schemes in the case of a random WMN topology with 30 nodes. In this scenario, the packing heuristic has been used as the scheduling technique for evaluating the frame lengths produced by the different routing schemes. The IAPR scheme requires 17 timeslots (Figure 8(a)), while the R-IAPR scheme requires 16 timeslots (Figure 8(b)). These should be compared to the MPR and the WPIR schemes, which both require 19 timeslots (Figures 8(c) and 8(b), resp.). Thus, the R-IARP scheme provides more than 15% improvement over the MPR. It is also interesting to note that for this scenario, the MNR scheme achieved the same frame length as the IARP. Note also that even though the spatial reuse for MPR, WPIR, and IR schemes is the same, the constructed spanning trees are very different.

8. CONCLUSIONS

In this paper, an interference aware pruning-based routing scheme has been proposed which strives to optimize path selection and STDMA scheduling. A randomized version of the pruning scheme has also been detailed and has been shown to improve the later heuristic. We have formulated the corresponding MILP to perform joint scheduling and routing, and used this formulation to compare the performance of the proposed scheme with the optimal joint scheduling and routing. The routing schemes were evaluated using both a greedy scheduling heuristic and optimal scheduling. Extensive performance evaluation in different network settings of the proposed scheme revealed that it outperforms the previously proposed routing schemes where interference and power consumption are used as a routing metric.

We should note that the general nature of the algorithms presented in this paper, including the proposed ones, is applicable to more general network settings that can include directional pattern transmissions and link gains that capture more precisely slow signal variations due to the physical terrain. Furthermore, we merely looked at the joint routing and scheduling problem without considering flows and the corresponding requirement of conserving them. This simplification allowed us to shed light into the interplay between the two engines and compare different schemes. We left the study of such augmented models that incorporate network flow requirements as an avenue of future research.

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