

Research Article

A Multiuser MIMO Transmit Beamformer Based on the Statistics of the Signal-to-Leakage Ratio

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Received 23 February 2009; Accepted 3 June 2009

Recommended by Alex Gershman

A multiuser multiple-input multiple-output (MIMO) downlink communication system is analyzed in a Rayleigh fading environment. The approximate closed-form expressions for the probability density function (PDF) of the signal-to-leakage ratio (SLR), its average, and the outage probability have been derived in terms of the transmit beamformer weight vector. With the help of some conservative derivations, it has been shown that the transmit beamformer which maximizes the average SLR also minimizes the outage probability of the SLR. Computer simulations are carried out to compare the theoretical and simulation results for the channels whose spatial correlations are modeled with different methods.

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1. Introduction

The capacity of a wireless cellular system is limited by the mutual interference among simultaneous users. Using multiple antenna systems, and in particular, the adaptive beamforming, this problem can be minimized, and the system capacity can be improved. In recent years, the optimum downlink beamforming problem (including power control) has been extensively studied in [1–3] where the signal-to-interference-plus-noise ratio (SINR) is used as a quality of service (QoS) criterion. After it has been found that the multiple-input multiple-output (MIMO) techniques significantly enhance the performance of wireless communication systems [4, 5], the joint optimization of the transmit and receive beamformers [6] has also been investigated for MIMO systems. Motivated by the fact that the optimum transmit beamformers [1–3] and the joint optimum transmit-receive beamformers [6] can be obtained only iteratively due to the coupled nature of the corresponding optimization problems, recently, the concept of leakage and subsequently the signal-to-leakage-plus-noise ratio (SLNR) as a figure of merit have been introduced in [7, 8]. (Note that SLNR as a performance criterion has been considered in [9–11] for multiple-

input-single-output (MISO) systems.) Although the latter approach only gives suboptimum solutions, it leads to a decoupled optimization problem and admits closed-form solutions for downlink beamforming in multiuser MIMO systems.

While investigating multiuser systems from a system level perspective, in many cases, the outage probability has also been widely used as a QoS parameter. The closed-form expressions of the outage probability with equal gain and optimum combining have been derived in [12, 13], respectively, in a flat-fading Rayleigh environment with cochannel interference. The latter work has been extended in [14] to a Rician-Rayleigh environment where the desired signal and interferers are subject to Rician and Rayleigh fading, respectively. However, in all of the above-mentioned papers, investigations have been limited to the derivations of the outage probability expressions for specific types of receivers. The outage probability of the signal-to-interference ratio is used to formulate the optimum power control problem for interference limited wireless systems in [15, 16] where the total transmit power is minimized subject to outage probability constraints. However, both of these works [15, 16] are limited to systems with single antenna at transmitters and receivers.

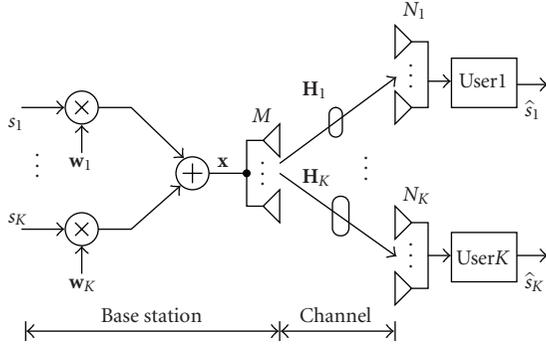


FIGURE 1: Multiuser MIMO downlink beamforming.

In this paper, we consider the downlink of a multiuser MIMO wireless communication system in a Rayleigh fading environment. The base station (BS) communicates with several cochannel users in the same time and frequency slots. In our method, we use the average signal-to-leakage ratio (SLR) and the outage probability of SLR as performance metrics which are based on the concept of leakage power [7, 8]. In particular, the novelty of our work lies on the facts that we first derive an approximation of the statistical distribution of SLR [7] for each cochannel user of the MIMO system in terms of transmit beamforming weight vector. Second, the approximate closed-form expression for the outage probability of SLR is derived. Then, we obtain the solution for the transmit beamformer that minimizes the aforementioned outage probability. According to our best source of knowledge, this approach has not been previously considered for the multiuser MIMO downlink beamforming. With some conservative derivations, we also demonstrate that the beamformer which minimizes the outage probability is same as the one which maximizes the average SLR. Note that similar conclusion has been made in [17] where the downlink beamforming for multiuser MISO systems is analyzed using the SINR and its outage probability as the performance criteria. In contrast to [7], we consider that the BS has only the knowledge of the second-order statistics such as the covariance matrix of the downlink user-channels. The motivation behind this assumption is that the knowledge of instantaneous channel information can be available at the BS only through the feedback from users. The drawbacks of the feedback approach are the reduction of the system capacity because of the frequent channel usage required for the transmission of the feedback information from users to the BS, and inherent time delays, errors, and extra costs associated with such a feedback. Furthermore, if the channel varies rapidly, it is not reasonable to acquire the instantaneous feedback at the transmitter, because the optimal transmitter designed on the basis of previously acquired information becomes outdated quickly (see [18] and the references therein). Thus, we consider that no full-rate feedback information is available at the BS.

The remainder of this paper is organized as follows. The system model is presented in Section 2. The probability density function (PDF) of SLR, its mean, and the outage

probability of SLR are derived in terms of the beamformer weight vector in Section 3. In Section 4, the transmit beamformer which maximizes the average SLR and minimizes the outage probability is obtained. In Section 5, analytical and numerical results are compared. Finally, conclusions are drawn in Section 6.

Notational conventions. Upper (lower) bold face letters will be used for matrices (vectors); $(\cdot)^H$, $E\{\cdot\}$, \mathbf{I}_n , $\|\cdot\|$, $\text{tr}(\cdot)$, and $\mathcal{C}^{M \times M}$ denote the Hermitian transpose, mathematical expectation, $n \times n$ identity matrix, Euclidean norm, trace operator, and the space of $M \times M$ matrices with complex entries, respectively.

2. System Model

Consider a downlink multiuser scenario with a multi-antenna BS of M antennas communicating with K multi-antenna users. (If there are multiple BSs and they have also the channel information of users assigned to other BSs, the SLR-based method needs to be modified in such a way that each BS takes into account the power leaked by it to the users of other BSs. The necessary modifications, in our case, can be done with some straightforward steps.) The block diagram is shown in Figure 1. The signal transmitted by the BS is given by

$$\mathbf{x} = \sum_{k=1}^K \mathbf{w}_k s_k \in \mathcal{C}^{M \times 1}, \quad (1)$$

where s_k and $\mathbf{w}_k \in \mathcal{C}^{M \times 1}$ are, respectively, the signal stream and the transmit beamformer weight vector for k th user. It is assumed that $E\{s_k\} = 0$ and $E\{|s_k|^2\} = 1$ for $k = 1, \dots, K$. (We consider equal power allocations to all users. Note that power control can be included in the design of beamformers by using a two-step approach, that is, by optimizing the beamformers first and then the powers or vice-versa [1, 2].) Moreover, following the spirit of [7], we consider that the beamformer weights are normalized, that is, $\|\mathbf{w}_k\|^2 = 1$. Let N_i denote the number of receive antennas at i th user. The signal vector received by i th user is

$$\mathbf{y}_i = \left[\sqrt{G_i} \mathbf{H}_i \mathbf{x} + \mathbf{n}_i \right] \in \mathcal{C}^{N_i \times 1}, \quad (2)$$

where G_i is a constant that includes the effect of distance-dependent path loss factor and the distance-independent mean-channel power gain, $\mathbf{H}_i \in \mathcal{C}^{N_i \times M}$ is the spatially correlated MIMO channel matrix, and $\mathbf{n}_i \in \mathcal{C}^{N_i \times 1}$ denotes the additive noise. It is assumed that each user is surrounded by a large number of scatterers whereas the BS, which is generally located at larger heights from the ground level, does not observe rich scattering. In this scenario, the MIMO channel as seen from the user/BS is spatially uncorrelated/correlated. Thus, the i th MIMO channel can be given by replacing the receive correlation matrix with an identity matrix in the famous Kronecker-model [19] which turns into the following form: $\mathbf{H}_i = \mathbf{H}_w^i \boldsymbol{\Sigma}_i^{1/2}$, where the entries of $\mathbf{H}_w^i \in \mathcal{C}^{N_i \times M}$ are assumed to be zero-mean

circularly symmetric complex Gaussian (ZMCSCG) random variables with unit variance such that $E\{\text{tr}((\mathbf{H}_i^H \mathbf{H}_i)^H \mathbf{H}_i^H)\} = N_i M$, and $\mathbf{\Sigma}_i \in \mathcal{C}^{M \times M}$ represents the spatial correlation matrix at the BS corresponding to the i th user channel. It is important to emphasize here that the derivations for the SLR mean and SLR outage probability can be easily extended to double-sided correlated MIMO channels (including the user side correlation), and thus, our main results are also valid for such MIMO channels. Note that $\mathbf{\Sigma}_i$ are symmetric positive semidefinite matrices and are a function of the antenna spacing, average direction of arrival of the scattered signal from i th user, and the corresponding angular spread [20]. We invite our readers to have a look at [20] and the references therein for determining $\mathbf{\Sigma}_i$. Furthermore, without loss of generality, the elements of \mathbf{n}_i in (2) are considered to be ZMCSCG with the variance σ_i^2 , that is, $\mathbf{n}_i \sim \mathcal{N}_C(\mathbf{0}, \mathbf{I}_{N_i} \sigma_i^2)$, where \mathbf{I}_{N_i} denotes $N_i \times N_i$ identity matrix. Inserting (1) into (2) and applying the statistical expectation over signal and noise realizations, the SLNR for i th user can be expressed as [7]

$$\text{SLNR}_i = \frac{G_i \|\mathbf{H}_i \mathbf{w}_i\|^2}{N_i \sigma_i^2 + \sum_{k=1, k \neq i}^K G_k \|\mathbf{H}_k \mathbf{w}_i\|^2}. \quad (3)$$

Note that, here, $G_i \|\mathbf{H}_i \mathbf{w}_i\|^2$ is the power of the desired signal for user i whereas $G_k \|\mathbf{H}_k \mathbf{w}_i\|^2$ is the power of interference that is caused by user i on the signal received by some other user k . The leakage for user i is thus the total power leaked from this user to all other users which is $\sum_{k=1, k \neq i}^K G_k \|\mathbf{H}_k \mathbf{w}_i\|^2$. The objective of beamformer is to make $G_i \|\mathbf{H}_i \mathbf{w}_i\|^2$ as large as possible when compared to the leakage power $\sum_{k=1, k \neq i}^K G_k \|\mathbf{H}_k \mathbf{w}_i\|^2$. (The performance of the beamformer can be boosted by taking into account the noise term $N_i \sigma_i^2$ which acts as a diagonal loading factor [21].) The main motivation behind this approach is that it results into a decoupled optimization problem and provides analytical closed-form solutions (see [7, Sections I-III] for more information), though they are not optimal relative to the SINR criterion [1–3]. Moreover, the SLNR as a performance criterion also allows the BS to work more independently from the receivers since the BS does not need the knowledge of receive beamformer or in general receiver's operator. Similarly, each user performs beamforming or any other linear operations to recover its signal without depending on transmit beamforming vectors of other users. Let i th user uses a matched filter to recover its signal. The detected signal of this user can be given by $\hat{s}_i = \mathbf{z}_i^H \mathbf{y}_i$ where $\mathbf{z}_i = (\mathbf{H}_i \mathbf{w}_i) / \|\mathbf{H}_i \mathbf{w}_i\| \in \mathcal{C}^{N_i \times 1}$ is the matched filter response. Then, using (1) and (2), \hat{s}_i can be written as

$$\begin{aligned} \hat{s}_i &= \frac{\mathbf{w}_i^H \mathbf{H}_i^H}{\|\mathbf{H}_i \mathbf{w}_i\|} \mathbf{H}_i \mathbf{w}_i \sqrt{G_i} s_i \\ &+ \sum_{k=1, k \neq i}^K \frac{\mathbf{w}_i^H \mathbf{H}_i^H}{\|\mathbf{H}_i \mathbf{w}_i\|} \mathbf{H}_k \mathbf{w}_k \sqrt{G_i} s_k + \frac{\mathbf{w}_i^H \mathbf{H}_i^H}{\|\mathbf{H}_i \mathbf{w}_i\|} \mathbf{n}_i. \end{aligned} \quad (4)$$

Applying mathematical expectation with respect to independent realizations of signals and noise, the SINR for i th user is

$$\text{SINR}_i = \frac{G_i \|\mathbf{w}_i^H \mathbf{H}_i^H\|^4}{\sigma_i^2 \|\mathbf{w}_i^H \mathbf{H}_i^H\|^4 + \sum_{k=1, k \neq i}^K G_k \|\mathbf{w}_i^H \mathbf{H}_i^H \mathbf{H}_k \mathbf{w}_k\|^2}. \quad (5)$$

It is considered that the transmitter (also the BS) does not know user's receiver, and thus, the SINR (5) is not available at the transmitter. In this case, the transmitter optimizes its beamforming vector to maximize the SLNR (3) thereby assisting the user's receiver in its task of improving the SINR (5). The latter fact can be verified numerically. Note that the beamformer based on maximization of (3) can also be designed for the cases where only the knowledge of second-order statistics of downlink channels is available at the BS. In such cases, the advantages are twofold; the BS and receivers can work in a distributed manner (since the criterion is SLNR), and the BS needs only a limited feedback information from the receivers. To facilitate the aforementioned scheme, we first analyze the statistics of SLNR (3) in the following section.

3. Average SLR and the Outage Probability

Using the notations $\mathbf{A}_i \triangleq \mathbf{H}_i^H \mathbf{H}_i$ for all i , and assuming that the leakage power (The derivation of outage probability expression and its minimization become too involved if the noise power is not negligible. However, noting that the cellular systems such as UMTS with beamforming techniques can support a significant number of cochannel users per cell [21] (this number can be further increased if more scrambling codes can be allocated for each cell [22]), the assumption that the multiuser leakage power dominates the thermal noise power at each user is not a stringent one.) is large compared to the noise power, we get the SLR from (3) as

$$\text{SLR}_i = \frac{G_i \mathbf{w}_i^H \mathbf{A}_i \mathbf{w}_i}{\sum_{k=1, k \neq i}^K G_k \mathbf{w}_i^H \mathbf{A}_k \mathbf{w}_i}. \quad (6)$$

We first note that the rows of \mathbf{H}_i are statistically independent, and each row has an M -variate complex Gaussian distribution with the mean vector $\mu = 0$ and the covariance matrix $\mathbf{\Sigma}_i$. According to [23], in this case, \mathbf{A}_i are complex Wishart distributed with the scaling matrix $\mathbf{\Sigma}_i$ and the degrees of freedom parameter N_i . For conciseness and simplified mathematical presentation, in the rest of this paper, we assume that $N_i = N$, for all i . Here, we also stress that our results can be easily extended to the general case where N_i are different. Mathematically, we can thus write $\mathbf{A}_i \sim \mathcal{C}\mathcal{W}_M(N, \mathbf{\Sigma}_i)$, where $\mathcal{C}\mathcal{W}_M(\cdot)$ represents the complex Wishart matrix of size $M \times M$. Let us use the notations $u \triangleq G_i \mathbf{w}_i^H \mathbf{A}_i \mathbf{w}_i$ and $v \triangleq \sum_{k=1, k \neq i}^{K-1} G_k \mathbf{w}_i^H \mathbf{A}_k \mathbf{w}_i$. According to the results of [14] and since $\mathbf{A}_i \sim \mathcal{C}\mathcal{W}_M(N, \mathbf{\Sigma}_i)$, we get $u \sim \mathcal{C}\mathcal{W}_1(N, G_i \mathbf{w}_i^H \mathbf{\Sigma}_i \mathbf{w}_i)$. We note that for any \mathbf{w}_i , $G_i \mathbf{w}_i^H \mathbf{\Sigma}_i \mathbf{w}_i \geq 0$, because $\mathbf{\Sigma}_i$ is a positive semidefinite matrix. Since $\mathcal{C}\mathcal{W}_1(\cdot)$ is

a Chi-square distribution, the random variable $u \geq 0$ has the following PDF:

$$f_U(u) = \frac{1}{c_i^N \Gamma(N)} u^{N-1} e^{-u/c_i} \quad (7)$$

where $f_U(u) = 0$, for $u \leq 0$, $c_i = G_i \mathbf{w}_i^H \boldsymbol{\Sigma}_i \mathbf{w}_i$, and $\Gamma(n) = \int_0^\infty x^{n-1} e^{-x} dx$ is the Gamma function. Comparing the PDF of (7) to the standard form of Chi-square PDF [23], u can be alternatively expressed as

$$u = \frac{1}{2} c_i \tilde{u}, \quad \text{where } \tilde{u} \sim \chi_{2N}^2, \quad (8)$$

where χ_{2N}^2 is the Chi-square distribution with degrees of freedom $2N$. Using (8), v can be written as

$$v = \sum_{k=1, k \neq i}^K \frac{1}{2} \left(G_k \mathbf{w}_i^H \boldsymbol{\Sigma}_k \mathbf{w}_i \right) \tilde{v}_k \quad \text{where } \tilde{v}_k \sim \chi_{2N}^2. \quad (9)$$

It can be observed from (9) that v is a weighted sum of statistically independent Chi-square random variables, where the weights $G_k \mathbf{w}_i^H \boldsymbol{\Sigma}_k \mathbf{w}_i \geq 0$ since $\boldsymbol{\Sigma}_k$ for all k are positive semidefinite. The exact and closed-form solution for the PDF of v is not known. However, according to [24] and the references therein, the PDF of v can be found by approximating v as a random variable with the Chi-square distribution having degrees of freedom 2β and the scaling factor $\alpha/2$ as

$$v \sim \sum_{k=1, k \neq i}^K \frac{1}{2} \left(G_k \mathbf{w}_i^H \boldsymbol{\Sigma}_k \mathbf{w}_i \right) \tilde{v}_k \sim \frac{\alpha}{2} \chi_{2\beta}^2 \quad (10)$$

where α and β can be determined by equating the first- and second-order moments of the left- and right-hand sides of relation (10). (This approximation is very accurate and widely adopted in statistics and engineering. The accuracy of the approximation will be confirmed later through numerical simulation results.) Evaluation of the first-order moment (mean) of the both sides of (10) gives

$$\sum_{k=1, k \neq i}^K \frac{1}{2} \left(G_k \mathbf{w}_i^H \boldsymbol{\Sigma}_k \mathbf{w}_i \right) \cdot 2N = \frac{\alpha}{2} \cdot 2\beta. \quad (11)$$

Similarly by equating the second-order moment (variance) of the both sides of (10), we get

$$\sum_{k=1, k \neq i}^K \frac{1}{4} \left(G_k \mathbf{w}_i^H \boldsymbol{\Sigma}_k \mathbf{w}_i \right)^2 \cdot 4N = \frac{1}{4} \alpha^2 \cdot 4\beta. \quad (12)$$

Solving (11) and (12), α and β can be expressed as

$$\alpha = \frac{\sum_{k=1, k \neq i}^K \left(G_k \mathbf{w}_i^H \boldsymbol{\Sigma}_k \mathbf{w}_i \right)^2}{\sum_{k=1, k \neq i}^K \left(G_k \mathbf{w}_i^H \boldsymbol{\Sigma}_k \mathbf{w}_i \right)}, \quad (13)$$

$$\beta = \frac{\left(\sum_{k=1, k \neq i}^K G_k \mathbf{w}_i^H \boldsymbol{\Sigma}_k \mathbf{w}_i \right)^2}{\sum_{k=1, k \neq i}^K \left(G_k \mathbf{w}_i^H \boldsymbol{\Sigma}_k \mathbf{w}_i \right)^2} N.$$

Like the PDF of u given in (7), the PDF of $v \geq 0$ is well known to be [23]

$$f_V(v) = \frac{1}{\alpha^\beta \Gamma(\beta)} v^{\beta-1} e^{-v/\alpha}, \quad (14)$$

where again $f_V(v) = 0$, for $v \leq 0$. For the sake of better exposition, let $\text{SLR}_i \triangleq z$, where $z = u/v$ is the ratio of two statistically independent random variables. The PDF of z can be thus written as

$$f_Z(z) = \int_0^\infty v f_U(zv) f_V(v) dv. \quad (15)$$

Applying (7) and (14) into (15) and after some steps, we get

$$f_Z(z) = \frac{z^{N-1}}{c_i^N \Gamma(N) \alpha^\beta \Gamma(\beta)} \int_0^\infty v^{N+\beta-1} e^{-[z/c_i + 1/\alpha]v} dv. \quad (16)$$

With the help of [25, equation 3.38.4], (16) can be written in the closed-form as

$$f_Z(z) = \frac{\Gamma(N+\beta)}{c_i^N \Gamma(N) \alpha^\beta \Gamma(\beta)} z^{N-1} \left[\frac{z}{c_i} + \frac{1}{\alpha} \right]^{-N-\beta}. \quad (17)$$

The average of the SLR is thus given by

$$E\{z\} = \int_0^\infty z f_Z(z) dz. \quad (18)$$

After substituting $f_Z(z)$ from (17), applying [25, equation 3.194.3], and after some steps of straightforward derivations, we get

$$E\{z\} = \frac{\Gamma(N+\beta) c_i}{\alpha \Gamma(\beta) \Gamma(N)} B(N+1, \beta-1), \quad (19)$$

where $B(x, y) = \Gamma(x)\Gamma(y)/\Gamma(x+y)$ is the Beta function. Noting that $\Gamma(x+1) = x\Gamma(x)$ and $\Gamma(x) = (x-1)!$, (19) can be further simplified as

$$E\{z\} = \frac{N c_i}{\alpha \beta - \alpha}. \quad (20)$$

The outage probability of SLR is a parameter that shows how often the transmit beamformer is not capable of maintaining the ratio of the signal power to the leakage power above a certain threshold value. The outage probability for the i th user is defined as

$$P_{\text{out}}(\gamma_0, \mathbf{w}_i) = \Pr\{\text{SLR}_i \triangleq z \leq \gamma_0\}, \quad (21)$$

where γ_0 is the system specific threshold value. Note that (21) represents the probability of the transmit beamformer failing to perform its beamforming task properly. Hence, the concept of the SLR outage is analogous to the probability of receiver failing to work properly but is only applicable from a transmitter's point of view. Since the PDF of SLR is already known, the outage probability of (21) can be expressed as

$$P_{\text{out}}(\gamma_0, \mathbf{w}_i) = \int_0^{\gamma_0} f_Z(z) dz. \quad (22)$$

Using (17) and applying [25, equation 3.194.1], it can be shown that the outage probability (22) can be expressed as

$$P_{\text{out}}(\gamma_0, \mathbf{w}_i) = \frac{1}{NB(\beta, N)} \cdot s_{\text{con}}^N \cdot {}_2F_1(N, \beta + N; N + 1; -s_{\text{con}}), \quad (23)$$

where $s_{\text{con}} \triangleq ((\alpha\gamma_0)/c_i)$ and ${}_2F_1(\cdot)$ is the Gauss hypergeometric function (see [25, equation 9.100]). Noting the transformation rule ${}_2F_1(a, b; c; x) = (1-x)^{-b} {}_2F_1(b, c-a; c; x/(x-1))$ (see [25, equation 9.131.1]) and the fact that ${}_2F_1(a, b; c; x) = {}_2F_1(b, a; c; x)$, and after some simple manipulations, (23) can also be expressed in the following alternative form:

$$P_{\text{out}}(\gamma_0, \mathbf{w}_i) = \frac{1}{NB(\beta, N)} \cdot \frac{s_{\text{con}}^N}{(1+s_{\text{con}})^{\beta+N}} \cdot {}_2F_1\left(1, \beta + N; N + 1; \frac{s_{\text{con}}}{1+s_{\text{con}}}\right). \quad (24)$$

Here, it is worthwhile to mention that for $N = 1$, u (7) becomes exponentially distributed whereas v (9) becomes a weighted sum of independent exponentially distributed random variables. In this case, the outage probability expression of [15] can be easily derived. However, it cannot be analytically obtained by substituting $N = 1$ in (23) due to the approximation (10). Also, note that the proposed outage probability analysis can be applied to frequency-selective fading channels where we can consider that the orthogonal frequency division multiplexing (OFDM) is used as a modulation technique. In this context, the MIMO channel for each subcarrier can be considered to be a flat-fading channel. Considering that all users can access a given subcarrier and that the lengths of channel impulse responses for all receive-transmit antenna combinations of all users are shorter than the cyclic prefix [26], the SLR for the i th user and s th subcarrier can be expressed as

$$\text{SLR}_{i,s} = \frac{G_{i,s} \|\mathbf{H}_i(s) \mathbf{w}_{i,s}\|^2}{\sum_{k=1, k \neq i}^K G_{k,s} \|\mathbf{H}_k(s) \mathbf{w}_{i,s}\|^2}, \quad (25)$$

where $\mathbf{H}_i(s) = \mathbf{H}_w^i(s) \boldsymbol{\Sigma}_i^{(1/2)}$ is the MIMO channel in frequency domain for the i th user and s th subcarrier, and $G_{i,s}$ is the corresponding gain. Let $[\mathbf{H}_w^i(s)]_{n,m}$ be the n th row and m th column entry of $\mathbf{H}_w^i(s)$, and be given by

$$[\mathbf{H}_w^i(s)]_{n,m} = \sum_{p=0}^{N_t} h_{n,m,i}^w(p) e^{-j(2\pi sp/N_c)}, \quad (26)$$

where N_c is the total number of subcarriers, $N_t + 1$ is the number of independently fading channel-taps, and $h_{n,m,i}^w(p)$ is the impulse response for p th tap of the channel between n th receive and m th transmit antenna. If $\{h_{n,m,i}^w(p)\}_{p=0}^{N_t}$ are ZMCSCG, it is very easy to note that $[\mathbf{H}_w^i(s)]_{n,m}$ is a ZMCSCG. Furthermore, if the average sum of the tap-powers for the channel between the n th receive and m th transmit antennas is same, that is, if $E\{\sum_{p=0}^{N_t} |h_{n,m,i}^w(p)|^2\} =$

a_i for all m, n , after some straightforward steps, we can easily verify that the distribution of $\{\mathbf{H}_i(s)^H \mathbf{H}_i(s)\}_{i=1}^K$ remains complex Wishart with the same scaling matrix $\{a_i \boldsymbol{\Sigma}_i\}_{i=1}^K$ and the degrees of freedom parameter N . This shows that the statistics of the signal and leakage powers for a given subcarrier and user remain unchanged.

4. Maximize the Average SLR and Minimize the Outage Probability

In this section, our objective is to find the optimum \mathbf{w}_i which maximizes the average SLR and minimizes the outage probability of the SLR observed by i th user. Note that due to the fact that we use the average SLR and SLR outage as the criteria, the beamformer design is a decoupled problem and can be carried out separately for each user.

4.1. Maximize the Average SLR. The beamformer which maximizes the average SLR is obtained by solving the problem $\max_{\mathbf{w}_i} E\{z\}$ which is a difficult optimization problem as α and β are complicated functions of \mathbf{w}_i , although c_i is a quadratic function of \mathbf{w}_i . In order to make this optimization problem tractable, we make certain assumptions which will be clear in the sequel. We can write (20) as

$$E\{z\} = \frac{Nc_i}{\alpha\beta} \cdot \frac{1}{1-1/\beta}. \quad (27)$$

Let us define $y_k \triangleq G_k \mathbf{w}_i^H \boldsymbol{\Sigma}_k \mathbf{w}_i$ for all $k \neq i$, where $y_k \geq 0$. Then, with the help of a well-known power-mean inequality, we can write

$$\frac{(\sum_{k=1, k \neq i}^K y_k)^2}{\sum_{k=1, k \neq i}^K y_k^2} \leq K - 1, \quad (28)$$

where the equality holds only if $\{y_k\}_{k=1, k \neq i}^K$ are all equal. Applying the above inequality to the expression of β in (13), we can get an upper bound for β and more specifically we can write $1/\beta \geq 1/N(K-1)$. With this observation, the average SLR (27) can be lowerbounded as

$$E\{z\} \geq \frac{Nc_i}{\alpha\beta} \cdot \frac{NK - N}{NK - N - 1}. \quad (29)$$

Here, an interesting observation is that though α and β are separately nonquadratic functions of \mathbf{w}_i , their products $\alpha\beta$ is quadratic in \mathbf{w}_i . The latter fact can be observed from (13), and thus the product $\alpha\beta$ can be expressed as

$$\alpha\beta = N \sum_{k=1, k \neq i}^K G_k \mathbf{w}_i^H \boldsymbol{\Sigma}_k \mathbf{w}_i. \quad (30)$$

Using (30) and resubstituting c_i in terms of \mathbf{w}_i , (29) can be expressed as

$$E\{z\} \geq \frac{G_i \mathbf{w}_i^H \boldsymbol{\Sigma}_i \mathbf{w}_i}{\sum_{k=1, k \neq i}^K G_k \mathbf{w}_i^H \boldsymbol{\Sigma}_k \mathbf{w}_i} \cdot \frac{NK - N}{NK - N - 1}. \quad (31)$$

Since the exact average SLR (27) is difficult to maximize, we maximize its lower bound (31) which has a Rayleigh quotient form. The latter can be maximized by maximizing the numerator $G_i \mathbf{w}_i^H \boldsymbol{\Sigma}_i \mathbf{w}_i$ (the useful power directed to the i th user) while keeping the denominator $\sum_{k=1, k \neq i}^K G_k \mathbf{w}_i^H \boldsymbol{\Sigma}_k \mathbf{w}_i$ (the leakage power) constant. This gives the well-known solution

$$(G_i \boldsymbol{\Sigma}_i) \mathbf{w}_i = \lambda \left(\sum_{k=1, k \neq i}^K G_k \boldsymbol{\Sigma}_k \right) \mathbf{w}_i. \quad (32)$$

Thus, the optimum weight vector \mathbf{w}_i^o is the eigenvector associated with the largest eigenvalue (generalized eigenvalue problem) of the characteristic equation given by (32). Later, our numerical results confirm the tightness of the lower bound (31) of average SLR for the weight obtained from (32).

4.2. Minimize the SLR Outage. Mathematically, this problem has the following unconstrained minimization form: $\min_{\mathbf{w}_i} P_{\text{out}}(\gamma_0, \mathbf{w}_i)$. We note that $P_{\text{out}}(\gamma_0, \mathbf{w}_i)$ is a complicated function of s_{con} and β which in turn depend on \mathbf{w}_i . Therefore, the standard way of finding the first-order derivative of the outage probability with respect to \mathbf{w}_i and equating the corresponding result to zero does not enable us to solve the problem in closed-form. Here, our approach is to first intuitively find the limiting values of s_{con} and β for which the outage in (24) approaches to zero. The second step is to find \mathbf{w}_i in order to achieve those limiting values of s_{con} and β . After simple manipulation, the outage probability (24) can also be written as

$$P_{\text{out}}(\gamma_0, \mathbf{w}_i) = \frac{1}{NB(\beta, N)} \cdot \frac{1}{(1 + 1/s_{\text{con}})^N (s_{\text{con}} + 1)^\beta} \cdot {}_2F_1\left(1, \beta + N; N + 1; \frac{1}{1 + 1/s_{\text{con}}}\right). \quad (33)$$

Note that the Gauss hypergeometric function ${}_2F_1(a, b; c; z)$ converges for arbitrary a , b and c if $|z| \leq 1$ (see [25, Section 9.1]). This is the case in (33) since $1/(1 + 1/s_{\text{con}}) \leq 1$ for any \mathbf{w}_i . It is also not difficult to see from the series form of ${}_2F_1(\cdot)$ (see [25, equation 9.100]) that its minimum in (33) is 1 which can be achieved if $s_{\text{con}} \rightarrow 0$ and $\beta \rightarrow 0$. As $\beta \rightarrow 0$, the term $1/B(\beta, N)$ approaches to zero whereas when $s_{\text{con}} \rightarrow 0$ and $\beta \rightarrow 0$, the term $1/(1 + 1/s_{\text{con}})^N (s_{\text{con}} + 1)^\beta$ tends to be zero. Hence, it can be concluded that if s_{con} and β can be minimized with respect to \mathbf{w}_i , the outage expression (33) can also be minimized. Here, we want to emphasize that the analytical proof for the optimality of the above mentioned approach is still an open issue. Now, the outage probability minimization problem can be turned to the problem of minimizing s_{con} and β simultaneously with respect to \mathbf{w}_i , that is, $\min_{\mathbf{w}_i} \{s_{\text{con}}, \beta\}$, which is a multicriterion optimization problem [27]. Using the notation $x_i \triangleq G_i \mathbf{w}_i^H \boldsymbol{\Sigma}_i \mathbf{w}_i$, this multicriterion minimization problem can be scalarized by forming the weighted objective function [27]

$$\min_{\mathbf{w}_i, t} \frac{1}{\gamma_0} s_{\text{con}} + \frac{1}{N} t \beta = \min_{\mathbf{w}_i, t} \frac{\sum_{k=1, k \neq i}^K \gamma_k^2}{x_i \sum_{k=1, k \neq i}^K \gamma_k} + t \frac{\left(\sum_{k=1, k \neq i}^K \gamma_k \right)^2}{\sum_{k=1, k \neq i}^K \gamma_k^2}, \quad (34)$$

where the weights for the first and second objective functions are 1 and $t \geq 0$, respectively. Here, we can interpret t as the relative importance of the second objective function with respect to the first one. Note that (34) is a difficult optimization problem. The following inequality can be easily shown:

$$\frac{\sum_{k=1, k \neq i}^K \gamma_k^2}{x_i \sum_{k=1, k \neq i}^K \gamma_k} \leq \frac{\sum_{k=1, k \neq i}^K \gamma_k}{x_i}. \quad (35)$$

Now using the upper bounds (35) and (28), the objective function in (34) can also be upperbounded as

$$\frac{1}{\gamma_0} s_{\text{con}} + \frac{1}{N} t \beta \leq \frac{\sum_{k=1, k \neq i}^K \gamma_k}{x_i} + t(K - 1), \quad (36)$$

where again equality holds if all $\{\gamma_k\}$ are equal. Using the above upper bound and resubstituting for x_i and γ_k , the minimization problem (34) takes the following form:

$$\min_{\mathbf{w}_i} \sum_{k=1, k \neq i}^K G_k \mathbf{w}_i^H \boldsymbol{\Sigma}_k \mathbf{w}_i \cdot \frac{1}{G_i \mathbf{w}_i^H \boldsymbol{\Sigma}_i \mathbf{w}_i} \quad (37)$$

which is also in the familiar Rayleigh quotient form. (Since we replace the exact cost function by its upper bound, the minimization problem becomes independent of t .) With the help of Lagrangian multiplier method, we can show that the optimum weight vector that minimizes (37) is given by (32) which is just the solution of the transmit beamformer that maximizes the average SLR. Hence, it is clear from (32) that the minimum outage probability and maximum average SLR transmit beamformer require only the knowledge of correlation matrices and average channel power gains. We will later demonstrate, with the numerical results, that the upper bounds in (35), (28), and (36) are relatively tight for the beamformer weight derived from (32).

5. Numerical Results and Discussions

In this section, we first verify the correctness of the analytically derived PDF (17) of SLR by comparing the analytical results with the Monte-Carlo simulation results. Next, we investigate the tightness of the bounds in (29) and (36). The outage probability of SLR for the i th user (for conciseness, the results are shown for $i = 1$) obtained via theory (23) and Monte-Carlo simulations are also shown for different parameters and correlation models. However, these results are not intended to illustrate the outage performance of a particular system. This would require additional assumptions regarding power control, modulation, and channel coding. Finally, we also demonstrate that the maximum average SLR or minimum outage probability transmit beamformer also helps to significantly improve the user SINR when the user employs linear operation such as matched filtering. We consider MIMO channels in which the transmit correlations are modeled with two different methods; exponential correlation and Gaussian angle of arrival (AoA) models. Throughout all examples, we take $M = 4$, $K = 3$, $G_1 = 0.5$, $G_2 = 0.1$, $G_3 = 0.2$, and

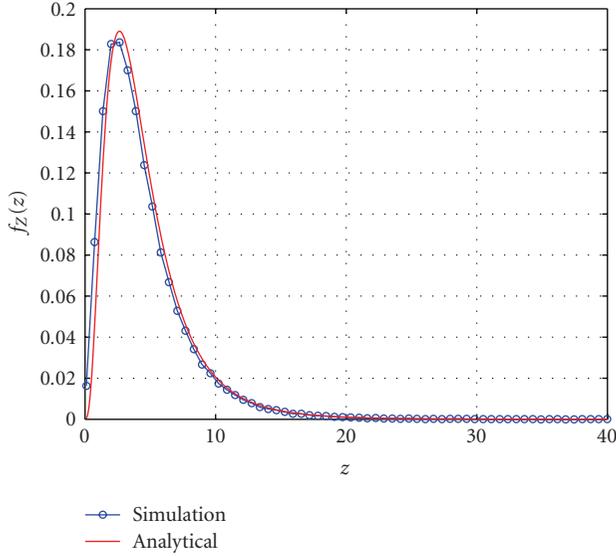


FIGURE 2: Comparison of analytical and simulated PDFs of SLR (\mathbf{w}_i is obtained from (32), and the exponential correlation model is used).

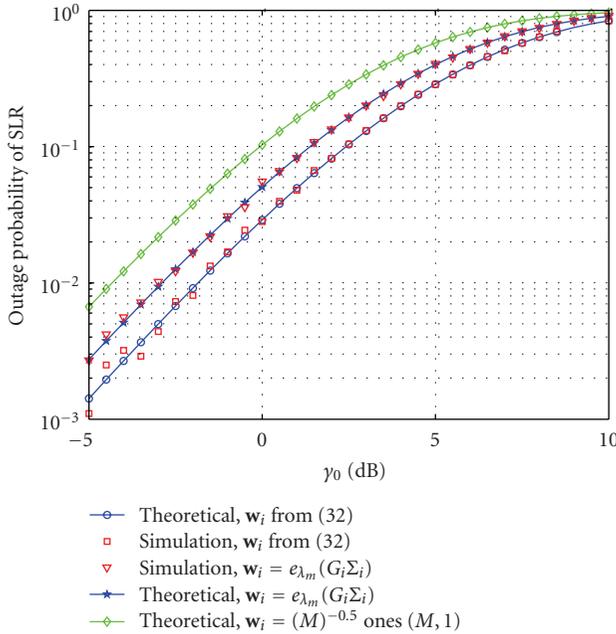


FIGURE 3: Comparison of outage probability with different weight vectors as a function of γ_0 for user $i = 1$ (exponential correlation model).

$N_i = N$ for all i . Note that this is purely by way of example, and other values could just have easily been considered. The outage probability of SLR is presented using Monte-Carlo simulation runs during which the channels ($\mathbf{H}_i, i = 1, \dots, K$) change independently and randomly. For each channel realization, the SLR for i th user is computed and compared with the threshold value γ_0 for determining the outage probability.

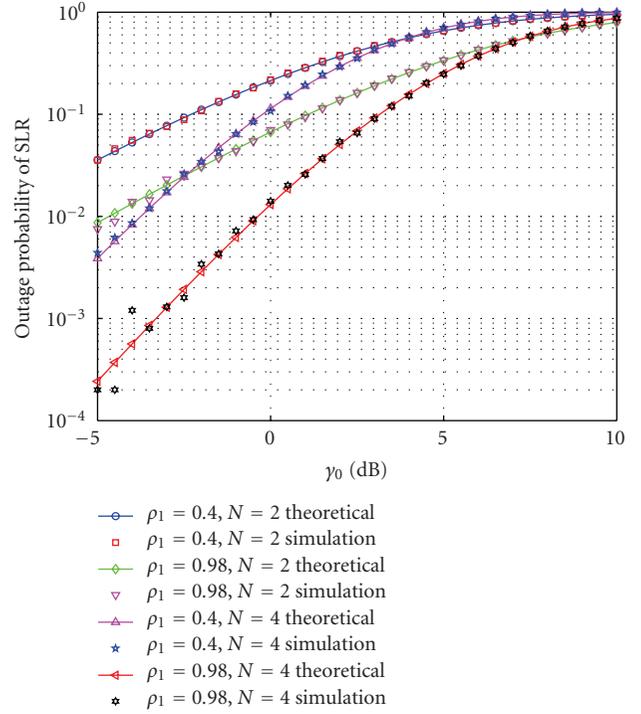


FIGURE 4: Comparison of theoretical and simulated outage probability as a function of γ_0 for the user $i = 1$ (exponential correlation model).

5.1. Exponential Correlation Model. In this example, the amplitudes of the spatial correlations among the elements of the BS antenna array are considered to be exponentially related. With this assumption, the correlation matrices are defined as

$$[\boldsymbol{\Sigma}_i]_{mn} = \rho_i^{|m-n|} e^{-j(m-n)\sin\theta_i}, \quad i = 1, \dots, K, \quad (38)$$

where $m, n = 1, \dots, M$ represent the m th row and n th column of $\boldsymbol{\Sigma}_i$, ρ_i are the amplitudes of correlation coefficients and θ_i is the AoA of the plane wave from the i th point source.

The analytically obtained PDF (17) of SLR is compared with the simulation results as shown in Figure 2. In this figure, the beamformer weights are optimized according to (32) for the exponential correlation model (38). It can be observed from Figure 2 that the analytical and simulation results are in fine agreement, and hence the accuracy of the derived PDF of SLR is validated. Figure 3 displays the analytical and simulated outage probabilities of SLR versus γ_0 for (a) the optimized \mathbf{w}_i from (32), (b) the non-optimized \mathbf{w}_i ($\mathbf{w}_i = (1/\sqrt{M}) \text{ones}(M, 1)$), and (c) \mathbf{w}_i which is the eigenvector corresponding to the maximum eigenvalue of $G_i \boldsymbol{\Sigma}_i$. Note that the last method simply tries to maximize the signal power toward the user of interest without even trying to suppress the leakage power toward the other users. Although this approach is highly suboptimal, it is very simple to implement, and its performance can be encouraging especially in UMTS cellular networks [28] where, due to downlink omnidirectional strong common pilot channels, the overall leakage power appears to be almost

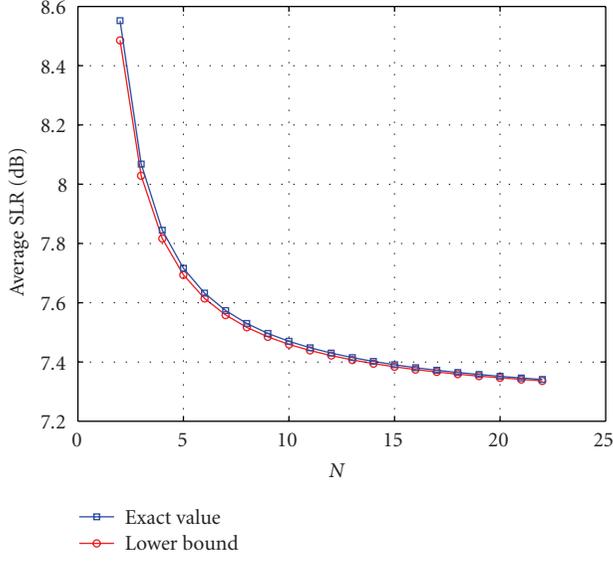


FIGURE 5: Exact average SLR and its lower bound in (31) as a function of N for the user $i = 1$ (\mathbf{w}_i is obtained from (32), and Gaussian AoA model is used).

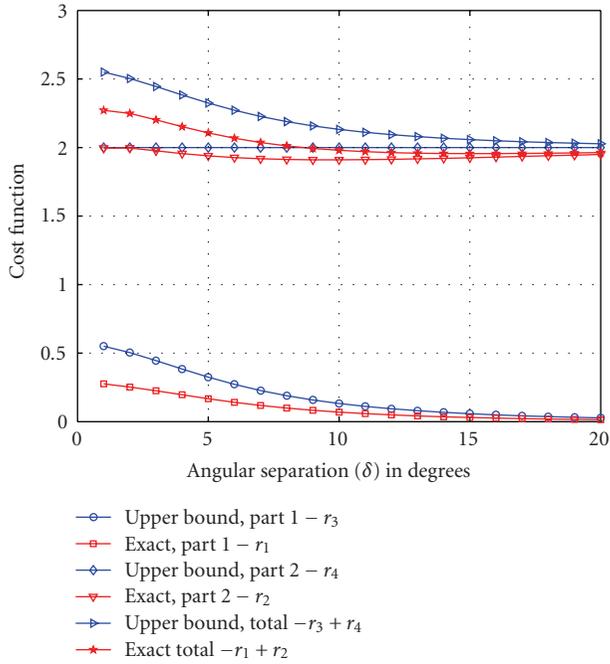


FIGURE 6: Exact cost function and its upper bound in (36) versus δ for the user $i = 1$ (\mathbf{w}_i is obtained from (32), and Gaussian AoA model is used, $r_1 = (1/\gamma_0) s_{\text{con}}$, $r_2 = (1/N)t\beta$, $r_3 = (\sum_{k=1, k \neq i}^K \gamma_k/x_i)$, and $r_4 = t(K-1)$).

white noise. As expected, it can be observed from Figure 3 that the method (32) outperforms the other two cases. The theoretical and numerical results for different values of ρ_1 and N are compared in Figure 4. In Figures 2 and 3, we take $\rho_1 = 0.8$, and in Figures 2, 3, and 4 we take $\rho_2 = 0.1$, $\rho_3 = 0.2$, $\theta_1 = 45^\circ$, $\theta_2 = 30^\circ$, and $\theta_3 = 60^\circ$.

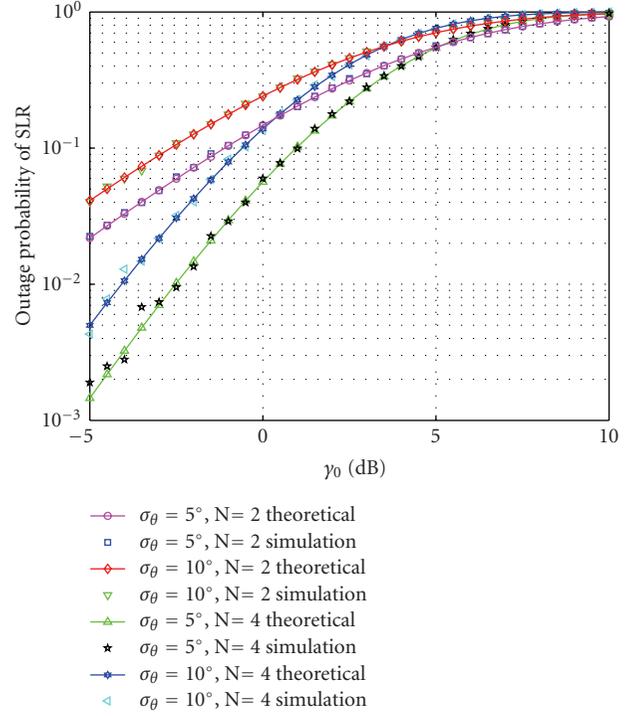


FIGURE 7: Comparison of theoretical and simulated outage probability as a function of γ_0 for user $i = 1$ (\mathbf{w}_i is obtained from (32) and Gaussian AoA model is used).

5.2. *Spatial Correlation Model-Gaussian Angle of Arrival (AoA)*. In this example, the spatial correlation among elements of the BS antenna array is modeled according to the distribution of the AoA of the incoming plane waves at the BS from the i th user. The AoA is assumed to be Gaussian distributed with a standard deviation σ_θ^i of angular spreading. For this case, we consider a uniform linear array with the half-wavelength spacing. The correlation is thus given by [3]

$$[\Sigma_i]_{mn} = e^{j\pi(m-n)\sin\theta_i} e^{-\pi(m-n)\sigma_\theta^i \cos\theta_i/2}, \quad i = 1, \dots, K, \quad (39)$$

where θ_i is the central angle of the incoming rays to the BS from the i th user. We assume that the first user is located at $\theta_1 = 10^\circ$ relative to the BS array broadside, and the other two users are located at $\theta_{2,3} = 10^\circ \pm \delta$ where we take $\delta = 8^\circ$ (except in Figure 6 where δ is varied) and $\sigma_\theta^i = \sigma_\theta$ for all i .

The exact average SLR (27) and its lower bound (31) both versus N are compared in Figure 5 where the optimum weight vector is chosen according to (32). We take $\sigma_\theta = 3^\circ$ for this figure. It can be seen from Figure 5, that the difference between the exact values of the average SLR and its lower bound is almost negligible for all N which in fact confirms that the beamformer (32) maximizes the average SLR with a very fine accuracy. The exact functions in (28) and (35), their corresponding upper bounds, the sum function (36) (with $t = 1$), and its upper bound are displayed in Figure 6 for different values of δ where the beamformer is derived from (32). It can be observed from this figure that

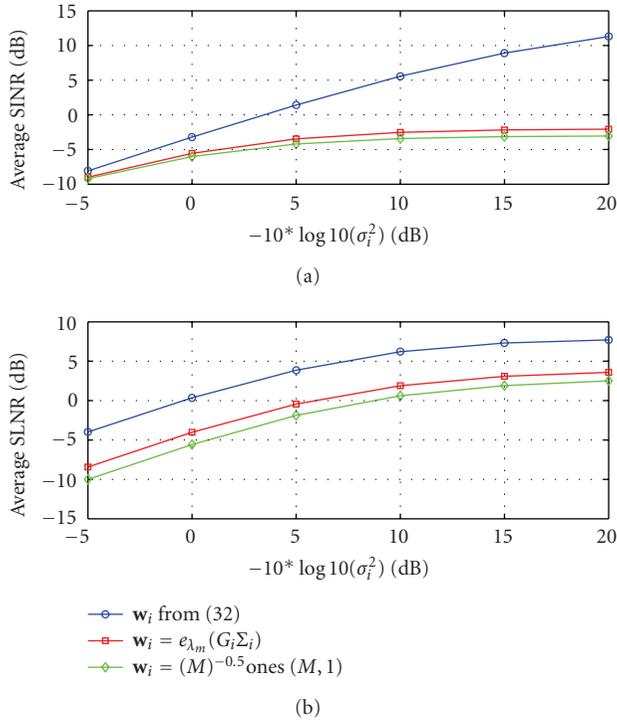


FIGURE 8: Average SINR and average SLNR versus noise power for the user $i = 1$ (Gaussian AoA model with $\sigma_\theta = 3^\circ$).

the bound in (28) is very tight for all values of δ whereas that in (35) is tight for the medium and larger values of δ . In fact, the gap between the overall exact function (36) and its upper bound is sufficiently small for all values of δ . Figure 7 shows the outage probability of SLR versus γ_0 obtained via theory and simulations for different values of σ_θ and N . The average SINR (5) and the average SLNR (3) of i th user versus the receiver noise power σ_r^2 are displayed in Figure 8 again for (a) the optimized w_i of (32), (b) the non-optimized w_i ($w_i = (1/\sqrt{M}) \text{ones}(M, 1)$), and (c) w_i which is the eigenvector corresponding to the maximum eigenvalue of $G_i \Sigma_i$. In this figure, the SINR and SLNR are averaged over 10^4 independent channel realizations, and it is considered that the receiver has perfect knowledge of instantaneous channels. It can be seen from Figure 8 that the transmit beamformer (32) based on maximization of SLR significantly helps to improve the receiver's SINR. Figures 3, 4, and 7 display that the matching between the theoretical and simulation results is very fine. This confirms the validity of the proposed theoretical expression for outage probability. It can be noticed (see Figures 3 and 8) that the beamformer, which tries to suppress the leakage power while maximizing the signal power (32), is better than the one which only maximizes the signal power of the user of interest by neglecting the leakage power (method (c)). The results (Figures 4 and 7) also show that as the spatial correlation between the antenna elements increases (correlation coefficient increases or angular spreading decreases), the outage probability decreases. The latter observation can be explained from the fact that

when the spatial correlation increases, the ranks of MIMO channels decrease, thereby allowing the beamformer to perform better. The best performance can even be obtained when the MIMO channels are fully correlated (i.e., channels become rank one). It can be also observed (see Figures 4 and 7) that by increasing the BS antenna correlation, the performance can be improved more effectively than just by increasing the number of user antennas while keeping the BS antenna correlation sufficiently low. Furthermore, as expected in Figures 3, 4, and 7, the outage probability increases with increasing γ_0 .

6. Conclusions

A fine agreement between the theoretical and simulation results for the PDF of SLR and its outage probability confirms the correctness of the proposed analysis for a multiuser MIMO downlink beamforming in a Rayleigh fading environment. The results also show that the spatial correlation between the antenna elements significantly helps to increase the performance of the SLR-based transmit beamformer in terms of the SLR outage probability. It has been found via some approximations that the transmit beamformer which maximizes the average SLR also minimizes the outage probability of the SLR.

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