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Research Article

An Empirical Model for Probability of Packet Reception in Vehicular Ad Hoc Networks

Moritz Killat and Hannes Hartenstein

Institute of Telematics, University of Karlsruhe, Zirkel 2, 76131 Karlsruhe, Germany

Correspondence should be addressed to Moritz Killat, killat@tm.uka.de

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Today's advanced simulators facilitate thorough studies on VANETs but are hampered by the computational effort required to consider all of the important influencing factors. In particular, large-scale simulations involving thousands of communicating vehicles cannot be served in reasonable simulation times with typical network simulation frameworks. A solution to this challenge might be found in hybrid simulations that encapsulate parts of a discrete-event simulation in an analytical model while maintaining the simulation's credibility. In this paper, we introduce a hybrid simulation model that analytically represents the probability of packet reception in an IEEE 802.11p network based on four inputs: the distance between sender and receiver, transmission power, transmission rate, and vehicular traffic density. We also describe the process of building our model which utilizes a large set of simulation traces and is based on general linear least squares approximation techniques. The model is then validated via the comparison of simulation results with the model output. In addition, we present a transmission power control problem in order to show the model's suitability for solving parameter optimization problems, which are of fundamental importance to VANETs.

1. Introduction

The forecasted rise in transport demand poses a huge challenge for intelligent transportation systems (ITSs). In Europe alone, road transport will have increased by 50% between 1990 and 2010 [1]. The increase in traffic will not only impact the road network itself, 10% of which is expected to experience jams once per day across the continent by 2010, but also traffic safety and the environment.

The application of emerging technologies like *vehicular ad hoc networks* (VANETs) might help to mitigate some of the adverse effects related to the rising demand. Presumably, the direct wireless exchange of information between vehicles will enable drivers to communicate and thus lead to improvements in traffic efficiency and safety. This technology will also enable vehicles to better exploit the capacity of the road network and to alert each other to oncoming dangers. However, as intuitive as the potential benefits may be, the implementation of the new technologies will require immense sophistication because wireless communication in a vehicular environment is subject to challenging radio conditions. The multitude of communicating nodes all accessing the limited communication channel and the high mobility of radio-signal-reflecting objects complicate the successful transmission of data.

With respect to the communication challenges that arise in VANETs, one-hop broadcast communication is a key component of inter-vehicle communication. In this paper, we will elaborate upon this key primitive and present an analytical model that gives the probability of successfully receiving a one-hop broadcast message based on the distance between sender and receiver, transmission power, transmission rate, and vehicular traffic density. The applicability of this model is at least twofold.

(1) *Hybrid simulations*. Nowadays, computer simulations are the primary means of studying the efficacy of VANETs. Although very advanced simulators are

currently available, the computational effort required to obtain credible simulation results limits the simulators' applicability, especially, for large-scale simulations involving thousands of communicating vehicles. By the 1970s, researchers were already proposing the use of hybrid simulations that combined mathematical modeling with discrete-event simulations; see, for example, [2]. This hybrid approach was able to reduce computation time by some orders of magnitude. Regarding simulations in VANETs, for instance, [3] have shown in a sample scenario that a hybrid approach accelerates the simulation study by a factor of 500. The basic building block of a hybrid approach, however, is a suitable model that maintains the accuracy of a pure discrete simulation.

(2) *Solving optimization problems*. On the road, vehicles need to autonomously choose the communication parameters that best fit an application's needs. The complexity of the numerous influencing factors, however, impedes the determination of an appropriate configuration. One could instead formulate and solve a corresponding optimization problem to achieve the same purpose; however, an appropriate model of the communication behavior is required.

In this paper, we address the development of just such a model. In detail, our contributions are as follows.

(1) *Model building*. From a large set of simulation traces we derive an empirical model that provides the probability of one-hop broadcast reception in an IEEE 802.11p network. The application of curve fitting techniques provides an analytical expression for the probability of packet reception that allows accurate interpolation between simulated data points. Our model holds for varying conditions in four dimensions: (i) distance between sender and receiver, (ii) transmission rate, (iii) transmission power, and (iv) vehicular traffic density.

(2) *Model validation*. We demonstrate the validity of the model by addressing the issue of optimal transmission power assignment from two perspectives: numerically gained solutions of optimization problems versus results of computer simulations.

The remaining part of this paper is structured as follows: in Section 2 we take a look at the literature and summarize previous papers on topics related to modeling issues in wireless communication systems. Section 3 deals with the model building process. We delineate our key assumptions and comprehensively describe our methodological approach, which employs analytical derivation and general linear least squares curve fitting. Section 4 addresses the validation of the derived model by statistical and application means. We consider the problem of optimal transmission power assignment and compare simulative results with numerically computed solutions. Finally, we conclude the paper in Section 5.

2. Related Work

To the best of our knowledge, no analytical model on communication characteristics has been published that considers all of the particularities of a vehicular environment. Wireless local area networks in general, however, have been exemplarily studied by Bianchi, who modeled the IEEE 802.11 standard by means of Markov Chains [4]. Although his analysis is restricted to certain assumptions, Bianchi has shown strong conformance between the model and simulative results. Bianchi's paper has spawned many follow-up papers that relax his assumptions, correct flaws, and propose extensions to his model. In [5], for instance, the authors corrected Bianchi's IEEE 802.11 modeling by allowing for packet drops caused by contention by revising the assumption of (potentially) infinite retransmissions by a node. The work in [6–8] incorporated a nonidealistic sensing of the radio channel and thus addressed the problem of hidden terminals. The work in [9, 10] digressed from the assumption of saturated conditions on the communication channel and [11] additionally introduced a probabilistic radio wave propagation. A deep mathematical analysis was proposed in [12] that studies a probabilistic radio wave propagation and the capturing effect. The complexity of a joint consideration of all effects, however, has thwarted the proposition of a complete model that reflects IEEE 802.11 communication behavior.

In [3] Killat et al. discussed empirical model building for the simulation of vehicular ad hoc networks. By exploiting simulation traces of an advanced communication simulator, the authors derived an empirical model that mirrored the communication behavior of the corresponding discrete-event simulations. Their model's limitation to tight scenario configurations is addressed and compensated for in the present paper.

A fundamental contribution to modeling in wireless communication networks has been made by Jiang et al. [13]. They defined the *communication density* as the product of transmission rate, transmission power, and traffic density and have shown that equal communication densities evince very similar communication characteristics. We will make use of this relationship in our model-building process.

Finally, we refer to advanced communication simulators for vehicular ad hoc networks that have thoroughly considered impacts on wireless communication systems. Employing the widely used network simulator ns-2 [14], Chen et al. provided emendations to address the effects that are especially relevant in a vehicular environment [15]. Their work has enabled researchers to improve existing analytical models. Chen et al.'s simulator will serve as the basis for the empirical model presented in the following.

3. Model Building

This section addresses the construction of an empirical model that represents the probability of successfully receiving one-hop packet transmissions under various circumstances. The model is conceived as a flexible tool for developers to use in the application design process and thus takes varying traffic conditions into account. Therefore, we span the problem space as follows: assuming a traffic density of δ vehicles per kilometer that all periodically broadcast messages with a certain transmission power ψ at a rate f and denoting the distance to the sender by x, the model $\mathcal{M}$ provides the corresponding probability of one-hop packet reception $\mathcal{P}_R(x, \delta, \psi, f)$. While the distance as input factor can naturally be explained by an attenuated radio signal over distance, we consider the remaining three input dimensions for the following reason. First of all, because all vehicles communicate over a shared medium, communicating nodes in close proximity need to cooperatively agree on adequately time-separated transmissions if packet collisions are to be avoided. As the number of neighboring nodes and quantity of packets to be served increase, the coordination of transmissions becomes more stressed. Hence, the frequency of transmissions (and thus the amount of packets) and the product of traffic density and communication distance (and thus the number of nodes) become major indicators for the challenge of collision-free distributed channel allocation.

Clearly, the presented four input dimensions do not completely cover the entire parameter space of the problem at hand. Hence, we begin the following section by presenting our key assumptions and delineate the model generation process, which involves analytical and simulative derivations and general linear least squares curve fitting techniques.

3.1. Assumptions. Our model assumes all nodes in the network to communicate according to the IEEE 802.11p draft standard [16], which offers a range of data transmission rates from 3 Mbit/s to 27 Mbit/s. In [17] Maurer et al. argued that lower data rates facilitate a robust message exchange by offering better opportunities for countering noise and interferences. In consideration of safety applications, which are especially dependent on robust communication, we chose the lowest data rate of 3 Mbit/s. The minimum contention window of the IEEE 802.11 mechanisms was set to the standard size of 15.

Regarding packet sizes, we assume all vehicles in the scenario to transmit datagrams of equal size. Bearing in mind that data packets require security protection, we allocate 128 bytes for a certificate, 54 bytes for a signature, and 200 bytes of available payload, which adds up to a default packet size of 382 bytes.

A key decision in VANET research concerns the assumed radio wave propagation model. Early, simplistic proposals assumed a deterministic attenuation of the radio signal power over the transmission distance. However, as a successful packet reception is determined by the comparison of the received signal power to the noise level on the medium, a deterministic reception behavior is thereby induced. Figure 1 illustrates the deterministic characteristic in a scenario containing a single sender only; we do not consider interferences from simultaneous transmissions. Obviously, a packet is received with certainty within the configured communication range (here: 250 m). At any distance beyond the communication range, however, message receptions

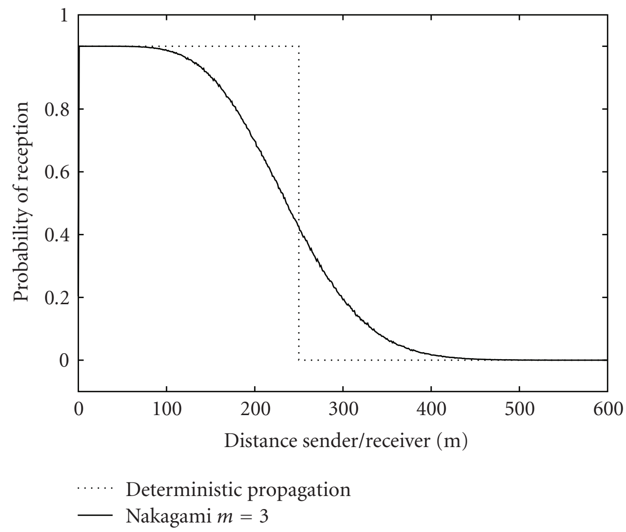


Figure 1: Probability of reception based on the distance between sender and receiver. Comparison of deterministic and probabilistic radio wave propagation models.

are ruled out. Clearly, this model hardly corresponds to behavior we would expect in reality. Indeed, measurements of inter-vehicle communications have shown a probabilistic character. Due to the highly mobile environment and to the multitude of reflecting objects, radio strengths vary at certain distances over time. Taliwal et al. have shown that the *Nakagami-m distribution* seems to suitably describe the radio wave propagation in vehicular networks on highways in the absence of interferences [18]. From the Nakagami distribution we can consequently infer a probabilistic reception behavior, depicted also in Figure 1. Apparently, in contrast to deterministic propagation models, we can no longer identify a "communication range". Nevertheless, a deterministic model will henceforth be assumed whenever we declare the transmission power for the Nakagami model, that is, the power necessary to reach a communication range of ψ meters. Additionally, in the following our study assumes moderate radio conditions, expressed in a relaxed fast-fading parameter ($m = 3$) of the Nakagami-m distribution. Varying channel conditions, that is, changing values of the m-parameters, were not considered in the model-building process.

Table 1 provides a detailed overview of the assumptions applicable to the following simulation study.

3.2. Model Generation. A successful wireless packet reception is determined by a number of influencing factors, such as radio wave propagation and interferences issuing from simultaneous transmissions. However, if only a single sender is considered, the effects reduce to the specifics of the environment and thus to the radio propagation. Under this assumption, it was shown in [3] that the probability of message reception can be analytically derived from the

Table 1: Simulation configuration parameters.

Parameter	Value
IEEE 802.11p data rate	3 Mbps
Frequency	5.9 GHz
Packet size	382 bytes
Antenna height	1.5 m
Antenna gain	4 dB
Minimum contention window	15 slots
Radio propagation model	Nakagami $m = 3$
Carrier sense threshold	−96 dBm
Noise floor	−99 dBm
SINR for preamble capture	4 dB
SINR for frame body capture	10 dB
Slot time	13 μs
SIFS time	32 μs
Preamble length	32 μs
PLCP header length	13 μs

Nakagami $m = 3$ model (for completeness, the derivations are given in the appendix) as follows:

$$\begin{aligned}\mathcal{P}_R(x, \delta, \psi, f) &= \mathcal{P}_R^{\text{single}}(x, \psi) \\ &= e^{-3(x/\psi)^2}\left(1 + 3\left(\frac{x}{\psi}\right)^2 + \frac{9}{2}\left(\frac{x}{\psi}\right)^4\right).\end{aligned} \tag{1}$$

In the most plausible scenario of many senders, the probability of reception is based on a plurality of factors. For example, IEEE 802.11p MAC contentions, the hidden terminal problem, and the capturing effect all effect message reception but are very difficult to express analytically. For this reason, we decided to pursue a hybrid approach: by applying linear least squares curve fitting techniques to an abundance of simulation traces, we obtain an analytical term for the probability of reception. Additionally, we reveal that dependencies exist between the varying input variables, which will allow us to infer nonsimulated parameter setting later on.

All of the simulations were conducted according to the following scenario setup. In order to mimic varying highway conditions, we pseudo-uniformly placed nodes on a straight line to reflect the simulated traffic density δ. In order to circumvent the "border effects" at the two ends of the road, we based distance calculations on the arc length of a circle. This approach essentially "eliminates" the two ends of the road by creating a torus. All of the nodes broadcast packets at the configured rate f with an equivalent transmission power of ψ meters (cf. Section 3.1).

In each simulation, the receptions of packets sent out by one specific node were recorded. To avoid correlations of subsequent transmissions triggered by the node of interest, we applied a relaxed transmission interval of one second to the selected node. With a scenario duration of 100 seconds and 30 seeds for each scenario, we thus captured up to 6000 packet receptions at each distance in every scenario (note that each distance exists on both sides of the sender). The number of simulated scenarios derives from all combinations in the three dimensions:

traffic density (veh/km), δ: 25, 50, 75, 100, 200, 300, 400, 500,

transmission power (m), ψ: 100, 200, 300, 400, 500,

transmission rate (Hz), f: 1, 2, 4, 5, 6, 8, 10,

which do not exceed a communication density of 500 packet transmissions per second. This limit is based on the work of Jiang et al., who defined communication density as the product of transmission power, transmission rate, and traffic density [13], which correspond to the number of packets per second. Assuming our default packet size of 382 bytes, a communication density of 500 corresponds to approximately 1.5 Mbit/s, or, when considering the entire neighborhood of a vehicle, to 3 Mbit/s. Hence, larger communication densities would exceed the capacity of available bandwidth and thus might cause irregularities that are not addressed by our model.

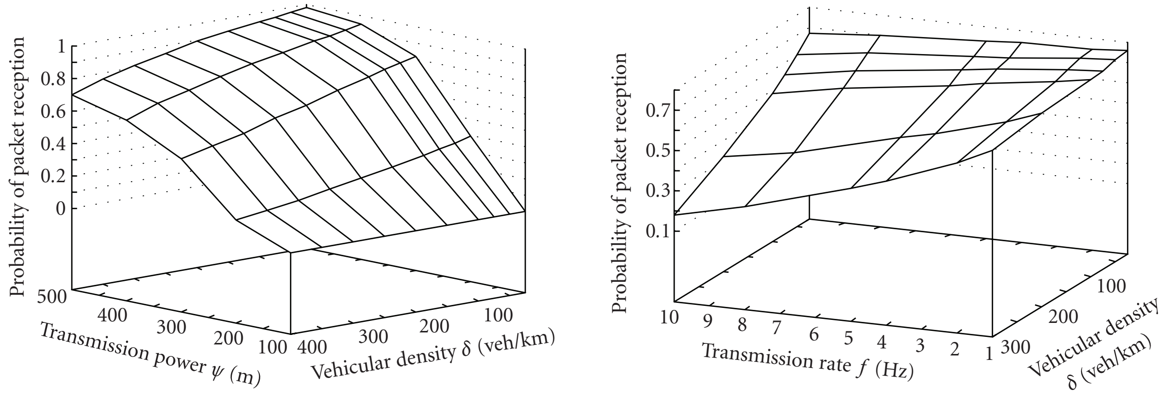
All simulations were run on an overhauled ns-2 network simulator that takes account of specifics especially relevant for inter-vehicle communications [15]. The system was modified to more accurately model physical conditions, considering the latest technology and improving flaws in existing simulation frameworks [19].

The results of the numerous simulation runs agree with the intuitive expectation that slight changes in the scenario setup lead to only minor deviations in the probability of reception. In more detail, when a single *configuration parameter* (transmission power, transmission rate, vehicular density) varies little, then we expect no abrupt deviation in the probability of one-hop broadcast packet reception. Mathematically speaking, we suppose the probability of reception to be partially differentiable on the three-dimensional interval that agrees with aforementioned restrictions on the configuration parameters (e.g., communication density not larger than 500). Figure 2 backs this supposition and exemplarily depicts how the probability of reception varies when configuration parameters change.

In order to obtain an empirical model from the simulation runs, we applied the technique of linear least squares curve fitting to each scenario trace. As suggested in [3], expression (1) serves as a starting point and is then extended with linear and cubic terms; in addition, fitting parameters a_1 through a_4 are introduced:

$$\widetilde{\mathcal{P}}_R(x, \psi) = e^{-3(x/\psi)^2}\left(1 + \sum_{i=1}^{4} a_i\left(\frac{x}{\psi}\right)^i\right). \tag{2}$$

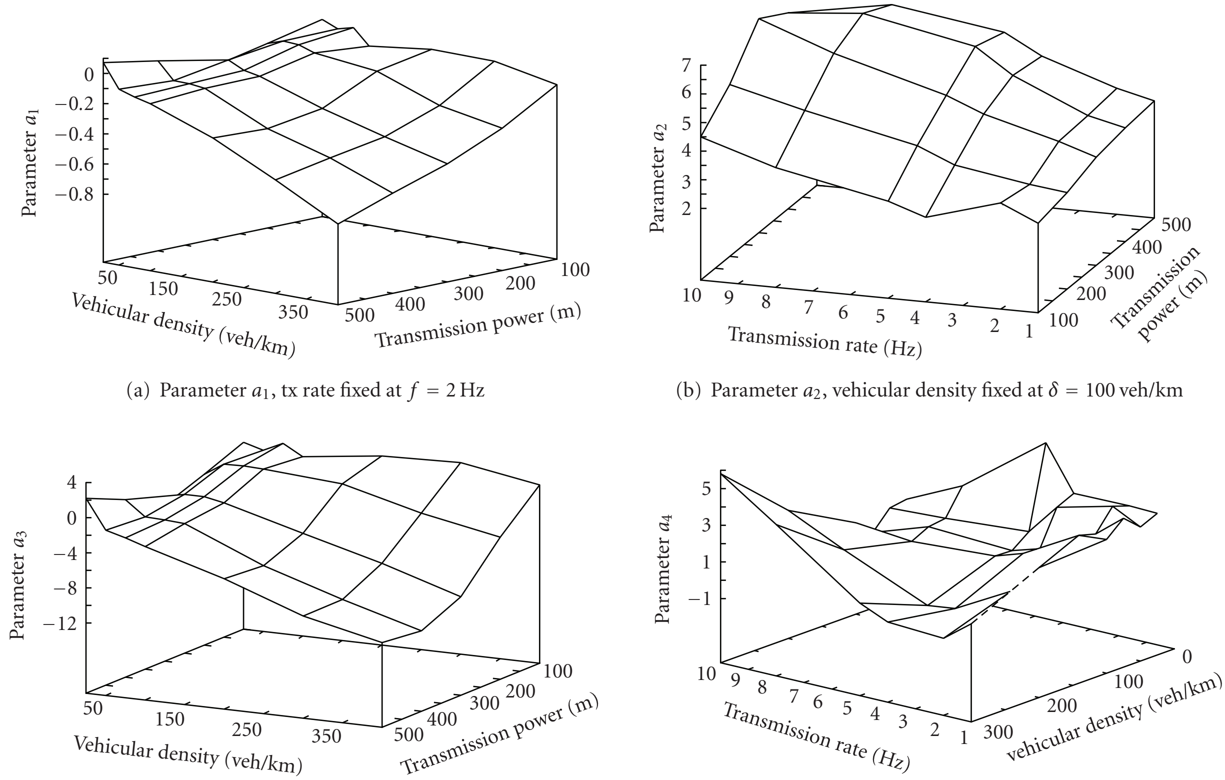
Following the curve fitting process, expression (2) proved to be an almost perfect match to the simulation traces for all ($\approx$190) simulated scenarios. As a result of the curve fits, we obtained a set of data points consisting of the determined fitting parameters a_1 through a_4 for all scenarios. Hence, we translated the dependencies pertinent to our objective (probability of reception) from the tuple (transmission power, transmission rate, vehicular density) to



(a) Probability of reception at a distance of 200 m. Transmission rate fixed at $f = 2$ Hz

(b) Probability of reception at a distance of 75 m. Transmission power fixed at $\psi = 100$ m

FIGURE 2: Probability of packet reception according to various configuration parameters.



(a) Parameter a_1, tx rate fixed at $f = 2$ Hz

(b) Parameter a_2, vehicular density fixed at $\delta = 100$ veh/km

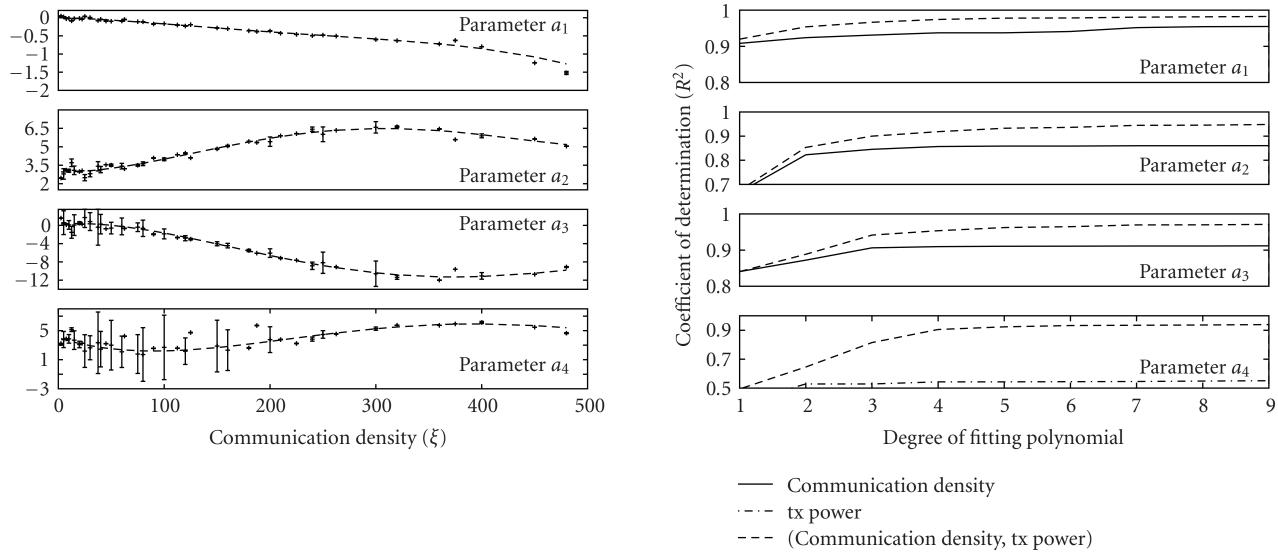
(c) Parameter a_3, tx rate fixed at $f = 2$ Hz

(d) Parameter a_4, tx power fixed at $\psi = 100$ m

FIGURE 3: Fitting parameters a_1 through a_4 based on configuration parameters.

the quadruple (a_1, a_2, a_3, a_4) and obtained the closed-form analytical expressions (2).

Now, knowing that the fitting parameters a_1 through a_4 only depend on the configuration parameters and assuming the differentiability of the probability of reception in the configuration parameters, we infer the differentiability of the fitting parameters in the configuration parameters. Again, this conclusion is supported by Figure 3, which illustrates the fitting parameters according to varying configuration parameters. At this point, our concern was to find a functional dependency that would allow us to choose the appropriate fitting parameters for a given scenario configuration. The assumed differentiability of the fitting parameters in the configuration parameters allowed us to apply *Taylor series* expansions to approximate the sought functional dependency by means of a polynomial. In mathematical



(a) Curve fit of a polynomial (fourth degree) to fitting parameters a_1 through a_4 based on the communication density ξ. a_4 in particular shows considerable deviations from the fitted curve

(b) Accuracy of fitting polynomials with varying degree to the configuration parameter a_1 through a_4

Figure 4: Approximating configuration parameters a_1 through a_4.

terms, we considered polynomial functions h_i, $i = 1, \ldots, 4$, which provide the corresponding fitting parameter a_i for any configuration parameter tuple, that is,

$$h_i(\delta, \psi, f) = \sum_{j,k,l \geq 0} h_i^{(j,k,l)} \delta^j \psi^k f^l \approx a_i, \quad i = 1, \ldots, 4, \tag{3}$$

where $h_i^{(j,k,l)}$ are the coefficients. We obtained h_i by approximating each a_i as closely as desired using a polynomial of appropriate degree N and a linear least squares approximation algorithm. However, due to the three-dimensional polynomial, the number of the coefficients $h_i^{(j,k,l)}$ for each a_i rapidly increases even for small N values because the number of coefficients $h_i^{(j,k,l)}$ is given by $\sum_{n=0}^{N} \binom{3+n-1}{3-1} = (1/6)(N+1)(N+2)(N+3)$. Hence, instead of achieving accuracy at the expense of complexity, in the resulting empirical model, we decided to reduce complexity by applying generalizations, such as communication density.

Inspired by Jiang et al.'s insights [13] of similar communication behavior in comparable communication densities, we downgraded the three-dimensional polynomial h_i to a one-dimensional polynomial depending only on the communication density $\xi = \delta \cdot \psi \cdot f$. Figure 4(a) illustrates that even a polynomial of the fourth degree can conveniently be fitted to a_1 through a_3, but difficulties (in terms of noisy behavior) obviously arise for the remaining parameter a_4. This observation is likewise reflected in Table 2, which provides the correlation coefficients of various configuration parameter combinations to the fitting parameters a_1 through a_4. In contrast to a_4, a_1 through a_3 shows a strong correlation to ξ. On the other hand, a_4 is considerably correlated to the transmission power parameter ψ. Thus, we decided to apply a two-dimensional polynomial h_i on the adjusted communication density, ξ, and the transmission power, ψ, which we fit to the dataset using the *Levenberg-Marquardt* (for the curve fitting process we utilized the open source software Gnu Regression, Econometric, and Time-series Library (GRETL) version 1.7.1.) algorithm:

$$a_i \approx h_i(\xi, \psi) = \sum_{j,k \geq 0} h_i^{(j,k)} \xi^j \psi^k, \qquad i = 1, \ldots, 4, \quad \text{with } \xi = \delta \cdot \psi \cdot f. \tag{4}$$

Regarding the degree of the two-dimensional polynomial, we analyzed the impact of an increasing degree on the accuracy of fitting parameter a_1 through a_4. Figure 4(b) illustrates the coefficient of determination R^2 of the fitting process for various degrees. For each parameter, Figure 4(b) depicts the fitting of a one-dimensional polynomial on the most correlated configuration parameter and of a two-dimensional polynomial on the most correlated parameter and the communication density. Based on Figure 4(b), we have chosen a two-dimensional polynomial of fourth degree for all fitting parameters in expression (4), that is, $j + k \leq 4$. We thus state the sought empirical model

$$\mathcal{M}\colon \widetilde{\mathcal{P}}_R(x, \delta, \psi, f) = e^{-3(x/\psi)^2} \left(1 + \sum_{i=1}^{4} h_i(\xi, \psi) \left(\frac{x}{\psi}\right)^i\right) \tag{5}$$

and list the coefficients obtained from the polynomial functions h_i (cf. expression (4)) in Table 3. Note that although some values in Table 3 seem to be negligible, a sensitivity analysis has shown that if a single of the $h_i^{(j,k)}$ parameter is omitted, deviations in the probability of reception from 8% to 100% can be observed.

Finally, Figure 5 shows that in scenarios which only contain a single sender ($f = \delta = 1$) the obtained fitting function h_1 through h_4 suitably conforms to the analytical

Table 2: Correlation of fitting parameters a_1 through a_4 and various configuration parameter combinations.

	ψ	δ	f	$\psi \cdot \delta$	$\psi \cdot f$	$\delta \cdot f$	ξ
a_1	−0.2564	−0.3983	−0.4125	−0.4655	−0.4728	−0.7449	**−0.9581**
a_2	0.2772	0.4163	0.4095	0.4690	0.4491	0.7019	**0.8255**
a_3	−0.3635	−0.3896	−0.3860	−0.5098	−0.5022	−0.6937	**−0.9179**
a_4	**0.7148**	−0.0667	−0.0351	0.3493	0.4060	0.0364	0.4712

Table 3: Coefficients $h_i^{(j,k)}$ subjected to the polynomials h_1 through h_4. Although some values seem to be negligible, all values significantly influence the resulting probability of reception.

	(j,k)				
	(0,0)	(1,0)	(2,0)	(3,0)	(4,0)
$h_1^{(j,k)}$	0.0209865	$-9.66304e-07$	$-1.72786e-11$	$5.09506e-17$	$-7.91921e-23$
$h_2^{(j,k)}$	2.24743	$7.84884e-07$	$2.28533e-10$	$-5.89802e-16$	$3.55262e-22$
$h_3^{(j,k)}$	2.56426	$2.82287e-05$	$-7.09939e-10$	$1.34371e-15$	$-3.01956e-22$
$h_4^{(j,k)}$	2.41146	$-9.32859e-05$	$6.77403e-10$	$-9.64188e-16$	$3.69652e-23$
	(3,1)	(2,1)	(2,2)	(1,1)	(1,2)
$h_1^{(j,k)}$	$3.16577e-20$	$2.13587e-14$	$-5.05716e-17$	$4.00928e-09$	$-1.88707e-11$
$h_2^{(j,k)}$	$4.07120e-19$	$-2.66510e-13$	$8.46273e-17$	$-7.31274e-08$	$2.98549e-10$
$h_3^{(j,k)}$	$-1.85451e-18$	$1.02847e-12$	$1.80250e-16$	$1.56259e-07$	$-8.50944e-10$
$h_4^{(j,k)}$	$1.85043e-18$	$-1.13894e-12$	$-4.05333e-16$	$-2.56738e-08$	$6.24415e-10$
	(1,3)	(0,1)	(0,2)	(0,3)	(0,4)
$h_1^{(j,k)}$	$3.25406e-14$	0.000418109	$-4.30875e-06$	$1.00775e-08$	$-7.32254e-12$
$h_2^{(j,k)}$	$-3.24982e-13$	0.00498750	$-7.22232e-06$	$1.69755e-08$	$-2.94381e-11$
$h_3^{(j,k)}$	$7.59094e-13$	−0.0227008	$7.50391e-05$	$-1.81469e-07$	$2.02182e-10$
$h_4^{(j,k)}$	$-3.57571e-13$	0.0191490	$-6.92678e-05$	$1.79917e-07$	$-2.07263e-10$

derivation as stated in expression (1) (i.e., $h_1 \sim 0$, $h_2 \sim 3$, $h_3 \sim 0$ and $h_4 \sim 4.5$) within the model's design restrictions.

4. Model Validation

In this section, we evaluate the quality of the empirical model derived in Section 3 from two perspectives: first we present statistical figures of merit, and then we address the problem of optimal transmission power assignment by making use of the derived model.

4.1. Statistical Evaluation. Our model, $\mathcal{M}$, is meant to be an analytical representation of the outcome of the numerous simulated scenarios generated in Section 3.2. In order to demonstrate the model's validity we (i) compare the model to the results of the simulated scenarios and (ii) demonstrate the model's ability to infer nonsimulated scenarios. The former is investigated in this subsection in which we study how accurately the model replaces a lookup table deduced from the simulation results. The latter is discussed in Section 4.2 by means of a transmission power control problem.

For each scenario, we determined the average probability of reception at each distance over all 30 seed values and computed the squared error compared to the model in each distance. Then, taking into consideration all of the distances in the investigated scenario, we summed up the squared errors (SSEs). Across all scenarios, the average value of these sums turned out to be $\mu_{\text{sse}} = 0.013$ (variance $\sigma^2_{\text{sse}} = 0.00041$). The largest SSE of $\mu^{\max}_{\text{sse}} = 0.15$ resulted from a scenario with a vehicular density of $\delta = 25$ all sending at a transmission rate of $f = 10$ Hz with a configured transmission power of $\psi = 500$ m (cf. Figure 6).

Regarding a comparison of the probability of reception determined in each distance and scenario by the model and simulation, respectively, we observed an average maximum deviation of $\mu_{\text{dev}} = 0.8\%$ ($\sigma^2_{\text{dev}} = 2.83e-05$) across all scenarios. The maximum deviation of 2.7% was encountered at a distance of 316 m in a scenario with a traffic density of $\delta = 25$ vehicles at a transmission rate of $f = 10$ Hz and a transmission power of $\psi = 400$ m.

4.2. Transmission Power Adjustment. A key problem in vehicular ad hoc networks concerns the optimal transmission power to be chosen by the vehicles. The single communication medium shared by all nodes requires a joint consideration of advantages from individual power increases, which induce interferences for surrounding nodes. An uncooperative choice of transmission power by each single vehicle, however, leads to "uncontrolled" load on the

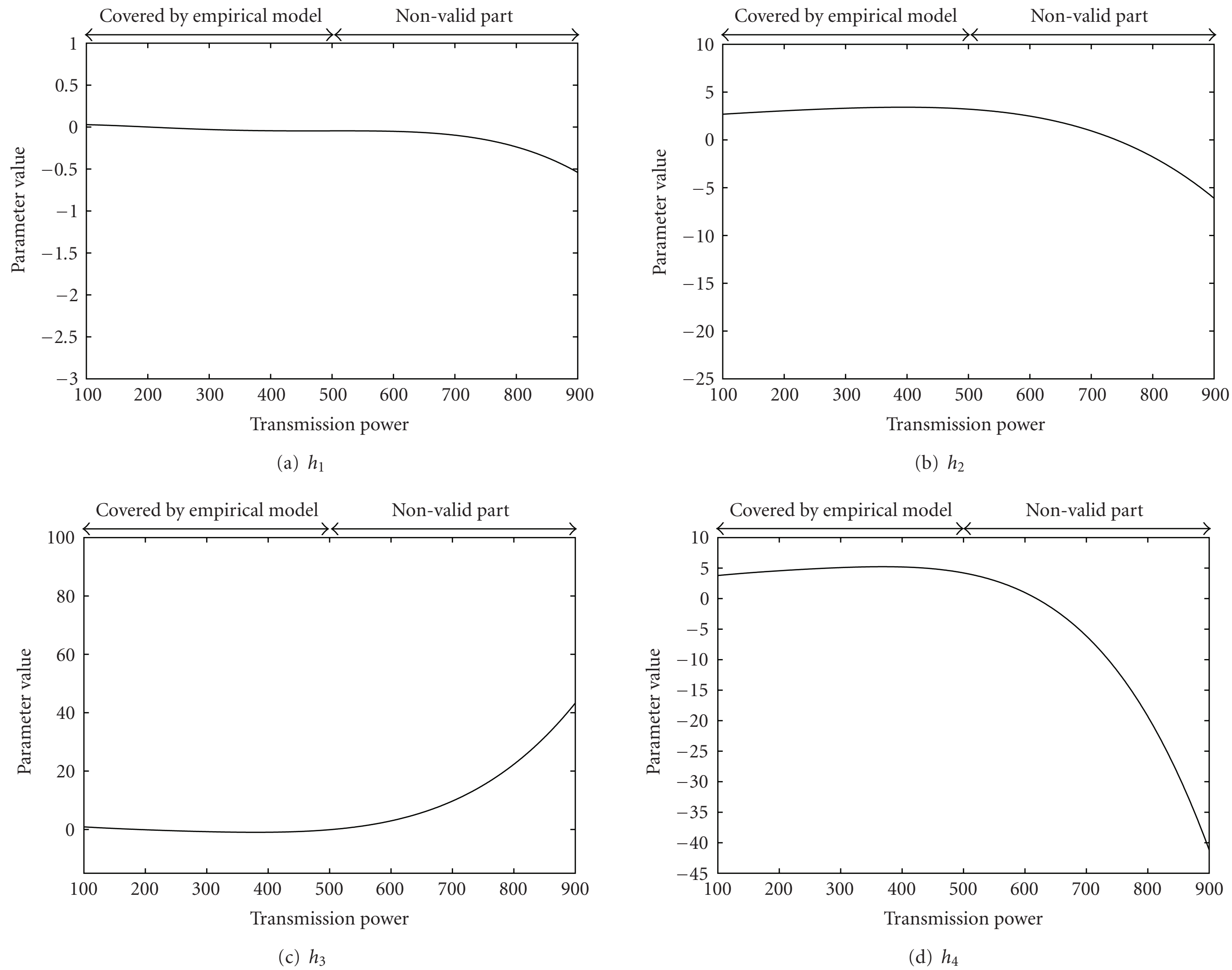


Figure 5: Values of the fitting functions h_1 through h_4 depending on the transmission power ψ (the remaining configuration parameters f and δ are both set to 1).

communication channel, thus, impairing the functionality of the communication system. For MAC fairness reasons, in general neighboring nodes should cooperatively decide on a common transmission power (see, e.g., [20]). From the perspective of a single application, an optimal power configuration is obtained when the application's constraints are fulfilled with a minimum amount of occupied resources in order to minimize the impact on surrounding vehicles. In the following, we assume an application $\mathcal{A}$ to run on all vehicles; the application periodically broadcasts packets as, that is, envisioned by beacon messages providing information on each vehicle's status. Let us assume that for a proper functionality $\mathcal{A}$ is constrained on certain probabilities of reception, q_i, at given distances, x_i. For the sake of simplicity, we treat transmission power adjustment as the only means of changing communication conditions.

Indeed, by utilizing the model, one can infer the minimum transmission power that satisfies $\mathcal{A}$'s constraints. Assuming the application is provided with enough knowledge on the current traffic conditions, the model allows one to assess the suitability of various transmission power configurations. The assessment could focus on either the *overall* influence or on the *selective* influence of the chosen transmission power. The former evaluates the impact on any potential recipient, that is, the probability of reception over all distances. Figure 7(a) exemplarily compares one simulated scenario under three various power configurations with the model $\mathcal{M}$. All of the scenario's trace files were not used in the model-building process presented in Section 3.2.

In contrast, a selective influence evaluation focuses on a specific distance for which the communication quality is studied. For the scenario underlying Figures 7(a) and 7(b) compares the probability of one-hop packet reception at distances of 100 m, 200 m, and 300 m for the simulation results and empirical model, respectively. Obviously, the divergence between the two approaches is kept within a small limit but starts increasing at larger transmission powers. Note that the model $\mathcal{M}$ was designed for communication densities that do not exceed 500; this parameter corresponds to a power level of $\psi = 555$ m for the scenario depicted in

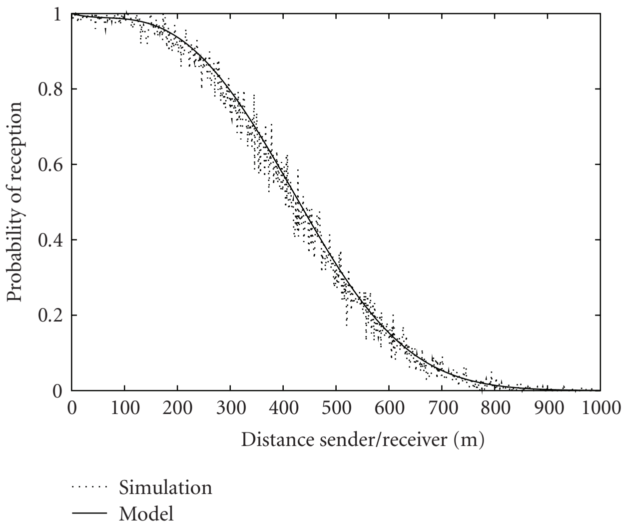


Figure 6: Comparison between model and simulation results: the figure illustrates the scenario ($\delta = 25$ veh/km, $f = 10$ Hz, $\psi = 500$ m) for which the maximum sum of squared error has been determined.

Figure 7(b). Hence, larger transmission powers increase the communication density and thus render the model invalid. Since the dataset used in the model-building process does not provide information for communication densities larger than 500, these data cannot be interpolated, and the model's predictions are no longer accurate.

We use the selective influence evaluation for determining the minimum transmission power that will meet the application's constraints. Typically, one of the constraints dominates the others, that is, the dominant constraint requires a certain (dominant) transmission power; however other constraints would likewise be satisfied under different power configurations. By utilizing the model $\mathcal{M}$, $\mathcal{A}$'s dominant transmission power is determined by numerically solving the optimization problem:

$$\begin{aligned} &\min \psi \\ &\text{subject to } q_i - \mathcal{P}_R(x_i, \delta, \psi, f) \leq 0, \quad \forall i, \end{aligned} \tag{6}$$

where q_i represents the aformentioned target probability of reception at distance x_i.

In the following, we compare the computed dominant transmission powers for the scenarios outlined in Table 4 with the simulation results. The simulations were not used in the model-building process and differ in that the reference vehicle also adapts to the commonly chosen transmission rate.

The three curves in Figure 8 represent the simulative probability of reception at distances of 100 m, 200 m, and 300 m in scenario A. Obviously, the analytical solutions to meet constraint 1 ($\psi_1 = 214$ m), constraint 2 ($\psi_2 = 352$ m), and constraint 3 ($\psi_3 = 511$ m) diverge only slightly from the simulative results. Note that increasing deviations at higher transmission powers are again ascribed to the model's aforementioned design restriction of a maximum communication density of 500. The computed solution to meet all constraints

Table 4: Scenario setups.

Scenario	δ (veh/km)	f (Hz)	q_1 at 100 m	q_2 at 200 m	q_3 at 300 m
A	150	6	80%	50%	33%
B	50	5	90%	90%	90%
C	300	2	95%	75%	60%

agrees with the dominant constraint 3 and corresponds to a communication density of 471.6.

Finally, Figure 10 depicts the results for scenario C. While the analytical solutions to meet each single constraint are quite precise, the optimization problem does not yield a solution for meeting all constraints at the same time. As indicated by the simulative results, constraints 1 and 3 are both dominant but contradictory, thus making compliance with all constraints at once impossible.

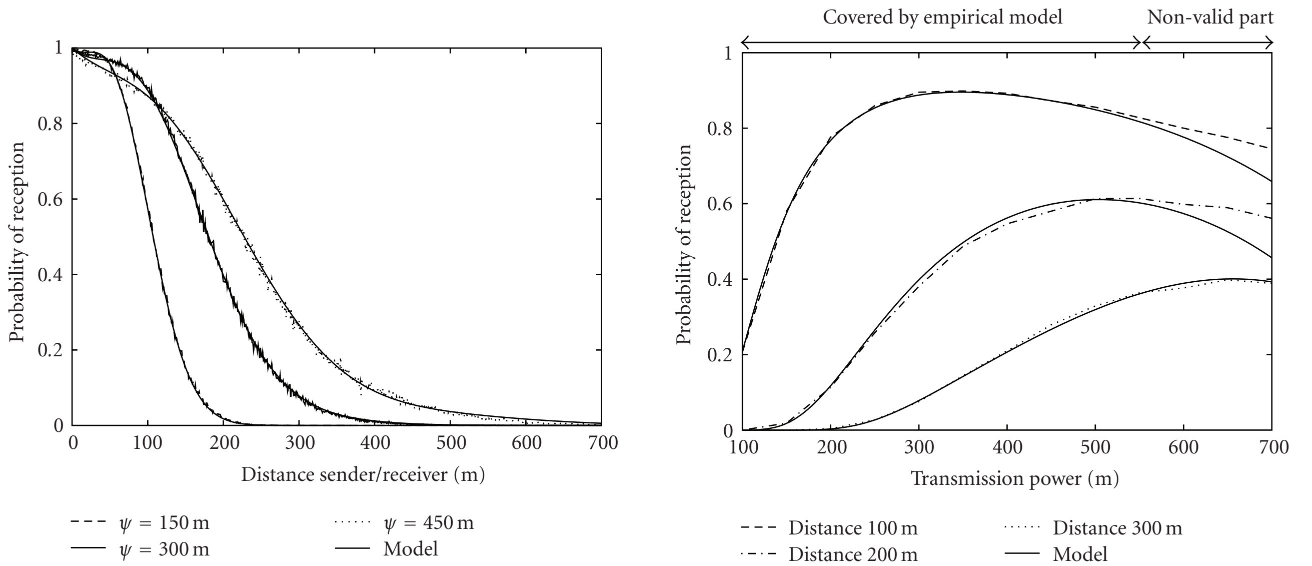
Figure 9 illustrates the results for scenario B with relaxed traffic conditions but tightened constraints. Again, only a small divergence between the analytical and simulative solutions is noticeable.

Although the three discussed scenarios have demonstrated the usefulness of the empirical model for dealing with the transmission power control problem, for best results, we highly recommend obtaining knowledge about the current traffic situation in advance. In reality, additional (communication) effort is required to provide vehicles with information in order to allow a precise estimation of the current communication density. If this condition is fulfilled, the presented approach may prove useful for choosing a suitable transmission power or other parameter optimization problems.

5. Conclusions

In this paper, we addressed the problem of determining an analytical expression for the probability of one-hop packet reception in vehicular ad hoc networks. While this probability is influenced by several parameters, including, for example, radio channel characteristics, IEEE 802.11 configurations, and vehicular conditions we reduced the number of varying parameters for simplicity's sake. Besides the distance between sender and receiver, three other variable input parameters were incorporated into our model: transmission power, transmission rate, and vehicular traffic density.

In our evaluation, we demonstrated the model's ability to represent numerous simulation traces as well as its power to predict the outcome of nonsimulated scenarios. In contrast to a huge database that serves as a lookup table for communication characteristics, the presented model can be utilized to solve analytical problems. In a sample application, we dealt with a configuration parameter optimization problem and determined minimum transmission powers to meet a VANET application's constraints. Future work needs to address the raised problem of providing nodes in the network with sufficient knowledge to precisely estimate the communication density, which is the key parameter of the empirical model.



(a) Varying distance between sender/receiver for three differing transmission powers ψ

(b) Varying transmission powers for three differing distances between sender/receiver

Figure 7: Comparison of model to simulated scenario. The scenario involves a traffic density of $\delta = 150$ veh/km all transmitting at a rate of $f = 6$ Hz. The simulation traces were not included in the model-building process.

Appendix

Analytical View on the Nakagami Model (Taken from [3])

The following discussion considers only a single transmitter. Hence, the noise level ν is assumed to be constant over the geographical space. In the following, we will make use of this assumption as we express a signal's power level σ in terms of its signal-to-noise ratio (SNR), $x = \sigma/\nu$.

According to the Nakagami-m distribution the following function f_d describes the *probability density function (pdf)* for a signal to be received with power x for a given average power strength Ω at distance d (see [21, Section 5.1.4, expression 5.14]):

$$f_d(x; m, \Omega) = \frac{m^m}{\Gamma(m)\Omega^m} x^{m-1} e^{-(mx/\Omega)}, \tag{A.1}$$

$$F_d(x; m, \Omega) = \frac{m^m}{\Gamma(m)\Omega^m} \int_0^x z^{m-1} e^{-(m/\Omega)z} dz. \tag{A.2}$$

F_d is the corresponding *cumulative density function (cdf)* and m denotes the fading parameter. It is known that the *pdf* of a *gamma distribution* $\Gamma(b, p)$ is given by

$$g(x) = \frac{b^p}{\Gamma(p)} x^{p-1} e^{-bx} \tag{A.3}$$

and hence, by setting $b := m/\Omega$ and $p := m$, one notes (A.1) being a gamma distribution with the according parameters.

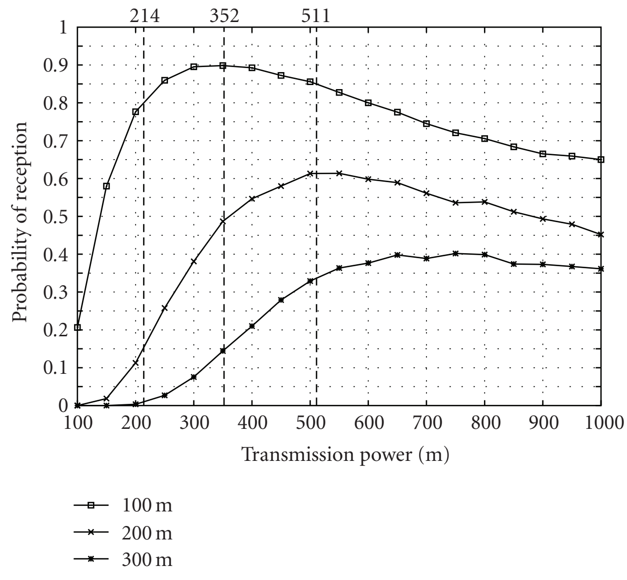


Figure 8: Scenario A (cf. Table 4). Probability of packet reception w.r.t. chosen power value: analytical computation (numbers) and simulative results (curves).

Moreover, for $p \in \mathbb{N}$ the gamma distribution matches an *Erlang distribution* $Erl(b, p)$ for which the following expression of the *cdf* is known:

$$F(x) = 1 - e^{-bx} \sum_{i=1}^{p} \frac{(bx)^{i-1}}{(i-1)!} i. \tag{A.4}$$

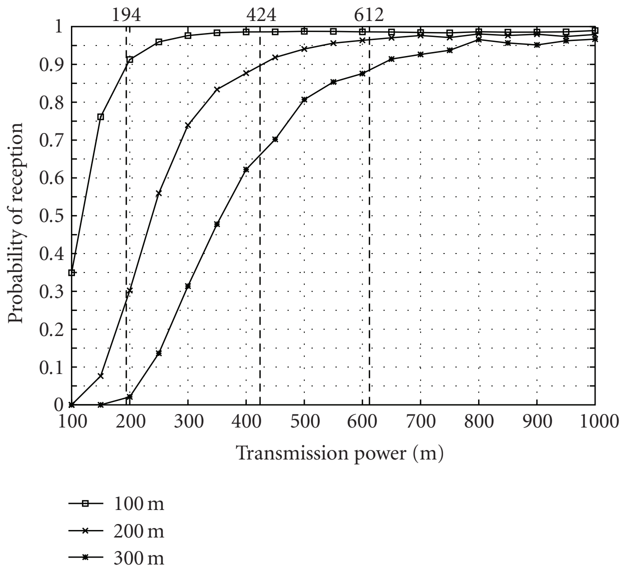


Figure 9: Scenario B (cf. Table 4). Probability of packet reception w.r.t. chosen power value: analytical computation (numbers) and simulative results (curves).

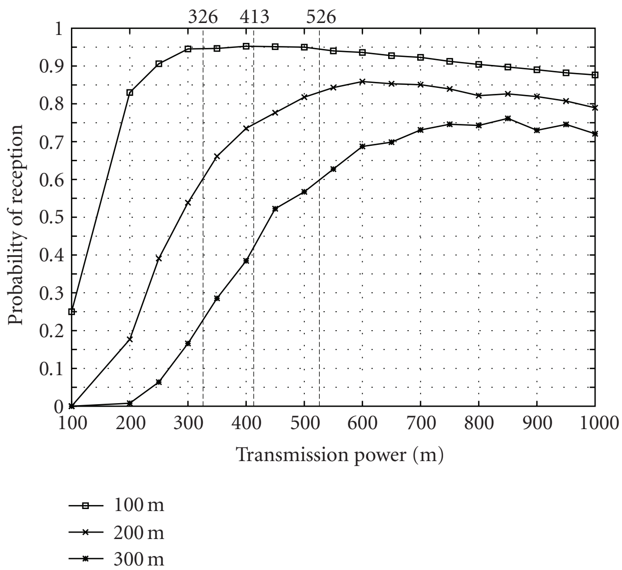


Figure 10: Scenario C (cf. Table 4). Probability of packet reception w.r.t. chosen power value: analytical computation (numbers) and simulative results (curves).

For the Nakagami distribution with a positive integer value for the fading parameter m we obtain

$$F_d(x; m, \Omega) = 1 - e^{-(mx/\Omega)} \sum_{i=1}^{m} \frac{((m/\Omega)x)^{i-1}}{(i-1)!}. \tag{A.5}$$

Then, the probability that a message is successfully received in the absence of interferers deduces from the probability that the message's signal is stronger than the *reception threshold* R_x, that is,

$$\begin{aligned} P_R(x > R_x) &= 1 - F_d(R_x; m, \Omega) \\ &= e^{-(mR_x/\Omega)} \sum_{i=1}^{m} \frac{((m/\Omega)R_x)^{i-1}}{(i-1)!}. \end{aligned} \tag{A.6}$$

For the Nakagami parameter $m = 3$ we derive the following:

$$P_R(x > R_x) = e^{-(3R_x/\Omega)} \left(1 + 3\frac{R_x}{\Omega} + \frac{9}{2}\left(\frac{R_x}{\Omega}\right)^2\right). \tag{A.7}$$

R_x should, in average, be detected in a distance equal to the "intended" communication range CR from the transmitter. Considering a quadratic path loss according to the *Friis*-model we get the relationship

$$R_x = \frac{T_P}{(CR)^2} G, \tag{A.8}$$

where T_P denotes the transmission power to be selected. G is a constant value defined as

$$G = \frac{G_t G_r \lambda^2}{(4\pi)^2 L}, \tag{A.9}$$

where G_t and G_r denote the antenna gains of transmitter and receiver, λ is the wavelength of the transmission, and L is the path loss factor, usually set to 1. We again apply the *Friis*-model to determine the average reception power $\Omega(d)$ at the distance d, that is,

$$\Omega(d) = \frac{T_P}{d^2} G. \tag{A.10}$$

By applying R_x and $\Omega(d)$ to (A.7) we obtain the expected probability of successfully receiving a message at distance d while considering an intended communication range of CR meter:

$$P_R(d, CR) = e^{-3(d/CR)^2} \left(1 + 3\left(\frac{d}{CR}\right)^2 + \frac{9}{2}\left(\frac{d}{CR}\right)^4\right). \tag{A.11}$$

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