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Research Article

Adaptive Resource Allocation with Strict Delay Constraints in OFDMA System

Naveed Ul Hassan and Mohamad Assaad

Department of Telecommunications, Ecole Supérieure d'Electricité (Supélec), Plateau de Moulon, 3 rue Joliot Curie, 91192 Gif-sur-Yvette Cedex, France

Correspondence should be addressed to Naveed Ul Hassan, naveed.hassan@yahoo.com

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We consider the adaptive resource allocation problem in downlink Orthogonal Frequency Division Multiple Access (OFDMA) system with strict packet delay constraints in the range of $1 < D < \infty$. In this range of delay constraints, resource optimization has to be simultaneously performed over multiple time slots. Thus optimal allocation decisions require future Channel State Information (CSI) and packet arrival rate information. The causal nature of CSI combined with the increase in the number of optimization variables makes it a very challenging problem. We propose a two-step solution by separating scheduling from subcarrier and power allocation. Our proposed causal scheduler ensures delay guarantees by deriving a minimum data rate out of the user queues while minimizing transmit power in every time slot. The output rates are fed to the resource allocation block and the problem is formulated as a convex optimization problem. The subcarrier and power allocation decisions are made in order to satisfy the demanded rates within the peak power constraint. We address the feasibility of the physical layer resource allocation problem and develop efficient algorithms. When the problem is infeasible we devise a strategy which incurs minimum deviation from the proposed rates for maximum number of users. We show by simulations that our proposed scheme can efficiently utilize time variations as well as multiuser diversity in the system.

1. Introduction

Harsh wireless channel conditions, scarce bandwidth, and limited power resources require intelligent allocation schemes which can efficiently exploit channel variations. OFDMA is a multicarrier modulation and multiplexing technique which divides the wideband frequency selective wireless channel into a set of orthogonal narrowband channels and provides immunity from Intersymbol Interference (ISI) [1]. In a multiuser system, different subcarriers can be allocated to different users without interference. Due to the multi-carrier nature of OFDMA systems, enormous opportunities exist for dynamic subcarrier and power allocation strategies [2–4].

Most of the existing work on adaptive resource allocation schemes in OFDMA systems has focused on traffic types with delay constraints of either $D = \infty$ or $D = 1$. $D = \infty$ represents the delay tolerant traffic while $D = 1$ represents the delay intolerant traffic. In both these cases ($D = 1$ and $D = \infty$), optimal subcarrier and power allocation decisions require instantaneous CSI only [5–7]. It is obvious that $D = \infty$ and $D = 1$ are in fact two extreme cases and do not represent practical service types. For all the practical service types, packet delay constraints are always in the range of $1 < D < \infty$. In this range of delay constraints, it is possible to exploit the short-term channel time variations. However, the resource allocation decisions depend on current as well as the future values of packet arrival and channel state information. The future traffic and channel state information is generally not available due to causality constraints. Moreover, this problem has a larger state space, an increased number of optimization variables, and the stochasticity in the arrival process and channel variations make it much harder to exploit time diversity and thus make this problem very challenging.

In this paper, we propose a two-step solution to sum-rate maximization problem with strict delay constraints in

the range $1 < D < \infty$. A Minimum Rate Scheduler (MRS) is developed which conceals the delay constraints in the form of data rate constraints while in the second step subcarrier and power allocation decisions are made based on the data rates proposed by the scheduler. We compare the performance of our physical layer resource allocation algorithm with the solution proposed in [8]. By using a greedy algorithm, the authors estimate the required resources based on average channel gains of the users in the first step while in the second step exclusive subcarrier assignments are made based on the Hungarian algorithm [9, 10].

1.1. Previous Work. When $D = 1$ scheduling is not required. Several schedulers have been developed in [11–15] for the case when $D = \infty$. Some of these schedulers [11, 12] base their scheduling decisions entirely on the current CSI and are known as the Channel-Aware Only (CAO) schedulers. CAO schedulers provide long-term fairness without ensuring strict delay guarantees for any packet in the system. The second class of schedulers employ both the channel and the queue state informations to incorporate fairness among users [13–15]. Modified Largest Weighted Delay First (M-LWDF) rule [14] and the Exponential (Exp) Rule [15] schedulers are Channel-Aware Queue-Aware (CAQA) schedulers. These schedulers perform significantly better than CAO schedulers but they are also unable to respect strict packet delay constraints. Scheduling is separated from subcarrier and power allocation in [16–18]. However, the objective of the authors in these papers is the average delay minimization rather than strict delay constraint achievement. In [19], the authors consider packet scheduling with strict delay constraints for AWGN channels and derive robust energy efficient schedulers. The authors in [20, 21] exploit energy delay tradeoff and propose strategies to minimize queuing delay for single-user single-carrier systems. Similarly dynamic programming was adopted in [22] for scheduling packets over a time-slotted single-user wireless link. Perhaps the work in [23] for TDMA systems is closest to our approach in terms of problem formulation. In this paper, the authors develop energy efficient scheduler with individual packet delay constraints by developing bounds on transmission rate and then write the optimization problem. However, their work is again limited to TDMA systems and there is no power control in their developed schemes.

We want to stress that the specific optimization problem of sum rate maximization subject to strict individual user delay constraints is largely ignored due to the larger state space size of the problem. In general, good schedulers to achieve strict delay constraints when $1 < D < \infty$ for multiuser OFDMA systems are largely missing and this paper is an effort to fill this gap.

1.2. Proposed Approach and Main Contributions. In this subsection we highlight our proposed solution and the main contributions of this paper.

(i) The problem of achieving strict target delay constraints is stretched in the past and in the future and we can capture this dependence by developing certain bounds on data rate transmission in each time slot. We develop two bounds (upper and lower bound constraints) which help us to write the resource allocation problem.

(ii) We write the adaptive resource allocation problem with strict delay constraints in OFDMA systems as an optimization problem. The objective of the problem is to maximize the sum-rate in $D = \max\{D_1, \ldots, D_K\}$ time slots. The problem formulation is flexible to accommodate different services for different users in the system. We develop a suboptimal two-step solution to solve this problem. We develop MRS which propose an instantaneous data rate for each user in each time slot and then we maximize the instantaneous sum-rate in the next step.

(iii) The objective of MRS is to propose a minimum data rate for each user which is just sufficient to attain strict delay constraints of the packets. If we can attain these data rates at the physical layer without violating the peak power constraint strict delays are guaranteed. In fact there are three possibilities as follows.

  (a) The proposed rates are achieved at the physical layer and all the available power gets consumed in achieving these rates.

  (b) The proposed rates are achieved and there is some power still left at the BS. In this case, the remaining power is allocated to the best users in the systems. Thus we maximize the sum-rate without fearing the delay violations of the packets.

  (c) The proposed rates cannot be achieved with the given amount of power. In this case, the data rates proposed by MRS are not feasible. We develop an algorithm where we decrease the data rates of some of the users. However, for such users the backlog is high for next time slots. Hence, MRS will adapt its decisions according to the backlog information and tries to compensates this loss in future time slots.

  MRS solves a sum-power minimization problem. The optimization problem for MRS is formulated over $T \gg D$ time slots. In order to reduce the complexity we take average power in future time slots. This is sub-optimal but the effects due to sub-optimality are corrected in the next time slot by utilizing the backlog information. Thus by using QSI, MRS tracks the actual channel conditions and corrects its decisions.

(iv) Once we have the target data rates proposed by the MRS, the remaining problem is an instantaneous optimization problem. However, since we have limited amount of transmission power available at the BS hence there is an issue of feasibility. We develop a method to detect the nonfeasibility in the problem. Then we propose algorithms for the feasible and nonfeasible cases. MRS decisions are thus corrected by the physical layer algorithms.

(v) It should be noted that we do not use Dynamic Programming (DP) or some other probabilistic optimization techniques in this paper. DP depends on the probability distribution function (pdf) of the arrivals (traffic and channels) and future allocations. The state space equation is easier to write for simple pdf functions like Bernouilli, Gaussian, or Brownian process. Since the future channels and traffic can have more elaborated pdf functions and it is hard to find the pdf of the allocation in the future time slots, thus it is difficult to write the problem using a state space equation. Moreover the use of DP is restricted by the number of variables in the optimization problem since they increase the number of state space variables. Since in our problem we have power allocation, plus exclusive subcarrier assignment constraint over D time slots thus the state space of our problem is very huge and dynamic programming techniques are not feasible.

The rest of the paper is organized as follows. In Section 2, system model is described and the problem is formulated. Section 3 details the causal scheduler which derives minimum data rates for the users. Physical layer resource allocation algorithm and feasibility issues are discussed in Section 4. Complexity analysis of the scheduler and the resource allocation algorithms is carried out in Section 5 while simulation results are presented in Section 6. Finally, the paper is concluded in Section 7.

2. System Model and Problem Formulation

We consider a downlink OFDMA system with K users and F subcarriers. We assume that the total transmit power from the BS is constrained to $P_{\max}$. Time is divided into slots and during each time slot a data frame consisting of M OFDM symbols is transmitted. User channels remain constant for the duration of a time slot but may change from one time slot to another. We assume that perfect Channel State Information (CSI) is available at the BS. The channel gain to noise ratio $g_{k,f}^t$ of user k on subcarrier f during time slot t is given by $g_{k,f}^t = |h_{k,f}^t|^2/N_0B$, where $|h_{k,f}^t|$ denotes the channel coefficient of user k on subcarrier f after Fast Fourier Transform (FFT), N_0 is the power spectral density (PSD) of white noise, and B denotes the bandwidth of single subcarrier. Each user maintains a separate queue at the BS which receives data from the higher layer. We assume that all the packets of user k have same delay constraint which is in terms of number of time slots and denoted by D_k (different users have different packet delay constraints). Therefore, packets of user k arriving at time t must be out of the queue before the start of time slot $t + D_k$, otherwise they are dropped. The system model is detailed in Figure 1. Each user has a separate MRS scheduler which derives a minimum rate based on the available channel and Queue State Information at the start of each time slot. Resource allocation algorithm is employed which allocates power and subcarriers according to the minimum rates and peak power constraint. This assignment information is sent to the users via separate control channels which allow the users to recover their data.

Let R_k^t and X_k^t be the output rate and the input arrival rate of user k at time t, the queue backlog B_k^t then evolves according to the following equation:

$$B_k^{t+1} = \left(B_k^t + X_k^{t+1} - R_k^t\right)^+, \quad \forall k. \tag{1}$$

Without any loss of generality we assume that we start at time $t = 1$ and that the initial backlog is zero. The backlog at the start of time slot t for user k is

$$\begin{aligned} B_k^t &= \left(X_k^1 - R_k^1\right) + \cdots + \left(X_k^{t-1} - R_k^{t-1}\right) \\ &= \sum_{i=1}^{t-1}\left(X_k^i - R_k^i\right), \quad \forall k. \end{aligned} \tag{2}$$

We assume that the packets are dropped if they cannot be delivered before their delay deadline which means that at time t, $B_k^{t+1-D_k} = 0$. In order to ensure strict delay constraint D_k for all the packets, we must impose certain conditions on the output data rate of each user k at each time slot t. Below we derive these necessary conditions,

2.1. Lower Bound Constraint. Since all the packets of user k have same delay constraint D_k, we have

$$X_k^1 + \cdots + X_k^t \le R_k^1 + \cdots + R_k^t + \cdots + R_k^{t+D_k-1}, \quad \forall k. \tag{3}$$

Therefore, at any time t the output rate R_k^t must satisfy the following constraint:

$$R_k^t \ge \sum_{i=1}^{t} X_k^i - \sum_{i=1}^{t-1} R_k^i - \sum_{i=t+1}^{t+D_k-1} R_k^i, \quad \forall k. \tag{4}$$

Finally, we can write that

$$R_k^t \ge B_k^t + X_k^t - \sum_{i=t+1}^{t+D_k-1} R_k^i, \quad \forall k. \tag{5}$$

This constraint ensures that a packet arriving at time t will be out of the buffer before $t + D_k - 1$. Equation (5) gives a lower bound on the output data rate. We have to proceed sequentially in time to derive the optimal output rates. Moreover, the dependence of R_k^t on future allocation decisions is explicit from this constraint.

2.2. Upper Bound Constraint. This constraint arises from the fact that a packet cannot be transmitted before its arrival. Therefore, packets arriving at time t should be transmitted either during this time slot or future time slots, that is,

$$R_k^1 + \cdots + R_k^t \le X_k^1 + \cdots + X_k^t, \quad \forall k. \tag{6}$$

The condition on output rate becomes

$$R_k^t \le B_k^t + X_k^t, \quad \forall k. \tag{7}$$

Equation (7) gives an upper bound on R_k^t.

2.3. Optimization Problem. Since this is an OFDMA problem we assume that I_k^t are the subcarriers allocated to user k during time slot t. By using the Shannon capacity formula, the data rate achieved by user k on its allocated subcarrier set I_k^t during time slot t is given as (data rates are expressed in nats for analytical convenience)

$$R_k^t\left(p_{k,f}^t, g_{k,f}^t\right) = \sum_{f\in I_k^t} \log\left(1 + p_{k,f}^t g_{k,f}^t\right) nats/s/Hz, \tag{8}$$

where $p_{k,f}^t$ is the power allocated to user k on subcarrier f during time slot t. We now write an optimization problem in order to determine the optimal output rates and to allocate the subcarriers and powers to different users. Let $D = \max\{D_1, \ldots, D_K\}$. In order to achieve the target packet delay constraints, we have the following optimization problem:

$$\max \sum_{i=t}^{t+D-1} \sum_{k=1}^{K} R_k^i\left(p_{k,f}^i, g_{k,f}^i\right) \tag{9}$$

subject to

$$R_k^i\left(p_{k,f}^i, g_{k,f}^i\right) \geq B_k^i + X_k^i - \sum_{\tilde{t}=i+1}^{i+D_k-1} R_k^{\tilde{t}}\left(p_{k,f}^{\tilde{t}}, g_{k,f}^{\tilde{t}}\right) \quad \forall k, i, \tag{10}$$

$$R_k^i\left(p_{k,f}^i, g_{k,f}^i\right) \leq B_k^i + X_k^i \quad \forall k, i, \tag{11}$$

$$\sum_{k=1}^{K} p_k^i \leq P_{\max}, \quad \forall i, \tag{12}$$

$$I_m^i \cap I_n^i = \Phi, \quad \forall m \neq n, \quad \forall i, \tag{13}$$

$$\bigcup_{k=1}^{K} I_k^i \subseteq \{1, \ldots, F\}, \quad \forall i. \tag{14}$$

The objective of the problem in (9) is the throughput maximization or system capacity which is the main goal of the network operators. Constraints (10) and (11) are the instantaneous constraints on the data rate of each user in order to ensure strict delay constraints. These constraints correspond to the lower bound (5) and the upper bound (7) on the output data rates, respectively. Constraint (12) demands that the total transmit power should always be less than the peak power constraint in each time slot. Constraints (13) and (14) are the OFDMA constraints which demand that at any time t each subcarrier should be allocated to no more than one user and that the sum of all the subcarriers should be equal to the total number of subcarriers in the system.

To get an optimal solution we need to find the optimal output rates R_k^t, for all t, k. This problem is nonconvex and is not easy to solve because the optimal value of R_k^t in (10) is bounded by unknown variables which depend on future allocation decisions as well as future channel gains and future input arrival rates. We develop a two-step solution to solve this problem. We develop MRS which propose a minimum data rate $R_{\min}^{t,k}$ for each user according to constraints (10) and (11). The instantaneous subcarrier and power allocation decisions are then made by solving a constrained instantaneous sum-rate maximization problem.(There are some instantaneous constraints in the above optimization problem. These constraints remain the same and do not affect the two-step approach. The peak power constraint has to be attained in each time slot as well as the OFDMA constraints. We replace the upper and lower bound constraints by the minimum data rate constraints. Now if the data rates proposed by the scheduler are optimal the two step approach is completely justified. Due to approximations and the complexity of our problem the scheduler is not optimal hence there is some performance loss. However, some of this performance loss is compensated by the physical layer algorithm.) In this problem, the proposed data rate vector by the scheduler is an additional constraint along with constraints (12), (13) and (14). The instantaneous sum-rate problem is as follows:

$$\max \sum_{k=1}^{K} \sum_{f\in I_k^t} R_{k,f}^t\left(p_{k,f}^t, g_{k,f}^t\right) \tag{15}$$

subject to

$$\sum_{f\in I_k^t} R_{k,f}^t\left(p_{k,f}^t, g_{k,f}^t\right) \geq R_{\min}^{t,k} \quad \forall t, k, \tag{16}$$

$$\sum_{k=1}^{K} \sum_{f\in I_k^t} p_{k,f}^t \leq P_{\max}, \quad \forall t, \tag{17}$$

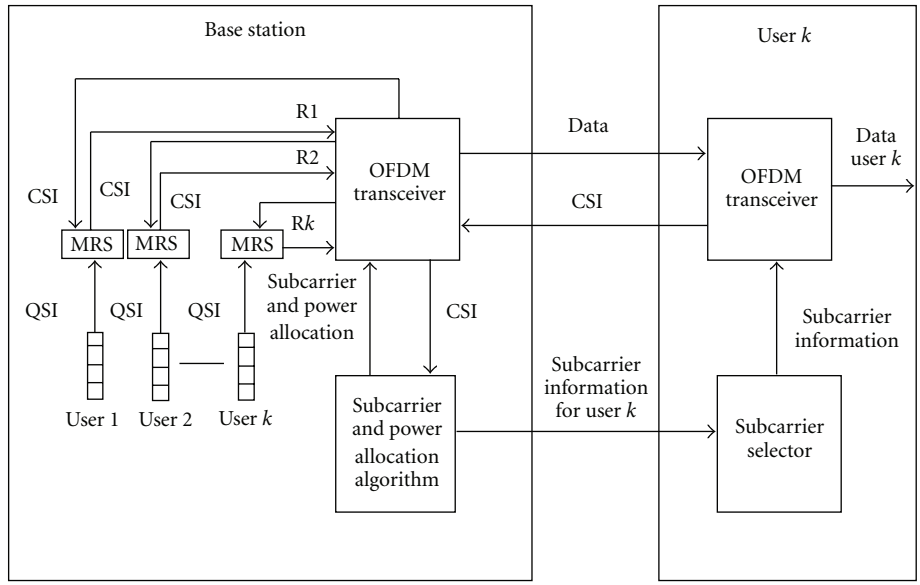
$$I_m^t \cap I_n^t = \Phi, \quad \forall m \neq n, \quad \forall t, \tag{18}$$

$$\bigcup_{k=1}^{K} I_k^t \subseteq \{1, \ldots, F\}, \quad \forall t. \tag{19}$$

Equation (16) is the instantaneous data rate constraint. If the data rates achieved by the resource allocation algorithm are equal to or greater than the proposed data rates $R_{\min}^{t,k}$, for all k, then delay constraints are satisfied. However, if in any time slot these data rates cannot be achieved due to bad channel conditions and power limitations then this loss is compensated for by the MRS in the next time slot. Hence, time diversity in the wireless channel is utilized since $D > 1$. Moreover, since we are maximizing the instantaneous sum-rate in (15) therefore the long term objective in (9) is also maximized. In the next section, we develop the Minimum Rate Scheduler to derive $R_{\min}^{t,k}$.

3. Minimum Rate Scheduler

We are interested in developing a scheduler which can propose minimum data rates such that strict delay constraints are guaranteed. From Figure 1, we can see that scheduler works in advance of subcarrier and power allocation block. Since actual transmitted power is not decided by the scheduler therefore we will base our scheduling decisions on transmit power minimization. Power minimization can be



QSI: queue state information
CSI: channel state information
MRS: minimum rate scheduler

Figure 1: System model.

seen as a useful way of enhancing sum-rate during resource allocation process. An optimal scheduler is able to to fully exploit the leverage provided by the delay constraints and at any time instant t it schedules a minimum rate out of the buffer which is able to satisfy all the delay constraints. If this is not the case then scheduling more packets than required will result in huge increase in power. So the name MRS comes from the fact that the scheduled rates are the lowest possible data rates which ensure strict delay guarantees while consuming the least amount of power. If these minimum rates can be achieved then the remaining power can be strictly utilized in enhancing the system capacity without worrying about delay violations. Thus the objective of MRS is the minimization of total transmit power subject to achieving strict delay constraints of the packets.

Since each user has a separate scheduler so in the subsequent analysis we will drop the user index for simplicity. During each time slot t we solve the optimization problem for MRS in a very large interval $[t, t+T]$. We call T the optimization interval where $T \gg 1$, $T \gg D$, for all k. The reason behind solving the optimization problem for T time slots is to make explicit the dependence of $R_{\min}^{t,k}$ on future arrival rates. Since the delay constraints of packets arriving at time slot $t+T$ is D, hence the summation is over $t+T+D-1$. In fact this formulation for MRS problem has been inspired from the work in [23]. The optimization problem for MRS is as follows:

$$\min p^t + \sum_{i=t+1}^{t+T+D-1} p^i \tag{20}$$

subject to

$$R^t(p^t, g^t) + \sum_{i=t+1}^{t+T+D-1} R^i\left(p^i, g^i\right) = B^t + X^t + \sum_{i=t+1}^{t+T} X^i, \tag{21}$$

$$R^t(p^t, g^t) \geq B^t + X^t - \sum_{d=t+1}^{t+D-1} R^d\left(p^d, g^d\right), \tag{22}$$

$$R^t(p^t, g^t) \leq B^t + X^t. \tag{23}$$

Due to causal nature of the scheduler and the fact that optimization interval T is assumed to be sufficiently large, at any time t the problem can be written as

$$\min p^t + (T + D - 1)p \tag{24}$$

subject to

$$R^t(p^t, g^t) + (T + D - 1)E[R(p, g)] = B^t + X^t + (T)X_0, \tag{25}$$

$$R^t(p^t, g^t) \geq B^t + X^t - (D - 1)E[R(p, g)], \tag{26}$$

$$R^t(p^t, g^t) \leq B^t + X^t. \tag{27}$$

The objective in (24) is the average power minimization in the optimization interval. Constraint (25) ensures that all the packets arriving in the optimization interval are transmitted before their delay deadlines. Constraints (26) and (27) are again the lower and upper bounds on the data rates. In this optimization problem, p and X_0 denote the mean power and the mean input arrival rate while $E[R(p, g)]$ is the

mean output rate estimated at time t. The above problem is not convex due to the presence of bounding constraints. We propose a heuristic solution where in order to get a good starting point we solve the problem by ignoring the bounding constraints (26) and (27). This problem results in output rate which may or may not be satisfying the bounding constraints. However, once this data rate is obtained, it is used in subsections A to C to get minimum output rate satisfying constraints (26) and (27). We observe that $E[R(p,g)]$ is a function of two random variables that is, g and p. By Jenson's inequality we have

$$E_{g,p}[R(p,g)] \geq E_g[R(p,g)] = E_g[\log(1+pg)], \tag{28}$$

where $E_g[R(p,g)]$ is the lower bound on the expected values of future output rates which will ensure that the minimum required rate over D time slots will be achieved. With this approximation, the relaxed optimization problem without constraints (26) and (27) can be written as

$$\min p^t + (T+D-1)p \tag{29}$$

subject to

$$\begin{aligned}\log(1+p^t g^t) + (T+D-1)E_g[\log(1+pg)] \\ = B^t + X^t + (T)X_0.\end{aligned} \tag{30}$$

This optimization problem can be solved using the Lagrange optimization techniques since the objective and the single constraint function are convex and KKT conditions are sufficient to arrive at the solution [24]. Let β be the Lagrange multiplier associated with the constraint, the Lagrangian is,

$$\begin{aligned}\mathcal{L}\{p^t, p\} = {} & p^t + (T+D-1)p \\ & - \beta\{\log(1+p^t g^t) \\ & \quad + (T+D-1)E_g[\log(1+pg)] \\ & \quad - B^t - X^t - (T)X_0\}.\end{aligned} \tag{31}$$

From KKT conditions $\partial\mathcal{L}\{p^t,p\}/\partial p^t = 0$ and $\partial\mathcal{L}\{p^t,p\}/\partial p = 0$, we get

$$\beta\left(\frac{g^t}{1+p^t g^t}\right) = 1, \tag{32}$$

$$\beta E_g\left[\frac{g}{1+pg}\right] = 1. \tag{33}$$

Let, $f_1(p) = E_g[g/1+pg]$ and $f_2(p) = E_g[\log(1+pg)]$. From (33), we have

$$\beta = \frac{1}{f_1(p)}. \tag{34}$$

Similarly, from (32), we have $\log(1+p^t g^t) = \log(\beta g^t)$ so we can rewrite (30) as

$$\log\left(\frac{g^t}{f_1(p)}\right) + (T+D-1)f_2(p) = B^t + X^t + (T)X_0. \tag{35}$$

Table 1: MRS algorithm.

(1) Numerically solve (35) to get the value of p.
(2) For this value of p, find β using (34).
(3) The output rate at time t is $R^t = \log(\beta g^t)$.
(4) The anticipated scheduled rates for future time slots at time t are, $\log(\beta g_0)$, where g_0 is the mean channel gain value.

Based on the previus equations we develop an algorithm which we will refer to as the MRS algorithm to find the value of R^t provided the probability density function (pdf) of the underlying physical channel is known. This algorithm is given in Table 1. It should be noted that the scheduled rates are not the actual future output rates because their exact values cannot be determined until that future time is reached. The value of interest is the current output rate R^t which may not be satisfying the two constraints given in (26) and (27).

Remark. $f_1(p)$ and $f_2(p)$ depend on the nature of the underlying physical channel and can be determined if the probability density function (pdf) of random channel variable g is known. Thus, the solution developed in this section is quite general and can be used for any type of channel as long as channel pdf is known and $f_1(p)$ and $f_2(p)$ are computable.

Example. As an example, we determine the values of $f_1(p)$ and $f_2(p)$ by assuming the underlying channel to be Rayleigh fading. In this case, random variable g is exponentially distributed with mean g_0 and probability density function given by $1/g_0 e^{-g/g_0}$. With $f_1(p)$ and $f_2(p)$ defined on the interval $[0,\infty)$, we have

$$\begin{aligned}f_1(p) &= \int \frac{g e^{-g/g_0} dg}{g_0(1+pg)} = \frac{g_0 p - e^{1/g_0 p} Ei(1/g_0 p)}{g_0 p^2}, \\ f_2(p) &= \int \frac{1}{g_0}\log(1+pg)e^{-g/g_0}dg = e^{1/g_0 p} Ei(1/g_0 p),\end{aligned} \tag{36}$$

where Ei is the exponential integral function, defined as $Ei(p) = \int_p^\infty e^{-p} dp/p,\ p > 0$.

Since the output rate has to satisfy both the upper and the lower bound constraints hence there are three possibilities for the value of R^t attained by the above algorithm. Let, $x = R^t$, $y = B^t + X^t$, and $z = (D-1)E[R(p,g)]$ in the constraint equations (26) and (27), then these three cases are as follows.

3.1. Case I: $x \leq y$ and $x \geq y - z$. In this case, both the constraints are satisfied so $R^{t,k}_{\min} = R^t$ is a valid minimum rate.

3.2. Case II: $x > y$. Constraint (27) is violated because the proposed output rate is higher than total number of packets available for transmission. The output rate is high because the channel is good. Therefore, valid strategy is to

transmit all the available packets in this time slot. We reduce the output rate and make it equal to y, that is, $R_{\min}^{t,k} = B^t + X^t$. It is obvious that by decreasing R^t, constraint (26) is not violated because all the packets are scheduled for instantaneous transmission.

3.3. Case III: $x \le y$ and $x < y - z$. In this case, constraint (26) is violated so the delay deadlines of the packets are not achieved. The output rate is less than what is required to ensure the delay constraints. Therefore, we have to increase x or z so that $x + z = y$. The problem can be viewed as rescheduling y packets over D time slots which is equivalent to the unconstrained problem in the optimization interval $[t, t + D - 1]$:

$$\min p^t + (D-1)p \tag{37}$$

subject to

$$R^t + (D-1)E[R(p,g)] = y. \tag{38}$$

This problem can be solved on similar lines to the relaxed problem discussed before and the same algorithm can be used. The resulting value of R^t is now a valid output rate which satisfies both the constraints. It is important to mention here that since we are proceeding sequentially in time so we are achieving delay constraint in every time slot. Since $x + z = y$, therefore, constraint (27) cannot be violated.

It should be noted that both constraints cannot be violated at the same time because they represent the upper and the lower bounds. After obtaining the minimum rates we pass them to the physical layer resource allocation block.

4. Physical Layer Resource Allocation

Let $R_{\min}^{t,k}$ be the data rate passed by each MRS to physical layer. The optimization problem during any time slot is,

$$\max \sum_{k=1}^{K} \sum_{f \in I_k^t} R_{k,f}^t \left(p_{k,f}^t, g_{k,f}^t\right) \tag{39}$$

subject to

$$\sum_{f \in I_k^t} R_{k,f}^t \left(p_{k,f}^t, g_{k,f}^t\right) \ge R_{\min}^{t,k} \quad \forall t, k, \tag{40}$$

$$\sum_{k=1}^{K} \sum_{f \in I_k^t} p_{k,f}^t \le P_{\max}, \quad \forall t, \tag{41}$$

$$I_m^t \cap I_n^t = \Phi, \quad \forall m \ne n, \quad \forall t, \tag{42}$$

$$\bigcup_{k=1}^{K} I_k^t \subseteq \{1, \ldots, F\}, \quad \forall t. \tag{43}$$

This problem can be viewed as a combination of margin-adaptive and rate-adaptive optimization problems. It is important to mention here that margin adaptive objective does not include power constraint as it tries to minimize total transmit power subject to minimum rate constraints. On the other hand, rate-adaptive objective has no minimum rate constraints as it maximizes the sum-rate subject to peak power constraint. Moreover, this optimization problem is a combinatorial problem due to the fact that users cannot share the same subcarrier. The combinatorial nature of the problem can be avoided by allowing the users to time-share each subcarrier over an OFDM symbol [2]. We introduce a time sharing factor $\gamma_{k,f}^t \in [0,1]$ for kth user on subcarrier f. During an OFDM symbol user k is allowed to transmit on subcarrier f for $\gamma_{k,f}^t$ percentage of time. This is possible from resource allocation point of view because we have assumed that channel remains constant in each time slot. This assumption on subcarrier sharing introduces the following constraint:

$$\sum_{k=1}^{K} \gamma_{k,f}^t \le 1, \quad \forall f. \tag{44}$$

As a result of time sharing, data rate achieved by user k on subcarrier f becomes $R_{k,f}^t(p_{k,f}^t, g_{k,f}^t) = \gamma_{k,f}^t \log(1 + p_{k,f}^t g_{k,f}^t)$. This function is neither convex nor concave. Therefore we define $\widetilde{p}_{k,f}^t = \gamma_{k,f}^t p_{k,f}^t$ as the average power allocated to user k on subcarrier f. With this change of variable, we have

$$R_{k,f}^t\left(\widetilde{p}_{k,f}^t, \gamma_{k,f}^t, g_{k,f}^t\right) = \gamma_{k,f}^t \log\left(1 + \frac{\widetilde{p}_{k,f}^t g_{k,f}^t}{\gamma_{k,f}^t}\right). \tag{45}$$

Equation (45) represents a concave function which can be verified from its Hessian which is negative semidefinite when $\gamma_{k,f}^t \ge 0$ and $\widetilde{p}_{k,f}^t \ge 0$. Finally, we can write the optimization problem as

$$\max \sum_{f=1}^{F} \sum_{k=1}^{K} \gamma_{k,f}^t \log\left(1 + \frac{\widetilde{p}_{k,f}^t g_{k,f}^t}{\gamma_{k,f}^t}\right) \tag{46}$$

subject to

$$\sum_{f=1}^{F} \gamma_{k,f}^t \log\left(1 + \frac{\widetilde{p}_{k,f}^t g_{k,f}^t)}{\gamma_{k,f}^t}\right) \ge R_{\min}^{t,k}, \quad \forall k, \tag{47}$$

$$\sum_{k=1}^{K} \gamma_{k,f}^t \le 1, \quad \forall f, \tag{48}$$

$$\sum_{f=1}^{F} \sum_{k=1}^{K} \widetilde{p}_{k,f}^t \le P_{\max}. \tag{49}$$

This is a convex optimization problem with linear and convex differentiable constraints. We can solve it by using convex optimization theory [24, 25]. Let $(\delta_k^t)_{k=1,\ldots,K}$, $(\mu_f^t)_{f=1,\ldots,F}$ and

α^t be the Lagrange multipliers associated with constraints (47), (48), and (49), respectively. The Lagrangian is

$$\mathcal{L}\left\{\widetilde{p}_{k,f}^t, \gamma_{k,f}^t\right\} = \sum_{k=1}^{K}\left(1+\delta_k^t\right)\left\{\sum_{f=1}^{F}\gamma_{k,f}^t \log\left(1+\frac{\widetilde{p}_{k,f}^t g_{k,f}^t}{\gamma_{k,f}^t}\right)\right\} - \sum_{k=1}^{K}\delta_k^t R_{\min}^{t,k} - \alpha^t\left(\sum_{k=1}^{K}\sum_{f=1}^{F}\widetilde{p}_{k,f}^t - P_{\max}\right) - \sum_{f=1}^{F}\mu_f^t\left(\sum_{k=1}^{K}\gamma_{k,f}^t\right) - 1). \tag{50}$$

Since the objective and constraint functions are convex the duality gap is zero and we can use Lagrange dual decomposition theory to solve this problem. The dual problem is to maximize

$$\mathcal{G}\left(\delta_k^t, \alpha^t, \mu_f^t\right) = \textbf{maximize } \mathcal{L}\left\{\widetilde{p}_{k,f}^t, \gamma_{k,f}^t\right\}. \tag{51}$$

We can readily decompose $\mathcal{G}(\delta_k^t, \alpha^t, \mu_f^t)$ on subcarriers and users to get KF subproblems

$$\mathcal{S}_{k,f}\left(\delta_k^t, \alpha^t, \mu_f^t\right) = \left(1+\delta_k^t\right)\left\{\gamma_{k,f}^t \log\left(1+\frac{\widetilde{p}_{k,f}^t g_{k,f}^t}{\gamma_{k,f}^t}\right)\right\} - \alpha^t \widetilde{p}_{k,f}^t - \mu_f^t \gamma_{k,f}^t \quad \forall k, f. \tag{52}$$

Since subproblems $\mathcal{S}_{k,f}(\delta_k^t, \alpha^t, \mu_f^t)$ are also convex, KKT conditions are sufficient to find a solution. From $\partial \mathcal{S}_{k,f}(\delta_k^t, \alpha^t, \mu_f^t)/\partial \widetilde{p}_{k,f}^t = 0$ and $\partial \mathcal{S}_{k,f}(\delta_k^t, \alpha^t, \mu_f^t)/\partial \gamma_{k,f}^t = 0$, we arrive at

$$p_{k,f}^t = \left(\frac{1+\delta_k^t}{\alpha^t} - \frac{1}{g_{k,f}^t}\right)^+, \quad \forall k, f, \tag{53}$$

$$\left(1+\delta_k^t\right)\left(\left(\log\left(\frac{\left(1+\delta_k^t\right)g_{k,f}^t}{\alpha^t}\right)\right)^+ - \left(1-\frac{\alpha^t}{\left(1+\delta_k^t\right)g_{k,f}^t}\right)^+\right) = \mu_f, \quad \forall f. \tag{54}$$

From (53) and (54), it is extremely difficult to develop an algorithm for subcarrier and power allocation. Moreover, there is also a question of feasibility because given a fixed total power, it might not be possible to support all the minimum rates during current time slot.

4.1. Feasibility of Physical Layer Optimization Problem. Since our problem is convex, a necessary and sufficient condition for feasibility is the nonemptiness of the feasible set. Let $\mathbf{C} = \{C_1, \ldots, C_k\}$ be the achieved data rate vector and $\mathbf{R} = \{R_1, \ldots, R_K\}$ be the rate constraint vector. Let P_t be the total power required in achieving $\mathbf{C}$. The feasible set can be defined as

$$\mathcal{R} = \{\mathbf{C} : (C_k \geq R_k, \ \forall k) \cap P_t \leq P_{\max}\}, \tag{55}$$

where $\mathcal{R}$ is the intersection of two sets. Let the set defined by rate constraint vector be denoted by $\mathcal{R}_1$ and peak power constraint by $\mathcal{R}_2$. Each rate constraint vector has an associated power region. Let $\mathcal{P}$ be the power region when $C_k = R_k$, for all k, that is,

$$\mathcal{P}(\mathbf{R}) = \{P_t : (C_k = R_k, \ \forall k)\}. \tag{56}$$

Let P_{mg} be the optimal point of the set (56). Our problem is feasible if $P_{mg} \cap \mathcal{R}_2$ is nonempty which is possible if P_{mg} lies inside or on the boundary of $\mathcal{R}_2$. Therefore, the feasibility issue is reduced to finding P_{mg} which can be obtained by solving the following margin adaptive problem:

$$\min \sum_{k=1}^{K}\sum_{f=1}^{F}\widetilde{p}_{k,f}^t \tag{57}$$

subject to

$$\sum_{f=1}^{F}\gamma_{k,f}^t \log\left(1+\frac{\widetilde{p}_{k,f}^t g_{k,f}^t}{\gamma_{k,f}^t}\right) \geq R_{\min}^{t,k}, \quad \forall \mathrm{k}, \tag{58}$$

$$\sum_{k=1}^{K}\gamma_{k,f}^t \leq 1, \quad \forall f. \tag{59}$$

From our previous analysis it is evident that this is also a convex optimization problem, therefore, with $(\delta_k^t)_{k=1,\ldots,K}$ and $(\mu_f^t)_{f=1,\ldots,F}$ as the Lagrange multipliers associated with the constraints (58) and (59), respectively. Then by solving the Lagrange-KKT optimality conditions, we get

$$p_{k,f}^t = \left(\delta_k^t - \frac{1}{g_{k,f}^t}\right)^+, \tag{60}$$

$$\delta_k^t\left(\left(\log\left(\delta_k^t g_{k,f}^t\right)\right)^+ - \left(1-\frac{1}{\delta_k^t g_{k,f}^t}\right)^+\right) = \mu_f^t. \tag{61}$$

Using (60) and (61), we can develop a margin adaptive algorithm. This algorithm is very similar to the one given in [2]. The algorithm is presented in Table 2. A set of subcarriers I'_k gets allocated to each user k and power is allocated on these subcarriers according to waterfilling principle. Step 1 of this algorithm can be used to get margin adaptive solution for single user OFDM system. For a small enough step size the convergence of above algorithm is surely attained [24]. There is a possibility that more than one user converge to the same value of μ_f^t. In this case, the optimal solution is attained by time sharing of a subcarrier between the tied users on each such subcarrier. These ties can be broken by randomly picking a single user for exclusive transmission on such subcarrier. Although this heuristic to break the ties will lead to small deviations in QoS requirements however it is adopted here to reduce the complexity of our proposed solution. Moreover, the probability of this event vanishes exponentially under some reasonable conditions as K and F increases. Further details can be found in [26, 27].

Comparing (60) and (61) with (53) and (54) we can see that the original problem with power constraint turns

Table 2: Margin adaptive algorithm.

(1) Initialization: $\delta_k^t = \min_{k,f} 1/g_{k,f}^t, \forall k, \phi_{k,f} = 0, \forall k, f, \gamma_{k,f}^t = 0, \forall k, f, \Gamma_k = 0, \forall k$.
(2) Repeat till all the rate constraints are achieved.
(3a) Repeat till k^{th} user rate constraint is achieved.
(3b) Increase waterlevel of user k, $\delta_k^t = \delta_k^t + \Delta_m$.
(3c) On all the subcarriers compute $\phi_{k,f} = \delta_k^t((\log(\delta_k^t g_{k,f}^t))^+ - (1 - 1/\delta_k^t g_{k,f}^t)^+)$.
(3d) Allocate subcarrier to this user if $\phi_{k,f}$ is maximum and set $\gamma_{k,f}^t = 1$ other wise $\gamma_{k,f}^t = 0$.
(4) Compute the achieved data rates according to, $\Gamma_k = \sum_{f=1}^{F} \gamma_{k,f}^t (\log(\delta_k^t g_{k,f}^t))^+ \ \forall k$.

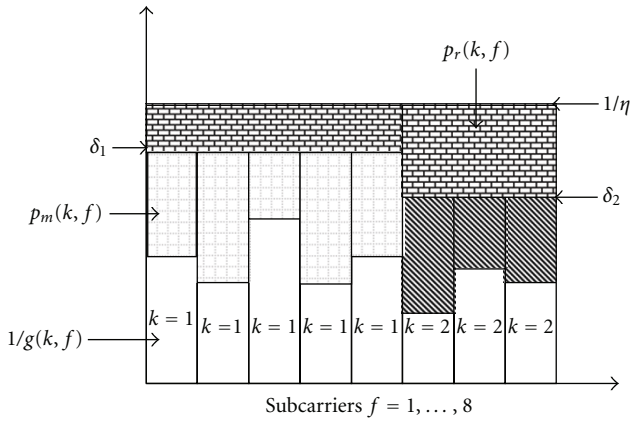


Figure 2: Illustration of multiuser waterfilling for 2 user 8 subcarrier system for feasible case.

into margin adaptive problem when $\alpha^t = 1$. Therefore, margin adaptive problem can be viewed as a special case of the original optimization problem. From (53) it is obvious that power is inversely proportional to α^t. Therefore, as α^t increases beyond zero total transmit power decreases and minimum power is attained for $\alpha^t = 1$. Since by definition $\alpha^t = 1$ corresponds to minimum total power therefore no solution exists for the original problem when $P_{mg} > P_{\max}$. Similarly it can be argued that when $P_{mg} > P_{\max}$ increasing α^t above one cannot attain $P_{\max}$. This argument is based on the observation that decrease in power when $\alpha^t > 1$ is compensated by the individual user waterlevels δ_k^t which are directly associated with the demanded rates and the resulting total power converges to P_{mg}. Therefore, if $P_{mg} < P_{\max}$, the original problem is feasible otherwise it is not.

4.2. Feasible Case: $P_{mg} \leq P_{\max}$. When the problem is feasible, minimum rates can be achieved under the peak power constraint. If $P_{mg} < P_{\max}$, there is more power than required to satisfy the minimum rates. We develop a scheme to allocate this additional power to the users. We use margin adaptive algorithm ($\alpha^t = 1$) to find subcarrier allocation. Then on the allocated subcarriers remaining power is utilized. Let Ω' be the set of users with non-empty queues after the transmission of $R_{\min}^{t,k}$, that is, $B_k^{t+1} > 0$, for all $k \in \Omega'$. Let $p_m^{t,k,f}$ be the margin adaptive power allocation and $\widetilde{I}_k^t$ be the set of subcarriers assigned by margin adaptive algorithm. We suppose that $p_r^{t,k,f}$ is the additional power allocated to user k on subcarrier f. We can write the following optimization problem to allocate additional power:

$$\max \sum_{k\in\Omega'} \sum_{f\in\widetilde{I}_k^t} \log\left(1 + \left(p_m^{t,k,f} + p_r^{t,k,f}\right) g_{k,f}^t\right) \tag{62}$$

subject to

$$\sum_{k\in\Omega'} \sum_{f\in\widetilde{I}_k^t} \left(p_m^{t,k,f} + p_r^{t,k,f}\right) + \sum_{k\notin\Omega'} \sum_{f\in\widetilde{I}_k^t} p_m^{t,k,f} = P_{\max}. \tag{63}$$

This problem is also convex, so with η^t as Lagrange multiplier associated with the constraint (63) and solving KKT conditions, we arrive at

$$p_r^{t,k,f} = \left(\frac{1}{\eta^t} - \left(p_m^{t,k,f} + \frac{1}{g_{k,f}^t}\right)\right)^+. \tag{64}$$

Since from (60) we have $p_m^{t,k,f} + 1/g_{k,f}^t = \delta_k^t$. therefore, we get

$$p_r^{t,k,f} = \left(\frac{1}{\eta^t} - \delta_k^t\right)^+. \tag{65}$$

In fact this solution has a very simple interpretation. The remaining power is waterfilled on top of the existing margin adaptive waterlevels of the users with non-empty queues. The additional power is strictly utilized in maximizing the system throughput without any fear of delay violations. Figure 2 explains the multi-level waterfilling in multiuser OFDMA system for the feasible case with $K = 2$ and $F = 8$. δ_1 and δ_2 are the margin adaptive waterlevels corresponding to users 1 and 2, respectively. For each user, channel gains $g_{k,f}^t$ on their allocated subcarriers are inverted which are represented by the blank regions. The amount of margin adaptive power allocated on each subcarrier is represented by the shaded portion and the additional power is waterfilled on top of margin adaptive water levels. Throughput is maximized because more power is allocated to the users which can achieve maximum data rates. Finally, the backlog values B_k^{t+1} are updated accordingly for the next time slot. Thus the additional power is now utilized in strictly increasing the sum-rate of the system without fearing about the packet drops and delay violations.

4.3. Nonfeasible Case: $P_{mg} > P_{\max}$. In this case, we cannot respect all the minimum rates proposed by MRS during current time slot. In order to respect the constraint on $P_{\max}$, we have to decrease the data rates of the users. We develop an algorithm where the data rates of some of the users are decreased in such a way that throughput is least sacrificed. Moreover, we ensure in this algorithm that minimum number of users are affected by the rate decrease so that the proposed rates of maximum number of users are attained. Again we use the subcarrier allocation as obtained by margin adaptive algorithm. Our algorithm is based on the observation that MRS propose higher data rates in following scenarios: (i) user channel is good compared to its mean channel gain, (ii) backlog value is high, and (iii) both (i) and (ii). Therefore, the user with maximum data rate constraint is the user with urgent need of data transmission. Decreasing its data rate will result in maximum delay violations. Let $R_m^{t,k}$ be the margin adaptive rate and $P_m^{t,k}$ be the margin adaptive power allocated to user k. We have Algorithm 1 for the nonfeasible case.

In step 1, we identify a data rate region $\mathcal{C}$. All the users whose demanded rates lie in this interval are the ones with urgent need of data rate transmission. In step 2, we form a set of users which will be considered for possible decrease in their data rate. We select a user k' which consumes maximum power to achieve its rate constraint. This user represents the worst user of the set Ω. Therefore, if we decrease its data rate by a small amount we will end up saving a huge amount of power. We repeat the process till $P_{\max}$ is achieved. This algorithm converges for small values of Δ. In step 1, $\overline{\phi}$ is used to determine the lower bound on interval $\mathcal{C}$. This parameter is adjusted in such a way that enough users are included in the set Ω for possible rate decrease.

Since we have decreased the data rates of some of the users, their backlog has increased. MRS utilizes the backlog information in its scheduling decisions hence it will propose a high data rate for such users in next time slots. Thus such users will get a higher data rate in future time slots in order to avoid packet drops, thereby decreasing the overall packet drop rate.

5. Complexity Analysis

In this section, we will separately analyze the complexity of the scheduler and the resource allocation algorithms.

5.1. MRS Complexity. The scheduler operates in two parts. In the first part, the MRS algorithm propose output rates without the bounding constraints (26) and (27) while in the second part these rates are adjusted in Case I to Case III. We separately analyze the complexity of these two parts.

(1) During each time slot, the MRS algorithm has four steps all of which involve mathematical operations. Let $\mathcal{C}_1$ denote the complexity of the mathematical operations involved in this algorithm. Since each user has a separate MRS, the total complexity of this part is $K\mathcal{C}_1$.

Table 3: Complexity order of different algorithms for K users and F subcarriers in the system.

Algorithm	Complexity Order	Required CSI (tti)
MRS scheduler	$O(2K)$	1
Margin Adaptive Algorithm	$O(\mathcal{I}_m FK)$	1
Feasible case	$O(K)$	1
Nonfeasible case	$O(\mathcal{I}_{nf} FK)$	1

(2) The additional complexity of the scheduler comes from the second step where the output rates are adjusted in Case I to Case III. Case I and Case II do not incur additional complexity. Case III can result in solving additional optimization problems by using the MRS algorithm. The maximum complexity of this step occurs when all the users require Case III. In this situation, the complexity of this part becomes equal to that of part 1.

Thus the maximum complexity of scheduling is $\mathcal{C}_S = 2K\mathcal{C}_1$. Since the complexity of mathematical operations can be ignored it can be concluded that the maximum complexity of the scheduler is of the order $O(2K)$.

5.2. Margin Adaptive Algorithm. The complexity of this algorithm depends on the number of iterations $\mathcal{I}_m$ required to update the waterlevels δ_k^t for a given step size Δ_m. Since the algorithm has to find the best user on each subcarrier by employing waterfilling power allocation, therefore, the complexity order of the sum-power minimization algorithm becomes $O(\mathcal{I}_m FK)$. The complexity of this algorithm is polynomial in number of users and subcarriers.

5.3. Feasible Case. This algorithm is not an iterative algorithm and like MRS only involves mathematical operations. Let $\mathcal{C}_2$ denote the complexity of the waterfilling operation in this case. Since additional power is allocated to the users with non-empty queues on top of the margin adaptive waterlevels, hence the complexity of this algorithm depends only on the number of users with non-empty queues. The number of such users can be less than or equal to the total number of users in the system. Therefore, the maximum complexity of this algorithm can be $\mathcal{C}_{FC} = K\mathcal{C}_2$ and the complexity order is $O(K)$.

5.4. Nonfeasible Case. The algorithm for the non-feasible case is an iterative algorithm. The complexity of this algorithm depends on the number of iterations required to decrease the data rate of the users for a given step size Δ till convergence. Since the algorithm achieves the new data rate by using waterfilling algorithm on the subcarriers allocated by the margin adaptive algorithm, therefore, the complexity order of this algorithm is $O(\mathcal{I}_{nf} FK)$. The complexity of this algorithm is also polynomial in number of users and subcarriers.

The complexity orders and the required CSI of these algorithms are given in Table 3.

Initialization: $P_{\text{rem}} = P_{mg} - P_{\max}$
While $P_{\text{rem}} > 0$
(1) $R_{ub} = \max\ R_m^{t,k}$, $R_{lb} = R_{ub} - \overline{\phi} R_{ub}$, $\mathcal{C} = [R_{lb}, R_{ub}]$
(2) $\Omega = \{\forall k \mid 0 < R_m^{t,k} < R_{lb}\}$
(3) $k' = \max_{k \in \Omega} P_m^{t,k} / R_m^{t,k}$
(4) $R_{\text{new}}^{t,k'} = R_m^{t,k'} - \Delta$, $B_{\text{new}}^{t+1,k'} = B_{k'}^{t+1} + \Delta$
(5) $R_{\text{new}}^{t,k'}$ is achieved by using step 1 of the margin adaptive algorithm.
(6) $P_{\text{new}}^{t,k'}$ is the power allocated to user k' by waterfilling over $\tilde{I}_k^t$.
(7) $P_{\text{rem}} = P_{\text{rem}} - \{P_m^{t,k'} - P_{\text{new}}^{t,k'}\}$
(8) $P_m^{t,k'} = P_{\text{new}}^{t,k'}$, $R_m^{t,k'} = R_{\text{new}}^{t,k'}$ and $B_{k'}^{t+1} = B_{\text{new}}^{t+1,k'}$.

Algorithm 1: $P_{mg} > P_{max}$.

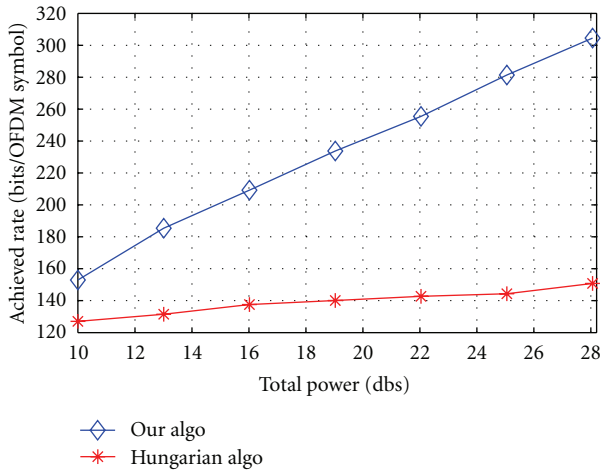


Figure 3: Total achieved rate versus $P_{\max}$ for 10 user 24 subcarrier system. Total demanded rate = 100 bits/OFDM symbol.

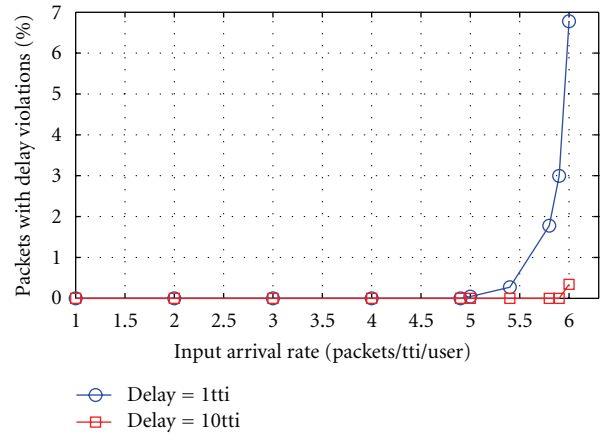


Figure 4: Percentage of packets with delay violations versus input arrival rate for all the users in the system. Users = 14, Subcarriers = 40, Cell Radius = 700 m, and tti = 1 ms.

6. Numerical Analysis

We consider a single cell downlink OFDMA system with perfect channel state information and a peak power constraint of 43 dBm. We consider a frequency selective Rayleigh fading channel with exponential delay profile. Path losses are calculated according to Cost-Hata Model [28]. The power spectral density of noise is −174 dBm/Hz. Time is divided into slots and duration of each time slot is 1 ms. A given number of packets are generated for each user every time slot. We assume that all the packets have same delay constraint and each packet has a size of 1 Kbits. The users are uniformly distributed in a cell of radius 700 m. Moreover, the bandwidth of each subcarrier is 375 KHz. The simulations are carried out for different scenarios. In each scenario, the distances of the users from BS remain constant which is a realistic assumption for low-speed mobile users. Each scenario is simulated for a total of 1000 tti which corresponds to 1 second of real time. Furthermore, in all the scenarios we assume that one user is always at a maximum distance from the BS in order to analyze the performance of our approach for worst user in the system.

In Figure 3, we compare the performance of our physical layer algorithm with the algorithm presented in [8] for 10 users and 24 subcarriers. It should be noted that this is the comparison of the physical layer resource allocation algorithms and not the comparison of our whole approach. Moreover, we have assumed in this simulation that the queues are backlogged and there are always packets available for transmission so that we are able to utilize all the available power. The authors in [8] solve the resource allocation problem presented in Section 4 for the feasible case by using hungarian algorithm. Although the authors do not consider scheduling and delay constraints, however, their algorithm can be considered to be applicable for feasible case assuming that the underlying application demands a delay of $D = 1$ and $R_{\min}^{t,k} = X_k^t$, for all t, k. From the simulations we can see that the total achieved rate by our approach is much higher because for the feasible case our algorithm gives the remaining power by waterfilling while the algorithm presented in [8] gives all the remaining resources to the user with highest mean channel gain value.

In order to evaluate the performance of our whole scheme (scheduling and resource allocation), we plot Figures 4, 5, 6, and 7. (The optimal solution for this problem is unknown in the literature. Moreover, the brute force method is also not applicable to this framework. In the brute

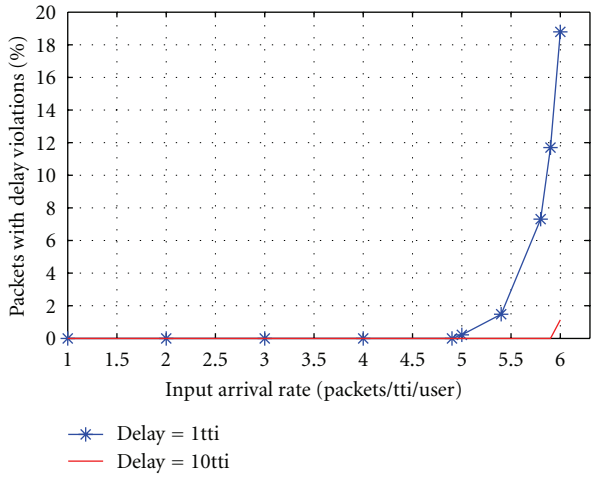


Figure 5: Percentage of packets with delay violations versus input arrival rate for worst user. Users = 14, Subcarriers = 40, Cell Radius = 700 m, and tti = 1 ms.

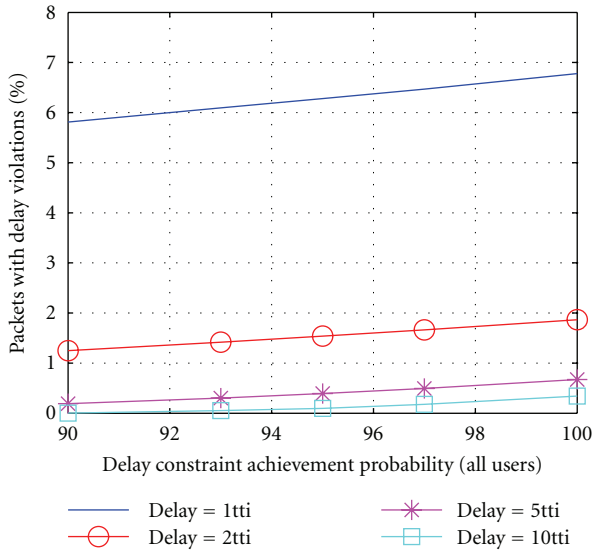


Figure 6: Percentage of packets with delay deadline violations for all user versus delay constraint achievement probability. Users = 14, Subcarriers = 40, Input arrival rate = 6 packets/tti, Cell Radius = 700 m,and tti = 1 ms.

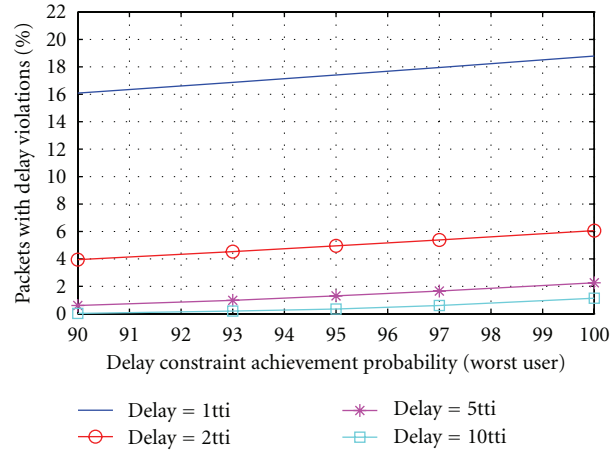


Figure 7: Percentage of packets with delay deadline violations for worst user versus delay constraint achievement probability. Users = 14, Subcarriers = 40, Input arrival rate = 6 packets/tti, Cell Radius = 700 m, and tti = 1 ms.

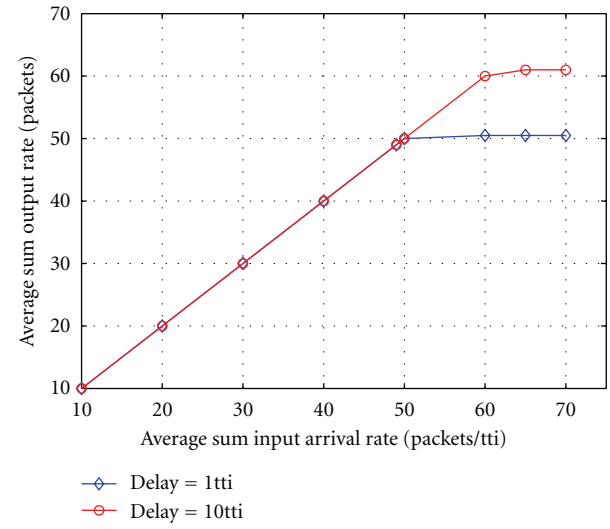


Figure 8: Average output sum rate versus average sum input arrival rate. Users = 14, Subcarriers = 40, Cell Radius = 700 m, and tti = 1 ms.

force method we have to try all the possible combinations. However, in this case since the rate is given by a continuous function w.r.t power there are infinite possibilities. It is therefore impossible to obtain the optimal solution using brute force method. Hence comparisons with optimal solution of this problem are not possible.) We plot these figures for 14 users and 40 subcarriers. Figure 4 shows the input arrival rate versus the percentage of packets whose delay constraints are violated for all the users in the system. We are interested in the maximum input arrival rate that can result in strict delay constraint achievement of all the packets in the system. When $D = 1$ tti, input arrival rate has to be constrained to 4.9 packets/tti/user for 0% packet delay violations. However, when $D = 10$ tti, our scheduling policy is able to deliver 5.9 packets/tti/user without any delay violations. This difference translates into achieving 14 Mbits/s higher transmission rate while achieving strict delay constraint for all the packets. As the input arrival rates are further increased, more and more packets are unable to achieve their delay constraints. In Figure 5, we plot the same parameters for worst user in the system.

Since the main objective of this work is a scheduling policy for delay constraints in the range of $1 < D < \infty$ therefore the performance of the scheduler has to be judged based on the number of packets which cannot achieve their delay constraints. We divide the total simulation interval into

subintervals of D time slots. The average achieved rate in each sub-interval should be greater than or equal to the average input arrival rate if all the packets are delivered successfully. If the achieved rate in a sub-interval is greater than or equal to the average input arrival rate, we term it as 100% delay achievement probability. However, if the achieved rate is, for example, 0.9 times the average input arrival rate then this results in 90% delay achievement probability. We plot in Figures 6 and 7 the delay constraint achievement probability versus the percentage of packets whose delay constraints are violated at the input arrival rate of 6 packets/tti/user. The figures are plotted for different values of delay constraints for the worst user and for all the users in the system. We can see that as D increases, more and more packets are able to achieve their respective delay constraints. In case of worst user when $D = 1$, at 100% delay achievement probability almost 80% of the packets are able to achieve their demanded delay constraint. However, when $D = 10$tti, the delay violations are less than 1% which goes to zero for 90% delay achievement probability. It is also evident that maximum improvement is achieved when delay is increased from 1tti to 2tti. In case of worst user, at 100% delay achievement probability only 6% of the packets are unable to achieve their delay constraints when $D = 2$ compared to more than 19% of the packets whose delay constraints are violated when $D = 1$. Therefore, by allowing a small delay tolerance huge performance gains can be made.

Finally in Figure 8 we plot the average output sum rate versus the average sum input arrival rate for $D = 1$ and $D = 10$ tti. It should be noted that the achieved data rates are the same till we reach the average sum input rate of 50 packets/tti. Since we do not consider infinite backlogged queues in our analysis and in the context of strict delay constraints, we drop the packets whose delay deadlines are not achieved thus all we can do is to transmit all the available packets in these queues. However, it is obvious that for lower values of sum input arrival rates there is some power available which is wasted if the user queues are empty and there are no more packets left for transmission. As the sum input arrival rate increase, we can transmit more packets till we reach the point where delay deadlines of the packets start getting violated. If we further increase the sum input arrival rate beyond this point the achieved sum rate becomes a flat curve since system capacity is reached and we cannot transmit more packets. However, for $D = 10$ tti the curve gets flat at input sum arrival rate of 60 packets/tti compared to 50 packets/tti for $D = 1$ tti.

7. Conclusion

In this paper, we have given a two-step solution to the sum-rate maximization problem with strict delay constraints on data transmission in OFDMA system. In the first step, we developed a causal Minimum Rate Scheduler for packet delays in the range of $1 < D < \infty$. The proposed data rates by the scheduler conceals the delay constraints from physical layer resource allocation block. Based on the minimum data rates and limited power budget, we studied the feasibility conditions of our resource allocation problem. We developed efficient algorithms for the feasible and the non-feasible cases. By separating scheduling from resource allocation, we achieved a significant reduction in complexity by solving a series of simple optimization problems. Simulation results revealed that by increasing packet delay constraint higher input arrival rates can be supported. The enhanced performance at higher values of delay constraint is due to better exploitation of time, frequency, and multiuser diversities.

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