

Research Article

Super-Orthogonal Block Codes with Multichannel Equalisation and OFDM in Frequency Selective Fading

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Super-orthogonal block codes in space-time domain (i.e., Super-orthogonal space-time trellis codes (SOSTTCs)) were initially designed for frequency nonselective (FNS) channels but in frequency selective (FS) channels these super-orthogonal block codes suffer performance degradation due to signal interference. To combat the effects of signal interference caused by the frequency selectivity of the fading channel, the authors employ two methods in this paper, namely, multichannel equalization (ME) and orthogonal frequency division multiplexing (OFDM). In spite of the increase complexity of the SOSTTC-ME optimum receiver design scheme, the SOSTTC-ME scheme maintains the same diversity advantage as compared to the SOSTTC scheme in FNS channel. In OFDM environments, the authors consider two forms of the super-orthogonal block codes, namely, super-orthogonal space-time trellis-coded OFDM and super-orthogonal space-frequency trellis-coded OFDM. The simulation results reveal that super-orthogonal space-frequency trellis-coded OFDM outperforms super-orthogonal space-time trellis-coded OFDM under various channel delay spreads.

1. Introduction

The use of channel codes in combination with multiple transmit antennas achieves diversity, but the drawback is loss in bandwidth efficiency. Diversity can be achieved without any sacrifice in bandwidth efficiency, if the channel codes are specifically designed for multiple transmit antennas. Space-time coding is a bandwidth and power efficient method of communication over fading channels. It combines, in its design, channel coding, modulation, transmit diversity, and receive diversity. Space-time codes provide better performance compared to an uncoded system. Some basic space-time coding techniques include layered space-time codes [1], space-time trellis codes (STTCs) [2, 3], space-time block codes (STBCs) [4, 5], and super-orthogonal space-time trellis codes (SOSTTCs) [6, 7]. SOSTTCs are a new class of space-time codes that combine set partitioning and a super set of orthogonal block codes in a systematic way, in order to provide full diversity and improved coding gain when compared with earlier space-time trellis constructions [2–5]. SOSTTCs, in a frequency nonselective (FNS) fading channel, do not only provide a scheme that has an improvement in

coding gain when compared with earlier constructions, but they also solve the problem of systematic design for any rate and number of states. The super-orthogonal block code transmission matrix used in the design of SOSTTCs is given in [6] as

$$A(x_1, x_2, \theta) = \begin{bmatrix} s_1 e^{j\theta} & s_2 \\ -s_2^* e^{j\theta} & s_1^* \end{bmatrix}. \quad (1)$$

For an M -Phase Shift Keying (PSK) modulation with constellation signal represented by $s_i \in e^{j2\pi a/M}$, $i = 1, 2, a = 0, 1, \dots, M - 1$, one can pick $\theta = 2\pi a'/M$, where $a' = 0, 1, \dots, M - 1$. In this case, the resulting transmitted signals of (1) are also members of the M -PSK constellation alphabet and thus no expansion of the constellation signals is required. Since the transmitted signals are from a PSK constellation, the peak-to-average power ratio of the transmitted signals is equal to one. The choice of θ that can be used in (1) for both Binary Phase Shift Keying (BPSK) and Quaternary Phase Shift Keying (QPSK) is given as $0, \pi$ and $0, \pi/2, \pi, 3\pi/2$, respectively.

It should be noted that when $\theta = 0$, (1) becomes the code presented in [4] (i.e., Alamouti code). The construction of

SOSTTCs is based on the expansion of a super-orthogonal block code transmission matrix using a unique method of set partitioning [8]. In [6], the set partitioning method applied to SOSTTCs is explained. These set partitioning methods maximize coding gain without sacrificing data rates.

However, the performance of super-orthogonal block code in space-time domain is based on two fundamental assumptions with regard to the fading channel, which are given as follows:

- (i) frequency nonselective channel, that is, the channel does not have multipath interference;
- (ii) the fading coefficients from each transmit antenna to any receive antenna are independently identically distributed (i.i.d.) random variables—this assumption is valid if the antennas are located far apart from each other (at least $\lambda/2$ separation between antennas).

The first assumption may not be guaranteed in outdoor settings where delay spreads are significantly large (i.e., occurrence of multipath) due to the frequency selectivity of the fading channel. Multipath interference can severely degrade the performance of space-time codes. Space-time codes typically suffer from an irreducible error-floor, both in terms of the frame error-rate and in terms of the bit-error rate [9]. The two main approaches that can be used to enhance the performance of space-time codes in frequency selective fading channels are the following:

- (i) orthogonal frequency division multiplexing, that is, multipath-induced intersymbol interference is reduced by converting the FS fading channel into parallel flat fading subchannels,
- (ii) employing maximum likelihood sequence estimation with multichannel equalization.

In [10], a multichannel equalizer with maximum likelihood sequence estimation was proposed to mitigate the effect of intersymbol interference for STTC in a multipath environment. Optimum receiver design was proposed for the STTC in the multipath environment. The number of states of the optimum receiver for the STTC in a multipath fading channel with L rays was given in [10] as $4^{L-1} * S$, where S is the original state number of the STTC. Alternatively, OFDM can also be used to mitigate the effects of intersymbol interference for space-time codes in multipath fading channels [11, 12]. In [12], the performance of space time trellis-coded OFDM was discussed and compared with Reed Solomon coded OFDM. The scheme in [12] is capable of providing reliable transmission at relatively low SNRs for a variety of power delay profiles, making it a robust solution. In [11], space-time trellis-coded OFDM systems, with no interleavers, over quasistatic FS fading channels were also considered. The performance of the code was analyzed under various channel conditions in terms of the coding gain. The work in [11] points out that the minimum determinant of the space-time-coded OFDM system increases with the maximum tap delay of the channel, thereby increasing coding gain.

The main contributions of this paper are as follows.

- (i) Multichannel equaliser was applied to the super-orthogonal block code in space-time domain and an optimum receiver design was proposed for the code.
- (ii) Coding in OFDM environment of the super-orthogonal block code in space-frequency domain was proposed.
- (iii) The performance comparison of using both ME and OFDM to mitigate the effects of signal interference for super-orthogonal block code in a multipath environment was presented.

The paper is organised as follows. Section 2 presents the system model for super-orthogonal block code in space-time domain designed for frequency nonselective fading channels. Section 3 presents the two main approaches (i.e., ME and OFDM) to mitigate the effects of intersymbol interference for super-orthogonal block codes in FS fading channels. Simulation results are presented in Section 4 and finally conclusions are drawn in Section 5.

2. System Model

A communication system equipped with N_t antennas at the transmitter and N_r antennas at the receiver is considered. The transmitter employs a concatenated coding scheme where a Multiple-Trellis-Coded Modulation (M-TCM) encoder with multiplicity of N_c is used as an outer code and an $N_c \times N_t$ super-orthogonal block code is used as the inner code. The transmitter encodes k_c information bits into $N_c N_t$ symbols (i.e., $N_c \times N_t$ in matrix dimension) corresponding to the edge in the trellis of the space-time code with 2^v states, where v is the memory of the space-time encoder. The encoded symbols are divided into N_t streams, where each stream is linearly modulated and simultaneously transmitted via each antenna using the super-orthogonal block transmission matrix in (1). The rate of this space-time code is defined as $R_c = k_c/N_c$ bits /symbol. For example, let us consider a transmitter that encodes 4 information bits into 4 symbols, that is, $N_t = 2$ and $N_c = 2$. This makes $R_c = 2$ bits/symbol which is the rate for a QPSK constellation. This shows that the transmission scheme employed is a full rate system. The transmission trellises for the two-state and four-state super-orthogonal block code in space-time domain (i.e., SOSTTC) scheme are given in Figure 1, which consist of eight parallel transitions per branch (A_i and B_i are transmission matrices of the form given in (1)) where

$$\begin{aligned} A_0 &\equiv \{(\pm 1, \pm 1, 0), (\pm j, \pm j, 0)\}, \\ A_1 &\equiv \{(\pm 1, \pm j, 0), (\pm j, \pm 1, 0)\}, \\ B_0 &\equiv \{(\pm 1, \pm 1, \pi), (\pm j, \pm j, \pi)\}, \\ B_1 &\equiv \{(\pm 1, \pm j, \pi), (\pm j, \pm 1, \pi)\}. \end{aligned} \quad (2)$$

3. Super-Orthogonal Block-Coded Schemes in FS Fading Channels

3.1. Multichannel Equalization with SOSTTC. To combat the distortive channel effects caused by frequency selectivity of

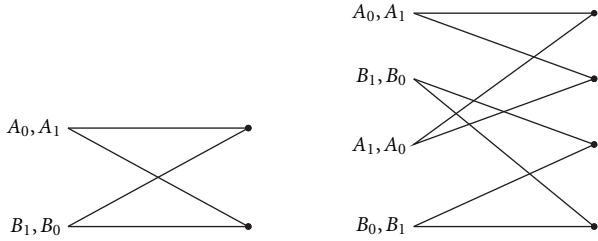


FIGURE 1: Two-State and Four-State 2-bits per symbol SOSTTC.

fading channel in a multiple-input multiple-output (MIMO) scheme, a multichannel equalization is needed. The purpose of the equalisation is to reduce the distortive channel effects as much as possible by maximising the probability of correct decision being made at the receiver. Figure 2 shows the block diagram of a super-orthogonal block coding scheme with multichannel equalisation.

Various equalisation techniques for MIMO schemes that is STBC have been proposed [13, 14]. The solution for multipath interference of STBC as stated in [13] assumes that the space-time coding is done over two large blocks of data symbols instead of just two symbols as in the original proposed scheme (i.e., [4]). At the receiver of the scheme proposed in [13] there is an increase in complexity due to the doubled front-end convolution of the overall system. In [14], a generalisation was proposed for the space time block code structure in [13]. The paper derived a receiver that consists of a frequency domain space time detector followed by a predictive decision feedback equaliser. In this paper, by assuming that the multipath interference of the super-orthogonal block code scheme in FS fading channels is over every two-symbol block as proposed in Asokan and Arslan [15] for [4], the authors design a new equalised trellis for the super-orthogonal block code in space-time domain at the receiver.

Based on the above assumptions and that the $N_r = 1$, the received samples over the t th coded super-orthogonal block transmission can be arranged in matrix form as

$$\begin{aligned} \begin{bmatrix} r_{11}(t) \\ r_{21}(t) \end{bmatrix} &= \begin{bmatrix} s_1(t)e^{j\theta} & s_2(t) \\ -s_2^*(t)e^{j\theta} & s_1^*(t) \end{bmatrix} \cdot \begin{bmatrix} h_{11}(0, t) \\ h_{21}(0, t) \end{bmatrix} \\ &+ \begin{bmatrix} -s_2^*(t-1)e^{j\theta} & s_1^*(t-1) \\ s_1(t)e^{j\theta} & s_2(t) \end{bmatrix} \cdot \begin{bmatrix} h_{11}(1, t) \\ h_{21}(1, t) \end{bmatrix} \quad (3) \\ &+ \cdots + \nu \cdot \begin{bmatrix} h_{11}(L-1, t) \\ h_{21}(L-1, t) \end{bmatrix} + \begin{bmatrix} \eta_{11}(t) \\ \eta_{21}(t) \end{bmatrix}, \end{aligned}$$

where L is the number of channel taps and the channel response $h_{ij}(l, t)$ stands for the l th channel tap at time t from

i th transmit to the j th receive antenna. From (3), one can write ν as

$$\nu = \begin{cases} \begin{bmatrix} s_1\left(\frac{(2t-L+1)}{2}\right)e^{j\theta} & s_2\left(\frac{(2t-L+1)}{2}\right) \\ \left(-s_2\left(\frac{(2t-L+1)}{2}\right)\right)^* e^{j\theta} & \left(s_1\left(\frac{(2t-L+1)}{2}\right)\right)^* \end{bmatrix} & \text{if } L \text{ is odd} \\ \begin{bmatrix} \left(-s_2\left(\frac{(2t-L)}{2}\right)\right)^* e^{j\theta} & \left(s_1\left(\frac{(2t-L)}{2}\right)\right)^* \\ s_1\left(\frac{(2t-L)}{2}\right)e^{j\theta} & s_2\left(\frac{(2t-L)}{2}\right) \end{bmatrix} & \text{if } L \text{ is even.} \end{cases} \quad (4)$$

The noise terms $\eta_{ij}(t)$ in (3) are independent identically distributed complex zero mean Gaussian samples, each with variance of $\sigma^2/2$ per dimension. It is assumed that the channel coefficients are Rayleigh distributed.

At the receiver, a resultant trellis (i.e., equalised trellis) that will take the multipath interference into account is needed for maximum likelihood decoding. As an example of the resultant trellis for the super-orthogonal block codes, the authors consider a scheme with only two rays in each subchannel and the multipath interference that spans two consecutive symbols. The numbers of states in the receiver structures for the two-state and four-state QPSK super-orthogonal block coding system increase to four and eight, respectively. The number of transition paths increases to 64 parallel paths. The resultant code trellis for the receiver structure is given in Figure 3.

The trellises in Figure 3 represent the multichannel equalised decoding trellis for the two-state and the four-state super-orthogonal block coding system in an FS fading channel with multipath interference that spans a two-symbol block. The transition per state contains 64 parallel paths of signal sets. In the trellises A_i (or B_i) - A_j (or B_j) represent the delayed version of two-symbol blocks, A_i (or B_i) affected by the second tap and A_j (or B_j) represent the two-symbol block affected by the first tap (our analysis assumes $L = 2$). By deduction the number of states in the received trellis, when the multipath interference spans two-symbol blocks with L rays, is given by $2 * (L-1) * S$, where S is the original number of states of the super-orthogonal block code in space-time domain.

3.2. Super-Orthogonal Block Codes for OFDM. The OFDM technique transforms an FS fading channel into parallel flat fading sub-channels and eliminates the signal interference caused by multipaths. OFDM can be implemented using inverse fast Fourier transform/fast Fourier transform-based multicarrier modulation and demodulation. The block diagram of a super-orthogonal block transmission in an OFDM environment is shown in Figure 4.

In this paper the authors consider two transmit diversity techniques that are possible for coded-OFDM schemes.

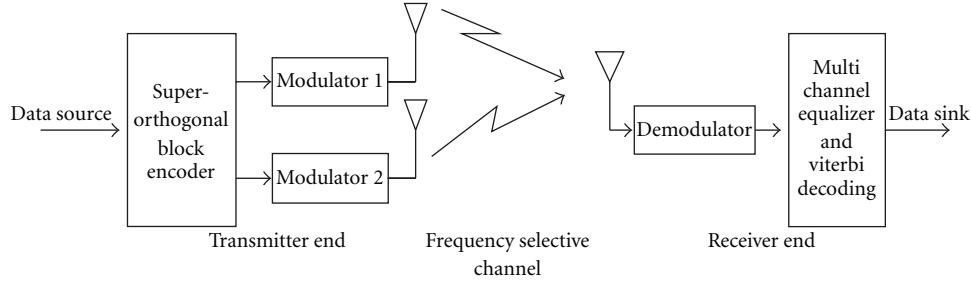


FIGURE 2: Block diagram of a super-orthogonal block coding scheme with multichannel equalisation with $N_t = 2$, $N_r = 1$.

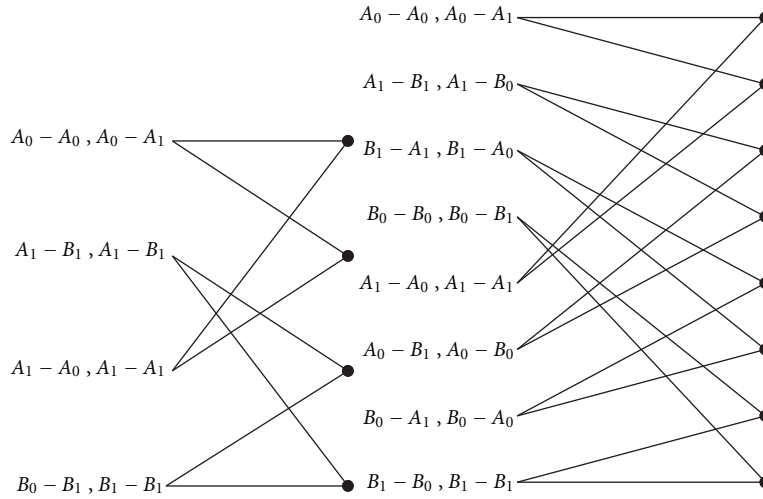


FIGURE 3: Decoding trellis for the Two-state and Four-state super-orthogonal block coding scheme in a frequency selective fading channel.

These are the following

- (i) Space-Time-Coded OFDM Schemes. These schemes are capable of realizing both spatial and Temporal diversity gains in MIMO fading channels [12].
- (ii) Space-Frequency-Coded OFDM schemes. These schemes are capable of realizing both spatial and Frequency diversity gain in multipath MIMO fading channels.

The time domain channel impulse representation between the i th transmit antenna and the j th receive antenna can be modeled as an L -tapped delay line. The channel response at time t with delay τ_s can be expressed as

$$h_{ij}(\tau_s, t) = \sum_{l=0}^{L-1} \hat{h}_{ij}(l, t) \delta\left(\tau_s - \frac{n_l}{N\Delta f}\right), \quad (5)$$

where $\delta(\cdot)$ is the Kronecker delta function, L denotes the number of nonzero taps, $\hat{h}_{ij}(l, t)$ is the complex amplitude of the l -th non-zero tap with delay of $n_l/N\Delta f$, n_l is an integer, and Δf is the tone spacing of the OFDM system. In (5) $\hat{h}_{ij}(l, t)$ is modeled by the wide-sense stationary narrowband complex Gaussian processes with power $E[|\hat{h}_{ij}(l, t)|^2] = \sigma_l^2$, which is normalized as $\sum_{l=0}^{L-1} \sigma_l^2 = 1$.

For an OFDM system with proper cyclic prefix, the channel frequency response is expressed as

$$\begin{aligned} H_{ij}(n) &= \sum_{l=0}^{L-1} \hat{h}_{ij}(l, t) \exp(-j2\pi n(l)/N) \\ &= \mathbf{h}_{ij} \mathbf{w}(n), \end{aligned} \quad (6)$$

where $\mathbf{h}_{ij} = [h_{ij}(0, t), h_{ij}(1, t), h_{ij}(2, t), \dots, h_{ij}(L-1, t)]$ consist of the channel vectors and $\mathbf{w}(n) = [1, \dots, \exp(-j2\pi n(L-2)/N), \exp(-j2\pi n(L-1)/N)]^T$ is the FFT coefficients. The time index t will be ignored in the rest of our analysis since the analysis is done for one OFDM frame.

Using the super-orthogonal block code transmission matrix given in (1) and assuming that the channel frequency response is constant for N_t consecutive symbol intervals, the scheme becomes an SOSTTC-OFDM scheme [16] (the received expression is given in (7)). Also we propose a case where the super-orthogonal block code takes advantage of the spatial and frequency diversity possible in the coded OFDM scheme by assuming that the channel frequency response is identical across the N_t adjacent subcarrier; the scheme becomes super-orthogonal space-frequency trellis-coded-OFDM (SOSFTC-OFDM) scheme (i.e., (8)). The

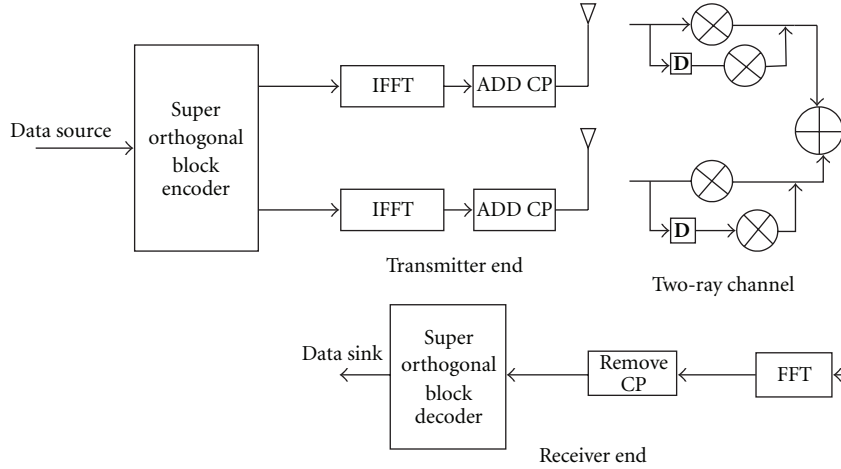


FIGURE 4: Block diagram of a Super-Orthogonal block coding scheme in OFDM environment with $N_t = 2$, $N_r = 1$, $L = 2$ and delay spread of D .

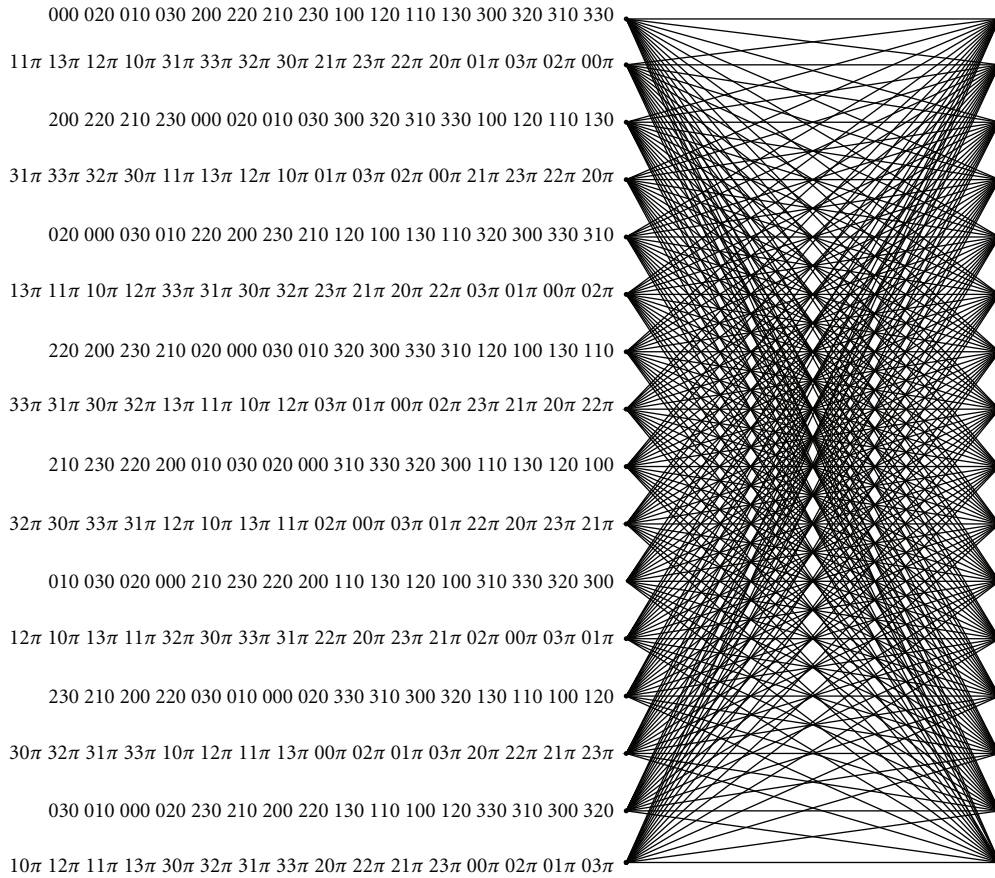


FIGURE 5: Sixteen-state QPSK SOSTTC-OFDM.

received signal at the j th received antenna for an SOSTTC-OFDM scheme ($N_t = 2$) and on the n th subcarrier is given as follows:

$$\begin{aligned} r_j(2n-1) &= s_1(n)e^{j\theta}\mathbf{H}_{1j}(n) + s_2(n)\mathbf{H}_{2j}(n) + \eta_j(2n-1), \\ r_j(2n) &= -s_2^*(n)e^{j\theta}\mathbf{H}_{1j}(n) + s_1^*(n)\mathbf{H}_{2j}(n) + \eta_j(2n). \end{aligned} \quad (7)$$

while the received signal at the j th received antenna for the SOSFTC-OFDM scheme ($N_t = 2$) and on the n th subcarrier is given as

$$\mathbf{R}_j = \mathbf{H}_{1j}\mathbf{S}_1 + \mathbf{H}_{2j}\mathbf{S}_2 + \mathbf{N}_j, \quad (8)$$

where $\mathbf{H}_{ij} = [H_{ij}(1), H_{ij}(2), H_{ij}(3), \dots, H_{ij}(N)]$ consist of channel frequency response vectors $H_{ij}(n)$ from the

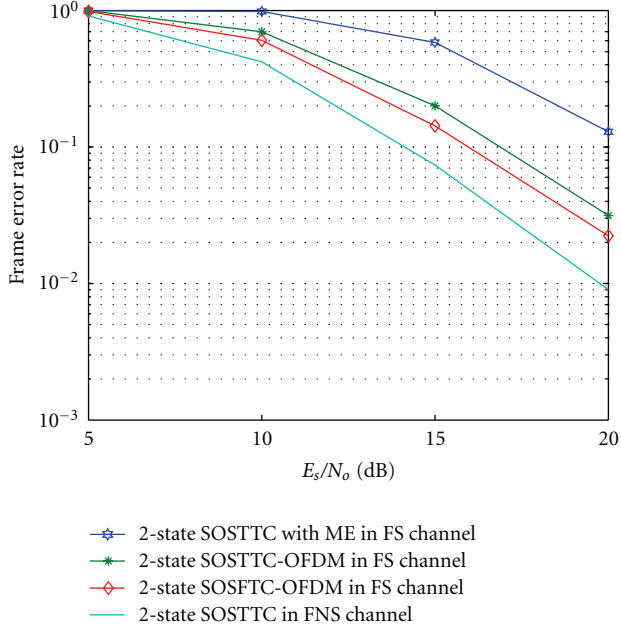


FIGURE 6: FER of Two-state SOSTTC schemes in fading channels.

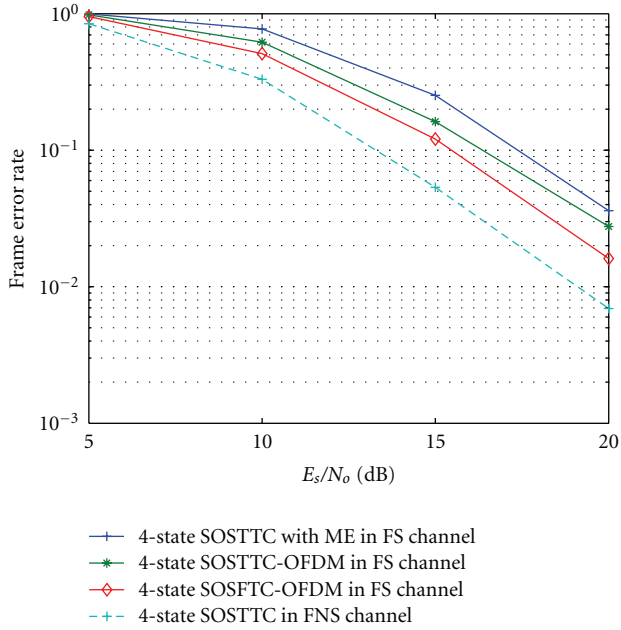


FIGURE 7: FER of Four-state SOSTTC schemes in fading channels.

i th transmit antenna to the j th receive antenna for the n th subcarrier and $\mathbf{N}_j = [\eta_j(1), \eta_j(2), \eta_j(3), \dots, \eta_j(N)]^T$ consists of the noise component $\eta_j(n)$ at the receive antenna j and subcarrier n . The noise components are independently identical complex Gaussian random variables with zero-mean and variance $N_o/2$ per dimension. The super-orthog-

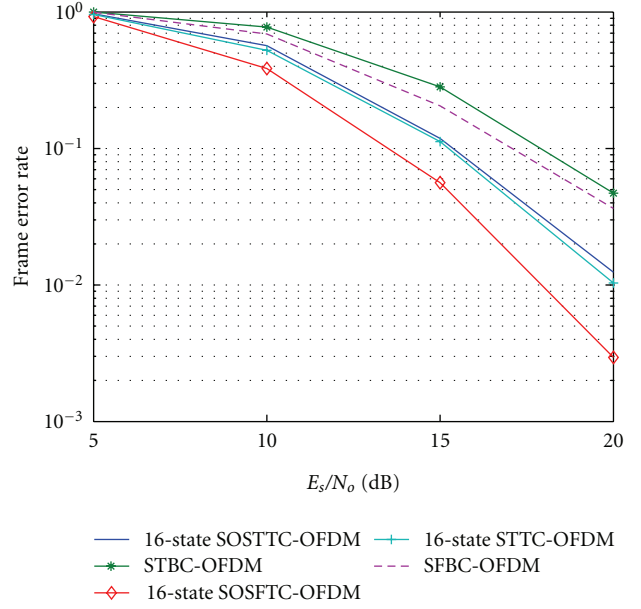


FIGURE 8: FER Performance of Space time-coded schemes with OFDM and 5 microseconds delay spread between the two paths.

onal block codes for the two transmit antennas in (8) are written:

$$\begin{aligned} \mathbf{S}_1 &= [s_1(1), s_1(2), s_1(3), \dots, s_1(N)]^T \\ &= [s(1)e^{j\theta}, -s^*(2)e^{j\theta}, s(3)e^{j\theta}, -s^*(4)e^{j\theta} \\ &\quad \dots, s(N-1)e^{j\theta}, -s^*(N)e^{j\theta}]^T, \end{aligned} \quad (9)$$

$$\begin{aligned} \mathbf{S}_2 &= [s_2(1), s_2(2), s_2(3), \dots, s_2(N)]^T \\ &= [s(2), s^*(1), s(4), s^*(3), \dots, s(N), s^*(N-1)]^T. \end{aligned}$$

The super-orthogonal block codes in space-time domain under FNS fading channel are designed by maximising the pairwise error probability (PEP), which is done by maximising the minimum rank of the codeword sequence matrix (equivalent to the diversity order) and the minimum determinant codeword sequence matrix (equivalent to the coding gain). Also to enumerate the design criteria of the SOSFTC-OFDM scheme, the authors consider the PEP of the scheme. To evaluate the PEP of an SOSFTC-OFDM scheme, that is, the probability of choosing the codeword is $\hat{\mathbf{S}} = [\hat{s}(1), \hat{s}(2), \hat{s}(3), \hat{s}(4), \hat{s}(5), \dots, \hat{s}(N)]$, where $\hat{s}(n) = [\hat{s}_1(n), \hat{s}_2(n)]$, when in fact the codeword $\mathbf{S} = [s(1), s(2), s(3), s(4), s(5), \dots, s(N)]$, where $s(n) = [s_1(n), s_2(n)]$ was transmitted, the maximum likelihood metric corresponding to the correct and the incorrect path will be used. The metric corresponding to the correct path and the incorrect path is given in (10):

$$\begin{aligned} m(\mathbf{R}, \mathbf{S}) &= \|\mathbf{R}_j - (\mathbf{H}_{1j}\mathbf{S}_1 + \mathbf{H}_{2j}\mathbf{S}_2)\|^2, \\ m(\mathbf{R}, \hat{\mathbf{S}}) &= \|\mathbf{R}_j - (\mathbf{H}_{1j}\hat{\mathbf{S}}_1 + \mathbf{H}_{2j}\hat{\mathbf{S}}_2)\|^2. \end{aligned} \quad (10)$$

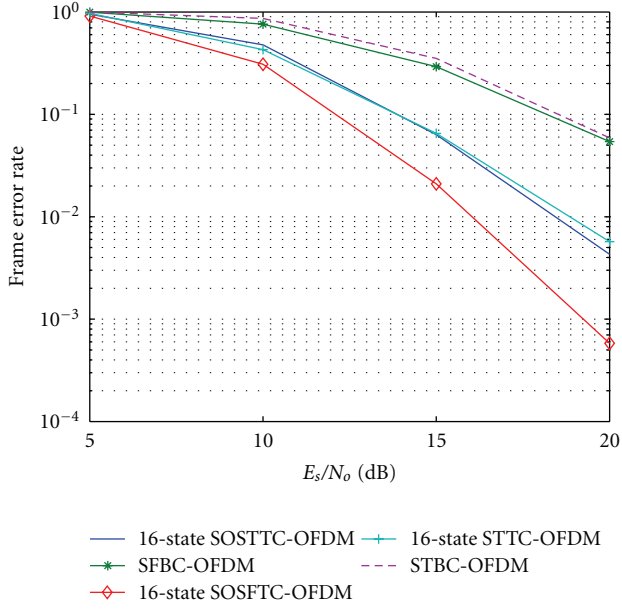


FIGURE 9: FER Performance of Space time-coded schemes with OFDM and 40 microseconds delay spread between the two paths.

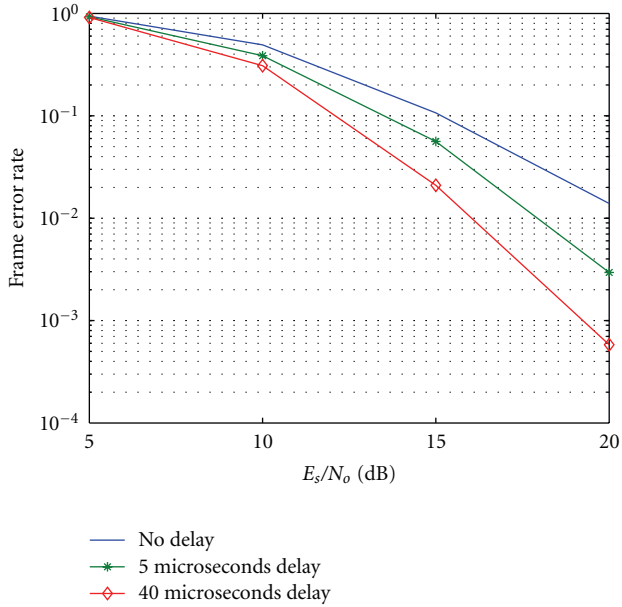


FIGURE 10: Effect delay spreads on Sixteen-state SOSFTC-OFDM system.

The realisation of the PEP over the entire frame length and for a given channel frequency response is given in (11):

$$\begin{aligned} P(\mathbf{S} \rightarrow \hat{\mathbf{S}} | \mathbf{H}) &= \Pr\{m(\mathbf{R}, \mathbf{S}) > m(\mathbf{R}, \hat{\mathbf{S}})\} \\ &= \Pr\{(m(\mathbf{R}, \mathbf{S}) - m(\mathbf{R}, \hat{\mathbf{S}})) > 0\}. \end{aligned} \quad (11)$$

Simplifying (10) and substituting it in (11) gives (12):

$$\begin{aligned} P(\mathbf{S} \rightarrow \hat{\mathbf{S}} | \mathbf{H}) &= \Pr\left\{\|\mathbf{H}_{1j}(\mathbf{S}_1 - \hat{\mathbf{S}}_1)\|^2 + \|\mathbf{H}_{2j}(\mathbf{S}_2 - \hat{\mathbf{S}}_2)\|^2\right\} \\ &= \Pr\left\{\|\mathbf{H}_j \Delta\|^2 > 0\right\}, \end{aligned} \quad (12)$$

where $\mathbf{H}_j = [\mathbf{H}_{1j} \mathbf{H}_{2j}]$, Δ is the block codeword matrix that characterise the SOSFTC-OFDM system, and $\|\cdot\|$ stands for the norm of the matrix element. The expression of Δ is given in (13):

$$\Delta = \begin{bmatrix} \mathbf{S}_1 - \hat{\mathbf{S}}_1 \\ \mathbf{S}_2 - \hat{\mathbf{S}}_2 \end{bmatrix} \quad (13)$$

The conditional PEP given in (12) can be expressed in terms of the Gaussian Q function [17] as

$$P(\mathbf{S} \rightarrow \hat{\mathbf{S}} | \mathbf{H}) = Q\left(\sqrt{\frac{E_s}{2N_o} \sum_{j=1}^{N_r} \mathbf{H}_j \Delta \Delta^H \mathbf{H}_j^H}\right). \quad (14)$$

The function $\Delta \Delta^H$ is a diagonal matrix of the form shown in (15), $(\cdot)^H$ represents the conjugate transpose of the matrix element, and E_s/N_o stands for the symbol SNR:

$$\Delta \Delta^H = \begin{bmatrix} \Delta(1)(\Delta(1))^H & 0 & \dots & 0 \\ 0 & \Delta(2)(\Delta(2))^H & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \Delta(N)(\Delta(N))^H \end{bmatrix}. \quad (15)$$

The diagonal element of (15) and further expansion of \mathbf{H}_j are given in (16) and (17):

$$\Delta(n) = \begin{bmatrix} s_1(n) - \hat{s}_1(n) \\ s_2(n) - \hat{s}_2(n) \end{bmatrix}, \quad (16)$$

$$\begin{aligned} \mathbf{H}_j &= [\mathbf{H}_{1j} \quad \mathbf{H}_{2j}]_{1 \times N_t} \\ &= [\hat{h}_{1j}(0) \dots \hat{h}_{1j}(L-1) \quad \hat{h}_{2j}(0) \dots \hat{h}_{2j}(L-1)]_{1 \times LN_t} \end{aligned}$$

$$\begin{bmatrix} \mathbf{w}(n) & 0 & \dots & 0 \\ 0 & \mathbf{w}(n) & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \mathbf{w}(n) \end{bmatrix}_{LN_t \times N_t}$$

$$\begin{aligned} &= [\mathbf{h}_{1j} \quad \mathbf{h}_{2j}] \mathbf{W}(n) \\ &= \mathbf{h}_j \mathbf{W}(n). \end{aligned} \quad (17)$$

The conditional PEP in (14) can now be written as (18) using the expanded form of the \mathbf{H}_j matrix (i.e., (17)) and the expression of $\Delta(n)$ given in (16):

$$P(\mathbf{S} \rightarrow \hat{\mathbf{S}} | \mathbf{H}) = Q \left(\sqrt{\frac{E_s}{2N_o} \sum_{j=1}^{N_r} \mathbf{h}_j \mathbf{W}(n) \Delta(n) (\Delta(n))^H (\mathbf{W}(n))^H \mathbf{h}_j^H} \right). \quad (18)$$

The Q function is defined by

$$Q = \frac{1}{\sqrt{2\pi}} \int_y^\infty e^{-t^2/2} dt, \quad (19)$$

and by using the inequality $Q(y) \leq 1/2 \exp(-y^2/2)$, $y \geq 0$, the PEP given in (18) can be upper bounded as

$$P(\mathbf{S} \rightarrow \hat{\mathbf{S}} | \mathbf{H}) \leq \exp \left(-\frac{E_s}{4N_o} \sum_{j=1}^{N_r} \mathbf{h}_j \mathbf{W}(n) \Delta(n) (\Delta(n))^H (\mathbf{W}(n))^H \mathbf{h}_j^H \right). \quad (20)$$

The PEP given in (20) can be averaged over all possible channel realisation as

$$P(\mathbf{S} \rightarrow \hat{\mathbf{S}}) \leq E \left\{ \exp \left(-\frac{E_s}{4N_o} \sum_{j=1}^{N_r} \mathbf{h}_j \mathbf{W}(n) \Delta(n) (\Delta(n))^H (\mathbf{W}(n))^H \mathbf{h}_j^H \right) \right\}. \quad (21)$$

The above expression (21) can be simplified further using the results in [18]. For a complex circularly distributed Gaussian random row vector \mathbf{z} with mean μ and covariance matrix $\sigma_z^2 = E[\mathbf{z}\mathbf{z}^*] - \mu\mu^*$, and a Hermitian matrix \mathbf{M} , we have

$$E_z \left[\exp(-\mathbf{z}\mathbf{M}(\mathbf{z}^*)^T) \right] = \frac{\exp[-\mu\mathbf{M}(\mathbf{I} + \sigma_z^2\mathbf{M})^{-1}1((\mu^*)^T)]}{\det(\mathbf{I} + \sigma_z^2\mathbf{M})}, \quad (22)$$

where \mathbf{I} is an identity matrix. Applying (22) in solving (21), (23) is obtained. Knowing that $\mathbf{z} = [\mathbf{h}_{1j}\mathbf{h}_{2j}]$, $M = -E_s/4N_o \mathbf{W}(n) \Delta(n) (\Delta(n))^H (\mathbf{W}(n))^H$ (It should be noted that since $\mathbf{W}(n) \Delta(n) (\Delta(n))^H (\mathbf{W}(n))^H$ is a diagonal matrix, \mathbf{M} is a Hermitian matrix, i.e., $\mathbf{M} = \mathbf{M}^T$), $\mu = 0$ ($[\mathbf{h}_{1j}\mathbf{h}_{2j}]$ has Rayleigh distribution), and $\sigma_z^2 = \sigma_{[\mathbf{h}_{1j}\mathbf{h}_{2j}]}^2 = \mathbf{I}_{LN_r}$:

$$P(\mathbf{S} \rightarrow \hat{\mathbf{S}}) \leq \prod_{j=1}^{N_r} \frac{1}{\det \left[(E_s/4N_o) \sum_{j=1}^{N_r} \mathbf{h}_j \mathbf{W}(n) \Delta(n) (\Delta(n))^H (\mathbf{W}(n))^H \mathbf{h}_j^H \right]}. \quad (23)$$

At high SNR (i.e., $E_s/N_o \gg 1$), the identity matrix at the denominator of (23) may be ignored and PEP upper bound

averaged over all possible channel realisations is derived as follows:

$$P(\mathbf{S} \rightarrow \hat{\mathbf{S}}) \leq \prod_{j=1}^{N_r} \frac{1}{\det \left[(E_s/4N_o) \sum_{j=1}^{N_r} \mathbf{h}_j \mathbf{W}(n) \Delta(n) (\Delta(n))^H (\mathbf{W}(n))^H \mathbf{h}_j^H \right]} \leq \left[\frac{E_s}{4N_o} \right]^{-\Omega N_r} \left[\prod_{k=1}^{\Omega} \lambda_k \right]^{-N_r}, \quad (24)$$

where Ω is the rank of matrix $\mathbf{W}(n) \Delta(n) (\Delta(n))^H (\mathbf{W}(n))^H$ and $\lambda_k, k \in (1, 2, \dots, \Omega)$ are the set of nonzero eigenvalues of matrix $\mathbf{W}(n) \Delta(n) (\Delta(n))^H (\mathbf{W}(n))^H$. From (24), the design criteria for an SOSFTC-OFDM scheme under the assumption of asymptotically high SNRs should be based on the rank and eigenvalue criterion. The rank criterion optimises the diversity of the SOSFTC-OFDM scheme while the eigenvalue criterion optimises the coding gain of the SOSFTC-OFDM scheme. The rank criterion is to maximise the minimum rank of $\mathbf{W}(n) \Delta(n) (\Delta(n))^H (\mathbf{W}(n))^H$ for any codeword \mathbf{S} and $\hat{\mathbf{S}}$, and the eigenvalue criterion is to maximise the minimum product of the nonzero eigenvalues of $\mathbf{W}(n) \Delta(n) (\Delta(n))^H (\mathbf{W}(n))^H$. As a comparison of both SOSTTC-OFDM and SOSFTC-OFDM systems we use a 16-state trellis (given in Figure 5) designed in [16] that maximises the space-frequency diversity and coding gain and minimises the number of parallel path for the SOSTTC-OFDM scheme. The QPSK symbols (x_1 and x_2) 0, 1, 2, and 3 correspond to the QPSK signal constellations and the value of the rotation angle is denoted by 0 and π .

4. Performance Results

Simulation results are shown to demonstrate the frame error rate (FER) performance of super-orthogonal block-coded schemes in both OFDM and multichannel equalisation environments. The wireless channels with two transmit antennas and one receive antenna are assumed to be quasistatic frequency selective Rayleigh fading channels with an average power of unity. The total power of the transmitted coded symbol was normalized to unity and the authors assumed an equal-power, two-path channel impulse response (CIR). The maximum Doppler frequency was 200 Hz. The entire multipath channel undergoes independent Rayleigh fading and the receiver is assumed to have perfect knowledge of the channel state information. The super-orthogonal block coding schemes with OFDM (i.e., SOSTTC-OFDM and SOSFTC-OFDM) are assumed to have a bandwidth of 1 MHz and 128 OFDM subcarriers (i.e., for SOSTTC-OFDM, the frame length equals 512 bits while for SOSFTC-OFDM, the frame length equals 256 bits), and with multichannel equalisation, the system is assumed to have 512 bits per frame (QPSK modulation assumed for all simulations). Cyclix prefixes that are equal to or greater than the delay spread of the channel are used for the OFDM-based schemes, to eliminate intersymbol interference.

Figures 6 and 7 show FER of two-state and four-state super-orthogonal block coding schemes in both FS and FNS fading channels when both OFDM and ME are employed for the FS fading channel scenario. From Figures 6 and 7 we can see that the two-state and four-state super-orthogonal block coding schemes with ME in FS channels achieve the same diversity order (i.e., slope of the error rate curve) compared with the scheme in the FNS fading scenario, although the scheme suffers some coding gain loss. The simulation results in Figures 6 and 7 also show that, in an FS channel, both the two-state and four-state SOSFTC-OFDM schemes outperform the two-state and four-state SOSTTC-OFDM schemes. However, the SOSTTC system in an FNS channel outperforms them all. The performance degradation in Figures 6 and 7 for the SOSTTC with ME in an FS fading channel can be attributed to the increase in parallel path transitions per state of the scheme. If one assumes that all the 64 transitions per branch in the decoding trellis (i.e., Figure 3) are equally likely to be decoded, the probability of correctly decoding a transmitter codeword per state is equal to $1/64 \times 1/64 = 1/4096$ while the probability of decoding a transmitted codeword in the scheme under the FNS fading assumption is $1/8 \times 1/8 = 1/64$. Hence the probability of decoding the transmitted codewords correctly is greater in the FNS fading case compared to the FS fading case. Although there is an increase in the number of decoder trellis states for super-orthogonal block coding scheme with ME in FS channels, compared with SOSTTC in an FNS channel, the overall probability of decoding correctly is still lower. This accounts for the performance loss obtained in terms of the coding advantage (i.e., shift in the error curve upward). The same argument goes for the four-state scenario in Figure 7. It should be noted that neither the two-state nor the four-state SOSTTC-OFDM schemes in FS channels are optimum, as the presence of parallel transitions degrades the code performance in the FS fading environment. This is due to the fact that they do not exploit the diversity order possible in such scenarios which is why the two-state SOSTTC in a FNS channel outperforms both of them. In Figures 8 and 9, the FER performance of, respectively, the sixteen-state super-orthogonal block coding OFDM schemes (i.e., SOSTTC-OFDM and SOSFTC-OFDM systems) sixteen-state STTC-OFDM and STBC OFDM schemes (i.e. SFBC and STBC OFDM systems) for $5 \mu\text{s}$ and $40 \mu\text{s}$ delay spreads between the two paths is shown. In both graphs the super-orthogonal space-frequency trellis-coded OFDM outperforms super-orthogonal space-time trellis coded OFDM under the two-channel delay spread scenario. It should be noted from Figure 10 that, for coded SOSFTC-OFDM schemes, a higher delay spread results in better performance.

5. Conclusion

The paper shows the performance of super-orthogonal block coding schemes in fading channels, that is, FS and FNS fading channels. The receiver structure of an SOSTTC in an FS channel is given so that multichannel equalisation is used to mitigate the effects of multipath interference.

New decoding trellises for two-state and four-state coding schemes are designed. The formula for the number of states of the SOSTTC in FS channels with ME equalisation was deduced as a function of the number of divergent paths per state, the multipath rays, and the original number of states of the super-orthogonal block coding scheme. The simulation results proved that although the code was designed for flat fading channels, it provides at least the same diversity advantage when applied to FS Rayleigh fading channels.

To demonstrate the performance of the super-orthogonal block coding schemes in OFDM environment (i.e, SOSTTC-OFDM, SOSFTC-OFDM) and the STTC-OFDM, trellises are used that have no parallel paths between transitions. FER performance shows that the SOSFTC-OFDM scheme outperforms the SOSTTC-OFDM scheme, the STBC-OFDM scheme, and the STTC-OFDM scheme for both $5 \mu\text{s}$ and $40 \mu\text{s}$ delay spread scenarios. The results also show that an increase in coding gain is obtained when there is an increase in the delay spread of the channel.

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