

## Research Article

# Channel Estimation for Two-Way Relay OFDM Networks

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We consider the channel estimation for two-way relay OFDM networks. We proposed an LS-based channel estimation algorithm under block-based training schemes. Based on the mean square error (MSE) criterion, the condition and the design method for optimal training sequences are discussed. To reduce peak-to-average power ratio (PAPR), a special sequence called Zadoff-Chu sequence is employed to design the optimal training sequence, which can achieve both minimum MSE performance and excellent PAPR performance.

## 1. Introduction

Two-way communication is a popular type of modern communications, where two source terminals simultaneously communicate. The two-way channel was first considered by Shannon, who derived inner and outer bounds on the capacity region [1]. Recently, the two-way relay networks (TWRNs) have attracted a great deal of research interest [2–13], due to its potential application in cellular networks and peer-to-peer networks. Both amplify-and-forward (AF) and decode-and-forward (DF) protocols developed for one-way relay channels are extended to the half-duplex additive white Gaussian noise (AWGN) two-way relay channel (TWRC) in [2] and the general full-duplex discrete TWRC in [3]. Furthermore, physical layer network coding is considered in [5, 6] for AWGN TWRC. Cui et al. have studied a differential network coding at the physical layer in [7]. The optimized relaying strategies for TWRC have been proposed in [8]. Information-theoretic results on TWRN were also presented in [12, 13].

However, many studies have focused on narrowband systems and flat fading environments. Since broadband transmission, such as orthogonal frequency division multiplexing (OFDM) or single carrier cyclic prefix (SCCP), is likely to be a key element in future wireless communication systems, two-way relay techniques should be investigated for such systems. Compared to the narrowband case, the

problem at hand is different due to the increase in degrees of freedom brought by OFDM or SCCP. In [14], two-way relaying over parallel tones of OFDM systems is investigated. The throughput of TWRN is discussed for high-speed 802.11n WLANs in [15]. The-Hanh Pham extended two-way relay communication to the frequency selective fading environment where SCCP was employed in [16].

All the existing work has assumed perfect channel state information (CSI) at both the relay node and/or the two terminals. However, under the assumption of coherent detection, the fading channel coefficients need to be first estimated and then used in the detection process. The quality of channel estimates inevitably affects the overall performance of relay-assisted transmission and might become a performance limiting factor [17]. Although there have been some discussions on the channel estimation for one-way relay networks [18–20], little work has been done for the TWRN. Recently, Gao et al. have studied the maximum likelihood (ML) and the linear maximum SNR (LMSNR) channel estimation for two-way AF relay networks over the flat fading channels in [21], and pointed out that there exist many challenges since the channel estimation is required not only for data detection but also for self-cancellation at the two terminals. The optimal training sequence has been designed to minimize the mean square error of the channel estimation for two-way SCCP systems according to zero-forcing criterion over frequency selective fading channels

in [16]. Two different types of training based on pilot-tone and block, respectively, were proposed to develop their corresponding channel estimation algorithms for OFDM modulated TWRN in [22].

In this work, we consider the channel estimation for two-way relay OFDM networks. Due to the nature of the signaling, we estimate two composite channels instead of separately estimating the two links from the source nodes to the relay node. Based on the mean square error (MSE) criterion, a design method for training sequences is proposed. Different from [22], to reduce peak-to-average power ratio (PAPR), a special sequence called Zadoff-Chu sequence is employed to design the optimal training sequence for TWRN, which can achieve the same minimum MSE performance as the training sequences in [22] with better PAPR performance.

This paper is organized as follows. Section 2 introduces the two-way relay OFDM system. Least-square channel estimation is developed in Section 3, and the optimal training sequence to reduce PAPR is also designed in this section. Section 4 discusses the numerical results. Finally, Section 5 concludes the paper.

*Notations.* The capital bold letters denote matrices and the small bold letters denote row/column vectors. Transpose, Hermitian transpose of a vector/matrix are denoted by  $(\cdot)^T$  and  $(\cdot)^H$ , respectively. The identity matrix of size  $N$  is denoted by  $I_N$ .  $0_{m,n}$  stands for a zero matrix of size  $m \times n$ . For a matrix  $\mathbf{B}$ ,  $[\mathbf{B}]_{m,n}$  is the  $(m, n)$ th element of  $\mathbf{B}$ .  $\text{diag}(\mathbf{b})$  is a diagonal matrix whose diagonal entries are from vector  $\mathbf{b}$ . The Hadamard product and convolution of two vectors  $\mathbf{a}$  and  $\mathbf{b}$  are denoted as  $\mathbf{a} \circ \mathbf{b}$  and  $\mathbf{a} \otimes \mathbf{b}$ , respectively.  $E\{\cdot\}$  denotes expectation operation.

## 2. Two-Way Relay OFDM Model

Consider a two-way relay network where the two source nodes, Tx1 and Tx2, exchange information relying on the help of a relay node R. The relay node and the two source nodes are assumed to equip single antenna each. Here, we employ the two-way relay protocol the same way as proposed in [10]. The bidirectional communication is performed slot-wise, while one time-slot is divided into two phases of equal duration, namely, the multiple access (MAC) and broadcast (BC) phase. During the MAC phase, both source nodes, Tx1 and Tx2, send one signal frame to the relay node R whereas during the BC phase, R processes the received signals and broadcasts them to Tx1 and Tx2. We assume perfect synchronization for both transmission phases.

For a given time-slot, the fading channels between Tx1 and Tx2 to R are assumed to be quasistatic frequency selective fading, that is, they do not change within one time-slot of transmission but can vary from one time-slot to another independently. Let  $\mathbf{h}_1 = [h_1(0), \dots, h_1(L)]^T$  and  $\mathbf{h}_2 = [h_2(0), \dots, h_2(L)]^T$  be the discrete-time baseband equivalent impulse response vectors of the frequency selective fading channels between Tx1 and Tx2 to R, respectively, where  $L$  represents the number of the taps of the corresponding channel. The channel impulse response includes the effects of

transmit receive filters, physical multipath, and relative delays among antennas. Each element of  $\mathbf{h}_1$  or  $\mathbf{h}_2$  is modeled as a zero-mean complex Gaussian random variable with variance  $\sigma_{i,l}^2$ ,  $i = 1, 2$ ,  $l = 0, \dots, L$ . These elements are also assumed to be independent of one another. The time-division duplexing (TDD) is a generally adopted assumption for TWRN, where the channels can be considered reciprocal so that the channel from R to Tx1 is still  $\mathbf{h}_1$  and the channel from R to Tx2 is still  $\mathbf{h}_2$ . The frequency-domain channel coefficient matrix is  $\mathbf{H}_i = \text{diag}\{H_i(0), \dots, H_i(N-1)\}$ ,  $i = 1, 2$  where  $H_i(n) = \sum_{l=0}^L h_i(l)e^{-j2\pi nl/N}$  is the channel frequency response on the  $n$ th subcarrier and  $N$  is the number of subcarriers.

For a given time-slot, the signal frame contains multiple OFDM blocks, while each OFDM block contains information symbols and a cyclic prefix (CP) of length  $L_{CP}$ . The length of the CP,  $L_{CP}$ , is greater than or equal to the channel memory to avoid the interblock interference (IBI),  $L_{CP} \geq L$ .

At the MAC phase, the input data bits are first mapped to complex symbols drawn from a signal constellation such as phase shift keying (PSK) or quadrature amplitude modulation (QAM). We use  $\mathbf{s}_1^k = [s_1^k(0), \dots, s_1^k(N-1)]^T$  and  $\mathbf{s}_2^k = [s_2^k(0), \dots, s_2^k(N-1)]^T$  to denote the  $k$ th frequency-domain symbol vectors transmitted from Tx1 and Tx2, respectively. The cyclic prefix (CP) is added to the top of each signal vector after taking IFFT, respectively. The power constraints of the transmission is  $E\{\mathbf{s}_1^{kH} \mathbf{s}_1^k\} = E\{\mathbf{s}_2^{kH} \mathbf{s}_2^k\} = P$ , where  $P$  is the average transmitting power of Tx1 and Tx2 without loss of generality.

These signal frames are transmitted simultaneously from Tx1 and Tx2 to R. At R, the received signals associated with the CP portion are discarded first. Then the remaining received signals are scaled by a real factor  $\alpha$  to keep the average power of R to be  $P_r$ . The resultant signals are CP-added and broadcasted to Tx1 and Tx2 during the second phase.

The  $k$ th received signal vector at R after the MAC phase is

$$\tilde{\mathbf{r}}_1^k = \tilde{\mathbf{H}}_1 \mathbf{F}^H \mathbf{s}_1^k + \tilde{\mathbf{H}}_2 \mathbf{F}^H \mathbf{s}_2^k + \tilde{\mathbf{w}}_1^k, \quad (1)$$

where  $\tilde{\mathbf{H}}_1$  and  $\tilde{\mathbf{H}}_2$  are two  $N \times N$  circulant matrices with  $[\mathbf{h}_1^T, \mathbf{0}_{1 \times (N-L-1)}]^T$  and  $[\mathbf{h}_2^T, \mathbf{0}_{1 \times (N-L-1)}]^T$  as their first columns, respectively;  $\mathbf{F}$  is the unitary discrete Fourier transform matrix with  $[\mathbf{F}]_{m,n} = (1/\sqrt{N})e^{-j2\pi mn/N}$ ;  $\tilde{\mathbf{w}}_1^k$  is an additive white Gaussian noise (AWGN) vector with zero mean and covariance matrix  $E\{\tilde{\mathbf{w}}_1^{kH} \tilde{\mathbf{w}}_1^k\} = \sigma_w^2 I_N$ . The received signal vector in (1) is then amplified by a real coefficient which is given by

$$\alpha = \sqrt{\frac{P_r}{\left(\sum_{l=0}^L \sigma_{1,l}^2 + \sum_{l=0}^L \sigma_{2,l}^2\right)P + \sigma_w^2}}. \quad (2)$$

The last  $L_{CP}$  components of the vector are appended to the top of itself and the resultant vector is broadcasted to both Tx1 and Tx2. Without loss of generality, we only consider the channel estimation problem at Tx1. A similar operation can be applied at Tx2.

The  $k$ th received signal vector at Tx1 after removing CP and taking FFT can be written as follows:

$$\begin{aligned} \mathbf{r}_2^k &= \alpha \mathbf{F} \tilde{\mathbf{H}}_1 \tilde{\mathbf{r}}_1^k + \mathbf{F} \tilde{\mathbf{w}}_2^k \\ &= \alpha \tilde{\mathbf{F}} \tilde{\mathbf{H}}_1 \tilde{\mathbf{H}}_1^H \mathbf{F}^H \mathbf{s}_1^k + \alpha \tilde{\mathbf{F}} \tilde{\mathbf{H}}_1 \tilde{\mathbf{H}}_2^H \mathbf{F}^H \mathbf{s}_1^k + \mathbf{w}_2^k, \end{aligned} \quad (3)$$

where  $\mathbf{w}_2^k = \alpha \tilde{\mathbf{F}} \tilde{\mathbf{H}}_1 \tilde{\mathbf{w}}_1^k + \mathbf{F} \tilde{\mathbf{w}}_2^k$ ,  $\tilde{\mathbf{w}}_2^k$  is an AWGN vector with zero mean and covariance matrix  $E\{\tilde{\mathbf{w}}_2^k \tilde{\mathbf{w}}_2^{kH}\} = \sigma_w^2 \mathbf{I}_N$ . Based on the DFT theory, with a condition of  $(2L+1) \leq N$ , let  $\mathbf{h} = \mathbf{h}_1 \otimes \mathbf{h}_1 = [h(0), h(1), \dots, h(2L)]^T$  and  $\mathbf{g} = \mathbf{h}_1 \otimes \mathbf{h}_2 = [g(0), g(1), \dots, g(2L)]^T$ , we obtain that

$$\mathbf{r}_2^k = \alpha \text{diag}(\mathbf{s}_1^k) \mathbf{F}_L \mathbf{h} + \alpha \text{diag}(\mathbf{s}_2^k) \mathbf{F}_L \mathbf{g} + \mathbf{w}_2^k, \quad (4)$$

where  $\mathbf{F}_L$  is the first  $2L+1$  columns of  $\mathbf{F}$ , and  $\mathbf{w}_2^k$  is a AWGN vector with zero mean and covariance matrix  $E\{\mathbf{w}_2^k \mathbf{w}_2^{kH}\} = \sigma_w^2 (\alpha^2 (\sum_{l=0}^L \sigma_{1,l}^2) + 1) \mathbf{I}_N$ .

Equation (4) can be written in another form as

$$\begin{aligned} \mathbf{r}_2^k &= \begin{bmatrix} \alpha \text{diag}(\mathbf{s}_1^k) \mathbf{F}_L & \alpha \text{diag}(\mathbf{s}_2^k) \mathbf{F}_L \end{bmatrix} \begin{bmatrix} \mathbf{h} \\ \mathbf{g} \end{bmatrix} + \mathbf{w}_2^k \\ &= \mathbf{S}^k \boldsymbol{\theta} + \mathbf{w}_2^k, \end{aligned} \quad (5)$$

where  $\mathbf{S}^k = \alpha [\text{diag}(\mathbf{s}_1^k) \mathbf{F}_L \quad \text{diag}(\mathbf{s}_2^k) \mathbf{F}_L]$  and  $\boldsymbol{\theta} = [\mathbf{h}^T \mathbf{g}^T]^T$ .

In this paper, we use all the carriers in one or more OFDM blocks for channel estimation normally happening at the start of the transmission, where these OFDM blocks are known as the training sequence. In the next section, without loss of generality, we use one OFDM block as training to estimate  $\boldsymbol{\theta} = [\mathbf{h}^T \mathbf{g}^T]^T$ . For the notational convenience, we omit the time index  $k$  in the following.

### 3. Least-Square Channel Estimation and Optimal Training Sequence Design

From (5), the least-square estimation of the composite channel  $\boldsymbol{\theta}$  is given by

$$\hat{\boldsymbol{\theta}} \triangleq (\mathbf{S}^H \mathbf{S})^{-1} \mathbf{S}^H \mathbf{r}_2 = \boldsymbol{\theta} + (\mathbf{S}^H \mathbf{S})^{-1} \mathbf{S}^H \mathbf{w}_2'. \quad (6)$$

The MSE of the estimation is defined as

$$\text{MSE} \triangleq \frac{1}{2(2L+1)} E\left\{(\hat{\boldsymbol{\theta}} - \boldsymbol{\theta})^H (\hat{\boldsymbol{\theta}} - \boldsymbol{\theta})\right\}. \quad (7)$$

Substituting (6) to (7), we can obtain that

$$\begin{aligned} \text{MSE} &= \frac{1}{2(2L+1)} \text{tr}\left\{(\mathbf{S}^H \mathbf{S})^{-1} \mathbf{S}^H E\{\mathbf{w}_2' \mathbf{w}_2'^H\} \left((\mathbf{S}^H \mathbf{S})^{-1} \mathbf{S}^H\right)^H\right\} \\ &= \frac{\sigma_w^2 (\alpha^2 (\sum_{l=0}^L \sigma_{1,l}^2) + 1)}{2(2L+1)} \text{tr}\left\{(\mathbf{S}^H \mathbf{S})^{-1}\right\}. \end{aligned} \quad (8)$$

It is clear from (8) that minimizing MSE is equivalent to minimizing  $Q \triangleq \text{tr}\{(\mathbf{S}^H \mathbf{S})^{-1}\}$ . From the above, we design the training sequences  $\mathbf{s}_1$  and  $\mathbf{s}_2$  so that  $Q$  is minimized.

Let  $\mathbf{A} = (\mathbf{S}^H \mathbf{S})^{-1}$ , using (5), we can obtain that

$$\mathbf{A} = \frac{1}{\alpha^2} \underbrace{\begin{pmatrix} \mathbf{F}_L^H \text{diag}(\mathbf{s}_1^H \odot \mathbf{s}_1) \mathbf{F}_L & \mathbf{F}_L^H \text{diag}(\mathbf{s}_1^H \odot \mathbf{s}_2) \mathbf{F}_L \\ \mathbf{F}_L^H \text{diag}(\mathbf{s}_2^H \odot \mathbf{s}_1) \mathbf{F}_L & \mathbf{F}_L^H \text{diag}(\mathbf{s}_2^H \odot \mathbf{s}_2) \mathbf{F}_L \end{pmatrix}^{-1}}_{\mathbf{B}}. \quad (9)$$

For a  $2(2L+1) \times 2(2L+1)$  positive definite matrix  $\mathbf{A}$ , we have

$$Q = \text{tr}\{\mathbf{A}\} \geq \frac{1}{\alpha^2} \sum_{i=1}^{2(2L+1)} [\mathbf{B}]_{i,i}, \quad (10)$$

where the equality holds if and only if  $\mathbf{A}$  is diagonal. Applying the Cauchy-Schwartz inequality on the RHS of (10), we further obtain

$$Q \geq \frac{1}{\alpha^2} (2(2L+1))^{2(2L+1)} \sqrt{\prod_{i=1}^{2(2L+1)} [\mathbf{B}]_{i,i}} \quad (11)$$

where equality holds if and only if  $[\mathbf{B}]_{i,i}$ 's are equal.

From (11), we get that to achieve minimum MSE, the training signal vectors  $\mathbf{s}_1$  and  $\mathbf{s}_2$  must be designed to meet the following conditions:

- (1)  $\mathbf{F}_L^H \text{diag}(\mathbf{s}_2^H \odot \mathbf{s}_1) \mathbf{F}_L = \mathbf{F}_L^H \text{diag}(\mathbf{s}_1^H \odot \mathbf{s}_2) \mathbf{F}_L = \mathbf{0}_{2L+1}$ ;
- (2)  $\mathbf{F}_L^H \text{diag}(\mathbf{s}_1^H \odot \mathbf{s}_1) \mathbf{F}_L$  and  $\mathbf{F}_L^H \text{diag}(\mathbf{s}_2^H \odot \mathbf{s}_2) \mathbf{F}_L$

are two diagonal matrices with equal diagonal elements.

Based on conditions (1) and (2), we can achieve the minimum MSE:

$$\text{MSE}_{\min} = \frac{\sigma_w^2 (\alpha^2 (\sum_{l=0}^L \sigma_{1,l}^2) + 1)}{\alpha^2} [\mathbf{B}]_{p,p}, \quad (12)$$

where the arbitrary  $p$  belongs to the interval  $\{1, \dots, 2(2L+1)\}$ .

To meet conditions (1) and (2), we can design different kinds of the training  $\mathbf{s}_1$  and  $\mathbf{s}_2$ . For instance, we can orthogonally design  $\mathbf{s}_1$  and  $\mathbf{s}_2$  according to [22]

$$s_1(i) = 1, \quad s_2(i) = e^{j2\pi i w/N}, \quad i = 0, \dots, N-1, \quad (13)$$

where  $w \in \{2L+1, \dots, N-2L-1\}$ . Obviously, the training  $\mathbf{s}_1$  and  $\mathbf{s}_2$  according to (13) can meet conditions (1) and (2), and achieve the minimum MSE in (12). However, the training  $\mathbf{s}_1$  and  $\mathbf{s}_2$  according to (13) may have high peak-to-average power ratio (PAPR),  $\text{PAPR} = \max_{n=0, \dots, N-1} [\tilde{s}_i(n)] / E[\tilde{s}_i(n)] = N$ . The high PAPR brings signal distortion in the nonlinear region of high-power amplifier (HPA), and the signal distortion induces the degradation of the detection performance. Here, the high PAPR of the training sequence will result in channel estimation errors.

To reduce PAPR of the training sequence, the time-domain signals should have constant magnitude. In [23], a special sequence, called Zadoff-Chu sequence, was proposed. All elements of this sequence have the same magnitude in both time and frequency domain. In this paper, we can

design the optimal training sequence based on the Zadoff-Chu sequence. The general form of a Zadoff-Chu sequence of length  $M$  is given as follows:

$$m(n) = \begin{cases} e^{-j\pi Un(n+2d)/M}, & n = 0, \dots, M-1; M \text{ is even,} \\ e^{-j\pi Un(n+1+2d)/M}, & n = 0, \dots, M-1; M \text{ is odd,} \end{cases} \quad (14)$$

where  $d$  is an integer and  $U$  is an integer relatively prime to  $M$ .

Based on a Zadoff-Chu sequence  $\mathbf{m} = [m(0), \dots, m(N/2 - 1)]^T$  of length  $N/2$ , we repeat  $\mathbf{m}$  twice to construct  $\tilde{\mathbf{s}}_1 = [\mathbf{m}^T, \mathbf{m}^T]^T$ , and then element-wise multiply  $\tilde{\mathbf{s}}_1$  with  $\mathbf{e}_1$  to construct  $\tilde{\mathbf{s}}_2 = \tilde{\mathbf{s}}_1 \odot \mathbf{e}_1$ , where  $\mathbf{e}_1 = [1, e^{j2\pi/N}, \dots, e^{j2\pi(N-1)/N}]^T$ . So, the frequency transform  $\mathbf{s}_i = [s_i(0), \dots, s_i(N-1)]^T$  of  $\tilde{\mathbf{s}}_i, i = 1, 2$ , can be determined by

$$s_i(k) = \begin{cases} \sqrt{2}m'((k-i)/2), & k = I_i, \\ 0, & \text{else,} \end{cases} \quad (15)$$

where  $I_i = \{i-1, i+1, \dots, i+N-3\}$ , and  $m'(k) = \sum_{n=0}^{N/2-1} m(n)e^{-j2\pi nk/N}$  is the frequency transform of  $\mathbf{m}$ .

Based on (15), and  $I_1 \cap I_2 = \emptyset$ , we can obtain that  $\text{diag}(\mathbf{s}_2^H \odot \mathbf{s}_1) = \mathbf{0}$ , therefore condition (1) can be met.

And

$$\begin{aligned} \Pi_i &= \mathbf{F}_L^H \text{diag}(\mathbf{s}_i^H \odot \mathbf{s}_i) \mathbf{F}_L \\ &= \mathbf{F}_L^H \text{diag}\left(\left[|s_i(0)|^2, \dots, |s_i(N-1)|^2\right]^T\right) \mathbf{F}_L, \end{aligned} \quad (16)$$

where  $|s_i(k)|^2 = 2|m'((k-i)/2)|^2 = 2$  if  $k \in I_i$  and  $|s_i(k)|^2 = 0$  if  $k \notin I_i$ .

So,  $(\Pi_i)_{p,q}$  for  $p, q = 0, 1, \dots, 2L$  is determined as follows:

$$\begin{aligned} (\Pi_i)_{p,q} &= \sum_{k \in I_i} |s_i(k)|^2 e^{j2\pi kp/N} e^{-j2\pi kq/N} \\ &= 2N \sum_{k \in I_i} e^{j2\pi k(p-q)/N} \\ &= \begin{cases} N, & \text{if } q = p \\ \frac{2N e^{j2\pi i(p-q)/N} (1 - e^{j2\pi(p-q)})}{1 - e^{j4\pi(p-q)/N}}. & \text{if } q \neq p \end{cases} \end{aligned} \quad (17)$$

Considering  $0 \leq |p-q| \leq 2L$ , in order not to make the quantity  $1 - e^{j4\pi(p-q)/N}$  equal to 0, we have to restrict  $L$  as follows:

$$2L < \frac{N}{2} \iff L < \frac{N}{4}. \quad (18)$$

With the condition of (18), we have

$$(\Pi_i)_{p,q} = \begin{cases} N, & \text{if } p = q, \\ 0, & \text{if } p \neq q. \end{cases} \quad (19)$$

It is easy to see that  $\mathbf{A}$  in (10) is a diagonal matrix with equal diagonal elements of  $1/\alpha^2 N$ .

So, based on the Zadoff-Chu sequence, we can design the training sequences  $\mathbf{s}_1$  and  $\mathbf{s}_2$  to achieve minimum MSE:

$$\text{MSE}_{\min} = \frac{\sigma_w^2 (\alpha^2 (\sum_{l=0}^L \sigma_{1,l}^2) + 1)}{\alpha^2 N}. \quad (20)$$

On the basis of the previous assumption, the MSE performance of the training sequences  $\mathbf{s}_1$  and  $\mathbf{s}_2$  according to (20) is in direct proportion to the noise power  $\sigma_w^2$  and in inverse proportion to the number of subcarriers  $N$ . Because of one OFDM block as training to estimate the channels, larger  $N$  implies more transmitted power being employed by channel estimation, which can achieve better MSE performance.

Because of the property of the Zadoff-Chu sequence according to (14), the designed training sequences  $\mathbf{s}_1$  and  $\mathbf{s}_2$  can also achieve the minimum PAPR performance with PAPR = 1, in which the power of the time domain signal  $\tilde{\mathbf{s}}_1$  and  $\tilde{\mathbf{s}}_2$  of  $\mathbf{s}_1$  and  $\mathbf{s}_2$  keep constant, respectively. So, the proposed optimal training sequences can achieve the same minimum MSE performance as the orthogonal optimal training sequences according to (13) with better PAPR performance. Furthermore, from (18), we can obtain that the maximum channel length supported by the LS-based estimation method is  $N/4 + 1$ .

## 4. Simulation Results and Discussion

In this section, we present computer simulations to verify our theoretical analyses. We assume  $L = 4$ ,  $L_{CP} = 8$ , and  $N = 64$ . We further assume that the power delay profile of each channel is uniform, that is, each tap of  $\mathbf{h}_1$  or  $\mathbf{h}_2$  is modeled as a zero-mean Gaussian random variable with variance  $E\{|h_i(l)|^2\} = 1/(L+1)$ ,  $i = 1, 2$  and  $l = 0, 1, \dots, L$ . The channels are assumed to be static over 50 time slots. The first time slot is used to estimate the channel information and the remaining are used for data transmission. For data transmission, BPSK modulation is deployed.

The four different trainings are considered, including the random training sequences whose elements are randomly chosen from BPSK signaling, the orthogonal training sequences which are distinct columns of Hadamard matrix of size  $N$ , the orthogonal optimal training sequences according to (13), and the Zadoff-Chu-based optimal training sequences. We compare both MSE performance as well as symbol error rate (SER) performance for two-way relay OFDM networks.

Figure 1 shows the MSE performance between  $\hat{\boldsymbol{\theta}}$  and  $\boldsymbol{\theta}$  for different training sequences. Figure 2 shows the SER performance for different training sequences. The four kinds of training sequences have the same power. The MSE performances obtained using random sequences or orthogonal

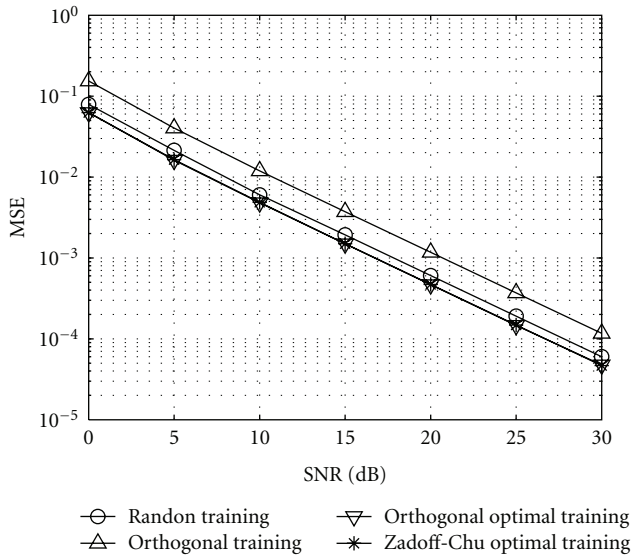


FIGURE 1: MSE performance for different SNR.

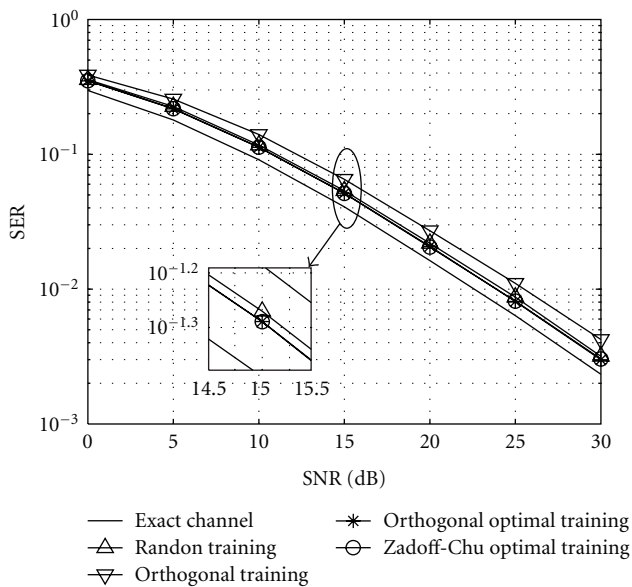


FIGURE 2: SER performance for different SNR.

sequences are worse compared with those of our proposed optimal design. From Figure 1, it is observed that both the orthogonal optimal training sequences and the Zadoff-Chu-based optimal training sequences can achieve the same and minimum MSE performance. From Figure 2, it is observed that the SER performance of the Zadoff-Chu-based optimal training sequences is same as that of the orthogonal optimal training sequences and much better that of the random training sequences and the orthogonal training sequences. However, the Zadoff-Chu-based optimal training sequences can also achieve better PAPR performance with PAPR = 1 than the orthogonal optimal training sequences according to [22], obviously.

### 5. Conclusion

In this paper, we proposed LS-based channel estimation algorithms under block-based training schemes for two-way relay OFDM networks. By minimizing MSE, the condition and design method of the optimal training sequences was discussed. The optimal training sequences based on a special sequence called Zadoff-Chu sequence are designed to achieve the same minimum MSE performance as the orthogonal optimal training sequences in [22], with better PAPR performance.

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