Research Article

Resource Allocation in MU-OFDM Cognitive Radio Systems with Partial Channel State Information

Dong Huang,[1] Zhiqi Shen,[2] Chunyan Miao,[1] and Cyril Leung[3]

[1] School of Computer Engineering, Nanyang Technological University, Singapore 639798
[2] School of Electrical and Electronic Engineering, Nanyang Technological University, Singapore 639798
[3] Department of Electrical and Computer Engineering, University of British Columbia, Vancouver, BC, Canada V6T1Z4

Correspondence should be addressed to Dong Huang, hu0013ng@ntu.edu.sg

In wireless communications, the assumption that the transmitter has perfect channel state information (CSI) is often unreasonable, due to feedback delays, estimation errors, and quantization errors. In order to accurately assess system performance, a more careful analysis with imperfect CSI is needed. In this paper, the impact of partial CSI due to feedback delays in a multiuser Orthogonal Frequency Division Multiplexing (MU-OFDM) cognitive radio (CR) system is investigated. The effect of partial CSI on the bit error rate (BER) is analyzed. A relationship between the transmit power and the number of bits loaded on a subcarrier is derived which takes into account the target BER requirement. With this relationship, existing resource allocation schemes which are based on perfect CSI being available can be applied when only partial CSI is available. Simulation results are provided to illustrate how the system performance degrades with increasingly poor CSI.

1. Introduction

In performance analyses of wireless communication systems, it is often assumed that perfect channel state information (CSI) is available at the transmitter. This assumption is often not valid due to channel estimation errors and/or feedback delays. To ensure that the system can satisfy target quality of service (QoS) requirements, a careful analysis which takes into account imperfect CSI is required [1].

Cognitive radio (CR) is a relatively new concept for improving the overall utilization of spectrum bands by allowing unlicensed secondary users (also referred to as CR users or CRUs) to access those frequency bands which are not currently being used by licensed primary users (PUs) in a given geographical area. In order to avoid causing unacceptable levels of interference to PUs, CRUs need to sense the radio environment and rapidly adapt their transmission parameter values [2–6].

Orthogonal frequency division multiplexing (OFDM) is a modulation scheme which is attractive for use in a CR system due to its flexibility in allocating resources among CRUs. The problem of optimal allocation of subcarriers, bits, and transmit powers among users in a multiuser-(MU-) OFDM system is a complex combinatorial optimization problem. In order to reduce the computational complexity, the problem is solved in two steps by many suboptimal algorithms [7–10]: (1) determine the allocation of subcarriers to users and (2) determine the allocation of bits and transmit powers to subcarriers. Resource allocation algorithms for MU-OFDM systems have been studied in [11–14]. These algorithms are designed for non-CR MU-OFDM systems in which there are no PUs.

In an MU-OFDM CR system, mutual interference between PUs and CRUs needs to be considered. The problem of optimal allocation of subcarriers, bits, and transmit powers among users in an MU-OFDM CR system is more complex. It is commonly assumed that perfect CSI is available at the transmitter [15, 16]. As noted earlier, this assumption is often not reasonable. In this paper, we investigate the problem of resource allocation in an MU-OFDM CR system when only partial CSI is available at the CR base station (CRBS). We assume that CSI is acquired perfectly at the CRUs and fed back to the CRBS with a delay of τ_d seconds. The channel experiences frequency-selective

fading. The objective is to maximize the total bit rate while satisfying BER, transmit power, and mutual interference constraints.

The rest of the paper is organized as follows. The system model is described in Section 2. Based on the system model, a constrained multiuser resource allocation problem is formulated in Section 3. A suboptimal algorithm for solving the problem is discussed in Section 4. Simulation results are presented in Section 5 and the main findings are summarized in Section 6.

2. System Model

We consider the problem of allocating resources on the downlink of an MU-OFDM CR system with one base station (BS) serving one PU and K CRUs. The basic system model is the same as that described in [15] and is summarized here for the convenience of the reader.

The PU channel is W_p Hz wide and the bandwidth of each OFDM subchannel is W_s Hz. On either side of the PU channel, there are $N/2$ OFDM subchannels. The BS has only partial CSI and allocates subcarriers, transmit powers, and bits to the CRUs once every OFDM symbol period. The channel gain of each subcarrier is assumed to be constant during an OFDM symbol duration.

Suppose that P_n is the transmit power allocated on subcarrier n and g_n is the channel gain of subcarrier n from the BS to the PU. The resulting interference power spilling into the PU channel is given by

$$I_n(d_n, P_n) = P_n \cdot IF_n, \tag{1}$$

where

$$IF_n \triangleq \int_{d_n - W_p/2}^{d_n + W_p/2} |g_n|^2 \Phi(f) df \tag{2}$$

represents the interference factor for subcarrier n, d_n is the spectral distance between the center frequency of subcarrier n and that of the PU channel, and $\Phi(f)$ denotes the normalized baseband power spectral density (PSD) of each subcarrier.

Let h_{nk} be the channel gain of subcarrier n from the BS to CRU k, and let $\Phi_{RR}(f)$ be the baseband PSD of the PU signal. The interference power to CRU k on subcarrier n is given by

$$S_{nk}(d_n) = \int_{d_n - W_s/2}^{d_n + W_s/2} |h_{nk}|^2 \Phi_{RR}(f) df. \tag{3}$$

Let P_{nk} denote the transmit power allocated to CRU k on subcarrier n. For QAM modulation, an approximation for the BER on subcarrier n of CRU k is [13]

$$\text{BER}[n] \approx 0.2 \exp\left(\frac{-1.5|h_{nk}|^2 P_{nk}}{(2^{b_{nk}} - 1)(N_0 W_s + S_{nk})}\right), \tag{4}$$

where N_0 is the one-sided noise PSD and S_{nk} is given by (3). Rearranging (4), the maximum number of bits per OFDM symbol period that can be transmitted on this subcarrier is given by

$$b_{nk} = \left\lfloor \log_2\left(1 + \frac{|h_{nk}|^2 P_{nk}}{\Gamma(N_0 W_s + S_{nk})}\right)\right\rfloor, \tag{5}$$

where $\Gamma \triangleq -\ln(5\text{BER}[n])/1.5$ and $\lfloor \cdot \rfloor$ denotes the floor function.

Equation (4) shows the relationship between the transmit power and the number of bits loaded on the subcarrier for a given BER requirement when perfect CSI is available at the transmitter. We now establish an analogous relationship when only partial CSI is available.

The imperfect CSI that is available to the BS is modeled as follows. We assume that perfect CSI is available at the receiver. The channel gain, hnk, for subcarrier n and CRU k is the outcome of an independent complex Gaussian random variable, that is, $H_{nk} \sim \mathcal{CN}(0, \sigma_h^2)$ [17], corresponding to Rayleigh fading. For clarity, we will denote random variables and their outcomes by uppercase and lowercase letters, respectively.

For notational simplicity, we will use h to denote an arbitrary channel gain. The BS receives the CSI after a feedback delay $\tau_d = dT_s$, where T_s is the OFDM symbol duration. We assume that the noise on the feedback link is negligible. Suppose that h_f is the channel gain information that is received at the BS, then $h_f(t) = h(t - \tau_d)$. From [18], the correlation between H and H_f is given by

$$E\left\{HH_f^H\right\} = \rho \sigma_h^2, \tag{6}$$

where the correlation coefficient, ρ, is given by

$$\rho = J_0(2\pi f_d dT_s). \tag{7}$$

In (6) and (7), $J_0(\cdot)$ denotes the zeroth-order Bessel function of the first kind, f_d is the Doppler frequency, $E\{\cdot\}$ is the expectation operator, and H_f^H denotes the complex conjugate of H_f.

The minimum mean square error (MMSE) estimator of H based on $H_f = h_f$ is given by [19]

$$\overline{H} = E\left\{H \mid H_f = h_f\right\} = \rho h_f. \tag{8}$$

From (6), the actual channel gain can be written as [20] follows:

$$h = \overline{H} + \epsilon, \tag{9}$$

where $\epsilon \sim \mathcal{CN}(0, \sigma_\epsilon^2)$ with $\sigma_\epsilon^2 = \sigma_h^2(1 - |\rho|^2)$.

3. Formulation of the Multiuser Resource Allocation Problem

Based on the partial CSI available at the BS, we wish to maximize the total CRU transmission rate while maintaining a target BER performance on each subcarrier and satisfying PU interference and total BS CRU transmit power constraints. Let $\overline{\mathrm{BER}}[n]$ denote the average BER on subcarrier n, and let $\overline{\mathrm{BER}}_0$ represent the prescribed target BER. The optimization problem can be expressed as follows:

$$\max R_s \stackrel{\Delta}{=} W_s \sum_{n=1}^{N} \sum_{k=1}^{K} a_{nk} b_{nk}, \qquad (10)$$

subject to

$$\overline{\mathrm{BER}}[n] \le \overline{\mathrm{BER}}_0, \quad \forall n \qquad (11)$$

$$\sum_{k=1}^{K} \sum_{n=1}^{N} a_{nk} P_{nk} \le P_{\mathrm{total}}, \qquad (12)$$

$$P_{nk} \ge 0, \quad \forall n, k \qquad (13)$$

$$\sum_{k=1}^{K} \sum_{n=1}^{N} a_{nk} P_{nk} IF_n \le I_{\mathrm{total}}, \qquad (14)$$

$$\sum_{k=1}^{K} a_{nk} \le 1, \quad \forall n \qquad (15)$$

$$a_{nk} \in \{0, 1\}, \quad \forall n, k \qquad (16)$$

$$R_1 : R_2 : \cdots : R_K = \lambda_1 : \lambda_2 : \cdots : \lambda_K, \qquad (17)$$

where P_{total} is the total power budget for all CRUs, I_{total} is the maximum interference power that can be tolerated by the PU, and $a_{nk} \in \{0, 1\}$ is a subcarrier assignment indicator, that is, $a_{nk} = 1$ if and only if subcarrier n is allocated to CRU k. The term λ_k represents the nominal bit rate weight (NBRW) for CRU k, and

$$R_k = W_s \sum_{n=1}^{N} a_{nk} b_{nk}, \quad \forall k = 1, 2, \ldots, K \qquad (18)$$

denotes the total bit rate achieved by CRU k. Constraint (11) ensures that the average BER for each subcarrier is below the given BER target. Constraint (12) states that the total power allocated to all CRUs cannot exceed P_{total}, while constraint (14) ensures that the interference power to the PU is maintained below an acceptable level I_{total}. Constraint (15) results from the assumption that each subcarrier can be assigned to at most one CRU. Constraint (17) ensures that the bit rate achieved by a CRU satisfies a proportional fairness condition.

Based on (9), we calculate the average of the right-hand side (RHS) of (4), treating h_{nk} as an outcome of an independent complex Gaussian variable. For an arbitrary vector $\boldsymbol{\alpha} \sim \mathcal{CN}(\boldsymbol{\mu}, \boldsymbol{\Sigma})$, we have [21] the following:

$$E\left\{\exp\left(-\boldsymbol{\alpha}^H \boldsymbol{\alpha}\right)\right\} = \frac{\exp\left(-\boldsymbol{\mu}^H (\mathbf{I} + \boldsymbol{\Sigma})^{-1} \boldsymbol{\mu}\right)}{\det(\mathbf{I} + \boldsymbol{\Sigma})}, \qquad (19)$$

where $\mathbf{I}$ denotes the identity matrix. Applying (19) to (4), we obtain

$$\overline{\mathrm{BER}}[n] \approx 0.2 \frac{1}{1 + \Psi \sigma_\epsilon^2} \exp\left(-\frac{\Psi \left|\overline{H}_{nk}\right|^2}{1 + \Psi \sigma_\epsilon^2}\right), \qquad (20)$$

where $\overline{H}_{nk} = \rho h_{nk}^f$, $\Psi = 1.5 P_{nk} / \{(2^{b_{nk}} - 1)(N_0 W_s + S_{nk})\}$, and h_{nk}^f denotes the channel gain that is fedback to the BS.

From (20), an explicit relationship between minimum transmit power and number of transmitted bits cannot be easily derived. However, since $\overline{\mathrm{BER}}[n]$ in (20) is a monotonically decreasing function of P_{nk}, we obtain the minimum power requirement while satisfying the constraint in (11) by setting $\overline{\mathrm{BER}}[n] = \overline{\mathrm{BER}}_0$.

We now derive a simpler, albeit approximate, relationship between the required transmit power, $\overline{\mathrm{BER}}$, and the number of loaded bits.

When setting $\mathcal{K}_\mu = |\overline{H}_{nk}|^2 / \sigma_\epsilon^2$, $r = 1.5 P_{nk} / (N_0 W_s + S_{nk})$, $g = 1/(2^{b_{nk}} - 1)$, and $\overline{\gamma} = (1 + \mathcal{K}_\mu) \sigma_\epsilon^2 r$, the RHS of (20) has the form

$$
\begin{aligned}
&I_\mu(\overline{\gamma}, g, \theta) \\
&= \frac{(1 + \mathcal{K}_\mu) \sin^2\theta}{(1 + \mathcal{K}_\mu) \sin^2\theta + g\overline{\gamma}} \exp\left(-\frac{\mathcal{K}_\mu g\overline{\gamma}}{(1 + \mathcal{K}_\mu) \sin^2\theta + g\overline{\gamma}}\right),
\end{aligned} \qquad (21)
$$

with $\theta = \pi/2$. The function $I_\mu(\overline{\gamma}, g, \theta)$ is Rician distributed with Rician factor $\mathcal{K}_\mu$ [20]. A Rician distribution with $\mathcal{K}_\mu$ can be approximated by a Nakagami-m distribution [22] as follows:

$$\tilde{I}_\mu(\overline{\gamma}, g, \theta) = \left(1 + \frac{g\overline{\gamma}}{m_\mu \sin^2\theta}\right)^{-m_\mu}, \qquad (22)$$

with $\theta = \pi/2$, where $m_\mu = (1 + \mathcal{K}_\mu)^2 / 1 + 2\mathcal{K}_\mu$. Therefore, we approximate the RHS of (20) by

$$\overline{\mathrm{BER}}[n] \approx 0.2 \left(1 + \frac{\left(\sigma_\epsilon^2 + \left|\overline{h}_{nk}\right|^2\right) \Psi}{m_\mu}\right)^{-m_\mu}. \qquad (23)$$

Then, from (23), we obtain

$$P_{nk} \approx \frac{\left(\left(5\overline{\mathrm{BER}}[n]\right)^{-(1/m_\mu)} - 1\right) m_\mu}{\sigma_\epsilon^2 + \left|\overline{h}_{nk}\right|^2} \cdot \Upsilon, \qquad (24)$$

where $\Upsilon = (2^{b_{nk}} - 1)(N_0 W_s + S_{nk})/1.5$. From (24), we obtain

$$b_{nk} = \left\lfloor \log_2\left(1 + \frac{P_{nk}\left(\sigma_\epsilon^2 + \left|\overline{h}_{nk}\right|^2\right)}{\Gamma'(N_0 W_s + S_{nk})}\right)\right\rfloor, \qquad (25)$$

where $\Gamma' = m_\mu((5\overline{\mathrm{BER}}_0)^{-1/m_\mu} - 1)/1.5$.

4. Resource Allocation with Partial Csi

Note that the joint subcarrier, bit, and power allocation problem in (10)–(17) belongs to the mixed integer nonlinear programming (MINP) class [23]. For brevity, we use the term "bit allocation" to denote both bit and power allocation. Since the optimization problem in (10)–(17) is generally computationally complex, we first use a suboptimal algorithm, which is based on a greedy approach, to solve the subcarrier allocation problem in Section 4.1. After subcarriers are allocated to CRUs, we apply a memetic algorithm (MA) to solve the bit allocation problem in Section 4.2.

4.1. Subcarrier Allocation. From (17), it can be seen that the subcarrier allocation depends not only on the channel gains, but also on the number of bits allocated to each subcarrier. Moreover, allocation of subcarriers close to the PU band should be avoided in order to reduce the interference power to the PU to a tolerable level. Therefore, we use a threshold scheme to select subcarriers for CRUs.

Suppose that $\hat{N}$ subcarriers are available for allocating to CRUs. We assume equal transmit power for each subcarrier. Let

$$\Psi_k = \frac{1}{\hat{N}} \sum_{n=1}^{\hat{N}} \frac{\left|\overline{H}_{nk}\right|^2 + \sigma_\epsilon^2}{\Gamma'(N_0 W_s + S_{nk})}, \quad \forall k = 1, 2, \dots, K \quad (26)$$

$$\overline{IF} = \frac{1}{\hat{N}} \sum_{n=1}^{\hat{N}} IF_n. \quad (27)$$

If a subcarrier is assigned to CRU k, the maximum number of bits which can be loaded on the subcarrier is given by

$$b_k = \min\left(\left\lfloor \log_2\left(1 + \frac{\Psi_k P_{\text{total}}}{\hat{N}}\right)\right\rfloor, \left\lfloor \log_2\left(1 + \frac{\Psi_k I_{\text{total}}}{\hat{N}\,\overline{IF}}\right)\right\rfloor\right),$$
$$\forall k = 1, 2, \dots, K. \quad (28)$$

Using (26)–(28), we can determine the number of subcarriers assigned to each CRU as follows. Let m_k be the number of subcarriers allocated to CRU k. Assuming that the same number of bits is loaded on every subcarrier assigned to a given CRU, the objective in (10) is equivalent to finding a set of $\{m_1, m_2, \dots, m_K\}$ subcarriers to maximize

$$\max R_s \triangleq W_s \sum_{k=1}^{K} m_k b_k, \quad (29)$$

subject to

$$m_1 b_1 : m_2 b_2 : \cdots : m_K b_K = \lambda_1 : \lambda_2 : \cdots : \lambda_K, \quad (30)$$

$$P \leq P_{\text{total}}, \quad (31)$$

$$I \leq I_{\text{total}}, \quad (32)$$

where P is the total transmit power allocated to all subcarriers and I is the total interference power experienced by the PU due to CRU signals. The subcarrier allocation problem

```
Algorithm: SA
    for n = 1 to number of subcarriers do
        find k* ∈ {1, 2, . . . , K} which maximizes
        (|H̄_nk|² + σ_ε²)/(Γ'(N₀W_s + S_nk));
        Using (25), calculate the number of bits loaded on
        Subcarrier
        n as b_nk* with P_nk* = P_total/N;
        initialize N̂ to 0;
        if b_nk* > 2 then
            subcarrier n is available; increment N̂ by 1;
        else
            subcarrier n is not available;
        end if
    end for
    For each k ∈ {1, 2, . . . , K}, initialize the number, m_k, of
    subcarriers allocated to CRU k to 0
    calculate b_k using (28);
    for n = 1 to N̂ do
        find the value, η, of k ∈ {1, 2, . . . , K} which minimizes
        m_k b_k / λ_k;
        allocate subcarrier n to CRU η;
        increment m_η by one.
    end for
```

PSEUDOCODE 1: Pseudocode for subcarrier allocation algorithm.

```
Algorithm: MA
    initialize Population P; {Input : x_i = [x_i1, x_i2, . . . , x_iN],
    i = 1, 2, . . . , pop_size}
    P = Local_Search(P);
    for i = 1 to Number_of_Generatio do
        S = selectForVariation(P);
        S' = crossover(S);
        S' = Local_Search(S');
        add S' to P;
        S'' = muation(S);
        S'' = Local_Search(S'');
        add S'' to P;
        P = selectForSurvival(P);
    end for
    return P. {Output : x_i = [x_i1, x_i2, . . . , x_iN], i =
    1, 2, . . . , pop_size}
```

PSEUDOCODE 2: Pseudocode for the memetic algorithm.

in (29)–(32) can be solved using the SA algorithm proposed in [24]. Note that we need to make use of (24) in the SA algorithm if only partial CSI is available. A pseudocode listing for the SA algorithm is shown in Pseudocode 1. The algorithm has a relatively low computational complexity $\mathcal{O}(KN)$. After subcarriers are allocated to CRUs, we then determine the number, b_n, of bits allocated to subcarrier n.

4.2. Bit Allocation. Memetic algorithm (MAs) are evolutionary algorithms which have been shown to be more efficient than standard genetic algorithms (GAs) for many combinatorial optimization problems [25–27]. Using (24),

the bit allocation problem can be solved using the MA algorithm proposed in [24]. It should be noted that the chosen genetic operators and local search methods greatly influence the performance of MAs. The selection of these parameters for the given optimization problem is based on the results in [24]. A pseudocode listing of the proposed memetic algorithm is shown in Pseudocode 2.

Let $\mathbf{x}_i$ be the chromosome of member i in a population, expressed as

$$\mathbf{x}_i = \begin{bmatrix} x_{i1} & x_{i2} & \cdots & x_{iN} \end{bmatrix}, \quad \forall i = 1, 2, \ldots, pop\_size, \quad (33)$$

where pop_size denotes the population size. A brief description of the MA algorithm in [24] is now provided.

(1) The *selectForVariation* function selects a set, $S = \{s_1, s_2, \ldots, s_{pop\_size}\}$, of chromosomes from P in a roulette wheel fashion, that is, selection with replacement.

(2) Crossover: suppose that $S = \{\mathbf{y}_1, \mathbf{y}_2, \ldots, \mathbf{y}_{pop\_size}\}$. Let P_{cross} denote the crossover probability, and let $u_i, i = 1, 2, \ldots, pop\_size$ denote the outcome of an independent random variable which is uniformly distributed in $[0, 1]$, then $\mathbf{y}_i$ is selected as a candidate for crossover if and only if $u_i \leq P_{cross}, i = 1, 2, \ldots, pop\_size$. Suppose that we have n_c such candidates, we then form $n_c/2$ disjoint pairs of candidates (parents).

For each pair of parents $\mathbf{y}_i$ and $\mathbf{y}_j$,

$$\begin{aligned} \mathbf{y}_i &= \begin{bmatrix} y_{i1} & y_{i2} & \cdots & y_{ip} & y_{i(p+1)} & \cdots & y_{iN} \end{bmatrix}, \\ \mathbf{y}_j &= \begin{bmatrix} y_{j1} & y_{j2} & \cdots & y_{jp} & y_{j(p+1)} & \cdots & y_{jN} \end{bmatrix}, \end{aligned} \quad (34)$$

we first generate a random integer $p \in [1, N-1]$, then we obtain the (possibly identical) chromosomes of two children as follows:

$$\begin{aligned} \mathbf{y}_i' &= \begin{bmatrix} y_{i1} & y_{i2} & \cdots & y_{ip} & y_{j(p+1)} & \cdots & y_{jN} \end{bmatrix}, \\ \mathbf{y}_j' &= \begin{bmatrix} y_{j1} & y_{j2} & \cdots & y_{jp} & y_{i(p+1)} & \cdots & y_{iN} \end{bmatrix}. \end{aligned} \quad (35)$$

(3) Mutation: let $P_{mutation}$ denote the mutation probability. For each chromosome in S, we generate $u_i, i = 1, 2, \ldots, N$, where u_i denotes the outcome of an independent random variable which is uniformly distributed in $[0, 1]$. Then for each component i for which $u_i \leq P_{mutation}$, we substitute the value with a randomly chosen admissible value.

(4) Selection of surviving chromosomes: we select the pop_size chromosomes of parents and offsprings with the best fitness values as input for the next generation.
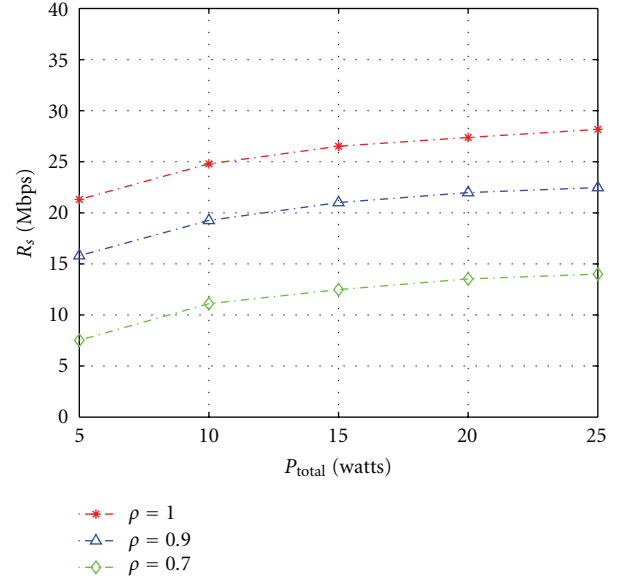

FIGURE 1: Average total CRU bit rate, R_s, versus total CRU transmit power, P_{total}, with $I_{total} = 0.02$ W, $P_m = 5$ W, and $\boldsymbol{\lambda} = [1\ 1\ 1\ 1]$.

5. Results

In this section, performance results for the proposed algorithm described in Section 4 are presented. In the simulation, the parameters of the MA algorithm were chosen as follows: population size, $pop\_size = 40$; number of generations = 20; crossover probability, $P_{cross} = 0.05$; mutation probability, $P_{mutation} = 0.7$.

We consider a system with one PU and $K = 4$ CRUs. The total available bandwidth for CRUs is 5 MHz and supports 16 subcarriers with $W_s = 0.3125$ MHz. We assume that $W_p = W_s$ and an OFDM symbol duration, T_s of $4\,\mu$s. In order to understand the impact of the fair bit rate constraint in (17) on the total bit rate, three cases of user bit rate requirements with $\boldsymbol{\lambda} = [1\ 1\ 1\ 1], [1\ 1\ 1\ 4], [1\ 1\ 1\ 8]$ were considered. In addition, three cases of partial CSI with $\rho = 1, 0.9$ and 0.7 were studied. It is assumed that the subcarrier gains h_{nk} and g_k, for $n \in \{1, 2, \ldots, N\}$, $k \in \{1, 2, \ldots, K\}$ are outcomes of independent identically distributed (i.i.d.) Rayleigh-distributed random variables (rvs) with mean square value $E(|H_{nk}|^2) = E(|G_k|^2) = 1$. The additive white Gaussian noise (AWGN) PSD, N_0, was set to 10^{-8} W/Hz. The PSD, $\Phi_{RR}(f)$, of the PU signal was assumed to be that of an elliptically filtered white noise process. The total CRU bit rate, R_s, results were obtained by averaging over 10,000 channel realizations. The 95% confidence intervals for the simulated R_s results are within $\pm 1\%$ of the average values shown.

Figure 1 shows the average total bit rate, R_s, as a function of the total CRU transmit power, P_{total}, for $\rho = 0.7, 0.9$, and 1 with $\boldsymbol{\lambda} = [1\ 1\ 1\ 1], I_{total} = 0.02$ W, and a PU transmit power, P_m, of 5 W. As expected, the average total bit rate increases with the maximum transmit power budget P_{total}. It can be seen that the average total bit rate, R_s, varies greatly with ρ.
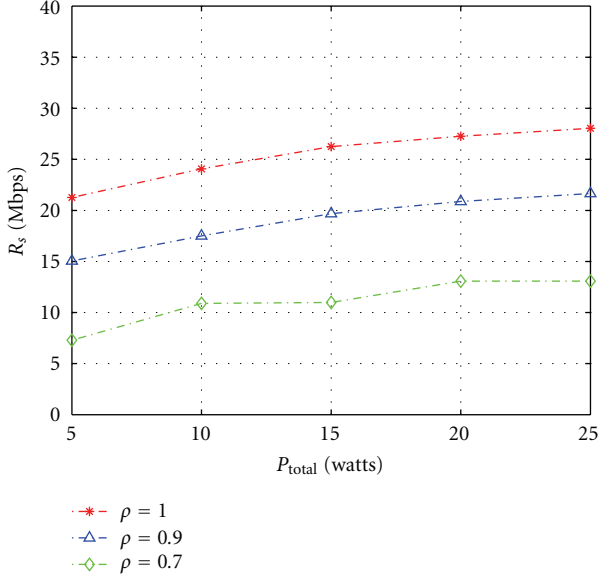
FIGURE 2: Average total CRU bit rate, R_s, versus total CRU transmit power, P_{total}, with $I_{total} = 0.02$ W, $P_m = 5$ W, and $\lambda = [1\ 1\ 1\ 4]$.

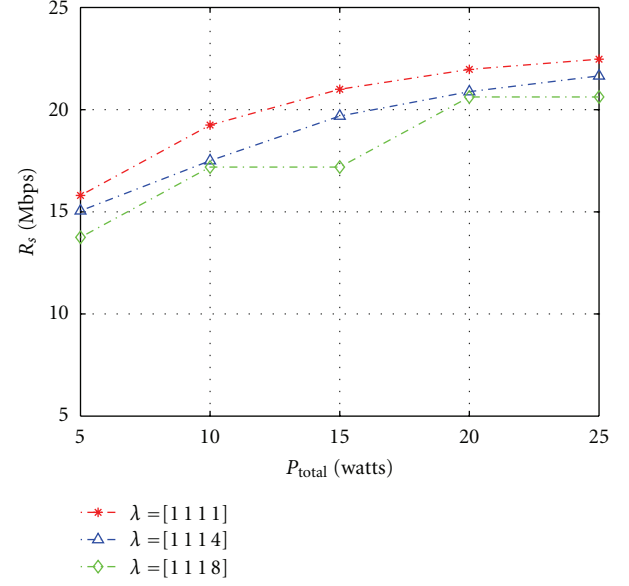FIGURE 4: Average total CRU bit rate, R_s, versus total CRU transmit power, P_{total}, with $I_{total} = 0.02$ W, $P_m = 5$ W, and $\rho = 0.9$.
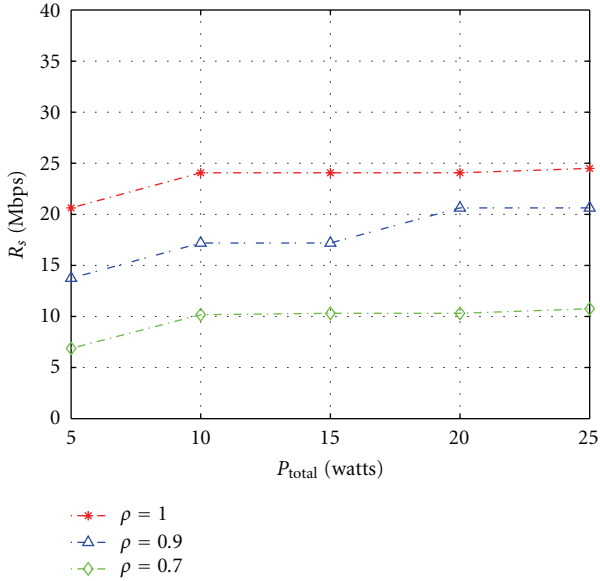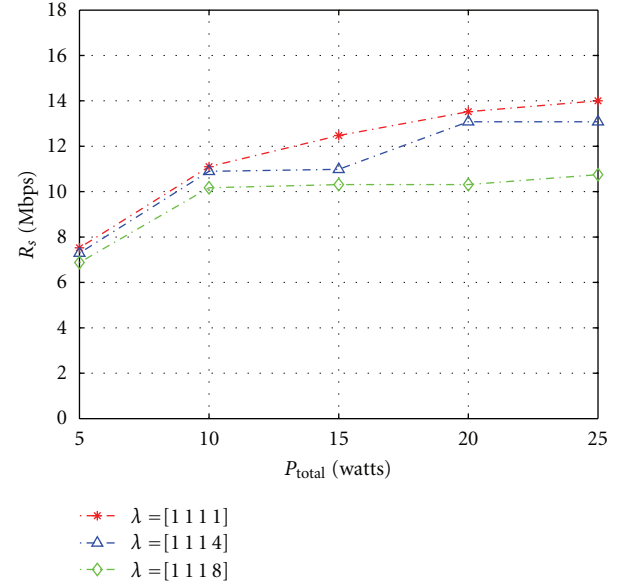

FIGURE 3: Average total CRU bit rate, R_s, versus total CRU transmit power, P_{total}, with $I_{total} = 0.02$ W, $P_m = 5$ W, and $\lambda = [1\ 1\ 1\ 8]$.

FIGURE 5: Average total CRU bit rate, R_s, versus total CRU transmit power, P_{total}, with $I_{total} = 0.02$ W, $P_m = 5$ W, and $\rho = 0.7$.

For example, at $P_{total} = 5$ W, R_s increases by a factor of 2 as ρ increases from 0.7 to 0.9. This illustrates the big impact that inaccurate CSI may have on system performance. The R_s curves level off as P_{total} increases due to the fixed value of the maximum interference power that can be tolerated by the PU.

Corresponding results for $\lambda = [1\ 1\ 1\ 4]$ and $\lambda = [1\ 1\ 1\ 8]$ are plotted in Figures 2 and 3, respectively. The average total bit rate, R_s, decreases as the NBRW distribution becomes less uniform; the reduction tends to increase with P_{total}.

Figure 4 shows R_s as a function of P_{total} for three different cases of λ with $\rho = 0.9$, $I_{total} = 0.02$ W, and $P_m = 5$ W. As to be expected, R_s increases with P_{total}. It can be seen that R_s for $\lambda = [1\ 1\ 1\ 1]$ is larger than for $\lambda = [1\ 1\ 1\ 4]$, and R_s for $\lambda = [1\ 1\ 1\ 4]$ is larger than for $\lambda = [1\ 1\ 1\ 8]$. When the bit rate requirements for CRUs become less uniform, R_s decreases due to a decrease in the benefits of user diversity. With $P_{total} = 15$ W, R_s increases by about 30% when λ changes from $[1\ 1\ 1\ 8]$ to $[1\ 1\ 1\ 1]$. Results for $\rho = 0.7$ are shown in Figure 5 and are qualitatively similar to those in Figure 4.
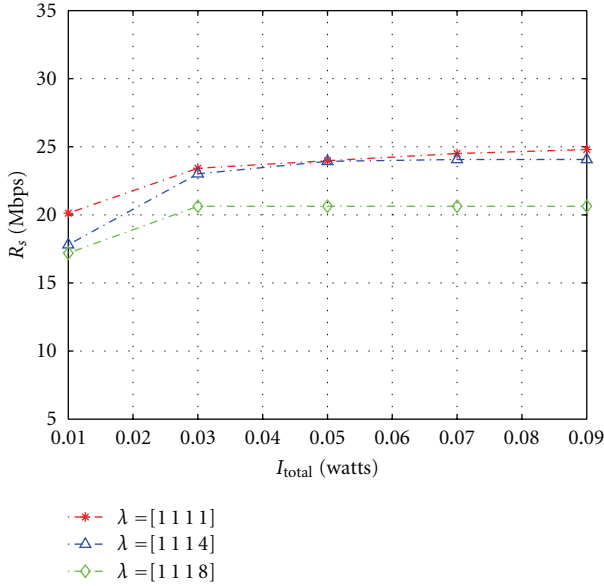
FIGURE 6: Average total CRU bit rate, R_s, versus maximum PU tolerable interference power, I_{total}, with $P_{total} = 25$ W, $P_m = 5$ W, and $\rho = 0.9$.
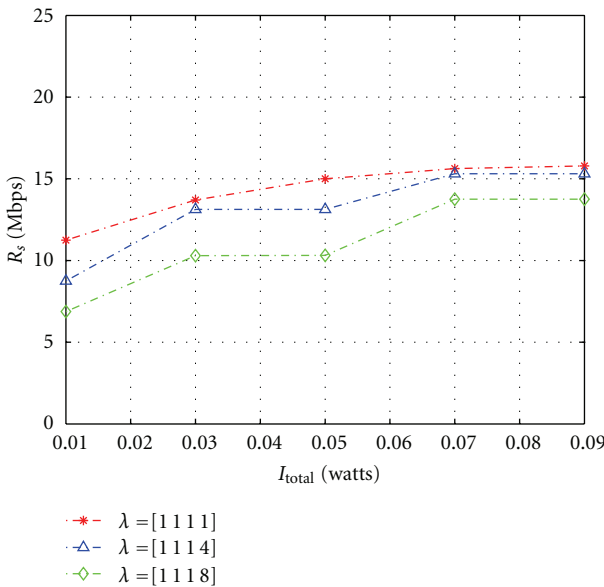

FIGURE 7: Average total CRU bit rate, R_s, versus maximum PU tolerable interference power, P_{total}, with $P_{total} = 25$ W, $P_m = 5$ W, and $\rho = 0.7$.

The average total bit rate, R_s, is plotted as a function of the maximum PU tolerable interference power, I_{total}, with $P_{total} = 25$ W and $P_m = 5$ W, for $\rho = 0.9$ and 0.7 in Figures 6 and 7, respectively. As expected, R_s increases with I_{total} and decreases as the CRU bit rate requirements become less uniform. The R_s curves level off as I_{total} increases due to the fixed value of the total CRU transmit power, P_{total}.

6. Conclusion

The assumption of perfect CSI being available at the transmitter is often unreasonable in a wireless communication system. In this paper, we studied an MU-OFDM CR system in which the available partial CSI is due to a delay in the feedback channel. The effect of partial CSI on the BER was investigated; a relationship between transmit power, number of bits loaded, and BER was derived. This relationship was used to study the performance of a resource allocation scheme when only partial CSI is available. It is found that the performance varies greatly with the quality of the partial CSI.

References

[1] S. Ye, R. S. Blum, and L. J. Cimini Jr., "Adaptive OFDM systems with imperfect channel state information," *IEEE Transactions on Wireless Communications*, vol. 5, no. 11, pp. 3255–3265, 2006.

[2] J. Mitola III and G. Q. Maguire Jr., "Cognitive radio: making software radios more personal," *IEEE Personal Communications*, vol. 6, no. 4, pp. 13–18, 1999.

[3] J. Mitola, *Cognitive Radio Architecture: The Engineering Foundations of Radio XML*, Wiley-Interscience, New York, NY, USA, 2006.

[4] S. Haykin, "Cognitive radio: brain-empowered wireless communications," *IEEE Journal on Selected Areas in Communications*, vol. 23, no. 2, pp. 201–220, 2005.

[5] T. Weiss, J. Hillenbrand, A. Krohn, and F. K. Jondral, "Mutual interference in OFDM-based spectrum pooling systems," in *Proceedings of the 59th IEEE Vehicular Technology Conference (VTC '04)*, vol. 4, pp. 1873–1877, May 2004.

[6] T. A. Weiss and F. K. Jondral, "Spectrum pooling: an innovative strategy for the enhancement of spectrum efficiency," *IEEE Communications Magazine*, vol. 42, no. 3, pp. S8–S14, 2004.

[7] R. F. H. Fischer and J. B. Huber, "A new loading algorithm for discrete multitone transmission," in *Proceedings of the IEEE Global Telecommunications Conference*, vol. 1, pp. 724–728, November 1996.

[8] R. V. Sonalkar and R. R. Shively, "An efficient bit-loading algorithm for DMT applications," in *Proceedings of the IEEE Global Telecommunications Conference (GLOBECOM '98)*, vol. 5, pp. 2683–2688, November 1998.

[9] C. Wong, C. Tsui, R. Cheng, and K. Letaief, "A real-time sub-carrier allocation scheme for multiple access downlink OFDM transmission," in *Proceedings of the 50th IEEE Vehicular Technology Conference (VTC '99)*, vol. 2, pp. 1124–1128, 1999.

[10] Y.-F. Chen, J.-W. Chen, and C.-P. Li, "A real-time joint subcarrier, bit, and power allocation scheme for multiuser OFDM-based systems," in *Proceedings of the 59th IEEE Vehicular Technology Conference (VTC '04)*, vol. 3, pp. 1826–1830, May 2004.

[11] Z. Shen, J. G. Andrews, and B. L. Evans, "Optimal power allocation in multiuser OFDM systems," in *Proceedings of the IEEE Global Telecommunications Conference (GLOBECOM '03)*, vol. 1, pp. 337–341, December 2003.

[12] C. Y. Wong, R. S. Cheng, K. B. Letaief, and R. D. Murch, "Multiuser OFDM with adaptive subcarrier, bit, and power allocation," *IEEE Journal on Selected Areas in Communications*, vol. 17, no. 10, pp. 1747–1758, 1999.

[13] J. Jang and K. B. Lee, "Transmit power adaptation for multiuser OFDM systems," *IEEE Journal on Selected Areas in Communications*, vol. 21, no. 2, pp. 171–178, 2003.

[14] D. W. K. Ng and R. Schober, "Cross-layer scheduling for OFDMA amplify-and-forward relay networks," in *Proceedings of the 70th IEEE Vehicular Technology Conference (VTC '09)*, pp. 1–5, 2009.

[15] T. Qin and C. Leung, "Fair adaptive resource allocation for multiuser OFDM cognitive radio systems," in *Proceedings of the 2nd International Conference on Communications and Networking in China (ChinaCom '07)*, pp. 115–119, Shanghai, China, August 2007.

[16] Y. Zhang and C. Leung, "Cross-layer resource allocation for mixed services in multiuser OFDM-based cognitive radio systems," *IEEE Transactions on Vehicular Technology*, vol. 58, no. 8, pp. 4605–4619, 2009.

[17] P. Xia, S. Zhou, and G. B. Giannakis, "Adaptive MIMO-OFDM based on partial channel state information," *IEEE Transactions on Signal Processing*, vol. 52, no. 1, pp. 202–213, 2004.

[18] W. Jakes, *Microwave Mobile Communications*, Wiley-IEEE Press, New York, NY, USA, 1994.

[19] S. Kay, *Fundamentals of Statistical Signal Processing: Estimation Theory*, Prentice-Hall, Upper Saddle River, NJ, USA, 1993.

[20] S. Zhou and G. B. Giannakis, "Optimal transmitter eigen-beamforming and space-time block coding based on channel mean feedback," *IEEE Transactions on Signal Processing*, vol. 50, no. 10, pp. 2599–2613, 2002.

[21] G. Taricco and E. Biglieri, "Exact pairwise error probability of space-time codes," *IEEE Transactions on Information Theory*, vol. 48, no. 2, pp. 510–513, 2002.

[22] M. K. Simon and M.-S. Alouini, "A unified approach to the performance analysis of digital communication over generalized fading channels," *Proceedings of the IEEE*, vol. 86, no. 9, pp. 1860–1877, 1998.

[23] D. Bertsekas, M. Homer, D. Logan, and S. Patek, *Nonlinear Programming*, Athena Scientific, 1995.

[24] D. Huang, Z. Shen, C. Miao, and C. Leung, "Fitness landscape analysis for resource allocation in multiuser OFDM based cognitive radio systems," *ACM SIGMOBILE Mobile Computing and Communications Review*, vol. 13, no. 2, pp. 26–36, 2009.

[25] P. Merz and B. Freisleben, "Memetic algorithms for the traveling salesman problem," *Complex Systems*, vol. 13, no. 4, pp. 297–345, 2001.

[26] P. Merz and K. Katayama, "Memetic algorithms for the unconstrained binary quadratic programming problem," *BioSystems*, vol. 78, no. 1–3, pp. 99–118, 2004.

[27] P. Merz, "On the performance of memetic algorithms in combinatorial optimization," in *Proceedings of the 2nd GECCO Workshop on Memetic Algorithms (WOMA '01)*, pp. 168–173, San Francisco, Calif, USA, 2001.