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Research Article

A Stochastic Multiobjective Optimization Framework for Wireless Sensor Networks

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In wireless sensor networks (WSNs), there generally exist many different objective functions to be optimized. In this paper, we propose a stochastic multiobjective optimization approach to solve such kind of problem. We first formulate a general multiobjective optimization problem. We then decompose the optimization formulation through Lagrange dual decomposition and adopt the stochastic quasigradient algorithm to solve the primal-dual problem in a distributed way. We show theoretically that our algorithm converges to the optimal solution of the primal problem by using the knowledge of stochastic programming. Furthermore, the formulation provides a general stochastic multiobjective optimization framework for WSNs. We illustrate how the general framework works by considering an example of the optimal rate allocation problem in multipath WSNs with timevarying channel. Extensive simulation results are given to demonstrate the effectiveness of our algorithm.

1. Introduction

The layered architecture approach has achieved great success in traditional wired network design by dividing the whole architecture into several modules, called layers, each of which performs a separate functionality. As each layer design only needs some interface variables from the layer below, the complexity of other layers can be hidden. The layered architecture approach suggests that the network design can be scalable, evolvable, and implementable. However, it may have limitations in improvement of efficiency and fairness, and suffer potential risks of manageability [1], which motivates the optimization of network design. Chiang et al. [1, 2] propose an optimization decomposition technique to systematically understand the network architecture, known as "layering as optimization decomposition". They model the network as an optimization problem and decompose the problem into many subproblems. They classify the decompositions into vertical decomposition and horizontal decomposition. Vertical decomposition layers the network architecture into several modules and horizontal decomposition provides distributed algorithms to fulfill the functionality within the modules.

According to the requirements of the applications, the decomposition may be different, yielding different layers and distributed algorithms. There are usually two steps in the process of layering as optimization decomposition: (1) modeling the network problem as a specific NUM problem, and (2) exploring the alternative decompositions to design different modules and distributed algorithms. Most existing efforts have been put to the second step and simply assume that the network problems can be modeled by a unified utility function at the first step [3–6]. However, not all network problems can be modeled by a unified utility function in a tractable way since there may exist many objectives to be achieved, such as guaranteeing fairness, maximizing throughput, reducing packet dropping and delay, prolonging the network lifetime, and so forth. It may not be possible to integrate all these objectives into a single unified utility function, that is, network problems should be formulated as multiobjective optimization problems.

While the performance of the network can be greatly enhanced by adopting the NUM approach, the corresponding cost of algorithm implementation also increases. As we usually design and implement an algorithm for a specific

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application from scratch, the implementation can hardly be transplantable to other applications. This is especially aggravated in wireless sensor networks (WSNs) due to the application-oriented and infrastructureless nature of these networks. For example, if we design an efficient algorithm for events monitoring, the network lifetime is the main concern and the propagation delay can be tolerant, but it is difficult to apply such algorithm to online query applications, where the query delay is the primary objective.

In this paper, we utilize the concept of multiobjective optimization and provide a general framework for a specific class of applications in WSNs. It is well known that the TCP/IP reference model, one of the most popular layered architectures, divides the whole architecture into five layers (modules), and each layer only communicates with the layers next to it, while recent work on the NUM approach divides the architectures according to applications. We first list all the constraints and objectives which the applications may have. Then the network architecture is divided into nmodules, each of which has several interface variables with other modules. In this way, we can inherit the advantages of both the layered architectures (as we have fixed modules) and the NUM approach (as each module can communicate with other modules). We illustrate this in Figure 1, in which λ and μ are the interface variables (see Section 3 for detailed definition of λ and μ), and O_i is the objective vector function in each module *i*. We transform the objectives of the network into specific modules through the interface variables. Each sensor optimizes its own objective vector function to achieve the global optimal solution of the whole network. In this way, for different requirements from the network, we do not redesign the framework, that is, the modules and interfaces in Figure 1 can be kept unchanged. We only need to introduce multiobjective methods to optimize the vector function \mathbf{O}_i in each module. This will greatly simplify network design for WSNs.

In WSNs, some parameters (e.g., the topology of the networks or channel condition) are time-varying. In [7], Lee et al. demonstrated that the state of the network can be more efficiently utilized to improve the performance of the network (e.g., increasing the throughput and reducing packet delay), by appropriately exploiting the variability of the time-varying channels. Also there are measurement errors in the implementation of distributed algorithms, such as the noisy feedback [8] or lossy links [9]. Therefore, we also characterize these random factors in our model. Our contributions in this paper are summarized as follows.

(1) We formulate a general multiobjective stochastic optimization problem for WSNs. We decompose the optimization problem through Lagrange dual decomposition and adopt the stochastic quasigradient method to solve the primal-dual problem. In other words, we transform the multiple objectives of WSNs into the multiple objectives of each individual sensor node. The global optimal solution can be obtained when each sensor node maximizes its own objective vector function. Therefore, our approach

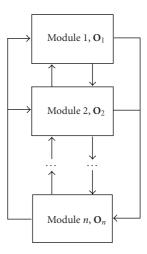


FIGURE 1: An illustration of our proposed framework.

provides a general framework for multiobjective optimization for WSNs.

- (2) We study the stability of the algorithm by using the knowledge of stochastic programming, and show that our algorithm for stochastic multiobjective optimization problem (ASMOP) can converge to the optimal solution of the primal problem.
- (3) We demonstrate how the general framework can be applied to different applications, by considering the rate allocation problem as an example. We introduce three multiobjective optimization methods: (1) constraint method, (2) linear weighted method, and (3) hierarchical sequence method. The three paradigms show that although different requirements may lead to different models [6, 10], we can solve them in the general framework.

The remainder of the paper is organized as follows: in Section 2, we discuss related work regarding the NUM problem and stochastic network utility maximization (SNUM). We formulate a general mathematical model and design a distributed algorithm to solve the problem in Section 3, and the stability of the algorithm is also discussed. We provide three paradigms in Section 4 to demonstrate the general framework for different applications. Simulation results are given in Section 5. We conclude the paper in Section 6.

2. Related Work

There are several research works in the literature studying the NUM problem [4, 11–14]. Kelly et al. were the first to propose the optimization approach, which provides a mathematical foundation for NUM problem [11]. In [12], Chiang adopted the NUM approach to obtain a cross-layer design including the physical layer and the transport layer. Zhu et al. [13] considered the energy model in the cross-layer design. In [1], Chiang et al. provided a mathematical theory of network architectures. Wang et al. studied joint interference-aware routing and TDMA link scheduling to

improve the throughput in multihop wireless networks [15]. Zhang et al. [8] elaborated on the impact of the feedback in the implementation of distributed NUM algorithms. Since feedback is often collected using error-prone measurement mechanisms, for example, biased estimator or unbiased estimator, they adopted the knowledge of stochastic approximation and proved stability of the algorithms of single-time scale and two-time scale. Lee et al. utilized the variation of channels to guide power and rate control in cross-layer design [7]. In this paper, we formulate a more general mathematical model by considering stochastic multiple objectives in objective functions. We apply our approach to rate allocation problem in multipath WSNs with time-varying channels. Rate allocation is a fundamental problem and has been extensively investigated [16–19]. Low and Lapsley [16] first introduced the Lagrange dual method to decompose the problem and proposed two algorithms under synchronous and asynchronous scenarios. A multipath formulation for rate control in multi-cast networks was proposed in [20], and three distributed algorithms were proposed to solve the problem. The goal is to maximize the aggregate utility. In [6], Srinivasan et al. considered two objectives: utility maximization and guaranteeing prespecified network lifetime for multipath wireless ad hoc networks. In [10], Zhu et al. also focused on the network lifetime and application performance (utility), and employed the linear weighted method from the multiobjective optimization to transform these two objective functions into a single one which was named to the utility-lifetime tradeoff function.

3. General Multiobjective Formulation and Solutions

Throughout the paper, we will denote sets by capital letters, variables by lowercase letters, vectors by bold lowercase letters, and matrices by bold capital letters. For a vector \mathbf{x} , we denote its *i*th component by x_i and its transpose by \mathbf{x}^T . We use capital letters for both the sets and the cardinality of sets

Consider S sensing nodes and N sink nodes in the region of interest. Let Ω be a probability space with a σ -algebra \mathcal{F} of random events, and have a finite set $\{\tau_m, m = 1, 2, ..., M\}$ with the corresponding probability $p(\tau_m) \geq 0$. There are n objective functions $P^{(i)}(\mathbf{x})$, i = 1, ..., n, defined on the subset χ of a Hilbert space. Let $\mathbf{P}(\mathbf{x}) = (P^{(1)}(\mathbf{x}), P^{(2)}(\mathbf{x}), ..., P^{(n)}(\mathbf{x}))$, $P^{(i)}(\mathbf{x}) = \sum_{s \in S} f_s^{(i)}(\mathbf{x})$. Then $\mathbf{P}_s(\mathbf{x}) = (f_s^{(1)}(\mathbf{x}), f_s^{(2)}(\mathbf{x}), ..., f_s^{(n)}(\mathbf{x}))$ is the objective vector function of sensing node s. Let $\mathbf{x}_s \in \chi_s$ be the column vector of variables of sensing node s, $\mathbf{x} = (\mathbf{x}_1^T, ..., \mathbf{x}_S^T)^T$, and $\mathbf{g}_s(\cdot)$ a column vector function. We can formulate the primal problem (\mathbf{PP}) as follows:

PP:
$$\mathbf{P}(\mathbf{x})$$
,
s.t.
$$\begin{cases} \sum_{m=1}^{M} p(\tau_m) \sum_{s \in S} \mathbf{g}_s(\mathbf{x}_s, \tau_m) \leq 0, \\ \mathbf{x}_s \in \chi_s, \quad \forall s \in S. \end{cases}$$
 (1)

The objective function, $f_s^{(i)}(\mathbf{x})$, may be a coupled one. In order to design a distributed algorithm, we introduce auxiliary variable \mathbf{y} to decouple it. Assume that the node set associated with coupled variables of $f_s^{(i)}(\mathbf{x})$ is $H^{(i)}(s)$, i = 1, 2, ..., n. Let $H(s) = \bigcup_{i=1}^n H^{(i)}(s)$, which denotes the node set associated with coupled variables of $\mathbf{P}_s(\mathbf{x})$ of sensing node s, then the decoupled primal problem (**DPP**) can be given by

DPP:
$$\mathbf{P}(\mathbf{x}, \mathbf{y}) = (P^{1}(\mathbf{x}, \mathbf{y}), \dots, P^{n}(\mathbf{x}, \mathbf{y})), \qquad (2)$$
s.t.
$$\begin{cases} \sum_{m=1}^{M} p(\tau_{m}) \sum_{s \in S} \mathbf{g}_{s}(\mathbf{x}_{s}, \tau_{m}) \leq 0, \\ \mathbf{y}_{ss'} = \mathbf{x}_{s'}, \\ \mathbf{x}_{s} \in \chi_{s}, \ \mathbf{y}_{ss'} \in \chi_{s'}, \\ \forall s \in S, \ s' \in H(s), \end{cases}$$
(3)

where $P^{(i)}(\mathbf{x}, \mathbf{y}) = \sum_{s \in S} f_s^{(i)}(\mathbf{x}_s, \mathbf{y}_s)$ is the *i*th decoupled objective function, and $\mathbf{y}_s = (\{\mathbf{y}_{ss'}\}_{s' \in H^{(i)}(s)})$ is the corresponding vector of auxiliary variables.

In our formulation, the objective functions are deterministic, taking the advantage that each sensing node can be obtained from the network. The constraint set contains the random factors of the networks, such as message exchange, and environmental effect. If we know the distribution, $p(\tau_m)$, $m=1,2,\ldots,M$, of τ , we can transform the problem into a deterministic one, by calculating the expectation. However, in WSNs, there is often no prior knowledge about the randomness from the networks themselves and the environmental effect. Therefore, we develop an algorithm without this prior knowledge, which can be achieved by the stochastic quasigradient method [7].

To decompose the problem, we take Lagrange dual approach. The Lagrange function [21] of (3) is given by

$$L(\mathbf{x}, \mathbf{y}, \boldsymbol{\lambda}, \boldsymbol{\mu}, \boldsymbol{\tau})$$

$$= \sum_{s \in S} \mathbf{P}_{s}(\mathbf{x}_{s}, \mathbf{y}_{s}) - \boldsymbol{\lambda}^{T} \sum_{m=1}^{M} p(\tau_{m}) \sum_{s \in S} \mathbf{g}_{s}(\mathbf{x}_{s}, \tau_{m})$$

$$+ \sum_{s \in S, s' \in H(s)} \mathbf{u}_{ss'}^{T} (\mathbf{x}_{s'} - \mathbf{y}_{ss'})$$

$$= \sum_{m=1}^{M} p(\tau_{m}) \left\{ \sum_{s \in S} \left\{ \mathbf{P}_{s}(\mathbf{x}_{s}, \mathbf{y}_{s}) - \boldsymbol{\lambda}^{T} \mathbf{g}_{s}(\mathbf{x}_{s}, \tau_{m}) + \mathbf{x}_{s'}^{T} \sum_{s': s \in H(s')} \mathbf{u}_{ss'} - \sum_{s' \in H(s)} \mathbf{u}_{ss'}^{T} \mathbf{y}_{ss'} \right\} \right\},$$

$$(4)$$

where $P_s(\mathbf{x}_s, \mathbf{y}_s)$ is the objective vector function of sensing node s. It is a formal expression which can be transformed into different objective functions for different applications.

We call λ and μ decoupled prices (λ is used to decouple the coupling of variables and μ is used to decouple the

coupling of objective functions). Since (4) is separable, we exploit the decomposable structure of Lagrangian function and decompose the problem into S subproblems. Maximization is achieved in each sensing node s, $s \in S$, with the knowledge of local variables $(\mathbf{x}_s, \mathbf{y}_s)$ and the current state τ , by solving the following optimization problem \mathbf{DP}_s .

$$\mathbf{DP}_s : \max \quad \mathbf{P}_s(\mathbf{x}_s, \mathbf{y}_s) - \boldsymbol{\lambda}^T \mathbf{g}_s(\mathbf{x}_s, \tau)$$

$$+ \left(\sum_{s':s \in H(s')} \mathbf{u}_{ss'} \right)^T \mathbf{x}_s - \sum_{s' \in H(s)} \mathbf{u}_{ss'}^T \mathbf{y}_{ss'}$$
 (5)

s.t.
$$\begin{cases} \mathbf{x}_s \in \boldsymbol{\chi}_s, \\ \mathbf{y}_{ss'} \in \boldsymbol{\chi}_{s'}. \end{cases}$$
 (6)

At iteration t, each sensing node s updates its resource variables \mathbf{x}_s and auxiliary variables \mathbf{y}_s according to

$$(\mathbf{x}_{s}, \mathbf{y}_{s}) = \underset{\substack{\mathbf{x}_{s} \in \mathcal{X}_{s} \\ \mathbf{y}_{ss'} \in \mathcal{X}_{s'}}}{\operatorname{max}} \mathbf{DP}_{s}.$$
(7)

We proceed to solve the dual problem. Let $D(\lambda, \mu, \tau) = \max_{\mathbf{x}, \mathbf{y}} L(\mathbf{x}, \mathbf{y}, \lambda, \mu, \tau)$. Then the dual problem (**DP**) is given by

$$\min_{\lambda \ge 0, \mu} D(\lambda, \mu, \tau).$$
(8)

At iteration t, each sensing node can acquire the state of random variables τ . The stochastic quasigradient method only needs this current state information of the system and utilizes it to form the stochastic subgradients of $D(\lambda, \mu, \tau)$ at iteration t. For the dual problem, **DP**, prices are updated according to

$$\lambda(t+1) = [\lambda(t) - \alpha(t)\vartheta(t)]^+, \tag{9}$$

$$\boldsymbol{\mu}_{ss'}(t+1) = \boldsymbol{\mu}_{ss'}(t) - \alpha(t)\boldsymbol{\nu}_{ss'}(t), \quad s' \in H(s), \tag{10}$$

where $\vartheta(t)$ and $\nu_{ss'}(t)$ are the stochastic quasignadients of $D(\lambda, \mu, \tau)$.

In our algorithm, we set

$$\alpha(t) = \frac{1}{t},\tag{11}$$

$$\vartheta(t) = -\sum_{s \in S} \mathbf{g}(\mathbf{x}_s(t), \tau(t)), \tag{12}$$

$$\mathbf{v}_{ss'}(t) = \mathbf{x}_{s'}(t) - \mathbf{v}_{ss'}(t), \quad s' \in H(s),$$
 (13)

where $\tau(t)$ is the state of τ at iteration t.

We summarize our algorithm for the general formulation of stochastic multiobjective optimization problem (ASMOP) in the Algorithm 1.

To prove that the algorithm can converge to the optimal solution of the primal problem, we make the following assumptions.

(1) $f_s^{(i)}(\cdot)$ as well as the objective vector function $\mathbf{P}_s(\cdot)$ (which can be transformed into a single function in applications), $s \in S$, i = 1, 2, ..., n, are twice continuous differentiable concave functions.

(2) $\mathbf{g}_s(\mathbf{x}, \tau)$, $s \in S$, are convex and twice continuous differentiable functions in \mathbf{x} , for all $\tau \in \Omega$.

Theorem 1. If (1) hold, then from an arbitrary point of $\mathbf{x}(0) \in \chi$, $\lambda(0) \geq 0$ and $\mathbf{y}_{ss'}(0) \in \chi_{ss'}, \boldsymbol{\mu}_{ss'}(0), s \in S, s' \in H(s)$, the sequence generated by (7), (9), and (10) converges. Every limit point $(\mathbf{x}^*, \mathbf{y}^*, \boldsymbol{\lambda}^*, \boldsymbol{\mu}^*)$ of the sequence $(\mathbf{x}(t), \mathbf{y}(t), \boldsymbol{\lambda}, \boldsymbol{\mu})$ is primal-dual optimal.

Proof. Let the sequences of iteration $\{\lambda(0), \lambda(1), \dots, \lambda(t)\}$ and $\{\mu(0), \mu(1), \dots, \mu(t)\}$ be generated by (9) and (10), respectively. Then to guarantee the convergence of the algorithm, according to [7, 22], the current stepsize and quasigradients $\alpha(t)$, $\vartheta(t)$, and $\nu(t)$ should be chosen such that

$$\alpha(t) \ge 0,$$
 $\sum_{t=0}^{\infty} \alpha(t) = \infty,$ $\sum_{t=0}^{\infty} (\alpha(t))^2 < \infty,$ (14)

$$E\{\vartheta(t) \mid \lambda(0), \dots, \lambda(t), \mu(0), \dots, \mu(t)\} = \partial_{\lambda} D(\lambda(t), \mu(t), \tau(t)),$$
(15)

$$E\{\boldsymbol{\nu}(t) \mid \boldsymbol{\lambda}(0), \dots, \boldsymbol{\lambda}(t), \boldsymbol{\mu}(0), \dots, \boldsymbol{\mu}(t)\} = \partial_{\boldsymbol{\mu}} D(\boldsymbol{\lambda}(t), \boldsymbol{\mu}(t), \boldsymbol{\tau}(t)).$$
(16)

It can be seen that $\alpha(t) = 1/n$, t = 0, 1, ..., satisfy (14). From [22]; we know that $\vartheta(t)$ and $\upsilon_{ss'}(t)$ from (12) and (13) also satisfy (15) and (16).

From assumptions (1) and (2), the primal function is concave and the dual function D is convex in λ and μ for a fixed τ . From (7), (9), (10), (11), (12), and (13), we can conclude that the sequence converges to the optimal solution by solving the dual problem [22]. As the primal problem is a convex optimization problem, there is no gap between the primal and dual problems. So the sequence $(\mathbf{x}^*, \mathbf{y}^*, \lambda^*, \mu^*)$ generated by the algorithm is primal-dual optimal.

Remarks. Because of multipath routing, the problem, \mathbf{DP}_s , may not be strictly concave even if $\mathbf{P}_s(\cdot)$ is strictly concave. This may lead to oscillation of the sequences generated by the algorithms. There are several ways to cope with this problem. For example, we can first add some augmented variables to \mathbf{DP}_s and adopt the first-order Lagrangian method to solve it [23].

The main difference of our proposed approach is that we adopt the knowledge of multiobjective optimization and provide some potential interfaces for each layer. In this way, we can take the advantages of both the layered architectures and cross-layer design. In other words, we can implement different algorithms in each module according to specific applications. In Figure 1, λ and μ act as the interface variables between different modules and sensor nodes. Through λ and μ , the network architectures can be decomposed into different modules and each module fulfills corresponding functionality distributively. From (7), we can transform the multiple objectives of the whole network into the multiple objectives of each sensor node. Optimizing the objective vector function of each sensor node can achieve the global optimal solution. Therefore, it is very convenient to implement algorithms in each module i to solve the objective vector function independently according to different requirements.

4. Paradigms of Objective Optimization in WSNs

In the proposed general framework, $P_s(\mathbf{x}_s, \mathbf{y}_s)$ in \mathbf{DP}_s is a vector function and can be transformed into a single function according to different requirements. Therefore, solving \mathbf{DP}_s is application-dependent, which provides the flexibility of solving a class of applications by the general framework.

In this section, we consider the rate allocation problem as an example and show how the general framework works. Rate allocation problem is a well-investigated problem [24], and has different requirements for different applications. There are usually three methods to cope with the requirements: (1) Constraint Method [6], (2) Linear Weighted Method [10], and (3) Hierarchical Sequence Method. While these methods are extensively studied in existing works, we can integrate these methods together into the general framework. Hence, our approach can be applied to a class of applications with different background, which will offer significant convenience to the designers.

- 4.1. Preliminary Knowledge. In this section, we give a brief introduction to the three multiobjective methods.
 - (1) The constraint method tries to solve the multiobjective problems by placing the most important one in the objective function, while other objective functions are constrained within the constraint set. In other words, constraint method can solve the following \mathbf{DP}_s of each sensing node s. (without loss of generality, we assume that $f_s^{(1)}$ is the most important objective function)

$$\mathbf{DP}_{s} : \max \quad f_{s}^{(1)}(\mathbf{x}_{s}, \mathbf{y}_{s}) - \boldsymbol{\lambda}^{T} \mathbf{g}_{s}(\mathbf{x}_{s}, \tau)$$

$$+ \left(\sum_{s': s \in H(s')} \mathbf{u}_{ss'} \right)^{T} \mathbf{x}_{s} - \sum_{s' \in H(s)} \mathbf{u}_{ss'}^{T} \mathbf{y}_{ss'}$$

$$\text{s.t.} \quad \begin{cases} \mathbf{x}_{s} \in \boldsymbol{\chi}_{s}, \\ \mathbf{y}_{ss'} \in \boldsymbol{\chi}_{s'}, \\ f_{s}^{(j)}(\mathbf{x}_{s}, \mathbf{y}_{s}) \leq A_{s}^{(j)}, \quad j = 2, \dots, n, \end{cases}$$

$$(17)$$

where $A_s^{(j)}$, j = 2,...,n, are constant constraints imposed by applications.

(2) The linear weighted method focuses on solving the multiobjective optimization problems by first associating each objective function $f_i(\cdot)$ with a weight y_i and then taking the weighted sum as a new objective function. Using linear weighted method to solve \mathbf{DP}_s ,

the variables in each sensing node s are updated according to

$$\mathbf{DP}_{s} : \max \sum_{j=1}^{n} \gamma_{j} f_{s}^{(j)}(\mathbf{x}_{s}, \mathbf{y}_{s}) - \boldsymbol{\lambda}^{T} \mathbf{g}_{s}(\mathbf{x}_{s}, \tau)$$

$$+ \left(\sum_{s': s \in H(s')} \mathbf{u}_{ss'} \right)^{T} \mathbf{x}_{s} - \sum_{s' \in H(s)} \mathbf{u}_{ss'}^{T} \mathbf{y}_{ss'} \quad (18)$$
s.t.
$$\begin{cases} \mathbf{x}_{s} \in \boldsymbol{\chi}_{s}, \\ \mathbf{y}_{ss'} \in \boldsymbol{\chi}_{s'}, \end{cases}$$

where γ_j , j = 1, ..., n, are weight coefficients.

(3) The hierarchical sequence method is concerned with solving the multiobjective optimization problems sequentially, that is, solving the most important problem first and then the less important problems. Let $\mathcal{L} = \{\mathbf{x} \mid \mathbf{x}_s \in \chi_s, \mathbf{y}_{ss'} \in \chi_{s'}\}$ and divide (5) into n functions $F_j(\mathbf{x}_s, \mathbf{y}_s)$, j = 1, 2, ..., n. Maximization is achieved by solving the following n subproblems sequentially.

$$\mathbf{DP}_{s} : \begin{cases} (1) & F_{1}^{*} = \max_{\mathbf{x} \in J} F_{1}(\mathbf{x}), \\ (2) & F_{2}^{*} = \max_{\mathbf{x} \in J \cap \{\mathbf{x} \mid F_{1} \geq F_{1}^{*}\}} F_{2}(\mathbf{x}), \\ \vdots & \vdots \\ (n) & F_{n}^{*} = \max_{\mathbf{x} \in J \cap \{\mathbf{x} \mid F_{1} \geq F_{n}^{*}, i=1, \dots, n-1\}} F_{n}(\mathbf{x}), \end{cases}$$

$$(19)$$

4.2. Rate Allocation Problem under ASMOP Formulation. In this section, we consider the rate allocation problem with two objectives: (1) maximizing aggregate utility and (2) prolonging the network lifetime.

Assume the sensing nodes can transmit their rates to the sink nodes over a set $L = \{1, 2, ..., L\}$ of links, each of which has capacity $c_l, l \in L$. Each sensing node s can transmit its rate through $R(s) \subseteq R$ of the routes. Each route $r \in R(s)$ traverses over a set $L(r) \subseteq L$ of links with a rate x_{sr} . Let \mathbf{x}_s be the rate vector of sensing node s, $S(l) = \{s \in S \mid s \in S$ $l \in L(r), r \in R(s)$ the set of sensing nodes using link l, and R(s, l) the subset of routes R(s) used by sensing node s to traverse over link l. We denote the set of sensing nodes that use sensing node s as an interim relay node by S(s) (not including the sensing node s itself). Let R(s, s') be the subset of routes R(s') which use sensing node s as a relay node and S(r) the relay nodes used by route r. Let M be finite number of state that the channels have and $p(\tau_m)$ the probability of the state τ_m , $m \in M$. Each sensing node s is characterized by three parameters $(U_s(\cdot), \underline{b}_s, \overline{b}_s)$, where $U_s: \Re_+ \to \Re$ is a strictly concave utility function, $\underline{b}_s \ge 0$ and $\overline{b}_s < \infty$ which are the required minimum and maximum transmission rates for each sensing node s, respectively.

From [11, 16], we know utility maximization can be formulated as follows.

$$\max \sum_{s \in S} U_s(\mathbf{x}_s) = \sum_s U_s \left(\sum_{r \in R(s)} x_{sr} \right)$$
 (20)

s.t.
$$\begin{cases} \sum_{s \in S} \sum_{r \in R(s,l)} x_r \leq \sum_{m \in M} p(\tau_m) c_l^m, \\ \underline{b}_s \leq \sum_{r \in R(s)} x_{sr} \leq \overline{b}_s, \quad \forall s \in, \\ x_{sr} \geq 0, \quad \forall s \in S, \ r \in R(s), \end{cases}$$
(21)

where $(p(\tau_1), p(\tau_2), \dots, p(\tau_M))$ describes the distribution of the states of link channel condition and c_l^m is the capacity of link l under state τ_m . We will establish algorithms that can guarantee convergence without prior knowledge of the underlying probability distribution of the system channel

The network lifetime is often defined as the time interval between initialization of the network and the exhaustion of the battery of the first sensing node. The total power dissipation, w_s , at sensing node s is equal to

$$w_{s} = \sum_{s' \in S(s)} \sum_{r \in R(s,s')} (w^{re} + w_{sr}^{t}) x_{r} + \sum_{r \in R(s)} w_{sr}^{t} x_{r},$$
(22)

where w_{sr}^t and w^{re} are the energy consumptions at sensing node s for transmitting or receiving unit data flow over route

Let e_s denote the initial energy of sensing node s. Its lifetime T_s is $T_s = e_s/w_s$. Following [10], we have the energy model for the network lifetime:

$$\max \quad -\sum_{s \in S} \frac{1}{\beta - 1} z_s^{\beta - 1} \tag{23}$$

s.t.
$$w_s = e_s z_s$$
, $s \in S$, (24)

where $z_s = 1/T_s$.

We can have the multiobjective model for rate allocation problem:

$$\mathbf{PP} : \max \quad \mathbf{P} = \left(P^{(1)}(\mathbf{x}), P^{(2)}(\mathbf{x}) \right) \tag{25}$$

where

$$P^{(1)}(\mathbf{x}) = \sum_{s} U_{s}(x_{s}),$$

$$P^{(2)}(\mathbf{x}) = -\sum_{s} \frac{\omega}{\beta - 1} z_{s}^{\beta - 1}.$$
(27)

Here, parameter ω scales the values of the two objective functions into the same order of magnitude.

4.3. Algorithm Design. Similar to (3), the decoupled form of (25) is

$$\mathbf{DPP} : \max \quad \mathbf{P} = \left(P^{(1)}(\mathbf{x}), P^{(2)}(\mathbf{x}, \mathbf{y}) \right) \tag{28}$$

s.t.
$$\begin{cases} y_{ss'r} = x_{s'r}, \forall s' \in S(s), r \in R(s, s'), \\ \text{constraints} \quad (21), (22), (24), \end{cases}$$
 (29)

where $P^{(2)}(\mathbf{x}, \mathbf{y}) = -\sum_{s} (\omega/(\beta - 1)) z_s^{\beta - 1}(\mathbf{x}, \mathbf{y})$ and $z_s(\mathbf{x}, \mathbf{y})$ is

$$z_{s}e_{s} = \sum_{s' \in S(s)} \sum_{r \in R(s,s')} (w^{re} + w_{sr}^{t}) y_{ss'r} + \sum_{r \in R(s)} w_{sr}^{t} x_{sr}.$$
 (30)

Then we have the Lagrange function:

 $L(\mathbf{x}, \mathbf{y}, \lambda, \boldsymbol{\mu}, \tau)$

$$= \sum_{m \in M} p(\tau_m) \left\{ \sum_{s \in S} \left\{ \mathbf{P}_s(\mathbf{x}_s, \mathbf{y}_s) - \sum_{r \in R(s)} x_{sr} (\lambda^r - \mu^{sr}) - \sum_{s' \in S(s)} \sum_{r \in R(s, s')} \mu_{ss'r} y_{ss'r} \right\} + \sum_{l \in L} \lambda_l c_l^m \right\},$$
(31)

where $\lambda^r = \sum_{l \in L(r)} \lambda_l$ and $\mu^{sr} = \sum_{s' \in S(r)} u_{s'sr}$. Notice that for given λ^r , μ^{sr} , μ^{sr} , $\mu^{ss'}$, $s' \in S(s)$, $r \in R(s, s')$, the update for each sensing node s is deterministic, no matter what the channel condition is. So at iteration t, each sensing node s updates its rate \mathbf{x}_s and auxiliary variable \mathbf{y}_s by solving the optimization below

$$\mathbf{DP}_{s}: \max \quad \mathbf{P}_{s}(\mathbf{x}_{s}, \mathbf{y}_{s}) - \sum_{r \in R(s)} x_{sr} (\lambda^{r} - \mu^{sr})$$

$$- \sum_{s' \in S(s)} \sum_{r \in R(s, s')} \mu_{ss'r} y_{ss'r}$$

$$\text{s.t.} \quad \begin{cases} \underline{b}_{s} \leq \sum_{r \in R(s)} x_{sr} \leq \overline{b}_{s}, x_{sr} \geq 0, \\ y_{ss'r} \geq 0, \quad \forall s' \in H(s), \ r \in R(s), \end{cases}$$

$$(32)$$

where $\mathbf{P}_s(\mathbf{x}_s, \mathbf{y}_s) = (U_s(\mathbf{x}_s), -(\omega/(\beta-1))z_s^{\beta-1})$ is the objective vector function for sensing node s.

At iteration t, t = 1, 2, ..., sensing node s first receives λ^r , μ^{sr} and $\mu_{ss'r}$, $s' \in S(s)$, $r \in R(s,s')$, from the network, and then updates its rate and auxiliary variables according to

$$(\mathbf{x}_s(t), \mathbf{y}_s(t)) = \operatorname{Arg\,max} \mathbf{DP}_s. \tag{33}$$

Notice that the update of decoupled prices, μ , does not depend on the channel condition. At each iteration t, with $x_{s'r}$, $y_{ss'r}$, $s' \in S(s)$, $r \in R(s,s')$, being collected, the decoupled price μ is updated according to

$$\mu_{ss'r}(t+1) = \mu_{ss'r}(t) - \alpha(t) (x_{s'r}(t) - y_{ss'r}(t)).$$
 (34)

To update price λ , without prior knowledge about the distribution of the channel state, each sensing node s can measure channel condition and get the link capacity $\hat{c}_l(t)$ for each link l at time t. Then λ can be updated according to

$$\lambda_l(t+1) = \left[\lambda(t) - \alpha(t)\left(\hat{c}_l(t) - x^l(t)\right)\right]^+. \tag{35}$$

So far, we have introduced interface variables (μ and λ) for a fully distributed implementation and provided a framework for rate allocation in WSNs with time-varying channels. Next, we show how to apply the framework to applications under different methods.

4.4. Paradigms: Applications under Different Methods. (1) Constraint Method. To maximize the utility of a network under the condition that the network lifetime exceeds a prespecified threshold time *T*, the constraint method can be used to solve the optimization problem below of each sensing node *s*.

$$\mathbf{DP}_{s} : \max \quad U_{s}(\mathbf{x}_{s}) - \sum_{r \in R(s)} x_{sr} (\lambda^{r} - \mu^{sr})$$

$$- \sum_{s' \in S(s)} \sum_{r \in R(s,s')} \mu_{ss'r} y_{ss'r}$$
(36)

s.t.
$$\begin{cases} z_s \le \frac{1}{T}, \\ \underline{b}_s \le \sum_{r \in R(s)} x_{sr} \le \overline{b}_s, \quad x_{sr} \ge 0, \end{cases}$$
(37)

(2) Linear Weighted Method. In [10, 13], Zhu et al. considered the tradeoff between lifetime and rate allocation. By introducing the weight parameter, γ , to evaluate the importance of the two objectives, they can be combined into a single one. For (29), the desired tradeoff between network utility and lifetime can be achieved by solving the optimization problem below.

$$\mathbf{DP}_{s} : \max \quad \gamma U_{s}(\mathbf{x}_{s}) - (1 - \gamma)\omega \frac{1}{\beta - 1} z_{s}^{\beta - 1}(\mathbf{x}, \mathbf{y})$$

$$- \sum_{r \in R(s)} x_{sr} (\lambda^{r} - \mu^{sr}) - \sum_{s' \in S(s)} \sum_{r \in R(s, s')} \mu_{ss'r} y_{ss'r}$$
(38)

s.t.
$$\begin{cases} \underline{b}_{s} \leq \sum_{r \in R(s)} x_{sr} \leq \overline{b}_{s}, & \forall s \in S, \\ x_{sr} \geq 0, \ y_{ss'r} \geq 0, \ s' \in H(s), \ r \in R(s), \end{cases}$$
(39)

where $y \in [0, 1]$, is the weight coefficient.

(3) Hierarchical Sequence Method. In our rate allocation paradigm, we have two objectives: (1) find a rate allocation strategy to maximize the total utility and (2) prolong the networks lifetime. To the best of our knowledge, this method has not been applied to the rate allocation problem before. We can achieve the two objectives by solving the two subproblems below sequentially.

(i):
$$\mathbf{l}^* = \arg\max U_s(\mathbf{x}_s) - \sum_{r \in R(s)} x_{sr} \lambda^r$$
s.t.
$$\begin{cases} \underline{b}_s \le \sum_{r \in R(s)} x_{sr} \le \overline{b}_s, & \forall s \in S, \\ x_{sr} \ge 0, & \forall s \in S, \ r \in R(s), \end{cases}$$
(ii):
$$\max \quad -\frac{\omega}{\beta - 1} z_s^{\beta - 1}(\mathbf{x}_s, \mathbf{y}_s) + \sum_{r \in R(s)} x_{sr} \mu^{sr}$$

$$-\sum_{s' \in S(s)} \sum_{r \in R(s, s')} \mu_{ss'r} y_{ss'r}$$

$$\text{s.t.} \quad \mathbf{x}_s \in \mathbf{l}^*, \qquad y_{ss'r} \ge 0.$$
(40)

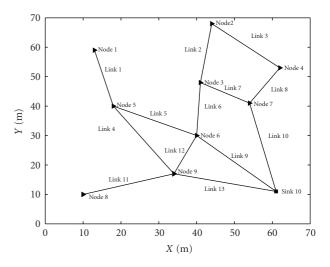


FIGURE 2: Topology of the simulated WSN.

It is sufficient to employ optimization methods to solve (37), (39), or (40), and (41) for different applications, while λ and μ updates are kept unchanged (according to (34) and (35)).

5. Simulation Results

5.1. Simulation Setting. We use 9 sensing nodes and 1 sink node in our simulations. These sensor nodes are randomly deployed in an area of size 70 × 70 to perform a sensing task. The randomly generated topology of the sensor nodes is shown in Figure 2, in which sensing nodes are marked by triangle icons and the sink node is marked by a square icon. In our simulations, we only focus on the rate allocation problem. The study of the routing in the network layer is beyond the scope of our paper. We assume that there are 15 routes available for data transmission. $L(r_1) = \{l_1, l_5, l_9\},\$ $L(r_2) = \{l_1, l_4, l_{13}\}, L(r_3) = \{l_2, l_6, l_9\}, L(r_4) = \{l_2, l_7, l_{10}\},$ $L(r_5) = \{l_3, l_8, l_{10}\}, L(r_6) = \{l_6, l_{12}, l_{13}\}, L(r_7) = \{l_7, l_{10}\},$ $L(r_8) = \{l_8, l_{10}\}, L(r_9) = \{l_5, l_9\}, L(r_{10}) = \{l_4, l_{13}\}, L(r_{11}) =$ $\{l_9\}, L(r_{12}) = \{l_{12}, l_{13}\}, L(r_{13}) = \{l_{10}\}, L(r_{14}) = \{l_{11}, l_{13}\},$ $L(r_{15}) = \{l_{13}\}$. Every sensing node s can transmit its sensing data to the sink node from a set of routes: $R(s_1) = \{r_1, r_2\},\$ $R(s_2) = \{r_3, r_4, r_5\}, R(s_3) = \{r_6, r_7\}, R(s_4) = \{r_8\}, R(s_5) =$ $\{r_9, r_{10}\}, R(s_6) = \{r_{11}, r_{12}\}, R(s_7) = \{r_{13}\}, R(s_8) = \{r_{14}\}, \text{ and }$ $R(s_9) = \{r_{15}\}.$

We have two objectives: maximizing the aggregate utility and the network lifetime. For the utility objective, we set $U_s(\cdot) = \xi_s \log(\cdot)$ for each sensor node s, where $\xi = (52, 54, 56, 58, 60, 62, 64, 66, 68)$. From [10], the function $-\sum_s (\omega/(\beta-1)) z_s^{\beta-1}$ can have a ratio higher than 0.95 to approximate the original lifetime problem T when $\beta \geq 8$. In our simulations, we use $\beta = 9$. The link capacities vary from time to time according to a uniform distribution with the expected capacities of links 1–13 to be $\mathbf{c} = (2000, 2000, 2200, 2000, 2000, 2500, 2800, 3500, 3500, 2000, 3000, 2800)$ (bit/s). For the energy consumption model, from [10], w^{re} is a constant and $w_{sr} = \rho + \sigma d_{sr}^{\eta}$,

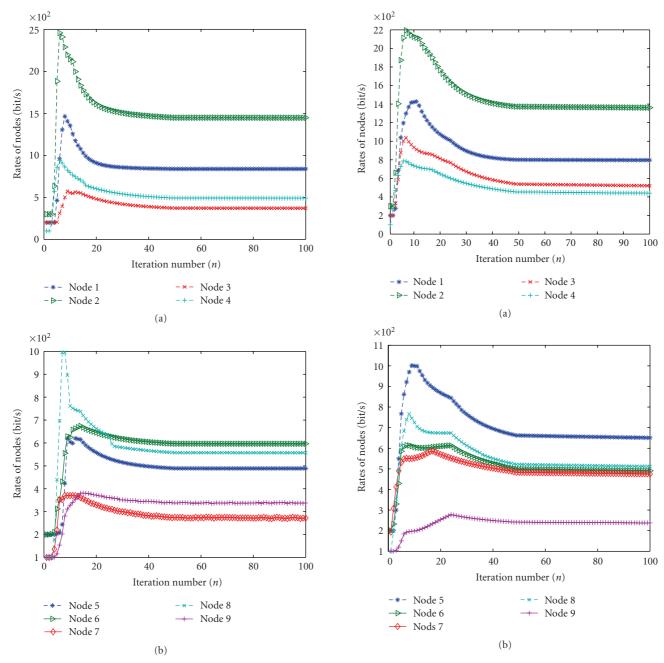


FIGURE 3: Convergence of the ASMOP algorithm by using constraint method.

FIGURE 4: Convergence of the ASMOP algorithm by using linear weighted method.

where d_{sr} is the length of the outgoing link of sensing node s for transmitting rate of route r. We set $\rho = 50$ nJ/bit, $\sigma = 0.0013$ pJ/bit/m⁴, $\eta = 4$, $w^{re} = 50$ nJ/bit. The initial energy of the sensing nodes 1–9 is set to be $\mathbf{e} = (450, 450, 475, 475, 450, 500, 500, 450, 500)$ (J) and the sink node (node 10) is assumed to have enough energy. The minimum and maximum rates of each sensing node are set to be $\underline{b}_s = 100$ and $\overline{b}_s = 2500$, respectively.

5.2. Simulation Results for Paradigms. First, we show the performance of the ASMOP algorithm by using the constraint method. Let the threshold of the network lifetime $T=\frac{1}{2}$

800 h. We collect the rate of each route r at each iteration. For each sensing node s, the rates are updated according to (37) and the decoupled price μ is updated according to (34). Each link l first collects information about the channel condition and computes its corresponding capacity, then updates its link price according to (35). The results are shown in Figure 3. Since there are 9 sensing nodes and 15 routes in our simulations, due to space limitation, we only show the aggregate rate of each sensing node. In Figure 3, we can be see that the rates first change sharply and then converge to the optimal one, which indicates the effectiveness of our ASMOP algorithm.

(1) Price update algorithm: At times t = 1, 2, ..., decoupled prices are updated according to $\lambda(t+1) = [\lambda(t) - \alpha(t)\theta(t)]^{+}$

 $\boldsymbol{\mu}_{ss'}(t+1) = \boldsymbol{\mu}_{ss'}(t) - \alpha(t)\boldsymbol{\nu}_{ss'}(t), \ s' \in H(s),$

(2) Sensor node s's Algorithm: At time $t=1,2,\ldots$, each sensing node s updates its variables according to

$$(\mathbf{x}_s, \mathbf{y}_s) = \underset{\mathbf{y}_{ss'} \in \mathbf{\chi}_{s'}}{\operatorname{Arg}} \underset{\mathbf{y}_{ss'} \in \mathbf{\chi}_{s'}}{\operatorname{DP}_s}$$

ALGORITHM 1: ASMOP.

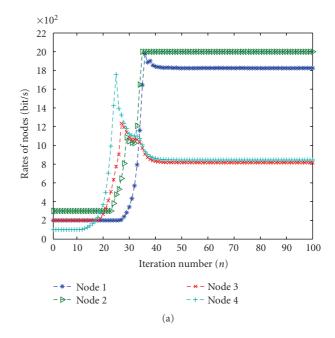
Next, we show the simulation results of the ASMOP algorithm using the linear weighted method. Here we set $\omega=8.17\times10^{26}$ and $\gamma=0.7$. At each iteration t, each sensing node s updates its rates according to (39), and the decoupled price λ and μ are updated as the same in paradigm I. The results are shown in Figure 4, and similar conclusions can be made as in the paradigm I.

A similar simulation is performed for the hierarchical sequence method and the corresponding results are shown in Figure 5. We can see that the rates change sharply at the beginning of each iteration, and then converge to the optimal one in Figure 5.

We further set the threshold of the network lifetime to be 800 h in the simulation for the constraint method and $\gamma = 0.7$ for linear weighted method. These two methods mainly target the energy-constraint in WSNs. For the hierarchical sequence method, we focus on the utility of the network. From Figures 3, 4, and 5, it can seen that the rates in Figure 5 are much larger than those in Figures 3 and 4. On the other hand, as the rates in each sensor node become large, the energy consumption increases. So the network lifetime under the hierarchical sequence method is less than that under the constraint method or linear weighted method. In addition, different multiobjective methods obtain different network performances. The results of the three simulations also demonstrate the efficiency and convenience of our proposed framework.

6. Conclusions

In this paper, we have proposed a general stochastic multiobjective optimization framework for WSNs. Our approach inherits advantages of both layered architectures and cross-layer design. Therefore, even the requirements and objectives are changed, it is not necessary to redesign the optimization framework but to have minor modifications of specific modules to meet the corresponding requirements. Although there may be uncertainty in WSNs, our approach can still achieve desired performance. In our future work, we will focus on investigating the general multiobjective optimization problem, instead of transforming the multiple objectives into a single one. We will study the distributed algorithms to optimize the objective vector function under some criteria, for example, Pareto optimality.



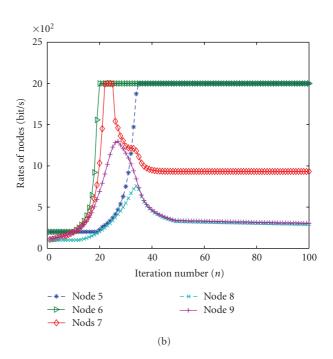


FIGURE 5: Convergence of the ASMOP algorithm by using hierarchical sequence method.

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