

## Research Article

# Resource Allocation with EGOS Constraint in Multicell OFDMA Communication Systems: Combating Intercell Interference

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Resource (power and bandwidth) allocation is an important issue in orthogonal frequency division multiple access (OFDMA) systems. For multicell systems, the interference across different cells makes the optimization of resource allocation difficult. For finite systems, a constraint on the rise over thermal (ROT) is placed to alleviate the intercell interference. A hybrid scheme with equal receive power and peak transmit power is shown to be optimal for the ROT constrained case. Large system analysis is applied for multi-cell OFDMA systems with the fairness constraint of equal grade of service (EGOS). An interference function is defined to model the intercell interference. Variational analysis is used to compute the optimal profile of transmit power and bandwidth. The optimal resource allocation is then computed using numerical simulations.

## 1. Introduction

Orthogonal frequency division multiple access (OFDMA) has become the fundamental signaling technique for uplink 4G wireless communication systems (e.g., Worldwide Interoperability for Microwave Access (WiMAX); although single-carrier frequency division multiple access (SC-FDMA) is used in uplink Long-Term Evolution (LTE) systems, the structure is still similar to OFDMA). One key issue in OFDMA systems is resource allocation, mainly the allocation of transmit power and subcarriers (or equivalently bandwidth), such that a tradeoff between the total throughput and fairness can be achieved. Plenty of studies have been done in this area [1–6], most of which are focused on single-cell systems. Since the transmissions of different users within the same cell are nearly orthogonal in OFDMA systems, many researches ignore the interference or consider only intersubcarrier interference within the cell. However, as illustrated in Figure 1, there are typical multiple cells in practical systems and the intercell interference incurs substantial performance degradation, thus being the bottleneck of system performance. Therefore, the resource allocation scheme obtained from the single-cell case may not apply for practical systems.

In contrast to the analysis on single-cell OFDMA systems, the study on multi-cell OFDMA systems is much less [7–12]. Most studies focus on the collaboration across different cells, for example, using noncooperative game theory. However, in practical systems, it incurs much overhead to maintain frequent coordinations across different cells. In [13, 14], the power control of multiple cells is studied without the coordination among multiple cells. Different from this paper, they focus on the down link and do not consider the allocation of bandwidth.

Therefore, in this paper, we study *the situation where there is no explicit cooperation across different cells while each user's resource (power and bandwidth) is stationarily allocated with the awareness of inter-cell interference*. It can be applied in practical OFDMA systems as an open-loop control strategy and provides reasonably good initial values for inter-cell coordination-based dynamic algorithms. The key difficulty is how to model and handle the intercell interference. One approach is to place a constraint on the total received power at the base station, which is widely used in industry. Then, the problem of resource allocation is converted into a constrained optimization. Such an approach is applicable in finite systems. An alternative approach is to apply the large system analysis,

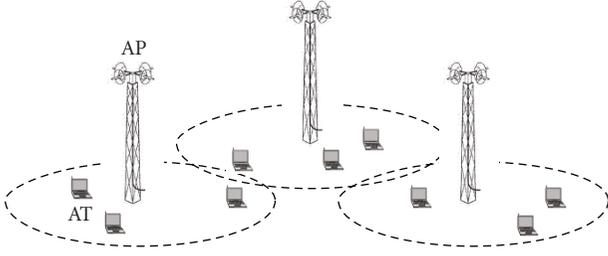


FIGURE 1: Illustration of multi-cell OFDMA systems.

that is, the number of active users tends to infinity, to alleviate this difficulty by defining an interference function. Variational analysis is then used to obtain a functional equation characterizing the optimal transmit power profile. Note that a fairness criterion is necessary for the resource allocation. For simplicity, we used the criterion equal grade of service (EGOS), that is, the throughput of each user within the same cell should be the same. It is straightforward to extend the framework of analysis proposed in this paper to other criteria like proportional fairness.

The remainder of this paper is organized as follows. The system model is introduced in Section 2. The resource allocation in finite systems is discussed in Section 3. The optimal resource allocation is derived for large systems in Section 4. Numerical results and conclusions are provided in Sections 5 and 6, respectively.

## 2. System Model

We consider the uplink of a multicell OFDMA system. For simplicity of analysis, the following assumptions are placed.

- (i) Suppose that each cell has  $J$  neighboring cells. We assume that there are  $K$  access terminals (ATs), namely users, and one access point (AP), namely base station, in each cell. It is straightforward to extend the discussion to the case of different numbers of ATs in different cells. We denote by  $g_k^j$  the channel power gain of AT  $k$  in cell  $j$ , whose cumulative distribution is denoted by  $F$ . We assume that the channel state information is perfectly known, which can be achieved by letting ATs sending pilots for channel estimation.
- (ii) The communication of ATs is confined by peak transmit power  $P_{\max}$  and total bandwidth  $W$ . We denote by  $P_k^j$  and  $W_k^j$  the transmit power and bandwidth allocated to AT  $k$  at cell  $j$ , which satisfies  $\sum_{k=1}^K W_k^j = W$  and  $0 \leq P_k^j \leq P_{\max}$ . We assume that the bandwidth is sufficiently large such that we can consider the allocated bandwidth as a continuous real number, thus simplifying the analysis.
- (iii) We denote by  $R_k^j$  the transmission rate of AT  $k$  at cell  $j$  and assume the EGOS fairness within the same cell, that is,  $R_1^j = R_2^j = \dots = R_K^j = R^j$ .

- (iv) We assume that the transmitters carry out frequency hopping in every OFDM symbol period and the hopping sequence is pseudo random. Therefore, the inter-cell interference is averaged and can be considered as Gaussian noise. Note that, if opportunistic subcarrier allocation is used, the assumption random frequency hopping is no longer valid. Then, the inter-cell interference is not averaged and thus becomes frequency selective (e.g., an edge user will cause more interference to the neighboring base station in the subcarriers allocated to this user). In such a scenario, the analysis becomes more complicated since the interference power spectral density is a function of the resource allocation. A new approach is needed for finding the optimal or near-optimal scheduling policy, which is beyond the scope of this paper.

- (v) For theoretical analysis, we use Shannon capacity to evaluate the reliable transmission rate, that is,

$$R_k^j = W_k^j \log \left( 1 + \frac{g_k^j P_k^j}{(N_0 + \mathcal{I}_j) W_k^j} \right), \quad (1)$$

where  $N_0$  and  $\mathcal{I}_j$  are the power spectral densities (PSDs) of additive white Gaussian noise and interference. Note that the interference PSD is identical for all ATs with the same cell due to the assumption of frequency hopping. For suboptimal coding schemes, we can use the following expression to approximate the reliable transmission rate:

$$R_k^j = W_k^j \log \left( 1 + \frac{g_k^j P_k^j}{G(N_0 + \mathcal{I}_j) W_k^j} \right), \quad (2)$$

where  $G > 1$  is the gap to Shannon capacity [15].

- (vi) Each cell has no information about the ATs of other cells, for example, locations, channel gains, and transmit powers. Therefore, the resource allocation is carried out within individual cells separately.

Based on the above assumptions, the optimal resource allocation can be formulated as an optimization problem, which is given by

$$\begin{aligned} & \max_{\{P_k^j\}, \{R_k^j\}} \sum_{j=1}^N R^j, \\ & \text{s.t. } P_k^j \leq P_{\max}, \quad \forall k, j, \\ & \sum_{k=1}^K W_k^j \leq W, \quad \forall j, \\ & R_1^j = R_2^j = \dots = R^j, \quad \forall j, \end{aligned} \quad (3)$$

where  $N$  is the total number of cells under consideration. Essentially, the optimization maximizes the total throughput of all cells, under the constraint of limited transmit power and limited total bandwidth. It is difficult to solve the optimization problem via explicit analysis for finite systems.

Moreover, this is not a convex optimization problem since the equality constraint is not affine. Therefore, we propose a heuristic approach for finite systems in the next section. In Section 4, we will use large system analysis to alleviate the key difficulties in finite systems.

### 3. Finite System Analysis: Constrained ROT

For finite systems, that is,  $K \leq \infty$ , it is computationally prohibitive to carry out the precise optimization across different cells, particularly when  $K$  is large. Moreover, it requires coordinations cross different cells, which contradicts the assumption of no explicit inter-cell coordinations. Therefore, we apply a heuristic approach for the resource allocation in each cell without the information of other cells. For simplicity of notation, we ignore the cell index,  $j$ , in all notations since we focus on only one cell.

The heuristic approach is to confine the total received power at AP (Rise Over Thermal, ROT) (the standard definition for ROT should be the ratio of the total received power over the thermal noise power. For simplicity, we define ROT as the total received power since the noise power is fixed and is known), which is based on the intuition that ROT is roughly proportional to the transmit powers of ATs within the desired cell and also positively correlated with the interference to other cells. Note that the metric of ROT is used to measure the congestion level of the cell in code-division multiple access (CDMA) systems [16]. Meanwhile, there is also requirement for ROT from the hardware viewpoint; if the ROT is too high, that is, the total received power is too high, weak signals may be lost due to the limited number of levels of analog-digital converter (ADC).

Then, the optimization problem in (3) can be approximated by

$$\begin{aligned}
 & \max_{\{P_k\}, \{R_k\}} R, \\
 \text{s.t. } & P_k \leq P_{\max}, \quad \forall k, \\
 & \sum_{k=1}^K W_k \leq W, \\
 & R_1 = R_2 = \dots = R, \\
 & \sum_k^K P_k g_k \leq \text{ROT}_{\max}.
 \end{aligned} \tag{4}$$

For simplicity, we assume that

$$P_{\max} \sum_{k=1}^K g_k > \text{ROT}_{\max}. \tag{5}$$

Otherwise the ROT constraint is useless.

**3.1. Feasibility.** First, we need to study the feasibility of the optimization, which is assured by the following lemma. The proof is given in Appendix A.

**Lemma 1.** *There exist feasible resource allocation schemes satisfying the constraints.*

The following lemma shows that, if we can find a new resource allocation scheme (not necessarily satisfying the EGOS constraint) such that the transmission rates are improved or remain unchanged for every AT, compared to an old EGOS allocation scheme, we can always find an EGOS resource allocation scheme that is better than the old one. The proof is given in Appendix B.

**Lemma 2.** *For a  $K$ -AT cell, suppose that a resource allocation  $\{P_k\}_{k=1,\dots,K}$  and  $\{W_k\}$  satisfies the EGOS constraint. If there exist a different resource allocation scheme  $\{P'_k\}_{k=1,\dots,K}$  and  $\{W'_k\}_{k=1,\dots,K}$  such that there exists a  $k$  such that  $R'_k > R_k$  and  $R'_i \geq R_i$  for  $i \neq k$  ( $R'_i$  and  $R'_j$  are not necessarily equal), we can always find an EGOS allocation scheme  $\{P''_k\}_{k=1,\dots,K}$  and  $\{W''_k\}_{k=1,\dots,K}$  such that  $R''_k > R_k$ .*

**3.2. Property.** Intuitively, the resource allocation should fully utilize the budget of total bandwidth and ROT. This intuition is proved rigorously in the following lemma. The proof is given in Appendix C.

**Lemma 3.** *For an optimal resource allocation, the ROT and total bandwidth constraints should be equalities.*

For exploiting the property of the solution to the optimization problem in (4), we show the following lemma, which states that when there are only two ATs, the AT with a better channel condition should use a larger receive power. The proof is given in Appendix D.

**Lemma 4.** *For a two-AT cell ( $K = 2$ ) and constrained ROT case, suppose that an allocation  $(P_1, P_2)$  and  $(W_1, W_2)$  satisfies  $g_1 P_1 > g_2 P_2$ ,  $P_1 < P_{\max}$ , and  $P_2 < P_{\max}$ . Then, we can always find a better allocation scheme yielding higher throughput and satisfying the EGOS and ROT constraints.*

Based on Lemma 4, we obtain the following proposition, which discloses a necessary condition of optimal ROT constraint-based resource allocation. The proof is given in Appendix E.

**Proposition 1.** *For the ROT constrained system and the corresponding optimal resource allocation, there exists a  $g_{\text{cut}}$  such that*

- (i) *all ATs having channel gains smaller than or equal to  $g_{\text{cut}}$  should transmit with peak power;*
- (ii) *all users with channel gains larger than  $g_{\text{cut}}$  should transmit with equal receive power  $P_r$ .*

**3.3. Algorithm.** Due to Proposition 1, the optimization is simplified to the task of finding the two optimal parameters, namely  $g_{\text{cut}}$  and  $P_r$ , which can be determined by using the following steps (without loss of generality, we assume that  $g_1 \leq g_2 \leq \dots \leq g_K$ ):

Step 1: set  $k = 0$ .

Step 2: suppose that ATs  $1, \dots, k$  transmit with peak power while other ATs transmit with the same received power  $P_r$ , which is given by

$$P_r = \frac{\text{ROT}_{\max} - \sum_{i=1}^k g_i P_{\max}}{K - k}. \quad (6)$$

Step 3: Compute the corresponding transmit power

$$P_i = \begin{cases} P_{\max}, & i = 1, \dots, k, \\ \frac{P_r}{g_i}, & i = k + 1, \dots, K. \end{cases} \quad (7)$$

If there exists an  $i$  such that  $P_i > P_{\max}$ , go to Step 5.

Step 4: Solve the EGOS enforcing bandwidth allocation, which satisfies (note that  $W_{k+1} = \dots = W_K$ )

$$\begin{aligned} W_1 \log\left(1 + \frac{g_1 P_{\max}}{N_0 W_1}\right) \\ = \dots = W_k \log\left(1 + \frac{g_k P_{\max}}{N_0 W_k}\right) \\ = W_{k+1} \log\left(1 + \frac{P_r}{N_0 W_{k+1}}\right), \end{aligned} \quad (8)$$

$$\sum_{i=1}^k W_k + (K - k) W_{k+1} = W.$$

Then, record the EGOS transmission rate  $R(k)$ . Let  $k = k + 1$  and go to Step 2.

Step 5: Compare  $\{R(i)\}_{i=1, \dots, k}$  and choose the scheme yielding the highest EGOS transmission rate.

Note that the quantity  $R_{\max}$  can also be optimized using the algorithm: different feasible  $R_{\max}$  can be evaluated using the above algorithm; then the  $R_{\max}$  achieving the highest EGOS rate should be adopted.

## 4. Large System Analysis

In this section, we study the resource allocation for large systems. We first explain the large system analysis and apply variational analysis to obtain a condition for optimal resource allocation. Then, we propose an algorithm for computing it and discuss the optimality of all peak power scheme.

**4.1. Large System.** As mentioned in Section 1, the difficulty of the optimal resource allocation is that the interference PSD at one cell is determined by other cells and thus cross-cell optimization is required. However, there is no explicit information exchange between neighboring cells. Mathematically, this difficulty can be alleviated by large system analysis, namely letting the number of ATs,  $K$ , and the total bandwidth,  $W$ , tend to infinity while keeping their radio  $w \triangleq W/K$ , which means the average bandwidth per AT, a constant. When the system size tends to infinity, the

interference PSD will converge to a constant, which is equal for all cells.

The following two points are key to the large system analysis:

- (i) since there is no explicit coordination across different cells, the resource allocation to an AT is determined by only the channel gain of the AT to the serving AP, which can be denoted by  $P(g)$  and  $W(g)$ . As  $K, W \rightarrow \infty$ , the resource allocation can be considered as two functions of the channel gain,
- (ii) for evaluating the interference of an AT to the AP of a neighboring cell, we define the corresponding *interference function* as

$$I(g) = E[\tilde{g} | g], \quad (9)$$

where  $g$  is the channel power gain to its serving AP and  $\tilde{g}$  is the channel power gain to the neighboring AP. It is impossible to derive an explicit expression for the interference function. However, it can be approximated by simulations or field experiment results. In Section 5, we will evaluate the interference function by adopting a certain wireless channel model.

Then, in the large system limit, the interference PSD is given by

$$\begin{aligned} \mathcal{I} &= \frac{1}{W} \sum_{j=1}^J \sum_{k=1}^K P_k^j I_k^j \\ &\rightarrow \frac{J}{w} \int_0^\infty I(g) P(g) dF(g), \quad \text{as } J, K \rightarrow \infty. \end{aligned} \quad (10)$$

**4.2. Variational Analysis.** We analyze the optimal resource allocation in the spirit of variational analysis via the following steps.

- (1) Change the power allocation function  $P(g)$  by a sufficiently small  $\delta P(g)$ .
- (2) We adjust the bandwidth allocation such that all ATs still keep the original EGOS rate corresponding to  $P(g)$  in the following way: for ATs with  $\delta P(g) > 0$ , they donate some bandwidth to a “bandwidth bank” (illustrated in Figure 2) to decrease their transmission rates; for ATs with  $\delta P(g) < 0$ , they borrow some bandwidth from the “bandwidth bank” to improve their transmission rates.
- (3) Then, we check the net income of the “bandwidth bank”: if it is positive, the remaining bandwidth can be distributed to all ATs to improve the EGOS rate, thus finding a better EGOS resource allocation scheme.

The details of the above steps will be discussed in the remainder of this subsection.

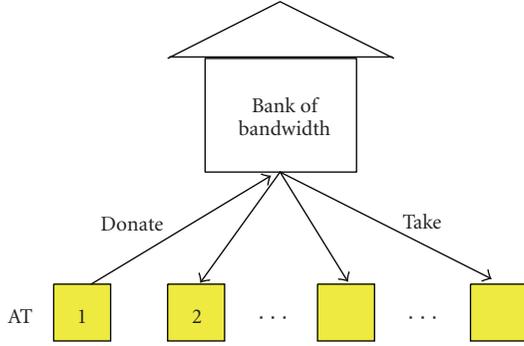


FIGURE 2: An illustration of the bank of bandwidth.

**4.2.1. Perturbation.** We assume that the resource allocations  $P(g)$  and  $W(g)$  are increased by sufficiently small  $\delta_P(g)$  and  $-\delta_W(g)$  (both are of the same order). By applying the Taylor's expansion, we can obtain the change of transmission rate of ATs having channel gain  $g$ , which is given by

$$\delta_R(g) = a(g)\delta_P(g) - b(g)\delta_W(g) + c(g)\delta\mathcal{I} + o(\delta_P^2(g)), \quad (11)$$

where  $\delta\mathcal{I}$  represents the change of interference PSD, which is given by

$$\delta\mathcal{I} = \frac{J}{w} \int_0^\infty I(g)\delta_P(g)dF(g), \quad (12)$$

and the coefficients  $a(g)$ ,  $b(g)$ , and  $c(g)$  are given by

$$\begin{aligned} a(g) &= \frac{g/(\mathcal{I} + N_0)}{1 + (gP(g))/((\mathcal{I} + N_0)W(g))}, \\ b(g) &= \log\left(1 + \frac{gP(g)}{(\mathcal{I} + N_0)W(g)}\right) \\ &\quad - \frac{(gP(g))/((\mathcal{I} + N_0)W(g))}{1 + (gP(g))/((\mathcal{I} + N_0)W(g))} \\ c(g) &= -\frac{(gP(g))/(\mathcal{I} + N_0)^2}{1 + (gP(g))/((\mathcal{I} + N_0)W(g))}. \end{aligned} \quad (13)$$

**4.2.2. Adjustment.** Then, we carry out the second step, that is, finding a suitable  $\delta_W(g)$  to recover the original transmission rate. Substituting  $\delta_R(g)$  into (11), we have

$$\delta_W(g) = \frac{a(g)\delta_P(g) + c(g)\delta\mathcal{I}}{b(g)} + o(\delta_P^2(g)). \quad (14)$$

When  $\delta_W(g) > 0$ , an AT having channel gain  $g$  can “donate” bandwidth  $\delta_W(g)$  to a virtual “bandwidth bank” while keeping its transmission rate unchanged; when  $\delta_W(g) < 0$ , an AT having channel gain  $g$  can “borrow” bandwidth  $\delta_W(g)$  from the virtual “bandwidth bank” to keep its transmission rate unchanged. Then, the net income of the

“bandwidth bank”, normalized by the total number of ATs, is given by

$$\begin{aligned} w_{\text{spare}} &= \int_0^\infty \delta_W(g)dF(g) \\ &= \int_0^\infty \frac{a(g)\delta_P(g) + c(g)\delta\mathcal{I}}{b(g)}dF(g) \\ &= \int_0^\infty \frac{a(g)\delta_P(g)}{b(g)}dF(g) \\ &\quad + \frac{J}{w} \int_0^\infty \frac{c(g)}{b(g)}dF(g) \int_0^\infty I(g)\delta_P(g)dF(g) \\ &= \int_0^\infty \Theta(g)\delta_P(g)dF(g), \end{aligned} \quad (15)$$

where

$$\Theta(g) = \frac{a(g)}{b(g)} + \frac{JI(g)}{w} \int_0^\infty \frac{c(x)}{b(x)}dF(x). \quad (16)$$

Then, for a sufficiently small  $\epsilon > 0$ , if we set

$$\delta_P(g) = \epsilon 1(\Theta(g) > 0) + P(g) - P_{\max} - \epsilon 1(\Theta(g) < 0), \quad (17)$$

where  $1(\cdot)$  is the characteristic function, then we can obtain  $w_{\text{spare}} > 0$  (it is easy to verify by substituting (17) into (15)), that is, we can distribute the spared bandwidth to all ATs and improve the EGOS throughput.

**4.2.3. Optimality.** Based on the above analysis, we obtain the following lemma (we coin the condition “ $\Theta$  condition”).

**Lemma 5.** *The optimal resource allocation  $P(g)$  and  $W(g)$  satisfies the following two conditions:*

- (i) for all  $g$ ,  $\Theta(g) \geq 0$ ;
- (ii) for any  $g$  such that  $P(g) < P_{\max}$ ,  $\Theta(g) = 0$ .

The second item implies that, if  $\Theta(g) > 0$ ,  $P(g) = P_{\max}$ .

In Lemma 5, we derived only a necessary condition for the optimal resource allocation. The following proposition shows that these conditions are also sufficient for a locally optimal resource allocation. (A locally optimal resource allocation means that all other resource allocation schemes within a neighborhood of it achieve worse or equal EGOS transmission rate.) The proof is given in Appendix F.

**Proposition 2.** *A resource allocation is locally optimal if and only if the  $\Theta$  condition holds.*

**4.3. Algorithm.** Proposition 2 also provides an efficient approach to compute the optimal resource allocation scheme, which can be iteratively done using the following steps.

- Step 1: discretize the channel gains and approximate the functions  $P(g)$  and  $W(g)$  by using a finite number of  $g$ s. Initialize all transmit powers and compute the corresponding bandwidth allocation  $W(g)$  that assures EGOS constraint.
- Step 2: Compute  $\Theta(g)$  according to (16) and the corresponding power change  $\delta_p(g)$  by using (17) and a sufficiently small  $\epsilon$ . Note that all integrals are approximated by discrete summations.
- Step 3: Update the power allocation to  $P(g) + \delta_p(g)$ . Check the stopping rule (either the maximum number of iterations or the difference between the resource allocations of successive iterations). If not stopping, go back to Step 2.

**4.4. Peak Power or Not.** Now we discuss whether the ATs should transmit in peak power or not. (If there is no inter-cell interference, the all peak power scheme is optimal.) From Proposition 2, we know that, if all ATs transmit in peak power in the optimal power allocation, we have  $\Theta(g) > 0$ , which is equivalent to

$$\begin{aligned} I(g) &< -\frac{(wa(g))/(Jb(g))}{\int_0^\infty (c(g)/b(g))dF(g)} \\ &= \frac{(w/J)((\mathcal{L} + N_0)/P_{\max})\theta(g)}{\int_0^\infty \theta(g)dF(g)}, \end{aligned} \quad (18)$$

where the function  $\theta(g)$  is defined as

$$\theta(g) = -\frac{c(g)(\mathcal{L} + N_0)}{b(g)}. \quad (19)$$

It is easy to verify that  $\theta(g)$  decreases as  $g$  is increased.

Then, we discuss the following special cases.

- (i) Thermal limited ( $N_0 \gg \mathcal{L}$ ): the right-hand side (RHS) in (18) is larger than the left-hand side (LHS). Therefore, the inequality in (18) holds and all ATs should transmit in peak power. This also coincides with our intuition.
- (ii) Broadband ( $w$  is sufficiently large): the inequality in (18) also holds and all ATs should transmit in peak power.
- (iii) Large SNR (sufficiently small  $N_0$ ): by ignoring  $N_0$ , (18) becomes

$$\frac{I(g)}{\int_0^\infty I(g)dF(g)} < \frac{\theta(g)}{\int_0^\infty \theta(g)dF(g)}. \quad (20)$$

Notice that integrating over the probability measure  $F(g)$  yields 1 on both sides. Then, there exists a  $g$  such that the inequality does not hold, unless  $I(g) = \theta(g)$ ,  $\forall g$ . Therefore, the all peak power allocation scheme is not optimal.

- (iv) Rapid change of interference function: we can rewrite (18) as

$$\frac{I(g)}{\int_0^\infty I(g)dF(g)} < \frac{\theta(g)}{\int_0^\infty \theta(g)dF(g)} \left(1 + \frac{wN_0}{J\mathcal{L}}\right). \quad (21)$$

When  $I(g)$  changes sufficiently rapidly, the LHS of (21) is larger for some  $g$ , which implies that the all peak power scheme is suboptimal.

Now, we assume that ATs transmitting with peak power with probability zero. Then, almost all ATs transmit with power less than  $P_{\max}$  and the inequality (21) becomes an equation, which is given by

$$\frac{I(g)}{\int_0^\infty I(g)dF(g)} = \frac{\theta(g)}{\int_0^\infty \theta(g)dF(g)} \left(1 + \frac{wN_0}{J\mathcal{L}}\right). \quad (22)$$

By integrating over  $F(g)$  on both sides, we obtain  $1 = 1 + (wN_0)/J\mathcal{L}$ , which is impossible. Therefore, we obtain the following proposition.

**Proposition 3.** *In any locally optimal power allocation, the proportion of ATs transmitting with peak power is nonzero.*

## 5. Numerical Results

We now illustrate the analytical results in this paper via simulations. For comparison, we also simulated the performance of two alternative resource allocation schemes, namely all peak power and equal receive power schemes.

**5.1. Interference Function.** We consider hexagonal cells and define cell radius as the distance from the center of hexagon to its vertices. We assume that an AP is located at the cell center and is interfered by only neighboring cells (thus  $J = 6$ ). For channel gains between ATs and APs, we consider pathloss, which is given by  $28.6 + 35 \log_{10}(d)$  (dB) ( $d$  is the distance measured by meters), and shadow fading, which is normally distributed (in dB scale) with expectation zero and variance 8.9 (dB). Note that, we use a correlated model for the shadow fading, namely the shadow fading between AP  $i$  and AT  $j$  is given by

$$F_{ij} = \alpha_1 S_i + \alpha_2 S_{ij} + \alpha_3 S_j, \quad (23)$$

where  $S_i$ ,  $S_{ij}$ , and  $S_j$  are i.i.d. Gaussian random variables (expectation zero and variance 8.9 dB) and  $\alpha_1^2 + \alpha_2^2 + \alpha_3^2 = 1$ . The physical meaning of (23) is as follows: the shadow fading between an AP and an AT consists of three independent components;  $S_i$  represents the shadowing effect around AP  $i$  is common for AP  $i$ ;  $S_j$  stands for the shadowing effect around AT  $j$ ; similarly,  $S_j$  represents the shadowing effect around AT  $j$  is common for AP  $j$ . The requirement  $\alpha_1^2 + \alpha_2^2 + \alpha_3^2 = 1$  normalizes the shadowing effect. For simplicity, we assume  $\alpha_1 = \alpha_2 = \alpha_3$ . This formulation is widely used in numerical simulations in industry.

In Figure 3, the interference functions,  $I(g)$ , obtained from simulations and discretized into 100 intervals, are shown for cell radiuses of 500 m, 1000 m, 1500 m, and 2000 m, each of which is obtained from  $5 \times 10^5$  realizations of node dropping. An interesting observation is that the interference function  $I(g)$  is not a monotonic function of  $g$ . A possible explanation is given as follows: when the channel gain is very small, the user is close to the cell edge; therefore, it could be very far away or very close to an AP in a neighboring

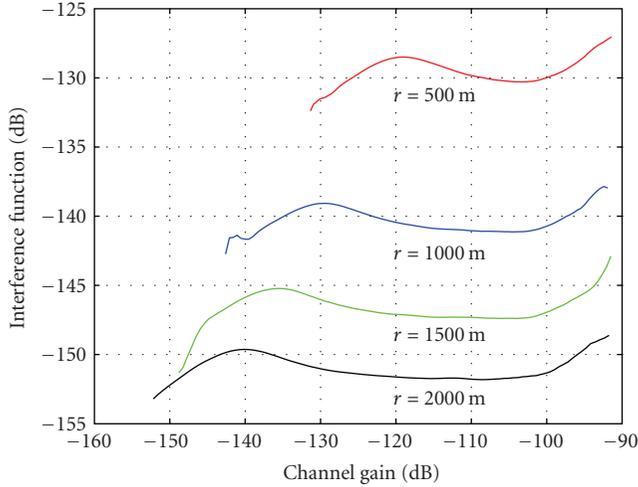


FIGURE 3: Interference functions for different cell radiuses.

cell; the average will make the interference function small; as the channel gain increases, the user is closer to the serving AP, thus making the distances to APs in neighboring cells more even; the effect of the more even distance is not monotonic.

**5.2. Optimal Transmit Power.** We assume that an AT is claimed to be in outage when the channel gain to its serving AP is smaller than a cutoff channel gain  $g_{\text{cut}}$ , which is chosen such that the outage probability is 5%. Then, we ignore the outage users in the resource allocation. We apply the algorithm in Section 4.3 to compute the optimal resource allocation by considering two types of initializations:

- (i) peak power: every AT transmits with peak power  $P_{\text{max}}$  (we assume that peak power is 200 mW);
- (ii) equal receive power: every AT transmits with equal received power such that the AT having the lowest channel gain transmits with peak power  $P_{\text{max}}$ .

When the average bandwidth,  $w$ , is 20 kHz, the three types of power allocations are shown in Figure 4. Note that both initializations result in very similar results for optimizing the resource allocation. We observe that the optimal power allocation (although we cannot show the rigorous optimality of the numerical result, we still use the term “optimal” for simplicity) is very similar to the power allocation with equal receive power.

We also computed the ROT-based resource allocation using the algorithm proposed in Section 3. The computation results in the scheme of equal receive power.

**5.3. EGOS Transmission Rate.** Using the power allocation obtained from the algorithm in Section 4.3, we obtain the asymptotic EGOS transmission rates corresponding to the optimal, equal receive power, and peak power allocation schemes, which are shown in Figure 5. We observe that the EGOS rate obtained from the equal receive power is only marginally worse than that of the optimal power allocation while the peak power allocation performs much worse.

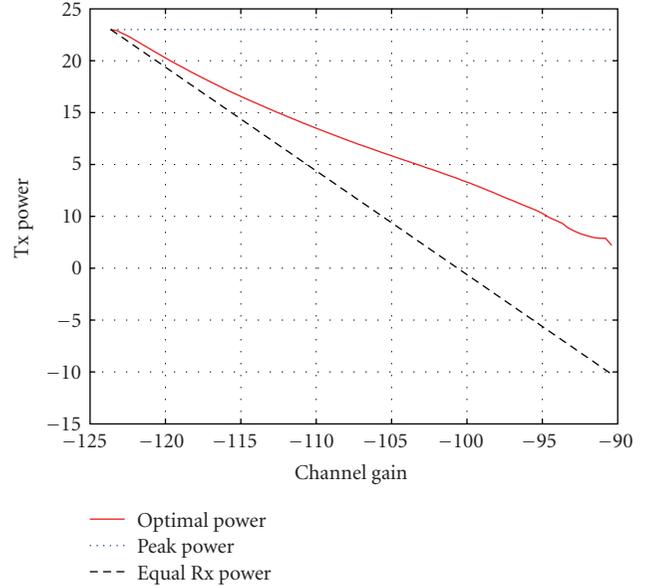


FIGURE 4: Optimal power allocation when  $w = 20$  kHz.

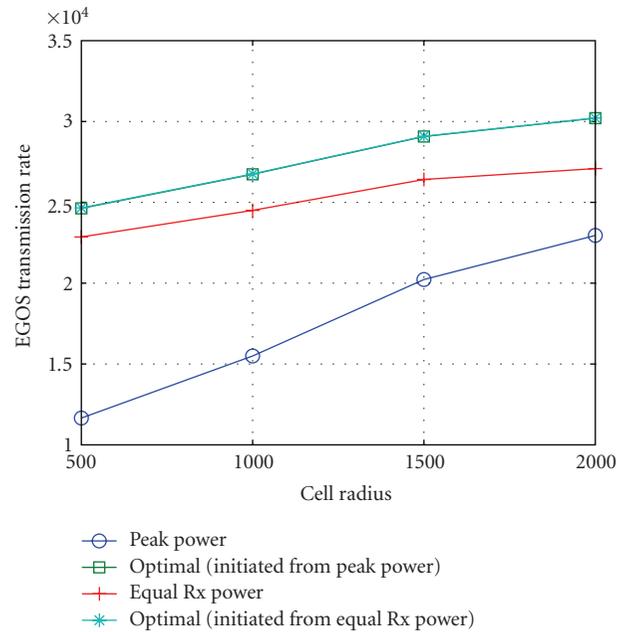


FIGURE 5: Asymptotic EGOS rates for different cell radiuses when  $w = 20$  kHz.

Figure 6 shows the EGOS rates obtained from finite systems (40 ATs in each cell). Each rate is obtained from 100 random drops of ATs. We observe that the asymptotic results can predict the finite-system results with marginal error.

Figure 7 shows the asymptotic EGOS rate when using a much larger average bandwidth  $w = 400$  kHz. For such a configuration, the ROT-based algorithm in Section 3 results in the scheme of all peak power. Comparing with Figure 5, we notice that the EGOS rate is reduced as cell radius increases. When the cell radius is large (2000 m), the peak power

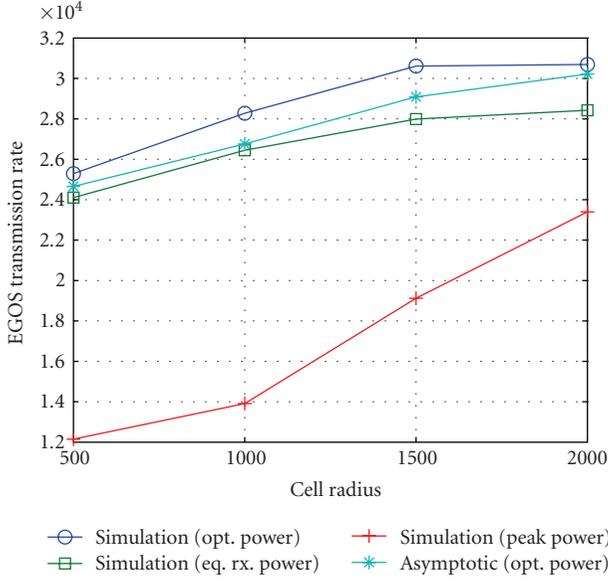


FIGURE 6: Finite-system EGOS rates for different cell radiuses when  $w = 20$  kHz.

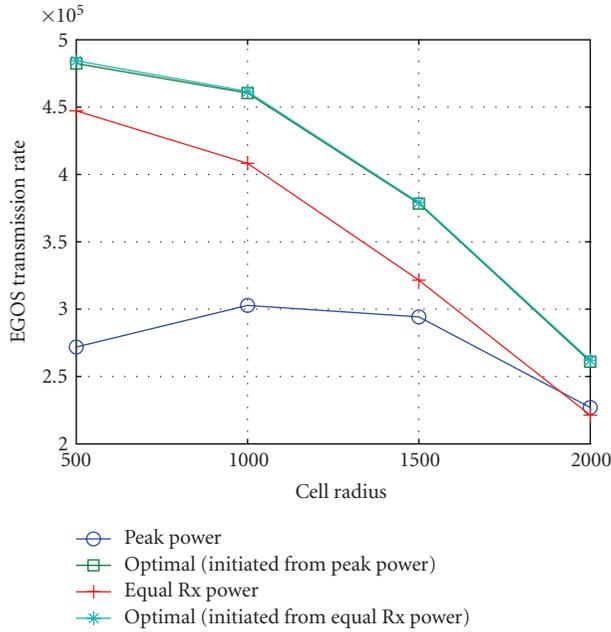


FIGURE 7: Asymptotic EGOS rates for different cell radiuses when  $w = 400$  kHz.

allocation has similar performance to that of the equal receive power scheme and is close to optimal.  $\square$

## 6. Conclusions

We have discussed the resource allocation (power and bandwidth) of multi-cell OFDMA communication systems under the fairness constraint of EGOS. For finite systems, we have applied the heuristic ROT constraint to derive the resource allocation scheme. On assuming sufficiently many users

within each cell, we have applied the large system analysis and variational analysis to obtain the optimal power and bandwidth profiles. To model the intercell interference, we have defined an interference function which is obtained from numerical simulations. We have used numerical simulations to demonstrate the effectiveness of the proposed algorithm.

## Appendices

### A. Proof of Lemma 1

*Proof.* It is easy to find a resource allocation satisfying the constraints of peak power, total bandwidth, and ROT. The only thing we need to check is the EGOS constraint.

We can do inductions on  $K$ . When  $K = 2$ , suppose that we have found  $(P_1, P_2)$  and  $(W_1, W - W_1)$  satisfying the constraints except the EGOS one. Then, we fix the transmit powers and vary  $W_1$  from 0 to  $W$ . The corresponding  $R_1$  ( $R_2$ ) ranges from 0 to  $W_1 \log(1 + (g_1 P_1)/(W_1 N_0))$  (from  $(W - W_1) \log(1 + (g_2 P_2)/((W - W_1) N_0))$  to 0). Since  $R_1$  and  $R_2$  are continuous and monotonic functions of  $W_1$ , there exists a unique  $W_1$  such that  $R_1$  and  $R_2$  are equal. We denote by  $R^{(2)}(W)$  this equal transmission rate as a function of  $W$  (suppose that  $P_1$  and  $P_2$  are fixed). Then, by applying the implicit function theorem, it is easy to check that  $R^{(2)}(W)$  is a continuous and monotonically increasing function of  $W$ , since  $R^{(2)}$  is a continuous function of  $W_1$ , which is determined by the following equation and is also continuous in  $W$ :

$$\begin{aligned} & W_1 \log\left(1 + \frac{g_1 P_1}{W_1 N_0}\right) \\ &= (W - W_1) \log\left(1 + \frac{g_2 P_2}{(W - W_1) N_0}\right). \end{aligned} \quad (\text{A.1})$$

We assume that when  $K = k$ , when  $\{P_i\}_{i=1,\dots,k}$  are fixed, there exists a unique  $\{W_i\}_{i=1,\dots,k}$  such that  $\sum_{i=1}^k W_i = W R^{(k)} \triangleq R_1 = R_2 = \dots = R_k$  and  $R^{(k)}$  is a continuous function of  $W$ .

Now, we consider the case  $K = k + 1$ . Again, we fix  $\{P_i\}_{i=1,\dots,k+1}$ . We range the bandwidth allocated to AT1,  $W_1$ , from 0 to  $W$  and the corresponding total bandwidth allocated to the remaining  $k$  ATs,  $W - W_1$ , is ranged from  $W$  to 0. Similar to the 2-AT case,  $R_1$  ranges from 0 to  $W_1 \log(1 + (g_1 P_1)/(W_1 N_0))$  and the EGOS transmission rate for the remaining  $k$  ATs ranges from  $R^{(k)}(W - W_1)$  to 0. Due to the continuity and monotonicity of  $R_1$  and  $R^{(k)}$  (due to the induction assumption), there exists a unique  $W_1$  such that  $R^{(k+1)}(W) \triangleq R_1(W_1) = R^{(k)}(W - W_1)$ . It is easy to verify the continuity of  $R^{(k+1)}(W)$  using the same argument as the 2-AT case. This concludes the proof.  $\square$

### B. Proof of Lemma 2

*Proof.* We fix  $P'_i = P_i$  and change only the bandwidth allocation to improve the performance. Since  $R'_k > R_k$ , we can find a sufficiently small  $\delta W_k > 0$  such that when  $W'_k = W'_k - \delta W_k$ , we have  $R'_k > R_k$ . Then, the spared bandwidth  $\delta W_k > 0$  can be allocated to the remaining  $K - 1$  ATs. Using a similar

argument to that in Lemma 1, we can find an allocation  $\{W_i''\}_{i \neq k}$  such that  $R_i'' = R_j''$ ,  $\forall i, j \neq k$ , and the equal rate is a continuous and monotonically increasing function of  $\delta W_k$ . When ranging  $\delta W_k$  from 0 to  $W_k'$ ,  $R_k''$  ranges from  $R_k'$  to 0 while  $R_i''$  ( $i \neq k$ ) ranges from being less than  $R_k'$  to being larger than  $R_k'$ . Due to the continuity, we can find a unique  $\delta W_k$  such that EGOS is satisfied and  $R_k'' = R_i'' > R_i$ . This concludes the proof.  $\square$

### C. Proof of Lemma 3

*Proof.* Suppose that the ROT constraint is an inequality for an optimal resource allocation. Then, we can increase the transmit power of an AT, for example, AT  $k$ , whose current transmit power is below the peak power (we can always find such an AT due to the assumption  $P_{\max} \sum_{k=1}^K g_k > ROT_{\max}$ ). Then, the transmission rate of AT  $k$  is increased while all other ATs remain the same rate. By applying Lemma 2, we can always find a better EGOS allocation scheme, which contradicts the optimality.

Suppose that the total bandwidth constraint is an inequality. Then, we can allocate the unused bandwidth to an arbitrary AT. A new EGOS allocation scheme that achieves a better performance exists by applying Lemma 2. The contradiction concludes the proof.  $\square$

### D. Proof of Lemma 4

*Proof.* Due to the assumption that  $g_1 P_1 > g_2 P_2$  and  $R_1 = R_2$ , we have

$$W_1 < W_2. \quad (\text{D.1})$$

We can choose a sufficiently small  $\delta P > 0$  and change the transmit powers to

$$\begin{aligned} P_1' &= P_1 - \frac{\delta P}{g_1}, \\ P_2' &= P_2 + \frac{\delta P}{g_2}. \end{aligned} \quad (\text{D.2})$$

Note that  $P_2$  can be increased since  $P_2 < P_{\max}$ . We also add a sufficiently small bandwidth  $\delta W > 0$  to AT 1 and reduce  $\delta W$  from AT 2. Notice that the changes in power and bandwidth still satisfy the bandwidth, ROT and peak power constraints.

Due to the perturbation on power and bandwidth allocation, the transmission rates of both ATs are changed to

$$\begin{aligned} R_1' &= -a_1 \delta P + b_1 \delta W + o(\delta P, \delta W), \\ R_2' &= a_2 \delta P - b_2 \delta W + o(\delta P, \delta W), \end{aligned} \quad (\text{D.3})$$

where

$$\begin{aligned} a_i &= \frac{1/(N_0 + I)}{1 + (g_i P_i)/((N_0 + I)W_i)}, \\ b_i &= \log\left(1 + \frac{P_i}{(N_0 + I)W_i}\right) - \frac{(g_i P_i)/((N_0 + I)W_i)}{1 + (g_i P_i)/((N_0 + I)W_i)}, \end{aligned} \quad \forall i = 1, 2, \quad (\text{D.4})$$

which are obtained simply by Taylor's expansion.

For assuring the improvement of performance, that is,  $R_1' > R_1$  and  $R_2' > R_2$ , we require that  $\delta P$  and  $\delta W$  satisfy

$$\begin{aligned} \frac{\delta P}{\delta W} &< \frac{b_1}{a_1}, \\ \frac{\delta P}{\delta W} &> \frac{b_2}{a_2}. \end{aligned} \quad (\text{D.5})$$

Applying the fact that  $g_1 P_1 > g_2 P_2$  and  $W_1 < W_2$ , it is easy to check that  $a_1 < a_2$  and  $b_1 > b_2$ . Therefore, it is always possible to find sufficiently small  $\delta P$  and  $\delta W$  satisfying (D.5) and yielding better performances for both ATs. The proof is concluded by applying Lemma 2.  $\square$

### E. Proof of Proposition 1

*Proof.* First, we prove that ATs not transmitting with peak power must transmit with equal receive power. Suppose that two such ATs have different channel gains and transmit with different receive power. Applying Lemma 4, we can find a resource allocation scheme for these two ATs such that the transmission rates of these two ATs are increased while keeping the total ROT and bandwidth unchanged. By applying Lemma 2, we can always find a better EGOS resource allocation scheme, which contradicts the assumption of optimality.

Then, we prove that ATs transmitting with peak power must have smaller channel gains than ATs not transmitting with peak power (as we have shown, these ATs must transmit with equal receive power). Suppose that, in the optimal scheme, AT  $i$  transmits with peak power, AT  $j$ 's transmit power is less than  $P_{\max}$ , and  $g_i > g_j$ . Then, the receive power of user  $i$  is higher than that of user  $j$ . By applying Lemma 4, we can change the power and bandwidth allocation of these two ATs to improve their performance without violating the constraints. The conclusion is obtained by applying Lemma 2.

Based on the above argument, we have

$$\tilde{g} = \max\{g_k \mid P_k = P_{\max}\}. \quad (\text{E.1})$$

This concludes the proof.  $\square$

### F. Proof of Proposition 2

*Proof.* The necessity has been established in the proof of Lemma 5. Therefore, we focus on the proof of sufficiency. Suppose that a resource allocation scheme  $P(g)$  and  $W(g)$ , yielding EGOS transmission rate  $R$ , satisfies the  $\Theta$  condition and there exists a better resource allocation scheme in each neighborhood. Then, for a sufficiently small neighborhood, we denote a better resource allocation scheme by  $P'(g)$  and  $W'(g)$ , which yields higher EGOS transmission rate  $R' > R$ . We define  $\delta_P(g) = P'(g) - P(g)$ . Now, we consider two approaches to recover the original EGOS rate  $R$  when the power allocation is changed to  $P'(g)$  and find contradiction.

- (i) Change  $P(g)$  and  $W(g)$  to  $P'(g)$  and  $W(g)$  to achieve higher EGOS rate  $R'$ . Then, each AT can discard some bandwidth such that the EGOS rate is decreased from  $R'$  to  $R$ . Via this approach, we obtain the total spared bandwidth  $\delta W_1 > 0$ .
- (ii) Change  $P(g)$  to  $P'(g)$  and then change  $W(g)$  to recover the EGOS rate  $R$ . Applying Lemma 5, we obtain that the total change of bandwidth  $\delta W_2$  is negative.

Notice that, in both approaches, the power allocations and EGOS rates are changed to  $P'(g)$  and  $R$ . Due to the bijective mapping between power allocation and bandwidth conditioned on a fixed EGOS rate (this can be easily shown by verifying that the transmission rate is a rigorous monotonically increasing function in both power and bandwidth), the final total used bandwidth should be the same in both approaches, which implies  $\delta W_1 = \delta W_2$ . This contradicts the fact that  $\delta W_1 > 0 > \delta W_2$  and concludes the proof.  $\square$

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