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Research Article

Trade-Offs between Energy Saving and Reliability in Low Duty Cycle Wireless Sensor Networks Using a Packet Splitting Forwarding Technique

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One of the challenging topics and design constraints in Wireless Sensor Networks (WSNs) is the reduction of energy consumption because, in most application scenarios, replacement of power resources in sensor devices might be unfeasible. In order to minimize the power consumption, some nodes can be put to sleep during idle times and wake up only when needed. Although it seems the best way to limit the consumption of energy, other performance parameters such as network reliability have to be considered. In a recent paper, we introduced a new forwarding algorithm for WSNs based on a simple splitting procedure able to increase the network lifetime. The forwarding technique is based on the Chinese Remainder Theorem and exhibits very good results in terms of energy efficiency and complexity. In this paper, we intend to investigate a trade-off between energy efficiency and reliability of the proposed forwarding scheme when duty-cycling techniques are considered too.

1. Introduction

The recent years have witnessed a large diffusion of wireless sensor networks in different application scenarios: agricultural fields monitoring, environmental pollution monitoring, search and rescue operations in contaminated areas, antimining operations, and so forth. Sensor networks are composed of several low-cost devices with limited processing and storage capabilities, consequently, one of the hot topics in wireless sensor networks is the reduction of energy consumption.

Due to the growing gap between application requirements and the slow progress in battery capacity, several techniques have been proposed in the literature which put nodes periodically into sleep whenever communications are not needed. Although this is the most effective way to reduce energy consumption, depending on the forwarding technique used, a sleep/wake up scheduling algorithm is sometime required which implies solving critical synchronization issues.

In [1], we have presented a forwarding technique based on the Chinese Remainder Theorem (CRT) which splits the original packets into several packets such that each node of the network will forward only small subpackets. The sink, once all subpackets are received correctly, will recombine them reconstructing the original message.

The proposed technique, investigated through analytical models and simulations, shows very good results in terms of energy efficiency and appears particularly suitable for those forwarding nodes that are more solicited than others due to their position into the sensor network. Moreover, we have investigated how the original packet can be also reconstructed even if not all the subpackets are received by the sink, in order to increase the network reliability.

However, previous results do not consider the possibility that sensor nodes can be in a sleep state due to a duty-cycling technique employed. Obviously, if the packet is splitted and some of the next-hop nodes are switched off, the probability that the splitted message is not forwarded increases if compared to the unsplitted case. On the other hand, duty-cycling techniques are needed to effectively reduce energy consumption. Therefore, it is important to investigate the performance of the proposed algorithm when duty-cycling techniques are also considered.

In this paper, we show that, with a proper choice of the duty-cycle period, the advantages of the CRT with duty-cycling are the same of those reported in [1], where the nodes are always active. Moreover, we investigate a trade-off between reliability and energy efficiency when the nodes are not perfectly synchronized.

The rest of the paper is organized as follows. Section 2 presents a brief summary of related works already existing in the literature and highlights the distinguished approach of our solution as compared to them. Section 3 describes the basic idea with the help of some examples. Section 4 resumes the duty-cycle technique adopted in this work. Sections 5 and 6 describe the initialization and forwarding procedure. In Sections 7 and 8, some analytical results are derived. In Section 9, the performance of the CRT-based forwarding approach is discussed and the analytical model is validated. Finally, in Section 10, some concluding remarks are drawn.

2. Related Works

When using duty-cycle techniques, the active and sleep states of the network nodes should be carefully designed in order to maintain the network connected and guarantee the delivery of the packets.

In the literature, several approaches have been proposed, most of them regarding the MAC layer.

A well-known MAC protocol is S-MAC [2], where, to maintain the synchronization, each node periodically broadcasts its schedule in a control message, so that the neighbors can update this information in their schedule tables. Other approaches are TRAMA [3], which reduces the energy consumption by ensuring that communications are collision free and by placing nodes in sleep mode when they are not communicating; T-MAC [4], which uses an adaptive duty-cycle, where the active part of it is dynamically changed. This reduces the amount of energy wasted on idle listening.

Other approaches regard a cross-layer interaction between MAC and routing, for example, [5, 6]. Both approaches exploit information at routing layer in order to deliver data packets much faster, without sacrificing the energy efficiency achieved by the duty-cycle mechanism.

In most cases the previous approaches require a very tight synchronization which is difficult to achieve in a sensor network composed by simple devices. Moreover, these approaches introduce delivery latency and do not work well when there are frequent changes in the network topologies and in the radio links conditions, causing serious problems related to the network reliability. An interesting example of using a multipath approach together with erasure codes to increase the reliability of a WSN has been proposed in [7]. However, that work suggested the use of disjoint paths. As compared to our proposed forwarding technique, the use of disjoint paths has two main drawbacks. First of all, a route discovery mechanism is needed. Secondly, as the number of disjoint paths are limited, the number of splits (and therefore the achievable energy reduction factor) is limited too.

Another similar work is [8], where the authors have proposed a protocol called ReInForM (Reliable Information Forwarding using Multiple Paths in Sensor Networks). The main idea investigated in this paper, is the introduction of redundancy in data to increase the probability of data delivery. The redundancy adopted is in the form of multiple copies of the same packet which travel to the destination along multiple paths. However, as shown in [9], multiple paths could remarkably consume more energy than the single shortest path because several copies of the same packet have to be sent. Furthermore, in all the papers mentioned above, the authors do not consider the splitting procedure as a method for reducing energy consumption.

An attempt to guarantee reliability, while minimizing the energy consumption and, at the same time, considering a packet splitting procedure, has been made in [10]. As in [7], the authors use disjoint paths and erasure codes to provide reliability in the network. However, the algorithm proposed is a centralized one based on convex programming which is not suitable for WSNs.

In this paper we show that, by using the CRT-based approach also in a network where nodes alternate between sleep and awake state, both reliability and energy saving can be achieved with a moderate increase in the overall complexity and with very low overhead as compared to the commonly used forwarding techniques.

3. The Forwarding Algorithm Based on the Chinese Remainder Theorem

The basic idea of the proposed forwarding technique [1] is to split the messages sent by the source node of a wireless sensor network so that the maximum number of bits per packet that a node has to forward is reduced, increasing in this way the network lifetime.

Consider the example in Figure 1. Nodes A and B have to forward a packet to the sink S. If a normal forwarding scheme is adopted, two cases can be distinguished: A and B select different next-hop nodes (see Figure 1(a)), this happens with probability 2/3 (case (a)); A and B select the same next-hop node (see Figure 1(b)), this happens with probability 1/3 (case (b)).

If there are w bits for each packet, the maximum number of bits transmitted by a node belonging to the set $\{X, Y, Z\}$ is w bits in the case (a), and $2w$ bits in the case (b). Let us now assume that each node in the set $\{X, Y, Z\}$ knows that A and B have three possible next-hops and that a different forwarding scheme is adopted, as shown in Figure 1(c). In particular, when X, Y, and Z receive a packet, they split it and send to the sink only a part (e.g., $w/3$ bits each). In this case, X, Y, and Z have to transmit at most $(2/3)w$ bits each. If we compare the two forwarding methods we can conclude that the last one reduces the maximum number of bits transmitted by a node belonging to the set $\{X, Y, Z\}$. More precisely, the reduction factor is $1 - 2/3 = 1/3$ when we compare the splitting procedure with the procedure

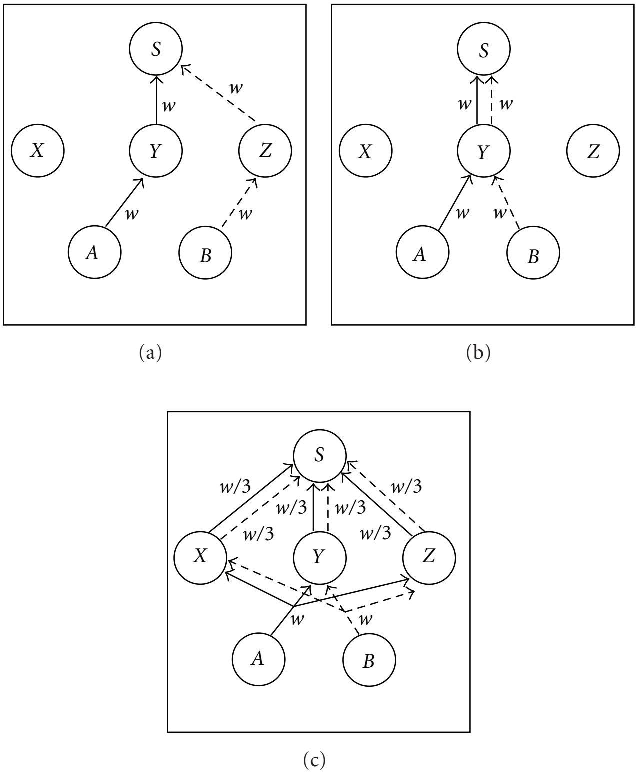

Figure 1: Forwarding examples: (a) normal forwarding with different next-hops; (b) normal forwarding with the same next-hop; (c) forwarding after splitting.

shown in case (a), and $(2 - 2/3) \cdot 1/2 = 2/3$ when the splitting procedure is compared to the procedure shown in case (b). Summarizing, an average reduction factor of 4/9 is obtained.

This example shows that by splitting a packet, it is possible to reduce the maximum number of transmitted bits per node, and therefore the energy that a node consumes for the transmission.

The splitting procedure is achieved applying the Chinese Remainder Theorem (CRT) which represents a low-complexity approach requiring only a modular division between integers and consequently it can be performed by very simple devices as sensor nodes.

Basically, the CRT can be formulated as follows [11].

Given N primes $p_i > 1$, with $i \in \{1 \cdots N\}$, by considering their product $M = \Pi_i p_i$, then for any set of given integers $\{m_1, m_2, \ldots, m_N\}$ there exists a unique integer $m < M$ that solves the system of simultaneous congruences $m = m_i (\bmod p_i)$, and it can be obtained by $m = (\sum_{i=1}^{N} c_i \cdot m_i)(\bmod M)$. The coefficients c_i are given by $c_i = Q_i q_i$, where $Q_i = M/p_i$, and q_i is its modular inverse, that is, q_i solves $q_i Q_i = 1(\bmod p_i)$.

For instance, let us consider the system $m = 1(\bmod 3)$; $m = 4(\bmod 5)$; $m = 1(\bmod 7)$.

It is simple to prove that $m = 64$ solves the system and that it can be obtained through the above equations (in fact we have $M = 105$; $c_1 = 70, c_2 = 21, c_3 = 15$, and $m = 64$).

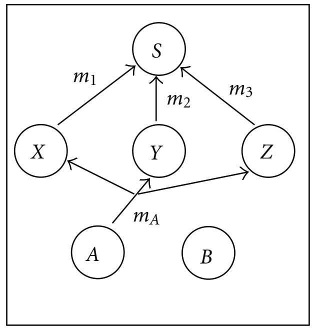

Figure 2: Example of forwarding after splitting.

As an example of application, consider Figure 2. If X, Y, and Z receive a message m_A broadcast from A, each of them, applying the procedure shown above, can transmit a message m_i, with $i \in \{1, 2, 3\}$ (called CRT components), to the sink instead of m_A. Furthermore, the sink, knowing p_i, with $i \in \{1, 2, 3\}$, and using the CRT approach, will be able to reconstruct m_A.

In order to apply the previous technique two questions must be answered: how to obtain the prime numbers in a distributed manner, and how to cope with packet loss.

In [1], we have presented a solution to the previous problems. In particular, we have discussed how to choose the set of prime numbers $p_i > 1$, with $i \in \{1 \cdots N\}$, in a distributed manner so that the message can be reconstructed by the sink, even if f CRT components are lost. For sake of completeness an example is reported in Section 6.

Basically, f is the number of admissible failures, that is, the maximum number of CRT components that can be lost (for each packet) without decreasing the network reliability, and is the main design parameter of the proposed algorithm. However, as already stated in Section 1, if duty-cycle techniques are adopted within the proposed CRT-based scheme (or any other splitting techniques) without modifications, the number of packets lost greatly increases. This loss cannot be compensated by increasing f because large values of f reduces the energy efficiency and therefore the network lifetime, that is, a trade-off between energy consumption and reliability exists.

This paper provides a solution to the above problem. As a major result we prove that, under proper conditions, the performance of the proposed CRT-based forwarding algorithm are the same with and without duty-cycle techniques. Furthermore, we investigate how energy consumption and reliability are related to the parameter f and other common parameters of duty-cycling techniques. In particular, we show how the parameter f can be properly chosen in order to cope with possible duty-cycle mismatching.

4. Duty-Cycling Parameters

When a duty-cycle technique is adopted, a node periodically switches from an active state to a power saving state (idle state) on the basis of a clock signal (see Figure 3). Throughout the paper we indicate with T_C the switching

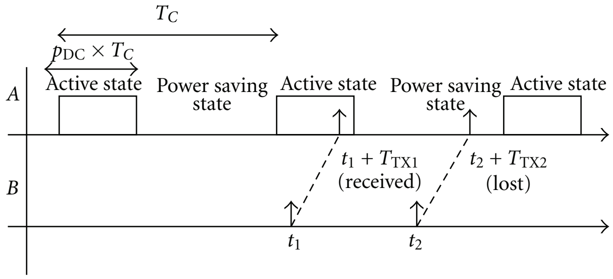

FIGURE 3: Duty-cycle parameters.

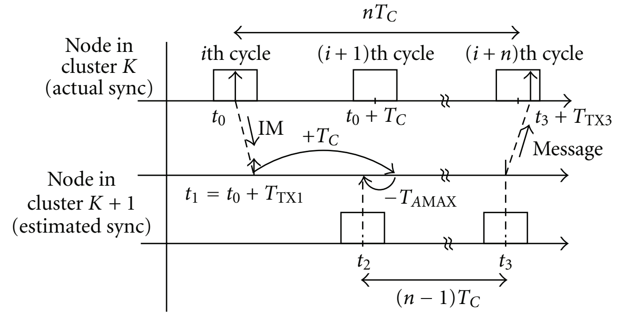

FIGURE 4: Duty-cycles synchronization.

period (or cycle time) and with p_{DC} the duty-cycle, that is, the fraction of time when a node is in active state.

Obviously, a low dutycycle is desirable in order to reduce the power consumption and increase the network lifetime.

Duty-cycle techniques impose a proper synchronization scheme to avoid that messages are received while a node is in a power saving state with the effect of increasing the packet loss and reducing the network reliability.

For instance, let us consider Figure 3, where the first time-line represents the clock signal of a generic node A which waits to receive a message, while the second time-line represents the time instants when a generic source node B generates a message. Let us assume that the first message, generated at the time t_1 from B, and after a transmission time equal to T_{TX1}, is received at the time $t_1 + T_{TX1}$. This time instant belongs to an active state for node A and therefore the message will be correctly received. On the other hand, the second message, generated at the time instant t_2, and requiring a transmission time T_{TX2}, is received during a power saving state of A and consequently it will be lost.

Throughout the paper we indicate with $T_{AMAX} = \max_j\{T_{TXj}\}$ the maximum transmission time which includes propagation delay, packet duration, maximum backoff, and time to receive an acknowledge (if an ARQ technique is used). It is worth mentioning that T_{AMAX} can be evaluated taking the specific MAC protocol into account.

For instance, in the case of the IEEE 802.15.4 standard [12], the maximum backoff time is 27.4 ms and assuming a negligible propagation delay (usually less than 1 μs), a packet duration of 1.8 ms (i.e., a 56-byte packet at a bitrate of 250 Kbps), and operating without ARQ, it follows that $T_{AMAX} = 27.4 + 1.8 = 29.2$ ms.

We show in the next sections how nodes can be synchronized on the basis of the knowledge of the parameters p_{DC}, T_C, and T_{AMAX}. Furthermore, we show how CRT allows to achieve high reliability even under an imperfect synchronization.

5. Initialization Procedure

An initialization procedure for the proposed CRT-based forwarding technique has been extensively described in [1]. The above mentioned procedure is mainly based on the exchange of Initialization Messages (IMs) and allows to organize the network in clusters. The sink is supposed to belong to the cluster 1 and generates a first IM with its own address and a sequence number SN = 2. Each node which receives an IM from its neighbors, with a sequence number SN = h, will belong to cluster h and will retransmit the IM with an increased SN value together with its own address and the list of the nodes that will be used as forwarders (which it knows according to the source addresses specified in the received IMs).

On the basis of the received IMs, at the end of the above procedure, each node in the network will know its own next-hops, which other nodes will use it as a next-hop, and into how many parts the received packets can be split. Further details on the initialization procedure are reported in [1].

We show below how nodes can be synchronized using the same IMs seen above.

It is worth mentioning that, using the proposed CRT-based scheme, a perfect synchronization among all the nodes of the network is not needed and we will demonstrate that a synchronization among consecutive clusters is sufficient.

Synchronization of the nodes belonging to cluster 2 is straightforward. In fact, we can consider that all the nodes in cluster 2 (i.e., the nodes that receive the first IM from the sink) start their synchronization signals when receiving the first IM. If the time needed to process the IM is negligible, with respect to the duration of an active state, we can assume that all nodes in cluster 2 are perfectly synchronized.

Now we consider synchronization for successive clusters. We suppose that all nodes know T_C and T_{AMAX} and that the IMs start being sent in the middle of an active state.

Let us consider that, during the initialization phase, a node in cluster K sends its IM at time t_0 and that a second node receives this IM at the time $t_1 = t_0 + T_{TX1}$ (see Figure 4). According to our initialization procedure, the latter node belongs to cluster $K + 1$. Furthermore, we assume that the node configures its clock signal so that the center of one of its active states coincides with the time $t_2 = t_1 + T_C - T_{AMAX}$ (as shown in Figure 4).

It is worth mentioning that for a perfect synchronization, the clock signal of the node in cluster $K + 1$ should be set to be in phase with the clock signal of the node in cluster K, so that the active states can overlap. However, due to the fact that T_{TX} is unknown, this is not possible. Therefore, using the previous procedure, the clock signal of nodes in cluster $K + 1$

is only roughly estimated (on the basis of the time t_2). Despite this fact, we demonstrate that the previous estimation, under proper conditions derived below, is sufficient.

In fact, let us suppose that in the forwarding phase, for instance, after $n-1$ clock cycles, a node in cluster $K+1$ wishes to send a message to one of the nodes in cluster K, so that it sends the message at the time $t_3 = t_2+(n-1)T_C$. The message will be received by nodes in cluster K at the time $t_3 + T_{TX3}$.

Obviously, the message will be properly received if $t_3 + T_{TX3}$ belongs to an active state of the node in cluster K, that is, if $t_0+T_{TX1}-T_{AMAX}+nT_C+T_{TX3} \in [t_0+nT_C-(p_{DC}/2)T_C, t_0+nT_C+(p_{DC}/2T_C)]$ which can be rewritten as:

$$-\frac{p_{DC}}{2}T_C < T_{TX1} - T_{AMAX} + T_{TX3} < \frac{p_{DC}}{2}T_C. \tag{1}$$

Considering the definition of T_{AMAX}, we have $\max\{T_{TX1}, T_{TX3}\} \leq T_{AMAX}$ and the previous condition is satisfied if

$$T_{AMAX} < \frac{p_{DC}}{2}T_C. \tag{2}$$

In fact, if the previous condition is respected, we have $T_{TX1} - T_{AMAX} + T_{TX3} \leq T_{TX3} \leq T_{AMAX} < (p_{DC}/2)T_C$ and $T_{TX1} - T_{AMAX} + T_{TX3} > -T_{AMAX} > -(p_{DC}/2)T_C$.

Simulation results confirm that, if the condition given by (2) is respected, all the messages sent in active states will reach the sink correctly, that is, the loss probability due to the duty-cycle is zero.

It is worth noting that a node in cluster $K + 1$ can receive more IMs from different nodes in cluster K. However, if we assume that IMs are processed by nodes belonging to the same cluster in almost the same time, we can use only the first message for synchronization purpose, and possible processing time differences can be easily taken into account by a small increasing of T_{AMAX}.

In Figure 4, a single message per cycle has been considered. However, multiple transmissions (or retransmissions of the same message) in the same cycle are possible and desirable.

The previous considerations can be easily extended in order to consider M transmissions per cycle, by replacing T_{AMAX} with $M \cdot T_{AMAX}$ so that the synchronization condition becomes

$$M \cdot T_{AMAX} < \frac{p_{DC}}{2}T_C. \tag{3}$$

In this case, only the first message is sent in the center of the active state (i.e., t_3 in Figure 4) while the other messages follow (in the same cycle).

Obviously, a maximum value of M exists in order to respect (3). Nevertheless, we can choose a low value of p_{DC} to reduce the power consumption, and a large value of T_C to have a large number of transmissions per cycle.

For instance, the IEEE 802.15.4 standard [12] provides a power-saving mechanism by setting two system parameters, *macBeaconOrder (BO)* and *macSuperFrameOrder (SO)*, able to achieve low duty-cycle operations. In this case, the duration of the cycle time is defined as

$$T_C = 2^{BO} \cdot 15.36\,\text{ms}, \quad 0 \leq BO \leq 14 \tag{4}$$

while the length of the active period is

$$T_{ON} = 2^{SO} \cdot 15.36\,\text{ms}, \quad 0 \leq SO \leq BO. \tag{5}$$

The duty-cycle is derived as the ratio between the length of an active period, and the length of a cycle time, and can be calculated as

$$p_{DC} = \frac{1}{2^{(BO-SO)}}. \tag{6}$$

Consequently, the condition in (3), becomes

$$SO \geq \log_2\left(2 \cdot M \cdot \frac{T_{AMAX}}{15.36}\right) \tag{7}$$

and the desired p_{DC} can be achieved by choosing

$$BO = SO - \log_2(p_{DC}). \tag{8}$$

If we consider a value of $T_{AMAX} = 30$ ms and the desired duty cycle is $p_{DC} = 1/16$, we can choose SO = 3, BO = 7 so that $T_C = 2$ s and $T_{ON} = 123$ ms. In this case, the condition in (3) is verified also with $M = 2$.

If we reduce $p_{DC} = 1/32$, we can choose SO = 4 and BO = 9, in order to have $T_C = 8$ s and $T_{ON} = 245$ ms. In this case, the condition in (3) is verified also with $M = 6$.

We remark that, in IEEE 802.15.4 WSNs, the fact that the standard is based on a cluster-tree topology [13] may make easier the integration of the proposed CRT-based forwarding technique. In fact, in this case some information needed for performing the splitting procedure are already in the nodes (each node knows how many children it has) and the different branches of the cluster-tree can be straightforwardly used for sending the CRT components.

6. Forwarding

In this section, we report an example of the proposed forwarding algorithm. Let us consider the network shown in Figure 5 where clusters are obtained according to the initialization procedure already described in the previous section. The figure shows the messages sent by each node when the source node H sends a message m to the sink S.

According to the initialization procedure, node G knows that it is the only next-hop of node H and therefore it must forward the packet without performing a splitting procedure. It is worth highlighting that it is not necessary for G to specify the list of the destination addresses $\{C, D, E, F\}$ in the packet. In fact, in the initialization phase, nodes $\{C, D, E, F\}$ have already received the IM message IM:[SN = 5, G, $\{C, D, E, F\}$], and therefore they know that node G has 4 next-hops and that all of them have to split into $N_G = 4$ parts the messages received from G. Therefore, when C, D, E, F receive the packet, they proceed as follows: (1) according to both the packet size, w, and the number of next-hops, N_G, they independently obtain the set of prime

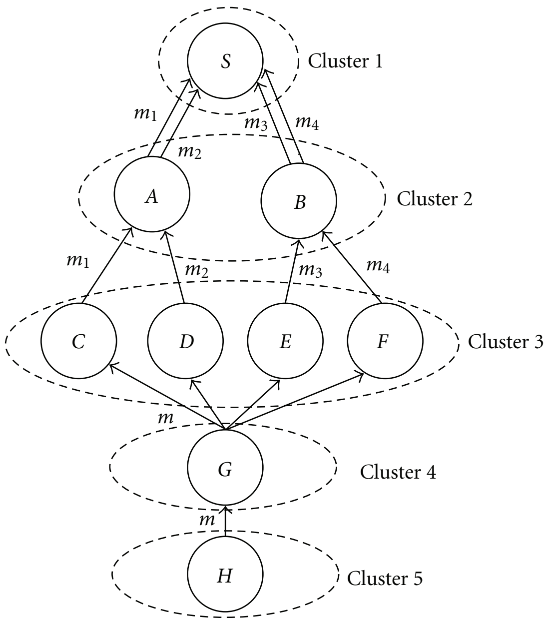

Figure 5: Forwarding example.

numbers (as explained below); (2) they select one of the prime numbers, each of them on the basis of their position in the list of addresses $\{C, D, E, F\}$ specified in the previously mentioned IM; (3) they send the components $m_i = m(\text{mod } p_i)$ (one each), together with a proper mask, to one of the possible next-hops (A or B in the example). The mask is needed to identify the component, i.e., its index i. For instance, it could be the binary representation of the index i followed by the number of components. In particular, in the example we considered, without loss of generality, that only node A is in the coverage range of nodes C and D and only node B is in the coverage range of nodes E and F.

Nodes A and B simply forward the CRT components. Finally, when the sink receives a component m_i, it identifies the number of expected components on the basis of the mask, and therefore it calculates the set of prime numbers, and the coefficients c_i needed to reconstruct the original message. Finally, when the sink receives at least $N - f$ components of the original message, it can reconstruct the message by $m = \sum_i c_i m_i (\text{mod } M')$ (where M' is the product of the prime numbers related to the received components).

It is worth noting that nodes $\{C, D, E, F\}$ can easily obtain the set of prime numbers by considering the smallest consecutive primes that satisfy $M' > 2^w$. For instance, if $N_G = 4$, $w = 40$, and $f = 1$, the set $\{10313, 10321, 10331, 10333\}$ is the set of smallest consecutive primes that guarantees $M' = \Pi_{i=1, i \neq \{j_1, \dots, j_f\}}^{N_G} p_i > 2^w$ whatever is the component (in general the set of components $\{j_1, \dots, j_f\}$) that is not received by the sink. Let us observe that, by fixing w, N, and f, the set is unique so that all the nodes obtain the same set in a stand-alone manner. We point out that the values of w and f can be preprogrammed in the sensor nodes or sent in the IM packets.

7. Energy Reduction Factor

For comparison purposes, we have considered the Shortest Path with Load Balancing (SP), which is very similar to the probabilistic routing. A sensor node having a packet to forward, randomly chooses a neighbor node as next-hop so that the number of hops needed to reach the sink is minimized. Load balancing (i.e., a random choice of the next hop) allows to prolong the network lifetime avoiding that some nodes can be overloaded.

Throughout the paper we consider that an SP packet is composed by K words of w-bits each and that the CRT-based splitting procedure can be applied to each word by considering that the same prime number is used for all the words of the same packet.

As already described in [1], the expected energy reduction factor can be expressed by considering the mean energy consumed by a node in the case of the proposed CRT-based and the SP forwarding technique, that is, $E_{\text{CRT}} = n_c K \overline{w}_{\text{CRT}} \cdot \epsilon_b$ and $E_{\text{SP}} = n_p K w \cdot \epsilon_b$, respectively, where n_c and n_p are the mean number of forwarded packets with the above forwarding schemes, $\overline{w}_{\text{CRT}}$ is the mean number of bits needed to represent the CRT components, and ϵ_b is the energy needed to transmit a bit. More precisely, the expected energy reduction factor can be defined as follows:

$$\text{ERF} = \frac{E_{\text{SP}} - E_{\text{CRT}}}{E_{\text{SP}}} = 1 - \frac{n_c \overline{w}_{\text{CRT}}}{n_p w}. \tag{9}$$

It is worth noting that we are considering the average value of the components, $\overline{w}_{\text{CRT}}$, because in the case of CRT, a node transmits packets which can have components of different length, w_i. However, if a large number of packets are considered, the expected total number of bits is $\sum_{i=1}^{n_c} K w_i \approx n_c K \overline{w}_{\text{CRT}}$ and the previous equation is still valid.

In (9), we have not explicitly considered the effect of packet header. However, it is straightforward to prove that when the length of the header is negligible in comparison to the total packet length (or if the CRT is applied to the header too), (9) is still valid.

Equation (9) can be rewritten by considering that n_c and n_p can be expressed according to the number of sent messages N_m and the mean number of nodes that receive the above messages in the case of CRT and SP schemes, $N_{H\text{crt}}$ and $N_{H\text{sp}}$, respectively. In fact, the mean number of packets forwarded by a node is $n_p = N_m / N_{H\text{sp}}$ for the SP forwarding algorithm, and $n_c = N_m N_{\text{CRT}} / N_{H\text{crt}}$ for the proposed CRT-based forwarding algorithm (if we consider N_{CRT} packets for each message), so that $n_c / n_p = N_{\text{CRT}} N_{H\text{sp}} / N_{H\text{crt}}$. Accordingly, the ERF is

$$\text{ERF} = 1 - N_{\text{CRT}} \frac{N_{H\text{sp}}}{N_{H\text{crt}}} \frac{\overline{w}_{\text{CRT}}}{w}. \tag{10}$$

In [1], we have shown that $N_{H\text{crt}}$ and $N_{H\text{sp}}$ can be expressed in terms of the number of possible nodes that can be used as next-hops, N_T, and the number of messages N_m.

Accordingly, the ERF is

$$\text{ERF} = 1 - N_{\text{CRT}} \frac{1 - (1 - 1/N_T)^{N_m}}{1 - (1 - N_{\text{CRT}}/N_T)^{N_m}} \frac{\overline{w}_{\text{CRT}}}{w}. \tag{11}$$

In particular, we proved that the above equation is valid also if the CRT components are forwarded independently and do not follow distinct paths.

Both N_T and N_m are related to the network, density, ρ. As regards N_T, if we restrict our analysis to the nodes of the second cluster, it can be easily obtained by $N_T = \rho\pi R^2$, where R is the transmission range of the sink. These nodes are the most critical because they represent the sink's neighbors, and if these nodes run out of energy, the sink remains isolated.

As regards N_m, we consider that a certain number of events E_v, randomly occurs in the sensor network and that for each event, all the nodes that recognize the event, generate a message having the sink node as destination. More precisely, we assume that only nodes inside the circular area of radius r, with center in the location of the event, will send a message to the sink. For each event, the number of messages generated is in the order of $\rho\pi r^2$, so $N_m \approx E_v \cdot \rho\pi r^2$.

According to the above relations, considering $N_{\text{CRT}}\overline{w}_{\text{CRT}}/w \approx 1$ and using $(1 - a/b)^c \approx e^{-ac/b}$, it is possible to prove that the ERF falls below a given threshold ERF_T when

$$E_v = \frac{R^2}{r^2}\left|\log(\text{ERF}_T)\right|. \tag{12}$$

On the basis of the previous equation, we can state that the number of events that a WSN can handle before that the ERF falls below ERF_T is not dependent from the density of the WSN, and that for a desired ERF a large number of events can be handled if the transmission range is large enough in comparison to the event range.

8. Reliability

Basically, the reliability of a WSN can be defined as the probability P_R that the sink is able to reconstruct the message.

In this section, we introduce an analytical framework which allows to relate P_R with the probability of erasure for a single hop, p_e. Moreover, we investigate the relation between P_R and a possible duty-cycle mismatch.

These relationships allows us to obtain the value of f (the number of admissible failures) to achieve a target P_R.

It is worth noting that the possibility to obtain different trade-offs between energy saving and reliability by choosing different values of f is one of the main advantages of using the CRT as splitting technique, and that this is not possible with other simple splitting techniques (e.g., simple chunk). Furthermore, considering the limited energy and computation capability of sensor nodes, the very low complexity of the CRT allows it to be more suitable to achieve reliability in WSNs in comparison to other techniques (e.g., FEC techniques based on RS and LT codes) commonly used for other types of wireless networks.

8.1. Reliability and Admissible Failures. Let us assume that, after the splitting procedure starts, each node fails to forward a packet (i.e., a CRT component) due to channel errors or other impairments, with a known probability, p_e. Therefore, if L is the number of hops needed to reach the sink, the probability that a CRT component is not received successfully is $p_n = 1 - (1 - p_e)^L$.

According to the proposed forwarding algorithm, the sink will not be able to reconstruct the original message if more than f components are not received. If we consider N_{CRT} components, this happens with probability

$$P_{NR} = \sum_{i=f+1}^{N_{\text{CRT}}} \binom{N_{\text{CRT}}}{i} p_n^i (1 - p_n)^{N_{\text{CRT}}-i}. \tag{13}$$

Therefore, the reliability can be related to both the erasure probability, p_e, and the number of failures, f, as follows:

$$P_R = 1 - P_{NR} = \sum_{i=0}^{f} \binom{N_{\text{CRT}}}{i} p_n^i (1 - p_n)^{N_{\text{CRT}}-i}. \tag{14}$$

Equation (14) can be read as the cumulative distribution function of a binomially distributed random variable [14]. It is well known that for a large number of trials (i.e., when N_{CRT} increases) the binomial distribution can be approximated by a normal distribution. Therefore, we can coarsely state that by fixing f so that

$$f = \mu + x\sigma, \tag{15}$$

where $\mu = N_{\text{CRT}} \cdot p_n$ and $\sigma^2 = N_{\text{CRT}} \cdot p_n(1 - p_n)$, we can obtain a reliability

$$P_R \approx \Phi(x) = \frac{1}{2} + \frac{1}{2}\operatorname{erf}\left(\frac{x}{\sqrt{2}}\right) \tag{16}$$

which is the cumulative distribution function of the normal variable x. For instance, by choosing $f = \mu + 2\sigma$, we can obtain a reliability of about 0.98.

This allows us, knowing p_n (i.e., p_e and L) and N_{CRT}, to select in a simple manner an appropriate value of f so that the desired value of P_R can be achieved. Once f is known, it is possible to calculate the appropriate set of primes $p_i > 1$, with $i \in \{1\}$ so that the splitting procedure can be performed correctly [1].

8.2. Reliability and Duty Cycle. In this subsection, we introduce a model for the reliability in order to take into account possible duty-cycle mismatching. In particular, we evaluate the probability $p_{n\text{DC}}$ that a CRT component is not received successfully due to the fact that condition in (2) is not satisfied.

On the basis of such a probability, the results previously obtained can be extended. In particular, the new reliability can be obtained by (14) considering $p_{n\text{DC}}$ instead of p_n, and the proper value of f to obtain a desired reliability can be evaluated on the basis of (15).

The condition (2) has been obtained starting from

$$-\frac{p_{\text{DC}}}{2} T_C < T_{\text{TX1}} - T_{\text{AMAX}} + T_{\text{TX3}} < \frac{p_{\text{DC}}}{2} T_C, \tag{17}$$

therefore, if we consider the random variable $z = T_{\text{TX1}} - T_{\text{AMAX}} + T_{\text{TX3}}$, then the probability that condition (2) is not satisfied, $p_{e\text{DC}}$, is the probability that $|z| \geq (p_{\text{DC}}/2)T_C$.

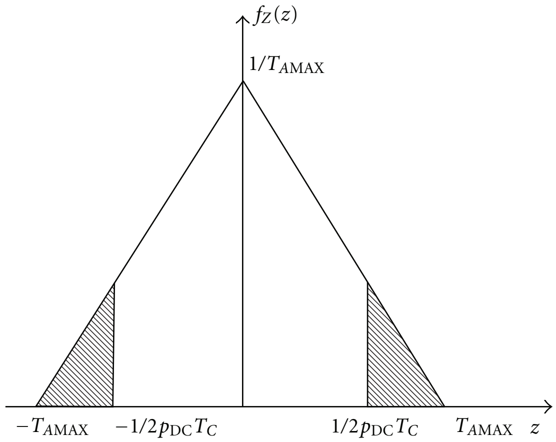

Figure 6: Probability distribution function of $z = T_{TX1} - T_{AMAX} + T_{TX3}$ and $p_{eDC} = P(|z| \geq p_{DC}/2T_C)$ (area of the shadow regions).

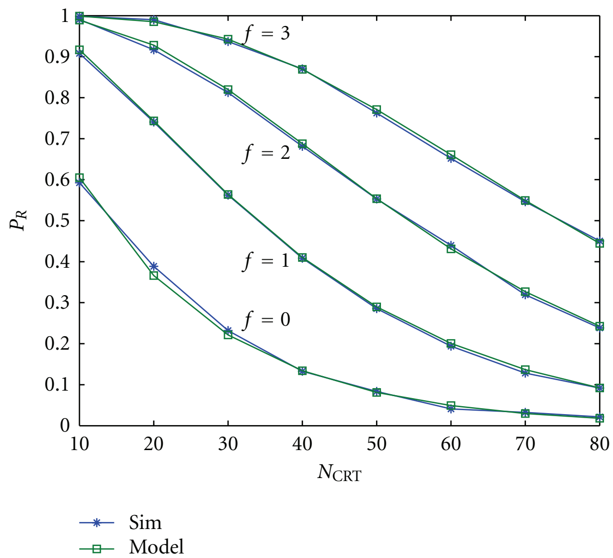

Figure 7: Comparison between P_R calculated through analytical model and simulations.

If we consider that T_{TXj} are uniformly distributed between 0 and T_{AMAX}, the random variable z has a triangular distribution function over the range $[-T_{AMAX}, +T_{AMAX}]$, and the probability that condition (2) is not satisfied coincides with the area of the shadow region shown in Figure 6, that is,

$$p_{eDC} = \frac{(T_{AMAX} - p_{DC}(T_C/2))^2}{T_{AMAX}^2}. \quad (18)$$

The above probability is the probability that two successive nodes are not synchronized. Therefore, if we consider that the sink is always in the active state and that there are L hops to reach the sink, we can evaluate the probability that a CRT component is not received successfully as

$$p_{nDC} = 1 - (1 - p_{eDC})^{L-1}. \quad (19)$$

Hence, the reliability due to possible duty cycle mismatching can be obtained using (14) by replacing p_n with p_{nDC}.

9. Performance Evaluation

In this section, we evaluate the performance of CRT in terms of energy consumption and reliability and validate our analytical model. Let us consider a sensor network where nodes are randomly distributed in a square area of size GridSize [m^2], with density ρ [nodes/m^2]. Sensor nodes are assumed to be static, the sink node is located in the center of the square grid in the first cluster (so that its cluster identifier is $CL_{ID} = 1$), and each sensor node has a transmission range equal to R [m]. Clusters have been obtained according to the initialization procedure described in Section 5. Furthermore, to model erasure channels we considered that each node fails to forward a packet or a CRT component with a known probability, p_e. Instead, issues like packet retransmissions and memory management are not considered here for sake of simplicity.

We also assume that E_v events randomly occur in faraway clusters such that $CL_{ID} \geq 5$. If not already specified, in the following we consider the condition of synchronization obtained through (2).

In Figure 7, we assess the accuracy of the proposed model comparing the analytical results obtained through eq. (14) related to the reliability, with those obtained with the simulator.

In particular, we have evaluated the number of packets lost, N_{PL}, when the following values are considered: $w \in [100, 200]$, $N_{CRT} \in [10, 80]$, $\rho = 0.05$, $R = 60$ m, $r = 10$ m, GridSize = [300 m × 300 m], $p_e = 0.01$, $L = 5$, and $f \in \{0, \ldots, 3\}$. From the number of packets lost we have obtained the P_R as $P_R = 1 - N_{PL}/N_m$ where N_m is the number of messages sent by the sources.

As can be observed, low values of f are sufficient to increase the reliability. For instance, when $N_{CRT} = 20$ and $f = 0$, we have a reliability value of about 0.36, but it is sufficient to choose $f = 2$ to increase the reliability to 0.92. Moreover, it is possible to observe that the results obtained through the analytical model in (14), and those reported by the simulator are very close to each other, for all the values of f considered. In particular, simulations show that, when the condition of perfect synchronization in (2) is satisfied, the loss is only due to channel errors.

In Figure 8, we show the reliability P_R versus the values of f, when $p_e = 0.01$, $L = 5$, $N_{CRT} = 21$, $E_v = 60$, and $r = 10$ m. If not already specified, we consider these values of parameters for all the following plots. Analytical results have been obtained according to (15)-(16).

The results obtained confirm the model. In particular, (15)-(16) correctly predict that to achieve a reliability of 0.98 for $N_{CRT} = 21$ and $p_n = 1 - 0.99^5 = 0.049$ it is necessary to choose $f = \mu + 2\sigma = 3$.

Figure 9(a) shows that the reliability P_R is not related to the event range r and therefore to the number of sensor nodes which detect the event. Same considerations can be

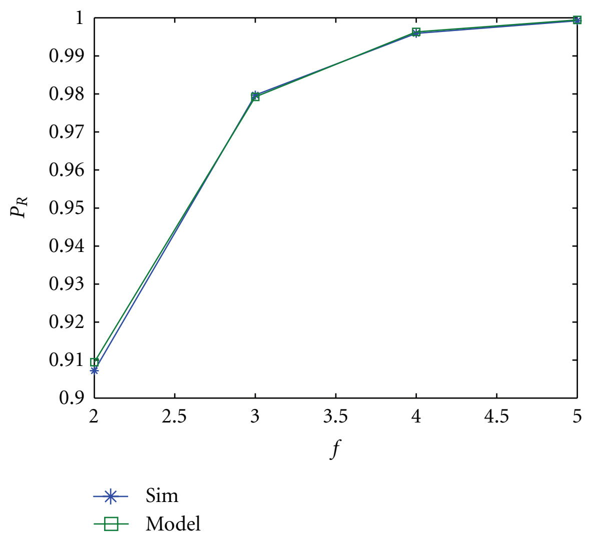

FIGURE 8: Analytical and simulation results of P_R versus f when P = 0.01 and r = 10 m.

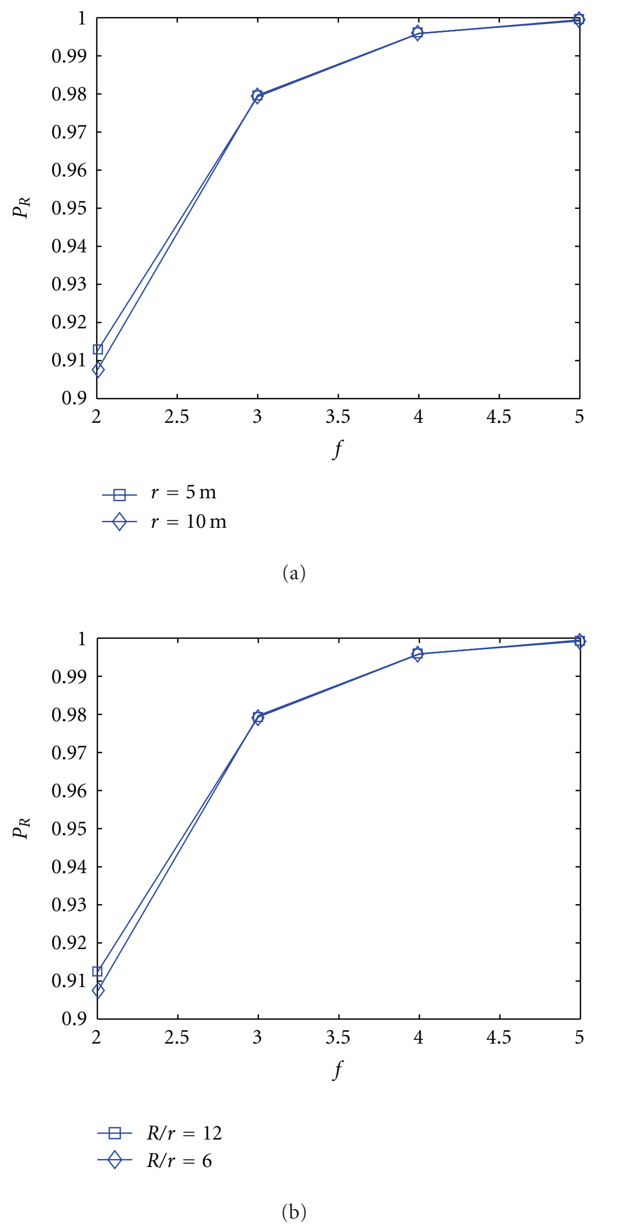

FIGURE 9: P_R versus f when r = 5 m and r = 10 m (a), and for different values of R/r (b).

obtained for the transmission range, R. Simulation results shown in Figure 9(b) confirm that the reliability is not related to the ratio R/r.

Instead, the above mentioned ratio greatly impact on the ERF.

In particular, it is possible to observe that, according to (12), when the ratio R/r increases, the ERF increases as well (see Figure 10(a)), while when the ratio R/r is constant, the ERF remains almost the same (see Figure 10(b)). Note that the curves in Figure 10(b) are not identical because to obtain the expression in (12), we have considered several approximations: $N_m \approx E_v \cdot \rho\pi r^2, N_{\text{CRT}} w_{\text{CRT}} \approx w, (1-a/b)^c \approx e^{-ac/b}$.

Previous results show that the parameters ERF, P_R, f are related. In particular, when f increases, the ERF decreases and P_R increases. Therefore, it is important to select f so that a desired trade-off between reliability and energy reduction can be achieved.

It is worth mentioning that the previous results have also been obtained by simulating also a duty-cycle technique under the synchronization condition given by (2). This allows us to state that performance of the proposed method and its analytical model derived in [1] are valid also if a duty-cycle technique is adopted.

Now, we consider the effect of small duty-cycle mismatching (i.e., synchronization faults). Duty-cycles mismatching are possible, for instance, if T_{AMAX} (i.e., the maximum transmission time) is not perfectly estimated or if small variations happen during the actual network operations.

In Figures 11 and 12 we report the results related to a scenario where we have simulated a perturbation in the value of T_{AMAX} for two values of P_{DC} = 1/16 and 1/32, assuming a cycle time equal to T_C = 1 s in both cases. Both values of P_{DC} have been calculated taking into account the IEEE 802.15.4 guidelines and are less than 10%. The maximum nominal values of T_{AMAX} that can be used to achieve the synchronization can be calculated according to (2), and are T_{AMAX} = 31.25 ms when P_{DC} = 1/16, and T_{AMAX} = 15.63 ms when P_{DC} = 1/32. We have simulated reliability for two values of T_{AMAX} greater than nominal values. More precisely, we considered T_{AMAX} = 36 ms when P_{DC} = 1/16, and T_{AMAX} = 17.2 ms when P_{DC} = 1/32, that is, a perturbation of 15% when P_{DC} = 1/16, and 10% in the case P_{DC} = 1/32.

Figure 11 shows the impact of the redundancy factor f over the reliability P_R. It is possible to see that the value of P_R goes down to 0.58 (for P_{DC} = 1/32) and 0.35 (for P_{DC} = 1/16) when f = 0, that is, when the number of admissible failures is zero. Increasing the value of f, it is possible to increase P_R

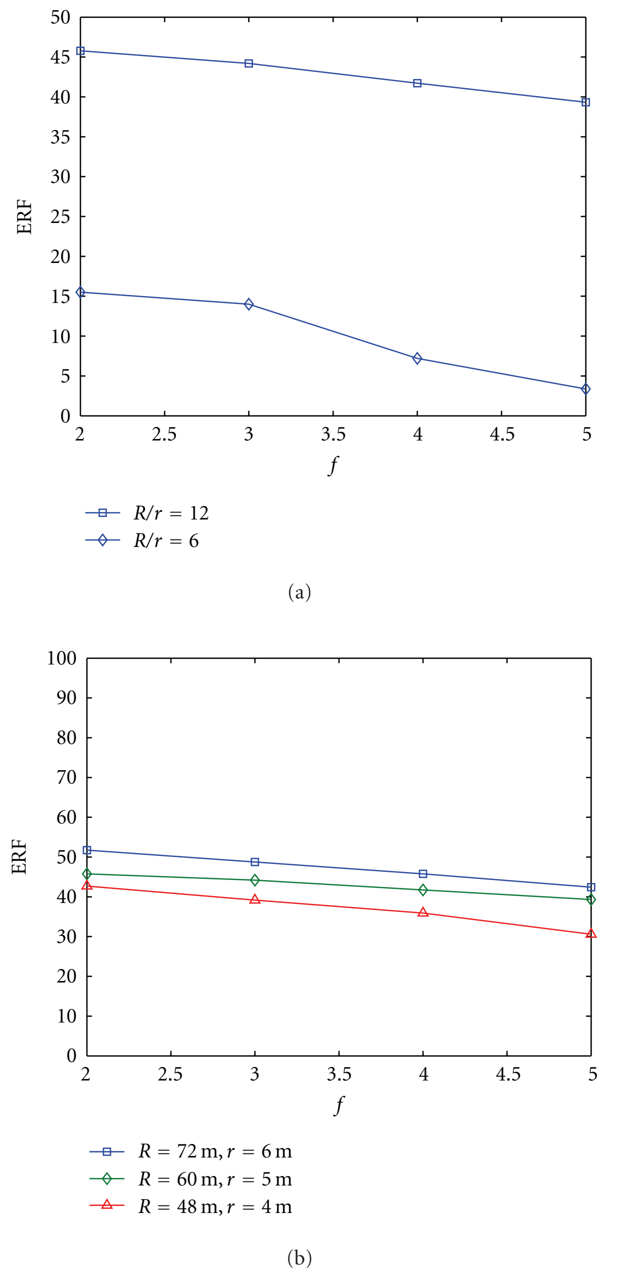

FIGURE 10: ERF versus f for different values of R/r (a) and when R/r=12 (b).

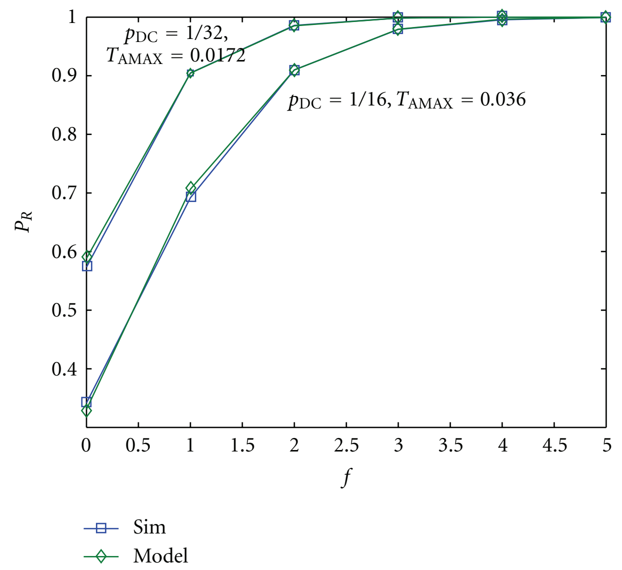

FIGURE 11: P_R versus f when $P_{\mathrm{DC}} = 1/32$ and $T_{\mathrm{AMAX}} = 17.2$ ms, and when $P_{\mathrm{DC}} = 1/16$ and $T_{\mathrm{AMAX}} = 36$ ms.

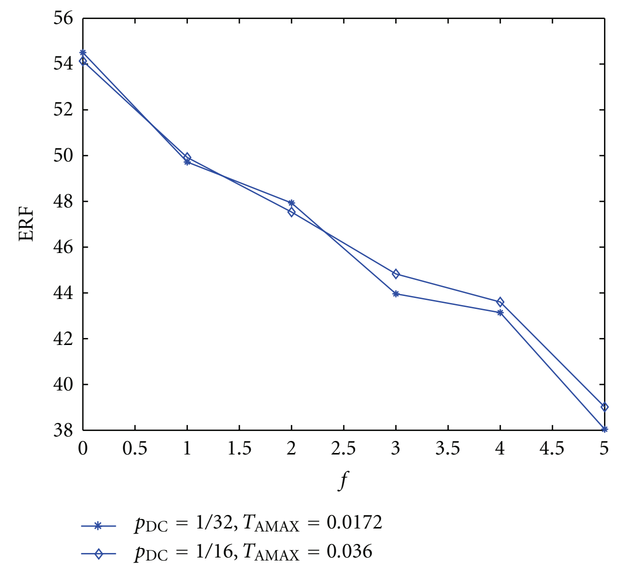

FIGURE 12: ERF versus f when $P_{\mathrm{DC}} = 1/32$ and $T_{\mathrm{AMAX}} = 17.2$ ms, and when $P_{\mathrm{DC}} = 1/16$ and $T_{\mathrm{AMAX}} = 36$ ms.

in both cases. In particular, $f = 2$ (resp. $f = 3$) is sufficient to achieve $P_R = 0.98$ when the perturbation is 10% (resp. 15%). Moreover, it is possible to observe that, as expected, when $P_{\mathrm{DC}} = 1/16$, the values of P_R are lower than the values of P_R when $P_{\mathrm{DC}} = 1/32$. This happen because, for the same value of T_C, the mismatch on the duty-cycle synchronization in the first case is higher than the second case. Finally, Figure 11 allow us to state that the developed model (i.e., (18)-(19)) is able to accurately predict the effect of a possible duty-cycle mismatching.

The increase in the reliability has as a counter-effect, namely, a decrease in the value of ERF. In Figure 12, we have reported the values of ERF versus the values of f. First of all, it is possible to see that ERF decreases when f increases, but its values are always greater than zero for both values of P_{DC}. This means that with the CRT-based forwarding technique we have an improvement with respect to the shortest path, for all the values of f considered. Secondly, it is possible to observe that the variation of ERF related to different values of T_{AMAX} is very small.

In Figure 13, we show the results obtained for different values of T_C when $P_{\mathrm{DC}} = 1/16$. We have considered a perturbation in the value of T_{AMAX} of 20% when $T_C = 1$ s, that is, $T_{\mathrm{AMAX}} = 37.5$ ms. As already shown in the previous

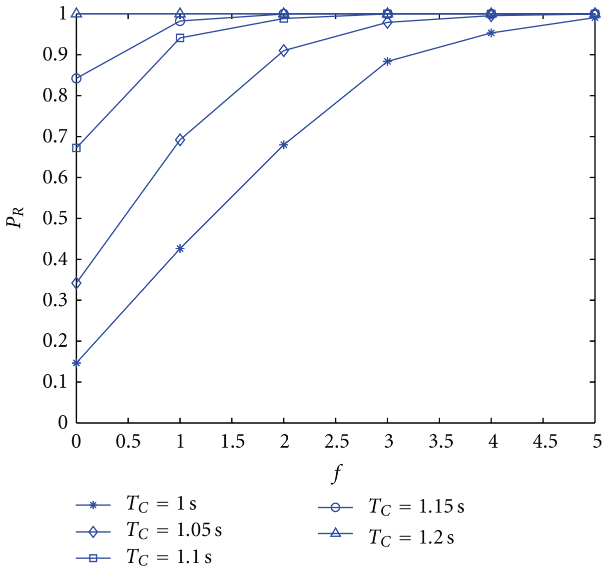

FIGURE 13: P_R versus f when $P_{DC} = 1/16$ and $T_{AMAX} = 37.5$ ms, for different values of T_C.

figures, an increase in the redundancy factor f makes the values of P_R increase as well in all cases. Moreover, it is possible to observe that, when T_C increases of small values, P_R increases significantly. In particular, when $T_C = 1.2$ s, the reliability (without considering channel errors) is 1. This result is in accordance with (2).

Results obtained allow us to assert that, thanks to the CRT approach, we can reduce the energy consumption so prolonging the network lifetime. Moreover, in case of failures due to channel errors or synchronization mismatches, it is possible to find a trade-off between the increase in reliability of the network and the energy efficiency.

10. Conclusions

In this paper we have discussed the trade-off conditions between energy consumption and reliability of a novel forwarding technique for WSNs, based on the Chinese Remainder Theorem (CRT).

In particular, first, we have derived an analytical model able to predict the energy efficiency of the method when applied in a sensor network where node use a duty-cycle technique in order to reduce the energy consumption. Then, we have discussed how to select the CRT algorithm parameters in order to obtain a reasonably trade-off between energy consumption and reliability.

Finally, through simulations, we have assessed the results obtained analytically, and we have shown that the CRT-based forwarding technique works well also in networks with duty cycle. Moreover, we have shown that possible synchronization mismatches can be compensated by choosing the CRT parameters accordingly.

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