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Research Article

Prevoting Cancellation-Based Detection for Underdetermined MIMO Systems

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Various detection methods including the maximum likelihood (ML) detection have been studied for multiple-input multiple-output (MIMO) systems. While it is usually assumed that the number of independent data symbols, M, to be transmitted by multiple antennas simultaneously is smaller than or equal to that of the receive antennas, N, in most cases, there could be cases where $M > N$, which results in underdetermined MIMO systems. In this paper, we employ the prevoting cancellation based detection for underdetermined MIMO systems and show that the proposed detectors can exploit a full receive diversity. Furthermore, the prevoting vector selection criteria for the proposed detectors are taken into account to improve performance further. We also show that our proposed scheme has a lower computational complexity compared to existing approaches, in particular when slow fading MIMO channels are considered.

1. Introduction

The use of multiple antennas in wireless communications, where the resulting system is called the multiple input multiple output (MIMO) system [1], can increase spectral efficiency [2]. For the symbol detection in MIMO systems, it is usually required to jointly detect the received signals at a receive antenna array as multiple signals are transmitted through a transmit antenna array consisting of multiple antenna elements. For the maximum likelihood (ML) detection, an exhaustive search can be used. In general, however, since the complexity of the ML detection grows exponentially with the number of independent data symbols transmitted, an exhaustive search for the ML detection may not be used in practical systems. Thus, for the MIMO detection, computationally efficient linear detectors, such as the minimum mean square error (MMSE) and zero-forcing (ZF) detectors [3], could be used in practical systems. Other low complexity approaches, including the successive interference cancellation (SIC) technique [1, 4], are also well investigated. With ordered signal detection and cancellation, the ZF-SIC and MMSE-SIC detectors perform better than linear detectors.

Taking the channel matrix as a basis for a lattice, lattice reduction-(LR-) based detectors have been proposed in [5, 6]. Since a lattice can be generated by different bases or channel matrices, in order to mitigate the interference from other signals, we can find a matrix (or basis) whose column vectors are nearly orthogonal to generate the same lattice. Based on the Lenstra-Lenstra-Loávsz (LLL) algorithm [7], LLL-LR-based detectors are proposed in [8], and complex-valued LLL-LR based detectors are also proposed in [9] by performing LR with a complex-valued channel matrix. Their performance is analyzed in [9–11]. LR-based detectors have a good performance, and their complexity is significantly, lower than that of the ML detector using an exhaustive search. Furthermore, it is shown in [11, 12] that the LR-based detectors can fully exploit a receive diversity.

In MIMO systems, the channel matrix is called fat, square, or tall matrix if the number of transmit antennas M is greater than, equal to, or smaller than the number of receive antennas N. According to [2], the MIMO channel capacity can be approximated as $C_{\mathrm{MIMO}} \simeq \min(M, N) C_{\mathrm{SISO}}$, where C_{SISO} denotes the channel capacity of single-input single-output channels. Thus, with regard to capacity, we may prefer a square channel matrix (i.e., $M = N$). However, if we need to employ a lower order modulation due to a limited receiver's complexity, we can consider a fat channel matrix (i.e., $M > N$), because the spectral efficiency per transmit antenna can be lower as $C_{\text{MIMO}}/M = (N/M)C_{\text{SISO}} < C_{\text{SISO}}$. For this reason, in this paper, we focus on underdetermined MIMO systems. (Throughout this paper, it is assumed that different symbols transmitted by M transmit antennas are linear independent with others within a time slot.)

For the detection in underdetermined MIMO systems, various techniques can be considered. Instead of exhaustive searching for all the possible decision vectors as in the ML detection, list-based detectors [13–18] create a list of candidate decision vectors and then choose the best candidate as their final decision. In [19–24], a family of list-based Chase detectors are proposed. Since the Chase detection cannot achieve a full receive diversity order, especially when underdetermined MIMO systems are considered, generalized sphere decoding (GSD) approaches [25–30] were developed. In [31], two suboptimal group detectors are introduced, and a geometrical approach-based detection for underdertermined MIMO systems is studied in [32]. To further reduce the complexity, a computationally efficient GSD-based detector with column reordering is proposed in [33], namely, "tree search decoder—column reordering" (TSD-CR). However, their complexity is still high. Moreover, the LR-based detector is only applicable to the case of tall or square channel matrices [8, 11]. Hence, we need to develop a detector that can be employed for fat channel matrices and has a near optimal performance with a reasonably low complexity, especially for a low-order modulation.

To apply MIMO detectors to underdertermined MIMO systems, in this paper, a prevoting cancellation-(PVC-) based MIMO (PVC-MIMO) detection approach is proposed. (This work is an extension of [34]. In [34], the PVC-MIMO is considered with the LR-based subdetectors. In this paper, we extend the PVC-MIMO with various subdetectors, including linear detectors, LR-based linear, and SIC detectors.) The main idea of the proposed detector is to divide the transmitted symbols into two groups. First, one or more reference symbols are selected out of all the transmitted symbols as the prevoting vector (the residual symbols from the postvoting vector), and all the possible candidate symbols for the prevoting vector are considered (e.g., for 2 symbols that are selected for the prevoting vector and 4-quadratic-amplitude modulation (4-QAM) method being used, there are $4 \times 4 = 16$ possible candidate symbol vectors to be considered). Then, for each candidate prevoting vector, its contribution (as the interference) is canceled from the received signal, and the remaining symbol estimates are obtained by a subdetector (which could be a linear detector or LR-based detector) operating on size-reduced square subchannels. The final hard-decision symbol vector is obtained by taking the one that minimizes the Euclidean distance metric among the candidate vectors. Note that the size of prevoting vectors is determined to generate square subchannels (e.g., for a 2×4 channel matrix, 2 symbols are selected for the prevoting vectors, and the size of subchannel matrix is 2×2 square matrix). With an LR-based detector for the sub-detection, theoretical and numerical results show that the proposed approach can achieve a full receive diversity order.

In [35], user selection criteria are considered for multiuser MIMO systems, where a single user is selected to transmit signals to a base station (BS) at a time. By viewing multiuser MIMO as virtual antennas in a single user MIMO system, the user selection problems can be regarded as the transmit antenna selection problems. In this paper, we extend the selection criteria in [35] to support multiple antennas (transmit symbols) at a time for the PVC-MIMO detection where there are more transmit antennas than receive antennas. This extension of the antenna selection, namely, the postvoting vector selection (PVS), becomes a combinatorial problem. Using low complexity suboptimal detectors (LR-based detectors or linear detectors) for the sub-detection, with an optimal PVS, it is also shown that a near ML performance can be achieved. For slow fading MIMO channels, through simulations, we show that the computational complexity of the proposed PVC-MIMO detection with PVS is lower than that of TSD-CR.

The rest of the paper is organized as follows. The system model and our proposed prevoting cancellation-based MIMO detection are presented in Section 2. The optimal PVS is discussed in Section 3. The performance of the proposed PVC-MIMO detectors is analyzed in Section 4. Simulation results and some further discussions are presented in Section 5. Finally, we conclude this paper in Section 6 with some remarks.

Throughout the paper, complex-valued vectors and matrices are represented by bold letters. We use Round-Gothic symbols to represent real-valued vectors and matrices. For a matrix $\mathbf{A}$, $\mathbf{A}^{\mathrm{T}}$, $\mathbf{A}^{\mathrm{H}}$, and $\mathbf{A}^{\dagger}$ denote its transpose, Hermitian transpose, and pseudo-inverse, respectively. $E[\cdot]$ denotes the statistical expectation. In addition, for a vector or matrix, $\|\cdot\|$ denotes the 2-norm. $\lceil \beta \rfloor$ denotes the nearest integer to β. Denote by $\setminus$ the set minus, by $\mathbf{I}_n$ an $n \times n$ identity matrix, and by $\mathcal{K} = \{k_{(1)}, k_{(2)}, \ldots\}$ the collection set of $k_{(1)}, k_{(2)}, \ldots$. The (p, q)th element of a matrix $\mathbf{R}$ is denoted by $[\mathbf{R}]_{p,q}$.

2. Joint Detection for Underdetermined MIMO Systems

We consider underdetermined MIMO systems with a receiver of limited complexity, where low-order modulation is employed as mentioned earlier. This would be the case for downlink channels in cellular systems where the transmitter is a base station and the receiver is a mobile terminal which usually has a small number of receive antennas and a limited computing power for detection. In this section, we present the system model for this underdetermined MIMO system and introduce our PVC-MIMO detection.

2.1. System Model. Consider a MIMO system with M transmit and N receive antennas. Let s_m denote the data symbol to be transmitted by the mth transmit antenna. Assume that a common signal alphabet, denoted by $\mathcal{S}$, is used for all s_m. That is, $s_m \in \mathcal{S}$, $m = 1, 2, \ldots, M$. Furthermore, let

$\mathcal{S}^A$ and $|\mathcal{S}|$ represent the A-dimensional Cartesian product and cardinality of $\mathcal{S}$, respectively. Denote by y_n the received signal at the nth receive antenna, $n = 1, 2, \ldots, N$. Then, the received signal vector over a flat-fading MIMO channel is given by

$$\begin{aligned} \mathbf{y} &= [y_1, y_2, \ldots, y_N]^{\mathrm{T}} \\ &= \mathbf{H}\mathbf{s} + \mathbf{n}, \end{aligned} \tag{1}$$

where $\mathbf{s} = [s_1, s_2, \ldots, s_M]^{\mathrm{T}}$ is the transmit signal vector and $\mathbf{n} = [n_1, n_2, \ldots, n_N]^{\mathrm{T}}$ is the noise vector which is assumed to be a zero-mean circular symmetric complex Gaussian (CSCG) random vector with $E[\mathbf{n}\mathbf{n}^{\mathrm{H}}] = N_0\mathbf{I}$. Here, $\mathbf{H}$ is the channel matrix which can also be written as

$$\mathbf{H} = [\mathbf{h}_1, \mathbf{h}_2, \ldots, \mathbf{h}_M], \tag{2}$$

where $\mathbf{h}_m$ denotes the mth column vector of $\mathbf{H}$. Throughout this paper, we assume that the channel state information (CSI) is perfectly known at the receiver. The impact of channel estimation error on the performance will be discussed in Section 5.2.

2.2. Proposed Approach: Prevoting Cancellation-Based MIMO Detection. For underdetermined MIMO systems, since a sufficiently low complexity and a near optimal performance cannot be obtained by existing MIMO detectors (i.e., MMSE detector, ML detector, list-based detectors [13–24], and GSD-based detectors [25–30]) at the same time, in this subsection, we propose the PVC-MIMO detection.

Let $R = M - N$, and denote by $\mathcal{P} = \{p_1, p_2, \ldots, p_R\}$ the index set for the prevoting signal vector (the selection of this vector will be discussed in Section 3), which is denoted by $\mathbf{s}_{\mathcal{P}} = [s_{p_1}, \ldots, s_{p_R}]^{\mathrm{T}}$. Then, (1) is rewritten as

$$\mathbf{y} = \mathbf{H}_{\mathcal{P}}\mathbf{s}_{\mathcal{P}} + \mathbf{H}_{\mathcal{Q}}\mathbf{s}_{\mathcal{Q}} + \mathbf{n}, \tag{3}$$

where $\mathbf{H}_{\mathcal{P}} = [\mathbf{h}_{p_1}, \ldots, \mathbf{h}_{p_R}]$ is a submatrix of $\mathbf{H}$ associated with $\mathbf{s}_{\mathcal{P}}$, $\mathbf{s}_{\mathcal{Q}} = [s_{q_1}, \ldots, s_{q_N}]^{\mathrm{T}}$ the postvoting signal vector and $\mathbf{H}_{\mathcal{Q}} = [\mathbf{h}_{q_1}, \ldots, \mathbf{h}_{q_N}]$ a submatrix of $\mathbf{H}$ associated with $\mathbf{s}_{\mathcal{Q}}$. Here, the index set $\mathcal{Q}$ is given by $\mathcal{Q} = \{1, \ldots, M\} \setminus \mathcal{P}$. Note that $\mathbf{H}_{\mathcal{Q}}$ is square and $\mathbf{s}_{\mathcal{P}} \in \mathcal{S}^R$ and $\mathbf{s}_{\mathcal{Q}} \in \mathcal{S}^N$.

Define the finite set of all the possible candidate vectors for $\mathbf{s}_{\mathcal{P}}$ as $\{\mathbf{s}_{\mathcal{P}}^1, \mathbf{s}_{\mathcal{P}}^2, \ldots, \mathbf{s}_{\mathcal{P}}^K\}$, where $K = |\mathcal{S}|^R$ (for example, $K = 4^2$ if the size of $\mathbf{s}_{\mathcal{P}}$ is 2×1 and 4-QAM is used). Assuming that $\mathbf{s}_{\mathcal{P}} = \mathbf{s}_{\mathcal{P}}^k$, $k \in \{1, \ldots, K\}$, (3) is rewritten as

$$\mathbf{r}^k = \mathbf{H}_{\mathcal{Q}}\mathbf{s}_{\mathcal{Q}} + \mathbf{n}, \tag{4}$$

where $\mathbf{r}^k = \mathbf{y} - \mathbf{H}_{\mathcal{P}}\mathbf{s}_{\mathcal{P}}^k$. After the PVC in (4), we can apply any conventional MIMO detector that works for a square MIMO channel for the detection of $\mathbf{s}_{\mathcal{Q}}$. For convenience, denote by $\hat{\mathbf{s}}_{\mathcal{Q}}^k$ the detected symbol vector of $\mathbf{s}_{\mathcal{Q}}$ (by any means) for given $\mathbf{s}_{\mathcal{P}} = \mathbf{s}_{\mathcal{P}}^k$. Let

$$\mathbf{s}^k = \begin{bmatrix} \mathbf{s}_{\mathcal{P}} \\ \hat{\mathbf{s}}_{\mathcal{Q}}^k \end{bmatrix}. \tag{5}$$

With K candidates of $\mathbf{s}^k$, that is, $\{\mathbf{s}^1, \ldots, \mathbf{s}^K\}$, based on the ML detection principle, the solution of the PVC-MIMO detection is given by

$$\hat{\mathbf{s}} = \arg \min_{\mathbf{s}^k \in \{\mathbf{s}^1, \ldots, \mathbf{s}^K\}} \|\mathbf{y} - \mathbf{H}'\mathbf{s}^k\|^2, \tag{6}$$

where $k \in \{1, \ldots, K\}$ and $\mathbf{H}' = [\mathbf{H}_{\mathcal{P}}\mathbf{H}_{\mathcal{Q}}]$.

3. Selection for Postvoting Vectors Depending on Subdetectors

In the PVC-MIMO detection, we note different postvoting vector results in different $\mathbf{H}_{\mathcal{Q}}$ which may lead to different performance of sub-detection. In order to exploit the performance of the PVC-MIMO detection, in this section, we focus on the selection of the postvoting vector. For the sub-detection, we consider a few low complexity detectors including linear and LR-based detectors. Note that since a number of the sub-detection operations are to be repeatedly performed, the complexity of sub-detection should be low.

3.1. Selection Criterion with Linear Detector. As a linear detector, for example, we consider the MMSE detector in this subsection. Under the assumption that the prevoting vector is correct, from (4), the output of the MMSE detector is given by

$$\hat{\mathbf{s}}_{\mathrm{Q}}^k = \mathbf{W}_{\mathrm{mmse}}^{\mathrm{H}}\mathbf{r}^k, \tag{7}$$

where $\mathbf{W}_{\mathrm{mmse}}$ is the MMSE filter that is given by $\mathbf{W}_{\mathrm{mmse}} = (\mathbf{H}_{\mathcal{Q}}\mathbf{H}_{\mathcal{Q}}^{\mathrm{H}} + (N_0/E_s)\mathbf{I}_N)^{-1}\mathbf{H}_{\mathcal{Q}}$. Here, E_s represents the symbol energy with $\mathcal{S}$.

The detection performance depends on the channel matrix. For a given channel matrix, as discussed in [35, 36], we can have the max-min eigenvalue (ME) selection criterion for the selection of $\mathcal{Q}$. Since $\mathcal{Q} \in \{1, \ldots, M\}$, the optimal set $\mathcal{Q}$ can be found by using the ME criterion as

$$\mathcal{Q}_{\mathrm{ME}} = \arg\max_{\mathcal{Q}} \lambda_{\min}\left(\mathbf{H}_{\mathcal{Q}}^{\mathrm{H}} \ \mathbf{H}_{\mathcal{Q}}\right), \tag{8}$$

where $\lambda_{\min}(\mathbf{A})$ denotes the minimum eigenvalue of $\mathbf{A}$.

3.2. Selection Criteria with LR-Based Linear and SIC Detectors. To determine $\mathcal{Q}$ for the PVS, we consider the case where LR-based MIMO detectors, which can provide a near ML performance with low complexity [6, 8], are employed for the sub-detection.

Without loss of generality, we assume that the elements of $\mathbf{s}$ are complex integers [6, 8]. For the LR-based linear detection, from (4), the received signal vector can be rewritten as

$$\mathbf{r}^k = \mathbf{G}\mathbf{c} + \mathbf{n}, \tag{9}$$

where $\mathbf{G} = \mathbf{H}_{\mathcal{Q}}\mathbf{U}^{-1}$ and $\mathbf{c} = \mathbf{U}\mathbf{s}_{\mathcal{Q}}$, while $\mathbf{U}$ is an integer unimodular matrix and $\mathbf{G}$ is an LR matrix of a nearly orthogonal basis. The LR-based linear detection is carried out to detect $\mathbf{c}$ as $\hat{\mathbf{c}} = \lfloor \widetilde{\mathbf{W}}\mathbf{r}^k \rceil$, where $\widetilde{\mathbf{W}} = \mathbf{G}^{\dagger}$ for the ZF detector and $\widetilde{\mathbf{W}} = \mathbf{G}^{\mathrm{H}}(\mathbf{G}\mathbf{G}^{\mathrm{H}} + (N_0/E_s)\mathbf{I}_N)^{-1}$ for the MMSE detection.

For the LR-based MMSE-SIC detector, $\mathbf{H}_{\mathcal{Q}}$ is replaced by an extended channel matrix defined as $\mathbf{H}_{\mathrm{ex}} = [\mathbf{H}_{\mathcal{Q}}^{\mathrm{T}} \;\; \sqrt{(N_0/E_s)}\mathbf{I}_N]^{\mathrm{T}}$, while $\mathbf{r}^k$ and $\mathbf{n}$ are replaced by $\mathbf{r}_{\mathrm{ex}} = [(\mathbf{r}^k)^{\mathrm{T}} \;\; \mathbf{0}]^{\mathrm{T}}$ and $\mathbf{n}_{\mathrm{ex}} = [\mathbf{n}^{\mathrm{T}} - \sqrt{(N_0/E_s)}\mathbf{s}_{\mathcal{Q}}^{\mathrm{T}}]^{\mathrm{T}}$, respectively. Using the LR with $\mathbf{H}_{\mathrm{ex}}$, the lattice-reduced matrix $\mathbf{G}_{\mathrm{ex}}$ can be found as $\mathbf{H}_{\mathrm{ex}} = \mathbf{G}_{\mathrm{ex}}\mathbf{U}_{\mathrm{ex}}$, where $\mathbf{U}_{\mathrm{ex}}$ is an integer unimodular matrix. The LR-based MMSE-SIC detection is carried out using the QR factorization of $\mathbf{G}_{\mathrm{ex}} = \mathbf{QR}$, where $\mathbf{R}$ is upper triangular. Multiplying $\mathbf{Q}^{\mathrm{H}}$ to $\mathbf{y}$ results in

$$\mathbf{Q}^{\mathrm{H}}\mathbf{r}_{\mathrm{ex}} = \mathbf{R}\overline{\mathbf{c}} + \overline{\mathbf{n}}, \tag{10}$$

where $\overline{\mathbf{c}} = \mathbf{U}_{\mathrm{ex}}\mathbf{s}_{\mathcal{Q}}$ and $\overline{\mathbf{n}} = \mathbf{Q}^{\mathrm{H}}\mathbf{n}_{\mathrm{ex}}$. The SIC is performed with (10). With the upper triangular matrix $\mathbf{R}$, the last element of $\overline{\mathbf{c}}$, that is, the Nth layer, is detected first. Then, in the detection of the $(N-1)$th layer, the contribution of the last element of $\overline{\mathbf{c}}$ is canceled, and the signal of the $(N-1)$th layer is detected. This operation is terminated when all the layers are detected.

With the LR-based MMSE and MMSE-SIC detectors performed on $\mathbf{H}_{\mathcal{Q}}$, where $\mathcal{Q} \in \{1,\ldots,M\}$, the optimal set $\mathcal{Q}$ can be found by using the ME and the max-min diagonal (MD) selection criteria [35], which are shown as

$$\mathcal{Q}_{\mathrm{ME}} = \arg\max_{\mathcal{Q}} \lambda_{\min}\left(\mathbf{G}_{\mathcal{Q}}^{\mathrm{H}}\mathbf{G}_{\mathcal{Q}}\right), \tag{11}$$

$$\mathcal{Q}_{\mathrm{MD}} = \arg\max_{\mathcal{Q}} \left\{\min_{r}\left| r_{r,r}^{(\mathcal{Q})} \right|\right\}, \tag{12}$$

respectively, where $\mathbf{G}_{\mathcal{Q}}$ is the lattice-reduced basis from $\mathbf{H}_{\mathcal{Q}}$ and $r_{r,r}^{(\mathcal{Q})}$ denotes the (r,r)th element of $\mathbf{R}$ from $\mathbf{H}_{\mathcal{Q}}$ in (10).

4. Performance Analysis

In this section, we consider the diversity gain of the proposed PVC-MIMO detector through the error probability under the assumption that the elements of $\mathbf{H}$ are independent CSCG random variables with mean zero and unit variance, that is, Rayleigh MIMO channels. We also discuss the complexity of the PVC-MIMO detection.

4.1. Diversity Analysis. In order to characterize the error probability of the PVC-MIMO detection, let $\mathbf{s}^o$ represent the original transmitted vectors and $\mathbb{S} = \{\mathbf{s}^1,\ldots,\mathbf{s}^K\}$ represent the set of the candidate solutions provided by the PVC, where each $\mathbf{s}^k$ is generated from (5), that is, $\mathbf{s}^k = [\mathbf{s}_{\mathcal{P}}^{k\mathrm{T}} \;\; \hat{\mathbf{s}}_{\mathcal{Q}}^{k\mathrm{T}}]^{\mathrm{T}}$, $k = 1,2,\ldots,K$. Let $\hat{\mathbf{s}}$ represent the final decision of the detector selected from the candidate solutions in $\mathbb{S}$ obtained in (6). Then, we can define two error probabilities as follows.

Definition 1. One defines the probability that the transmitted symbol vector does not belong to the set of candidate solutions as $P_{e,\mathrm{PV}} = \Pr(\mathbf{s}^o \notin \mathbb{S}) = 1 - \Pr(\mathbf{s}^o \in \mathbb{S})$, that is, $\Pr(\mathbf{s}^o \in \mathbb{S}) = \Pr(\exists \mathbf{s}^{k'} \in \mathbb{S} : \mathbf{s}^{k'} = \mathbf{s}^o)$, $k' = 1,2,\ldots,K$, where the event of $\{\exists x : f(x)\}$ denotes that there is at least one x such that a function of x, $f(x)$, is true.

Definition 2. One defines the probability that the final decision is not the transmitted one provided that the transmitted vector belongs to the set of candidate solutions as $P_{e,\mathrm{SEL}}$. In other words, $P_{e,\mathrm{SEL}}$ is the probability that the final decision is not correct conditioned on $\mathbf{s}^o \in \mathbb{S}$, that is, $P_{e,\mathrm{SEL}} = \Pr(\hat{\mathbf{s}} \neq \mathbf{s}^o \mid \mathbf{s}^o \in \mathbb{S})$.

Using these two probabilities, the error probability of the PVC-MIMO detection can be given by

$$P_e = 1 - \left(1 - P_{e,\mathrm{PV}}\right)\left(1 - P_{e,\mathrm{SEL}}\right) = P_{e,\mathrm{PV}} + P_{e,\mathrm{SEL}} - P_{e,\mathrm{PV}}P_{e,\mathrm{SEL}}. \tag{13}$$

We will first discuss the error probability when an LR-based detector is employed for the sub-detection of PVC-MIMO without PVS. Since an LR-based detector can provide a full receive diversity [11, 12], the PVC-MIMO detection can provide a reasonably good performance even without PVS. Next, we will consider the error probability when a linear detector is employed. In this case, the PVS plays a crucial role in achieving a good performance.

4.1.1. Error Probability with LR-Based Detectors. Let us consider the case where LR-based detectors are used for the sub-detection of PVC-MIMO *without* PVS.

A sufficient and necessary condition for $\mathbf{s}^o \in \mathbb{S}$ is given by $\{\exists \mathbf{s}^{k'} \in \mathbb{S} : \mathbf{s}^{k'} = \mathbf{s}^o\}$. In the proposed PVC approach, noting that $\mathbf{s}^k = [\mathbf{s}_{\mathcal{P}}^{k\mathrm{T}} \;\; \hat{\mathbf{s}}_{\mathcal{Q}}^{k\mathrm{T}}]^{\mathrm{T}}$ and $\mathbf{s}^o = [\mathbf{s}_{\mathcal{P}}^{o\mathrm{T}} \;\; \mathbf{s}_{\mathcal{Q}}^{o\mathrm{T}}]^{\mathrm{T}}$, we have $\Pr(\mathbf{s}^o \in \mathbb{S}) = \Pr(\exists \mathbf{s}^{k'} \in \mathbb{S} : \mathbf{s}_{\mathcal{P}}^{k'} = \mathbf{s}_{\mathcal{P}}^o, \hat{\mathbf{s}}_{\mathcal{Q}}^{k'} = \mathbf{s}_{\mathcal{Q}}^o)$. That is, we have $\mathbf{s}^o \in \mathbb{S}$ if and only if there exists a candidate solution $\mathbf{s}^{k'}$ ($\mathbf{s}^{k'} \in \mathbb{S}$ and $\mathbf{s}^{k'} = [\mathbf{s}_{\mathcal{P}}^{k'\mathrm{T}} \;\; \hat{\mathbf{s}}_{\mathcal{Q}}^{k'\mathrm{T}}]^{\mathrm{T}}$), where the selected $\mathbf{s}_{\mathcal{P}}$ by the PVC approach, that is, $\mathbf{s}_{\mathcal{P}}^{k'}$ in $\mathbf{s}^{k'}$, satisfies $\mathbf{s}_{\mathcal{P}}^{k'} = \mathbf{s}_{\mathcal{P}}^o$ and the detected postvoting vector (see (4)) after this PVC, that is, $\hat{\mathbf{s}}_{\mathcal{Q}}^{k'}$ in $\mathbf{s}^{k'}$, also satisfies $\hat{\mathbf{s}}_{\mathcal{Q}}^{k'} = \mathbf{s}_{\mathcal{Q}}^o$. Note that with the exhaustive search approach of PVC, we have $\Pr(\exists \mathbf{s}^{k'} \in \mathbb{S} : \mathbf{s}_{\mathcal{P}}^{k'} = \mathbf{s}_{\mathcal{P}}^o) = 1$. Hence, we have

$$\begin{aligned} P_{e,\mathrm{PV}} &= 1 - \Pr(\mathbf{s}^o \in \mathbb{S}) = 1 - \Pr\left(\exists \mathbf{s}^{k'} \in \mathbb{S} : \mathbf{s}_{\mathcal{P}}^{k'} = \mathbf{s}_{\mathcal{P}}^o, \hat{\mathbf{s}}_{\mathcal{Q}}^{k'} = \mathbf{s}_{\mathcal{Q}}^o\right) \\ &= 1 - \Pr\left(\hat{\mathbf{s}}_{\mathcal{Q}}^{k'} = \mathbf{s}_{\mathcal{Q}}^o \mid \mathbf{s}_{\mathcal{P}}^{k'} = \mathbf{s}_{\mathcal{P}}^o\right) = E_{\mathbf{H}_{\mathcal{Q}}}\left[P_{e|\mathbf{H}_{\mathcal{Q}}}\right], \end{aligned} \tag{14}$$

where $P_{e|\mathbf{H}_{\mathcal{Q}}}$ denotes the error probability of the sub-detection that detects $\mathbf{s}_{\mathcal{Q}}$ for given $\mathbf{H}_{\mathcal{Q}}$. That is, $P_{e,\mathrm{PV}}$ in (14) is equivalent to the (average) error probability of the sub-detection performed on the square submatrix, $\mathbf{H}_{\mathcal{Q}}$.

Based on the principle of LR, we derive $P_{e,\mathrm{PV}}$ for LR-based detectors. LR-based detectors can achieve a full receive diversity with a relative low complexity by generating a *nearly* orthogonal basis for a given channel matrix [8] to mitigate the effect of (multiple antenna) interference. In the LLL-LR [7] algorithm, we transform $\mathbf{H}_{\mathcal{Q}}$ into a new basis, for example, denoted by $\mathbf{G}$ in (9). Here, we have $\mathcal{L}(\mathbf{G}) = \mathcal{L}(\mathbf{H}_{\mathcal{Q}}) \Leftrightarrow \mathbf{G} = \mathbf{H}_{\mathcal{Q}}\mathbf{T}$, where $\mathbf{T}$ is an integer unimodular matrix and $\mathcal{L}(\mathbf{A})$ denotes a basis of lattice generated by $\mathbf{A}$. Then, $\mathbf{G}$ is called LLL-reduced with parameter δ if $\mathbf{G}$ is QR factorized as

$$\mathbf{G} = \mathbf{QR}, \tag{15}$$

where $\mathbf{Q}$ is unitary ($\mathbf{Q}^T\mathbf{Q} = \mathbf{I}_N$), $\mathbf{R}$ is upper triangular, and the elements of $\mathbf{R}$ satisfy the following inequalities:

$$\begin{aligned} \left|[\mathbf{R}]_{\ell,\rho}\right| &\leq \frac{1}{2}\left|[\mathbf{R}]_{\ell,\ell}\right|, \quad \text{with } 1 \leq \ell < \rho \leq N, \\ \delta[\mathbf{R}]^2_{\rho-1,\rho-1} &\leq [\mathbf{R}]^2_{\rho,\rho} + [\mathbf{R}]^2_{\rho-1,\rho}, \quad \text{with } \rho = 2, \ldots, N, \end{aligned} \tag{16}$$

where δ is a given parameter ($\delta \in (1/2, 1)$) [11].

From [11], the error probability of the LR-based MMSE detection is almost equivalent to that of the LR-based ZF detection. From (9), with the LR-based ZF detection, let $\mathbf{x} = \mathbf{G}^\dagger \mathbf{r}^k$. Then, it follows that

$$\mathbf{x} = \mathbf{U}\mathbf{s}_{\mathcal{Q}} + \mathbf{G}^\dagger \mathbf{n}. \tag{17}$$

The estimation of $\mathbf{s}_{\mathcal{Q}}$ can be expressed as

$$\hat{\mathbf{s}}_{\mathcal{Q}} = \mathbf{U}^{-1}\lfloor \mathbf{x} \rceil = \mathbf{s}_{\mathcal{Q}} + \mathbf{U}^{-1}\left\lfloor \mathbf{G}^\dagger \mathbf{n} \right\rceil. \tag{18}$$

Thus, the error probability of detecting $\mathbf{s}_{\mathcal{Q}}$ for given $\mathbf{H}_{\mathcal{Q}}$ with the LR-based MMSE detectors can be deduced from [11] (for details, see Section 4.3 in [11]). We have

$$P_{e,\text{PV}} \leq c_{NN}\left(\frac{2}{c_\delta^2}\right)^N \frac{(2N-1)!}{(N-1)!}\left(\frac{1}{N_0}\right)^{-N}, \tag{19}$$

where c_{NN} and c_δ^2 are constants and $c_\delta := 2^{N/2}(2/(2\delta - 1))^{-N(N+1)/4} < 1$. The upper bound on $P_{e,\text{PV}}$ in (19) results from the Nth moment of Chi-square random variable, $\|\mathbf{n}\|^2$.

In addition, for LR-based SIC detection, it can be deduced from [12] that the bound of its error probability results from the same moment of $\|\mathbf{n}\|^2$ as the LR-based linear detection. Thus, for LR-based detectors, the upper bound on $P_{e,\text{PV}}$ in (19) results from the Nth moment of $\|\mathbf{n}\|^2$.

Next, we consider $P_{e,\text{SEL}}$. Note that if the ML detector can choose the correct transmitted symbol vector, $\mathbf{s}$, among all the possible candidate vectors in their alphabet $\mathcal{S}$, the detection in (6) can also choose $\mathbf{s}$ (provided that $\mathbf{s} \in \mathbb{S}$ and $\mathbb{S} \subset \mathcal{S}$), and it is obvious to show that

$$P_{e,\text{SEL}} \leq P_{e,\text{ML}}, \tag{20}$$

where $P_{e,\text{ML}}$ is the error probability of the ML detection employed with an $N \times M$ MIMO system. (Inequality (20) is correct if $\mathbf{s}^o \in \mathbb{S}$. Note that the definition of $P_{e,\text{SEL}}$ is the selection error probability when there is one correct candidate in the set $\mathbb{S}$. We can use (20) to calculate $P_{e,\text{SEL}}$, while the error probability if $\mathbf{s}^o$ is not in $\mathbb{S}$ is already calculated by $P_{e,\text{PV}}$.) It is well known that a full receive diversity gain is achieved by this ML detector, which is N [2]. That is, the upper bound on $P_{e,\text{SEL}}$ can also be obtained from the Nth moment of Chi-square random variable, $\|\mathbf{n}\|^2$.

From (13), when the LR-based detectors are employed for the sub-detection, the error probability of the PVC-MIMO detection is given by

$$\begin{aligned} P_e &= P_{e,\text{PV}} + P_{e,\text{SEL}} - P_{e,\text{PV}}P_{e,\text{SEL}} \\ &\leq P_{e,\text{PV}} + P_{e,\text{ML}} - P_{e,\text{PV}}P_{e,\text{SEL}} \leq P_{e,\text{PV}} + P_{e,\text{ML}}. \end{aligned} \tag{21}$$

Since $P_{e,\text{PV}}$, $P_{e,\text{SEL}}$, and $P_{e,\text{ML}}$ in (21) are tail probabilities of a Chi-square random variable with $2N$ degrees of freedom, $\|\mathbf{n}\|^2$, we can see that $P_e \approx c(1/N_0)^{-N}$ as $N_0 \to 0$, where c is a constant that is independent of N_0. Note that N is also the maximum receive diversity order for an underdetermined $N \times M$ MIMO system. Thus, a full receive diversity can be achieved by the proposed PVC-MIMO detection with LR-based subdetectors.

4.1.2. Error Probability with Linear Detectors. If a linear detector (e.g., the MMSE detector introduced in Section 3.1) is used for the sub-detection, the ME criterion can be employed for PVS. Since a linear detector cannot exploit a full receive diversity, the diversity order of the PVC-MIMO detection is less than N. However, if the PVS is employed, the PVC-MIMO detection can achieve a higher diversity order.

It can be shown that for a given set $\mathcal{Q}$, the error probability of the linear sub-detection that detects $\mathbf{s}_{\mathcal{Q}}$ for a given square submatrix $\mathbf{H}_{\mathcal{Q}}$ is expressed as [35]

$$P_{e|\mathbf{H}_{\mathcal{Q}}} \leq \frac{1}{2}\operatorname{erfc}\left(\sqrt{\frac{\lambda_{\min}\left(\mathbf{H}_{\mathcal{Q}}^{\text{H}}\mathbf{H}_{\mathcal{Q}}\right)\|\boldsymbol{\Delta}\|^2}{4N_0}}\right), \tag{22}$$

where $\boldsymbol{\Delta} = \mathbf{s}_{\mathcal{Q}(1)} - \mathbf{s}_{\mathcal{Q}(2)}$ (suppose that $\mathbf{s}_{\mathcal{Q}(1)}$ is transmitted, while $\mathbf{s}_{\mathcal{Q}(2)}$ is erroneously detected) and $\operatorname{erfc}(x)$ is the complementary error function of x, that is, $\operatorname{erfc}(x) = (2/\sqrt{\pi})\int_x^{+\infty} e^{-z^2}dz$. Thus, under the ME selection criterion, the pairwise error probability (PEP) for detecting $\mathbf{s}_{\mathcal{Q}}$ becomes

$$\begin{aligned} P\left(\mathbf{s}_{\mathcal{Q}(1)} \longrightarrow \mathbf{s}_{\mathcal{Q}(2)}\right) &= P_{e|\mathbf{H}_{\mathcal{Q}_{\text{ME}}}} \\ &\leq \frac{1}{2}\operatorname{erfc}\left(\sqrt{\frac{\max_{\mathcal{Q}}\lambda_{\min}\left(\mathbf{H}_{\mathcal{Q}}^{\text{H}}\mathbf{H}_{\mathcal{Q}}\right)\|\boldsymbol{\Delta}\|^2}{4N_0}}\right) \\ &= \frac{1}{2}\operatorname{erfc}\left(\sqrt{\frac{\sigma_h^2\|\boldsymbol{\Delta}\|^2\max_{\mathcal{Q}}X_{\mathcal{Q}}}{4N_0}}\right) \\ &= \frac{1}{2}\operatorname{erfc}\left(\sqrt{\frac{\sigma_h^2\|\boldsymbol{\Delta}\|^2 V}{4N_0}}\right), \end{aligned} \tag{23}$$

where $X_{\mathcal{Q}} = \lambda_{\min}(\mathbf{H}_{\mathcal{Q}}^{\text{H}}\mathbf{H}_{\mathcal{Q}})/\sigma_h^2$, $V = \max_{\mathcal{Q}}X_{\mathcal{Q}}$ and σ_h^2 is the variance of the elements in channel matrix $\mathbf{H}_{\mathcal{Q}}$.

Similar to (14), we have

$$\begin{aligned} P_{e,\text{PV}} &= 1 - \Pr\left(\exists \mathbf{s}^{k'} \in \mathbb{S} : \mathbf{s}_{\mathcal{P}}^{k'} = \mathbf{s}_{\mathcal{P}}^o, \hat{\mathbf{s}}_{\mathcal{Q}}^{k'} = \mathbf{s}_{\mathcal{Q}}^o\right) \\ &= 1 - \Pr\left(\hat{\mathbf{s}}_{\mathcal{Q}}^{k'} = \mathbf{s}_{\mathcal{Q}}^o \mid \mathbf{s}_{\mathcal{P}}^{k'} = \mathbf{s}_{\mathcal{P}}^o\right) = E_{\mathbf{H}_{\mathcal{Q}}}\left[P_{e|\mathbf{H}_{\mathcal{Q}_{\text{ME}}}}\right]. \end{aligned} \tag{24}$$

Then from (23), we can obtain that

$$P_{e,\text{PV}} = E_{\mathbf{H}_{\mathcal{Q}}}\left[P_{e|\mathbf{H}_{\mathcal{Q}_{\text{ME}}}}\right] \leq E_V\left[\frac{1}{2}\operatorname{erfc}\left(\sqrt{\frac{\sigma_h^2\|\boldsymbol{\Delta}\|^2 V}{4N_0}}\right)\right]. \tag{25}$$

For the random matrix $\mathbf{H}_{\mathcal{Q}}$, the probability density function (pdf) of $X_{\mathcal{Q}}$ is given by [37]

$$f_x(x) = Ne^{-Nx}. \tag{26}$$

If all the possible submatrices $\mathbf{H}_{\mathcal{Q}}$ (after PVS), which are the candidate channel matrices for the sub-detection, are assumed to be independent, the pdf of V is

$$\begin{aligned} f_V(v) &= LN\left(1 - e^{-Nv}\right)^{L-1} e^{-Nv} \\ &= LN^L v^{L-1} + o\left(v^{L-1+\epsilon}\right) \quad (v \to 0^+), \end{aligned} \tag{27}$$

where $\epsilon > 0$ and $L = C_M^N$ denotes the number of possible candidates for $\mathcal{Q}$ (C_M^N is the number of combinations for selecting N items in M items). (This assumption does not hold in practical situation (the last paragraph of this subsection addresses the practical situation).)

The relation between the PEP in (23) and the pdf of variable V can be deduced by Wang and Giannakis in [38]. Thus, according to [38], we can show that

$$\begin{aligned} P_{e,\text{PV}} &\le E_V\left[\frac{1}{2}\operatorname{erfc}\left(\sqrt{\frac{\sigma_h^2\|\mathbf{\Delta}\|^2 V}{4N_0}}\right)\right] \\ &\le \int_0^{+\infty} \frac{1}{2}\operatorname{erfc}\left(\sqrt{\frac{\sigma_h^2\|\mathbf{\Delta}\|^2 v}{4N_0}}\right) f_V(v)\,dv \\ &= c_1\gamma_\Delta^{-L} + o\left(\gamma_\Delta^{-(L+1)}\right), \end{aligned} \tag{28}$$

where $\gamma_\Delta = \sigma_h^2\|\mathbf{\Delta}\|^2/N_0$ and $c_1 > 0$ is constant.

Note that for $M \ge N+1$, $C_M^N = M!/(M-N)!N! \ge M!/(M-N)!N(N-1)\cdots 2 \ge M!/(M-N)!(M-1)(M-2)\cdots(M-N+1) = M!/(M-1)! = M > N$, that is, $L > N$. In addition, (20) and (21) also hold for linear detectors. Hence, according to (28), a full diversity order N can be achieved by the proposed detectors when the ME criterion for index set $\mathcal{Q}$ selection is employed.

In practice, different $\mathbf{H}_{\mathcal{Q}}$'s are not independent (i.e., $X_{\mathcal{Q}}$ are correlated for different $\mathcal{Q}$), and the minimum eigenvalues of $\mathbf{H}_{\mathcal{Q}}^{\text{H}}\mathbf{H}_{\mathcal{Q}}$'s are correlated in the proposed detection after PVS. Thus, (27) may *not* be valid (but just an approximation), and a full diversity order N*cannot* be achieved. However, for a small-sized matrix $\mathbf{H}_{\mathcal{Q}}$, a near full diversity order may be achieved due to the low correlation of the minimum eigenvalues of different $\mathbf{H}_{\mathcal{Q}}^{\text{H}}\mathbf{H}_{\mathcal{Q}}$'s. The numerical results shown in the following section also confirm this observation. That is, with the optimal PVS, the linear detector-based PVC-MIMO detection can achieve a higher diversity; for a small matrix $\mathbf{H}_{\mathcal{Q}}$ (e.g., a 2×2 matrix), a near-full receive diversity is achieved by the proposed detection.

4.2. Complexity Analysis. Denote by $\mathcal{C}_{\text{Sub}}$ the complexity of the sub-detection with a square channel matrix of $N \times N$. Excluding the complexity of the PVS, the complexity of the PVC-MIMO detection is given by

$$\mathcal{C}_{\text{PVC}} = K\mathcal{C}_{\text{Sub}}. \tag{29}$$

If an exhaustive search is employed to determine $\mathcal{Q}$ in (8), (11), or (12), because there are $\prod_{i=0}^{M-N-1}(M-i)$ possible index sets, the complexity for building $\mathcal{Q}$ is $\prod_{i=0}^{M-N-1}(M-i)\mathcal{C}_{\text{Sel}}$, where $\mathcal{C}_{\text{Sel}}$ denotes the computational complexity for each possible index set. For example, if the MD selection criterion is used when $M = 4$ and $N = 2$, we need $4 \times 3 = 12$ LRs of 2×2 complex-valued channel matrices, and $\mathcal{C}_{\text{Sel}}$ becomes the complexity for each LR. We will list the complexity of $\mathcal{C}_{\text{Sel}}$ with different PVSs for their corresponding subdetectors in Section 5, empirically using the average number of floating point operation (flops).

For a block fading channel, assume that the channel is not varying for a duration of W symbol vectors transmitted. Note that PVS is only performed once for a channel matrix. Then, including the complexity of PVS, the overall computational complexity of the PVC-MIMO detection per each symbol vector is given by

$$\mathcal{C}_{\text{PVC}} = \frac{\prod_{i=0}^{M-N-1}(M-i)\mathcal{C}_{\text{Sel}}}{W} + K\mathcal{C}_{\text{Sub}}. \tag{30}$$

For slow fading channels, where the coherence time is long, W will be large. In this case, the extracomputational complexity required for PVS per each symbol detection would be negligible, where we have $\mathcal{C}_{\text{PVC}} \approx K\mathcal{C}_{\text{Sub}}$. In Section 5, we will compare the complexity of our proposed PVC-MIMO detectors to other MIMO detectors using flops.

5. Simulation Results and Discussions

5.1. Simulation Results. In this subsection, we present simulation results to compare our PVC-MIMO detectors with other detectors (including the MMSE (linear) detector, ML detector, the Chase detector, and the TSD-CR [33] which provides the ML performance) for underdetermined MIMO systems. (For the Chase detector [19–24], the subvector of sized $(M-N)\times 1$ to be detected in the first layer is selected from $\mathbf{s}$ as the one with the smallest MSE (i.e., equivalently the highest SNR), and a list of Q candidates for this subvector is constructed. In the second layer, the contribution from the detected subvector is treated as the interference and is canceled from the received signal. Then, the sub-detection is employed with the corresponding $N \times N$ subchannel matrix to detect the residual $N \times 1$ subvector. Two scenarios are considered for the Chase detection: (i) MMSE + Chase (MMSE subdetector used in Chase detection); (ii) LR-based MMSE-SIC + Chase (LR-based MMSE-SIC subdetector used in Chase detection).) Six combinations of the PVC-MIMO detectors are considered as follows: (a) MMSE + PVC-MIMO (MMSE subdetector used in PVC-MIMO); (b) LR-based MMSE + PVC-MIMO (LR-based MMSE subdetector used in PVC-MIMO); (c) LR-based MMSE-SIC + PVC-MIMO (LR-based MMSE-SIC subdetector used in PVC-MIMO); (d) MMSE + PVC-MIMO + PVS (MMSE subdetector used in PVC-MIMO with optimal PVS (ME criterion)); (e) LR-based MMSE + PVC-MIMO + PVS (LR-based MMSE subdetector used in PVC-MIMO with optimal PVS (ME criterion)); (f) LR-based MMSE-SIC + PVC-MIMO + PVS (LR-based MMSE-SIC subdetector used in PVC-MIMO with

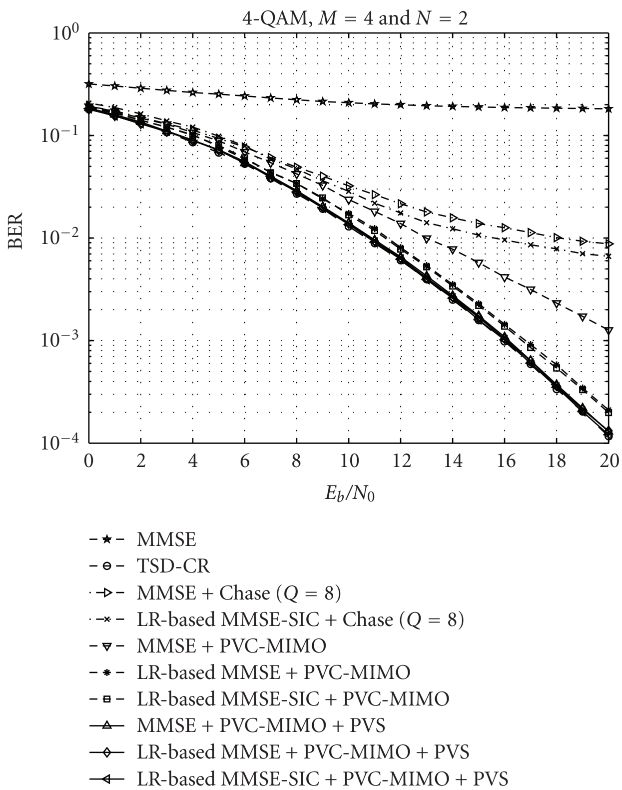

FIGURE 1: BER versus E_b/N_0 of different detectors represented in Section 5.1 for 4-QAM, $M = 4$, $N = 2$.

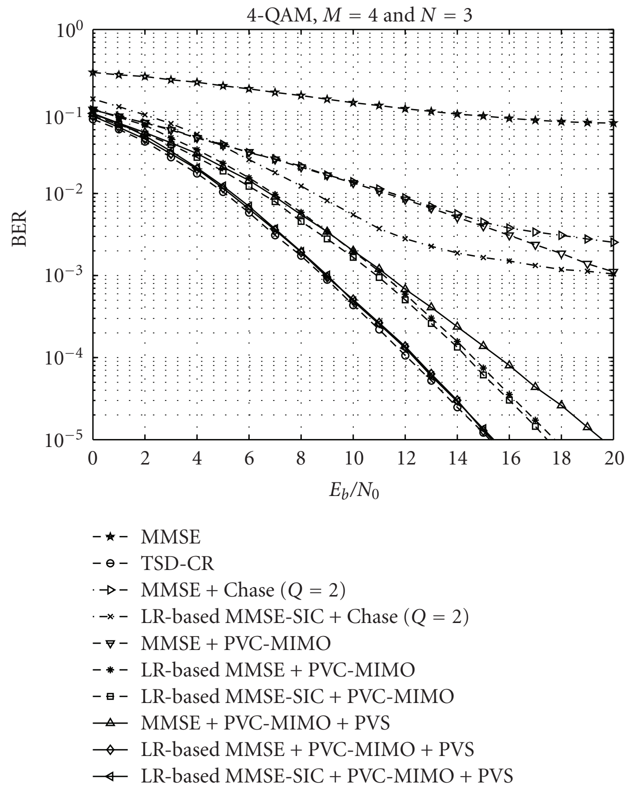

FIGURE 2: BER versus E_b/N_0 of different detectors represented in Section 5.1 for 4-QAM, $M = 4$, $N = 3$.

optimal PVS (MD criterion)). As we are interested in the case where the receiver's computational complexity is limited, we only consider the cases of $(M, N) \in \{(4, 2), (4, 3), (3, 2)\}$. (The case of a large $M - N$ is discussed in Section 5.2.). Note that elements of MIMO channel matrices in simulations are generated as independent CSCG random variables with mean zero and unit variance. The SNR is defined as the energy per bit to the noise power spectral density ratio, E_b/N_0. We assume that 4-QAM and 16-QAM are used for signaling with Gray mapping.

With 4-QAM modulation, in Figures 1 and 2, for channel matrices of size 2×4 and 3×4, respectively, we show simulation results of BER for various detectors. In Figures 3 and 4, with 16-QAM modulation, simulation results of BER for various detectors are presented for channel matrices of size 2×3 and 3×4, respectively.

From the simulation results, it is shown that a full receive diversity can be achieved by employing the PVC-MIMO detection approach with LR-based subdetectors. In Figures 1 and 3, we can see that "LR-based MMSE/MMSE-SIC + PVC-MIMO" has a slight performance degradation from the ML detector and the SNR loss is a half dB at a broad range of BER. In all the simulation results, it is also shown that "LR-based MMSE/MMSE-SIC + PVC-MIMO + PVS" has negligible performance degradation compared to the ML performance. Furthermore, we note that "MMSE + Chase" and "LR-based MMSE-SIC + Chase" cannot provide a full diversity and good performance, especially when SNR is high.

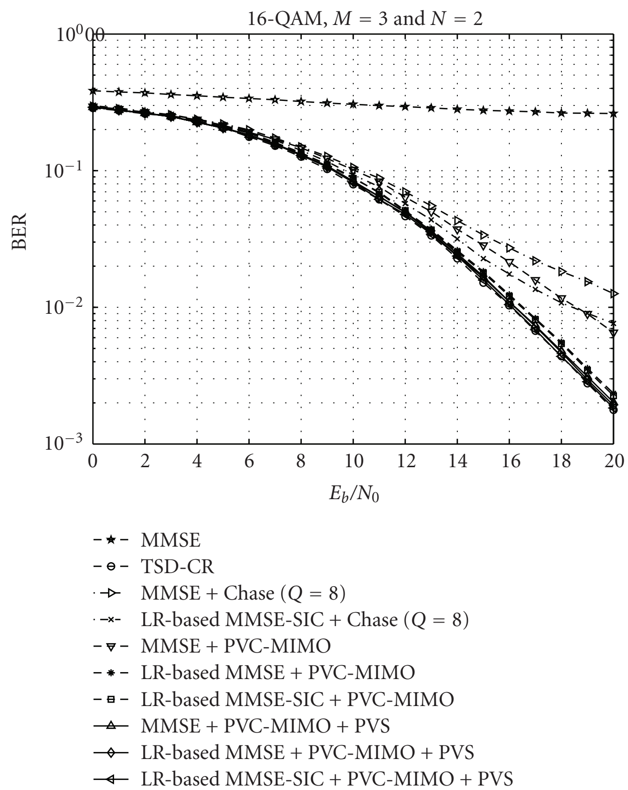

FIGURE 3: BER versus E_b/N_0 of different detectors represented in Section 5.1 for 16-QAM, $M = 3$, $N = 2$.

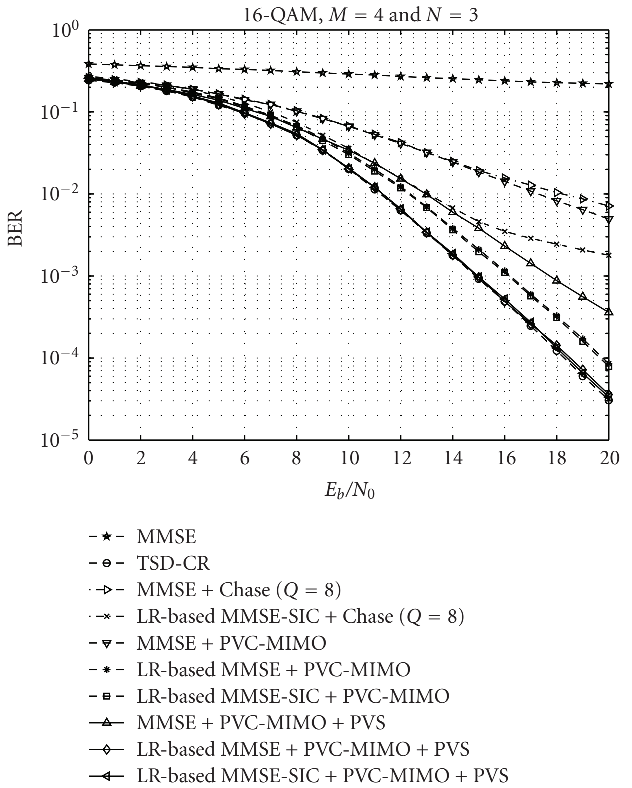

FIGURE 4: BER versus E_b/N_0 of different detectors represented in Section 5.1 for 16-QAM, $M = 4$, $N = 3$.

In Figures 2 and 4, we can see that "MMSE + PVC-MIMO + PVS" can provide a reasonably good performance. For a 2×2 submatrix, we can observe that "MMSE + PVC-MIMO + PVS" can provide a near ML performance from Figures 1 and 3, where the sizes of channel matrices are 2×4 and 2×3, respectively. We note that the performance of "MMSE + PVC-MIMO + PVS" with $N = 2$ is better than that with $N = 3$. Since a low correlation of the minimum eigenvalue of $\mathbf{H}_{\mathcal{Q}}^{\mathrm{H}}\mathbf{H}_{\mathcal{Q}}$ is obtained by employing a reduced-sized channel matrix $\mathbf{H}_{\mathcal{Q}}$, a less error propagation is expected. This confirms that the PVC-MIMO detection with MMSE subdetector could be effective when N is sufficiently small.

In Table 1, we list the complexity of $\mathcal{C}_{\text{Sel}}$ for different detectors (i.e., "MMSE + PVC-MIMO + PVS," "LR-based MMSE + PVC-MIMO + PVS," and "LR-based MMSE-SIC + PVC-MIMO + PVS") by using flops, for the case of $N = 2$ and $N = 3$, respectively. Since the computation for both LR and eigenvalue is considered in "LR-based MMSE + PVC-MIMO + PVS," the highest complexity is required.

Since the TSD-CR approach [33] can be applied to underdetermined MIMO systems with a reasonable low complexity and optimal performance, it is worthy to compare its complexity with our proposed schemes. In Table 2, we compare the complexity of our proposed PVC-MIMO detectors to other MIMO detectors including the ML detector (using an exhaustive search), MMSE detector, TSD-CR, and Chase detectors by using flops with $W = 1000$, where slow fading channels are considered. (The complexity of PVC-MIMO with fast fading channels is discussed in Section 5.2.). Note that for PVC-MIMO and TSD-CR, the PVS and Householder QR decomposition of channel matrix with minimum column pivoting are carried out once for 1000 symbol vectors transmitted, respectively, to make this comparison fair. The flops listed in Table 2 are obtained with $E_b/N_0 = 20$ dB.

TABLE 1: Complexity comparison of $\mathcal{C}_{\text{Sel}}$ for different detectors listed in Section 5.1.

	Average flops of $\mathcal{C}_{\text{Sel}}$	
Detector	$N = 2$	$N = 3$
MMSE + PVC-MIMO + PVS	258	1608
LR-based MMSE + PVC-MIMO + PVS	678	3070
LR-based MMSE-SIC + PVC-MIMO + PVS	473	1587

Although the MMSE and Chase detectors have a low complexity, they do not suit for underdetermined MIMO systems. It is shown that the computational complexity of the proposed PVC-MIMO detectors with optimal PVS for the case of $\{M, N\} = \{3, 2\}$, $\{M, N\} = \{4, 2\}$, and $\{M, N\} = \{4, 3\}$ with 4-QAM is significantly lower than that of ML and TSD-CR. It is also shown that, with 16-QAM, the proposed detectors can also provide a relatively lower complexity for the case of $\{M, N\} = \{3, 2\}$ and $\{M, N\} = \{4, 3\}$. In addition, for different PVC-MIMO detectors in the same MIMO system, "MMSE + PVC-MIMO + PVS" has the lowest computational complexity among the PVC-MIMO detectors, since no LR is used in PVS and sub-detection.

Overall, "LR-based MMSE-SIC + PVC-MIMO + PVS" is shown to be very attractive, because its performance is close to that of the ML detection and its complexity is low (the complexity is almost the same as that of "MMSE + PVC-MIMO + PVS", which is the lowest). From this, we can see that the combination of LR detector and optimal PVS is the key ingredient to build low complexity, but near ML performance, detection schemes for underdetermined MIMO systems.

5.2. Discussion. In Section 5.1, we have discussed the computational complexity of PVC-MIMO detection with slow fading MIMO channels, where $M - N$ is small (e.g., 1 or 2). In this subsection, we discuss the complexity of the PVC-MIMO detection for fast fading channels and a large $M - N$. Furthermore, the impact of channel estimation errors is considered.

5.2.1. Fast Fading Channels. Previously, we have analyzed the complexity of the PVC-MIMO detection with PVS for slow fading MIMO channels, where W is large (e.g., $W = 1000$). Note that fast fading channels lead to a small W. With the overall complexity per each symbol vector of the PVC-MIMO detection in (30), $\mathcal{C}_{\text{PVC}}$ would be high since the weight of $\mathcal{C}_{\text{Sel}}$ is high when W is small (i.e., the complexity of $\mathcal{C}_{\text{Sel}}$ is given in Table 1). Therefore, the PVC-MIMO detection with PVS could have a high complexity with a small W.

Table 2: Complexity comparison of different detectors listed in Section 5.1.

System	Average flops for each symbol vector detection				
	4-QAM			16-QAM	
	$\{M, N\} = \{3, 2\}$	$\{M, N\} = \{4, 2\}$	$\{M, N\} = \{4, 3\}$	$\{M, N\} = \{3, 2\}$	$\{M, N\} = \{4, 3\}$
MMSE	78	109	112	302	411
ML	4484	22021	32773	286724	8388613
TSD-CR	753	1296	1226	3467	5546
MMSE + Chase	168	623	239	1671	2479
LR-based MMSE-SIC + Chase	170	626	255	1673	2490
MMSE + PVC-MIMO + PVS	193	770	325	3056	4645
LR-based MMSE + PVC-MIMO + PVS	201	783	377	3074	4697
LR-based MMSE-SIC + PVC-MIMO + PVS	197	778	356	3060	4666

For the case of $W = 10$, where channel varies every 10 symbol vectors transmitted (i.e., reasonably fast fading channels), with $\{N, M\} = \{2, 3\}$ and $\{N, M\} = \{2, 4\}$, the average computational complexity per each symbol vector for PVS of "LR-based MMSE-SIC + PVC-MIMO + PVS" is 155 and 310, respectively, in terms of flops. In this case, compared to existing approaches (in Table 2), the complexity of the PVC-MIMO with PVS is still low.

5.2.2. Large $M - N$. Since there are underdetermined MIMO systems with a large $M - N$, it is worthy to discuss the complexity of PVC-MIMO detection employed in such MIMO systems. Considering a low-order modulation (4-QAM), by using the same method that obtains the flops in Table 2, we compare the computational complexity of "LR-based MMSE-SIC + PVC-MIMO + PVS" and TSD-CR [33] for the cases of $\{M, N\} = \{5, 2\}$ and $\{6, 2\}$, respectively, in terms of flops. For "LR-based MMSE-SIC + PVC-MIMO + PVS," the flops of $\{M, N\} = \{5, 2\}$ and $\{6, 2\}$ are 3106 and 12263, respectively. For TSD-CR, the flops of $\{M, N\} = \{5, 2\}$ and $\{6, 2\}$ are 5010 and 19564, respectively. It shows that the PVC-MIMO detection has a lower complexity than TSD-CR with a large $M - N$ and a low-order modulation.

We note that the PVC-MIMO detection is not suitable for the case of a large $M - N$ and a high-order modulation (16-QAM or 64-QAM) due to the exhaustive cancellation of prevoting vectors. However, it is noteworthy that the GSD-based detection (e.g., TSD-CR) has also high complexity [25–33].

5.2.3. Imperfect CSI Estimation. In practice, the channel matrix has to be estimated, and there could be estimation errors. Considering an $N \times M$ channel matrix $\mathbf{H}$ represented in (1), whose elements are generated as independent CSCG random variables with mean zero and unit variance, with an imperfect CSI estimation, the estimated channel matrix is given by $\hat{\mathbf{H}} = \mathbf{H} + \mathbf{E}$. Here, an $N \times M$ matrix $\mathbf{E}$ represents errors in the CSI estimation, whose elements are generated as independent zero-mean CSCG random variables with variance ν_e^2.

With $\{N, M\} = \{2, 4\}$ and 4-QAM modulation, in Figure 5, we present simulation results of BER for TSD-CR and "LR-based MMSE-SIC + PVC-MIMO + PVS" with different CSI errors, where $\nu_e = 0$, 0.02, and 0.05. Figure 5 shows that the performance of TSD-CR and "LR-based MMSE-SIC + PVC-MIMO + PVS" degrades when ν_e increases in general. Nevertheless, it shows that our proposed PVC-MIMO detection with PVS (i.e., "LR-based MMSE-SIC + PVC-MIMO + PVS") has a negligible performance gap from the ML performance (i.e., TSD-CR) with CSI estimation errors.

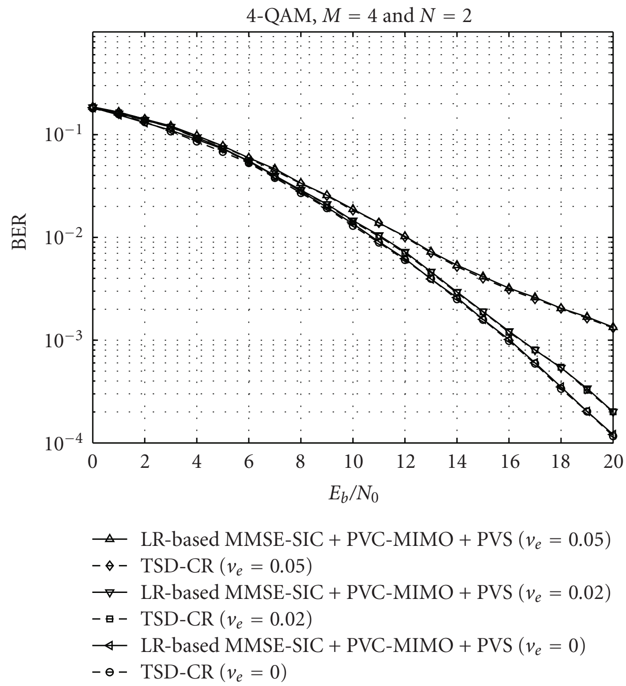

Figure 5: BER versus E_b/N_0 of "TSD-CR" and "LR-based MMSE-SIC + PVC-MIMO + PVS" represented in Section 5.1 for $\nu_e = \{0, 0.02, 0.05\}$ with 4-QAM, $M = 4$, $N = 2$.

6. Conclusion

For underdetermined MIMO systems where a lower-order modulation scheme can be employed, we considered low complexity MIMO detection approaches based on PVC in this paper. It was shown that if an LR-based detector is used for the sub-detection, the PVC-MIMO detection can achieve a full receive diversity order. We confirmed this through simulations. It was also shown that the complexity of the proposed PVC-MIMO detectors is low and comparable to that of the MMSE detector when 4-QAM is used. Therefore, the proposed detection approach can be employed for underdetermined MIMO systems where the receiver's computational complexity is limited such as mobile terminals.

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