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Research Article

Carrier Frequency Offset Estimation for Multiuser MIMO OFDM Uplink Using CAZAC Sequences: Performance and Sequence Optimization

Yan Wu,[1] J. W. M. Bergmans,[1] and Samir Attallah[2]

[1] *Signal Processing Systems Group, Department of Electrical Engineering, Technische Universiteit Eindhoven, P.O. Box 513, 5600 MB Eindhoven, The Netherlands*
[2] *School of Science and Technology, SIM University, Singapore 599491*

Correspondence should be addressed to Yan Wu, y.w.wu@tue.nl

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This paper studies carrier frequency offset (CFO) estimation in the uplink of multi-user multiple-input multiple-output (MIMO) orthogonal frequency division multiplexing (OFDM) systems. Conventional maximum likelihood estimator requires computational complexity that increases exponentially with the number of users. To reduce the complexity, we propose a sub-optimal estimation algorithm using constant amplitude zero autocorrelation (CAZAC) training sequences. The complexity of the proposed algorithm increases only linearly with the number of users. In this algorithm, the different CFOs from different users destroy the orthogonality among training sequences and introduce multiple access interference (MAI), which causes an irreducible error floor in the CFO estimation. To reduce the effect of the MAI, we find the CAZAC sequence that maximizes the signal to interference ratio (SIR). The optimal training sequence is dependent on the CFOs of all users, which are unknown. To solve this problem, we propose a new cost function which closely approximates the SIR-based cost function for small CFO values and is independent of the actual CFOs. Computer simulations show that the error floor in the CFO estimation can be significantly reduced by using the optimal sequences found with the new cost function compared to a randomly chosen CAZAC sequence.

1. Introduction

Compared to single-input single-output (SISO) systems, multiple-input multiple-output (MIMO) systems increase the capacity of rich scattering wireless fading channels enormously through employing multiple antennas at the transmitter and the receiver [1, 2]. Orthogonal Frequency Division Multiplexing (OFDM) is a widely used technology for wireless communication in frequency selective fading channels due to its high spectral efficiency and its ability to "divide" a frequency selective fading channel into multiple flat fading subchannels (subcarriers). Hence, MIMO-OFDM is an ideal combination for applying MIMO technology in frequency fading channels and has been included in various wireless standards such as IEEE 802.11n [3] and IEEE 802.16e [4]. An extension of the MIMO-OFDM system is the multiuser MIMO-OFDM system as illustrated in Figure 1. In such a system, multiple users, each with one or multiple antennas, transmit simultaneously using the same frequency band. The receiver is a base-station equipped with multiple antennas. It uses spatial processing techniques to separate the signals of different users. If we view the signals from different users as signals from different transmit antennas of a virtual transmitter, then the whole system can be viewed as a MIMO system. This system is also known as the virtual MIMO system [5].

Carrier frequency offset (CFO) is caused by the Doppler effect of the channel and the difference between the transmitter and receiver local oscillator (LO) frequencies. In OFDM systems, CFO destroys the orthogonality between subcarriers and causes intercarrier interference (ICI). To ensure good performance of OFDM systems, the CFO must be accurately estimated and compensated. For SISO-OFDM systems, periodic training sequences are used in

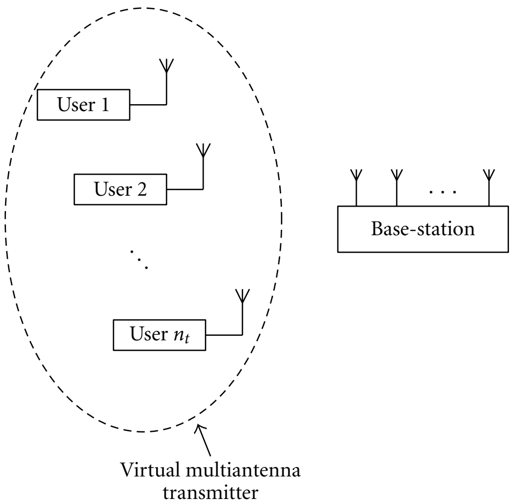

Figure 1: Overview of multiuser MIMO-OFDM systems.

[6, 7] to estimate the CFO. It is shown that these CFO estimators reach the Cramer-Rao bound (CRB) with low-computational complexity. A similar idea was extended to collocated MIMO-OFDM systems [8–10], where all the transmit antennas are driven by a centralized LO and so are all the receive antennas. In this case, the CFO is still a single parameter. For multiuser MIMO-OFDM systems, each user has its own LO, while the multiple antennas at the base-station (receiver) are driven by a centralized LO. Therefore, in the uplink, the receiver needs to estimate multiple CFO values for all the users. In [11, 12], methods were proposed to estimate multiple CFO values for MIMO systems in flat fading channels. In [13], a semiblind method was proposed to jointly estimate the CFO and channel for the uplink of multiuser MIMO-OFDM systems in frequency selective fading channels. An asymptotic Cramer-Rao bound for joint CFO and channel estimation in the uplink of MIMO-Orthogonal Frequency Division Multiple Access (OFDMA) system was derived in [14] and training strategies that minimize the asymptotic CRB were studied. In [15], a reduced-complexity CFO and channel estimator was proposed for the uplink of MIMO-OFDMA systems using an approximation of the ML cost function and a Newton search algorithm. It was also shown that the reduced-complexity method is asymptotically efficient. The joint CFO and channel estimation for multiuser MIMO-OFDM systems was studied in [16]. Training sequences that minimize the asymptotic CRB were also designed in [16].

It is known in the literature that the computational complexity for obtaining the ML CFO estimates in the uplink of multiuser MIMO-OFDM system grows exponentially with the number of users [15, 16]. A low-complexity algorithm was proposed in [16] for CFO estimation in the uplink of multiuser MIMO OFDM systems based on importance sampling. However, the complexity required to generate sufficient samples for importance sampling may still be high for practical implementations. In this paper, we study algorithms that can further reduce the computational complexity of the CFO estimation. Following a similar approach as in [17], we first derive the maximum likelihood (ML) estimator for the multiple CFO values in frequency selective fading channels. Obtaining the ML estimates requires a search over all possible CFO values and the computational complexity is prohibitive for practical implementations. To reduce the complexity, we propose a sub-optimal algorithm using constant amplitude zero autocorrelation (CAZAC) training sequences, which have zero autocorrelation for any nonzero circular shifts. Using the proposed algorithm, the CFO estimates can be obtained using simple correlation operations and the complexity of this algorithm grows only linearly with the number of users. However, the multiple CFO values destroy the orthogonality between the training sequences of different users. This introduces multiple access interference (MAI) and causes an irreducible error floor in the mean square error (MSE) of the CFO estimates. We derive an expression for the signal to interference ratio (SIR) in the presence of multiple CFO values. To reduce the MAI, we find the training sequence that maximizes the SIR. The optimal training sequence turns out to be dependent on the actual CFO values from different users. This is obviously not practical as it is not possible to know the CFO values and hence select the optimal training sequence in advance. To remove this dependency, we propose a new cost function, which is the Taylor's series approximation of the original cost function. The new cost function is independent of the actual CFO values and is an accurate approximation of the original SIR-based cost function for small CFO values. Using the new cost function, we obtain the optimal training sequences for the following three classes of CAZAC sequences:

(i) Frank and Zadoff Sequences [18],

(ii) Chu Sequences [19],

(iii) Polyphase Sequence by Sueshiro and Hatori (S&H Sequences) [20].

Both Frank and Zadoff sequences and S&H sequences exist for sequence length of $N = K^2$, where N is the length of the sequence and K is a positive integer, while Chu sequences exist for any integer length. For both Frank and Zadoff and Chu sequences, there are a finite number of sequences for each sequence length. Therefore, the optimal sequence can be obtained using a search among these sequences. However, for S&H sequences, there are infinitely many possible sequences. As the optimization problem for S&H sequences cannot be solved analytically, we resort to a numerical method to obtain a near-optimal solution. To this end, we use the adaptive simulated annealing (ASA) technique [21]. For small sequence lengths, for example, $N = 16$ and $N = 36$, we are able to use exhaustive search to verify that the solution obtained using ASA is globally optimal. (Because CFO values are continuous variables, theoretically, it is not possible to obtain the exact optimum using exhaustive computer search, which works in discrete variables. If we keep the step size in the search small enough, we can be sure that the obtained "optimum" is very close to the actual optimum and can be practically assumed to the actual optimum. In this way, we are able to verify the solution obtained by the ASA is "practically" optimal.) Computer simulations were conducted to evaluate the performance of the CFO estimation using CAZAC sequences. We first compare the performance using CAZAC sequences with the performance using two other sequences with good correlation properties, namely, the IEEE 802.11n short training field (STF) [3] and the m sequences [22]. The results show that the error floor using the CAZAC sequences is more than 10 times smaller compared to the other two sequences. Comparing the three classes of CAZAC sequences, we find that the performance of the Chu sequences is better than the Frank and Zadoff sequences due to the larger degree of freedom in the sequence construction. The S&H sequences have the largest number of degree of freedom in the construction of the CAZAC sequences. However, the simulation results show that they have only very marginal performance gain compared to the Chu sequences. This makes Chu sequences a good choice for practical implementation due to its simple construction and flexibility in sequence lengths. By using the identified optimal sequences, the error floor in the CFO estimation is significantly lower compared to using a randomly selected CAZAC sequence.

The rest of the paper is organized as follows. In Section 2, we present the system model and derive the ML estimator for the multiple CFO values. The sub-optimal CFO estimation algorithm using CAZAC sequences is proposed in Section 3. The training sequence optimization problem is formulated in Section 4 and methods are given to obtain the optimal training sequence. In Section 5, we present the computer simulation results and Section 6 concludes the paper.

2. System Model

In this paper, we study a multiuser MIMO-OFDM system with n_t users. For simplicity of illustration and analysis, we assume that each user has a single transmit antenna. The base-station has n_r receive antennas, where $n_r \geq n_t$. The received signal at the ith receive antenna can be written as

$$r_i(k) = \sum_{m=1}^{n_t} \left(e^{j\phi_m k} \sum_{d=0}^{L-1} h_{i,m}(d) s_m(k-d) \right) + n_i(k), \tag{1}$$

where ϕ_m is the CFO of the mth user, k is the time index, and L is the number of multipath components in the channel. The dth tab of the channel impulse response between the mth user and the ith receive antenna is denoted as $h_{i,m}(d)$, s_m denotes the transmitted signal from the mth user and n_i is the additive white Gaussian noise at the ith receive antenna. Here we assume the initial phase for each user is absorbed in the channel impulse response. From (1), we can see that we have n_t different CFO values (ϕ_m's) to estimate. We consider a training sequence of length N and cyclic prefix (CP) of length L. The received signal after removal of CP can be written in an equivalent matrix form

$$\mathbf{r}_i = \sum_{m=1}^{n_t} \mathbf{E}(\phi_m) \mathbf{S}_m \mathbf{h}_{i,m} + \mathbf{n}_i, \tag{2}$$

where $\mathbf{r}_i = [r_i(0), \ldots, r_i(N-1)]^T$ and superscript T denotes vector transpose. The CFO matrix of user m is denoted $\mathbf{E}(\phi_m)$ and is a diagonal matrix with diagonal elements equal to $[1, \exp(j\phi_m), \ldots, \exp(j(N-1)\phi_m)]$. We use $\mathbf{S}_m$ to denote the transmitted signal matrix for the mth user, which is an $N \times N$ circulant matrix with the first column defined by $[s_m(0), s_m(1), s_m(2), \ldots, s_m(N-1)]^T$. Here we assume $N > L$ so the channel vector between the mth user and the ith receive antenna $\mathbf{h}_{i,m}$ is an $N \times 1$ vector by appending the $L \times 1$ channel impulse response $[h_{i,m}(0), \ldots, h_{i,m}(L-1)]^T$ vector with $N - L$ zeros.

Using this system model, the received signals from all n_r receive antennas can be written as

$$\boldsymbol{\mathcal{R}} = \boldsymbol{\mathcal{A}}(\phi) \boldsymbol{\mathcal{H}} + \boldsymbol{\mathcal{N}}, \tag{3}$$

where

$$\begin{aligned} \boldsymbol{\mathcal{R}} &= [\mathbf{r}_1, \ldots, \mathbf{r}_{n_r}]_{N \times n_r}, \\ \boldsymbol{\mathcal{A}}(\phi) &= [\mathbf{E}(\phi_1)\mathbf{S}_1, \ldots, \mathbf{E}(\phi_{n_t})\mathbf{S}_{n_t}]_{N \times (N \times n_t)}. \end{aligned} \tag{4}$$

For clearness of presentation, we use subscripts under the square bracket to denote the size of the corresponding matrix. The vector $\phi = [\phi_1, \ldots, \phi_{n_t}]$ is the CFO vector containing the CFO values from all users, and the channels of all users are stacked into the channel matrix $\boldsymbol{\mathcal{H}}$ given as

$$\boldsymbol{\mathcal{H}} = \begin{bmatrix} \boldsymbol{\mathcal{H}}_1 \\ \vdots \\ \boldsymbol{\mathcal{H}}_{n_t} \end{bmatrix}_{(N \times n_t) \times n_r}, \tag{5}$$

with $\boldsymbol{\mathcal{H}}_i = [\mathbf{h}_{1,i}, \ldots, \mathbf{h}_{n_r,i}]_{N \times n_r}$ being the channel matrix for the ith user. The noise matrix is given by $\boldsymbol{\mathcal{N}} = [\mathbf{n}_1, \ldots, \mathbf{n}_{n_r}]$.

Because the noise is Gaussian and uncorrelated, the likelihood function for the channel $\mathcal{H}$ and CFO values ϕ can be written as

$$\Lambda\left(\widetilde{\boldsymbol{\mathcal{H}}}, \widetilde{\phi}\right) = \frac{1}{\left(\pi\sigma_n^2\right)^{N \times n_r}} \exp\left\{ -\frac{1}{\sigma_n^2} \left\| \boldsymbol{\mathcal{R}} - \boldsymbol{\mathcal{A}}(\widetilde{\phi}) \widetilde{\boldsymbol{\mathcal{H}}} \right\|^2 \right\}, \tag{6}$$

where $\widetilde{\boldsymbol{\mathcal{H}}}$ and $\widetilde{\phi}$ are trial values for $\boldsymbol{\mathcal{H}}$ and ϕ and σ_n^2 is the variance of the AWGN noise. Following a similar approach as in [17], we find that for a fixed CFO vector ϕ, the ML estimate of the channel matrix is given by

$$\widehat{\boldsymbol{\mathcal{H}}}\left(\widetilde{\phi}\right) = \left[\boldsymbol{\mathcal{A}}^H\left(\widetilde{\phi}\right) \boldsymbol{\mathcal{A}}\left(\widetilde{\phi}\right) \right]^{-1} \boldsymbol{\mathcal{A}}^H\left(\widetilde{\phi}\right) \boldsymbol{\mathcal{R}}, \tag{7}$$

where superscript H denotes matrix Hermitian. Substituting (7) into (6) and after some algebraic manipulations, we obtain that the ML estimate of the CFO vector ϕ is given by

$$\widehat{\phi} = \arg\max_{\widetilde{\phi}} \left\{ \operatorname{tr}\left(\boldsymbol{\mathcal{R}}^H \boldsymbol{\mathcal{B}}\left(\widetilde{\phi}\right) \boldsymbol{\mathcal{R}} \right) \right\}, \tag{8}$$

with

$$\boldsymbol{\mathcal{B}}\left(\widetilde{\phi}\right) = \boldsymbol{\mathcal{A}}\left(\widetilde{\phi}\right) \left[\boldsymbol{\mathcal{A}}^H\left(\widetilde{\phi}\right) \boldsymbol{\mathcal{A}}\left(\widetilde{\phi}\right) \right]^{-1} \boldsymbol{\mathcal{A}}^H\left(\widetilde{\phi}\right), \tag{9}$$

and $\operatorname{tr}(\bullet)$ denotes the trace of a matrix. To obtain the ML estimate of the CFO vector ϕ, a search needs to be performed over the possible ranges of CFO values of all the users. The complexity of this search grows exponentially with the number of users and hence the search is not practical.

3. CAZAC Sequences for Multiple CFOs Estimation

To reduce the complexity of the CFO estimation for multiuser MIMO-OFDM systems, in this section, we propose a sub-optimal algorithm using CAZAC sequences as training sequences. CAZAC sequences are special sequences with constant amplitude elements and zero autocorrelation for any nonzero circular shifts. This means for a length-N CAZAC sequence, we have $s(n) = \exp(j\theta_n)$ and the auto-correlation

$$R(k) = \sum_{n=1}^{N} s(n)s^*(n \ominus k) = \begin{cases} N, & k = 0, \\ 0, & k \neq 0, \end{cases} \tag{10}$$

for all values of $k = 0, 1, \ldots, N-1$. Here we use $\ominus$ to denote circular subtraction. Let $\mathbf{S}$ be a circulant matrix with the first column equal to $[s(0), s(1), \ldots, s(N-1)]^T$. The autocorrelation property of CAZAC sequences can be written in equivalent matrix form as

$$\mathbf{S}^H\mathbf{S} = N\mathbf{I}_N, \tag{11}$$

where $\mathbf{I}_N$ is the identity matrix of size $N \times N$. This means that $\mathbf{S}$ is both a unitary (up to a normalization factor of N) and a circulant matrix.

In [23], we showed that for collocated MIMO-OFDM systems, using CAZAC sequences as training sequences reduces overhead for channel estimation while achieving Cramer Rao Bound (CRB) performance in the CFO estimation. Here, we extend the idea to the estimation of multiple CFO values in the uplink of multiuser MIMO-OFDM systems. Let the training sequence of the first user be $\mathbf{s}_1$. The training sequence of the mth user is the cyclic shifted version of the first user, that is, $\mathbf{s}_m(n) = [\mathbf{s}_1(n \ominus \tau_m)]^T$, where τ_m denotes the shift value. It is straightforward to show that the training sequences between different users have the following properties.

(i) The autocorrelation of the training sequence for the ith user satisfies

$$\mathbf{S}_i^H\mathbf{S}_i = N\mathbf{I}_N, \tag{12}$$

for $i = 1, \ldots, n_t$.

(ii) The cross correlation between training sequences of the ith and jth users satisfies

$$\mathbf{S}_i^H\mathbf{S}_j = N\mathfrak{J}^{\tau_j - \tau_i}, \tag{13}$$

where $\mathfrak{J}^{\tau_j - \tau_i}$ denotes a matrix which results from cyclically shifting the one elements of the identify matrix to the right by $\tau_j - \tau_i$ positions.

For SISO-OFDM systems, an efficient CFO estimation technique is to use periodic training sequences [6, 7]. In this paper, we extend the idea to multiuser MIMO-OFDM systems. In this case, each user transmit two periods of the same training sequences and the received signal over two periods can be written as (We assume here timing synchronization is perfect. We also assume a cyclic prefix with length L is appended to the training sequence during transmission and removed at the receiver.)

$$\mathcal{R} = \begin{bmatrix} \mathbf{E}(\phi_1)\mathbf{S}_1 & \cdots & \mathbf{E}(\phi_{n_t})\mathbf{S}_{n_t} \\ e^{jN\phi_1}\mathbf{E}(\phi_1)\mathbf{S}_1 & \cdots & e^{jN\phi_{n_t}}\mathbf{E}(\phi_{n_t})\mathbf{S}_{n_t} \end{bmatrix}\mathcal{H} + \mathcal{N}. \tag{14}$$

Without loss of generality, we show how to estimate the CFO of the first user and the same procedure is applied to all the other users to estimate the other CFO values. Since same procedure is applied to all the users, the complexity of this CFO estimation method increases linearly with the number of users.

We first consider a special case when there are no CFOs for all the other uses except user one, that is, $\phi_m = 0$ for $m = 2, \ldots, n_t$. In this case, we cross correlate the training sequence of the first user with the received signal as shown below

$$\begin{aligned}
\mathcal{Y}_1' &= \mathbf{W}_1\mathcal{R} \\
&= \begin{bmatrix} \mathbf{S}_1^H & 0 \\ 0 & \mathbf{S}_1^H \end{bmatrix}\begin{bmatrix} \mathbf{E}(\phi_1)\mathbf{S}_1 & \cdots & \mathbf{S}_{n_t} \\ e^{jN\phi_1}\mathbf{E}(\phi_1)\mathbf{S}_1 & \cdots & \mathbf{S}_{n_t} \end{bmatrix}\mathcal{H} + \mathcal{N}' \\
&= \begin{bmatrix} \mathbf{S}_1^H\mathbf{E}(\phi_1)\mathbf{S}_1\mathcal{H}_1 + \sum_{m=2}^{n_t}\mathbf{S}_1^H\mathbf{S}_m\mathcal{H}_m \\ e^{jN\phi_1}\mathbf{S}_1^H\mathbf{E}(\phi_1)\mathbf{S}_1\mathcal{H}_1 + \sum_{m=2}^{n_t}\mathbf{S}_1^H\mathbf{S}_m\mathcal{H}_m \end{bmatrix} + \mathcal{N}' \\
&= \begin{bmatrix} \mathbf{S}_1^H\mathbf{E}(\phi_1)\mathbf{S}_1\mathcal{H}_1 + \sum_{m=2}^{n_t}\mathfrak{J}^{\tau_m}\mathcal{H}_m \\ e^{jN\phi_1}\mathbf{S}_1^H\mathbf{E}(\phi_1)\mathbf{S}_1\mathcal{H}_1 + \sum_{m=2}^{n_t}\mathfrak{J}^{\tau_m}\mathcal{H}_m \end{bmatrix} + \mathcal{N}'.
\end{aligned} \tag{15}$$

Because $\mathfrak{J}^{\tau_m}$ is a matrix resulting from cyclic shifting the identity matrix to the right by τ_m elements, $\mathfrak{J}^{\tau_m}\mathcal{H}_m$ produces a matrix resulting by cyclic shifting the rows of $\mathcal{H}_m$ by τ_m elements downwards.

We make sure that the cyclic shift between the $m-1$th and mth users is not smaller than the length of the channel impulse response, that is, $\tau_m - \tau_{m-1} \geq L$. Since the channel has only L multipath components, only the first L rows in the $N \times n_r$ matrix $\mathcal{H}_m$ are nonzero. Therefore, $\mathfrak{J}^{\tau_m}\mathcal{H}_m$ has all zero elements in the first L rows when $\tau_m - \tau_{m-1} \geq L$ for $m = 2, \ldots, n_t$ and $N - \tau_{n_t} \geq L$ (notice that to ensure these conditions hold, we need to have the training sequence length $N \geq n_t L$). Hence, the first L rows of $\mathcal{Y}_1$ will be free of the interference from all the other users. Let us define $\mathcal{I}_L$ as the first L rows of the $N \times N$ identity matrix; we have

$$\mathcal{Y}_1 = \begin{bmatrix} \mathcal{I}_L & \mathbf{0} \\ \mathbf{0} & \mathcal{I}_L \end{bmatrix}\mathcal{Y}_1' = \begin{bmatrix} \mathcal{I}_L\mathbf{S}_1^H\mathbf{E}(\phi_1)\mathbf{S}_1\mathcal{H}_1 \\ e^{jN\phi_1}\mathcal{I}_L\mathbf{S}_1^H\mathbf{E}(\phi_1)\mathbf{S}_1\mathcal{H}_1 \end{bmatrix} + \mathcal{N}''. \tag{16}$$

The multiplication of $\mathcal{I}_L$ is to select the first L rows from the matrix $\mathbf{S}_1^H\mathbf{E}(\phi_1)\mathbf{S}_1\mathcal{H}_1$. Because the CFOs of all the other users are 0, the shift orthogonality between their training sequences and user 1's training sequence is maintained. In this case, $\boldsymbol{\mathcal{Y}}_1$ is free of interferences from the other users. Following the similar approach as in [23], we can show that the ML estimate of user 1's CFO given $\boldsymbol{\mathcal{Y}}_1$ can be obtained as

$$\hat{\phi}_1 = \frac{1}{N}\angle\left\{\sum_{k=1}^{L}\sum_{m=1}^{n_r}\mathcal{Y}_1^*(k,m)\mathcal{Y}_1(k+N,m)\right\}, \tag{17}$$

where $\angle(\bullet)$ denotes the angle of a complex number. The computational complexity of this estimator is low.

When the other users' CFO values are not zero, $\boldsymbol{\mathcal{Y}}_1$ is given by

$$\begin{aligned}\boldsymbol{\mathcal{Y}}_1 &= \begin{bmatrix}\boldsymbol{\mathcal{I}}_L\mathbf{S}_1^H\mathbf{E}(\phi_1)\mathbf{S}_1\boldsymbol{\mathcal{H}}_1\\ e^{jN\phi_1}\boldsymbol{\mathcal{I}}_L\mathbf{S}_1^H\mathbf{E}(\phi_1)\mathbf{S}_1\boldsymbol{\mathcal{H}}_1\end{bmatrix}\\ &\quad+\begin{bmatrix}\boldsymbol{\mathcal{I}}_L\sum_{m=2}^{n_t}\mathbf{S}_1^H\mathbf{E}(\phi_m)\mathbf{S}_m\boldsymbol{\mathcal{H}}_m\\ \boldsymbol{\mathcal{I}}_L\sum_{m=2}^{n_t}e^{jN\phi_m}\mathbf{S}_1^H\mathbf{E}(\phi_m)\mathbf{S}_m\boldsymbol{\mathcal{H}}_m\end{bmatrix}+\boldsymbol{\mathcal{N}}'' \\ &= \begin{bmatrix}\boldsymbol{\mathcal{I}}_L\mathbf{S}_1^H\mathbf{E}(\phi_1)\mathbf{S}_1\boldsymbol{\mathcal{H}}_1\\ e^{jN\phi_1}\boldsymbol{\mathcal{I}}_L\mathbf{S}_1^H\mathbf{E}(\phi_1)\mathbf{S}_1\boldsymbol{\mathcal{H}}_1\end{bmatrix}+\boldsymbol{\mathcal{V}}+\boldsymbol{\mathcal{N}}''.\end{aligned} \tag{18}$$

From (18), we can see that the orthogonality between the training sequences from different users is destroyed by the non-zero CFO values ϕ_m. As a result, there is an extra Multiple Access Interference (MAI) term $\boldsymbol{\mathcal{V}}$ in the correlation output $\boldsymbol{\mathcal{Y}}_1$. This interference is independent of the noise and therefore it will cause an irreducible error floor in MSE of the CFO estimator in (17). The covariance matrix of the MAI can be expressed as

$$\begin{aligned}E\left\{\boldsymbol{\mathcal{V}}\boldsymbol{\mathcal{V}}^H\right\} = E\Bigg\{&\begin{bmatrix}\boldsymbol{\mathcal{I}}_L\sum_{m=2}^{n_t}\mathbf{S}_1^H\mathbf{E}(\phi_m)\mathbf{S}_m\boldsymbol{\mathcal{H}}_m\\ \boldsymbol{\mathcal{I}}_L\sum_{m=2}^{n_t}e^{jN\phi_m}\mathbf{S}_1^H\mathbf{E}(\phi_m)\mathbf{S}_m\boldsymbol{\mathcal{H}}_m\end{bmatrix}\\ &\times\Bigg[\sum_{m=2}^{n_t}\boldsymbol{\mathcal{H}}_m^H\mathbf{S}_m^H\mathbf{E}^H(\phi_m)\mathbf{S}_1\boldsymbol{\mathcal{I}}_L^H,\\ &\qquad\sum_{m=2}^{n_t}e^{-jN\phi_m}\boldsymbol{\mathcal{H}}_m^H\mathbf{S}_m^H\mathbf{E}^H(\phi_m)\mathbf{S}_1\boldsymbol{\mathcal{I}}_L^H\Bigg]\Bigg\}.\end{aligned} \tag{19}$$

We assume the channels between different transmit and receive antennas are uncorrelated in space and different paths in the multipath channel are also uncorrelated. We define $\mathbf{p}_{i,m} = [p_{i,m}(0),\ldots, p_{i,m}(L-1),0,\ldots 0]^T_{(N\times 1)}$ as the power delay profile (PDP) of the channel between the mth user and the ith receive antenna and we have

$$E\left\{\boldsymbol{\mathcal{H}}_m\boldsymbol{\mathcal{H}}_n^H\right\} = \begin{cases}\mathbf{0}, & m\neq n,\\ \operatorname{diag}\left(\sum_{i=1}^{n_r}\mathbf{p}_{i,m}\right), & n=m.\end{cases} \tag{20}$$

Defining $\mathbf{P}_m = \operatorname{diag}(\sum_{i=1}^{n_r}\mathbf{p}_{i,m})$, we can rewrite the covariance matrix of the interference as

$$\mathrm{E}\left\{\boldsymbol{\mathcal{V}}\boldsymbol{\mathcal{V}}^H\right\} = \begin{bmatrix}\mathbf{C} & \mathbf{D}\\ \mathbf{D}^H & \mathbf{C}\end{bmatrix}, \tag{21}$$

where

$$\begin{aligned}\mathbf{C} &= \boldsymbol{\mathcal{I}}_L\left\{\sum_{m=2}^{n_t}\mathbf{S}_1^H\mathbf{E}(\phi_m)\mathbf{S}_m\mathbf{P}_m\mathbf{S}_m^H\mathbf{E}^H(\phi_m)\mathbf{S}_1\right\}\boldsymbol{\mathcal{I}}_L^H,\\ \mathbf{D} &= \boldsymbol{\mathcal{I}}_L\left\{\sum_{m=2}^{n_t}e^{-jN2\phi_m}\mathbf{S}_1^H\mathbf{E}(\phi_m)\mathbf{S}_m\mathbf{P}_m\mathbf{S}_m^H\mathbf{E}^H(\phi_m)\mathbf{S}_1\right\}\boldsymbol{\mathcal{I}}_L^H.\end{aligned} \tag{22}$$

We can see that the interference power is a function of the training sequence $\mathbf{S}_m$, the channel delay power profile $\mathbf{P}_m$, and the CFO matrices $\mathbf{E}(\phi_m)$.

4. Training Sequence Optimization

In the previous section, we showed that the multiple CFO values destroy the orthogonality among the training sequences of different users and introduces MAI. In this section, we study how to find the training sequence such that the signal to interference ratio (SIR) is maximized.

4.1. Cost Function Based on SIR. From the signal model in (18), we can define the SIR of the first user as

$$\mathrm{SIR}_1 = \frac{\operatorname{tr}\left[\boldsymbol{\mathcal{I}}_L\left\{\mathbf{S}_1^H\mathbf{E}(\phi_1)\mathbf{S}_1\mathbf{P}_1\mathbf{S}_1^H\mathbf{E}^H(\phi_1)\mathbf{S}_1\right\}\boldsymbol{\mathcal{I}}_L^H\right]}{\operatorname{tr}\left[\boldsymbol{\mathcal{I}}_L\left\{\sum_{m=2}^{n_t}\mathbf{S}_1^H\mathbf{E}(\phi_m)\mathbf{S}_m\mathbf{P}_m\mathbf{S}_m^H\mathbf{E}^H(\phi_m)\mathbf{S}_1\right\}\boldsymbol{\mathcal{I}}_L^H\right]}. \tag{23}$$

From the denominator of (23), we can see that the total interference power depends on the CFO values ϕ_m of all the other users. As a result, the optimal training sequence that maximizes the SIR is also dependent on ϕ_m for $m = 1,\ldots,m$. In this case, even if we can find the optimal training sequences for different values of ϕ_m, we still do not know which one to choose during the actual transmission as the values ϕ_m are not available before transmission. This makes (23) an unpractical cost function.

Let us look at user 1 again. In the absence of the CFO, the signal from user 1 is contained in the first L rows of the received signal $\widetilde{\mathcal{Y}}_1$. When the CFO is present, such orthogonality is destroyed and some information from user 1 will be "spilled" to the other rows of $\widetilde{\mathcal{Y}}_1$, thus causing interference to the other users. For user 1, therefore, to keep the interference to the other users small, such "spilled" signal power should be minimized. On the other hand, the useful signal we used to estimate the CFO of user 1 is contained in the first L rows of $\widetilde{\mathcal{Y}}_1$ and such signal power should be maximized. Therefore, considering user 1 alone, we can define the signal to "spilled" interference (to other users) ratio for user 1 as

$$\mathrm{SIR}_1 = \frac{\mathrm{tr}\Big[\mathcal{I}_L\Big\{\mathbf{S}_1^H\mathbf{E}(\phi_1)\mathbf{S}_1\mathbf{P}_1\mathbf{S}_1^H\mathbf{E}^H(\phi_1)\mathbf{S}_1\Big\}\mathcal{I}_L^H\Big]}{\mathrm{tr}\Big[\overline{\mathcal{I}_L}\Big\{\mathbf{S}_1^H\mathbf{E}(\phi_1)\mathbf{S}_1\mathbf{P}_1\mathbf{S}_1^H\mathbf{E}^H(\phi_1)\mathbf{S}_1\Big\}\overline{\mathcal{I}_L}^H\Big]}, \quad (24)$$

where $\overline{\mathcal{I}_L}$ is the complement of $\mathcal{I}_L$, that is, $\overline{\mathcal{I}_L}$ is the last $N-L$ rows of the $N \times N$ identity matrix.

The denominator in (24) can be expressed as

$$\begin{aligned}
&\mathrm{tr}\Big[\overline{\mathcal{I}_L}\Big\{\mathbf{S}_1^H\mathbf{E}(\phi_1)\mathbf{S}_1\mathbf{P}_1\mathbf{S}_1^H\mathbf{E}^H(\phi_1)\mathbf{S}_1\Big\}\overline{\mathcal{I}_L}^H\Big] \\
&\quad = N\,\mathrm{tr}\Big[\mathbf{S}_1\mathbf{P}_1\mathbf{S}_1^H\Big] \\
&\qquad - \mathrm{tr}\Big[\mathcal{I}_L\Big\{\mathbf{S}_1^H\mathbf{E}(\phi_1)\mathbf{S}_1\mathbf{P}_1\mathbf{S}_1^H\mathbf{E}^H(\phi_1)\mathbf{S}_1\Big\}\mathcal{I}_L{}^H\Big] \\
&\quad = N^2\,\mathrm{tr}[\mathbf{P}_1] - \mathrm{tr}\Big[\mathcal{I}_L\Big\{\mathbf{S}_1^H\mathbf{E}(\phi_1)\mathbf{S}_1\mathbf{P}_1\mathbf{S}_1^H\mathbf{E}^H(\phi_1)\mathbf{S}_1\Big\}\mathcal{I}_L{}^H\Big].
\end{aligned} \quad (25)$$

Substituting this into (24), we have

$$\mathrm{SIR}_1 = \frac{\mathrm{tr}\Big[\mathcal{I}_L\Big\{\mathbf{S}_1^H\mathbf{E}(\phi_1)\mathbf{S}_1\mathbf{P}_1\mathbf{S}_1^H\mathbf{E}^H(\phi_1)\mathbf{S}_1\Big\}\mathcal{I}_L^H\Big]}{N^2\,\mathrm{tr}[\mathbf{P}_1] - \mathrm{tr}\Big[\mathcal{I}_L\Big\{\mathbf{S}_1^H\mathbf{E}(\phi_1)\mathbf{S}_1\mathbf{P}_1\mathbf{S}_1^H\mathbf{E}^H(\phi_1)\mathbf{S}_1\Big\}\mathcal{I}_L^H\Big]}. \quad (26)$$

Now we can define the training sequence optimization problem as

$$\begin{aligned}
\mathbf{S}_{\mathrm{opt}} &= \arg\max_{\widetilde{\mathbf{S}}_1} \mathrm{SIR}_1' \\
&= \arg\max_{\widetilde{\mathbf{S}}_1} \frac{\mathrm{tr}\Big[\mathcal{I}_L\Big\{\widetilde{\mathbf{S}}_1^H\mathbf{E}(\phi_1)\widetilde{\mathbf{S}}_1\mathbf{P}_1\mathbf{S}_1^H\mathbf{E}^H(\phi_1)\widetilde{\mathbf{S}}_1\Big\}\mathcal{I}_L^H\Big]}{\mathfrak{r} - \mathrm{tr}\Big[\mathcal{I}_L\Big\{\widetilde{\mathbf{S}}_1^H\mathbf{E}(\phi_1)\widetilde{\mathbf{S}}_1\mathbf{P}_1\widetilde{\mathbf{S}}_1^H\mathbf{E}^H(\phi_1)\widetilde{\mathbf{S}}_1\Big\}\mathcal{I}_L^H\Big]} \\
&= \arg\min_{\widetilde{\mathbf{S}}_1} \frac{\mathfrak{r} - \mathrm{tr}\Big[\mathcal{I}_L\Big\{\widetilde{\mathbf{S}}_1^H\mathbf{E}(\phi_1)\widetilde{\mathbf{S}}_1\mathbf{P}_1\widetilde{\mathbf{S}}_1^H\mathbf{E}^H(\phi_1)\widetilde{\mathbf{S}}_1\Big\}\mathcal{I}_L^H\Big]}{\mathrm{tr}\Big[\mathcal{I}_L\Big\{\widetilde{\mathbf{S}}_1^H\mathbf{E}(\phi_1)\widetilde{\mathbf{S}}_1\mathbf{P}_1\widetilde{\mathbf{S}}_1^H\mathbf{E}^H(\phi_1)\widetilde{\mathbf{S}}_1\Big\}\mathcal{I}_L^H\Big]} \\
&= \arg\min_{\widetilde{\mathbf{S}}_1} \left\{\frac{\mathfrak{r}}{\mathrm{tr}\Big[\mathcal{I}_L\Big\{\widetilde{\mathbf{S}}_1^H\mathbf{E}(\phi_1)\widetilde{\mathbf{S}}_1\mathbf{P}_1\widetilde{\mathbf{S}}_1^H\mathbf{E}^H(\phi_1)\widetilde{\mathbf{S}}_1\Big\}\mathcal{I}_L^H\Big]} - 1\right\} \\
&= \arg\max_{\widetilde{\mathbf{S}}_1} \Big\{\mathrm{tr}\Big[\mathcal{I}_L\Big\{\widetilde{\mathbf{S}}_1^H\mathbf{E}(\phi_1)\widetilde{\mathbf{S}}_1\mathbf{P}_1\widetilde{\mathbf{S}}_1^H\mathbf{E}^H(\phi_1)\widetilde{\mathbf{S}}_1\Big\}\mathcal{I}_L^H\Big]\Big\},
\end{aligned} \quad (27)$$

where $\mathfrak{r}$ denotes $N^2\,\mathrm{tr}[\mathbf{P}_1]$.

From (27), we can see that the optimal training sequence depends on the power delay profile $\mathbf{P}_1$ and the actual CFO value ϕ_1. The channel delay profile is an environment-dependent statistical property that does not change very frequently. Therefore, in practice, we can store a few training sequences for different typical power delay profiles at the transmitter and select the one that matches the actual channel delay profile. On the other hand, it is impossible to know the actual CFO ϕ in advance to select the optimal training sequence. In the following, we will propose a new cost function based on SIR approximation which can remove the dependency on the actual CFO ϕ_1 in the optimization.

Table 1: Number of possible Frank-Zadoff and Chu sequences for different sequence lengths.

N	Frank-Zadoff Sequence	Chu Sequence
16	2	8
36	2	12
64	4	32

4.2. CFO Independent Cost Function. Let us assume that the CFO value ϕ is small. In this case, we can approximate the exponential function in the original cost function by its first-order Taylor series expansion, that is, $\exp(j\phi) \approx 1 + j\phi$. Therefore, we have

$$\mathbf{E}(\phi_1) \approx \mathbf{I}_N + j\phi_1\mathbf{N}, \quad (28)$$

where $\mathbf{N}$ is a diagonal matrix given by $\mathbf{N} = \mathrm{diag}[0, 1, 2, \ldots, N-1]$. Using this approximation, we get

$$\begin{aligned}
\mathbf{S}^H\mathbf{E}(\phi)\mathbf{SPS}^H\mathbf{E}^H(\phi)\mathbf{S} &\approx \mathbf{S}^H(\mathbf{I} + j\phi\mathbf{N})\mathbf{SPS}^H(\mathbf{I} - j\phi\mathbf{N})\mathbf{S} \\
&= \mathbf{P} + j\phi\mathbf{S}^H\mathbf{NSP} - j\phi\mathbf{PS}^H\mathbf{NS} \\
&\quad + \phi^2\mathbf{S}^H\mathbf{NSPS}^H\mathbf{NS}.
\end{aligned} \quad (29)$$

Here we omitted the subscript 1 for the clearness of the presentation. Therefore, the optimization problem can be approximated as

$$\begin{aligned}
\mathbf{S}_{\mathrm{opt}} = \arg\max_{\widetilde{\mathbf{S}}} \Big\{\mathrm{tr}\Big[\mathcal{I}_L\Big(&\mathbf{P} + j\phi\widetilde{\mathbf{S}}^H\mathbf{N}\widetilde{\mathbf{S}}\mathbf{P} - j\phi\mathbf{P}\widetilde{\mathbf{S}}^H\mathbf{N}\widetilde{\mathbf{S}} \\
&+\phi^2\widetilde{\mathbf{S}}^H\mathbf{N}\widetilde{\mathbf{S}}\mathbf{P}\widetilde{\mathbf{S}}^H\mathbf{N}\widetilde{\mathbf{S}}\Big)\mathcal{I}_L^H\Big]\Big\}.
\end{aligned} \quad (30)$$

Notice that the first term $\mathbf{P}$ in the summation is independent of $\mathbf{S}$ and hence can be dropped. It can be shown that the diagonal elements of the second term $j\phi\mathbf{S}^H\mathbf{NSP}$ are constant and independent of $\mathbf{S}$. Therefore, $\mathrm{tr}[\mathcal{I}_L(j\phi\mathbf{S}^H\mathbf{NSP})\mathcal{I}_L^H]$ is also independent of $\mathbf{S}$ and hence can be dropped from the cost function. The same applies to the third term $-j\phi\mathbf{PS}^H\mathbf{NS}$, which is the conjugate of the second term. Therefore, the final form of the optimization using Taylor's series approximation can be written as

$$\mathbf{S}_{\mathrm{opt}} = \arg\max_{\widetilde{\mathbf{S}}} \Big\{\mathrm{tr}\Big[\mathcal{I}_L\Big(\widetilde{\mathbf{S}}^H\mathbf{N}\widetilde{\mathbf{S}}\mathbf{P}\widetilde{\mathbf{S}}^H\mathbf{N}\widetilde{\mathbf{S}}\Big)\mathcal{I}_L^H\Big]\Big\}. \quad (31)$$

The advantage of (31) is that the optimization problem is independent of the actual CFO value ϕ as long as the value of ϕ is small enough to ensure the accuracy of the Taylor's series approximation in (28).

Now we look at how we can obtain the optimal CAZAC training sequences for the cost function (31). In particular, we look at three classes of CAZAC sequences, namely, the Frank-Zadoff sequences [18], the Chu sequences [19], and the S&H sequences [20]. The Frank-Zadoff sequences exist for sequence length $N = K^2$ where K is any positive integer. For $N = 16$, all elements of the Frank-Zadoff sequences are BPSK symbols while for $N = 64$, all elements are BPSK and QPSK symbols. Therefore, the advantage of the Frank-Zadoff sequences is that they are simple for practical implementation. The disadvantage is that there are limited numbers of sequences available for each sequence length as shown in Table 1. The advantage of Chu sequences is that the length of the sequence can be an arbitrary integer N. Compared to Frank-Zadoff sequences, there are more sequences available for the same sequence length as shown in Table 1. For both Frank-Zadoff and Chu sequences, there are a finite number of possible sequences for each N. The optimal sequence can be found by using a computer search using the cost function (31). The S&H sequences only exist for sequence length $N = K^2$. The sequences are constructed using a size K phase vector $\exp(j\boldsymbol{\theta}) = [e^{j\theta_1}, \ldots, e^{j\theta_K}]^T$. Therefore, the optimization of training sequence $\mathbf{S}$ is equivalent to the optimization on the phase vector $\boldsymbol{\theta}$ given by

$$\boldsymbol{\theta} = \arg\max_{\tilde{\boldsymbol{\theta}}}\left\{\mathcal{J}\left(\tilde{\boldsymbol{\theta}}\right)\right\} \text{ with}$$
$$\mathcal{J}\left(\tilde{\boldsymbol{\theta}}\right) = \operatorname{tr}\left[\boldsymbol{\mathcal{I}}_L\left(\mathbf{S}^H\left(\tilde{\boldsymbol{\theta}}\right)\mathbf{N}\mathbf{S}\left(\tilde{\boldsymbol{\theta}}\right)\mathbf{P}\mathbf{S}^H\left(\tilde{\boldsymbol{\theta}}\right)\mathbf{N}\mathbf{S}\left(\tilde{\boldsymbol{\theta}}\right)\boldsymbol{\mathcal{I}}_L^H\right)\right]. \quad (32)$$

Notice that this is an unconstrained optimization problem and each element of the phase vector can take any values in the interval $[0, 2\pi)$. From the construction of the S&H sequence [20], it can be easily shown that $\mathbf{S}(\boldsymbol{\theta}+\psi) = e^{j\psi}\mathbf{S}(\boldsymbol{\theta})$, where $\boldsymbol{\theta} + \psi = [\theta_1 + \psi, \ldots, \theta_K + \psi]^T$. Hence, from (32), we can get $\mathcal{J}(\boldsymbol{\theta}) = \mathcal{J}(\boldsymbol{\theta} + \psi)$. By letting $\psi = -\theta_1$, the original optimization problem over the K-dimension phase vector $\boldsymbol{\theta} = [\theta_1, \theta_2, \ldots, \theta_K]^T$ can be simplified to the optimization over a $(K-1)$-dimension phase vector $\boldsymbol{\theta}' = [0, \theta'_1, \ldots, \theta'_{K-1}]^T$ where $\theta'_k = \theta_{k+1} - \theta_1$.

There are an infinite number of possible S&H sequences for each sequence length; it is impossible to use exhaustive computer search to obtain the optimal sequence. We resort to numerical methods and use the adaptive simulated annealing (ASA) method [21] to find a near-optimal sequence. To test the near-optimality of the sequence obtained using the ASA, for smaller sequence lengths of $N = 16$ and $N = 36$, we use exhaustive computer search to obtain the globally optimal S&H sequence. The obtained sequence through computer search is consistent with the sequence obtained using ASA and this proves the effectiveness of the ASA in approaching the globally optimal sequence.

5. Simulation Results

In this section, we use computer simulations to study the performance of the CFO estimation using CAZAC sequences and demonstrate the performance gain achieved by using the optimal training sequences. In the simulations, we assume a multiuser MIMO-OFDM systems with two users. (In multiuser MIMO-OFDM systems, the number of receive antennas has to be no less than the number of transmit antennas from all users. Due to the practical limitations, it is not possible to implement too many base-station antennas. Therefore, to accommodate more users, the multiuser MIMO-OFDM systems can be used in conjunction with other multiple access schemes such as TDMA and FDMA.) Each user has one transmit antenna and the base-station has two receive antennas. We simulate an OFDM system with 128 subcarriers. The CFO is normalized with respect to the subcarrier spacing. Unless otherwise stated, the actual CFO values for the two users are modeled as random

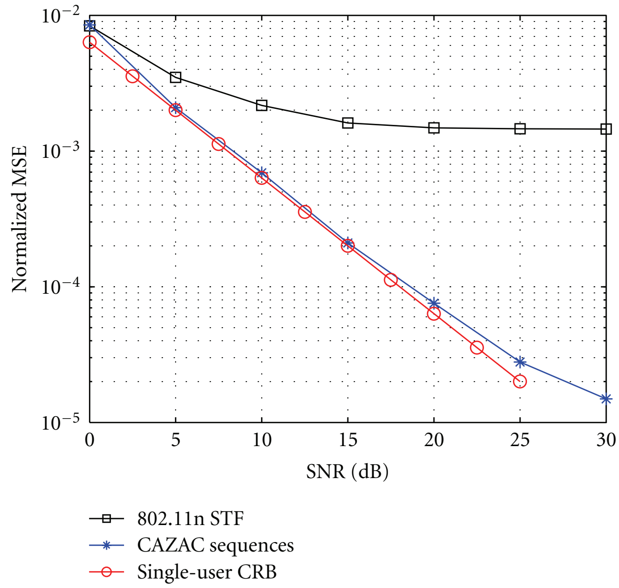

FIGURE 2: MSE of CFO estimation using $N = 32$ Chu sequences and IEEE 802.11n STF for uniform power delay profile.

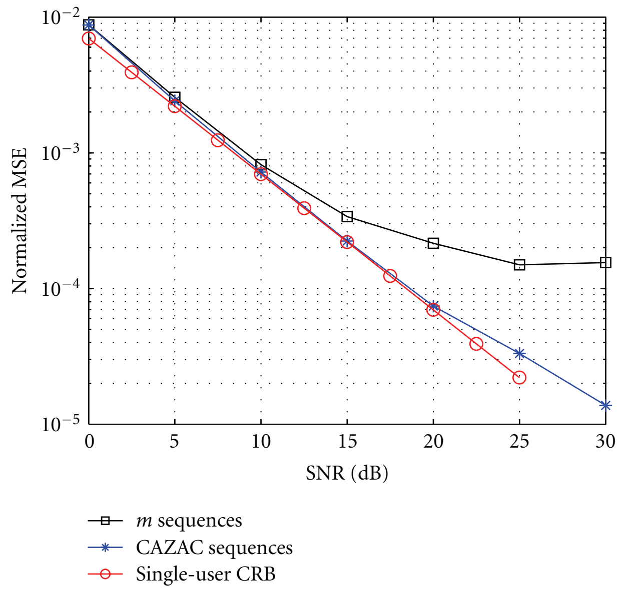

FIGURE 3: Comparison of CFO estimation using $N = 31$ Chu sequences and m sequence for uniform power delay profile.

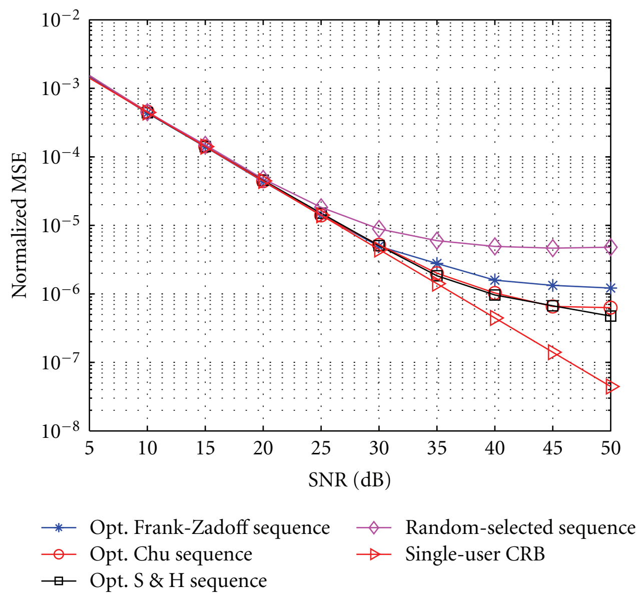

Figure 4: Comparison of CFO estimation using different $N = 36$ CAZAC sequences for $L = 18$ channel for uniform power delay profile.

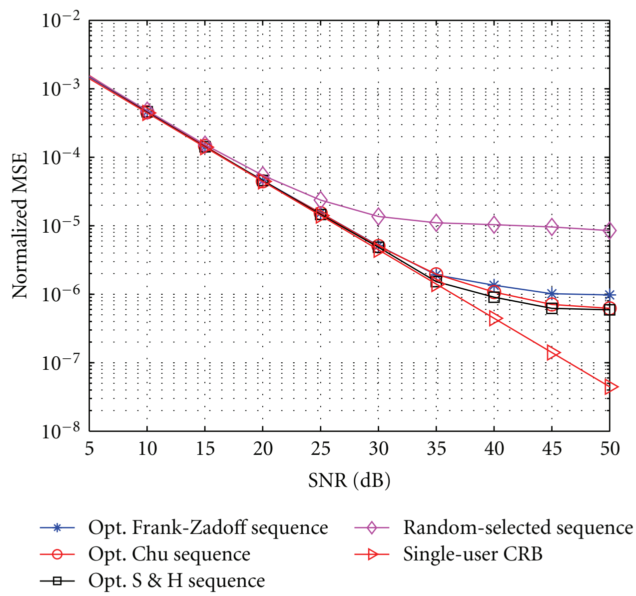

Figure 5: Comparison of CFO estimation using different $N = 36$ CAZAC sequences for $L = 18$ channel for exponential power delay profile.

variables uniformly distributed between $[-0.5, 0.5]$. The mean square error (MSE) of the CFO estimation is defined as

$$\mathrm{MSE} = \frac{1}{N_s}\sum_{i=1}^{N_s}\left(\frac{\hat{\phi}-\phi}{2\pi/M}\right)^2, \tag{33}$$

where $\hat{\phi}$ and ϕ represent the estimated and true CFO's, respectively, M is the number of subcarriers, and N_s denotes the total number of Monte Carlo trials.

First we compare the performance of CFO estimation using CAZAC sequences with the following two sequences which also have good autocorrelation properties:

(1) IEEE 802.11n short training field [3],

(2) m sequences [22].

In the simulations, we use the 802.11n STF for 40 MHz operations which has a length of 32. For the m sequence, we use a sequence length of 31. To provide a fair comparison, we compare the performance using the 802.11n STF with a length-32 Chu (CAZAC) sequence generated by [19]

$$s(n) = \exp\left[j\pi\frac{(n-1)^2}{N}\right], \tag{34}$$

and we compare the performance with the m sequence using a length-31 Chu sequence generated by [19]

$$s(n) = \exp\left[j\pi\frac{(n-1)n}{N}\right]. \tag{35}$$

The performance of CFO estimation using the 802.11n STF and $N = 32$ Chu sequence is shown in Figure 2. Here we

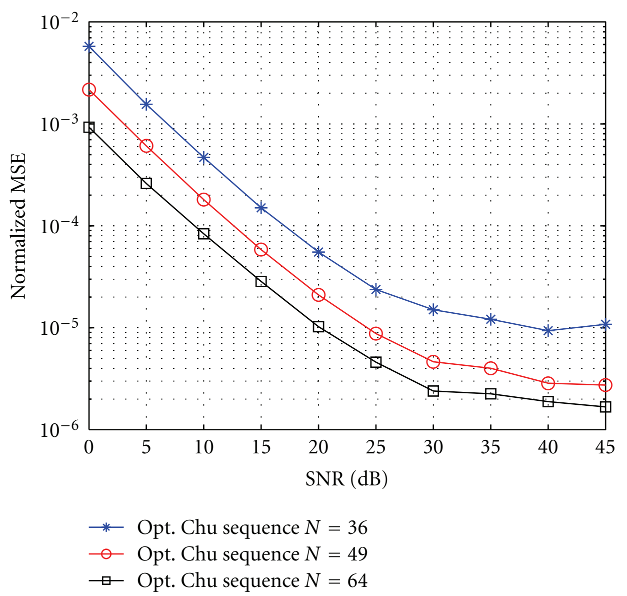

Figure 6: Comparison of CFO estimation using different length of optimal Chu sequences for $L = 18$ channel for uniform power delay profile.

use 16-tab multipath channels and the circular shift between the training sequences of the two users $\tau_2 = 16$. To gauge the performance of the CFO estimation, we also included the single-user CRB in the comparison. The single-user CRB is obtained by assuming no MAI and can be shown to be [24]

$$\mathrm{CRB} = \frac{M^2}{4\pi^2 n_r N^3 \gamma}, \tag{36}$$

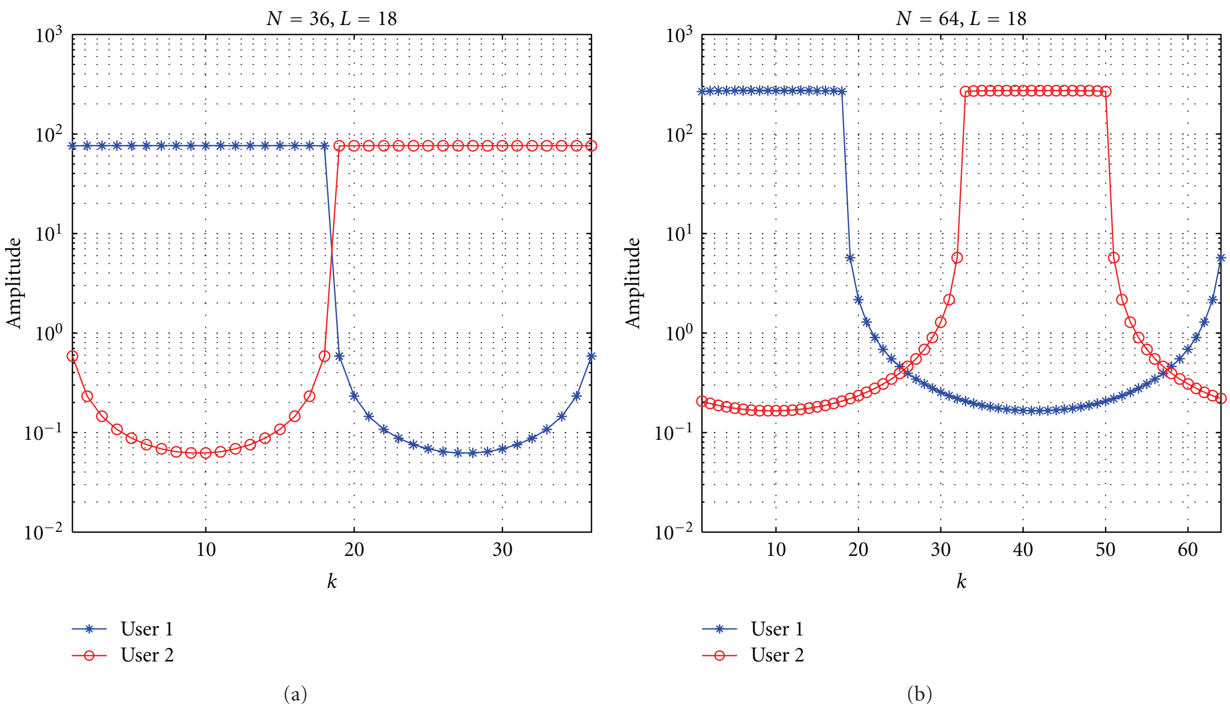

FIGURE 7: Comparison of useful signal and interference power for different sequence lengths (uniform power delay profile).

where γ is the SNR per receive antenna and M is the number of subcarriers. From the results, we can see that the CFO estimation using the 802.11n STF has a very high error floor above MSE of 10^{-3}. The performance using CAZAC sequences is much better. In low to medium SNR regions, the performance is very close to the single-user CRB. An error floor starts to appear at SNR of about 25 dB. The error floor is around 100 times smaller compared to the error floor using the 802.11n STF.

The performance of the CFO estimation using the $N = 31$ m sequence and Chu sequence is shown in Figure 3. Here to satisfy the condition of $N \geq n_t L$, we use 15-tab multipath fading channels and the circular shift between user 1 and 2's training sequence is also set to 15. Again using CAZAC sequences leads to a much better performance. We can see that in low to medium SNR regions, their performance is very close to the single-user CRB. The error floor using CAZAC sequences is more than 10 times smaller than that using the m sequence.

The performance of CFO estimation using different CAZAC sequences is compared in Figure 4. Here we fix the sequence length to 36 and the multipath channel has $L = 18$ tabs with uniform power delay profile. Comparing the performances of optimal Chu sequence and the optimal Frank-Zadoff sequence, we can see that the error floor of the Chu sequence is smaller. This is because there are more possible Chu sequences compared to Frank-Zadoff sequences and hence more degrees of freedom in the optimization. However, comparing the performance of optimal Chu sequence with that of the optimal S&H sequence, we can see that the additional degrees of freedom in the S&H sequence do not lead to significant performance gain. Compared to the performance using a randomly selected CAZAC sequence, we can see that the error floor using an optimized sequence is significantly smaller. Simulations were also performed in multipath channels with exponential power delay profile and root mean square delay spread equal to 2 sampling intervals. The other simulation parameters are the same as in the uniform power delay profile simulations. Simulation results in Figure 5 show again that the error floor in CFO estimation can be significantly reduced when using the optimized training sequence.

From both Figures 4 and 5, we can see that the gain of using S&H sequences compared to Chu sequences is really small. Therefore, in practical implementation, it is better to use the Chu sequence because it is simple to generate and it is available for all sequence lengths. Another advantage of the Chu sequence is that the optimal Chu sequence obtained using cost function (31) is the same for the uniform power delay profile and some exponential power delay profiles we tested. Hence, a common optimal Chu sequence can be used for both channel PDP's. This is not the case for the S&H sequences.

Figure 6 shows the performance of CFO estimation for different lengths of optimal Chu sequences. Here we fix the channel length to $L = 18$. From the previous sections, to accommodate two users, the minimum sequence length is $n_t L$. Therefore, we need Chu sequences of length at least 36. We compare the performance of the optimal length-36 sequence with that of optimal length-49 and length-64 sequences. For the length-49 sequence, the cyclic shift between training sequence of two users is 24, while for length-64 sequence, the cyclic shift is 32. From the comparison, we can see that there are two advantages using a longer sequence. Firstly, in the low to medium SNR regions, there is SNR gain in the CFO estimation due to the longer sequences length. Secondly, in the high SNR regions, the error floor using longer sequences is much smaller. This can be explained using Figure 7. In Figure 7, we plotted the signal power for user 1 and user 2 after the correlation operation in (15) for sequence length of 36 and 64. In the absence of the CFO, user 1's signal should be contained in the first 18 samples ($L = 18$). However, due to CFO, some signal components are leaked into the other samples and become interference to user 2. For the case of $L = 18$ and $N = 36$, all the leaked signals from user 1 become interference to user 2 and vice versa. If we use a longer training sequence, there is some "guard time" between the useful signals of the two users as shown in Figure 7 for the $N = 64$ case. As we only take the useful L samples for CFO estimation (16), only part of the leaked signal becomes interference. Hence, the overall SIR is improved. The cost of using longer sequences is the additional training overhead that is required. Therefore, based on the requirement on the precision of CFO estimation, the system design should choose the best sequence length that achieves the best compromise between performance and overhead.

6. Conclusions

In this paper, we studied the CFO estimation algorithm in the uplink of the multiuser MIMO-OFDM systems. We proposed a low-complexity sub-optimal CFO estimation methods using CAZAC sequences. The complexity of the proposed algorithm grows only linearly with the number of users. We showed that in this algorithm, multiple CFO values from multiple users cause MAI in the CFO estimation. To reduce such detrimental effect, we formulated an optimization problem based on the maximization of the SIR. However, the optimization problem is dependent on the actual CFO values which are not known in advance. To remove such dependency, we proposed a new cost function which closely approximate the SIR for small CFO values. Using the new cost function, we can obtain optimal training sequences for a different class of CAZAC sequences. Computer simulations show that the performance of the CFO estimation using CAZAC sequence is very close to the single-user CRB for low to medium SNR values. For high SNR, there is an error floor due to the MAI. By using the obtained optimal CAZAC sequence, such error floor can be significantly reduced compared to using a randomly chosen CAZAC sequence.

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