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Research Article

Fast Signal Recovery in the Presence of Mutual Coupling Based on New 2-D Direct Data Domain Approach

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The performance of adaptive algorithms, including direct data domain least square, can be significantly degraded in the presence of mutual coupling among array elements. In this paper, a new adaptive algorithm was proposed for the fast recovery of the signal with one snapshot of receiving signals in the presence of mutual coupling, based on the two-dimensional direct data domain least squares (2-D D^3LS) for uniform rectangular array (URA). In this method, inverse mutual coupling matrix was not computed. Thus, the computation was reduced and the signal recovery was very fast. Taking mutual coupling into account, a method was derived for estimation of the coupling coefficient which can accurately estimate the coupling coefficient without any auxiliary sensors. Numerical simulations show that recovery of the desired signal is accurate in the presence of mutual coupling.

1. Introduction

Adaptive antenna arrays are strongly affected by the existence of mutual coupling (MC) effect between antenna elements; thus, if the effects of MC are ignored, the system performance will not be accurate [1, 2]. Research into compensation for the MC has been mainly based on the idea of using open circuit voltages, firstly proposed by Gupta and Ksienski [2]. While this method has calculated the mutual impedance, the presence of other antenna elements has been ignored and a very simplified current distribution has been assumed for each antenna elements. Many efforts have been made to compensate for the MC effect for uniform linear array (ULA) and uniform circular array (UCA) [2–9]. In [3], an adaptive algorithm was used to compensate for the MC effect in a ULA. In [7], the authors introduced a minimum norm technique MC compensation method, which is based on the technique in [2] for general arrays with arbitrary elements and more accurate. In [9], a new method was proposed to compensate for the MC effect which relied on the calculation of a new definition of mutual impedance. however, the authors did not deal with 2-D DOA estimation problem.

On the other hand, many algorithms of the 1-D DOA estimation have been extended to solve the 2-D cases [10, 11]; however, a few have considered the effect of mutual coupling or any other array errors [12]. Besides, most of these proposed adaptive algorithms are based on the covariance matrix of the interference. However, these statistical algorithms suffer from two major drawbacks. First, they require independent identically-distributed secondary data in order to estimate the covariance matrix of the interference. Unfortunately, the statistics of the interference may fluctuate rapidly over a short distance, limiting the availability of homogeneous secondary data. The resulting errors in the covariance matrix reduce the ability to suppress the interference. The second drawback is that the estimation of the covariance matrix requires the storage and processing of the secondary data. This is computationally intensive, requiring many calculations in real-time. Recently, direct data domain algorithms have been proposed to overcome these drawbacks of statistical techniques [13–16]. The approach is to adaptively minimize the interference power while maintaining the array gain in the direction of the signal. The sample support problem is eliminated by

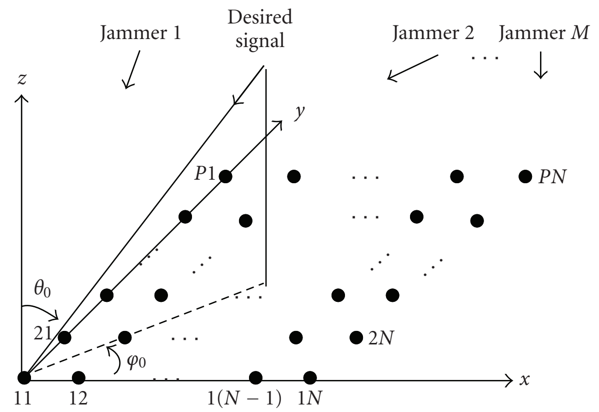

Figure 1: URA with $N \times P$ elements.

avoiding the estimation of a covariance matrix which leads to enormous savings in the required real-time computations. The performance of this algorithm is affected by the MC effect, too [17] and must be compensated.

Unfortunately, the MC matrix tends to change with time due to environmental factors, so full elimination of its effect and prediction of its variability are impossible. Therefore, calibration procedures based upon signal processing algorithms are needed to estimate and compensate for the effect of the MC. The most likely way is to carry out some measurements for calibration. However, this procedure has the drawbacks of being time-consuming and very expensive [18]. Some other researches suggested self-calibration adaptive algorithms for damping the MC effect [19–21].

In this paper, a new adaptive algorithm was proposed for the fast recovery of the signal with one snapshot of receiving signals in the presence of mutual coupling, based on 2-D D^{3}LS algorithm for URA. Then, utilizing the 2-D D^{3}LS algorithm properties, a novel technique for the coupling coefficients estimation, without using any auxiliary sensors is presented.

This paper is organized as follows. Section 2, conventional 2-D D^{3}LS algorithm is reviewed. In Section 3, a fast adaptive algorithm of direct data domain including mutual coupling effect is presented. In Section 4, a new technique is presented for compensation of the MC effect. In Section 5, numerical simulations illustrate these proposed techniques which can accurately recover the desired signal in the presence of MC.

2. 2-D Direct Data Domain Algorithm

Consider a URA consisting of $N \times P$ equally spaced elements with the spacing of d_x in rows (in the x direction) and d_y in columns (in the y direction). The array receives a signal from a known direction (θ_0, φ_0) and M interferers from unknown directions (θ_m, φ_m), $m = 1, 2, \ldots, M$ as shown in Figure 1.

The output of the array voltage can be expressed as

$$\mathbf{x} = \mathbf{A}\mathbf{s} + \mathbf{n}, \tag{1}$$

where $\mathbf{x}$, $\mathbf{A}$, $\mathbf{s}$, and $\mathbf{n}$ denote the received signal vector, steering matrix, signal plus jammers vector and additive white Gaussian noise vector, respectively, defined as:

$$\begin{aligned}
\mathbf{x} &= [x_{11}(t), x_{12}(t), \ldots, x_{1N}(t), x_{21}(t), \ldots, \\
&\qquad x_{2N}(t), \ldots, x_{P1}(t), \ldots, x_{PN}(t)]^T, \\
\mathbf{s} &= [s(t), J_1(t), J_2(t), \ldots, J_M(t)]^T, \\
\mathbf{n} &= [n_{11}(t), n_{12}(t), \ldots, n_{1N}(t), n_{21}(t), \ldots, \\
&\qquad n_{2N}(t), \ldots, n_{P1}(t), \ldots, n_{PN}(t)]^T, \\
\mathbf{A} &= [\mathbf{a}(\theta_0, \varphi_0), \mathbf{a}(\theta_1, \varphi_1), \ldots, \mathbf{a}(\theta_M, \varphi_M)],
\end{aligned} \tag{2}$$

where

$$\begin{aligned}
\mathbf{a}(\theta_m, \varphi_m) &= \mathbf{a}_y(\theta_m, \varphi_m) \otimes \mathbf{a}_x(\theta_m, \varphi_m), \quad m = 0, 1, 2, \ldots, M, \\
\mathbf{a}_x(\theta_m, \varphi_m) &= \left[1, \beta(\theta_m, \varphi_m), \ldots, \beta^{N-1}(\theta_m, \varphi_m)\right]^T, \\
\mathbf{a}_y(\theta_m, \varphi_m) &= \left[1, \alpha(\theta_m, \varphi_m), \ldots, \alpha^{P-1}(\theta_m, \varphi_m)\right]^T.
\end{aligned} \tag{3}$$

We define $\beta(\theta_m, \varphi_m) = \exp(j2\pi(d_x/\lambda)\sin\theta_m \cos\varphi_m)$ and $\alpha(\theta_m, \varphi_m) = \exp(j2\pi(d_y/\lambda)\sin\theta_m \sin\varphi_m)$ which represent the phase progression of the signal between one element and the next in the row and column, respectively. The $\mathbf{a}(\theta_m, \varphi_m)$ is mth signal's direction manifold vector, superscript $(\cdot)^T$ is the transpose operation and the symbol $\otimes$ denotes the Kronecker tensor. Therefore, by suppression of time dependence in the phasor notation, complex vector of phasor voltage is:

$$\mathbf{x} = s_0 \mathbf{a}(\theta_0, \varphi_0) + \left(\sum_{m=1}^{M} J_m \mathbf{a}(\theta_m, \varphi_m) \right) + \mathbf{n}, \tag{4}$$

where s_0 and J_m are the complex amplitude of the desired signal and mth interferers, respectively. Next, the first row from each column is multiplied by β and subtracted from the second row; then the result of each column is multiplied by α and subtracted from the next column. This cancels out all the signals and only noise and interferers are left. The first row of the matrix in (5) is the constraint to the desired signal which produces a gain factor of Q. For a conventional adaptive array system, the K weights w_k are used and the relationship between K with P and N can be chosen as $K = K_1 \cdot K_2$, $K_1 = (N+1)/2$, $K_2 = (P+1)/2$ [16]. Matrix equation can be constructed as:

$$\begin{bmatrix}
\mathbf{b}_1 & \mathbf{b}_2 & \cdots & \mathbf{b}_{K_2-1} & \mathbf{b}_{K_2} \\
\mathbf{D}_1 & \mathbf{D}_2 & \cdots & \mathbf{D}_{(K_2-1)} & \mathbf{D}_{K_2} \\
\mathbf{D}_2 & \mathbf{D}_3 & \cdots & \mathbf{D}_{K_2} & \mathbf{D}_{(K_2+1)} \\
\vdots & & & & \\
\mathbf{D}_{(K_2-1)} & \mathbf{D}_{K_2} & \cdots & \mathbf{D}_{(P-2)} & \mathbf{D}_{(P-1)}
\end{bmatrix} \times \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_K \end{bmatrix} = \begin{bmatrix} Q \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \tag{5}$$

where

$$\mathbf{b}_1 = \left[1 \;\; \beta \;\; \cdots \;\; \beta^{K_1-1}\right], \qquad \mathbf{b}_i = \alpha^{i-1}\mathbf{b}_1, \tag{6}$$

$$\mathbf{D}_i = \begin{bmatrix} (x_{i1} - \beta^{-1}x_{i2}) - \alpha^{-1}(x_{(i+1)1} - \beta^{-1}x_{(i+1)2}) & \cdots & (x_{iK_1} - \beta^{-1}x_{i(K_1+1)}) - \alpha^{-1}(x_{(i+1)K_1} - \beta^{-1}x_{(i+1)(K_1+1)}) \\ \vdots & & \vdots \\ (x_{i(K_1-1)} - \beta^{-1}x_{iK_1}) - \alpha^{-1}(x_{(i+1)(K_1-1)} - \beta^{-1}x_{(i+1)K_1}) & \cdots & (x_{i(N-1)} - \beta^{-1}x_{iN}) - \alpha^{-1}(x_{(i+1)(N-1)} - \beta^{-1}x_{(i+1)N}) \end{bmatrix}. \quad (7)$$

For simplicity $\beta(\theta_0, \varphi_0) = \beta$ and $\alpha(\theta_0, \varphi_0) = \alpha$. Because the matrix in (5) is not square, the conjugate gradient method (CGM) is used to solve the matrix equation and to obtain the weighting solution. It has a good convergence characteristic and converges to the minimum norm solution, even for the singular problem [13]. Now, the amplitude of the recovered signal is as [16]:

$$s_0 = \frac{1}{Q}\sum_{i=1}^{K_1K_2} w_i x_{i+[(i-1)/K_1](K_1-1)}, \quad (8)$$

where $\mathbf{w} = [w_1, w_2, \ldots, w_K]^T$ is the weights vector in the absence of coupling and subscript, $[\cdot]$, denotes rounding down to the integer:

$$Q = \sum_{i=1}^{K_1K_2} \alpha^{[(i-1)/K_1]}\beta^{i-1-[(i-1)/K_1]K_1} w_i. \quad (9)$$

3. 2-D Fast Signal Recovery Algorithm in the Presence of Mutual Coupling

If one assumes that $\mathbf{C}$ denotes the mutual coupling matrix (MCM) of the array, the output will be as:

$$\mathbf{x} = \mathbf{CAs} + \mathbf{n}, \quad (10)$$

$$\mathbf{x} = s_0\mathbf{Ca}(\theta_0, \varphi_0) + \left(\sum_{m=1}^{M} J_m \mathbf{Ca}(\theta_m, \varphi_m)\right) + \mathbf{n}. \quad (11)$$

Svantesson [6] showed that the coupling between the neighboring elements with the same interspace is almost the same and the magnitude of the mutual coupling coefficient between two far apart elements is so small that can be approximated to zero. Thus, a banded symmetric Toeplitz matrix can be used as a model for the mutual coupling of ULA and URA. In this paper, each sensor is assumed to be affected by the coupling of the 8 sensors around it, which is shown in Figure 2.

We define MCM as [12]:

$$\mathbf{C} = \begin{bmatrix} \mathbf{C}_1 & \mathbf{C}_2 & 0 & \cdots & 0 & 0 & 0 \\ \mathbf{C}_2 & \mathbf{C}_1 & \mathbf{C}_2 & \cdots & 0 & 0 & 0 \\ \vdots & \ddots & & \ddots & \vdots & & \\ 0 & 0 & 0 & \cdots & \mathbf{C}_2 & \mathbf{C}_1 & \mathbf{C}_2 \\ 0 & 0 & 0 & \cdots & 0 & \mathbf{C}_2 & \mathbf{C}_1 \end{bmatrix}_{PN\times PN}, \quad (12)$$

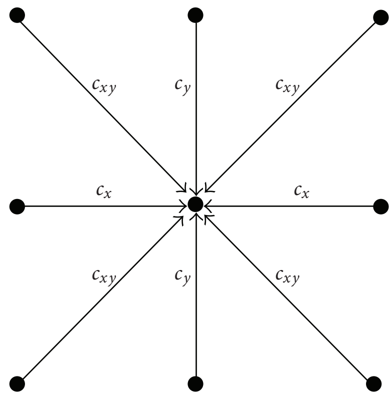

Figure 2: Map of mutual coupling.

where $\mathbf{C}_1$ and $\mathbf{C}_2$ are $N \times N$ submatrices of $\mathbf{C}$ and can be given by

$$\mathbf{C}_1 = \text{Toeplitz}([1, c_x, 0, \ldots, 0]),$$
$$\mathbf{C}_2 = \text{Toeplitz}\left(\left[c_y, c_{xy}, 0, \ldots, 0\right]\right). \quad (13)$$

Then, the following equation is derived to recover the desired signal in the presence of mutual coupling (Proof in the appendix), notwithstanding to compute the inverse matrix of MC. Hence, this equation could be reduced the computation of the algorithm

$$s_0 = \frac{1}{Q_c}\sum_{i=1}^{K_1K_2} wc_i \cdot x_{i+[(i-1)/K_1](K_1-1)}, \quad (14)$$

where $\mathbf{w_c} = [wc_1, wc_2, \ldots, wc_K]^T$ is the weights vector when coupling is known and

$$Q_c = \left(1 + \beta c_x + \alpha c_y + \alpha\beta c_{xy}\right)\sum_{i=1}^{K_1K_2} \alpha^{[(i-1)/K_1]}\beta^{i-1-[(i-1)/K_1]K_1} wc_i$$
$$+ \left(c_x + \alpha c_{xy}\right)\sum_{i=1}^{(K_1-1)K_2} \alpha^{[(i-1)/(K_1-1)]}\beta^{i-1-[(i-1)/(K_1-1)](K_1-1)}$$
$$\times wc_{i+1+[(i-1)/(K_1-1)]}$$

$$+\left(c_y+\beta c_{xy}\right)\sum_{i=1}^{K_1(K_2-1)}\alpha^{[(i-1)/K_1]}\beta^{i-1-[(i-1)/K_1]K_1}wc_{i+K_1}$$
$$+c_{xy}\sum_{i=1}^{(K_1-1)(K_2-1)}\alpha^{[(i-1)/(K_1-1)]}\beta^{i-1-[(i-1)/(K_1-1)](K_1-1)}$$
$$\times wc_{i+K_1+1+[(i-1)/(K_1-1)]}. \quad (15)$$

The conventional recovering of the signal is as the following:

$$s_0=\frac{1}{Q}\left(\mathbf{w}^T[\mathbf{C}^{-1}\mathbf{x}]_K\right), \quad (16)$$

where $[\cdot]_K$ denotes, K rows from the vector. $\mathbf{C}^{-1}$ is computationally intensive and requires many calculations in the real-time because evaluation of the inverse requires an $\Theta([PN]^3)$process (here $\Theta(\cdot)$ denotes "on the order of"). Therefore, (14) can be replaced with (16) and the number of processes would be an $\Theta(K_1K_2)$.

4. Mutual Coupling Compensation

In this section, a new method is presented to estimate the coupling coefficients from the properties of the 2-D D^3LS algorithm. If the mutual coupling effect is ignored, the term $(x_{ij}-\beta^{-1}x_{i(j+1)})-\alpha^{-1}(x_{(i+1)j}-\beta^{-1}x_{(i+1)(j+1)})$, for $i=1,2,\ldots,P-1$ and $j=1,2,\ldots,N-1$ will have no signal components. However, in the presence of MC, for the edge elements in the URA, the above term can be written as the following:

$$\begin{aligned}
(x_{11}-\beta^{-1}x_{12})-\alpha^{-1}(x_{21}-\beta^{-1}x_{22}) &= s_0\alpha^{-1}\beta^{-1}c_{xy}+\text{Interferers},\\
(x_{11}-\beta^{-1}x_{12}) &= -\left(\beta^{-1}c_x+\alpha\beta^{-1}c_{xy}\right)s_0+\text{Interferers},\\
(x_{11}-\alpha^{-1}x_{21}) &= -\left(\alpha^{-1}c_y+\alpha^{-1}\beta c_{xy}\right)s_0+\text{Interferers},
\end{aligned} \quad (17)$$

As is seen in (17), when there are no interferers, the equations can be solved. In this paper, it is assumed that $d_x=d_y=d$; so $c_x=c_y$. The above equations can be solved in order to estimate $\hat{c}_x$, $\hat{c}_y$, and $\hat{c}_{xy}$. Once the system estimates the coupling coefficient, it needs only one snapshot of the data in order to obtain an acceptable solution. So, when the coupling is unknown, first we can estimate mutual coupling from (17) and then, the fast recovering of the signal is as the following:

$$\hat{s}_0=\frac{1}{\hat{Q}_c}\sum_{i=1}^{K_1K_2}wc_i\cdot x_{i+[(i-1)/K_1](K_1-1)}, \quad (18)$$

where $\hat{s}_0$ is the estimation of s_0 and $\hat{Q}_c$ is Q_c with replacement of c_x, c_y, c_{xy} with $\hat{c}_x$, $\hat{c}_y$, $\hat{c}_{xy}$.

5. Numerical Examples

In this section, the capability of MC compensation for the proposed algorithm will be tested with two examples.

Table 1: Parameters for the desired signal and interferer.

	Magnitude	Phase	θ_s	φ_s
Signal	1–10 V/m	0	75°	45°
Jammer1	1000 V/m	0	43°	−77°

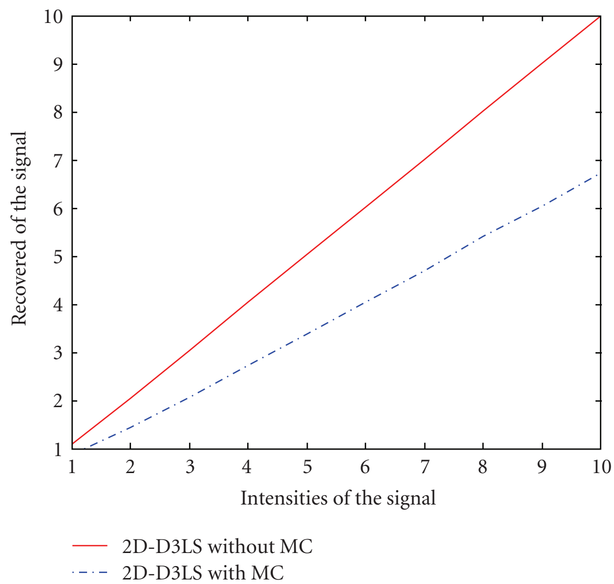

Figure 3: Recovered strength of the desired signal in the absence and presence of mutual coupling.

Consider a URA with 7×7 elements in which the spacing between each two elements in rows and columns is $\lambda/2$. The array receives the desired signal with one jammer. The signal to noise ratio is 20 dB and other parameters are listed in Table 1.

The number of adaptive weights chosen for our simulation will be 16 [16]. Jammer is 60 dB stronger than the intensity of the desired signal. The magnitude of incident signal varies from 1 V/m to 10 V/m; but jammer intensities are constant as given in Table 1. Figure 3 shows the accuracy of the recovered signal in the presence of MC using new formulation (18) with comparison to the ideal recovering. Figure 4 shows the result of the recovered signal in the presence of MC, using a new proposed algorithm with comparison to the ideal recovering. The expected linear relationship is clearly seen and the jammer has been nulled and signal recovered correctly.

Later on, the performance of the proposed method is illustrated by the various simulations. The amplitude of the desired signal accuracy is measured by the root mean-squared error (RMSE), and $L=100$ is the number of Monte Carlo runs.

Figure 5 shows the RMSE of the estimated coupling coefficients versus signal-to-noise ratio (SNR). Figure 6 shows the RMSE of the estimated amplitude of the desired signal, versus SNR. For high SNR, error is very low and in case there is no noise, new formulation is equal to the ideal.

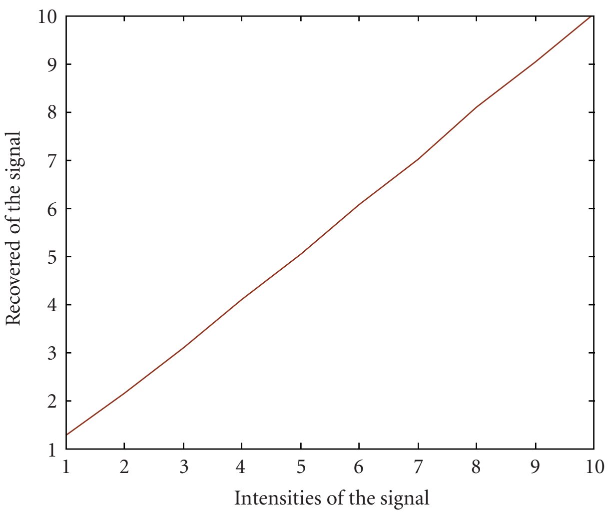

FIGURE 4: Recovered strength of the desired signal with the proposed algorithm in the presence of mutual coupling.

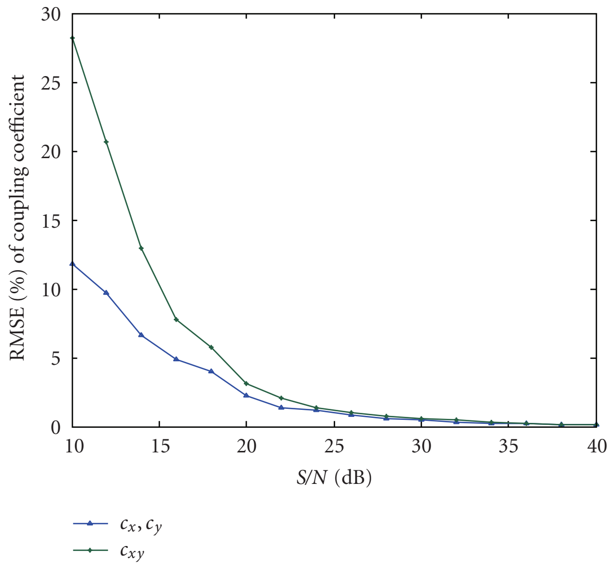

FIGURE 5: RMSE of the coupling coefficients versus the SNR.

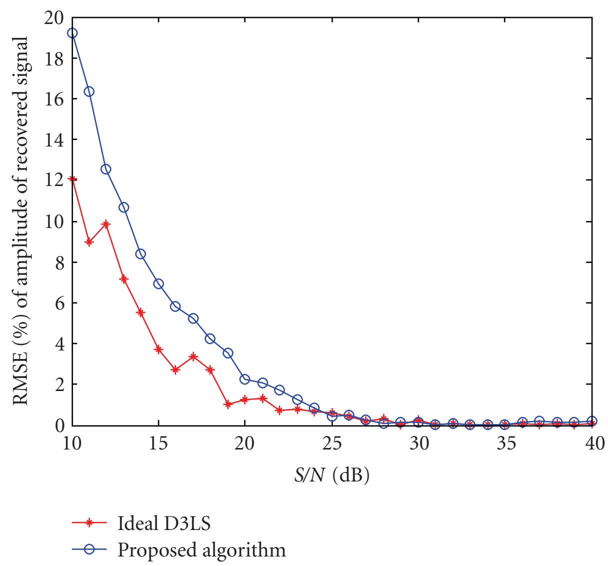

FIGURE 6: RMSE of the recovered amplitude versus the SNR.

6. Conclusion

In this paper, the problems of 2-D D^{3}LS algorithms were studied for recovering of the signal in the presence of mutual coupling and driving a new formulation to recover the signal in the presence of MC. Without using the moment of method and impedance matrix calculation, coupling coefficients can be automatically estimated and without computing the inverse matrix, the desired signal can be recovered. Because we did not use the inverse MC matrix, the amount of computation would be reduced. Moreover, simulation results were confirmed when SNR was high and the RMSE of the method was very close to the ideal D^{3}LS in the absence of MC.

Appendix

In this appendix, (8) and (14) are proved. Consider a URA consisting of 5×5 elements. The array receives one signal (s) from a known direction (θ_0, φ_0) and one interferer (j) (this proof can be extended similarly). From (1), let the received signal at the array in the presence of mutual coupling for each element be

$$x_{np} = s_{np} + j_{np}, \quad \text{for } (n = 1, \ldots, 5,\ p = 1, \ldots, 5), \quad \text{(A.1)}$$

where s_{np}, j_{np} are the received signal and jammer at the npth element, expressed as

$$s_{11} = s = g_s e^{jwt}, \qquad s_{n(p+1)} = \beta s_{np}, \qquad s_{(n+1)p} = \alpha s_{np}. \quad \text{(A.2)}$$

By taking mutual coupling into account, from (11) for each column

first column:

$$s_{11} = \left(1 + \beta c_x + \alpha c_y + \alpha\beta c_{xy}\right) s,$$

$$s_{12} = \beta s_{11} + \left(c_x + \alpha c_{xy}\right) s,$$

$$s_{1p} = \beta s_{1(p-1)}, \quad \text{for } p = 3, 4, 5,$$

2nd column:

$$s_{21} = \alpha s_{11} + \left(c_y + \beta c_{xy}\right) s,$$

$$s_{22} = \beta s_{21} + \left(c_{xy} + \alpha c_x + \alpha^2 c_{xy}\right)s,$$

$$s_{2p} = \beta s_{2(p-1)}, \quad \text{for } p = 3,4,5,$$

3 rd column:

$$s_{31} = \alpha s_{21},$$

$$s_{32} = \beta s_{31} + \alpha\left(c_{xy} + \alpha c_x + \alpha^2 c_{xy}\right)s,$$

$$s_{3p} = \beta s_{3(p-1)}, \quad \text{for } p = 3,4,5,$$

4th column:

$$s_{41} = \alpha s_{31},$$

$$s_{42} = \beta s_{41} + \alpha^2\left(c_{xy} + \alpha c_x + \alpha^2 c_{xy}\right)s,$$

$$s_{4p} = \beta s_{4(p-1)}, \quad \text{for } p = 3,4,5. \tag{A.3}$$

(a) Absence of the Mutual Coupling. If the one row from each column is multiplied by β and subtracted from the next row and then the result of each column is multiplied by α and subtracted from the next column, in the absence of mutual coupling, this will cancel out all the signals and only noise and interferer will be left

$$\left(x_{np} - \beta^{-1}x_{n(p+1)}\right) - \alpha^{-1}\left(x_{(n+1)p} - \beta^{-1}x_{(n+1)(p+1)}\right), \quad \text{for } n = 1,2,\ldots,4, \quad p = 1,2,\ldots,4. \tag{A.4}$$

The weight vectors should be in a way that produces zero output; therefore, a reduced rank matrix is formed in which the weighted sum of all its elements would be zero. In order to make the matrix not singular, the additional equation is introduced through the constraint that the same weights when operating on the signal produced a gain factor Q, which is the first equation. Therefore, (5) will be

$$\begin{bmatrix} \mathbf{b}_1 & \mathbf{b}_2 & \mathbf{b}_3 \\ \mathbf{D}_1 & \mathbf{D}_2 & \mathbf{D}_3 \\ \mathbf{D}_2 & \mathbf{D}_3 & \mathbf{D}_4 \end{bmatrix} \times \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_9 \end{bmatrix} = \begin{bmatrix} Q \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \tag{A.5}$$

$$\left[\begin{matrix} 1 & \cdots & \beta^2 \\ \left[(x_{11} - \beta^{-1}x_{12}) - \alpha^{-1}(x_{21} - \beta^{-1}x_{22})\right] & \cdots & \left[(x_{13} - \beta^{-1}x_{14}) - \alpha^{-1}(x_{23} - \beta^{-1}x_{24})\right] \\ \left[(x_{12} - \beta^{-1}x_{13}) - \alpha^{-1}(x_{22} - \beta^{-1}x_{23})\right] & \cdots & \left[(x_{14} - \beta^{-1}x_{15}) - \alpha^{-1}(x_{24} - \beta^{-1}x_{25})\right] \\ \left[(x_{21} - \beta^{-1}x_{22}) - \alpha^{-1}(x_{31} - \beta^{-1}x_{32})\right] & \cdots & \left[(x_{23} - \beta^{-1}x_{24}) - \alpha^{-1}(x_{33} - \beta^{-1}x_{34})\right] \\ \left[(x_{22} - \beta^{-1}x_{23}) - \alpha^{-1}(x_{32} - \beta^{-1}x_{33})\right] & \cdots & \left[(x_{24} - \beta^{-1}x_{25}) - \alpha^{-1}(x_{34} - \beta^{-1}x_{35})\right] \\ \alpha & \cdots & \alpha\beta^2 \\ \left[(x_{21} - \beta^{-1}x_{22}) - \alpha^{-1}(x_{31} - \beta^{-1}x_{32})\right] & \cdots & \left[(x_{23} - \beta^{-1}x_{24}) - \alpha^{-1}(x_{33} - \beta^{-1}x_{34})\right] \\ \left[(x_{22} - \beta^{-1}x_{23}) - \alpha^{-1}(x_{32} - \beta^{-1}x_{33})\right] & \cdots & \left[(x_{24} - \beta^{-1}x_{25}) - \alpha^{-1}(x_{34} - \beta^{-1}x_{35})\right] \\ \left[(x_{31} - \beta^{-1}x_{32}) - \alpha^{-1}(x_{41} - \beta^{-1}x_{42})\right] & \cdots & \left[(x_{33} - \beta^{-1}x_{34}) - \alpha^{-1}(x_{43} - \beta^{-1}x_{44})\right] \\ \left[(x_{32} - \beta^{-1}x_{33}) - \alpha^{-1}(x_{42} - \beta^{-1}x_{43})\right] & \cdots & \left[(x_{34} - \beta^{-1}x_{35}) - \alpha^{-1}(x_{44} - \beta^{-1}x_{45})\right] \\ \alpha^2 & \cdots & \alpha^2\beta^2 \\ \left[(x_{31} - \beta^{-1}x_{32}) - \alpha^{-1}(x_{41} - \beta^{-1}x_{42})\right] & \cdots & \left[(x_{33} - \beta^{-1}x_{34}) - \alpha^{-1}(x_{43} - \beta^{-1}x_{44})\right] \\ \left[(x_{32} - \beta^{-1}x_{33}) - \alpha^{-1}(x_{42} - \beta^{-1}x_{43})\right] & \cdots & \left[(x_{34} - \beta^{-1}x_{35}) - \alpha^{-1}(x_{44} - \beta^{-1}x_{45})\right] \\ \left[(x_{41} - \beta^{-1}x_{42}) - \alpha^{-1}(x_{51} - \beta^{-1}x_{52})\right] & \cdots & \left[(x_{43} - \beta^{-1}x_{44}) - \alpha^{-1}(x_{53} - \beta^{-1}x_{54})\right] \\ \left[(x_{42} - \beta^{-1}x_{43}) - \alpha^{-1}(x_{52} - \beta^{-1}x_{53})\right] & \cdots & \left[(x_{44} - \beta^{-1}x_{45}) - \alpha^{-1}(x_{54} - \beta^{-1}x_{55})\right] \end{matrix}\right] \times \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_9 \end{bmatrix} = \begin{bmatrix} Q \\ 0 \\ \vdots \\ 0 \end{bmatrix}. \tag{A.6}$$

Then, performing the matrix multiplication in (A.6) for the first row of the matrix will give

$$w_1+\beta w_2+\beta^2 w_3+\alpha w_4+\alpha\beta w_5+\alpha\beta^2 w_6 + \alpha^2 w_7+\alpha^2\beta w_8+\alpha^2\beta^2 w_9 = Q. \tag{A.7}$$

With performing the matrix multiplication in (A.6) for the second row of the matrix the following is obtained:

$$\begin{aligned}
&[(x_{11}-\beta^{-1}x_{12})-\alpha^{-1}(x_{21}-\beta^{-1}x_{22})]w_1\\
&\quad+[(x_{12}-\beta^{-1}x_{13})-\alpha^{-1}(x_{22}-\beta^{-1}x_{23})]w_2\\
&\quad+[(x_{13}-\beta^{-1}x_{14})-\alpha^{-1}(x_{23}-\beta^{-1}x_{24})]w_3\\
&\quad+[(x_{21}-\beta^{-1}x_{22})-\alpha^{-1}(x_{31}-\beta^{-1}x_{32})]w_4\\
&\quad+[(x_{22}-\beta^{-1}x_{23})-\alpha^{-1}(x_{32}-\beta^{-1}x_{33})]w_5\\
&\quad+[(x_{23}-\beta^{-1}x_{24})-\alpha^{-1}(x_{33}-\beta^{-1}x_{34})]w_6\\
&\quad+[(x_{31}-\beta^{-1}x_{32})-\alpha^{-1}(x_{41}-\beta^{-1}x_{42})]w_7\\
&\quad+[(x_{32}-\beta^{-1}x_{33})-\alpha^{-1}(x_{42}-\beta^{-1}x_{43})]w_8\\
&\quad+[(x_{33}-\beta^{-1}x_{34})-\alpha^{-1}(x_{43}-\beta^{-1}x_{44})]w_9=0.
\end{aligned} \tag{A.8}$$

So

$$\begin{aligned}
&(j_{11}w_1+j_{12}w_2+j_{13}w_3+j_{21}w_4+j_{22}w_5\\
&\quad+j_{23}w_6+j_{31}w_7+j_{32}w_8+j_{33}w_9)\\
&\qquad-\beta^{-1}(j_{12}w_1+j_{13}w_2+j_{14}w_3+j_{22}w_4+j_{23}w_5\\
&\qquad\quad+j_{24}w_6+j_{32}w_7+j_{33}w_8+j_{34}w_9)\\
&\qquad-\alpha^{-1}(j_{21}w_1+j_{22}w_2+j_{23}w_3+j_{31}w_4+j_{32}w_5\\
&\qquad\quad+j_{33}w_6+j_{41}w_7+j_{42}w_8+j_{43}w_9)\\
&\qquad+\alpha^{-1}\beta^{-1}(j_{22}w_1+j_{23}w_2+j_{24}w_3+j_{32}w_4+j_{33}w_5\\
&\qquad\quad+j_{34}w_6+j_{42}w_7+j_{43}w_8+j_{44}w_9)=0.
\end{aligned} \tag{A.9}$$

Az $\alpha^{-1}\neq 0$, $\beta^{-1}\neq 0$, and $w_i\neq 0$, (A.9) will be true for all $\mathbf{w}$ if and only if each summation in the parenthesis is equal to zero. Therefore, the first summation will be used

$$j_{11}w_1+j_{12}w_2+j_{13}w_3+j_{21}w_4+j_{22}w_5 + j_{23}w_6+j_{31}w_7+j_{32}w_8+j_{33}w_9=0. \tag{A.10}$$

Similarly, the same can be done for the third row of the matrix (A.5), and so forth. In the absence of mutual coupling ($c_x=c_y=c_{xy}=0$). From (A.3) and (A.10)

$$\begin{aligned}
&(x_{11}-s_{11})\cdot w_1+(x_{12}-\beta s_{11})\cdot w_2+(x_{13}-\beta^2 s_{11})\cdot w_3\\
&\quad+(x_{21}-\alpha s_{11})\cdot w_4+(x_{22}-\alpha\beta s_{11})\cdot w_5\\
&\quad+(x_{23}-\alpha\beta^2 s_{11})\cdot w_6+(x_{31}-\alpha^2 s_{11})\cdot w_7\\
&\quad+(x_{32}-\alpha^2\beta s_{11})\cdot w_8+(x_{33}-\alpha^2\beta^2 s_{11})\cdot w_9=0.
\end{aligned} \tag{A.11}$$

Then, (A.11) will be as simple as

$$\begin{aligned}
&(x_{11}w_1+x_{12}w_2+x_{13}w_3)+(x_{21}w_4+x_{22}w_5+x_{23}w_6)\\
&\quad+(x_{31}w_7+x_{32}w_8+x_{33}w_9)\\
&\quad=s\{(w_1+\beta w_2+\beta^2 w_3)+(\alpha w_4+\alpha\beta w_5+\alpha\beta^2 w_6)\\
&\qquad+(\alpha^2 w_7+\alpha^2\beta w_8+\alpha^2\beta^2 w_9)\}\\
&\quad\Longrightarrow\sum_{i=1}^{9} w_i x_{i+2[(i-1)/3]} = sQ\,.
\end{aligned} \tag{A.12}$$

Therefore, the desired signal can be recovered by

$$s=\frac{1}{Q}\sum_{i=1}^{K_2K_1} w_i x_{i+[(i-1)/K_1](K_1-1)}. \tag{A.13}$$

(b) Presence of the Mutual Coupling. When there is mutual coupling, the matrix (A.5) can be formed and the (A.3) and (A.10) can be written in a similar way

$$\begin{aligned}
&(x_{11}-s_{11})\cdot w_1+\Big(x_{12}-\beta s_{11}-\big(c_x+\alpha c_{xy}\big)s\Big)\cdot w_2\\
&\quad+\Big(x_{13}-\beta^2 s_{11}-\beta\big(c_x+\alpha c_{xy}\big)s\Big)\cdot w_3\\
&\quad+\Big(x_{21}-\alpha s_{11}-\big(c_y+\beta c_{xy}\big)s\Big)\cdot w_4\\
&\quad+\Big(x_{22}-\alpha\beta s_{11}-\beta\big(c_y+\beta c_{xy}\big)s\\
&\qquad-\big(c_{xy}+\alpha c_x+\alpha^2 c_{xy}\big)s\Big)\cdot w_5\\
&\quad+\Big(x_{23}-\alpha\beta^2 s_{11}-\beta^2\big(c_y+\beta c_{xy}\big)s\\
&\qquad-\beta\big(c_{xy}+\alpha c_x+\alpha^2 c_{xy}\big)s\Big)\cdot w_6\\
&\quad+\Big(x_{21}-\alpha^2 s_{11}-\alpha\big(c_y+\beta c_{xy}\big)s\Big)\cdot w_7\\
&\quad+\Big(x_{22}-\alpha^2\beta s_{11}-\alpha\beta\big(c_y+\beta c_{xy}\big)s\\
&\qquad-\alpha\big(c_{xy}+\alpha c_x+\alpha^2 c_{xy}\big)s\Big)\cdot w_8\\
&\quad+\Big(x_{23}-\alpha^2\beta^2 s_{11}-\alpha\beta^2\big(c_y+\beta c_{xy}\big)s\\
&\qquad-\alpha\beta\big(c_{xy}+\alpha c_x+\alpha^2 c_{xy}\big)s\Big)\cdot w_9=0.
\end{aligned} \tag{A.14}$$

Similar to (A.11), the following can be presented

$$
\begin{aligned}
&(x_{11}w_1 + x_{12}w_2 + x_{13}w_3) + (x_{21}w_4 + x_{22}w_5 + x_{23}w_6) \\
&\quad + (x_{31}w_7 + x_{32}w_8 + x_{33}w_9) \\
&= \left(1 + \beta c_x + \alpha c_y + \alpha\beta c_{xy}\right) \\
&\quad \times s\{(w_1 + \beta w_2 + \beta^2 w_3) + (\alpha w_4 + \alpha\beta w_5 + \alpha\beta^2 w_6) \\
&\qquad + (\alpha^2 w_7 + \alpha^2\beta w_8 + \alpha^2\beta^2 w_9)\} + \left(c_x + \alpha c_{xy}\right) \\
&\quad \times s(w_2 + \beta w_3 + \alpha w_5 + \alpha\beta w_6 + \alpha^2 w_8 + \alpha^2\beta w_9) \\
&\quad + \left(c_y + \beta c_{xy}\right) s(w_4 + \beta w_5 + \beta^2 w_6 + \alpha w_7 + \alpha\beta w_8 + \alpha\beta^2 w_9) \\
&\quad + \left(c_{xy}\right) s(w_5 + \beta w_6 + \alpha w_8 + \alpha\beta w_9).
\end{aligned} \tag{A.15}
$$

The recovered signal will be as follows:

$$
\begin{aligned}
&\Longrightarrow \sum_{i=1}^{9} w_i x_{i+2[(i-1)/3]} \\
&= s\Bigg[\left(1 + \beta c_x + \alpha c_y + \alpha\beta c_{xy}\right)\sum_{i=1}^{9}\alpha^{[(i-1)/3]}\beta^{i-1-3[(i-1)/3]} wc_i \\
&\qquad + \left(c_x + \alpha c_{xy}\right)\sum_{i=1}^{6}\alpha^{[(i-1)/2]}\beta^{i-1-2[(i-1)/2]} wc_{i+1+[(i-1)/2]} \\
&\qquad + \left(c_y + \beta c_{xy}\right)\sum_{i=1}^{6}\alpha^{[(i-1)/3]}\beta^{i-1-[(i-1)/3]K_1} wc_{i+3} \\
&\qquad + c_{xy}\sum_{i=1}^{4}\alpha^{[(i-1)/2]}\beta^{i-1-2[(i-1)/2]} wc_{i+4+[(i-1)/2]}\Bigg].
\end{aligned} \tag{A.16}
$$

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