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Joint Performance Study of Channel Estimation and Multiuser Detection for Uplink Long-Code CDMA Systems

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Although numerous channel estimation and multiuser detection approaches have appeared for long-code uplink CDMA systems, joint performance study of channel estimators and symbol detectors remains largely open. In this paper, we construct three typical symbol-level linear receivers upon existing channel estimation method, known as zero-forcing (ZF), minimum mean-square-error (MMSE), and RAKE receivers, for symbol detection. Since the channel estimation error is rippled to the linear receivers, performance of all receivers is thus jointly analyzed with the channel estimator from a perturbation perspective. Extensive simulation examples involving different communication environments demonstrate high consistency between our analysis and experimental results.

Keywords and phrases: long codes, channel estimation, multiuser detection, perturbation analysis.

1. INTRODUCTION

Direct sequence (DS) code division multiple access (CDMA) technology has become an appealing solution to third-generation wireless systems [1, 2]. It provides a communication-system unique capabilities of simultaneous spectrum sharing, mitigation of jamming, interception, and multipath fading [3], and can easily support transmission of multirate information streams.

Multiuser interference (MUI) is a typical obstacle to be obviated in detection of input signals in a DS/CDMA system, which has attracted substantial efforts in recent years [4]. In DS/CDMA systems, the bandwidth of the input signal is spread by a sequence with a much higher rate in order to effectively suppress the interference. There are two kinds of spreading codes: the periodic spreading sequence (short codes) which repeats from symbol to symbol, and the aperiodic spreading sequence (long codes) which has a much longer period compared with the symbol duration. In recent years, many efforts have been focused on the CDMA system with short codes, resulting in simplified methods, and very importantly, tractable analysis of the system performance. However, long spreading codes feature the new CDMA-based wireless standards [5]. Employment of long codes exhibits certain superiority to short codes in terms of increasing immunity of the system to MUI and channel fading on the average [6], improving the spectrum efficiency through the uniform distribution of the signal bandwidth, and moreover ensuring a secure communication link in a hostile environment, protecting users' information against intentional interception.

Despite various advantages, adopted long spreading codes inevitably destroy cyclostationarity of CDMA signals, making many of the existing channel estimation and detection approaches for short-code CDMA systems not directly applicable. Therefore, solutions for long-code CDMA systems are still under extensive investigation. It has been witnessed that various solutions for downlink communications have been developed [7, 8, 9, 10]. However, uplink communications incur new problems due to asynchronism and particular code assignment strategies. Given pilot symbols of all users, least squares (LS)-fitting or iterative maximum likelihood (ML) approaches have been reported [6, 11, 12]. Blind methods have also been derived. Correlation matching techniques have been successfully applied for channel estimation [13, 14]. Employing a space-time

2D RAKE receiver structure to maximize the output signal-to-interference-plus-noise ratio (SINR), both channel estimation and minimum mean-square-error (MMSE)/zero-forcing (ZF) receivers are presented in [15, 16]. Built upon the structure of convolutive channels, a Toeplitz displacement channel estimation method has been proposed [17]. A novel parallel factor analysis technique is applied to multiuser detection [18]. Even in the presence of colored Gaussian noise in addition to unknown multipath fading and MUI, turbo multiuser detectors can be developed [19]. Based on LS fitting, a blind decorrelating RAKE receiver is recently proposed [20]. If all users' spreading codes and propagation delays are known, a subspace technique can be applied to estimate multipath parameters [21]. Performance of long-code CDMA systems and detection methods has also been studied under the assumption of perfect channel estimate. A heuristic approximation of SINR for the decorrelator and simulation results for the MMSE receiver are provided [22]. Spectral efficiency of some linear receivers are explored [23, 24]. Near-far resistance of MMSE detection is investigated [25] and trade-offs of long-versus-short spreading sequences are demonstrated [26, 27]. Performance of ML detectors is studied for multirate CDMA systems [28].

In this paper, we study joint performance of the channel estimator and multiuser receivers for long-code uplink CDMA systems. The channel estimator considered in our analysis is based on [21], which extends [10] to uplink communications with slight modification. It is also closely related to [20] where a low-complexity algorithm is further developed. Based on the estimated channel, different linear detection techniques such as ZF, MMSE, and RAKE are applied. Both ZF and MMSE receivers have complexity which is cubic in the processing gain. Due to the time-varying property of a long-code system, designing new low-complexity multiuser detectors remains a challenging issue. Since in practice, the channel estimation is based on processing of finite data samples, it will deviate from its theoretical value, resulting in estimation errors. Perturbation in channel estimate will be further carried on to the receivers, causing receivers' output SINRs and BERs all perturbed. To quantify the effect of sample size, we apply perturbation theory to obtain the channel mean-square error (MSE), detectors' perturbed SINR, and finite-data-based bit-error rate (BER). Unlike previous work [22, 23], which studied the effect of randomness of codes on the fluctuation of the SINR of receivers constructed from perfect signature vectors, our analysis considers the effect of imperfect signature waveforms caused by finite data samples on the performance of receivers. Although joint performance has been studied for short-code DS/CDMA systems [29] and long-code downlink CDMA systems [30], long-code uplink communications adopt completely different channel estimation and symbol detection schemes and the results in [29, 30] are not applicable. Therefore, new joint performance analysis for long-code uplink systems is necessary and important. Moreover, high consistency is observed between our experimental results and analytical results.

This paper is organized as follows. A long-code CDMA uplink system model is described in Section 2. Subspace-based channel estimation method is reviewed, together with addition of noise-power estimation method and implementation of three typical linear detectors are presented in Section 3. In Section 4, joint performance of channel estimator and detectors in terms of channel MSE, receivers' SINR and BER is evaluated. Finally, various simulation examples are provided in Section 5 and conclusions are drawn in Section 6.

2. CDMA UPLINK WITH LONG CODES

Consider an uplink CDMA system [14], where J mobile stations are communicating with a base station. The jth user's bit $w_j(n)$ is first spread by aperiodic codes $c_{j,n}(k)$ ($k = 0,\dots,P-1$), and then transmitted through a multipath channel $g_j(m)$. All channels are assumed to have maximum order q ($q \ll P$). The signal from user j is assumed to arrive at the base station with delay d_j. Then, after considering rectangle pulse shaping, the received chip-rate signal is a superposition of signals from J users corrupted by noise [13]

$$y(k) = \sum_{j=1}^{J}\sum_{m=0}^{q} g_j(m)s_j(k-m-d_j) + v(k), \tag{1}$$

where

$$s_j(k) = \sum_{n=-\infty}^{\infty} w_j(n)c_{j,n}(k-nP), \tag{2}$$

$v(n)$ is zero-mean AWGN with variance $\sigma_v^2 = E\{|v(n)|^2\}$. If we assume that the system is quasi-synchronous, where transmission delays from all users to the base station are within a small fraction of a chip duration, that is, $0 \le d_j \ll P$. As a result, the intersymbol interference could be eliminated if we collect only $L = P-\mu$ samples in the nth bit interval into a vector $\mathbf{y}(n) = [y(nP+\mu),\dots,y(nP+P-1)]^T$ with $\mu = \max\{q+d_j\}$. Let $\mathbf{C}_j(n)$ be the truncated version of the following code filtering matrix of dimensions $(P+q)$ by $(q+1)$ for user j:

$$\bar{\mathbf{C}}_j(n) = \begin{bmatrix} c_{j,n}(0) & & \mathbf{0} \\ \vdots & \ddots & c_{j,n}(0) \\ c_{j,n}(P-1) & & \vdots \\ \mathbf{0} & \ddots & c_{j,n}(P-1) \end{bmatrix}, \tag{3}$$

that is, $\mathbf{C}_j(n) = [\bar{\mathbf{C}}_j(n)]_{\mu+1:P,1:q+1}$. Then according to (1), a simple matrix form follows:

$$\begin{aligned} \mathbf{y}(n) &= \sum_{j=1}^{J}\mathbf{C}_j(n)\mathbf{g}_j w_j(n) + \mathbf{v}(n) \\ &= \mathcal{C}(n)\mathcal{G}\mathbf{w}(n) + \mathbf{v}(n) = \mathbf{H}(n)\mathbf{w}(n) + \mathbf{v}(n), \end{aligned} \tag{4}$$

where $\mathcal{C}(n)_{(P-\mu)\times(J(q+1))}$, $\mathcal{G}_{(J(q+1))\times J}$, and $\mathbf{H}(n)_{(P-\mu)\times J}$ are defined as

$$\begin{aligned}\mathcal{C}(n) &= [\mathbf{C}_1(n),\dots,\mathbf{C}_J(n)],\\ \mathcal{G} &= \operatorname{diag}\{\mathbf{g}_1,\dots,\mathbf{g}_J\},\\ \mathbf{H}(n) &= [\mathbf{h}_1(n),\dots,\mathbf{h}_J(n)]\\ &= [\mathbf{C}_1(n)\mathbf{g}_1,\dots,\mathbf{C}_J(n)\mathbf{g}_J] = \mathcal{C}(n)\mathcal{G},\end{aligned} \tag{5}$$

$\mathbf{g}_j$ with $(q+1)$ elements is the channel vector of user j, $\mathbf{w}(n)_{J\times 1}$ and $\mathbf{v}(n)_{(P-\mu)\times 1}$ are given by

$$\begin{aligned}\mathbf{w}(n) &= [w_1(n),\dots,w_J(n)]^T,\\ \mathbf{v}(n) &= [v(nP+\mu),\dots,v(nP+P-1)]^T.\end{aligned} \tag{6}$$

Throughout this paper, we make the following assumptions.

(AS1) All users' information sequences are mutually independent and temporally i.i.d. with unit power.
(AS2) Each user's codes and delay are known.
(AS3) All users' long codes are pseudorandom variables.
(AS4) The number of active users in the system satisfies $J < (P-q)/(q+1)$.

The first assumption is adopted for most systems and convenient for analysis. However, the effect of different interfering power is studied in the simulation in the paper. For the second assumption, it is reasonable to assume that the base station knows all users' spreading codes. On the other hand, all users' delays are assumed to be estimated in advance by some delay estimator such as [16]. However, if they are not available, we can over-parameterize the channel vector to absorb the delay ambiguity [20]. Long-code sequence can be categorized as deterministic or stochastic. To facilitate our analysis, the latter is adopted as (AS3). (AS4) is the condition required for channel estimation in [10, 21] upon which our receivers are built.

3. CHANNEL ESTIMATION AND MULTIUSER DETECTION IN CDMA UPLINK

Before we start joint performance analysis, we first briefly present the channel estimation method, which extends [10] to CDMA uplink with some alterations. The tailored method, instead of using the whole code-decorrelated data vector, utilizes only a portion pertaining to the desired user to estimate channel in a reduced complexity. Whitening is performed before subspace method is applied, and noise power is estimated from the whitened data. Then, three most commonly used symbol-level linear receivers, ZF, MMSE, and RAKE receivers, are constructed in order to detect the desired user's symbols.

3.1. Subspace-based channel and noise-power estimation

According to (4), the original data vector $\mathbf{y}(n)$ contains nonstationary signal contribution and stationary noise contribution. Consequently, the conventional unconditional covariance of $\mathbf{y}(n)$, which is estimated from sample average of $\mathbf{y}(n)\mathbf{y}(n)^H$, contains no noise subspace. The signal subspace in fact spans the whole operational space, as mentioned in [30]. Therefore, subspace approach cannot be applied to the unconditional covariance of $\mathbf{y}(n)$.

As in [10], we first decorrelate the data vector $\mathbf{y}(n)$ in (4) using the pseudoinverse of code matrix $\mathcal{C}(n)$ at each symbol to obtain an approximately stationary sequence of $J(q+1)$ elements as

$$\begin{aligned}\mathbf{u}(n) &= \mathcal{C}(n)^\dagger\mathbf{y}(n)\\ &= \mathcal{G}\mathbf{w}(n) + \mathcal{C}(n)^\dagger\mathbf{v}(n).\end{aligned} \tag{7}$$

It is observed that after code decorrelation, $\mathbf{u}(n)$ contains a stationary signal process and a nonstationary noise process. The unconditional covariance of the decorrelated sequence becomes

$$\bar{\mathbf{R}} = \mathcal{G}\mathcal{G}^H + \sigma_v^2\mathbf{A}, \tag{8}$$

where

$$\begin{aligned}\mathbf{A}_{J(q+1)\times J(q+1)} &\triangleq E\{\mathcal{C}(n)^\dagger(\mathcal{C}(n)^\dagger)^H\}\\ &= E\{[\mathcal{C}(n)^H\mathcal{C}(n)]^{-1}\}.\end{aligned} \tag{9}$$

Noticing that $\mathcal{G}$ contains J users' channel vector as its diagonal block, we then partition both $\bar{\mathbf{R}}$ and $\mathbf{A}$ into $J\times J$ submatrices of dimensions $(q+1)\times(q+1)$. If we denote their (j,j)th submatrices as $\bar{\mathbf{R}}_j$ and $\mathbf{A}_j$, respectively, then

$$\bar{\mathbf{R}}_j \triangleq \mathbf{S}_j^T\bar{\mathbf{R}}\mathbf{S}_j = \mathbf{g}_j\mathbf{g}_j^H + \sigma_v^2\mathbf{A}_j, \quad \text{where } j = 1,\dots,J, \tag{10}$$

and $\mathbf{A}_j \triangleq \mathbf{S}_j^T\mathbf{A}\mathbf{S}_j$, where $\mathbf{S}_j$ is a selection matrix of dimensions $(q+1)$ by $J(q+1)$ and defined as

$$\mathbf{S}_j = [\mathbf{0}_{(j-1)(q+1)\times(q+1)};\mathbf{I}_{(q+1)\times(q+1)};\mathbf{0}_{(J-j)(q+1)\times(q+1)}]^T. \tag{11}$$

The correlation matrix $\bar{\mathbf{R}}_j$ after decorrelation now contains the desired subspace spanned by the jth user's channel vector, but it is corrupted by a colored matrix $\mathbf{A}_j$. Therefore, whitening is performed to yield

$$\mathbf{R}_j \triangleq \mathbf{A}_j^{-1/2}\bar{\mathbf{R}}_j\mathbf{A}_j^{-1/2} = \mathbf{A}_j^{-1/2}\mathbf{g}_j\mathbf{g}_j^H\mathbf{A}_j^{-1/2} + \sigma_v^2\mathbf{I}. \tag{12}$$

It is observed that $\mathbf{A}_j^{-1/2}\mathbf{g}_j$ constitutes the unique signal subspace of $\mathbf{R}_j$, and σ_v^2 is the least eigenvalue of $\mathbf{R}_j$ with multiplicity q. For convenience, denote a new column vector of $(q+1)$ elements as $\boldsymbol{\chi}_j \triangleq \mathbf{A}_j^{-1/2}\mathbf{g}_j/\|\mathbf{A}_j^{-1/2}\mathbf{g}_j\|$, the subspace orthogonal to $\boldsymbol{\chi}_j$ as the noise subspace $\mathbf{U}_n^j$, and the eigenvalues of $\mathbf{R}_j$ as λ_i $(i = 1,\dots,q+1)$ in a descending order. Applying the subspace technique to $\mathbf{R}_j$ immediately yields the following channel estimation method for the channel of user j and the noise power

$$\chi_j = \arg\max_{\|\boldsymbol{\beta}\|=1} \boldsymbol{\beta}^H \mathbf{R}_j \boldsymbol{\beta},$$
$$\mathbf{g}_j = \frac{\mathbf{A}_j^{1/2}\chi_j}{\|\mathbf{A}_j^{1/2}\chi_j\|}, \qquad \sigma_v^2 = \frac{1}{q}\sum_{i=2}^{q+1}\lambda_i. \tag{13}$$

It can easily be verified from (12) that

$$\sigma_v^2 = \frac{1}{q}\operatorname{tr}\left\{(\mathbf{U}_n^j)^H \mathbf{R}_j \mathbf{U}_n^j\right\}, \tag{14}$$

where "tr" is a trace operator.

It is necessary to mention that the above derivations use (AS3). However, without any assumption on codes, the averaged covariance of the decorrelated sequence $\mathbf{u}(n)$ can still be obtained as (8), if we define $\mathbf{A}_{J(q+1)\times J(q+1)} \triangleq (1/N)\sum_n [\mathcal{C}(n)^H\mathcal{C}(n)]^{-1}$. Clearly, in this situation, $\mathbf{A}$ can be calculated in advance given all sets of codes, and therefore modeled as a deterministic quantity. As a result, the above mentioned approach is still applicable.

3.2. Symbol detection

We now turn to symbol detection based on estimated channel vectors. Without loss of generality, we focus on our desired user, user 1, and typical linear receivers to detect the desired user's symbols $w_1(n)$.

3.2.1. ZF receiver

Once all users' channel vectors are estimated, the signature matrix can be constructed at each time instance n using all users' spreading codes as (5). Then, the ZF receiver at time instant n is defined as a column vector with $(P-q)$ elements

$$\mathbf{f}_{\mathrm{ZF}}(n) = \mathbf{H}(n)\left[\mathbf{H}(n)^H\mathbf{H}(n)\right]^{-1}\mathbf{e}, \tag{15}$$

where $\mathbf{e}$ is a unitary vector with the first element as 1. Then $w_1(n)$ is estimated by

$$\hat{w}_{1,\mathrm{ZF}}(n) = \mathbf{f}_{\mathrm{ZF}}^H(n)\mathbf{y}(n). \tag{16}$$

3.2.2. MMSE receiver

The MMSE receiver, which is a column vector with $(P-q)$ elements, can be defined as

$$\mathbf{f}_{\mathrm{MMSE}}(n) = \mathcal{R}(n)^{-1}\mathbf{C}_1(n)\mathbf{g}_1, \tag{17}$$

where $\mathcal{R}(n)$ is the conditional correlation matrix of $\mathbf{y}(n)$ at time n, which is conditioned on all users' codes. Notice that $\mathcal{R}(n)$ cannot be estimated by conventional sample average. However, it can be constructed by

$$\mathcal{R}(n) = \mathbf{H}(n)\mathbf{H}(n)^H + \sigma_v^2\mathbf{I}, \tag{18}$$

once $H(n)$ is constructed as (5) from all estimated channel vectors and σ_v^2 is estimated according to (14). Correspondingly, the detected symbol is given by

$$\hat{w}_{1,\mathrm{MMSE}}(n) = \mathbf{f}_{\mathrm{MMSE}}^H(n)\mathbf{y}(n). \tag{19}$$

3.2.3. RAKE receiver

The RAKE receiver at time n is constructed as the following column vector with $(P-q)$ elements [8]:

$$\mathbf{f}_{\mathrm{RAKE}}(n) = \mathbf{h}_1(n) = \mathbf{C}_1(n)\mathbf{g}_1, \tag{20}$$

and the desired user's symbol is estimated as

$$\hat{w}_{1,\mathrm{RAKE}}(n) = \mathbf{f}_{\mathrm{RAKE}}^H(n)\mathbf{y}(n). \tag{21}$$

Linear receivers are coupled with estimated channel vectors. Their performance will be investigated jointly with channel estimators next.

4. PERFORMANCE ANALYSIS

In this section, we will apply perturbation theory to analyze the channel estimation MSE for each user. Performance of three linear receivers for the desired user is then evaluated in terms of SINR and BER. Before we proceed, we clarify some notations used in the following analysis. First, the perturbation is defined as the difference between the estimated quantity based on finite data samples and its asymptotic value based on infinite large data samples. For our subspace-based channel estimation, its perturbation is given by the difference between the estimated channel based on N data samples and the true channel vector, after noticing that the true channel vector can be obtained when infinite data samples are applied. For receivers, since they are constructed based on estimated channels, their perturbations thus depend on channel perturbations, which will be derived later. Moreover, as we can see later, all perturbations originate from the perturbation of data-covariance matrix, which is estimated from finite data samples. Since the inputs and noise are all random, therefore, the perturbation of data covariance, and thus all other perturbations, can be modeled as random process. In the sequel, the perturbation is denoted by preceding the corresponding quantity by δ, and the perturbed quantity with $\tilde{}$. For example, $\delta\mathbf{g}_j = \tilde{\mathbf{g}}_j - \mathbf{g}_j$, $\delta\mathbf{R}_j = \tilde{\mathbf{R}}_j - \mathbf{R}_j$. Assume the number of data samples N is sufficiently large such that perturbation technique is applicable.

4.1. Channel mean-square error

When estimated from finite data samples, data-covariance matrix gets perturbed. Its perturbation will be carried over to channel estimate. In the sequel, we will study the statistical performance of the jth user's channel estimator under a large sample size assumption by applying perturbation techniques. It is observed from (13) that the jth user's channel estimate depends on $\mathbf{R}_j$ and therefore $\bar{\mathbf{R}}$. When $\bar{\mathbf{R}}$ is estimated from N decorrelated data samples as

$$\tilde{\bar{\mathbf{R}}} = \frac{1}{N}\sum_{n=1}^{N}\mathbf{u}(n)\mathbf{u}(n)^H, \tag{22}$$

$\mathbf{R}_j$ is perturbed as the following according to (10), (11), and (12):

$$\tilde{\mathbf{R}}_j = \mathbf{A}_j^{-1/2}\mathbf{S}_j^T\tilde{\bar{\mathbf{R}}}\mathbf{S}_j\mathbf{A}_j^{-1/2}. \tag{23}$$

Using (22) and (23), and replacing $\mathbf{u}(n)$ with (7), we can further express $\tilde{\mathbf{R}}_j$ as an estimate from N decorrelated data samples as

$$\tilde{\mathbf{R}}_j = \frac{1}{N}\sum_{i=1}^{N}\left[\mathbf{x}_j(n) + \nu_j(n)\right]\left[\mathbf{x}_j(n) + \nu_j(n)\right]^H, \tag{24}$$

where

$$\begin{aligned} \mathbf{x}_j(n) &= \mathbf{A}_j^{-1/2}\mathbf{g}_j w_j(n), \\ \nu_j(n) &= \mathbf{A}_j^{-1/2}\mathbf{S}_j^T \mathcal{C}(n)^{\dagger}\mathbf{v}(n). \end{aligned} \tag{25}$$

Noticing that $\nu_j(n)$ is a Gaussian vector whose covariance $E\{\mathbf{A}_j^{-1/2}\mathbf{S}_j^T\mathcal{C}(n)^{\dagger}\mathbf{v}(n)\mathbf{v}(n)^H(\mathcal{C}(n)^{\dagger})^H\mathbf{S}_j\mathbf{A}_j^{-1/2}\}$ can be approximated by $\sigma_v^2\mathbf{I}$, we can view $\tilde{\mathbf{R}}_j$ to be estimated from N samples of $\mathbf{x}_j(n)$ corrupted by AWGN $\nu_j(n)$, and perturbation analysis can be readily conducted to $\mathbf{R}_j$. The Gaussian property on the noise $\nu_j(n)$ is necessary for deriving some statistics later. According to [31], the first-order perturbation of the signal subspace χ_j is given by

$$\delta\chi_j = (1/\gamma_j^2)\mathbf{U}_n^j(\mathbf{U}_n^j)^H\delta\mathbf{R}_j\chi_j, \tag{26}$$

where $\gamma_j^2 = \mathbf{g}_j^H\mathbf{A}_j^{-1}\mathbf{g}_j$. According to (13), $\delta\chi_j$ will cause $\mathbf{g}_j$ perturbed as $\tilde{\mathbf{g}}_j = (\tilde{\chi}_j^H\mathbf{A}_j\tilde{\chi}_j)^{-1/2}\mathbf{A}_j^{1/2}\tilde{\chi}_j$. Substituting $\delta\chi_j$ into $\tilde{\chi}_j$, expanding the power term using Taylor series, and keeping only the first order terms, we obtain the jth user's channel perturbation as

$$\begin{aligned} \delta\mathbf{g}_j \approx{}& (\chi_j^H\mathbf{A}_j\chi_j)^{-1/2}\mathbf{A}_j^{1/2}\delta\chi_j \\ &- \frac{1}{2}(\chi_j^H\mathbf{A}_j\chi_j)^{-3/2}(\delta\chi_j^H\mathbf{A}_j\chi_j + \chi_j^H\mathbf{A}_j\delta\chi_j)\mathbf{A}_j^{1/2}\chi_j. \end{aligned} \tag{27}$$

Since in-space error is much smaller than the orthogonal space error by [31], and noticing $\mathbf{g}_j = (\chi_j^H\mathbf{A}_j\chi_j)^{-1/2}\mathbf{A}_j^{1/2}\chi_j$, we further simply (27) to the following by keeping only the orthogonal space error:

$$\delta\mathbf{g}_j \approx \mathbf{\Pi}_{\mathbf{g}_j}^{\perp}(\chi_j^H\mathbf{A}_j\chi_j)^{-1/2}\mathbf{A}_j^{1/2}\delta\chi_j, \tag{28}$$

where $\mathbf{\Pi}_{\mathbf{g}_j}^{\perp} \triangleq (\mathbf{I} - \mathbf{g}_j\mathbf{g}_j^H/\mathbf{g}_j^H\mathbf{g}_j)$. On the other hand, $(\mathbf{g}_j^H\mathbf{A}_j^{-1}\mathbf{g}_j)(\chi_j^H\mathbf{A}_j\chi_j) = 1$ by (13), which implies $\chi_j^H\mathbf{A}_j\chi_j = 1/\gamma_j^2$. Using these results and replacing $\delta\chi_j$ with (26), (28) becomes

$$\delta\mathbf{g}_j \approx \frac{1}{\gamma_j}\mathbf{\Pi}_{\mathbf{g}_j}^{\perp}\mathbf{A}_j^{1/2}\mathbf{U}_n^j(\mathbf{U}_n^j)^H\delta\mathbf{R}_j\chi_j. \tag{29}$$

Then the covariance of channel estimation error is evaluated according to

$$\begin{aligned} &E\{\delta\mathbf{g}_j\delta\mathbf{g}_j^H\} \\ &\quad\approx \frac{1}{\gamma_j^2}\mathbf{\Pi}_{\mathbf{g}_j}^{\perp}\mathbf{A}_j^{1/2}\mathbf{U}_n^j(\mathbf{U}_n^j)^H E\{\delta\mathbf{R}_j\chi_j\chi_j^H\delta\mathbf{R}_j\}\mathbf{U}_n^j(\mathbf{U}_n^j)^H\mathbf{A}_j^{1/2}\mathbf{\Pi}_{\mathbf{g}_j}^{\perp}. \end{aligned} \tag{30}$$

Clearly, the covariance depends on the term $E\{\delta\mathbf{R}_j\chi_j\chi_j^H\delta\mathbf{R}_j\}$. Hence, it suffices to determine a $(q+1)$ by $(q+1)$ general-form matrix that is also required later:

$$\mathbf{T}_j(\mathbf{D}) = E\{\delta\mathbf{R}_j\mathbf{D}\delta\mathbf{R}_j\}, \tag{31}$$

where $\mathbf{D}$ can be replaced by corresponding deterministic matrices of dimensions $(q+1)$ by $(q+1)$, respectively, later. Noticing (12) and (24), and using the results in [32], we have that if all quantities are real, and the noise is Gaussian, then

$$\begin{aligned} \mathbf{T}_j(\mathbf{D}) = {}& \frac{\gamma_j^4\kappa_{4w}}{N}\chi_j[\mathbf{I}\odot(\chi_j^T\mathbf{D}\chi_j)]\chi_j^T \\ &+ \frac{1}{N}\operatorname{tr}(\mathbf{R}_j\mathbf{D})\mathbf{R}_j + \frac{1}{N}\mathbf{R}_j\mathbf{D}^T\mathbf{R}_j, \end{aligned} \tag{32}$$

where $\odot$ represents elementwise multiplication, and κ_{4w} is the fourth-order cumulant of $w_j(n)$. For a complex system,

$$\mathbf{T}_j(\mathbf{D}) = \frac{\gamma_j^4\kappa_{4w}}{N}\chi_j[\mathbf{I}\odot(\chi_j^H\mathbf{D}\chi_j)]\chi_j^H + \frac{1}{N}\operatorname{tr}(\mathbf{R}_j\mathbf{D})\mathbf{R}_j. \tag{33}$$

Therefore, for a given data model, statistical properties of the inputs and additive noise, $\mathbf{T}_j(\mathbf{D})$ can always be evaluated. Applying (32) or (33) to (30), setting $\mathbf{D} = \chi_j\chi_j^H$, and noticing $\chi_j^H\mathbf{U}_n^j = \mathbf{0}$, $\operatorname{tr}\{\mathbf{R}_j\mathbf{D}\} = \chi_j^H\mathbf{R}_j\chi_j = \gamma_j^2 + \sigma_v^2$, one can verify that in both cases (30) reduces to

$$E\{\delta\mathbf{g}_j\delta\mathbf{g}_j^H\} \approx \frac{\sigma_v^2(\gamma_j^2 + \sigma_v^2)}{N\gamma_j^2}\mathbf{\Pi}_{\mathbf{g}_j}^{\perp}\mathbf{A}_j^{1/2}\mathbf{U}_n^j(\mathbf{U}_n^j)^H\mathbf{A}_j^{1/2}\mathbf{\Pi}_{\mathbf{g}_j}^{\perp}. \tag{34}$$

It is further simplified if noise is small

$$E\{\delta\mathbf{g}_j\delta\mathbf{g}_j^H\} \approx \frac{\sigma_v^2}{N}\mathbf{\Pi}_{\mathbf{g}_j}^{\perp}\mathbf{A}_j^{1/2}\mathbf{U}_n^j(\mathbf{U}_n^j)^H\mathbf{A}_j^{1/2}\mathbf{\Pi}_{\mathbf{g}_j}^{\perp}. \tag{35}$$

The channel MSE is then given by the trace of (35). It can be seen that the jth user's channel MSE is proportional to noise power and inversely proportional to data length N. It also depends on its own channel condition and all users' long codes.

4.2. SINRs of different receivers

SINR is an important performance indicator for receivers. The average SINR can be defined as

$$\text{SINR} = \frac{E\{\mathbf{f}(n)^H\mathcal{R}_1(n)\mathbf{f}(n)\}}{E\{\mathbf{f}(n)^H\mathcal{R}_{\text{int}}(n)\mathbf{f}(n)\}}, \tag{36}$$

where $\mathbf{f}(n)$ is any symbol-level receiver,

$$\begin{aligned} \mathcal{R}_1(n) &= \mathbf{h}_1(n)\mathbf{h}_1(n)^H, \\ \mathcal{R}_{\text{int}}(n) &= \mathcal{R}(n) - \mathcal{R}_1(n). \end{aligned} \tag{37}$$

Although fluctuation of SINR can be analyzed under perfect conditions, perturbation in channel estimation induced by finite data samples inevitably causes the receiver to be perturbed as $\tilde{\mathbf{f}}(n) = \mathbf{f}(n) + \delta\mathbf{f}(n)$, where the first-order perturbation $\delta\mathbf{f}(n)$ is assumed to have zero mean. This is a reasonable assumption. According to [32], it can be assumed that $E\{\delta\mathbf{R}_j\} = 0$ and $E\{\delta\sigma_v^2\} = 0$, when they are estimated from finite data. Therefore one can verify that $E\{\delta\mathbf{g}_j\} = 0$, and then $E\{\delta\mathbf{H}\} = 0$. Consequently, from later analysis, we have $E\{\delta\mathbf{f}(n)\} = 0$. Based on it, the perturbed SINR has the following form:

$$\widetilde{\text{SINR}} = \frac{E\{\mathbf{f}(n)^H\mathcal{R}_1(n)\mathbf{f}(n)\} + E\{\delta\mathbf{f}(n)^H\mathcal{R}_1(n)\delta\mathbf{f}(n)\}}{E\{\mathbf{f}(n)^H\mathcal{R}_{\text{int}}(n)\mathbf{f}(n)\} + E\{\delta\mathbf{f}(n)^H\mathcal{R}_{\text{int}}(n)\delta\mathbf{f}(n)\}}. \tag{38}$$

It depends on both unperturbed terms (signal power, interference-plus-noise power) and corresponding perturbed terms. They follow particular forms of $\Phi(\mathbf{X}) = E\{\mathbf{f}(n)^H\mathbf{X}\mathbf{f}(n)\}$ for unperturbed terms and of $\Psi(\mathbf{X}) = E\{\delta\mathbf{f}(n)^H\mathbf{X}\delta\mathbf{f}(n)\}$ for perturbed terms, where $\mathbf{X}$ can be replaced by $\mathcal{R}_1(n)$ or $\mathcal{R}_{\text{int}}(n)$. Since different receivers take different forms, these quantities need to be evaluated for each receiver respectively. For shorter notations, all receiver subscripts are dropped later.

4.2.1. SINR of the ZF receiver

First, replacing the ZF receiver with (15), it can be shown that

$$\Phi(\mathcal{R}_1(n)) = 1 \tag{39}$$

and $\Phi(\mathcal{R}_{\text{int}}(n)) = \sigma_v^2\|\mathbf{f}(n)\|^2 = \sigma_v^2\mathbf{e}^H E\{[\mathbf{H}(n)^H\mathbf{H}(n)]^{-1}\}\mathbf{e}$. It is much involved to further simplify $\Phi(\mathcal{R}_{\text{int}}(n))$. Therefore, time average over codes is performed for approximation:

$$\Phi(\mathcal{R}_{\text{int}}(n)) \approx \sigma_v^2\mathbf{e}^H\frac{1}{N}\sum_{n=1}^{N}[\mathbf{H}(n)^H\mathbf{H}(n)]^{-1}\mathbf{e}. \tag{40}$$

To evaluate the perturbation term, that is, $\Psi(\mathbf{X})$, we first obtain $\delta\mathbf{f}(n)$ according to (15). Since perturbation in the signature matrix $\mathbf{H}(n)$ is given by

$$\delta\mathbf{H}(n) = [\mathbf{C}_1(n)\delta\mathbf{g}_1, \ldots, \mathbf{C}_J(n)\delta\mathbf{g}_J], \tag{41}$$

according to (5), noticing $\tilde{\mathbf{H}}(n) = \mathbf{H}(n) + \delta\mathbf{H}(n)$, expanding $[\tilde{\mathbf{H}}(n)^H\tilde{\mathbf{H}}(n)]^{-1}$ using Taylor series, and keeping only the first-order terms, we obtain

$$\begin{aligned}\delta\mathbf{f}(n) &\approx \mathbf{\Gamma}(n)\delta\mathbf{H}(n)(\mathbf{H}(n)^H\mathbf{H}(n))^{-1}\mathbf{e} \\ &\quad - (\mathbf{H}(n)^\dagger)^H\delta\mathbf{H}(n)^H(\mathbf{H}(n)^\dagger)^H\mathbf{e},\end{aligned} \tag{42}$$

where

$$\begin{aligned}\mathbf{\Gamma}(n) &= \mathbf{I} - \mathbf{H}(n)\mathbf{H}(n)^\dagger, \\ \mathbf{H}(n)^\dagger &= (\mathbf{H}(n)^H\mathbf{H}(n))^{-1}\mathbf{H}(n)^H.\end{aligned} \tag{43}$$

Then we obtain

$$\begin{aligned}\Psi(\mathbf{X}) \approx\ & E\Big\{\mathbf{e}^H[\mathbf{H}(n)^H\mathbf{H}(n)]^{-1}\delta\mathbf{H}(n)^H\mathbf{\Gamma}(n) \\ &\quad \times\mathbf{X}\mathbf{\Gamma}(n)\delta\mathbf{H}(n)[\mathbf{H}(n)^H\mathbf{H}(n)]^{-1}\mathbf{e}\Big\} \\ &- E\Big\{\mathbf{e}^H[\mathbf{H}(n)^H\mathbf{H}(n)]^{-1}\delta\mathbf{H}(n)^H\mathbf{\Gamma}(n) \\ &\quad \times\mathbf{X}(\mathbf{H}(n)^\dagger)^H\delta\mathbf{H}(n)^H(\mathbf{H}(n)^\dagger)^H\mathbf{e}\Big\} \\ &- E\Big\{\mathbf{e}^H\mathbf{H}(n)^\dagger\delta\mathbf{H}(n)\mathbf{H}(n)^\dagger\mathbf{X}\mathbf{\Gamma}(n) \\ &\quad \times\delta\mathbf{H}(n)[\mathbf{H}(n)^H\mathbf{H}(n)]^{-1}\mathbf{e}\Big\} \\ &+ E\Big\{\mathbf{e}^H\mathbf{H}(n)^\dagger\delta\mathbf{H}(n)\mathbf{H}(n)^\dagger\mathbf{X}(\mathbf{H}(n)^\dagger)^H \\ &\quad \times\delta\mathbf{H}(n)^H(\mathbf{H}(n)^\dagger)^H\mathbf{e}\Big\}.\end{aligned} \tag{44}$$

$\delta\mathbf{g}_j$ could be regarded as independent of $\mathbf{C}_j(n)$ for $j = 1, \ldots, J$, since the former depends on $\mathbf{A}_j$ which can be viewed as independent of $\mathbf{C}_j(n)$ when the number of samples used to estimate $\mathbf{A}_j$ is sufficiently large. As a result, the expectation in (44) will be performed in two steps: first with respect to $\delta\mathbf{g}_j$ while regarding each $\mathbf{C}_j(n)$ as a constant, and then with respect to $\mathbf{C}_j(n)$ for the result obtained in the first step. Similarly, ensemble average can be replaced by sample average for large N, resulting in the approximation of (44) by

$$\begin{aligned}&\Psi(\mathbf{X}) \\ &\approx \frac{1}{N}\sum_{n=1}^{N}\Big\{\mathbf{e}^H[\mathbf{H}(n)^H\mathbf{H}(n)]^{-1}\underline{E\{\delta\mathbf{H}(n)^H\mathbf{\Gamma}(n)\mathbf{X}\mathbf{\Gamma}(n)\delta\mathbf{H}(n)\}} \\ &\qquad \times[\mathbf{H}(n)^H\mathbf{H}(n)]^{-1}\mathbf{e} - \mathbf{e}^H[\mathbf{H}(n)^H\mathbf{H}(n)]^{-1} \\ &\qquad \times\underline{E\{\delta\mathbf{H}(n)^H\mathbf{\Gamma}(n)\mathbf{X}(\mathbf{H}(n)^\dagger)^H\delta\mathbf{H}(n)^H\}}[\mathbf{H}(n)^\dagger]^H\mathbf{e} \\ &\qquad - \mathbf{e}^H\mathbf{H}(n)^\dagger\underline{E\{\delta\mathbf{H}(n)\mathbf{H}(n)^\dagger\mathbf{X}\mathbf{\Gamma}(n)\delta\mathbf{H}(n)\}} \\ &\qquad \times[\mathbf{H}(n)^H\mathbf{H}(n)]^{-1}\mathbf{e} + \mathbf{e}^H\mathbf{H}(n)^\dagger \\ &\qquad \times\underline{E\Big\{\delta\mathbf{H}(n)\mathbf{H}(n)^\dagger\mathbf{X}(\mathbf{H}(n)^\dagger)^H\delta\mathbf{H}(n)^H\Big\}} \\ &\qquad \times(\mathbf{H}(n)^\dagger)^H\mathbf{e}\Big\},\end{aligned} \tag{45}$$

where each unperturbed quantity inside the expectation (underlined) terms is regarded as deterministic. Noticing that $\mathbf{\Gamma}(n)$ is orthogonal to $\mathcal{R}_1(n)$ and $\mathbf{H}(n)^\dagger\mathcal{R}_1(n)(\mathbf{H}(n)^\dagger)^H = \mathbf{e}\mathbf{e}^H$, the perturbation of the desired power is then simplified as

$$\Psi(\mathcal{R}_1(n)) = \mathbf{e}^H\mathbf{H}(n)^\dagger\underline{E\{\delta\mathbf{H}(n)\mathbf{e}\mathbf{e}^H\delta\mathbf{H}(n)^H\}}(\mathbf{H}(n)^\dagger)^H\mathbf{e}. \tag{46}$$

Computation of the underlined term is given by Appendix A. If we apply results there, (46) becomes

$$\begin{aligned}&\Psi(\mathcal{R}_1(n)) \\ &\quad \approx \frac{1}{N}\sum_{n=1}^{N}\mathbf{e}^H\mathbf{H}(n)^\dagger\mathbf{C}_1(n)E\{\delta\mathbf{g}_1\delta\mathbf{g}_1^H\}\mathbf{C}_1(n)^H(\mathbf{H}(n)^\dagger)^H\mathbf{e}.\end{aligned} \tag{47}$$

Similarly, if we decompose $\mathcal{R}_{\text{int}}(n) = \mathbf{H}(n)(\mathbf{I} - \mathbf{e}\mathbf{e}^H)\mathbf{H}(n)^H + \sigma_v^2\mathbf{I}$ and apply the orthogonality between $\mathbf{\Gamma}(n)$ and $\mathbf{H}(n)\mathbf{H}(n)^H$, the second and third terms in (45) for computing the perturbation of noise power disappear. Moreover, it can be shown that $\boldsymbol{\Gamma}(n)\mathcal{R}_{\text{int}}(n)\boldsymbol{\Gamma}(n) = \sigma_v^2\boldsymbol{\Gamma}(n)$, and

$$\mathbf{H}(n)^{\dagger}\mathcal{R}_{\text{int}}(n)(\mathbf{H}(n)^{\dagger})^H = \mathbf{I} - \mathbf{e}\mathbf{e}^H + \sigma_v^2[\mathbf{H}(n)^H\mathbf{H}(n)]^{-1}. \tag{48}$$

Applying the above results and using Appendix A, we obtain

$$\begin{aligned}\Psi(\mathcal{R}_{\text{int}}(n)) &\approx \frac{1}{N}\sum_{n=1}^{N}\Big\{\sigma_v^2\mathbf{e}^H[\mathbf{H}(n)^H\mathbf{H}(n)]^{-1}\underline{E\{\delta\mathbf{H}(n)^H\boldsymbol{\Gamma}(n)\delta\mathbf{H}(n)\}}\\ &\quad\times[\mathbf{H}(n)^H\mathbf{H}(n)]^{-1}\mathbf{e} + \sigma_v^2\mathbf{e}^H\mathbf{H}(n)^{\dagger}\\ &\quad\times\underline{E\Big\{\delta\mathbf{H}(n)[\mathbf{H}(n)^H\mathbf{H}(n)]^{-1}\delta\mathbf{H}(n)^H\Big\}}(\mathbf{H}(n)^{\dagger})^H\mathbf{e}\\ &\quad+\mathbf{e}^H\mathbf{H}(n)^{\dagger}\sum_{j=2}^{J}\mathbf{C}_j(n)E\Big\{\delta\mathbf{g}_j\delta\mathbf{g}_j^H\Big\}\\ &\quad\times\mathbf{C}_j(n)^H(\mathbf{H}(n)^{\dagger})^H\mathbf{e}\Big\}.\end{aligned} \tag{49}$$

Since the underlined terms computed by Appendix A are at the order of $O(\sigma_v^2/N)$, the first and second terms in (49) are at the order of $O(\sigma_v^4/N)$ and can be omitted in the presence of very small noise power. Then (49) reduces to

$$\Psi(\mathcal{R}_{\text{int}}(n)) \approx \frac{1}{N}\sum_{n=1}^{N}\mathbf{e}^H\mathbf{H}(n)^{\dagger}\left[\sum_{j=2}^{J}\mathbf{C}_j(n)E\{\delta\mathbf{g}_j\delta\mathbf{g}_j^H\}\mathbf{C}_j(n)^H\right](\mathbf{H}(n)^{\dagger})^H\mathbf{e}. \tag{50}$$

To summarize, the perturbed SINR can be predicted based on (38), where unperturbed terms are computed by (39) and (40), and perturbed terms are computed by (47) and (50). According to (35), (47) and (50), both $\Psi(\mathcal{R}_1(n))$ and $\Psi(\mathcal{R}_{\text{int}}(n))$ are approximated at the order of $O(\sigma_v^2/N)$, which is inversely proportional to N and proportional to σ_v^2. Moreover, it is observed that $\Psi(\mathcal{R}_1(n))$ is caused by the desired user's channel perturbation, while $\Psi(\mathcal{R}_{\text{int}}(n))$ is caused by all interfering users' channel perturbations.

4.2.2. SINR of the MMSE receiver

Substituting (17) for the receiver, the unperturbed term can be shown to be $\Phi(\mathbf{X}) = \mathbf{g}_1^H E\{\mathbf{C}_1(n)^H\mathcal{R}(n)^{-1}\mathbf{X}\mathcal{R}(n)^{-1}\mathbf{C}_1(n)\}\mathbf{g}_1$, which is unobtainable in a closed form due to the inverse of $\mathcal{R}(n)$. Therefore, approximation of expectation by time average is applied here as for the ZF receiver,

$$\Phi(\mathbf{X}) = \mathbf{g}_1^H\left[\frac{1}{N}\sum_{n=1}^{N}\mathbf{C}_1(n)^H\mathcal{R}(n)^{-1}\mathbf{X}\mathcal{R}(n)^{-1}\mathbf{C}_1(n)\right]\mathbf{g}_1. \tag{51}$$

To evaluate the perturbed term, perturbation of the receiver is necessary. Obtaining $\delta\mathcal{R}(n)$ from (18) and then using (17), one can verify the perturbation of the MMSE receiver by

$$\begin{aligned}\delta\mathbf{f}(n) &= \mathcal{R}(n)^{-1}\delta\mathbf{H}\mathbf{e}\\ &\quad-\mathcal{R}(n)^{-1}[\delta\mathbf{H}(n)\mathbf{H}(n)^H+\mathbf{H}(n)\delta\mathbf{H}(n)^H+\delta\sigma_v^2\mathbf{I}]\mathbf{f}(n).\end{aligned} \tag{52}$$

Correspondingly, $\Psi(\mathbf{X})$ can be approximated by two-step expectations in the same way as for the ZF receiver as

$$\begin{aligned}\Psi(\mathbf{X}) = \frac{1}{N}\sum_{n=1}^{N}\Big\{&\mathbf{e}^H\underline{E\{\delta\mathbf{H}(n)^H\mathcal{R}(n)^{-1}\mathbf{X}\mathcal{R}(n)^{-1}\delta\mathbf{H}(n)\}}\mathbf{e}\\ &+\mathbf{f}(n)^H\mathbf{H}(n)\underline{E\{\delta\mathbf{H}(n)^H\mathcal{R}(n)^{-1}\mathbf{X}\mathcal{R}(n)^{-1}\delta\mathbf{H}(n)\}}\mathbf{H}(n)^H\mathbf{f}(n)\\ &+\mathbf{f}(n)^H\underline{E\{\delta\mathbf{H}(n)\mathbf{H}(n)^H\mathcal{R}(n)^{-1}\mathbf{X}\mathcal{R}(n)^{-1}\mathbf{H}(n)\delta\mathbf{H}(n)^H\}}\mathbf{f}(n)\\ &+\mathbf{f}(n)^H\mathcal{R}(n)^{-1}\mathbf{X}\mathcal{R}(n)^{-1}\mathbf{f}(n)\underline{E\{\delta\sigma_v^2\delta\sigma_v^2\}}\\ &+\underbrace{\mathbf{f}(n)^H\underline{E\{\delta\mathbf{H}(n)\mathbf{H}(n)^H\mathcal{R}(n)^{-1}\mathbf{X}\mathcal{R}(n)^{-1}\delta\mathbf{H}(n)\}}\mathbf{H}(n)\mathbf{f}(n)}_{a_1}+a_1^*\\ &+\underbrace{\mathbf{f}(n)^H\mathcal{R}(n)^{-1}\mathbf{X}\mathcal{R}(n)^{-1}\underline{E\{\delta\mathbf{H}(n)\delta\sigma_v^2\}}\mathbf{H}(n)^H\mathbf{f}(n)}_{a_2}+a_2^*\\ &+\underbrace{\mathbf{f}(n)^H\underline{E\{\delta\mathbf{H}(n)\delta\sigma_v^2\}}\mathbf{H}(n)^H\mathcal{R}(n)^{-1}\mathbf{X}\mathcal{R}(n)^{-1}\mathbf{f}(n)}_{a_3}\,a_3^*\\ &-\underbrace{\mathbf{e}^H\underline{E\{\delta\mathbf{H}(n)^H\mathcal{R}(n)^{-1}\mathbf{X}\mathcal{R}(n)^{-1}\delta\mathbf{H}(n)\}}\mathbf{H}(n)^H\mathbf{f}(n)}_{a_4}-a_4^*\\ &-\underbrace{\mathbf{e}^H\underline{E\{\delta\mathbf{H}(n)^H\mathcal{R}(n)^{-1}\mathbf{X}\mathcal{R}(n)^{-1}\mathbf{H}(n)\delta\mathbf{H}(n)^H\}}\mathbf{f}(n)}_{a_5}-a_5^*\\ &-\underbrace{\mathbf{e}^H\underline{E\{\delta\mathbf{H}(n)^H\delta\sigma_v^2\}}\mathcal{R}(n)^{-1}\mathbf{X}\mathcal{R}(n)^{-1}\mathbf{f}(n)}_{a_6}-a_6^*\Big\},\end{aligned} \tag{53}$$

where for shorter expression a_1 up to a_6 have been defined. All underlined terms follow general forms derived in Appendix A except $E\{\delta\sigma_v^2\delta\sigma_v^2\}$ and $E\{\delta\mathbf{H}(n)\delta\sigma_v^2\}$ which are derived in Appendix B. Since each expectation term is shown to be at the order of $O(\sigma_v^2/N)$, the final perturbations for both the desired signal and interference plus noise are thus all at the order of $O(\sigma_v^2/N)$. To summarize, the perturbed SINR for the MMSE receiver can be predicted based on (38), where unperturbed terms are computed by (51), and perturbed terms are computed by (53), with $\mathbf{X}$ replaced by $\mathcal{R}_1(n)$ or $\mathcal{R}_{\text{int}}(n)$ respectively.

4.2.3. *SINR of the RAKE receiver*

Closed-form SINR of the RAKE receiver will be derived here due to the simplicity of the receiver. We first rewrite the signature matrix as the following:

$$\begin{aligned}\mathbf{H}(n) &= [\mathbf{C}_1(n)\mathbf{g}_1, \ldots, \mathbf{C}_J(n)\mathbf{g}_J] \\ &= [\mathbf{G}_1\mathbf{c}_{1,n}, \ldots, \mathbf{G}_J\mathbf{c}_{J,n}],\end{aligned} \tag{54}$$

where

$$\begin{aligned}\mathbf{c}_{j,n} &= [c_{j,n}(0), \ldots, c_{j,n}(P-1)]^T, \\ \mathbf{G}_j &= [\bar{\mathbf{G}}_j]_{\mu+1-d_j:P-d_j,1:P} = \mathcal{S}_j\bar{\mathbf{G}}_j,\end{aligned} \tag{55}$$

$\bar{\mathbf{G}}_j$ with dimensions of $(P+q)\times P$ is defined as

$$\bar{\mathbf{G}}_j(n) = \begin{bmatrix} g_j(0) & & \mathbf{0} \\ \vdots & \ddots & g_j(0) \\ g_j(q+1) & & \vdots \\ \mathbf{0} & \ddots & g_j(q+1) \end{bmatrix}, \tag{56}$$

and $\mathcal{S}_j = [\mathbf{0}_{(p-\mu)\times(\mu-d_j)}; \mathbf{I}_{(P-\mu)\times(P-\mu)}; \mathbf{0}_{(P-\mu)\times(q+d_j)}]$. Then all subsequent analysis will be based on (54). First, the unperturbed desired power is derived after replacing the RAKE receiver with (20) and $\mathbf{h}_1(n)$ with $\mathbf{G}_1\mathbf{c}_{1,n}$, and applying trace, vec, and kronecker product operations [33]. These steps are summarized by

$$\begin{aligned}&\Phi(\mathcal{R}_1(n)) \\ &\quad= E\{\mathbf{c}_{1,n}^H\mathbf{G}_1^H\mathbf{G}_1\mathbf{c}_{1,n}\mathbf{c}_{1,n}^H\mathbf{G}_1^H\mathbf{G}_1\mathbf{c}_{1,n}\} \\ &\quad= E\{\operatorname{tr}\{\mathbf{G}_1^H\mathbf{G}_1\mathbf{c}_{1,n}\mathbf{c}_{1,n}^H\mathbf{G}_1^H\mathbf{G}_1\mathbf{c}_{1,n}\mathbf{c}_{1,n}^H\}\} \\ &\quad= \operatorname{vec}^H(\mathbf{G}_1^H\mathbf{G}_1)\operatorname{vec}(E\{\mathbf{c}_{1,n}\mathbf{c}_{1,n}^H\mathbf{G}_1^H\mathbf{G}_1\mathbf{c}_{1,n}\mathbf{c}_{1,n}^H\}) \\ &\quad= \operatorname{vec}^H(\mathbf{G}_1^H\mathbf{G}_1)\underline{E\{(\mathbf{c}_{1,n}^*\otimes\mathbf{c}_{1,n})(\mathbf{c}_{1,n}^T\otimes\mathbf{c}_{1,n}^H)\}}\operatorname{vec}(\mathbf{G}_1^H\mathbf{G}_1).\end{aligned} \tag{57}$$

The underlined term is given by [34] for real codes as

$$\begin{aligned}&E\{(\mathbf{c}_{1,n}^*\otimes\mathbf{c}_{1,n})(\mathbf{c}_{1,n}^T\otimes\mathbf{c}_{1,n}^H)\} \\ &\quad= \kappa_{4c}\mathbf{X}_1 + \sigma_c^4\mathbf{X}_2 + \sigma_c^4\operatorname{vec}(\mathbf{I})\operatorname{vec}(\mathbf{I})^H + \sigma_c^4\mathbf{I},\end{aligned} \tag{58}$$

and for complex codes as

$$\begin{aligned}&E\{(\mathbf{c}_{1,n}^*\otimes\mathbf{c}_{1,n})(\mathbf{c}_{1,n}^T\otimes\mathbf{c}_{1,n}^H)\} \\ &\quad= \kappa_{4c}\mathbf{X}_1 + \sigma_c^4\operatorname{vec}(\mathbf{I})\operatorname{vec}(\mathbf{I})^H + \sigma_c^4\mathbf{I},\end{aligned} \tag{59}$$

where κ_{4c} is the fourth-order cumulant of the spreading codes, $\sigma_c^2 = E\{c_j(n)^2\}$,

$$\begin{aligned}\mathbf{X}_1 &= \operatorname{diag}\{\mathbf{a}_1\mathbf{a}_1^T, \ldots, \mathbf{a}_P\mathbf{a}_P^T\}, \\ \mathbf{a}_i &= \Big[\underbrace{0,\ldots,0}_{i-1},1,\underbrace{0,\ldots,0}_{P-i}\Big]^T,\end{aligned} \tag{60}$$

and $\mathbf{X}_2$ is partitioned into $P\times P$ subblocks with the (i,j)th subblock $\mathbf{a}_j\mathbf{a}_i^T$.

Similarly, the unperturbed interference-plus-noise power can be computed as the following after noticing that $\mathbf{c}_{1,n}$ is independent of $\mathbf{c}_{j,n}$, and $E\{\mathbf{c}_{j,n}\mathbf{c}_{j,n}^H\} = \sigma_c^2\mathbf{I}$ for $j = 1,\ldots,J$:

$$\begin{aligned}\Phi(\mathcal{R}_{\text{int}}(n)) &= \sum_{j=2}^{J} E\{\mathbf{c}_{1,n}^H\mathbf{G}_1^H\mathbf{G}_j\mathbf{c}_{j,n}\mathbf{c}_{j,n}^H\mathbf{G}_j^H\mathbf{G}_1\mathbf{c}_{1,n}\} \\ &\quad + E\{\sigma_v^2\mathbf{c}_{1,n}^H\mathbf{G}_1^H\mathbf{G}_1\mathbf{c}_{1,n}\} \\ &= \sigma_c^4\sum_{j=2}^{J}\operatorname{tr}\{\mathbf{G}_1^H\mathbf{G}_j\mathbf{G}_j^H\mathbf{G}_1\} + \sigma_c^2\sigma_v^2\operatorname{tr}\{\mathbf{G}_1^H\mathbf{G}_1\}.\end{aligned} \tag{61}$$

We now proceed to evaluate the perturbed term $\Psi(\mathbf{X})$ by first obtaining the perturbation of the RAKE receiver. According to (20) and (54), we have $\delta\mathbf{f}(n) = \delta\mathbf{G}_1\mathbf{c}_{1,n}$. Noticing (56), $\delta\mathbf{G}_1$ is calculated as

$$\delta\mathbf{G}_1 = [\mathbf{B}_0\delta\mathbf{g}_1, \ldots, \mathbf{B}_{P-1}\delta\mathbf{g}_1], \tag{62}$$

where

$$\mathbf{B}_j \triangleq \mathcal{S}_1\mathbf{\Omega}^j\mathbf{B}, \qquad \mathbf{B} = \begin{bmatrix}\mathbf{I}_{M(q+1)} \\ \mathbf{0}\end{bmatrix}, \tag{63}$$

$\mathbf{\Omega}$ is a shifting matrix with all 1's in the first subdiagonal, and $\mathbf{\Omega}^0$ is defined as an identity matrix for convenience. Based on the above results, following the same steps for deriving (57), and noticing that $\delta\mathbf{G}_1$ is independent of codes, then the perturbed signal power is first obtained as

$$\begin{aligned}\Psi(\mathcal{R}_1(n)) &= E\{\mathbf{c}_{1,n}^H\delta\mathbf{G}_1^H\mathbf{G}_1\mathbf{c}_{1,n}\mathbf{c}_{1,n}^H\mathbf{G}_1^H\delta\mathbf{G}_1\mathbf{c}_{1,n}\} \\ &= \operatorname{tr}\Big\{\underline{E\{(\mathbf{c}_{1,n}^*\otimes\mathbf{c}_{1,n})(\mathbf{c}_{1,n}^T\otimes\mathbf{c}_{1,n}^H)\}} \\ &\qquad \times\underline{E\{\operatorname{vec}(\mathbf{G}_1^H\delta\mathbf{G}_1)\operatorname{vec}^H(\mathbf{G}_1^H\delta\mathbf{G}_1)\}}\Big\},\end{aligned} \tag{64}$$

where the first underlined term is given by (58) or (59) as explained before, and the second underlined term is given in Appendix C. Finally, the perturbed interference-plus-noise power is computed by

$$\begin{aligned}\Psi(\mathcal{R}_{\text{int}}(n)) &= \sum_{j=2}^{J} E\{\mathbf{c}_{1,n}^H \delta\mathbf{G}_1^H \mathbf{G}_j \mathbf{c}_{j,n} \mathbf{c}_{j,n}^H \mathbf{G}_j^H \delta\mathbf{G}_1 \mathbf{c}_{1,n}\} \\ &\quad + E\{\sigma_v^2 \mathbf{c}_{1,n}^H \delta\mathbf{G}_1^H \delta\mathbf{G}_1 \mathbf{c}_{1,n}\} \\ &= \sigma_c^4 \sum_{j=2}^{J} \operatorname{tr}\left\{\mathbf{G}_j \mathbf{G}_j^H \underline{E\{\delta\mathbf{G}_1 \delta\mathbf{G}_1^H\}}\right\} \\ &\quad + \sigma_c^2 \sigma_v^2 \operatorname{tr}\left\{\underline{E\{\delta\mathbf{G}_1 \delta\mathbf{G}_1^H\}}\right\}\end{aligned} \tag{65}$$

with $E\{\delta\mathbf{G}_1 \delta\mathbf{G}_1^H\}$ given in Appendix C.

To summarize, the perturbed SINR for the RAKE receiver can be obtained analytically based on (38), where the unperturbed signal power of the RAKE receiver, given by (57), depends on the desired user's channel conditions as well as the second- and fourth-order statistics of its own code sequence, while the unperturbed interference plus noise power given by (61) depends on the second-order moment of the codes, and cross channel conditions between each interfering user and the desired user. Finally, perturbed signal power and interference are given by (64) and (65), respectively. From them and Appendix C, we can conclude that both the desired user's power and interference-plus-noise power are perturbed at the order of $O(\sigma_v^2/N)$.

4.3. BER performance of different receivers

For each receiver, once its output SINR is evaluated, BER can be obtained by assuming that the interference is Gaussian distributed. This may not necessarily be correct, but this approximation has been shown to be relatively good [8, 35], especially when the number of interfering users is large. The BER for BPSK information symbol is

$$\text{BER} = Q\left(\sqrt{\widetilde{\text{SINR}}}\right), \tag{66}$$

where $Q(x) = (1/\sqrt{2\pi}) \int_x^\infty e^{-t^2/2} dt$.

5. SIMULATION EXAMPLES

In this section, we verify our performance analysis by simulation examples. The average channel MSE, and the average SINR and BER of each receiver over 100 independent realizations are used as performance indicators. All analytical results are obtained based on the results in Section 4. As theoretical limits, SINRs and BERs of all ideal linear receivers are also presented. Those ideal receivers are constructed from all users' perfect channel vectors as well as noise power. In the simulation setup, we consider an uplink CDMA system, where each user transmit BPSK signals through a respective multipath channel, whose parameters are randomly generated with equal power for each path according to Gaussian distribution. All users are assumed to have multipath delay spread as $q = 2$. All users are assumed to have the same transmission power, if not stated otherwise. The spreading factor is set to be 32 for all simulations. To validate our analysis, we study joint performance of the channel estimator and detectors under the effect of different system parameters, such as different data length N, various input signal-to-noise ratio SNR, varied number of users J, and signal to each interfering user's power ratio (SIR).

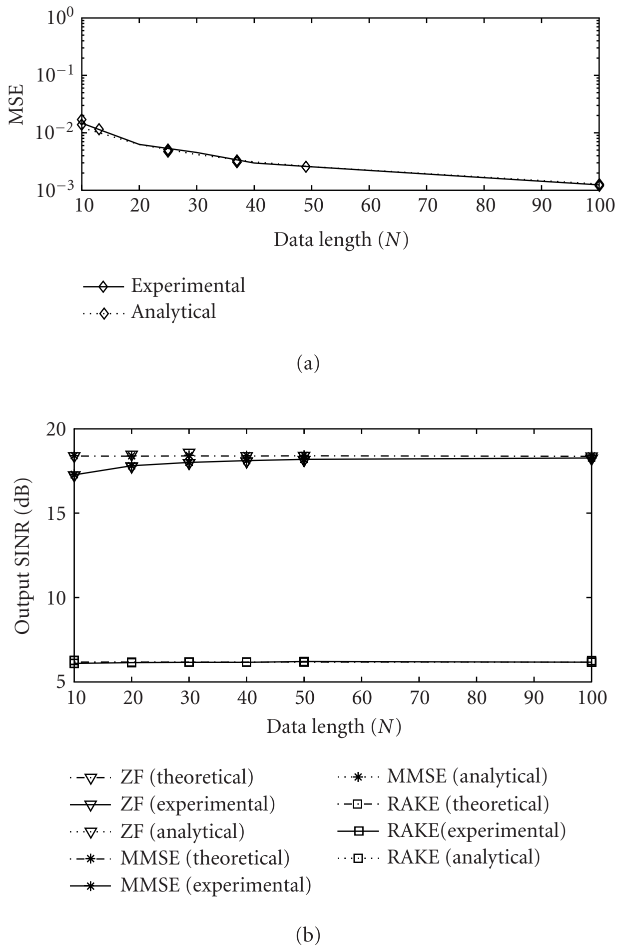


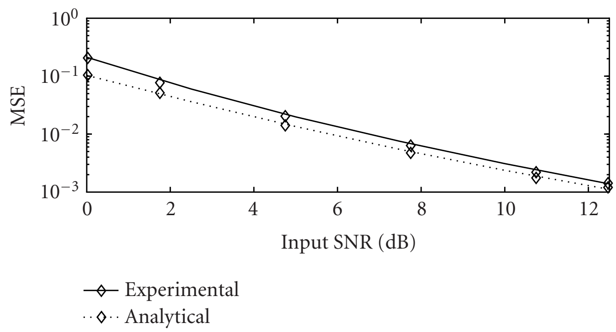
Figure 1: Effect of data length. (a) MSE. (b) Output SINR.

Effect of N*,* $J = 8$*, SNR* $= 20$ *dB, SIR* $= 0$ *dB*

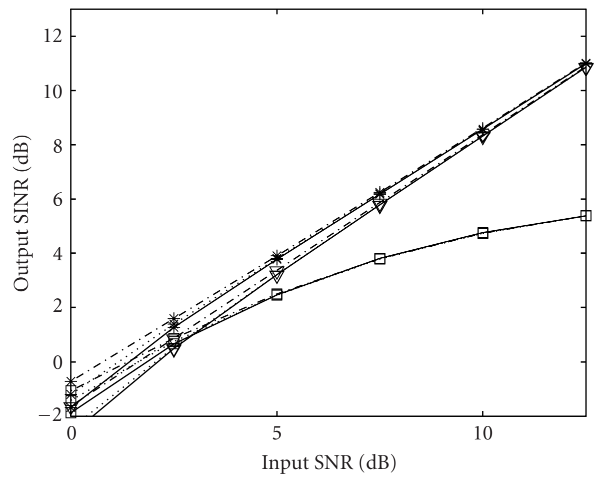
MSEs of channel estimation over different N are plotted in Figure 1a. Experimental and analytical curves are highly consistent for all examined N, and as expected, the MSE level decreases when N becomes large. SINRs of all receivers are illuminated in Figure 1b, where experimental, analytical, and theoretical SINRs start to overlap from $N = 50$, indicating that the proposed receivers require very small data size to achieve their theoretical limit at 20 dB SNR environment. On the other hand, the ZF and MMSE receivers show much better performance than the RAKE receiver.

Effect of SNR, $J = 8$*,* $N = 500$*, SIR* $= 0$ *dB*

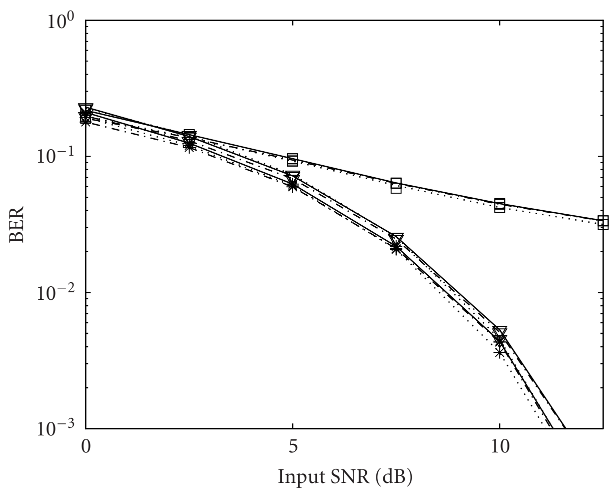
We consider various SNR from 0 dB to 12.5 dB at a step of 2.5 dB. Figure 2a illustrates the channel estimation MSE, which monotonically decreases as SNR increases. It is also



(a)



(b)



(c)

Figure 2: Effect of SNR. (a) MSE. (b) Output SINR. (c) BER. The legend of (b) and (c) is the same as Figure 1b.

seen that the experimental MSE converges to its analytical value at large SNR. Slight differences are caused by the first-order approximation error. Figures 2b and 2c plot SINRs and BERs of each receiver. The MMSE receiver shows slightly better performance than the ZF receiver. Both of them are significantly superior to the RAKE receiver. The convergence between the experimental and analytical values can also be observed for each receiver, indicating that our analytical SINR and BER can serve as good performance predictors. Moreover, the experimental values are found to be very close to their theoretical ones, showing that each proposed receiver constructed from the proposed channel estimator and estimated noise behaves as well as its ideal counterpart.

Effect of J, $N = 500$, SNR $= 10$ dB, SIR $= 0$ dB

The number of equally powered users in the system varies from 2 to 8. Figures 3a to 3c illustrate the MSE, SINR, and BER performance of the proposed methods, respectively. It can be observed that the MSE slightly degrades when J increases. Although an explicit relationship between the MSE and J is not obtained in (35), J's effect can still be found to be caused by the matrix $\mathbf{A}_j^{1/2}$. Due to the same reason, the SINRs and BERs of the ZF and MMSE receivers also slightly degrade for large J. The RAKE receiver's performance degrades drastically, which can easily be explained from our analysis that the interference-plus-noise power in (61) significantly increases with J. Again, for either MSE, SINR, or BER, experimental values are highly consistent with their corresponding analytical ones for large J where Gaussian assumption for interference signals is well suitable. Each receiver's experimental SINRs and BERs are overlapped with their theoretical values.

Near-far effect, $N = 500$, SNR $= 10$ dB and $J = 8$

The power ratio of each interfering user over the desired user ranges from 0 dB to 10 dB at a step of 2 dB. Analytical and experimental MSEs plotted in Figure 4a are found to be a constant, indicating that channel estimation MSE is not affected by interference power. This conclusion complies with (35), where $\mathbf{A}_j$, as the only term containing interfering users' effect, is independent of interfering user's power. The near-far resistance of the channel estimator can be accredited to the code correlation operation which significantly removes MUI in the partial data covariance matrix. Similar conclusion can be drawn to SINRs and BERs of both ZF and MMSE receivers since they explicitly remove MUI. As expected, RAKE receiver's performance degrades significantly when interference power increases.

6. CONCLUSION

We derived a closed form covariance for channel estimation error when channel is estimated using code decorrelation and subspace techniques as [10, 20, 21] based on finite data samples in long-code CDMA uplink. The performance of three symbol-level linear receivers constructed from the estimated channel, known as ZF, MMSE, and RAKE receivers, is also studied. Simulation examples involving different communication environments are provided and demonstrate high consistency between our analysis and experimental results.

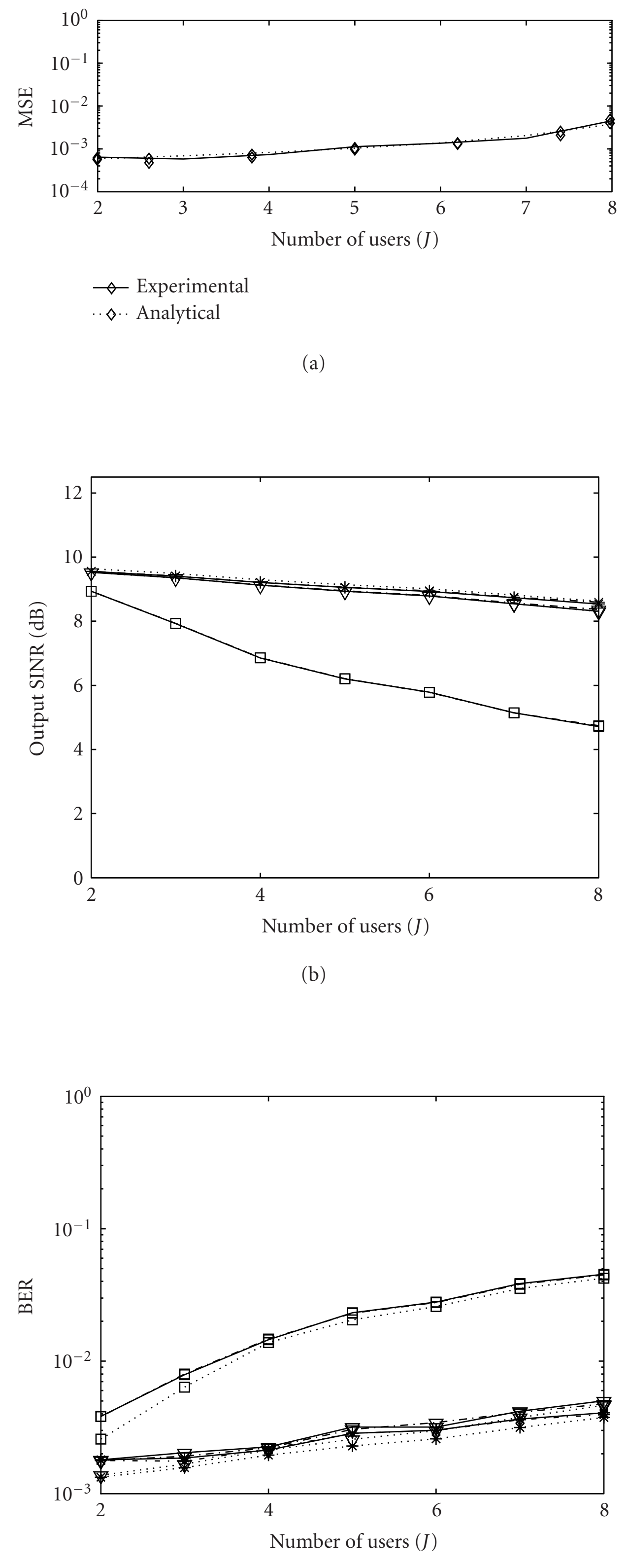


Figure 3: Effect of number of users. (a) MSE. (b) Output SINR. (c) BER. The legend of (b) and (c) is the same as Figure 1b.

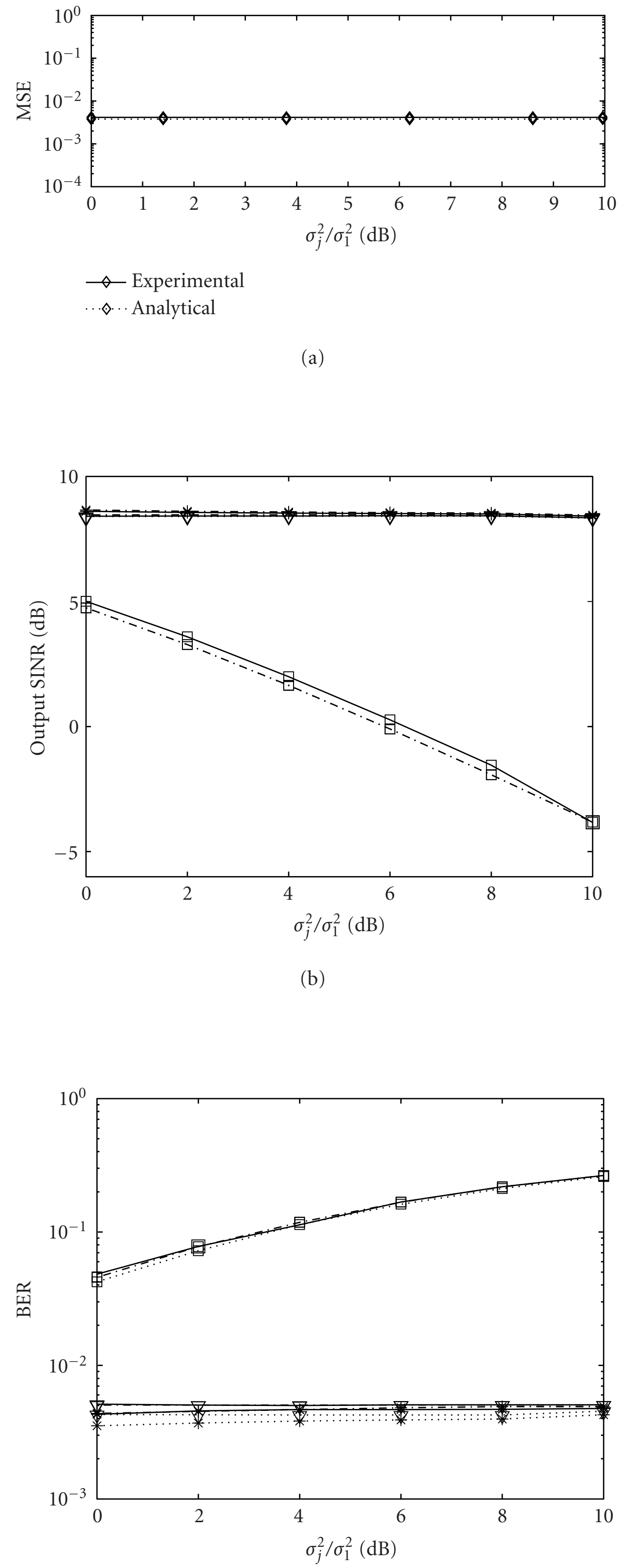


Figure 4: Near-far effect. (a) MSE. (b) Output SINR. (c) BER. The legend of (b) and (c) is the same as Figure 1b.

APPENDICES

A. DERIVATION OF EXPECTATION QUANTITIES FOR THE ZF RECEIVER

A.1. Derivation of $E\{\delta\mathbf{H}(n)^H\mathbf{Z}\delta\mathbf{H}(n)\}$

In this case, $\mathbf{Z}$ is a deterministic matrix of dimensions $(P-\mu)$ by $(P-\mu)$. By (41), the (i,j)th element of $E\{\delta\mathbf{H}(n)^H\mathbf{Z}\delta\mathbf{H}(n)\}$ is given by $E\{\delta\mathbf{g}_i^H\mathbf{C}_i(n)^H\mathbf{Z}\mathbf{C}_j(n)\delta\mathbf{g}_j\}$. The diagonal term (i.e., $i=j$) can readily be obtained as follows after replacing $E\{\delta\mathbf{g}_j\delta\mathbf{g}_j^H\}$ by (35):

$$\begin{aligned} &E\{\delta\mathbf{g}_j^H\mathbf{C}_j(n)^H\mathbf{Z}\mathbf{C}_j(n)\delta\mathbf{g}_j\} \\ &= \operatorname{tr}\{\mathbf{C}_j(n)^H\mathbf{Z}\mathbf{C}_j(n)E\{\delta\mathbf{g}_j\delta\mathbf{g}_j^H\}\} \\ &\approx \frac{\sigma_v^2}{N}\operatorname{tr}\left\{\mathbf{C}_j(n)^H\mathbf{Z}\mathbf{C}_j(n)\mathbf{\Pi}_{\mathbf{g}_j}^\perp\mathbf{A}_j^{1/2}\mathbf{U}_n^j(\mathbf{U}_n^j)^H\mathbf{A}_j^{1/2}\mathbf{\Pi}_{\mathbf{g}_j}^\perp\right\}\end{aligned} \tag{A.1}$$

which is clearly at the order of $O(\sigma_v^2/N)$. On the other hand, off-diagonal terms $E\{\delta\mathbf{g}_i^H\mathbf{C}_i(n)^H\mathbf{Z}\mathbf{C}_j(n)\delta\mathbf{g}_j\} = \operatorname{tr}(\mathbf{C}_i(n)^H\mathbf{Z}\mathbf{C}_j(n)E\{\delta\mathbf{g}_j\delta\mathbf{g}_i^H\})$ for $i\neq j$ depend on $E\{\delta\mathbf{g}_j\delta\mathbf{g}_i^H\}$ and will be shown next to be at the order of $O(\sigma_v^4/N)$ and thus are negligible.

Replacing $\delta\mathbf{g}_j$ by (29), the covariance between different channel perturbations is computed as

$$\begin{aligned} &E\{\delta\mathbf{g}_j\delta\mathbf{g}_i^H\} \\ &= \frac{1}{\gamma_i\gamma_j}\mathbf{\Pi}_{\mathbf{g}_j}^\perp\mathbf{A}_j^{1/2}\mathbf{U}_n^j(\mathbf{U}_n^j)^H E\{\delta\mathbf{R}_j\boldsymbol{\chi}_j\boldsymbol{\chi}_i^H\delta\mathbf{R}_i\}\mathbf{U}_n^i(\mathbf{U}_n^i)^H\mathbf{A}_i^{1/2}\mathbf{\Pi}_{\mathbf{g}_i}^\perp\end{aligned} \tag{A.2}$$

which depends on $E\{\delta\mathbf{R}_j\boldsymbol{\chi}_j\boldsymbol{\chi}_i^H\delta\mathbf{R}_i\}$. Following similar steps in [32], one can verify that

$$E\{\delta\mathbf{R}_j\mathbf{D}\delta\mathbf{R}_i\} = \frac{1}{N}[\mathbf{E}\{\nu_j(n)\nu_j(n)^H\mathbf{D}\nu_i(n)\nu_i(n)^H\} - \sigma_v^4\mathbf{D}]. \tag{A.3}$$

Since $\nu_j(n) = \mathbf{A}_j^{-1/2}\mathbf{S}_j^H\mathcal{C}(n)^\dagger\mathbf{v}(n)$ with $\mathbf{v}(n)$ independent of $\mathcal{C}(n)$, applying the results in [34] for $E\{\mathbf{v}(n)\mathbf{v}(n)^H\mathbf{D}\mathbf{v}(n)\mathbf{v}(n)^H\}$, it is found that $E\{\mathbf{v}(n)\mathbf{v}(n)^H\mathbf{D}\mathbf{v}(n)\mathbf{v}(n)^H\}$ can be approximated by $O(\sigma_v^4)$, causing $E\{\delta\mathbf{R}_j\mathbf{D}\delta\mathbf{R}_i\}$ in (A.3) at the order of $O(\sigma_v^4/N)$. As a result, $E\{\delta\mathbf{g}_i\delta\mathbf{g}_j^H\}$ in (A.2) can be approximated at most by the order of $O(\sigma_v^4/N)$, making all off-diagonal terms negligible.

To summarize, $E\{\delta\mathbf{H}(n)^H\mathbf{Z}\delta\mathbf{H}(n)\}$ can be approximated by a diagonal matrix with its (i,i)th element given by (A.1), which is at the order of $O(\sigma_v^2/N)$.

A.2. Derivation of $E\{\delta\mathbf{H}(n)\mathbf{Z}\delta\mathbf{H}(n)^H\}$

In this case, $\mathbf{Z}$ is a deterministic matrix of dimensions J by J. According to (41), we immediately have

$$\begin{aligned} E\{\delta\mathbf{H}(n)\mathbf{Z}\delta\mathbf{H}(n)^H\} &= \sum_{j=1}^{J} z_{j,j}\mathbf{C}_j(n)E\{\delta\mathbf{g}_j\delta\mathbf{g}_j^H\}\mathbf{C}_j(n)^H \\ &+ \sum_{i,j=1,\, i\neq j}^{J} z_{i,j}\mathbf{C}_i(n)E\{\delta\mathbf{g}_i\delta\mathbf{g}_j^H\}\mathbf{C}_j(n)^H,\end{aligned} \tag{A.4}$$

where $z_{i,j}$ denotes the (i,j)th entry of $\mathbf{Z}$. According to our previous analysis, the first term on the right-hand side of (A.4) involves the covariance of the perturbation between the same channel, which is at the order of $O(\sigma_v^2/N)$, while the second term involves cross covariance between two different channel perturbations, which is at the order of $O(\sigma_v^4/N)$, therefore, (A.4) can be simplified to (A.5), after applying (35), as follows:

$$\begin{aligned} &E\{\delta\mathbf{H}(n)\mathbf{Z}\delta\mathbf{H}(n)^H\} \\ &= \frac{\sigma_v^2}{N}\sum_{j=1}^{J} z_{j,j}\mathbf{C}_j(n)\mathbf{\Pi}_{\mathbf{g}_j}^\perp\mathbf{A}_j^{1/2}\mathbf{U}_n^j(\mathbf{U}_n^j)^H\mathbf{A}_j^{1/2}\mathbf{\Pi}_{\mathbf{g}_j}^\perp\mathbf{C}_j(n)^H.\end{aligned} \tag{A.5}$$

A.3. Derivation of $E\{\delta\mathbf{H}(n)\mathbf{Z}\delta\mathbf{H}(n)\}$ and $E\{\delta\mathbf{H}(n)^H\mathbf{Z}^H\delta\mathbf{H}(n)^H\}$

Rewriting (41) as $\delta\mathbf{H} = \mathcal{C}(n)\operatorname{diag}\{\delta\mathbf{g}_1,\dots,\delta\mathbf{g}_J\}$, then

$$\begin{aligned} &E\{\delta\mathbf{H}(n)\mathbf{Z}\delta\mathbf{H}(n)\} \\ &= \mathcal{C}(n)E\{\operatorname{diag}\{\delta\mathbf{g}_1,\dots,\delta\mathbf{g}_J\}\mathbf{Z}\mathcal{C}(n)\operatorname{diag}\{\delta\mathbf{g}_1,\dots,\delta\mathbf{g}_J\}\}.\end{aligned} \tag{A.6}$$

In this case, the matrix $\mathbf{Z}$ should have dimensions of $J\times(P-\mu)$, and $\mathbf{Z}\mathcal{C}(n)$ has dimensions of $J\times J(q+1)$. Partitioning each row of $\mathbf{Z}\mathcal{C}(n)$ into J subvectors of equal length of $q+1$ elements, and denoting the jth subvector at the ith row as $\mathbf{z}_{ij}^H$, we have

$$\begin{aligned} &E\{\delta\mathbf{H}(n)\mathbf{Z}\delta\mathbf{H}(n)\} \\ &= \mathcal{C}(n)E\left\{\operatorname{diag}\{\delta\mathbf{g}_1,\dots,\delta\mathbf{g}_J\}\begin{pmatrix}\mathbf{z}_{11}^H & \cdots & \mathbf{z}_{1J}^H\\ \vdots & \vdots & \vdots\\ \mathbf{z}_{J1}^H & \cdots & \mathbf{z}_{JJ}^H\end{pmatrix}\right. \\ &\qquad\left.\times\operatorname{diag}\{\delta\mathbf{g}_1,\dots,\delta\mathbf{g}_J\}\right\} \\ &= \mathcal{C}(n)\begin{pmatrix}E\{\delta\mathbf{g}_1\mathbf{g}_1^T\}\mathbf{z}_{11}^* & \cdots & E\{\delta\mathbf{g}_J\mathbf{g}_1^T\}\mathbf{z}_{1J}^*\\ \vdots & \vdots & \vdots\\ E\{\delta\mathbf{g}_1\mathbf{g}_J^T\}\mathbf{z}_{J1}^* & \cdots & E\{\delta\mathbf{g}_J\mathbf{g}_J^T\}\mathbf{z}_{JJ}^*\end{pmatrix}.\end{aligned} \tag{A.7}$$

For a complex system, (A.7) clearly becomes a zero matrix. For a real system, as before, if we ignore the cross covariance between different channel perturbations, then (A.7) reduces to the following after using (35):

$$\begin{aligned} &E\{\delta\mathbf{H}(n)\mathbf{Z}\delta\mathbf{H}(n)\} \\ &= \frac{\sigma_v^2}{N}\Big[\mathbf{C}_1(n)\mathbf{\Pi}_{\mathbf{g}_1}^\perp\mathbf{A}_1^{1/2}\mathbf{U}_n^1(\mathbf{U}_n^1)^H\mathbf{A}_1^{1/2}\mathbf{\Pi}_{\mathbf{g}_1}^\perp\mathbf{z}_{11}^*,\dots, \\ &\qquad \mathbf{C}_J(n)\mathbf{\Pi}_{\mathbf{g}_J}^\perp\mathbf{A}_J^{1/2}\mathbf{U}_n^j(\mathbf{U}_n^j)^H\mathbf{A}_J^{1/2}\mathbf{\Pi}_{\mathbf{g}_J}^\perp\mathbf{z}_{JJ}^*\Big].\end{aligned} \tag{A.8}$$

Once $E\{\delta\mathbf{H}(n)\mathbf{Z}\delta\mathbf{H}(n)\}$ is computed, $E\{\delta\mathbf{H}(n)^H\mathbf{Z}^H\delta\mathbf{H}(n)^H\}$ can be readily obtained as the Hermitian of $E\{\delta\mathbf{H}(n)\mathbf{Z}\delta\mathbf{H}(n)\}$. It can also be observed that both $E\{\delta\mathbf{H}(n)\mathbf{Z}\delta\mathbf{H}(n)\}$ and its Hermitian are at the order of $O(\sigma_v^2/N)$ for a real system and $\mathbf{0}$ for a complex system.

B. DERIVATION OF EXPECTATION QUANTITIES FOR THE MMSE RECEIVER

B.1. Derivation of $E\{\delta\sigma_v^2\delta\sigma_v^2\}$

The perturbation of the q least eigenvalues of $\mathbf{R}_1$ is given by $\delta\sigma_v^2 = (1/q)\operatorname{tr}\{(\mathbf{U}_n^1)^H\delta\mathbf{R}_1\mathbf{U}_n^1\}$ according to (14). Using vec and tr operations, it is straightforward to show

$$\begin{aligned} E\{\delta\sigma_v^2\delta\sigma_v^2\} &= \frac{1}{q^2}E\Big\{\operatorname{tr}\left\{(\mathbf{U}_n^1)^H\delta\mathbf{R}_1\mathbf{U}_n^1\right\}\operatorname{tr}\left\{(\mathbf{U}_n^1)^H\delta\mathbf{R}_1\mathbf{U}_n^1\right\}\Big\} \\ &= \frac{1}{q^2}\operatorname{vec}^H\left(\mathbf{U}_n^1(\mathbf{U}_n^1)^H\right)E\{\operatorname{vec}(\delta\mathbf{R}_1)\operatorname{vec}^H(\delta\mathbf{R}_1)\} \\ &\quad\times\operatorname{vec}\left(\mathbf{U}_n^1(\mathbf{U}_n^1)^H\right) \end{aligned} \tag{B.1}$$

which depends on $E\{\operatorname{vec}(\delta\mathbf{R}_1)\operatorname{vec}^H(\delta\mathbf{R}_1)\}$. If we denote the ith column of $\delta\mathbf{R}_1$ as $\delta\mathbf{r}_i$, then we have $\delta\mathbf{r}_i = \delta\mathbf{R}_1\boldsymbol{\theta}_i$, where $\boldsymbol{\theta}_i$ is a unit vector with its ith element as 1. Correspondingly, the (i,j)th submatrix of $E\{\operatorname{vec}(\delta\mathbf{R}_1)\operatorname{vec}^H(\delta\mathbf{R}_1)\}$, which has size $(q+1)\times(q+1)$, becomes $E\{\delta\mathbf{R}_1\boldsymbol{\theta}_i\boldsymbol{\theta}_j^H\delta\mathbf{R}_1\} = \mathbf{T}_1(\boldsymbol{\theta}_i\boldsymbol{\theta}_j^H)$ according to our definition (31). Applying those results, and noticing that

$$\operatorname{vec}\left(\mathbf{U}_n^1(\mathbf{U}_n^1)^H\right) = \left[\boldsymbol{\theta}_1^T\mathbf{U}_n^1(\mathbf{U}_n^1)^H,\ldots,\boldsymbol{\theta}_{q+1}^T\mathbf{U}_n^1(\mathbf{U}_n^1)^H\right]^H, \tag{B.2}$$

(B.1) reduces to

$$E\{\delta\sigma_v^2\delta\sigma_v^2\} = \frac{1}{q^2}\sum_{i,j=1}^{q+1}\boldsymbol{\theta}_i^T\mathbf{U}_n^1(\mathbf{U}_n^1)^H\mathbf{T}_1(\boldsymbol{\theta}_i\boldsymbol{\theta}_j^H)\mathbf{U}_n^1(\mathbf{U}_n^1)^H\boldsymbol{\theta}_j. \tag{B.3}$$

Substituting $\mathbf{T}_1(\boldsymbol{\theta}_i\boldsymbol{\theta}_j^H)$ by (32) or (33), and noticing that $(\mathbf{U}_n^1)^H\boldsymbol{\chi}_1 = 0$, and $(\mathbf{U}_n^1)^H\mathbf{R}_1 = \sigma_v^2(\mathbf{U}_n^1)^H$, we have that for a real system,

$$\begin{aligned} E\{\delta\sigma_v^2\delta\sigma_v^2\} = \frac{\sigma_v^2}{Nq^2}\Bigg[&\sum_{i,j=1}^{q+1}(\boldsymbol{\theta}_j^T\mathbf{R}_1\boldsymbol{\theta}_i)\boldsymbol{\theta}_i^T\mathbf{U}_n^1(\mathbf{U}_n^1)^H\boldsymbol{\theta}_j \\ &+\sigma_v^2\sum_{i,j=1}^{q+1}\left(\boldsymbol{\theta}_i^T\mathbf{U}_n^1(\mathbf{U}_n^1)^H\boldsymbol{\theta}_j\right)^2\Bigg], \end{aligned} \tag{B.4}$$

and for a complex system

$$E\{\delta\sigma_v^2\delta\sigma_v^2\} = \frac{\sigma_v^2}{Nq^2}\sum_{i,j=1}^{q+1}(\boldsymbol{\theta}_j^T\mathbf{R}_1\boldsymbol{\theta}_i)\boldsymbol{\theta}_i^T\mathbf{U}_n^1(\mathbf{U}_n^1)^H\boldsymbol{\theta}_j. \tag{B.5}$$

Clearly, $E\{\delta\sigma_v^2\delta\sigma_v^2\}$ is at the order of $O(\sigma_v^2/Nq^2)$ for either a real or complex system.

B.2. Derivation of $E\{\delta\mathbf{H}(n)\delta\sigma_v^2\}$

Replacing $\delta\sigma_v^2$ with $(1/q)\operatorname{tr}\{(\mathbf{U}_n^1)^H\delta\mathbf{R}_1\mathbf{U}_n^1\}$, and expressing $E\{\delta\mathbf{H}(n)\delta\sigma_v^2\}$ in columns, we obtain

$$\begin{aligned} E\{\delta\mathbf{H}(n)\delta\sigma_v^2\} = \frac{1}{q}\Big[&E\Big\{\mathbf{C}_1(n)\delta\mathbf{g}_1\operatorname{tr}\left\{(\mathbf{U}_n^1)^H\delta\mathbf{R}_1\mathbf{U}_n^1\right\}\Big\},\ldots, \\ &E\Big\{\mathbf{C}_J(n)\delta\mathbf{g}_J\operatorname{tr}\left\{(\mathbf{U}_n^1)^H\delta\mathbf{R}_1\mathbf{U}_n^1\right\}\Big\}\Big]. \end{aligned} \tag{B.6}$$

It is observed that except the first column, all other ones involve cross-correlation between $\delta\mathbf{g}_j$ and $\delta\mathbf{R}_1$. That cross-correlation depends on the cross-correlation between $\delta\mathbf{R}_1$ and $\delta\mathbf{R}_j$ for $j \neq 1$, and has been shown to be at the order of $O(\sigma_v^4/N)$ by our previous analysis. Then we only focus on the first column next, and approximate all the other columns as $\mathbf{0}$. Replacing $\delta\mathbf{g}_1$ by (29) and applying trace property, the first column of (B.6) becomes

$$\begin{aligned} E\{\delta\mathbf{h}_1(n)\delta\sigma_v^2\} = \frac{1}{q\gamma_1}&\mathbf{C}_1(n)\boldsymbol{\Pi}_{\mathbf{g}_1}^{\perp}\mathbf{A}_1^{1/2}\mathbf{U}_n^1(\mathbf{U}_n^1)^H \\ &\times E\Big\{\delta\mathbf{R}_1\operatorname{vec}^H(\delta\mathbf{R}_1)\operatorname{vec}\left(\mathbf{U}_n^1(\mathbf{U}_n^1)^H\right)\Big\}\boldsymbol{\chi}_1 \end{aligned} \tag{B.7}$$

which depends on $E\{\delta\mathbf{R}_1\operatorname{vec}^H(\delta\mathbf{R}_1)\operatorname{vec}(\mathbf{U}_n^1(\mathbf{U}_n^1)^H)\}$. Noticing $\delta\mathbf{R}_1 = [\delta\mathbf{r}_1,\ldots,\delta\mathbf{r}_{q+1}]$, we have $\operatorname{vec}(\delta\mathbf{R}_1) = [\delta\mathbf{r}_1^H,\ldots,\delta\mathbf{r}_{q+1}^H]^H$. Applying (B.2), it can be shown that the following holds:

$$\begin{aligned} &E\Big\{\delta\mathbf{R}_1\operatorname{vec}^H(\delta\mathbf{R}_1)\operatorname{vec}\left(\mathbf{U}_n^1(\mathbf{U}_n^1)^H\right)\Big\} \\ &\quad=\Bigg[\sum_{j=1}^{q+1}\mathbf{T}_1(\boldsymbol{\theta}_1\boldsymbol{\theta}_j^H)\mathbf{U}_n^1(\mathbf{U}_n^1)^H\boldsymbol{\theta}_j,\ldots, \\ &\qquad\sum_{j=1}^{q+1}\mathbf{T}_1(\boldsymbol{\theta}_{q+1}\boldsymbol{\theta}_j^H)\mathbf{U}_n^1(\mathbf{U}_n^1)^H\boldsymbol{\theta}_j\Bigg]. \end{aligned} \tag{B.8}$$

Replacing (B.8) back into (B.7), substituting $\mathbf{T}_1(\boldsymbol{\theta}_i\boldsymbol{\theta}_j^H)$ by (32) or (33), and noticing that $(\mathbf{U}_n^1)^H\boldsymbol{\chi}_1 = 0$ and $(\mathbf{U}_n^1)^H\mathbf{R}_1 = \sigma_v^2(\mathbf{U}_n^1)^H$, (B.7) reduces to the following after neglecting the higher order terms of $O(\sigma_v^4/N)$:

$$\begin{aligned} &E\{\delta\mathbf{h}_1(n)\delta\sigma_v^2\} \\ &\approx\frac{\sigma_v^2}{Nq\gamma_1}\mathbf{C}_1(n)\boldsymbol{\Pi}_{\mathbf{g}_1}^{\perp}\mathbf{A}_1^{1/2}\Bigg[\sum_{j=1}^{q+1}(\boldsymbol{\theta}_j^H\mathbf{R}_1\boldsymbol{\theta}_1)\mathbf{U}_n^1(\mathbf{U}_n^1)^H\boldsymbol{\theta}_j,\ldots, \\ &\qquad\sum_{j=1}^{q+1}(\boldsymbol{\theta}_j^H\mathbf{R}_1\boldsymbol{\theta}_{q+1})\mathbf{U}_n^1(\mathbf{U}_n^1)^H\boldsymbol{\theta}_j\Bigg]\boldsymbol{\chi}_1 \end{aligned} \tag{B.9}$$

which is clearly at the order of $O(\sigma_v^2/Nq)$.

To summarize, $E\{\delta\mathbf{H}(n)\delta\sigma_v^2\}$ can be approximated by a matrix with its first column given by (B.9) and all the other columns by zeros.

C. DERIVATION OF EXPECTATION QUANTITIES FOR THE RAKE RECEIVER

C.1. Derivation of $E\{\mathrm{vec}(\mathbf{G}_1^H\delta\mathbf{G}_1)\,\mathrm{vec}^H(\mathbf{G}_1^H\delta\mathbf{G}_1)\}$

Rewriting $\mathrm{vec}(\mathbf{G}_1^H\delta\mathbf{G}_1)$ as $\mathrm{vec}(\mathbf{G}_1^H\delta\mathbf{G}_1\mathbf{I})$ and using vec operations, then the following holds:

$$\begin{aligned} &E\{\,\mathrm{vec}\,(\mathbf{G}_1^H\delta\mathbf{G}_1)\,\mathrm{vec}^H\,(\mathbf{G}_1^H\delta\mathbf{G}_1)\}\\ &\quad = (\mathbf{I}\otimes\mathbf{G}_1^H)E\Big\{\,\mathrm{vec}\,(\delta\mathbf{G}_1)\,\mathrm{vec}\,(\delta\mathbf{G}_1)^H\Big\}(\mathbf{I}\otimes\mathbf{G}_1). \end{aligned} \tag{C.1}$$

Directly replacing $\delta\mathbf{G}_1$ by (62), applying the result in (35), (C.1) is further computed as

$$\begin{aligned} &E\{\,\mathrm{vec}\,(\mathbf{G}_1^H\delta\mathbf{G}_1)\,\mathrm{vec}^H\,(\mathbf{G}_1^H\delta\mathbf{G}_1)\}\\ &\quad = (\mathbf{I}\otimes\mathbf{G}_1^H)\mathcal{B}E\{\delta\mathbf{g}_1\delta\mathbf{g}_1^H\}\mathcal{B}^H(\mathbf{I}\otimes\mathbf{G}_1)\\ &\quad = \frac{\sigma_v^2}{N}(\mathbf{I}\otimes\mathbf{G}_1^H)\mathcal{B}\mathbf{\Pi}_{\mathbf{g}_1}^{\perp}\mathbf{A}_1^{1/2}\mathbf{U}_n^1(\mathbf{U}_n^1)^H\mathbf{A}_1^{1/2}\mathbf{\Pi}_{\mathbf{g}_1}^{\perp}\mathcal{B}^H(\mathbf{I}\otimes\mathbf{G}_1), \end{aligned} \tag{C.2}$$

where $\mathcal{B}$ is defined as

$$\mathcal{B} = \left[\mathbf{B}_0^T,\ldots,\mathbf{B}_{P-1}^T\right]^T. \tag{C.3}$$

C.2. Derivation of $E\{\delta\mathbf{G}_1\delta\mathbf{G}_1^H\}$

Noticing (62), applying the result in (34), and ignoring $O(\sigma_v^4/N)$ terms, we have

$$\begin{aligned} E\{\delta\mathbf{G}_1\delta\mathbf{G}_1^H\} &= \sum_{i=0}^{P-1}\mathbf{B}_iE\{\delta\mathbf{g}_1\delta\mathbf{g}_1^H\}\mathbf{B}_i^H\\ &= \frac{\sigma_v^2}{N}\sum_{i=0}^{P-1}\mathbf{B}_i\mathbf{\Pi}_{\mathbf{g}_1}^{\perp}\mathbf{A}_1^{1/2}\mathbf{U}_n^1(\mathbf{U}_n^1)^H\mathbf{A}_1^{1/2}\mathbf{\Pi}_{\mathbf{g}_1}^{\perp}\mathbf{B}_i^H. \end{aligned} \tag{C.4}$$

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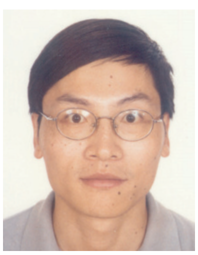

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