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Greedy SINR Maximization in Collaborative Multibase Wireless Systems

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We present a codeword adaptation algorithm for collaborative multibase wireless systems. The system is modeled with multiple inputs and multiple outputs (MIMO) in which information is transmitted using multicode CDMA, and codewords are adapted based on greedy maximization of the signal-to-interference-plus-noise ratio. The procedure monotonically increases the sum capacity and, when repeated iteratively for all codewords in the system, converges to a fixed point. Fixed-point properties and a connection with sum capacity maximization, along with a discussion of simulations that corroborate the basic analytic results, are included in the paper.

Keywords and phrases: multibase systems, collaboration, MIMO, sum capacity, interference avoidance.

1. INTRODUCTION

Code division multiple access (CDMA) schemes have been proposed for use in future generation wireless systems due to the fact that they enable efficient utilization of communication resources like available spectrum and transmitted power, and have received increased attention from the wireless research community lately. In particular, the problem of optimizing CDMA codewords has been addressed by several researchers which have established algorithms that yield optimal codeword ensembles for CDMA systems [1, 2, 3, 4, 5, 6]. Recently, optimal codeword ensembles for CDMA systems have also been obtained by application of interference avoidance methods [7, 8, 9, 10, 11, 12] which provide distributed algorithms for codeword optimization in CDMA systems by which users independently adjust codewords in response to changing patterns of interference.

Optimal codewords provide all users in a CDMA system with uniform signal-to-interference-plus-noise ratio (SINR), and imply that the optimal linear receiver is a matched filter for each codeword [6]. In addition, optimal codeword ensembles also maximize sum capacity [4]. We note that codeword optimization algorithms available in the literature have been defined in the context of single-cell CDMA systems in which all users communicate with a single base station which knows codewords for all users in the system and uses them to decode the transmitted information symbols. In general, wireless systems consist of a collection of users and base stations dispersed over a given geographical area, in which individual users are interested in sending information to a particular base station and each base cares only about decoding the users assigned to it. When no cooperation among users/bases is assumed, the problem of decoding one user at its associated base station under interference generated by all the other users in the system is an instance of the interference channel [13, Chapter 14], and is still a mostly open research problem.

We consider a wireless communication system with multiple transmitters and receivers geographically distributed over some area as described schematically in Figure 1. We assume that the available spectrum is shared by all users and bases, as is the case in unlicensed bands, and make the simplifying assumption that receivers are allowed to share information. This implies that the system under consideration can be regarded as a system with multiple inputs and multiple outputs (MIMO), unlike the usual cellular scenario in which users are assigned to bases based on a quality of service criterion (like, e.g., the SINR). We note that such a collaborative scenario may provide upper bounds on various measures of interest as one can do no better than jointly decode, and has been considered by researchers in previous works dealing with systems with multiple transmitters and receivers

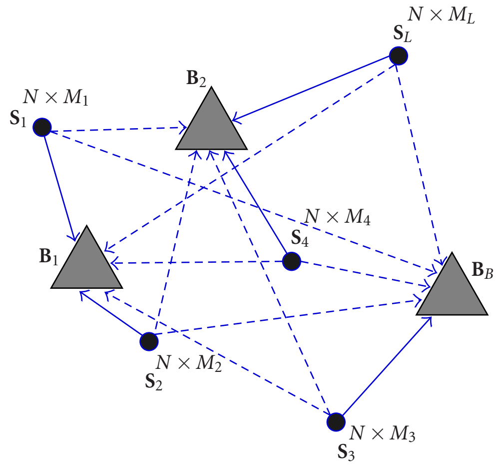

Figure 1: A multibase system with B receiving bases and L transmitting locations, with each location k using M_k signatures. Triangles denote receivers and circles denote transmitters/users.

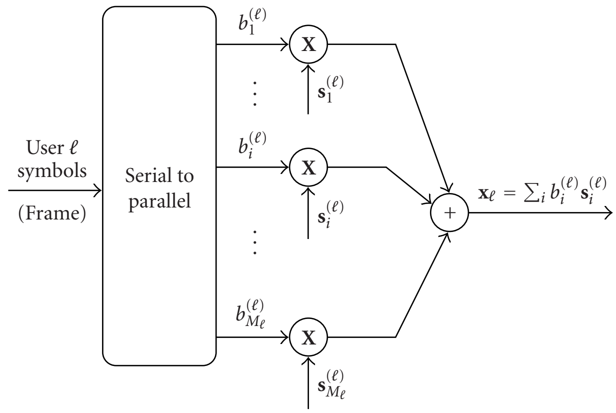

Figure 2: Multicode CDMA approach for sending frames of information.

[14, 15, 16, 17]. We also note that, while unusual in the context of current multiple-base cellular communications systems where, in general, bases do not share information, the availability of relatively low-cost high-speed terrestrial links makes collaboration possibly practicable in future generation wireless systems.

We assume that a multicode CDMA approach is used for transmitting information by users in the collaborative multi-base system, in which users transmit frames of data by assigning each symbol in a given user's frame a distinct codeword for transmission, and we present a greedy algorithm for codeword adaptation based on selfish optimization of individual SINRs. We note that this is different from [18] which deals with optimization of transmit covariance matrices in multiuser MIMO systems, and from [19] which presents joint transmitter-receiver optimization algorithms for multiuser MIMO systems.

In addition to maximizing SINRs at each step, the proposed algorithm also monotonically increases the sum capacity, which is a global criterion. Fixed-point properties of the proposed algorithm are investigated, and the connection with sum capacity maximization is made. We note that the optimal fixed point of the algorithm corresponds to a maximum sum capacity, and optimal codeword ensembles satisfy a simultaneous water-filling solution [20]. However, the algorithm is not a water-filling procedure, but a codeword adaptation one, and in the most general scenario, replacement of one codeword of one user is followed by replacement of another codeword of a different user. In fact, when the number of codewords assigned to users is such that the transmit covariance matrices do not have full rank, then water-filling schemes may not even be applicable, as maximization of the sum capacity under trace *and* rank constraints on user transmit covariance matrices is no longer a convex optimization problem and does not enjoy global convergence properties [21].

We have also performed simulations to corroborate our analytical results, and discuss their results in the paper.

2. SYSTEM MODEL AND PROBLEM STATEMENT

The system model is depicted in Figure 1 and consists of B base stations that are situated in a given geographical area, and L users transmitting from various locations within the same region. We assume a common signal space representation of dimension N for all users/bases implied by finite bandwidth and finite signaling interval constraints [22]. In this signal space setting, we assume that during each signaling interval of duration, T users transmit "frames" of data using a multicode CDMA approach in which each symbol in a given user's frame is assigned a distinct signature (codeword), and the transmitted signal is a superposition of all the codewords scaled by their corresponding information symbols, as described schematically in Figure 2. Thus, each user ℓ at a given location transmits the signal

$$\mathbf{x}_\ell = \sum_{m=1}^{M_\ell} \mathbf{s}_m^{(\ell)} b_m^{(\ell)} = \mathbf{S}_\ell \mathbf{b}_\ell \quad \forall \ell = 1, \ldots, L, \tag{1}$$

where

$$\mathbf{S}_\ell = \begin{bmatrix} | & & | & & | \\ \mathbf{s}_1^{(\ell)} & \cdots & \mathbf{s}_m^{(\ell)} & \cdots & \mathbf{s}_{M_\ell}^{(\ell)} \\ | & & | & & | \end{bmatrix} \tag{2}$$

is the $N \times M_\ell$ codeword matrix corresponding to user ℓ, and $\mathbf{b}_\ell = [b_1^{(\ell)} \cdots b_{M_\ell}^{(\ell)}]^\top$ is the vector containing the information symbols transmitted by user ℓ that are assumed uncorrelated, with zero mean and unit variance. We note that the model is general and allows coexistence of users with different data rates, since the number of symbols in a frame transmitted during a signaling interval of duration T may not be the same for all users. All signature sequences are assumed to have unit energy, $\|\mathbf{s}_m^{(\ell)}\| = 1$, $m = 1, \ldots, M_\ell$, $\ell = 1, \ldots, L$, and the user transmit power, which is the same for all symbols in the frame and should appear as a scalar multiplying the codeword matrix, is incorporated in the $N \times N$ gain matrix $\mathbf{G}_{\ell j}$ which characterizes the vector channel between user

ℓ and base station j. In general, this gain matrix incorporates channel attenuation and multipath [20].

We assume complete synchronization at all bases among all users, which may be justified by assuming sufficiently long signaling intervals relative to the communication bandwidth alloted. Under this assumption, the received signal at base station j is

$$\mathbf{r}_j = \sum_{\ell=1}^{L} \mathbf{G}_{\ell j}\mathbf{S}_\ell \mathbf{b}_\ell + \mathbf{w}_j \quad \forall j = 1,\dots,B, \tag{3}$$

where $\mathbf{w}_j$ is an additive Gaussian noise term with covariance matrix $\mathbf{W}_j$. Assuming collaboration, the information received at all base stations is pooled, forming a BN-dimensional received vector

$$\underbrace{\begin{bmatrix}\mathbf{r}_1\\ \vdots \\ \mathbf{r}_B\end{bmatrix}}_{\mathbf{r}} = \sum_{\ell=1}^{L} \underbrace{\begin{bmatrix}\mathbf{G}_{\ell 1}\\ \vdots \\ \mathbf{G}_{\ell B}\end{bmatrix}}_{\mathbf{G}_\ell} \mathbf{S}_\ell \mathbf{b}_\ell + \underbrace{\begin{bmatrix}\mathbf{w}_1\\ \vdots \\ \mathbf{w}_B\end{bmatrix}}_{\mathbf{w}} \tag{4}$$

with correlation matrix

$$\mathbf{R} = E\left[\mathbf{r}\mathbf{r}^\top\right] = \sum_{\ell=1}^{L} \mathbf{R}(\ell) + \mathbf{W}, \tag{5}$$

where matrix $\mathbf{R}(\ell)$ represents user ℓ's contribution to $\mathbf{R}$ and is expressed in terms of its codeword and corresponding gain matrices as

$$\mathbf{R}(\ell) = \mathbf{G}_\ell \mathbf{S}_\ell \mathbf{S}_\ell^\top \mathbf{G}_\ell^\top, \tag{6}$$

and $\mathbf{W}$ is the covariance matrix of the resulting noise vector $\mathbf{w}$. Thus we have

$$\mathbf{R} = \sum_{\ell=1}^{L} \mathbf{G}_\ell \mathbf{S}_\ell \mathbf{S}_\ell^\top \mathbf{G}_\ell^\top + \mathbf{W}. \tag{7}$$

Under Gaussian signaling and noise assumptions, the sum capacity for the considered multibase system with collaboration in (4) is given by [5, 20]

$$C_{\text{sum}} = \frac{1}{2}\log|\mathbf{R}| - \frac{1}{2}\log|\mathbf{W}|. \tag{8}$$

We note that the sum capacity expression in (8) has been used in the context of previous works on multiuser MIMO systems [19, 23, 24].

Our objective is to define a codeword adaptation algorithm for the system described by (4) which iteratively updates codewords of all users until an optimal ensemble of codewords is obtained. Forestalling the formal definition of the proposed algorithm which is given in the following section, we note that it is an interference avoidance algorithm based on greedy SINR maximization for individual codewords, which monotonically increases the sum capacity and leads the system toward the socially optimal ensemble corresponding to maximum sum capacity.

3. GREEDY SINR MAXIMIZATION THROUGH DISTRIBUTED CODEWORD ADAPTATION

For the MIMO system in (4), we assume that linear receivers are used, and would like to adjust codewords in the system, one at a time, so that their SINR is maximized. This is a two-step process in which we first derive the expression of the linear receiver which yields the maximum SINR for a given codeword, and then look to replace the considered codeword with a new one which will increase the SINR. A similar approach was used for a single-base system in [12]. However, the algorithm in [12] updates codewords by replacing them with the current maximum SINR receiver, as opposed to choosing a codeword and a receiver which absolutely maximize the SINR as we are doing here.

We simplify our notation and express the covariance matrix in (7) in terms of individual codewords $\mathbf{s}_i$, rather than user codeword matrices, as

$$\mathbf{R} = \sum_{i=1}^{M} \underbrace{\mathbf{G}_i \mathbf{s}_i}_{\mathbf{y}_i} \mathbf{s}_i^\top \mathbf{G}_i^\top + \mathbf{W} = \sum_{i=1}^{M} \mathbf{y}_i \mathbf{y}_i^\top + \mathbf{W}, \tag{9}$$

where $M = \sum_{\ell=1}^{L} M_\ell$ is the total number of codewords in the ensemble, and the gain matrix $\mathbf{G}_i$ will be equal for all the codewords of a given user under the multicode assumption. This is because in the most general scenario, one may replace one codeword of a given user and follow it by the replacement of one codeword of a different user. Thus, it is not the user index which is relevant in the update process, but the codeword index in the ensemble.

We denote by $\mathbf{c}_i$ the NB-dimensional linear receiver associated with the received vector $\mathbf{y}_i = \mathbf{G}_i\mathbf{s}_i$ which implies that the SINR for codeword $\mathbf{s}_i$ is

$$\begin{aligned}\gamma_i &= \frac{\left(\mathbf{c}_i^\top \mathbf{y}_i\right)^2}{\sum_{k\neq i}^{M}\left(\mathbf{c}_i^\top \mathbf{y}_k\right)^2 + E\left[\left(\mathbf{c}_i^\top \mathbf{n}\right)^2\right]} \\ &= \frac{\mathbf{c}_i^\top \mathbf{y}_i \mathbf{y}_i^\top \mathbf{c}_i}{\mathbf{c}_i^\top\left[\mathbf{R} - \mathbf{y}_i\mathbf{y}_i^\top\right]\mathbf{c}_i} = \frac{\mathbf{c}_i^\top \mathbf{y}_i \mathbf{y}_i^\top \mathbf{c}_i}{\mathbf{c}_i^\top \mathbf{R}_i \mathbf{c}_i},\end{aligned} \tag{10}$$

where $\mathbf{R}_i = \mathbf{R} - \mathbf{y}_i\mathbf{y}_i^\top$.

Since $\mathbf{R}_i$ is positive definite,[1] we can consider an eigendecomposition $\mathbf{R}_i = \mathbf{\Phi}_i \mathbf{\Lambda}_i \mathbf{\Phi}_i^\top$, and define a new vector $\mathbf{z}_i = \mathbf{\Lambda}_i^{1/2}\mathbf{\Phi}_i^\top \mathbf{c}_i$ such that $\mathbf{c}_i = \mathbf{\Phi}\mathbf{\Lambda}_i^{-1/2}\mathbf{z}_i$. This implies that the SINR for codeword $\mathbf{s}_i$ can be rewritten as

$$\gamma_i = \frac{\mathbf{z}_i^\top \mathbf{\Lambda}_i^{-1/2}\mathbf{\Phi}_i^\top \mathbf{y}_i \mathbf{y}_i^\top \mathbf{\Phi}_i \mathbf{\Lambda}_i^{-1/2}\mathbf{z}_i}{\mathbf{z}_i^\top \mathbf{z}_i} \tag{11}$$

which represents the Rayleigh quotient of a rank-one matrix and is maximized when $\mathbf{z}_i = \mathbf{\Lambda}_i^{-1/2}\mathbf{\Phi}_i^\top \mathbf{y}_i$. Thus, the SINR maximizing linear receiver $\mathbf{c}_i$ is

$$\mathbf{c}_i = \mathbf{R}_i^{-1}\mathbf{y}_i = \mathbf{R}_i^{-1}\mathbf{G}_i\mathbf{s}_i \tag{12}$$

[1] $\mathbf{R}_i$ contains the noise covariance matrix $\mathbf{W}$ which is positive definite; otherwise, the capacity will be infinite.

which is an MMSE-type receiver [25]. We note that MMSE receivers have been previously used for transmitter and receiver optimization in MIMO systems with multiple users [18, 19]. For the MMSE receiver in (12), the SINR value is

$$\gamma_i = \mathbf{y}_i^\top \mathbf{R}_i^{-1} \mathbf{y}_i = \mathbf{s}_i^\top \mathbf{G}_i^\top \mathbf{R}_i^{-1} \mathbf{G}_i \mathbf{s}_i \tag{13}$$

and is maximized when $\mathbf{s}_i$ is the eigenvector $\mathbf{x}_i$ corresponding to the maximum eigenvalue (maximum eigenvector for short) of $\mathbf{G}_i^\top \mathbf{R}_i^{-1} \mathbf{G}_i$. Thus, replacement of codeword $\mathbf{s}_i$ by $\mathbf{x}_i$ maximizes the SINR of codeword i which becomes

$$\gamma_i' = \mathbf{x}_i^\top \mathbf{G}_i^\top \mathbf{R}_i^{-1} \mathbf{G}_i \mathbf{x}_i \geq \gamma_i. \tag{14}$$

Applying this procedure iteratively for all codewords in the ensemble defines a codeword adaptation algorithm which is formally stated below.

Greedy SINR Maximization Algorithm

(1) Initialize codewords $\{\mathbf{s}_i\}$ and gain matrices $\mathbf{G}_i$.
(2) For each codeword in the ensemble $i = 1, \ldots, M$, do.
  (i) Replace codeword $\mathbf{s}_i$ with the maximum eigenvector of matrix $\mathbf{G}_i^\top \mathbf{R}_i^{-1} \mathbf{G}_i$
(3) Repeat step (2) until a fixed point is reached.

Numerically, a fixed point of the algorithm is defined with respect to a stopping criterion. That is, we say that a fixed point is reached when the difference between two consecutive values of the stopping criterion is within a specified tolerance ϵ. The stopping criterion can be an individual one, like the codeword SINR or the Euclidean distance between codewords and their corresponding replacements, or a global one like the sum capacity. We note that in the case of individual stopping criteria, all values corresponding to all codewords must be within the specified tolerance for the algorithm to stop.

Mathematically, convergence of the greedy SINR maximization algorithm to a fixed point is ensured by the fact that the algorithm monotonically increases the sum capacity, and that the sum capacity is upper bounded. This does not necessarily imply that the fixed point is unique, and theoretically, many fixed points of this algorithm are possible. However, extensive simulations have shown that the algorithm has always reached the maximum sum capacity point when starting with randomly selected codewords, though we were not able to prove this result in general.

In order to see that the proposed greedy SINR maximization procedure monotonically increases the sum capacity, we consider the determinant of $\mathbf{R}$ in the sum capacity expression in (8) which we write in terms of codeword $\mathbf{s}_i$ as

$$|\mathbf{R}| = \left| \sum_{k\neq i}^{M} \mathbf{G}_k \mathbf{s}_k \mathbf{s}_k^\top \mathbf{G}_k^\top + \mathbf{W} + \mathbf{G}_i \mathbf{s}_i \mathbf{s}_i^\top \mathbf{G}_i^\top \right| = \left| \mathbf{R}_i + \mathbf{G}_i \mathbf{s}_i \mathbf{s}_i^\top \mathbf{G}_i^\top \right|. \tag{15}$$

Since $\mathbf{R}_i$ is invertible, it can be factored out:

$$\mathbf{R} = \mathbf{R}_i \left[\mathbf{I}_{BN} + \mathbf{R}_i^{-1} \mathbf{G}_i \mathbf{s}_i \mathbf{s}_i^\top \mathbf{G}_i^\top \right], \tag{16}$$

which implies that $|\mathbf{R}|$ can also be written as

$$\begin{aligned} |\mathbf{R}| &= \left| \mathbf{R}_i \right| \left| \mathbf{I}_{BN} + \mathbf{R}_i^{-1} \mathbf{G}_i \mathbf{s}_i \mathbf{s}_i^\top \mathbf{G}_i^\top \right| \\ &= \left| \mathbf{R}_i \right| \left(1 + \mathbf{s}_i^\top \mathbf{G}_i^\top \mathbf{R}_i^{-1} \mathbf{G}_i \mathbf{s}_i \right), \end{aligned} \tag{17}$$

where the last equality follows from

$$\left| \mathbf{I}_k + \mathbf{A}\mathbf{B} \right| = \left| \mathbf{I}_m + \mathbf{B}\mathbf{A} \right|, \quad \mathbf{A} \in M_{k\times m}, \mathbf{B} \in M_{m\times k}. \tag{18}$$

From (14), we obtain

$$|\mathbf{R}| = \left| \mathbf{R}_i \right| \left(1 + \gamma_i \right) \tag{19}$$

which shows that each iteration monotonically increases $|\mathbf{R}|$. This in turn implies a monotonic increase in the sum capacity, and because the sum capacity is upper bounded, this proves our claim that the greedy SINR maximization algorithm will reach a fixed point. Properties of this fixed point as well as a connection with maximizing the sum capacity are presented in the following sections.

To conclude this section, we note that the formal statement of the greedy SINR maximization algorithm does not impose a particular order of codeword adaptation, and in the most general case, replacement of one codeword of one user is followed by replacement of one codeword of a different user. We also note that the proposed SINR maximization algorithm is a greedy interference avoidance algorithm [11] since at each step, it greedily maximizes the SINR of an individual codeword without paying attention to the consequences that this action may have on the ensemble of codewords. Different interference avoidance algorithms have been previously derived for general multiaccess vector channels [26] as well as for multiuser MIMO systems [8].

4. FIXED-POINT PROPERTIES OF THE GREEDY SINR MAXIMIZATION ALGORITHM

Let $\{\lambda_j^i\}$ be the set of eigenvalues for the matrix $\mathbf{G}_i^\top \mathbf{R}_i^{-1} \mathbf{G}_i$ in decreasing order, $\lambda_1^i \geq \lambda_2^i \geq \cdots \geq \lambda_N^i$, with $\{\mathbf{x}_j^i\}$ being the corresponding eigenvectors. At a fixed point of the greedy SINR maximization algorithm, any further change in user codewords brings no improvement in the SINR values, which will be equal to the maximum eigenvalues: $\gamma_i = \lambda_1^i$. The codewords will be the maximum eigenvectors of $\mathbf{G}_i^\top \mathbf{R}_i^{-1} \mathbf{G}_i$, that is, $\mathbf{s}_i = \mathbf{x}_1^i$, and we can write

$$\mathbf{G}_i^\top \mathbf{R}_i^{-1} \mathbf{G}_i \mathbf{s}_i = \lambda_1^i \mathbf{s}_i = \gamma_i \mathbf{s}_i. \tag{20}$$

In addition, any eigenvector of $\mathbf{G}_i^\top \mathbf{R}_i^{-1} \mathbf{G}_i$ is also an eigenvector of $\mathbf{G}_i^\top \mathbf{R}^{-1} \mathbf{G}_i$. In order to see this, we start by writing

$$\mathbf{R} = \mathbf{R}_i + \mathbf{G}_i \mathbf{s}_i \mathbf{s}_i^\top \mathbf{G}_i^\top = \mathbf{R}_i + \mathbf{G}_i \mathbf{x}_1^i \left(\mathbf{x}_1^i\right)^\top \mathbf{G}_i^\top \tag{21}$$

and using the matrix inversion lemma [27, page 19]

$$\mathbf{R}^{-1} = \mathbf{R}_i^{-1} - \mathbf{R}_i^{-1} \mathbf{G}_i \mathbf{x}_1^i \left(1 + \left(\mathbf{x}_1^i\right)^\top \mathbf{G}_i^\top \mathbf{R}_i^{-1} \mathbf{G}_i \mathbf{x}_1^i \right)^{-1} \left(\mathbf{x}_1^i\right)^\top \mathbf{G}_i^\top \mathbf{R}_i^{-1}, \tag{22}$$

we get

$$\begin{aligned}\mathbf{G}_i^\top \mathbf{R}^{-1}\mathbf{G}_i\mathbf{x}_j^i &= \lambda_j^i\mathbf{x}_j^i - \frac{\gamma_i^2}{1+\gamma_i}\mathbf{x}_1^i\left(\mathbf{x}_1^i\right)^\top\mathbf{x}_j^i \\ &= \left(\lambda_j^i - \frac{\left(\lambda_j^i\right)^2}{1+\lambda_j^i}\delta_{1j}\right)\mathbf{x}_j^i,\end{aligned} \tag{23}$$

where δ_{ij} is the Kronecker delta operator which is 1 for $i = j$ and 0 for $i \neq j$.

Thus, at a fixed point, matrices $\mathbf{G}_i^\top\mathbf{R}_i^{-1}\mathbf{G}_i$ and $\mathbf{G}_i^\top\mathbf{R}^{-1}\mathbf{G}_i$ share the same set of eigenvectors. In addition, they have the same eigenvalues with the exception of the one that corresponds to $\mathbf{s}_i$, for which we have

$$\mathbf{G}_i^\top\mathbf{R}_i^{-1}\mathbf{G}_i\mathbf{s}_i = \gamma_i\mathbf{s}_i, \qquad \mathbf{G}_i^\top\mathbf{R}^{-1}\mathbf{G}_i\mathbf{s}_i = \frac{\gamma_i}{1+\gamma_i}\mathbf{s}_i. \tag{24}$$

Using (24), we now note that at a fixed point of the algorithm, all codewords of a given user ℓ, which share the same gain matrix $\mathbf{G}_\ell$, must be orthogonal if they have different SINRs since they are eigenvectors of the same matrix $\mathbf{G}_\ell^\top\mathbf{R}^{-1}\mathbf{G}_\ell$ corresponding to different eigenvalues. More precisely, if $\mathbf{s}_m^{(\ell)}$ and $\mathbf{s}_n^{(\ell)}$ are two distinct codewords corresponding to user ℓ but with different SINRs, $\gamma_m^{(\ell)} \neq \gamma_n^{(\ell)}$, then $\mathbf{s}_m^{(\ell)} \perp \mathbf{s}_n^{(\ell)}$. Alternatively, when all codewords of a given user have the same SINR at a fixed point, then user ℓ's codeword matrix satisfies

$$\mathbf{G}_\ell^\top\mathbf{R}^{-1}\mathbf{G}_\ell\mathbf{S}_\ell = \frac{\gamma_\ell}{1+\gamma_\ell}\mathbf{S}_\ell \tag{25}$$

and $\gamma_\ell/(1+\gamma_\ell)$ is the maximum eigenvalue of $\mathbf{G}_\ell^\top\mathbf{R}^{-1}\mathbf{G}_\ell$. To conclude this section, we note that empirically, we have observed convergence of the algorithm from random initial points to a fixed point in which all codewords of a given user have the same SINR.

5. MAKING THE CONNECTION WITH SUM CAPACITY MAXIMIZATION

We have seen in Section 3 that the greedy SINR maximization algorithm for codeword adaptation monotonically increases the sum capacity. We have also noted in Section 3 that the algorithm is a greedy interference avoidance algorithm. Putting this observation together with the fact that previously proposed interference avoidance algorithms converge to a codeword ensemble that maximizes sum capacity [9] makes us suspect that our proposed greedy SINR maximization algorithm for collaborative multibase systems also yields sum-capacity-maximizing codeword ensembles. Thus, in this section, we examine some properties of codeword ensembles which maximize the sum capacity and relate them to the fixed-point properties of the proposed algorithm with the goal of supporting the claim that the maximum sum capacity point is among the fixed points of the algorithm.

Maximization of the sum capacity for a general multiaccess vector channel was solved in [20] as a spectral optimization problem, where it has been shown that optimal user transmit covariance matrices $\mathbf{X}_\ell$ can be obtained as solution of the following convex optimization problem:

$$\max_{\mathbf{X}_\ell} C_{\text{sum}} \quad \text{subject to Trace}\left[\mathbf{X}_\ell\right] = \text{const}, \quad \ell = 1,\dots,L. \tag{26}$$

If we assume that the noise is stationary with fixed covariance matrix $\mathbf{W}$, then (8) shows that maximizing the sum capacity is equivalent to maximizing $|\mathbf{R}|$. According to [20], this implies that a water filling condition is satisfied for all users simultaneously, for which eigenvectors of the interference-plus-noise covariance matrix seen by a given user ℓ align with those of its transmit covariance matrix $\mathbf{X}_\ell$ whose eigenvalues satisfy the well-known water-filling condition [13, page 253]. We note that optimal transmit covariance matrices may be obtained through an iterative water-filling procedure [20], and then subsequently used to construct optimal codeword ensembles that maximize the sum capacity using established algorithms [4, 5]. When additional rank constraints are imposed on user transmit covariance matrices, these can no longer be obtained by solving a convex optimization problem and using the iterative water-filling procedure, since maximizing the sum capacity subject to trace and rank constraints on user transmit covariance matrices is currently an open problem [21].

In our formulation, user transmit covariance matrices are expressed in terms of codeword matrices as $\mathbf{X}_\ell = \mathbf{S}_\ell\mathbf{S}_\ell^\top$, and when users are assigned at least $M_\ell = N$ codewords for transmission, then $\mathbf{S}_\ell\mathbf{S}_\ell^\top$ is not rank constrained and may span the entire signal space. In this case, optimal $\mathbf{S}_\ell\mathbf{S}_\ell^\top$ must satisfy a simultaneous water-filling condition as described in the previous paragraph. When users are assigned $M_\ell < N$ codewords for transmission, then $\mathbf{S}_\ell\mathbf{S}_\ell^\top$ is rank constrained. However, from the perspective of the simultaneous water-filling solution [20] at the maximum sum capacity point, $|\mathbf{R}|$ will be maximized when $\mathbf{S}_\ell\mathbf{S}_\ell^\top$ satisfies a water-filling condition on a lower-dimension eigensubspace containing the smallest eigenvalues of the interference-plus-noise covariance matrix seen by user ℓ. Regardless, in both cases, codewords which maximize the sum capacity are eigenvectors of their corresponding interference-plus-noise covariance matrix.

We will show that this feature is identical to that seen for the fixed-point user codeword matrices obtained via the SINR maximization described in Section 3. To begin, we rewrite $|\mathbf{R}|$ using (5) and (6) as

$$|\mathbf{R}| = \left|\mathbf{Q}_\ell + \mathbf{G}_\ell\mathbf{S}_\ell\mathbf{S}_\ell^\top\mathbf{G}_\ell^\top\right|, \tag{27}$$

where $\mathbf{S}_\ell$ is a matrix of codewords with identical gain matrices $\mathbf{G}_\ell$ corresponding to a given user ℓ and $\mathbf{Q}_\ell = \mathbf{R} - \mathbf{R}(\ell)$ is the covariance matrix of the interference plus noise seen by user ℓ. This implies that

$$\begin{aligned}|\mathbf{R}| &= \left|\mathbf{Q}_\ell\right|\left|\mathbf{I}_{BN} + \mathbf{Q}_\ell^{-1/2}\mathbf{G}_\ell\mathbf{S}_\ell\mathbf{S}_\ell^\top\mathbf{G}_\ell^\top\mathbf{Q}_\ell^{-1/2}\right| \\ &= \left|\mathbf{Q}_\ell\right|\left|\mathbf{I}_{BN} + \mathbf{H}_\ell\mathbf{S}_\ell\mathbf{S}_\ell^\top\mathbf{H}_\ell^\top\right| \\ &= \left|\mathbf{Q}_\ell\right|\left|\mathbf{I}_N + \mathbf{H}_\ell^\top\mathbf{H}_\ell\mathbf{S}_\ell\mathbf{S}_\ell^\top\right|,\end{aligned} \tag{28}$$

where $\mathbf{H}_\ell = \mathbf{Q}_\ell^{-1/2}\mathbf{G}_\ell$. Using the singular value decomposition [28, page 443] for matrix $\mathbf{H}_\ell$:

$$\mathbf{H}_\ell = \mathbf{U}_\ell \mathbf{D}_\ell \mathbf{V}_\ell^\top = \mathbf{U}_\ell \begin{bmatrix} \boldsymbol{\mathcal{D}}_\ell \\ \mathbf{0} \end{bmatrix} \mathbf{V}_\ell^\top, \tag{29}$$

we define

$$\bar{\mathbf{H}}_\ell = \mathbf{U}_\ell \mathbf{D}_\ell \mathbf{V}_\ell^\top = \mathbf{U}_\ell \begin{bmatrix} \boldsymbol{\mathcal{D}}_\ell^{-1} \\ \mathbf{0} \end{bmatrix} \mathbf{V}_\ell^\top \tag{30}$$

such that $\mathbf{H}_\ell^\top \bar{\mathbf{H}}_\ell = \bar{\mathbf{H}}_\ell^\top \mathbf{H}_\ell = \mathbf{I}_N$. Also $(\mathbf{H}_\ell^\top \mathbf{H}_\ell)(\bar{\mathbf{H}}_\ell^\top \bar{\mathbf{H}}_\ell) = \mathbf{I}_N$ and we may then write

$$|\mathbf{R}| = |\mathbf{Q}_\ell|\,|\mathbf{H}_\ell^\top \mathbf{H}_\ell|\,|\bar{\mathbf{H}}_\ell^\top \bar{\mathbf{H}}_\ell + \mathbf{S}_\ell \mathbf{S}_\ell^\top| \tag{31}$$

which can be rewritten using (29) and (30) as

$$|\mathbf{R}| = |\mathbf{Q}_\ell|\,|\mathbf{G}_\ell^\top \mathbf{Q}_\ell^{-1}\mathbf{G}_\ell|\,\left|(\mathbf{G}_\ell^\top \mathbf{Q}_\ell^{-1}\mathbf{G}_\ell)^{-1} + \mathbf{S}_\ell \mathbf{S}_\ell^\top\right|. \tag{32}$$

Each column of $\mathbf{S}_\ell$ is an eigenvector of $(\mathbf{G}_\ell^\top \mathbf{Q}_\ell^{-1}\mathbf{G}_\ell)^{-1}$ for the sum-capacity-maximizing codeword ensemble, and the water-filling solution dictates

$$\left[(\mathbf{G}_\ell^\top \mathbf{Q}_\ell^{-1}\mathbf{G}_\ell)^{-1} + \mathbf{S}_\ell \mathbf{S}_\ell^\top\right]\mathbf{S}_\ell = c\mathbf{S}_\ell, \tag{33}$$

where c denotes the corresponding "watermark."

We now recall (25) and will show that this implies that (33) is satisfied with $c = (1+\gamma_\ell)/\gamma_\ell$. To show this, we rewrite $\mathbf{R}$ as we did when we looked at its determinant in (27):

$$\begin{aligned} \mathbf{R} &= \mathbf{Q}_\ell + \mathbf{G}_\ell \mathbf{S}_\ell \mathbf{S}_\ell^\top \mathbf{G}_\ell^\top \\ &= \mathbf{Q}_\ell^{1/2}(\mathbf{I}_{BN} + \mathbf{Q}_\ell^{-1/2}\mathbf{G}_\ell \mathbf{S}_\ell \mathbf{S}_\ell^\top \mathbf{Q}_\ell^{-1/2})\mathbf{Q}_\ell^{1/2}. \end{aligned} \tag{34}$$

With $\mathbf{H}_\ell$ defined as in (29), we then have

$$\mathbf{R} = \mathbf{Q}_\ell^{1/2}\mathbf{U}_\ell \begin{bmatrix} \mathbf{I}_N + \boldsymbol{\mathcal{D}}_\ell \mathbf{V}_\ell^\top \mathbf{S}_\ell \mathbf{S}_\ell^\top \mathbf{V}_\ell \boldsymbol{\mathcal{D}}_\ell & \mathbf{0} \\ \mathbf{0} & \mathbf{I}_{(B-1)N} \end{bmatrix} \mathbf{U}_\ell^\top \mathbf{Q}_\ell^{1/2} \tag{35}$$

which implies that

$$\begin{aligned} &\mathbf{R}^{-1} \\ &= \mathbf{Q}_\ell^{-1/2}\mathbf{U}_\ell \begin{bmatrix} \boldsymbol{\mathcal{D}}_\ell^{-1}\mathbf{V}_\ell^\top (\mathbf{V}_\ell \boldsymbol{\mathcal{D}}_\ell^{-2}\mathbf{V}_\ell^\top + \mathbf{S}_\ell \mathbf{S}_\ell^\top)^{-1}\mathbf{V}_\ell \boldsymbol{\mathcal{D}}_\ell^{-1} & \mathbf{0} \\ \mathbf{0} & \mathbf{I}_{(B-1)N} \end{bmatrix} \\ &\quad \times \mathbf{U}_\ell^\top \mathbf{Q}_\ell^{-1/2} \end{aligned} \tag{36}$$

in which we have used the matrix inversion lemma to compute the inverse of the upper-left block in (35). Again using the definition of $\mathbf{H}$ in (29) and applying it to (36), we obtain

$$\mathbf{G}_\ell^\top \mathbf{R}^{-1}\mathbf{G}_\ell = \left[(\mathbf{G}_\ell^\top \mathbf{Q}_\ell^{-1}\mathbf{G}_\ell)^{-1} + \mathbf{S}_\ell \mathbf{S}_\ell^\top\right]^{-1}. \tag{37}$$

Combining (37) and (25) which is satisfied at a fixed point when all codewords of user ℓ have the same SINR, we obtain

$$\left[(\mathbf{G}_\ell^\top \mathbf{Q}_\ell^{-1}\mathbf{G}_\ell)^{-1} + \mathbf{S}_\ell \mathbf{S}_\ell^\top\right]^{-1}\mathbf{S}_\ell = \frac{\gamma_\ell}{1+\gamma_\ell}\mathbf{S}_\ell \tag{38}$$

which also implies that

$$\left[(\mathbf{G}_\ell^\top \mathbf{Q}_\ell^{-1}\mathbf{G}_\ell)^{-1} + \mathbf{S}_\ell \mathbf{S}_\ell^\top\right]\mathbf{S}_\ell = \frac{1+\gamma_\ell}{\gamma_\ell}\mathbf{S}_\ell \tag{39}$$

which is identical to (33) in which $c = (1+\gamma_\ell)/\gamma_\ell$. However, before deciding that this is a water-filling solution, we need to check that $(1+\gamma_\ell)/\gamma_\ell$ is the minimum eigenvalue of $(\mathbf{G}_\ell^\top \mathbf{Q}_\ell^{-1}\mathbf{G}_\ell)^{-1} + \mathbf{S}_\ell \mathbf{S}_\ell^\top$, since water filling requires that all the other eigenvalues be larger than c. This can be easily seen from (25) which shows that if all codewords of a given user have the same SINR γ_ℓ, then $\gamma_\ell/(1+\gamma_\ell)$ is the maximum eigenvalue of $\mathbf{G}_\ell^\top \mathbf{R}^{-1}\mathbf{G}_\ell$, which implies that it is the minimum eigenvalue of $(\mathbf{G}_\ell^\top \mathbf{R}^{-1}\mathbf{G}_\ell)^{-1}$. Thus, the optimal point which corresponds to the sum-capacity-maximizing codeword ensembles is a fixed point of the proposed algorithm, more precisely, that fixed point for which all codewords of a given user have the same SINR. Empirically, we have always observed convergence of the algorithm to this fixed point from random initializations.

We conclude this section by noting that optimal codewords which maximize the sum capacity could be found by solving the constrained optimization problem

$$\max_{\{\mathbf{s}_m^{(\ell)}\}} C_{\text{sum}} \text{ subject to } \|\mathbf{s}_m^{(\ell)}\| = 1, \quad m = 1,\dots,M_\ell,\ \ell = 1,\dots,L, \tag{40}$$

having the same objective function as the one in (26), but with different optimization variables and constraints. This is a convex optimization problem only in the special case when $M_\ell \geq N$, but in general, convex optimization methods [29] will not be applicable. From this perspective, the proposed algorithm for greedy SINR maximization can also be regarded as an attempt to provide a solution to the constrained optimization problem in (40).

6. NUMERICAL RESULTS AND DISCUSSION

While we have not been able to prove that, in general, the proposed algorithm for greedy SINR maximization converges to optimal fixed points, we have performed extensive simulations that corroborate our theoretical findings. To keep the results general, we have not restricted the simulations to particular modulation and propagation models, but rather performed them in the arbitrary signal space with gain matrices generated randomly. The noise covariance matrix at each base station was also a randomly generated positive-definite matrix.

6.1. *Full codeword complement (N codewords per user)*

We first assumed that the number of codewords for each user was equal to the number of signal space dimensions ($M_\ell = N$). Numerical studies with signal space dimensions ranging from 2 to 20 and different numbers of users indicate that the proposed algorithm seems to reach the optimal point corresponding to the maximum sum capacity from random initial codeword ensembles without special assistance, and

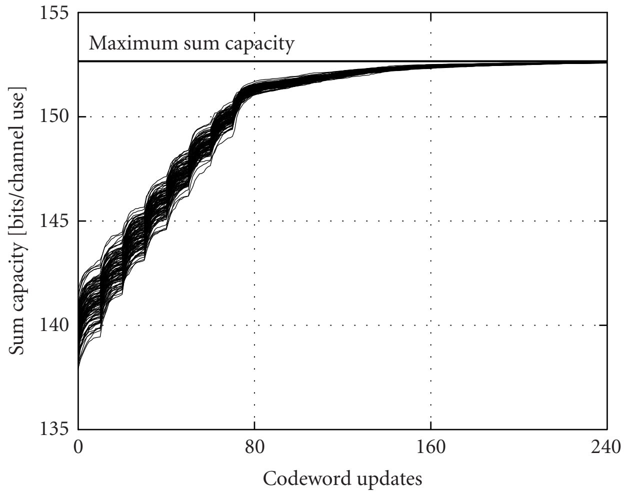

Figure 3: Convergence of the sum capacity for the greedy SINR maximization algorithm for a system with $L = 8$ users and $B = 4$ bases in a signal space of dimension $N = 10$ in 100 trials. One ensemble iteration consists of 80 codeword updates.

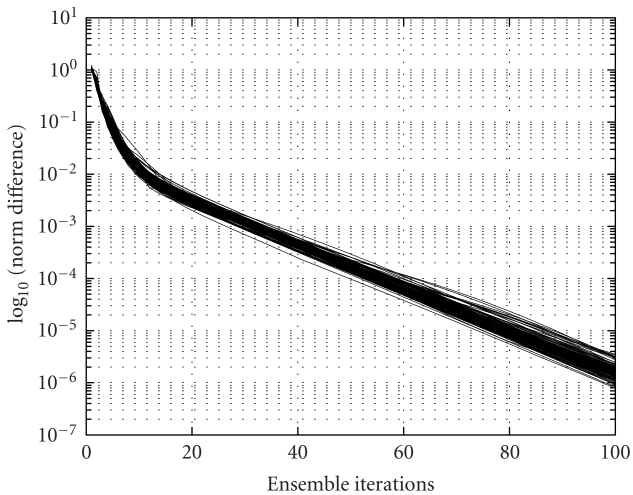

Figure 4: Codeword convergence for the greedy SINR maximization algorithm for a system with $L = 8$ users and $B = 4$ bases in a signal space of dimension $N = 10$ in 100 trials.

no suboptimal fixed points were observed. That is, we have invariably seen that the algorithm yielded ensembles where all the codewords of any given user k are eigenvectors of $\mathbf{G}_k\mathbf{R}^{-1}\mathbf{G}_k$ with identical eigenvalues $\gamma_k/(1+\gamma_k)$. Furthermore, at such fixed points, the remaining eigenvalues of $\mathbf{G}_k\mathbf{R}^{-1}\mathbf{G}_k$ are smaller than $\gamma_k/(1+\gamma_k)$ which shows that a simultaneously water-filling solution is satisfied. In addition, the sum capacity value at these points is identical to that obtained by applying iterative water filling [20].

Experiments have shown rapid convergence of the algorithm when the sum capacity was used as a global convergence metric, usually within 3–5 iteration cycles. This is illustrated in Figure 3 which is typical for all the simulations. In addition, codeword convergence was observed. That is, experiments have shown that codewords converged to within tight norm difference tolerances ($|\mathbf{s}_i(\kappa+1) - \mathbf{s}_i(\kappa)| \leq \varepsilon$) when starting from different random initializations. However, codeword convergence was much slower than convergence in the sum capacity as can be seen from Figure 4 (> 30 ensemble iterations for $\varepsilon = 10^{-3}$).

6.2. *One codeword per user*

We have also investigated the effect of using fewer codewords per user on the attainable sum capacity. The use of $M_\ell < N$ codewords by users for transmission restricts transmit covariance matrices to lower-dimensional subspaces, and imposes additional rank constraints in the sum capacity maximization problem. As previously noted, maximizing the sum capacity subject to rank constraints on transmit covariance matrices is not a convex optimization problem and is still an open research issue.

We have performed trials where each user's power budget could be applied to only a single codeword and calculated the sum capacity achieved after application of the greedy SINR maximization algorithm. In all cases, the sum capacity values obtained for given gain matrices and noise covariance were identical even when starting from different randomly chosen codeword ensembles. This might indicate that the algorithm attains the sum capacity maximum for this case and provides an analytic path for solution of the reduced-rank sum capacity maximization problem. We note that the final sum capacity value to which the algorithm converges in this case is smaller than that obtained when the available power budget is distributed equally to N codewords as can be observed from Figure 5. We also note an interesting trend: for a small number of users, the average penalty in the sum capacity associated with using a single codeword was pronounced—about 76% for one user per base with $N = 10$ dimensions and $B = 4$ bases. However, as the number of users was increased over 40, that is, more than 10 users per base, the difference between the attainable sum capacity and that achieved using single codewords decreases to under 2% on average. Thus, the addition of users seems to enable the ensemble of user codewords to get more closer to the maximum possible sum capacity, and eventually approximate a simultaneously water-filling solution.

7. CONCLUSIONS

We have proposed a codeword adaptation algorithm for wireless systems with multiple bases which pool information and jointly decode all users. The algorithm is based on greedy SINR maximization and is an interference-avoidance-type algorithm in which ensemble codewords are updated based on a selfish optimization of an individual criterion rather than on a socially optimal, global criterion. However, the proposed algorithm also monotonically increases the sum capacity, and usually converges to sum-capacity-maximizing codeword ensembles. While suboptimal ensembles of

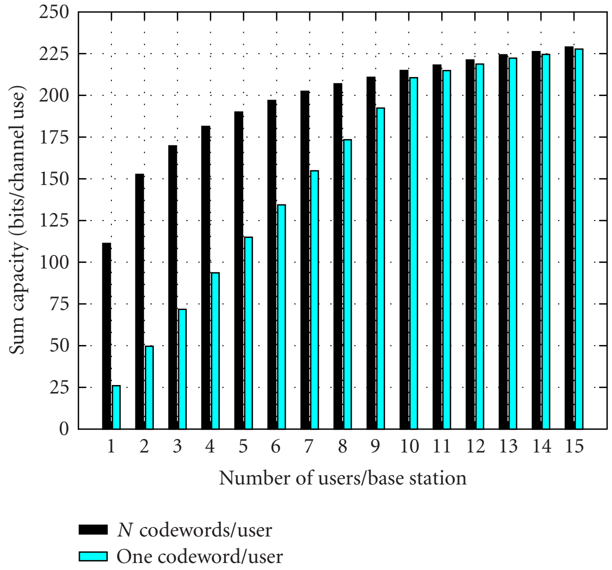

Figure 5: Sum capacity values achievable with one codeword per user and with N codewords per user for various number of users in the system for a system with $L = 8$ users and $B = 4$ bases in a signal space of dimension $N = 10$.

codewords are theoretically possible, numerical experiments have shown that these were never obtained when starting from randomly chosen initial codewords, and the algorithm has consistently yielded the sum-capacity-maximizing codeword ensembles. This is consistent with similar observations on related interference avoidance algorithms which showed convergence to the socially optimal point that corresponds to maximum sum capacity.

The algorithm requires knowledge of communication channels between users and base stations, which are considered stationary for the duration of the transmission. The algorithm is also applicable to fading channel scenarios [9, 30]. In such cases, the assumption of perfect channel knowledge made in the paper can be relaxed, and one can assume that the channel is either slowly varying, in which case channel estimates can be used for a relatively large number of transmission intervals, or that the average characteristics of the channel are known. These are reasonable assumptions for high data rate systems and environments with reduced degrees of mobility [31].

Our system model assumes that a multicode CDMA transmission scheme is employed by users, and we have also performed experiments which analyzed the penalty in terms of sum capacity that is paid when users transmit using only one codeword versus multiple codewords. We note that, when users are restricted to a single codeword and the rank of the transmit covariance matrix is one, a significant penalty in terms of sum capacity is paid when the number of users per base is small. However, increasing the number of users per base seems to allow the "full-codeword-complement" sum capacity bound to be closely approached. This suggests that for heavily loaded systems, each user need not carry a full complement of codewords, thus reducing modulation/demodulation complexity.

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REFERENCES

[1] T. Guess, "Optimal sequences for CDMA with decision-feedback receivers," *IEEE Transactions on Information Theory*, vol. 49, no. 4, pp. 886–900, 2003.

[2] G. S. Rajappan and M. L. Honig, "Signature sequence adaptation for DS-CDMA with multipath," *IEEE Journal on Selected Areas in Communications*, vol. 20, no. 2, pp. 384–395, 2002.

[3] M. Rupf and J. L. Massey, "Optimum sequence multisets for synchronous code-division multiple-access channels," *IEEE Transactions on Information Theory*, vol. 40, no. 4, pp. 1261–1266, 1994.

[4] P. Viswanath and V. Anantharam, "Optimal sequences and sum capacity of synchronous CDMA systems," *IEEE Transactions on Information Theory*, vol. 45, no. 6, pp. 1984–1991, 1999.

[5] P. Viswanath and V. Anantharam, "Optimal sequences for CDMA under colored noise: a Schur-Saddle function property," *IEEE Transactions on Information Theory*, vol. 48, no. 6, pp. 1295–1318, 2002.

[6] P. Viswanath, V. Anantharam, and D. Tse, "Optimal sequences, power control, and user capacity of synchronous CDMA systems with linear MMSE multiuser receivers," *IEEE Transactions on Information Theory*, vol. 45, no. 6, pp. 1968–1983, 1999.

[7] D. C. Popescu and C. Rose, "Codeword optimzation for uplink CDMA dispersive channels," to appear in *IEEE Transactions on Wireless Communications*.

[8] D. C. Popescu and C. Rose, "Interference avoidance and multiuser MIMO systems," *International Journal of Satellite Communications and Networking*, vol. 21, no. 1, pp. 143–161, 2003.

[9] D. C. Popescu and C. Rose, *Interference Avoidance Methods for Wireless Systems*, Kluwer Academic Publishers, New York, NY, USA, 2004.

[10] C. Rose, "CDMA codeword optimization: interference avoidance and convergence via class warfare," *IEEE Transactions on Information Theory*, vol. 47, no. 6, pp. 2368–2382, 2001.

[11] C. Rose, S. Ulukus, and R. Yates, "Wireless systems and interference avoidance," *IEEE Transactions on Wireless Communications*, vol. 1, no. 3, pp. 415–428, 2002.

[12] S. Ulukus and R. D. Yates, "Iterative construction of optimum signature sequence sets in synchronous CDMA systems," *IEEE Transactions on Information Theory*, vol. 47, no. 5, pp. 1989–1998, 2001.

[13] T. M. Cover and J. A. Thomas, *Elements of Information Theory*, Wiley-Interscience, New York, NY, USA, 1991.

[14] S. Jafar, G. J. Foschini, and A. Goldsmith, "PhantomNet: exploring optimal multicellular multiple antenna systems," in *Proc. IEEE Vehicular Technology Conference*, vol. 1, pp. 261–265, Vancouver, Canada, September 2002.

[15] S. V. Hanly and P. A. Whiting, "Information-theoretic capacity of multi-receiver networks," *Telecommunications Systems*, vol. 1, no. 1, pp. 1–42, 1993.

[16] O. Popescu and C. Rose, "Sum capacity and tsc bounds in collaborative multi-base wireless systems," to appear in *IEEE Transactions on Information Theory*.

[17] D. Tse and S. Hanly, "Multiaccess fading channels. I. polymatroid structure, optimal resource allocation and throughput capacities," *IEEE Transactions on Information Theory*, vol. 44, no. 7, pp. 2796–2815, 1998.

[18] E. A. Jorswieck and H. Boche, "Transmission strategies for the MIMO MAC with MMSE receiver: average MSE optimization and achievable individual MSE region," *IEEE Trans. Signal Processing*, vol. 51, no. 11, pp. 2872–2881, 2003.

[19] S. Serbetli and A. Yener, "Transceiver optimization for multiuser MIMO systems," *IEEE Trans. Signal Processing*, vol. 52, no. 1, pp. 214–226, 2004.

[20] W. Yu, W. Rhee, S. Boyd, and J. M. Cioffi, "Iterative waterfilling for Gaussian vector multiple-access channels," *IEEE Transactions on Information Theory*, vol. 50, no. 1, pp. 145–152, 2004.

[21] W. Yu, "Interference avoidance and iterative water filling: a connection," private communication, 2001.

[22] H. J. Landau and H. O. Pollak, "Prolate spheroidal wave functions, Fourier analysis and uncertainty—III. The dimension of the space of essentially time- and band-limited signals," *Bell System Technical Journal*, vol. 41, no. 4, pp. 1295–1336, 1962.

[23] H. Boche and E. A. Jorswieck, "Sum capacity optimization of the MIMO Gaussian MAC," in *Proc. 5th International Symposium on Wireless Personal Multimedia Communications*, vol. 1, pp. 130–134, Honolulu, Hawaii, USA, October 2002.

[24] W. Rhee, W. Yu, and J. M. Cioffi, "The optimality of beamforming in uplink multiuser wireless systems," *IEEE Transactions on Wireless Communications*, vol. 3, no. 1, pp. 86–96, 2004.

[25] U. Madhow and M. L. Honig, "MMSE interference suppression for direct-sequence spread-spectrum CDMA," *IEEE Trans. Communications*, vol. 42, no. 12, pp. 3178–3188, 1994.

[26] D. C. Popescu, O. Popescu, and C. Rose, "Interference avoidance for multiaccess vector channels," in *Proc. IEEE International Symposium on Information Theory*, p. 499, Lausanne, Switzerland, July 2002.

[27] R. A. Horn and C. R. Johnson, *Matrix Analysis*, Cambridge University Press, Cambridge, UK, 1985.

[28] G. Strang, *Linear Algebra and its Applications*, Harcourt Brace Jovanovich College Publishers, San Diego, Calif, USA, 3rd edition, 1988.

[29] S. Boyd and L. Vandenberghe, *Convex Optimization*, Cambridge University Press, Cambridge, UK, 2004.

[30] D. C. Popescu and C. Rose, "Fading channels and interference avoidance," in *Proc. Allerton Conference on Communication, Control, and Computing*, pp. 1073–1074, Monticello, Ill, USA, October 2001.

[31] H. Hashemi, "The indoor radio propagation channel," *Proceedings of the IEEE*, vol. 81, no. 7, pp. 943–968, 1993.

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