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Factor-Graph-Based Soft Self-Iterative Equalizer for Multipath Channels

Ben Lu
Silicon Laboratories, Inc., Austin, TX 78735, USA
Email: ben.lu@silabs.com

Guosen Yue
NEC Laboratories America, Inc., Princeton, NJ 08540, USA
Email: yueg@nec_labs.com

Xiaodong Wang
Department of Electrical Engineering, Columbia University, New York, NY 10027, USA
Email: wangx@ee.columbia.edu

Mohammad Madihian
NEC Laboratories America, Inc., Princeton, NJ 08540, USA
Email: madihian@nec-labs.com

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We consider factor-graph-based soft self-iterative equalization in wireless multipath channels. Since factor graphs are able to characterize multipath channels to per-path level, the corresponding soft self-iterative equalizer possesses reduced computational complexity in sparse multipath channels. The performance of the considered self-iterative equalizer is analyzed in both single-antenna and multiple-antenna multipath channels. When factor graphs of multipath channels have no cycles or mild cycle conditions, the considered self-iterative equalizer can converge to optimum performance after a few iterations; but it may suffer local convergence in channels with severe cycle conditions.

Keywords and phrases: factor graph, equalizer, iterative processing, multipath fading, MIMO.

1. INTRODUCTION

A multipath fading channel, which can be mathematically described by a convolution of transmitted signals and linear channel response, is one of many typical channel models occurring in digital communications. In general, an equalizer that makes detection based on a number of adjacent received symbols is necessary to achieve optimal or near-optimal performance in multipath channels. In classical communication theory, different representations of multipath channels have led to equalizers with different designs. By representing multipath channels as trellis structures, the optimum sequence detector can be computed by the Viterbi algorithm [1], and the optimum symbol detector can be computed by BCJR algorithm [2]. Starting from the transfer function representation of linear multipath systems, people proposed various low-complexity designs such as linear zero-forcing (ZF) equalizer, linear minimum mean-square-error (MMSE) equalizer, nonlinear zero-forcing decision feedback equalizer (ZF-DFE), non-linear MMSE-DFE, and so forth. [3]. In this work, the multipath channels are represented by factor graphs, and soft self-iterative equalizers that execute belief propagation algorithm on factor graphs are studied. (Please refer to [4] for an excellent tutorial on factor graph and its applications.)

One question might rise regarding the motivation of this work, since we have already had both Viterbi algorithm and BCJR algorithm as exact optimum equalizers. The answer to this question lies in the flexibility of factor graph in characterizing multipath channels to per-path level. As a well-known fact, the computational complexity of Viterbi and BCJR algorithms are exponential in the total number of multipaths L. In practice, there exist cases when only L' out of L paths (with $L' < L$) have significant channel gains and moreover the location of these significant L' paths can be slowly changing in time, for example, rural wireless channels. Then, a reduced-complexity equalizer that avoids or reduces the computations spent on those zero multipath taps is desirable. Some efforts along this direction have been made in earlier works, for example, parallel Viterbi and parallel BCJR algorithms in [5, 6], which however may require specifically designed control logic for a different multipath scenario. In the considered factor-graph-based soft iterative equalizer, the log-likelihood probabilities are passed as messages in factor graphs between channel nodes and information nodes only along the edges that correspond to paths with significant gain, thus it inherently results in a complexity reduction owing to the sparseness of multipath channels. In particular, we consider three schemes to compute the messages passed from channel nodes to information nodes, namely the scheme based on the a posteriori probability (APP) algorithm, the one based on the linear-MMSE-soft-interference-cancelation (LMMSE-SIC), and the one based on match-filter-soft-interference-cancelation (MF-SIC); and we analyze their performance and applicabilities in practical multipath channels.

One main focus of this paper is the effect of cycles that existed in factor graph on the equalization performance. As compared to the Viterbi and BCJR algorithms which themselves are belief propagation algorithms operating in trellis trees of multipath channels and guarantee the optimum performance, the belief propagation algorithm operating in factor graphs guarantee global optimality only if the underlying factor graph is a tree. Although the condition of factor graph being a tree (i.e., without cycles) is not always met in practice, the factor-graph-based belief propagation algorithm has achieved great success in decoding cycle-contained linear turbo codes and low-density parity-check (LDPC) codes. For the considered self-iterative equalizer, we quantitatively analyze the cycle effect in single-input single-output (SISO), multiple-input single-output (MISO), and multiple-input multiple-output (MIMO) wireless systems; and discuss an alternative representation of factor graphs that ameliorates the performance degradation due to cycle effects.

While it bears similarities to various iterative receivers developed earlier, for example, [7, 8, 9, 10], we highlight that the soft self-iterative equalizer is a *self-iterative* device which successively improves the equalization performance by taking advantage of the constraints in received signals due to multipaths, instead of other constraints for instance imposed by error-control coding. Moreover, since the considered equalizer inputs prior and outputs a posteriori probabilities of information symbols, it can easily concatenate with other receiver modules to achieve the turbo receiver processing gains [8].

The rest of this paper is organized as follows. In Section 2, the system model and factor graph representation of multipath channels are described. In Section 3, the factor-graph-based soft iterative equalizer is derived. In Section 4, the performance of the soft self-iterative equalizer is analyzed by numerical simulations for both single-antenna and multiple-antenna systems. Finally, Section 5 contains the conclusions.

2. SYSTEM MODEL AND FACTOR GRAPH REPRESENTATION

Assume match-filtering and symbol-rate sampling, the received signals of multipath channels are normally described by the following time-domain equation [11]:

$$y_t = \sum_{l=0}^{L-1} h_{t,l} x_{t-l} + n_t, \quad t = 1, 2 \ldots, T, \tag{1}$$

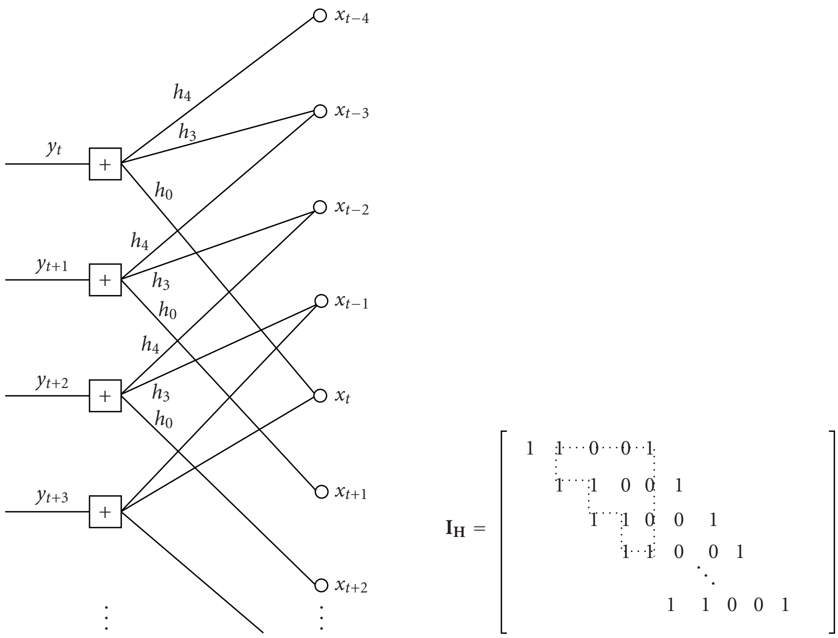
where $y_t \in \mathcal{C}$ and $x_t \in \Omega$ are the receive and transmit signals at time t, respectively; Ω is the modulation set; $h_{t,l} \in \mathcal{C}$ is the channel impulse response with delay of l times the symbol rate at time t; $n_t \in \mathcal{C} \sim \mathcal{N}(0, \sigma^2)$ is the zero-mean σ-variance circularly symmetrical Gaussian ambient noise that has been properly whitened and is independent of data; L is the total number of multipaths; T is the frame length. In this paper, we are concerned with block signal processing, and assume that zero prefix is inserted in each signal frame, that is, $x_t = 0$, $t = -L+1, \ldots, -1$. For ease of comparison, we also assume that channel gain is properly normalized: in static channels, $\sum_{l=0}^{L-1} |h_{t,l}|^2 = 1$; and in fading channels, $\sum_{l=0}^{L-1} \mathrm{E}(|h_{t,l}|^2) = 1$, where $\mathrm{E}(\cdot)$ denotes the expectation over random variables $h_{t,l}$, for all l. As mentioned earlier, we only consider uncoded systems in this work, thus x_t have equal prior probabilities and are assumed to be independent for different t.

Equivalently, (1) can be written in a matrix form as

$$\begin{bmatrix} y_1 \\ \vdots \\ y_t \\ \vdots \\ y_T \end{bmatrix} = \underbrace{\begin{bmatrix} h_{1,L-1} & h_{1,L-2} & \cdots & h_{1,0} & & & \\ & \ddots & \ddots & & \ddots & & \\ & & h_{t,L-1} & h_{t,L-2} & \cdots & h_{t,0} & \\ & & & \ddots & \ddots & & \ddots \\ & & & & h_{T,L-1} & h_{T,L-2} \cdots & h_{T,0} \end{bmatrix}}_{\mathbf{H}} \times \begin{bmatrix} x_{-L+2} \\ \vdots \\ x_{t-L+2} \\ \vdots \\ x_T \end{bmatrix} + \begin{bmatrix} n_1 \\ \vdots \\ n_t \\ \vdots \\ n_T \end{bmatrix}, \tag{2}$$

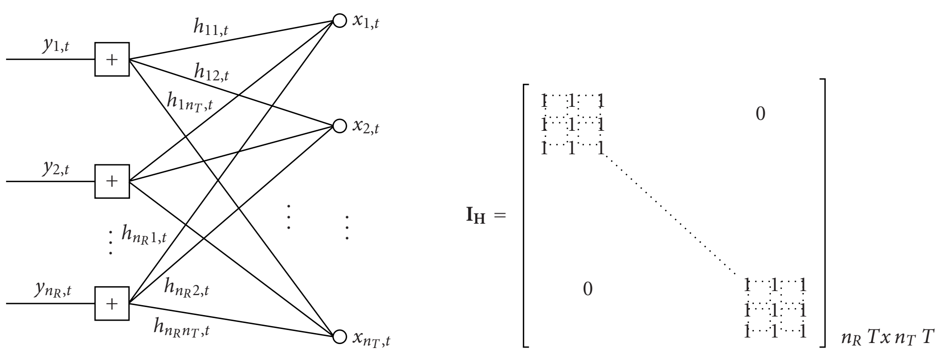
where $\mathbf{H}$ is a $T \times (T+L-1)$ Toeplitz matrix. Throughout this paper, we assume that $\mathbf{H}$ is perfectly known to the receiver, and $h_{t,l}$, for all t, l, can be either time invariant or time variant within each signal frame. In addition, we define $\mathbf{I}_\mathbf{H}$ as the incidence matrix of $\mathbf{H}$, such that $\{\mathbf{I}_\mathbf{H}\}_{i,j} = 1$, if $|\{\mathbf{H}\}_{i,j}|^2 > 0$; $\{\mathbf{I}_\mathbf{H}\}_{i,j} = 0$, otherwise. $\mathbf{I}_\mathbf{H}$ will later be used to help explain the cycle effects of the factor-graph-based soft equalizer.

In the above, we described single-input single-output (SISO) multipath systems. Without much difficulty, (1)



(a) (b)

Figure 1: (a) The factor graph representation and (b) the incidence matrix of a single-antenna multipath channel: $y_t = h_0 x_t + h_3 x_{t-3} + h_4 x_{t-4} + n_t$.



(a) (b)

Figure 2: (a) The factor graph representation and (b) the incidence matrix of an $n_T \times n_R$ single-path MIMO channel.

and (2) as well as $\mathbf{I_H}$ can be extended to multiple-input single-output (MISO) and multiple-output multiple-output (MIMO) cases, by simply replacing y_t, $h_{t,l}$, x_t, n_t with their matrix/vector counterparts $\mathbf{y}_t$, $\mathbf{h}_{t,l}$, $\mathbf{x}_t$, $\mathbf{n}_t$. As a result, $\mathbf{H}$ and $\mathbf{I_H}$ now become $N_r T \times N_t \cdot (T + L - 1)$ matrices, where N_r and N_t are the number of receive and transmit antennas, respectively.

The above multipath channels in (1) and (2) can also be depicted by factor graphs. The example of the factor graph representations of SISO multipath and MIMO single-path channels are given in Figures 1 and 2. There are two types of nodes in the factor graph: the channel nodes for y_t, for all t, and the information nodes x_t, for all t. An edge connects channel node t and information node t', only if the channel gain is significant, that is, $|h_{t-t',l}|^2 > 0$. We remark that by no means the factor graphs shown in the figures are unique representation of the corresponding multipath channels; indeed, different representations of the same multipath channel lead to different designs of the factor-graph-based soft iterative equalizer, which we will discuss in Section 4.3.

3. SOFT SELF-ITERATIVE EQUALIZER BASED ON FACTOR GRAPH

The considered soft self-iterative equalizer computes the marginal probabilities of information symbol $\{x_t\}_{t=0}^{T}$ based on prior probabilities of the receive signals $\{y_t\}_{t=0}^{T}$ and $\{x_t\}_{t=0}^{T}$, by executing belief propagations in factor graphs. (As a comparison, both Viterbi algorithm and BCJR algorithm execute belief propagation in trellis trees.)

The messages, defined as the log-likelihood ratio (LLR) of information symbols, are iteratively passed among the nodes in factor graphs, such as to compute the marginal probabilities of information symbols. For BPSK modulation, the message is 1-tuple. In this paper, we will mainly study the complex modulation schemes such as MPSK and MQAM for which the message is $\log_2|\Omega|$-tuple. Let $m_{ci}^{(p)}$ be the message passed from the channel node c to the information node i at the pth iteration, $m_{ci}^{(p)} \triangleq (m_{ci,0}^{(p)}, m_{ci,1}^{(p)}, \ldots, m_{ci,\log_2|\Omega|-1}^{(p)})$; and it is updated as

$$\begin{aligned} m_{ci,k}^{(p)} &\triangleq F_{ci,k}\left(y_c, m_{i'c}^{(p)},\ \forall i' \in \mathcal{U}_c\right) \\ &= \log \frac{\Pr\left[b_{i,k}=0 \mid y_c, m_{i'c}^{(p-1)},\ \forall i' \in \mathcal{U}_c \setminus \{i\},\ m_{ic,k'}^{(p-1)},\ \forall k' \neq k\right]}{\Pr\left[b_{i,k}=1 \mid y_c, m_{i'c}^{(p-1)},\ \forall i' \in \mathcal{U}_c \setminus \{i\},\ m_{ic,k'}^{(p-1)},\ \forall k' \neq k\right]}, \\ &\qquad \forall k, \end{aligned} \tag{3}$$

where the mapping function from $\log_2|\Omega|$-tuple $(b_{i,0}, \ldots, b_{i,\log_2|\Omega|-1})$ to complex symbol x_i is usually referred to as modulation format; $m_{i'c}$ is the message sent from information node i' to channel node c, as explained next; $\mathcal{U}_c$ denotes the set of all information nodes incident to channel node c, $\mathcal{U}_c \setminus \{i\}$ denotes $\mathcal{U}_c$ excluding information node i; and y_c is the received signal at time c. The message update rule in (3) follows the general principle of a belief propagation algorithm, that is, the component message $m_{ci,k}^{(p)}$ sent from channel node c to information node i is updated based on received signal y_c and all incident messages to channel node c except for the same incident component message $m_{ci,k}^{(p-1)}$. Similarly, we let $m_{ic}^{(p)}$ be the message passed from the information node i to the channel node c at the pth iteration, $m_{ic}^{(p)} \triangleq (m_{ic,0}^{(p)}, m_{ic,1}^{(p)}, \ldots, m_{ic,\log_2|\Omega|-1}^{(p)})$; and it is updated as

$$\begin{aligned} m_{ic,k}^{(p)} &\triangleq G_{ic,k}\left(m_i^{(0)}, m_{c'i}^{(p)},\ \forall c' \in \mathcal{V}_i\right) \\ &= \log \frac{\Pr\left[b_{i,k}=0 \mid m_i^{(0)}, m_{c'i}^{(p-1)},\ \forall c' \in \mathcal{V}_i \setminus \{c\},\ m_{ci,k'}^{(p-1)},\ \forall k' \neq k\right]}{\Pr\left[b_{i,k}=1 \mid m_i^{(0)}, m_{c'i}^{(p-1)},\ \forall c' \in \mathcal{V}_i \setminus \{c\},\ m_{ci,k'}^{(p-1)},\ \forall k' \neq k\right]}, \\ &\qquad \forall k, \end{aligned} \tag{4}$$

where m_i^0 denotes the prior probabilities of the ith information symbol, input from other receiver modules (e.g., a channel decoder); $\mathcal{V}_i$ denotes the set of all channel nodes incident to information node i.

In (4), assume that the messages $m_i^{(0)}$ and $m_{c'i}^{(p-1)}$, for all c' are independent random variables, then we have

$$G_{ic,k}\left(m_{i,k}^{(0)}, m_{c'i,k}^{(p)},\ \forall c' \in \mathcal{V}_i\right) = m_{i,k}^{(0)} + \sum_{c' \in \mathcal{V}_i \setminus \{c\}} m_{c'i,k}^{(p)}. \tag{5}$$

On the other hand, we have the following three different approaches, that is, *a-posteriori*-probability- (APP-) based scheme, linear-MMSE-soft-interference-cancellation- (LMMSE-SIC-)based scheme, and match-filter-soft-interference-cancellation- (MF-SIC-)based scheme, to compute (3), that is,

$$\begin{aligned} &F_{ci,k}\left(y_c, m_{i'c}^{(p)},\ \forall i' \in \mathcal{U}_c\right) \\ &= \begin{cases} \log \dfrac{\sum_{x_{i'} \in \mathcal{Q}_{i,k}^{+}} \exp\left(-\left|y_c - \sum_{i' \in \mathcal{U}_c} h_{c,c-i'} x_{i'}\right|^2/\sigma^2 + \sum_{k=0}^{\log_2|\Omega|-1} b_{i',k} \cdot m_{i'c,k}^{(p-1)}/2\right)}{\sum_{x_{i'} \in \mathcal{Q}_{i,k}^{-}} \exp\left(-\left|y_c - \sum_{i' \in \mathcal{U}_c} h_{c,c-i'} x_{i'}\right|^2/\sigma^2 + \sum_{k=0}^{\log_2|\Omega|-1} b_{i',k} \cdot m_{i'c,k}^{(p-1)}/2\right)} - m_{ic,k}^{(p-1)}, & \text{for APP}, \\ \log \dfrac{\sum_{x_i \in \mathcal{S}_{i,k}^{+}} \exp\left(-\left|w_{c,i}^*(y_c - \tilde{y}_c) - \mu_{c,i} x_i\right|^2/\nu_{c,i}^2 + \sum_{k=0}^{\log_2|\Omega|-1} b_{i,k} \cdot m_{ic,k}^{(p-1)}/2\right)}{\sum_{x_i \in \mathcal{S}_{i,k}^{-}} \exp\left(-\left|w_{c,i}^*(y_c - \tilde{y}_c) - \mu_{c,i} x_i\right|^2/\nu_{c,i}^2 + \sum_{k=0}^{\log_2|\Omega|-1} b_{i,k} \cdot m_{ic,k}^{(p-1)}/2\right)} - m_{ic,k}^{(p-1)}, & \text{for LMMSE-SIC, MF-SIC}, \end{cases} \end{aligned} \tag{6}$$

and for LMMSE-SIC,

$$\begin{aligned} w_{c,i}^* &= \frac{h_{c,c-i}^*}{\sum_{i' \in \mathcal{U}_c \setminus \{i\}} \left|h_{c,c-i'}\right|^2 \left(1 - \left|\tilde{x}_{c-i'}\right|^2\right) + \left|h_{c,c-i}\right|^2 + \sigma^2}, \\ \mu_{c,i} &= w_{c,i}^* h_{c,c-i}, \qquad \nu_{c,i}^2 = \mu_{c,i} - \mu_{c,i}^2, \end{aligned} \tag{7}$$

and for MF-SIC,

$$\begin{aligned} w_{c,i}^* &= \frac{h_{c,c-i}^*}{\left|h_{c,c-i}\right|^2}, \qquad \mu_{c,i} = 1, \\ \nu_{c,i}^2 &= \frac{\sum_{i' \in \mathcal{U}_c \setminus \{i\}} \left|h_{c,c-i'}\right|^2 \left(1 - \left|\tilde{x}_{c-i'}\right|^2\right) + \sigma^2}{\left|h_{c,c-i}\right|^2}, \end{aligned} \tag{8}$$

Initialize: for all edges
$m_{ic}^{(0)} = 0$
for all edges
$m_{ci}^{(0)} = F_{ci}(y_c, m_{i'c}^{(0)}, \ \forall i' \in \mathcal{U}_c)$

Self-iterative equalize:
for $p = 1$ to P
/* compute messages from channel nodes to
information nodes */ for all edges
$m_{ci}^{(p)} = F_{ci}(y_c, m_{i'c}^{(p)}, \ \forall i' \in \mathcal{U}_c)$
/* compute messages from
information nodes to channel
nodes */ for all edges
$m_{ic}^{(p)} = G_{ic}(m_i^{(0)}, m_{c'i}^{(p)}, \ \forall c' \in \mathcal{V}_i)$
end

Output: /* compute information symbols' a posteriori
probabilities $m_i^{(P)}$ */ for $i = 0$ to T
$m_i^{(P)} = \sum_{c' \in \mathcal{V}_i} m_{c'i}^{(p)}$
end

Algorithm 1: Algorithm description of the factor-graph-based soft self-iterative equalizer.

with $\tilde{y}_c = \sum_{i' \in \mathcal{U}_c \setminus \{i\}} h_{c,c-i'} \tilde{x}_{c-i'}$,

$$\tilde{x}_i = \sum_{x_i \in \Omega} x_i \prod_{k=0}^{\log_2 |\Omega| - 1} \frac{b_{i,k} m_{ic,k}^{(p-1)}}{1 + b_{i,k} m_{ic,k}^{(p-1)}}, \qquad (9)$$

where $\mathcal{S}_{i,k}^{+}$ is the set defined as $\{x_i \in \Omega \mid b_{i,k} = 0\}$, and similarly is $\mathcal{S}_{i,k}^{-}$; $\mathcal{Q}_{i,k}^{+}$ is the union of $\{x_{i'} \in \Omega \mid$ for all $i' \in \mathcal{U}_c \setminus \{i\}\}$ and $\mathcal{S}_{i,k}^{+}$, and similarly is $\mathcal{Q}_{i,k}^{-}$. The detailed derivation of (6) is shown in the appendix.

Finally, the whole steps of the proposed equalizer are given in Algorithm 1.

4. NUMERICAL SIMULATIONS AND ANALYSIS

In this section, we analyze the factor-graph-based soft self-iterative equalizer in sparse wireless multipath channels through numerical simulations. For simplicity, we assume that channel gains remain constant in one frame and change independently from one to the other. The modulator uses the QPSK constellation with Gray mapping. Each frame contains 128 QPSK symbols per transmit antenna; proper zero prefix information symbols are inserted in each frame. The soft equalizer is a self-iterative device; and we only study the uncoded system. The performance is evaluated in terms of frame error rate (FER) versus the signal-to-noise ratio (SNR).

4.1. *SISO multipath fading channels*

First, consider a sparse 4-path fading channel: $y_t = h_0 x_t + h_3 x_{t-3} + n_t$, with $\mathrm{E}\{|h_0|^2\} = 0.8$, $\mathrm{E}\{|h_3|^2\} = 0.2$; thus, $L = 4$ and $L' = 2$. In Figure 3, the performance of three different approaches, (i.e., APP, LMMSE-SIC, and MF-SIC), to computing the extrinsic messages passed from channel nodes to information nodes is presented. For each scheme, total six iterations, that is, $P = 6$, are conducted in the self-iterative equalizer. Serving as a benchmark, the performance of the optimum maximum likelihood equalizer based on BCJR algorithm is also included in the figure. Since the factor graph of this channel is cycle free, the belief propagation algorithm theoretically is able to achieve optimum performance. Indeed, the soft iterative equalizer using APP-based message update scheme achieves the optimum performance after a few iterations. On the contrary, two low-complexity schemes, LMMSE-SIC and MF-SIC, suffer error floors at high SNRs. We remark that the prior probability input from other receiver modules (e.g., channel decoder) can lower but never eradicate such error floors; henceforth we will only consider the APP-based scheme for channel node message updating.

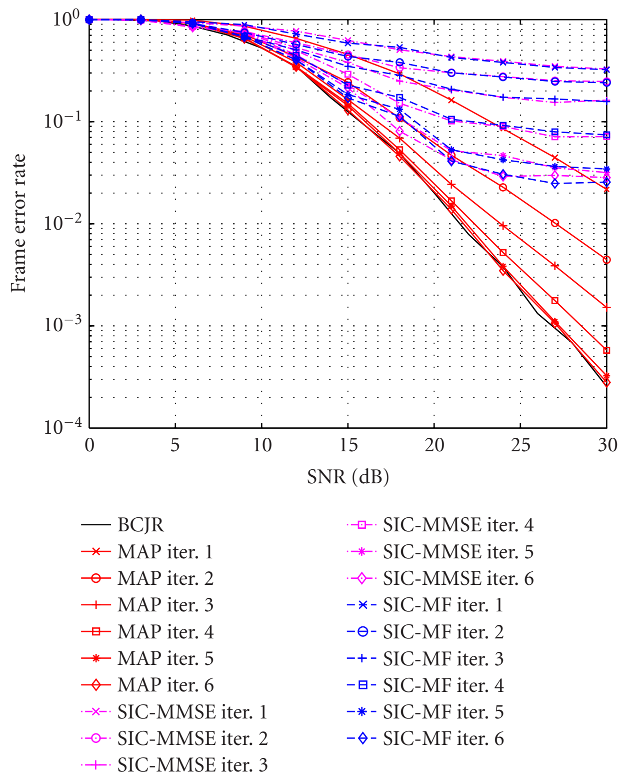


Figure 3: FER performance of the factor-graph-based soft iterative equalizer in SISO multipath fading channels ($n_T = 1$, $n_R = 1$, $L = 4$, $L' = 2$).

Now, consider a sparse 5-path fading channel: $y_t = h_0 x_t + h_3 x_{t-3} + h_4 x_{t-4} + n_t$, where $\mathrm{E}\{|h_0|^2\} = 0.7$, $\mathrm{E}\{|h_3|^2\} = 0.2$, and $\mathrm{E}\{|h_4|^2\} = 0.1$; thus, $L = 5$ and $L' = 3$. As seen in Figure 1, there exist a number of cycles with length 8 in the factor graph, where a "cycle" is defined as a close loop in the graph and its "length" is defined as the number of edges traversed by that loop. This cycle condition accounts for the marginal gap between the factor-graph-based equalization and the optimum performance, as shown in Figure 4.

4.2. *MISO multipath fading channels*

Equalization of MISO multipath channels falls into the group of "underdetermined" problems: at each time instance a mix-

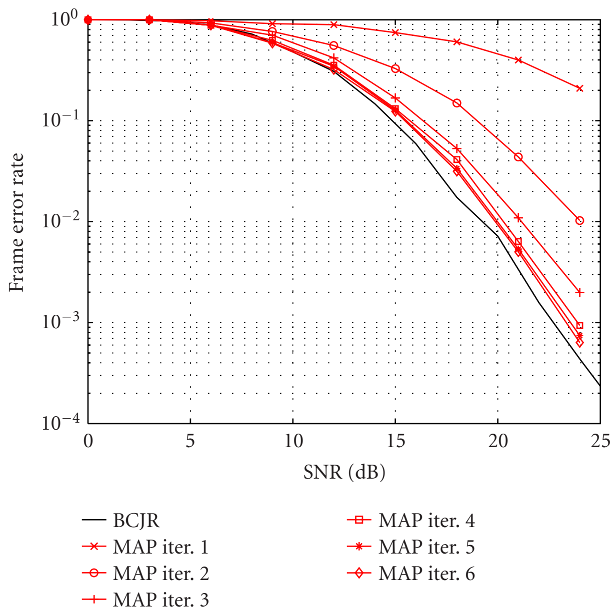


FIGURE 4: FER performance of the factor-graph-based soft iterative equalizer in SISO multipath fading channels ($n_T = 1, n_R = 1, L = 5, L' = 3$).

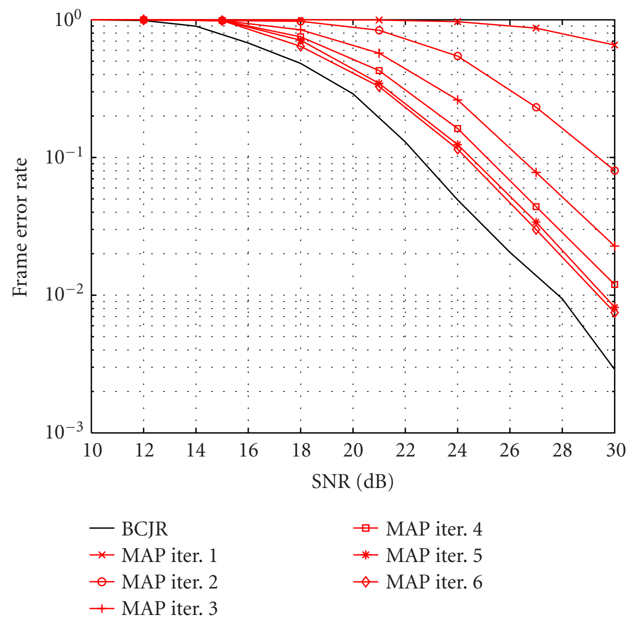


FIGURE 5: FER performance of the factor-graph-based soft iterative equalizer in MISO multipath fading channels ($n_T = 2, n_R = 1, L = 3$).

ture of plural information symbols that transmitted with different delays and from different antennas is to be detected from a single-receiver observation. Conventional linear equalization or decision-feedback-cancellation equalization schemes would lead to unsatisfactory performance, whereas an optimal equalizer has complexity exponential in $(L-1)\cdot n_T$. When MISO multipath channels exhibit sparseness, the factor-graph-based soft equalizer becomes potentially attractive, as it can reduce the complexity exponent to $(L'-1)\cdot n_T$.

We consider a two-transmit-one-receive-antenna (2×1) MISO system in a sparse 3-path fading. Every transmit-receive antenna pair follows the same multipath profile, that is, $\mathrm{E}\{|h_0|^2\} = 0.8$, and $\mathrm{E}\{|h_2|^2\} = 0.2$; fading coefficients for different paths and different antenna pairs are assumed to be mutually independent. The performance is illustrated in Figure 5. It is seen that after a few iterations the considered factor-graph-based equalizer performs slightly more than one dB away from the optimum equalizer. Again, this performance gap is due to the existence of length-4 cycles in the factor graphs. It is worth to remark that the complexity of optimum BCJR equalizer soon becomes prohibitive for (2×1) MISO systems with QPSK modulation and $L > 3$ multipaths; in comparison, the complexity exponent of factor-graph-based equalizer is proportional to L', hence in sparse channels it is strictly lower than the original L.

4.3. MIMO multipath fading channels

Recently, there has been increasing interest in developing MIMO equalization schemes in multipath channels. We analyze the performance of the factor-graph-based equalizer as below. First, we consider $(n_T \times n_R)$ MIMO systems in single path fading channels. It is easily seen from Figure 2 that the incidence matrix $\mathbf{I_H}$ contains length-4 cycles everywhere; and the cycle condition worsens as more antennas are employed. To the best of our knowledge, little efforts have been made to rigorously quantify the cycle condition of factor graphs. Empirically, the cycle condition is better, if the length of cycles is increased, or given the cycle length, the number of cycles is reduced, or the cycles have a larger number of edges connecting to rest of the graph. However, by and large, the combined effect of these empirical assertions is unclear; we then have to resort to numerical simulations. It is seen from Figures 6 and 7 that the considered self-iterative equalizer approaches optimum demodulation performance in (2×2) MIMO channels, but it suffers considerable performance loss in 4×4 MIMO channels. Especially from the (4×4) MIMO case, we conclude that the direct application of the factor-graph-based equalizer may not be a good option for MIMO channels. It is seen from Figure 8 that the above observation also holds for MIMO multipath channels—as much as 2.5 dB performance loss is seen in a (2×2) MIMO with 3 multipaths.

Alternative factor graph representation for MIMO multipath fading channels

The previous simulation results and analysis has identified the difficulty in directly applying the factor-graph-based equalizer in MIMO channels. An alternative way to ameliorate this problem is to reconstruct the underlying factor graphs. Shown in Figure 9 the idea is to glue all channel nodes in the original graph $\{y_{1,t},\ldots,y_{n_R,t}\}$ that corresponds to different receiver antennas at the same time instance t into

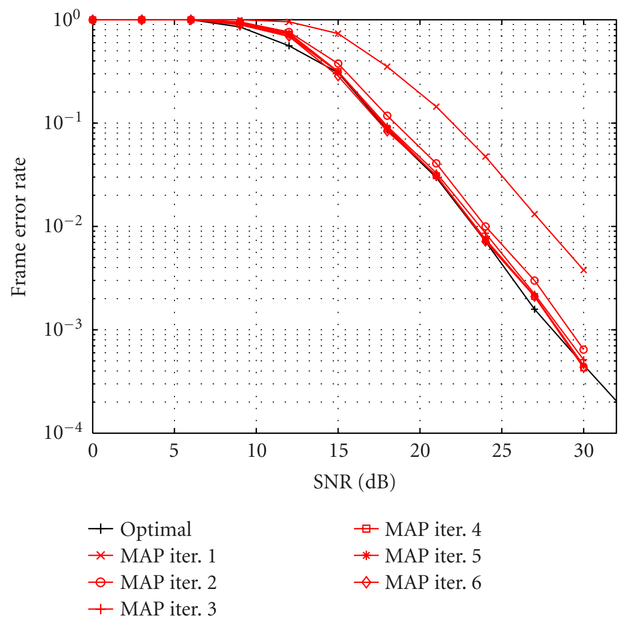


Figure 6: FER performance of the factor-graph-based soft iterative equalizer in MIMO multipath fading channels ($n_T = 2$, $n_R = 2$, $L = 1$).

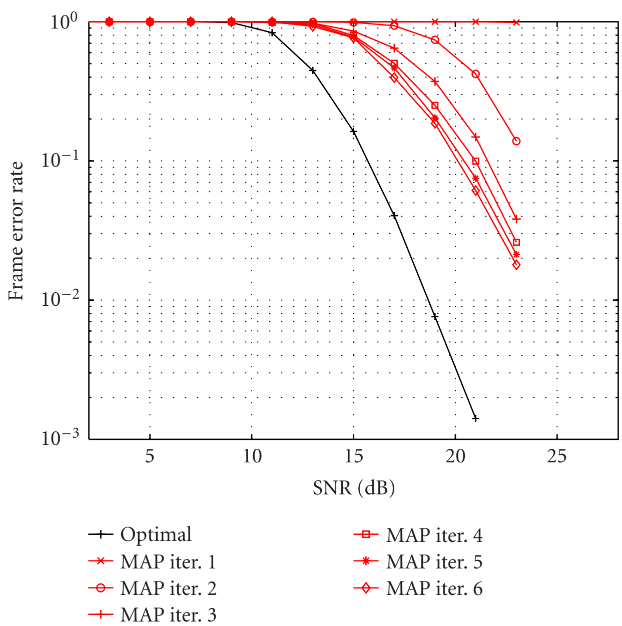


Figure 7: FER performance of the factor-graph-based soft iterative equalizer in MIMO multipath fading channels ($n_T = 4$, $n_R = 4$, $L = 1$).

a new channel node $\mathbf{y}_t \triangleq [y_{1,t}, \ldots, y_{n_R,t}]^T$; the channel coefficient on each edge is now an ($n_R \times 1$) vector instead of a scalar. In doing so, the alternative factor graph still represents the same MIMO multipath systems, but the extensive short cycles due to multiple receive antennas are systematically avoided. The belief propagation algorithm can be accordingly rederived; and in single-path channels, it converges

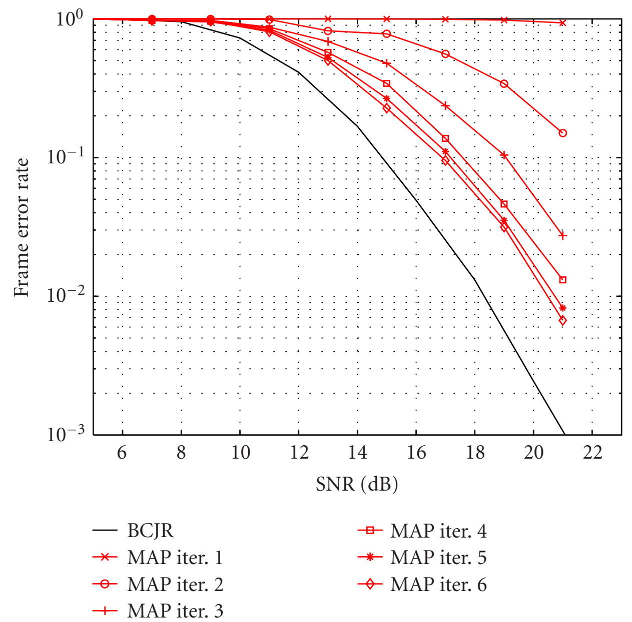


Figure 8: FER performance of the factor-graph-based soft iterative equalizer in MIMO multipath fading channels ($n_T = 2$, $n_R = 2$, $L = 3, L' = 2$).

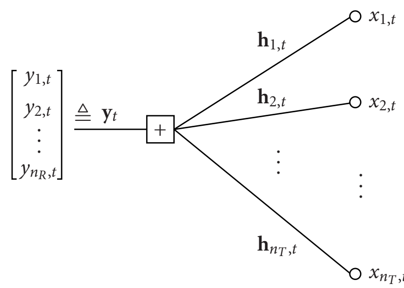


Figure 9: The alternative factor graph representation of an $n_T \times n_R$ single-path MIMO channel. Compared to Figure 2, here all channel nodes $\{y_{1,t}, y_{2,t}, \ldots, y_{n_R,t}\}$ that correspond to different receiver antennas at the same time instance t are glued to form a new channel node $\mathbf{y}_t$.

in one iteration and coincides with the optimal APP MIMO demodulator [12]. With this alternative factor graph representation, we can continue to apply the self-iterative equalizer for MIMO multipath fading channels to improve the performance. We now consider the case of (2×2) MIMO with 3 multipaths as an example. The FER curves are shown in Figure 10. It is seen that the resulting performance is significantly improved and approaches the performance from the optimum demodulation.

5. CONCLUSIONS

Since a factor graph is able to characterize multipath channels to per-path level, the factor-graph-based soft self-iterative equalizer with reduced computational complexity is a potential candidate for sparse multipath channel equalization. By numerical simulations, we have shown that the cycles in factor graphs are crucial to the convergence property of the considered soft self-iterative equalization. While being able to achieve near-optimum performance in single-input single-output (SISO) and multiple-input single-output (MISO) sparse multipath channels with mild cycle conditions, a factor-graph-based soft self-iterative equalizer may suffer noticeable performance loss in multiple-input multiple-output (MIMO) multipath channels, unless proper means is taken to ameliorate the cycle conditions in factor graphs.

APPENDIX

DERIVATION OF (6)

(i) For APP detection, we have

$$\begin{aligned}
&F_{ci,k}\left(y_c, m_{i'c}^{(p)},\ \forall i' \in \mathcal{U}_c\right)\\
&\quad = \log \frac{\sum_{x_{i'} \in \mathcal{Q}_{i,k}^{+}} P\left(x_{ci} = x_{i'} \mid y_c\right)}{\sum_{x_{i'} \in \mathcal{Q}_{i,k}^{-}} P\left(x_{ci} = x_{i'} \mid y_c\right)} - \underbrace{\log \frac{P\left(b_{i,k} = +1\right)}{P\left(b_{i,k} = -1\right)}}_{m_{ic,k}^{(p-1)}}\\
&\quad = \log \frac{\sum_{x_{i'} \in \mathcal{Q}_{i,k}^{+}} P\left(y_c \mid x_{ci} = x_{i'}\right) P\left(x_{ci} = x_{i'}\right)}{\sum_{x_{i'} \in \mathcal{Q}_{i,k}^{-}} P\left(y_c \mid x_{ci} = x_{i'}\right) P\left(x_{ci} = x_{i'}\right)} - m_{ic,k}^{(p-1)}\\
&\quad = \log \frac{\sum_{x_{i'} \in \mathcal{Q}_{i,k}^{+}} \exp\left(-\left| y_c - \sum_{i' \in \mathcal{U}_c} h_{c,c-i'} x_{i'}\right|^2/\sigma^2\right) \prod_{x_{i'} \in \mathcal{Q}_{i,k}^{+}} P\left(b_{i',k}^{(p-1)}\right)}{\sum_{x_{i'} \in \mathcal{Q}_{i,k}^{-}} \exp\left(-\left| y_c - \sum_{i' \in \mathcal{U}_c} h_{c,c-i'} x_{i'}\right|^2/\sigma^2\right) \prod_{x_{i'} \in \mathcal{Q}_{i,k}^{-}} P\left(b_{i',k}^{(p-1)}\right)} - m_{ic,k}^{(p-1)}\\
&\quad = \log \frac{\sum_{x_{i'} \in \mathcal{Q}_{i,k}^{+}} \exp\left(-\left| y_c - \sum_{i' \in \mathcal{U}_c} h_{c,c-i'} x_{i'}\right|^2/\sigma^2 + \sum_{k=0}^{\log_2|\Omega|-1} b_{i',k}\cdot m_{i'c,k}^{(p-1)}/2\right)}{\sum_{x_{i'} \in \mathcal{Q}_{i,k}^{-}} \exp\left(-\left| y_c - \sum_{i' \in \mathcal{U}_c} h_{c,c-i'} x_{i'}\right|^2/\sigma^2 + \sum_{k=0}^{\log_2|\Omega|-1} b_{i',k}\cdot m_{i'c,k}^{(p-1)}/2\right)} - m_{ic,k}^{(p-1)}.
\end{aligned} \tag{A.1}$$

(ii) For LMMSE-SIC detection, we first obtain the MMSE filtering output, given by

$$z_{c,i} = w_{c,i}^{*}\left(y_c - \tilde{y}_c\right). \tag{A.2}$$

Based on Gaussian approximation of z_{c_i}, the extrinsic messages can be computed by

$$\begin{aligned}
&F_{ci,k}\left(y_c, m_{i'c}^{(p)},\ \forall i' \in \mathcal{U}_c\right)\\
&\quad = \log \frac{\sum_{x_i \in \mathcal{S}_{i,k}^{+}} \exp\left(-\left| z_{c,i} - \mu_{c,i} x_i\right|^2/\nu_{c,i}^2\right) \prod_{x_i \in \mathcal{S}_{i,k}^{+}} P\left(b_{i,k}^{(p-1)}\right)}{\sum_{x_i \in \mathcal{S}_{i,k}^{-}} \exp\left(-\left| z_{c,i} - \mu_{c,i} x_i\right|^2/\nu_{c,i}^2\right) \prod_{x_i \in \mathcal{S}_{i,k}^{-}} P\left(b_{i,k}^{(p-1)}\right)} - m_{ic,k}^{(p-1)}\\
&\quad = \log \frac{\sum_{x_i \in \mathcal{S}_{i,k}^{+}} \exp\left(-\left| w_{c,i}^{*}\left(y_c - \tilde{y}_c\right) - \mu_{c,i} x_i\right|^2/\nu_{c,i}^2 + \sum_{k=0}^{\log_2|\Omega|-1} b_{i,k} m_{ic,k}^{(p-1)}/2\right)}{\sum_{x_i \in \mathcal{S}_{i,k}^{-}} \exp\left(-\left| w_{c,i}^{*}\left(y_c - \tilde{y}_c\right) - \mu_{c,i} x_i\right|^2/\nu_{c,i}^2 + \sum_{k=0}^{\log_2|\Omega|-1} b_{i,k} m_{ic,k}^{(p-1)}/2\right)} - m_{ic,k}^{(p-1)},
\end{aligned} \tag{A.3}$$

where

$$\begin{aligned}
w_{c,i}^{*} &= \frac{h_{c,c-i}^{*}}{\sum_{i' \in \mathcal{U}_c \setminus \{i\}} \left| h_{c,c-i'}\right|^2 \left(1 - \left|\tilde{x}_{c-i'}\right|^2\right) + \left| h_{c,c-i}\right|^2 + \sigma^2},\\
\mu_{c,i} &= w_{c,i}^{*} h_{c,c-i}, \qquad \nu_{c,i}^2 = \mu_{c,i} - \mu_{c,i}^2.
\end{aligned} \tag{A.4}$$

The details for obtaining $w_{c,i}^{*}$, $\mu_{c,i}$, and $\nu_{c,i}^2$ can be found in [7].

(iii) For MF-SIC, we simply apply the match filter to the soft interference canceled output, that is,

$$z_{c,i} = w_{c,i}^{*}\left(y_c - \tilde{y}_c\right), \quad w_{c,i}^{*} = \frac{h_{c,c-i}^{*}}{\left| h_{c,c-i}\right|^2}. \tag{A.5}$$

We then approximate the MF-SIC output as Gaussian distributed, and compute extrinsic message in the same form in

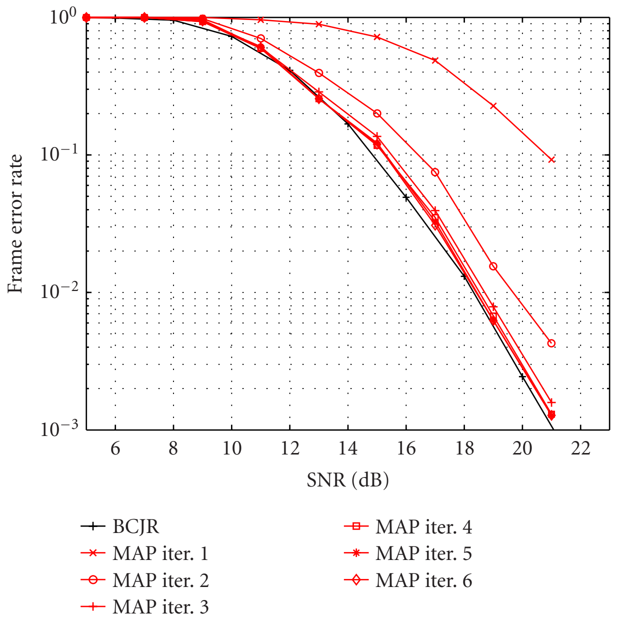


FIGURE 10: FER performance of the soft iterative equalizer based on alternative factor graph representation in MIMO multipath fading channels ($n_T = 2, n_R = 2, L = 3, L' = 2$).

(6) with mean and variance given by

$$\mu_{c,i} = 1,$$

$$\nu^2_{c,i} = \frac{\sum_{i' \in \mathcal{U}_c \setminus \{i\}} |h_{c,c-i'}|^2 (1 - |\tilde{x}_{c-i'}|^2) + \sigma^2}{|h_{c,c-i}|^2}. \quad \text{(A.6)}$$

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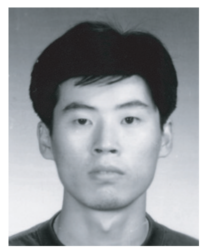
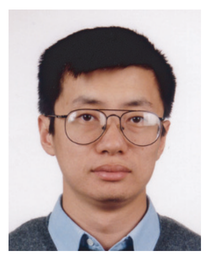
Ben Lu received the B.S. and M.S. degrees in electrical engineering from Southeast University, Nanjing, China, in 1994 and 1997, and the Ph.D. degree from Texas A&M University, in 2002. From 1994 to 1997, he was a Research Assistant with National Mobile Communications Research Laboratory at Southeast University, China. From 1997 to 1998, he was with the CDMA Research Department of Zhongxing Telecommunication Equipment Co., Shanghai, China. From 2002 to 2004, he worked for the project of high-speed wireless packet data transmission (4G prototype) at NEC Laboratories America, Inc., Princeton, New Jersey. He is now with Silicon Laboratories. His research interests include the signal processing and error-control coding for mobile and wireless communication systems.



Guosen Yue received the B.S. degree in physics and the M.S. degree in electrical engineering from Nanjing University, Nanjing, China, in 1994 and 1997, and the Ph.D. degree from Texas A&M University, College Station, Texas, in 2004. Since August 2004, he has been with NEC Laboratories America, Inc., Princeton, New Jersey, conducting research on broadband wireless systems and mobile networks. His research interests are in the area of advanced modulation and channel coding techniques for wireless communications.



Xiaodong Wang received the B.S. degree in electrical engineering and applied mathematics (with the highest honors) from Shanghai Jiao Tong University, Shanghai, China, in 1992; the M.S. degree in electrical and computer engineering from Purdue University, in 1995; and the Ph.D degree in electrical engineering from Princeton University, in 1998. From July 1998 to December 2001, he was an Assistant Professor in the Department of Electrical Engineering, Texas A&M University. In January 2002, he joined the faculty of the Department of Electrical Engineering, Columbia University. Dr. Wang's research interests fall in the general areas of computing, signal processing, and communications. He has worked in the areas of digital communications, digital signal processing, parallel and distributed computing, nanoelectronics, and bioinformatics, and has published extensively in these areas. Among his publications is a recent book entitled *Wireless Communication Systems: Advanced Techniques for Signal Reception,* published by Prentice Hall, Upper Saddle River, in 2003. His current research interests include wireless communications, Monte-Carlo-based statistical signal processing, and genomic signal processing. Dr. Wang received the 1999 NSF CAREER Award, and the 2001 IEEE Communications Society and Information Theory Society Joint Paper Award. He currently serves as an Associate Editor for the IEEE Transactions on Communications, the IEEE Transactions on Wireless Communications, the IEEE Transactions on Signal Processing, and the IEEE Transactions on Information Theory.

Mohammad Madihian received the Ph.D. degree in electronic engineering from Shizuoka University, Japan, in 1983. He joined NEC Central Research Laboratories, Kawasaki, Japan, where he worked on research and development of Si and GaAs device-based digital as well as microwave and millimeter-wave monolithic ICs. In 1999, he moved to NEC Laboratories America, Inc., Princeton, New Jersey, and is presently the Department Head of Microwave and Signal Processing and Chief Patent Officer. He conducts PHY/MAC layer signal processing activities for high-speed wireless networks and personal communication applications. He has authored or coauthored more than 130 scientific publications including 20 invited talks, and holds 35 Japan/US patents. Dr. Madihian has received the IEEE MTT-S Best Paper Microwave Prize in 1988, and the IEEE Fellow Award in 1998. He holds 8 NEC Distinguished R&D Achievement Awards. He has served as a Guest Editor for the IEEE Journal of Solid-State Circuits, Japan IEICE Transactions on Electronics, and IEEE Transactions on Microwave Theory and Techniques. He is presently serving on the IEEE Speaker's Bureau, IEEE Compound Semiconductor IC Symposium (CSICS) Executive Committee, IEEE Radio and Wireless Conference Steering Committee, IEEE International Microwave Symposium (IMS) Technical Program Committee, IEEE MTT-6 Subcommittee, IEEE MTT Editorial Board, and Technical Program Committee of the International Conference on Solid State Devices and Materials (SSDM). Dr. Madihian is an Adjunct Professor at the Electrical and Computer Engineering Department, Drexel University, Philadelphia, Pennsylvania.