Hindawi Publishing Corporation
EURASIP Journal on Wireless Communications and Networking
Volume 2006, Article ID 32097, Pages 1–11
DOI 10.1155/WCN/2006/32097

Transient and Steady-State Analysis of Nonlinear RF and Microwave Circuits

Lei (Lana) Zhu and Carlos E. Christoffersen

Department of Electrical Engineering, Lakehead University, Thunder Bay, ON, Canada P7B 5E1

Received 1 November 2005; Accepted 28 February 2006

This paper offers a review of simulation methods currently available for the transient and steady-state analysis of nonlinear RF and microwave circuits. The most general method continues to be the time-marching approach used in Spice, but more recent methods based on multiple time dimensions are particularly effective for RF and microwave circuits. We derive nodal formulations for the most widely used multiple time dimension methods. We put special emphasis on methods for the analysis of oscillators based in the warped multitime partial differential equations (WaMPDE) approach. Case studies of a Colpitts oscillator and a voltage controlled Clapp-Gouriet oscillator are presented and discussed. The accuracy of the amplitude and phase of these methods is investigated. It is shown that the exploitation of frequency-domain latency reduces the computational effort.

1. INTRODUCTION

The most common computer-aided circuit analysis technique is the time-domain transient analysis using the time-marching approach to solve the system of ordinary differential equations (ODEs) that represent the circuit. This approach is used in Spice and many other circuit simulators. Although still very useful, this technique presents a number of shortcomings for the analysis of RF and microwave circuits. Amongst these shortcomings is the huge amount of memory and computation time required for the analysis of circuits with widely separated time scales and/or excitations [1]. This is often the case for mixers, power amplifiers, and oscillators. Several techniques have been developed to overcome this problem. Some of them are available in modern circuit simulators and are widely used. In this paper we present an overview of these methods with special attention to the analysis of oscillators.

We consider transient analysis methods first. In Section 3 we focus on methods used to directly calculate the steady-state response of circuits. In Section 4 oscillator analysis is discussed with emphasis on methods based on multiple time dimensions. Case studies of a Colpitts oscillator and a voltage controlled Clapp-Gouriet oscillator are presented and discussed in Section 5.

2. TRANSIENT ANALYSIS

A generic circuit can be described by the following system of differential-algebraic equations:

$$Gu(t) + C\frac{du(t)}{dt} + \frac{dQ(u(t))}{dt} + I(u(t)) = S(t), \qquad (1)$$

here $u(t)$ is the vector of state variables (nodal voltages and selected branch currents), G is a matrix of conductances, C is the matrix representing the linear charge terms, $Q(u(t))$ and $I(u(t))$ are vector functions corresponding to the nonlinear devices, and $S(t)$ is a vector that represents the sources.

The traditional time-marching technique consists in replacing the derivatives of the state variables for an approximation using a numerical integration rule. For example, if the backward Euler rule is used, we have

$$\frac{du(t_{n+1})}{dt} \approx \frac{u(t_{n+1}) - u(t_n)}{t_{n+1} - t_n}. \qquad (2)$$

The same procedure is applied to the Q vector and the resulting nonlinear algebraic system of equations is solved using the Newton method for each time step:

$$Gu_{n+1} + C\frac{u_{n+1} - u_n}{h} + \frac{Q(u_{n+1}) - Q(u_n)}{h} + I(u_{n+1}) = S_{n+1}, \tag{3}$$

here $u_{n+1} = u(t_{n+1})$, $u_n = u(t_n)$, $S_{n+1} = S(t_{n+1})$, and $h = t_{n+1} - t_n$ for simplicity. This analysis is the most general nonlinear circuit analysis. It is applicable to autonomous or nonautonomous circuits with any kind of excitation.

If a circuit presents rapid and slow variations simultaneously, this analysis becomes inefficient because a small time-step is required to follow the fast variations with a long total simulation time. Consider, for example, a simulation to estimate the spectral regrowth in an amplifier driven by a digitally modulated carrier [2]. If the rapid variations are oscillatory, the circuit can be analysed using a more efficient approach. Different implementations of this approach exist (e.g., see [3–5]), but the underlying theory is common to all of them and is presented in [1]. Consider a voltage described by the following function:

$$v_1(t) = \left[1 - \exp\left(-\frac{t}{\tau_a}\right)\right]\left[1 + \sin\left(\frac{2\pi}{\tau_b}t\right)\right], \tag{4}$$

where τ_a and τ_b are time constants. A graphical representation of this function with $\tau_a = 5$ seconds and $\tau_b = 20$ ms is shown in Figure 1. Many sample points are required to represent this function. For example, 5000 samples were needed in Figure 1 for a total time of 5 seconds. Envelope-following methods [4, 5] take advantage on the fact that the oscillations do not change much in adjacent periods. They follow the shape of the envelope of signals using a time-step much greater than one period of the rapid oscillation and calculate the full response of the circuit with a small time-step only once in a while. Envelope-following methods have been recognised [6] as the most promising methods for the analysis of radio components such as mixers. It was proved in [1] that the idea in envelope-following methods can be seen as a particular case of a more general approach called multipartial differential equations (MPDE). The main idea in the time domain envelope-following (TD-ENV) method using the MPDE approach is to represent signals in more than one time dimension according to the scale of variation. The signals must be periodic in at least one of the dimensions and the period must be constant and known. Note that this condition excludes autonomous circuits. We consider oscillators in Section 4. For example, $v_1(t)$ is replaced by the following bidimensional function:

$$\hat{v}_1(t_1, t_2) = \left[1 - \exp\left(-\frac{t_1}{\tau_a}\right)\right]\left[1 + \sin\left(\frac{2\pi}{\tau_b}t_2\right)\right]. \tag{5}$$

This function is plotted in Figure 2, for the same values of τ_a and τ_b used before. Only 100 sample points were necessary to represent the waveform in the same time interval. The original function can be easily recovered by setting $t_1 = t$ and $t_2 = t$. Equation (1) must be modified as follows:

$$G\hat{u} + C\left(\frac{\partial \hat{u}}{\partial t_1} + \frac{\partial \hat{u}}{\partial t_2}\right) + \frac{\partial Q(\hat{u})}{\partial t_1} + \frac{\partial Q(\hat{u})}{\partial t_2} + I(\hat{u}) = \hat{S}(t_1, t_2), \tag{6}$$

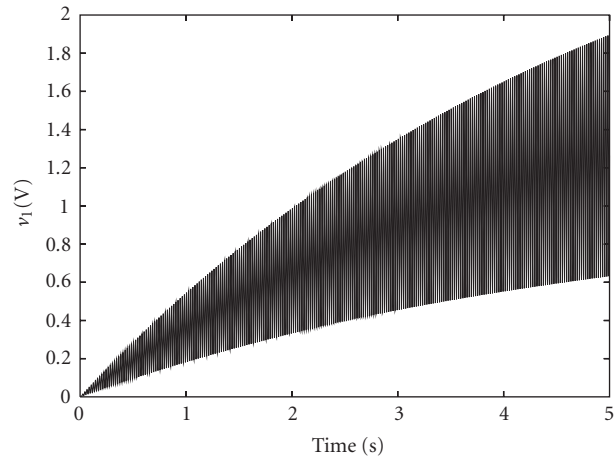


Figure 1: The voltage v_1 as a function of time.

where $\hat{u}$ and $\hat{S}$ represent the bivariate versions of the state variables and the source vector, respectively. The time plane with the boundaries is shown in Figure 3. In the figure, T is the period along the t_2 axis. It was demonstrated in [1] that a solution of (6) with t_1 set equal to t_2 (the diagonal lines in Figure 3) is also a solution to (1). To obtain the transient response of the circuit, a steady-state problem in the t_2 direction must be solved for every time-step along t_1. The efficiency of the method is better when the size of the time-step along t_1 is many times greater than T. Unfortunately this is not always possible because fast variations along t_1 occur frequently. This problem can be alleviated using an adaptive time-step in the direction of t_1. A time-step control algorithm for this purpose is presented in [7]. Another factor that has a great effect in the rate of variations along t_1 is the choice of boundary conditions. The boundary conditions of the MPDE are determined by the initial conditions of the original ODE only at $t_1 = 0$ and $t_2 = 0$. Work [8] presents a method to choose the boundary conditions in the rest of the $t_1 = 0$ line to avoid fast variations along the t_1 dimension. The solution of the steady-state problem along the t_2 dimension can be accomplished using several methods described in Section 3. One popular choice is to use harmonic balance. Recently an approach using wavelets in the t_2 dimension was presented [9].

The MPDE method was presented here for two time dimensions, but if the problem presents more than two rates, more time dimensions can be used as necessary. Work [10] considers a phase-locked loop (PLL) simulation using three time scales.

3. STEADY-STATE ANALYSIS

Steady-state methods are of great interest for the analysis of RF and microwave circuits. The most widely used steady-state analysis method is harmonic balance (HB). Some of the advantages of this method are that the solution directly gives the harmonic content of the circuit response and that it is

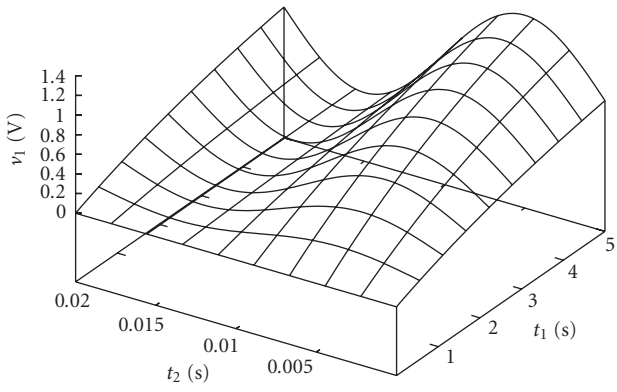


Figure 2: The bidimensional voltage $\hat{v}_1(t_1, t_2)$.

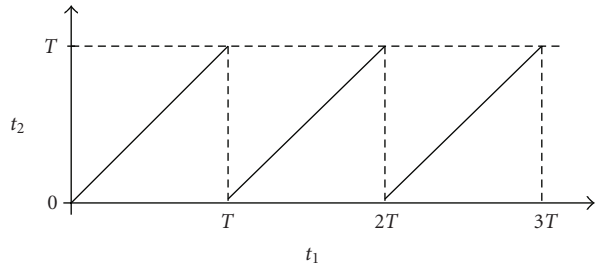


Figure 3: The time plane used for the MPDE method.

easier to include RF circuit components that are better modelled in frequency domain such as transmission lines, filters, and others.

In HB each element of the $u(t)$ vector in (1) is represented by a set of phasors,

$$u_n(t) = \Re\left\{\sum_{k=0}^{K} U_n^k e^{jk\omega t}\right\}, \tag{7}$$

where n is the state variable number, k is the harmonic number, and $\omega = 2\pi/T$. Equation (1) can now be expressed for each angular frequency $(k\omega)$ as follows:

$$\left(G + C\Omega^k\right)U^k + I^k + \Omega^k Q^k = S^k. \tag{8}$$

In this equation Ω^k is a diagonal matrix with nonzero elements equal to $jk\omega$. The S^k vector represents the sources at angular frequency $k\omega$. The nonlinear devices in the circuit are normally modelled in time domain and the I^k and Q^k vectors are formed taking the kth harmonic component of the Fourier transform of the respective time-domain vectors. The nonlinear devices could be modelled directly in the frequency domain using Volterra series [11]. For weekly nonlinear circuits Volterra series result in more accurate models and are also useful for analytical calculations for design [6].

The unknowns in (8) are the Fourier coefficients (U^k). The system of algebraic equations formed by applying (8) for all harmonics is normally solved using the Newton method. The number of simultaneous unknowns in this equation is equal to $(2K + 1)n$. In many cases this number can be very large in the order of several tens of thousands. In these cases the solution can still be found with a reasonable CPU time using inexact Newton methods [12] and Krylov subspace methods [2] (note that for smaller problems the regular Newton method is more efficient).

Another relevant development is the exploitation of frequency-domain latency in HB [13–15]. These techniques take advantage of the fact that in most circuits the number of harmonics necessary to represent each variable (voltage, current) is not the same. By using a different number of harmonics in each variable, a significant reduction in the computational effort is achieved.

There are many problems of interest where the signals are not strictly periodic but quasiperiodic. For example, consider the following voltage waveform:

$$v_2(t) = V_2 \cos(\omega_1 t)\cos(\omega_2 t). \tag{9}$$

No period can be defined for this signal if ω_1/ω_2 is not a rational number. Even in the case that the signal is strictly periodic, it is frequently more convenient to treat it as quasiperiodic. Two approaches exist to treat this problem in HB. The first of them is called artificial frequency mapping and the second is to use a multidimensional Fourier transform [16].

It is often acknowledged [17–19] that for strongly nonlinear circuits the HB method may not be the best because a large number of harmonics is necessary to represent the signals. The most common alternative to HB is the shooting method [20]. The shooting method works by solving (3) for one period of the excitation and finding what initial conditions result in the same state of the circuit at the end of the period. Other approaches using wavelets [17] and adaptive basis functions [18] are still in the research stage and are not widely used yet. The state variables in these methods are represented as a linear combination of a set of basis functions. Equation (1) can then be transformed into a nonlinear algebraic equation.

3.1. Steady-state analysis and the MPDE

The MPDE approach is also useful for steady-state analysis of quasiperiodic excitations by considering each period in a different time dimension. Equation (6) is applicable with periodic boundary conditions in both t_1 and t_2 dimensions. The MPDE approach is thus not only useful to analyse circuits with widely separated excitation frequencies but also for circuits with closely spaced excitation frequencies [19]. The HB with multidimensional Fourier transform approach can be seen as a particular case of the MPDE when both time dimensions are considered in the frequency domain.

4. ANALYSIS OF OSCILLATORS

Oscillator analysis is a difficult task [20–26]. Except for regular time-marching transient analysis (3), all the methods that were reviewed so far must be modified to analyse oscillators.

For transient analysis, an alternative to the traditional time-marching approach called warped multitime partial differential equation (WaMPDE) was presented in [22, 27,

28]. This approach deals with the fact that the period (or equivalently, the local frequency) in one of the time dimensions is no longer constant. In the WaMPDE this is solved by warping one of the time scales in MPDE to have a constant normalised period. As a result the local frequency is normalised to a constant value and the warped time becomes a function of time. The t_1 time axis in the MPDE is now renamed τ_2 (they are otherwise equivalent) and the warped time scale is named τ_1. The relation between τ_1 and τ_2 is given by

$$\tau_1 = \int_0^t \omega(\tau_2)\, d\tau_2, \tag{10}$$

where $\omega(\tau_2)$ is the unknown local frequency. Substituting t_1 and t_2 in (6) we obtain the WaMPDE nodal equation:

$$\begin{aligned} &G\hat{u} + C\left(\omega(\tau_2)\frac{\partial \hat{u}}{\partial \tau_1} + \frac{\partial \hat{u}}{\partial \tau_2}\right) \\ &\quad + \omega(\tau_2)\frac{\partial Q(\hat{u})}{\partial \tau_1} + \frac{\partial Q(\hat{u})}{\partial \tau_2} + I(\hat{u}) = \hat{S}(\tau_1, \tau_2). \end{aligned} \tag{11}$$

An additional equation is required to balance the introduction of the unknown local frequency. This equation is formed by imposing a smooth phase variation along τ_2 [22].

Often the HB method is used in the τ_1 dimension. Then each element of the $u(t)$ vector in (1) is represented by

$$\hat{u}_n(\tau_1, \tau_2) = \Re\left\{\sum_{k=0}^{K} U_n^k(\tau_2) e^{jk\tau_1}\right\}, \tag{12}$$

where again k is the harmonic number and the period in the warped time scale (τ_1) is normalised to 2π (*i.e.*, $\omega = 1$). We can reformulate now (11) for each harmonic (k):

$$(G + C\Omega^k)U^k + C\frac{\partial U^k}{\partial \tau_2} + \Omega^k Q^k + \frac{\partial Q^k}{\partial \tau_2} + I^k - S^k = 0. \tag{13}$$

As stated before, the phase of one of the variables must then be fixed to restore the number of unknowns to be equal to the number of equations. That can be achieved by setting the imaginary part of one of the variables to be zero,

$$\Im(U_n^1) = 0. \tag{14}$$

Equation (13) is discretised in the τ_2 direction using the backward Euler (BE) rule, trapezoidal rule, or any other numerical integration method. The resulting algebraic nonlinear system is then solved with the Newton-Raphson method for each value of τ_2. This technique is referred in this work as time-frequency envelope transient (TFET).

4.1. Transient

Given initial conditions for (1), the corresponding boundary conditions in the TFET analysis can be obtained from a short section of the transient response of the oscillator obtained from a time-marching simulation [29]. The choice of accurate boundary conditions is important if a good agreement between time-marching and TFET analyses is desired.

To improve the efficiency of the simulation, the number of harmonics for each variable u_n can be adaptively controlled [15]. This is sometimes referred as frequency-domain latency exploitation. This is achieved as follows. At the end of the calculation for each step of τ_2, the magnitudes of the last two harmonics are considered. If they are greater than some threshold value, then the number of harmonics for that variable (l) is increased by one. If they are smaller than another threshold, then l is decreased by one. Otherwise it is left unchanged. One advantage of this approach is that the number of harmonics is increased or reduced as needed. Each row of (13) (nodal equation at one frequency) is considered at a number of frequencies equal to the number of harmonics of the corresponding nodal voltage. In this way the number of equations is always kept equal to the number of unknowns.

An adaptive time-step control algorithm is used in order to minimise the number of time steps [29]. The time-step along τ_2 is adaptively changed according to a local truncation error estimation,

$$h_{\text{new}} = h_{\text{old}}\left(\frac{\epsilon_{\max}}{\epsilon}\right)^{1/m}, \tag{15}$$

where $\epsilon_{\max}$ is the maximum acceptable truncation error, ϵ is the current truncation error, and m is a number that depends on the integration method being used ($m = 3$ for trapezoidal integration). This substantially reduces the computational cost and improves the accuracy of the TFET.

4.2. Steady-state

For the steady-state analysis of oscillators, harmonic balance (HB) has been the dominant approach in recent years. Though HB is one of the most important frequency-domain techniques, it still has some limitations. In particular a good initial guess is needed to make HB converge to the desired oscillatory solution. It is especially difficult to get a good initial guess of the oscillator frequency. Several methods have been proposed to improve this limitation. For example, in [25] the HB equations are modified by including the Kurokawa condition to eliminate the DC solution. In [26] a voltage source probe at the fundamental frequency that is an open circuit at all other frequencies is inserted to avoid the DC solution. By means of an iterative process the amplitude and frequency of the probe are adjusted until there is no current through the probe. At this point the autonomous solution is found. A similar probe concept with the addition of a continuation method has been proposed more recently [23] to improve convergence. Accelerated transients have been used [15, 20, 30] to find the steady-state regime. In order to accelerate the finding of the steady-state regime, the transient behaviour of the circuit is artificially reduced in [30]. In [20] an envelope-transient analysis is used to improve the convergence of the shooting method.

Another new approach improving the convergence of the HB analysis of oscillators was presented in [15]. This approach is based on an accelerated TFET method and is

summarised here. A TFET simulation is started with the boundary conditions set to the DC bias point values. An excitation current,

$$i_S(\tau_1, \tau_2) = \begin{cases} I_0 \cos(\tau_1) & \text{if } \tau_2 \le t_a, \\ 0 & \text{otherwise,} \end{cases} \tag{16}$$

is injected from the ground node into one of the nodes where oscillations are expected. Here, I_0 is a small real number (normally a few μA) and t_a is set equal to the initial time-step size along τ_2 (h). The purpose of i_S is to start oscillations by moving the system away from the equilibrium point. The system will then naturally tend to reach the desired oscillatory steady state. The key of this work is to accelerate the TFET simulation to reach a point in τ_2 close to the steady state in the minimum possible number of Newton iterations and then use the state at that point as the initial guess of a regular autonomous HB analysis. Since the focus is on the steady state, it is not necessary to calculate the transient evolution with great precision as long as it converges close to the actual steady state of the circuit. Thus the time-step along τ_2 and other parameters in the simulation are controlled to minimise the number of Newton iterations [15]. When the local frequency function becomes constant and the difference between two periodic solutions along τ_2 becomes small, the accelerated TFET is stopped and a regular oscillator HB analysis is started using the last solution along τ_2 as the initial guess. Note that the regular oscillator HB analysis is obtained by setting all derivatives with respect to τ_2 to be zero in (13).

5. CASE STUDIES AND DISCUSSION

In this section we perform a transient and steady-state analysis of two oscillators: a Colpitts oscillator and voltage controlled oscillator (VCO). In each oscillator, the regular time domain simulation is provided followed by a WaMPDE-based simulation, and then both results are compared.

5.1. Colpitts oscillator

The Colpitts oscillator taken from [31] uses a capacitive voltage divider in the LC tank circuit. In the circuit shown in Figure 4, the component values are: $C_1 = C_2 = 2\,\text{pF}$, $C_c = 400\,\text{pF}$, $C_e = 100\,\text{pF}$, $L_1 = 1\,\mu\text{H}$, $R_1 = 8\,\text{k}\Omega$, $R_2 = 2\,\text{k}\Omega$, $R_c = 2.4\,\text{k}\Omega$, $R_e = 1.3\,\text{k}\Omega$, $V_{cc} = 11\,\text{V}$, $BF = 100$, and $BR = 1$.

5.1.1. Transient analysis

This oscillator exhibits an extremely long initial transient compared with the oscillation period. Part of the transient simulation is presented in Figure 5. The CPU time in a 1.5 GHz computer using Matlab was 57 seconds for a simulation stop time of 10 μs. Clearly time-marching simulation is very time-consuming for the analysis of this circuit. In order to obtain an accurate result, the acceptable truncation error in this simulation was reduced until no phase difference could be observed with further reductions. In this way we can take the result of this time-marching simulation as a reference for both magnitude and phase information.

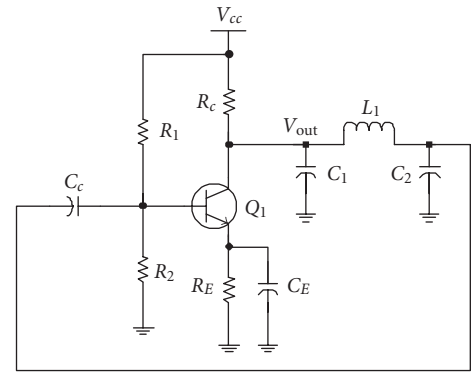


Figure 4: Schematic of a Colpitts oscillator.

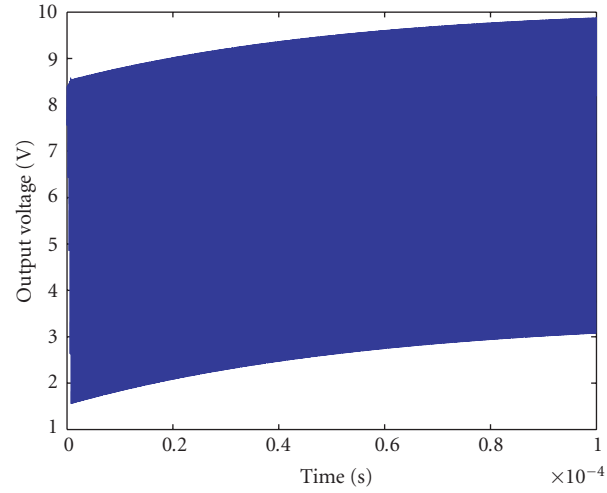


Figure 5: Transient response of the Colpitts oscillator.

We now discuss the simulation results using the TFET approach. The TFET simulation starts from the specified initial conditions and the stop time along τ_2 is set to 0.07 seconds. The initial number of harmonics is 11 and the adaptive harmonic balance automatically adjusts this number as required. The adaptive time-step algorithm increases the step size according to the local truncation error. These two provisions largely speed up the simulation. Total CPU time is 199 seconds. A time-marching simulation would require approximately 15 hours to produce the same result. Figure 6 shows the multitime expression of the output voltage.

Very good agreement between TFET simulation and the time-marching simulation has been achieved as shown in the top of Figure 7. There is a phase error accumulation along τ_2 shown at the bottom of Figure 7. The relative amplitude error and absolute phase error of the first harmonic in TFET and ODE simulations are presented in Figure 8. The amplitude error is very small, but the phase error may be important in

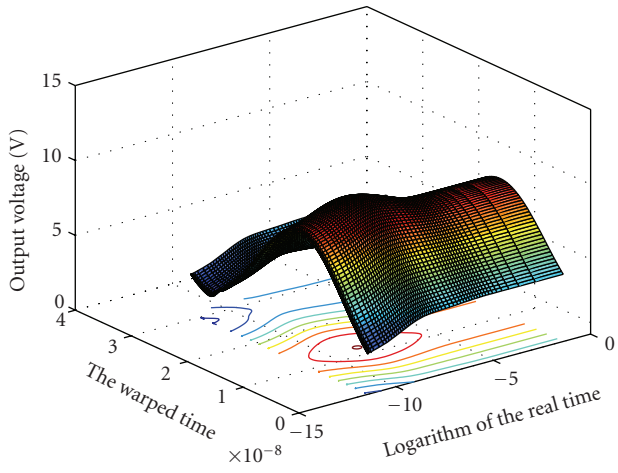


Figure 6: Bidimensional representation of output voltage.

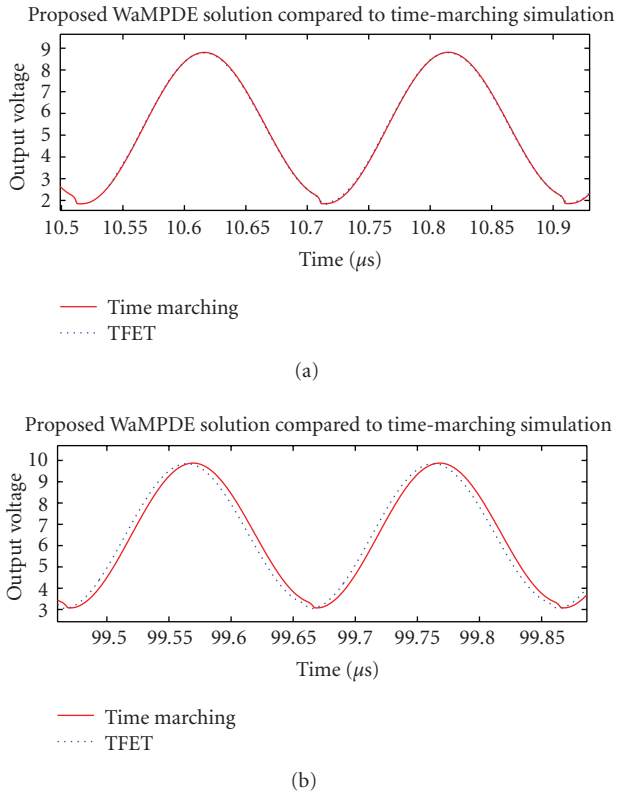


Figure 7: TFET solution compared to time-marching solution.

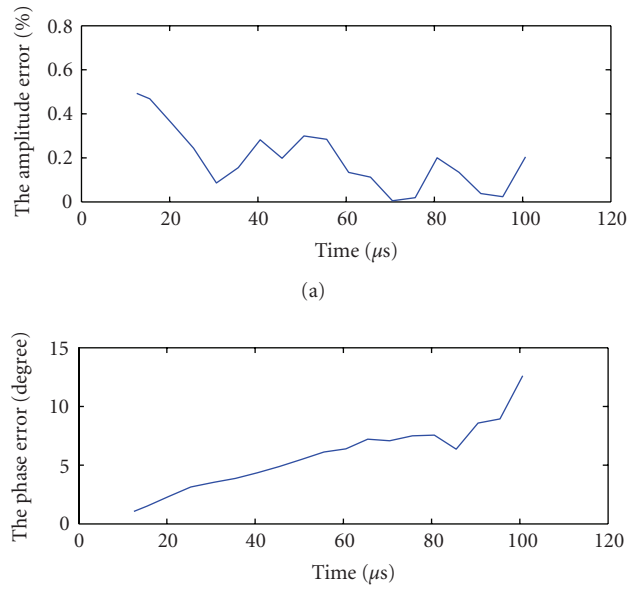


Figure 8: Relative error in the magnitude of the first harmonic.

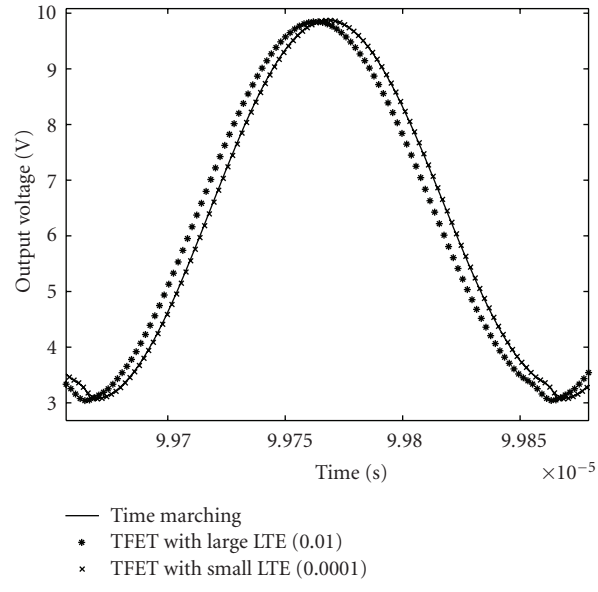


Figure 9: Comparison of time-marching and TFET with a smaller maximum local truncation error.

some cases. It should be noted that a comparable phase error would be obtained with a time-marching simulation if the acceptable truncation error is not set to a very small number. The phase error in the TFET simulation can be reduced if the acceptable local truncation error in the TFET simulation is reduced as the results of Figure 9 indicate. However, the reduction in the acceptable local truncation error increases the number of time steps along τ_2 and this results in a significantly longer simulation time. Finally, Figure 10 shows the size of the Jacobian matrix and the size of the time-step along τ_2 for each step in τ_2. The size of the Jacobian matrix is proportional to the total number of harmonics considered in the simulation. As the variations in τ_2 become smooth, the time-step is increased and the size of the Jacobian matrix is increased as more harmonics are generated in the nodal voltages.

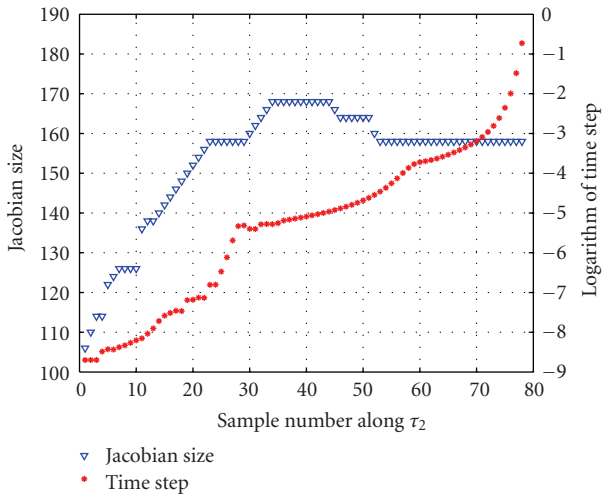


Figure 10: Size of the Jacobian matrix and the size of the time-step along τ_2 for each step in τ_2.

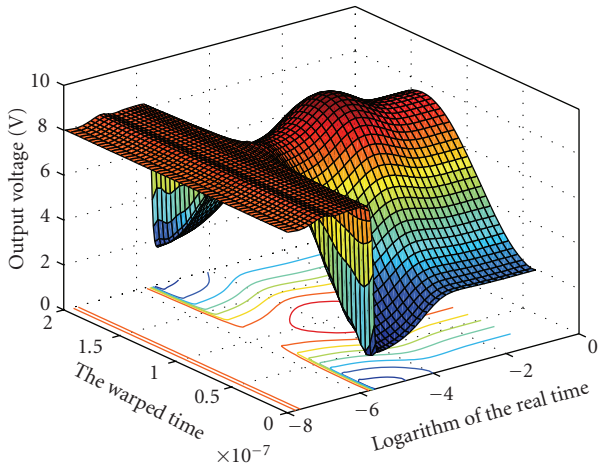


Figure 11: Bidimensional representation of output voltage.

5.1.2. Steady-state analysis

The excitation current (i_S) was applied to the base node with I_0 set to 10 μA. The bidimensional plot of the output voltage as a function of τ_1 and τ_2 is shown in Figure 11. The CPU time of the accelerated TFET and HB analyses combined is 52 seconds.

The oscillator frequency determined by the warped function $\omega(\tau_2)$ is 5.04 MHz as shown in Figure 12. This figure illustrates the robustness of the proposed method. Different initial frequencies converge to the correct value.

In Figure 13 the steady state is compared with the final line of the multitime simulation. This shows how close the result from the accelerated TFET to the steady state is. Figure 14 shows the Jacobian matrix size and the number of Newton iterations at each time-step (sample number).The

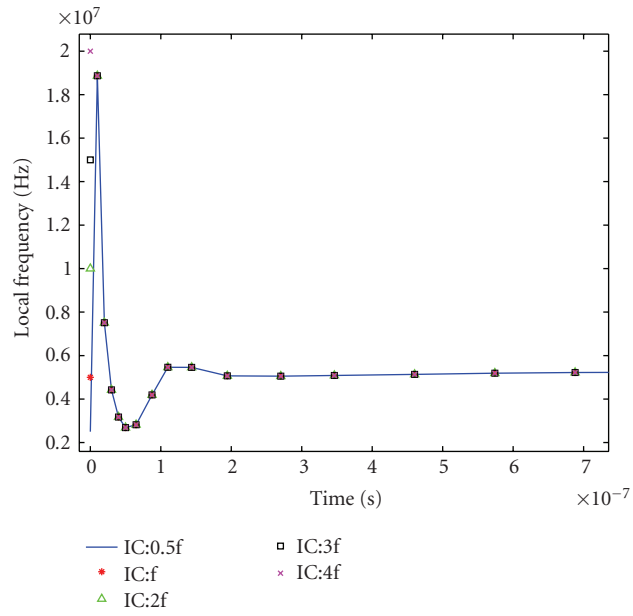


Figure 12: Fundamental frequency as a function of τ_2.

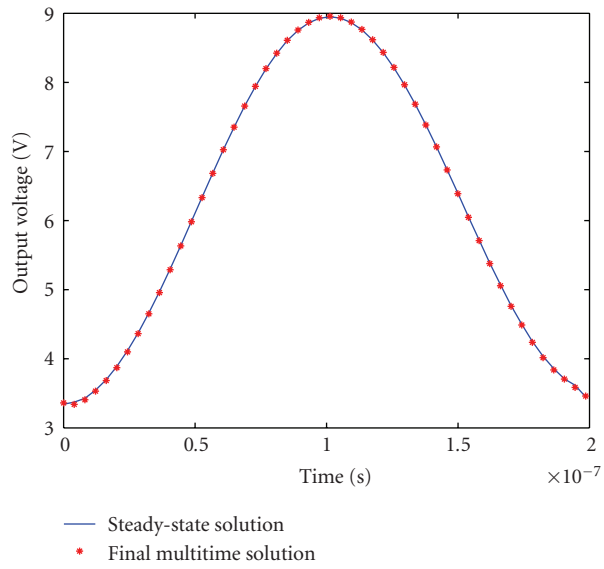


Figure 13: Steady-state solution compared to final line in multitime solution.

adaptive control of the time-step along τ_2 keeps the number of Newton iterations small for each value of τ_2. The Jacobian matrix size increases as the number of harmonics increases.

5.2. Clapp-Gouriet oscillator

In this section, we present the analysis of a VCO circuit based on the Clapp-Gouriet configuration [32]. Figure 15 shows the electrical schematic of the VCO. In this circuit

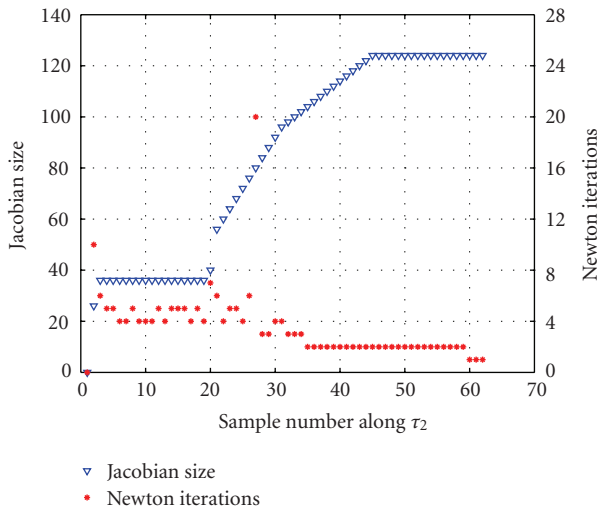


Figure 14: Jacobian size and number of Newton iterations.

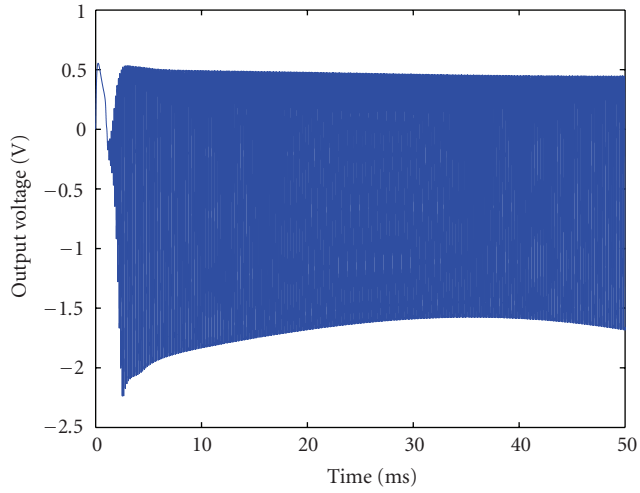


Figure 16: Transient response of the VCO.

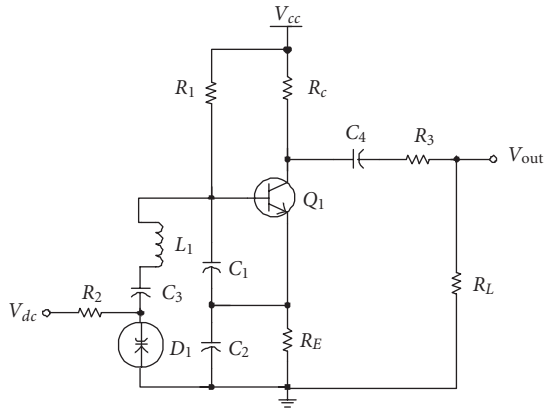


Figure 15: A VCO using Clapp-Gouriet configuration.

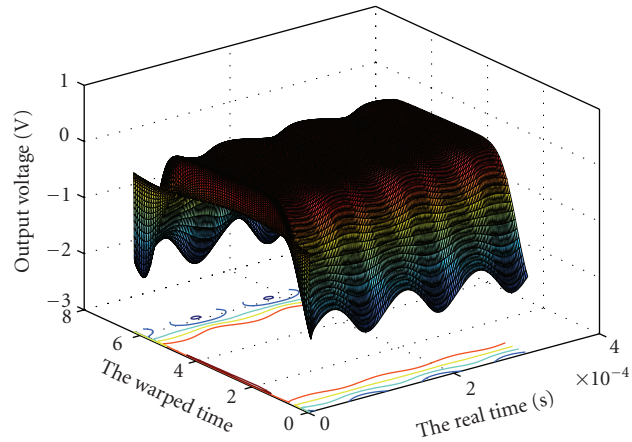


Figure 17: Bidimensional representation of output voltage of VCO.

$C_1 = 82\,\text{pF}$, $C_2 = 220\,\text{pF}$, $C_3 = 47\,\text{pF}$, $C_4 = 330\,\text{pF}$, $L_1 = 102.55\,\mu\text{H}$, $R_1 = 220\,\text{k}\Omega$, $R_2 = 22\,\text{k}\Omega$, $R_3 = 47\,\Omega$, $R_c = 2.2\,\text{k}\Omega$, $R_e = 220\,\Omega$, $R_l = 100\,\Omega$, $V_{cc} = 12\,\text{V}$, $BF = 70$, and $BR = 5$. The oscillator frequency is tuned by a diode connected to a control voltage, V_{dc}. The circuit is analysed with a sinusoidal control voltage,

$$V_{dc} = 3 + \sin\left(2\pi 10^4 t\right)\ \text{V}. \tag{17}$$

The initial transient is shown in Figure 16. In TFET analysis, the initial number of harmonics is set to 8 and the adaptive HB algorithm automatically adjusts this number for each node every time-step along τ_2. The bidimensional plot of the output voltage is shown in Figure 17.

Good agreement between time-marching and TFET analyses is achieved as shown in Figure 18. A plot showing the first harmonic magnitude and phase difference between the time-marching and TFET analyses is shown in Figure 19. As with the previous case study, there is some phase error that can be reduced by reducing the tolerance of the local truncation error. Finally, Figure 20 shows the size of the Jacobian matrix and the size of the time-step along τ_2 for each step in τ_2. It can be observed that both quantities follow the variations of the transient and the control voltage.

6. CONCLUSIONS

We have presented a review of simulation methods currently available for the transient and steady-state analysis of nonlinear RF and microwave circuits. Although the time-marching approach used in Spice will continue to be widely used, envelope-following methods are particularly effective for RF and microwave circuits and they are becoming a popular choice. Recent improvements to oscillator analysis were summarised and case studies of a Colpitts oscillator

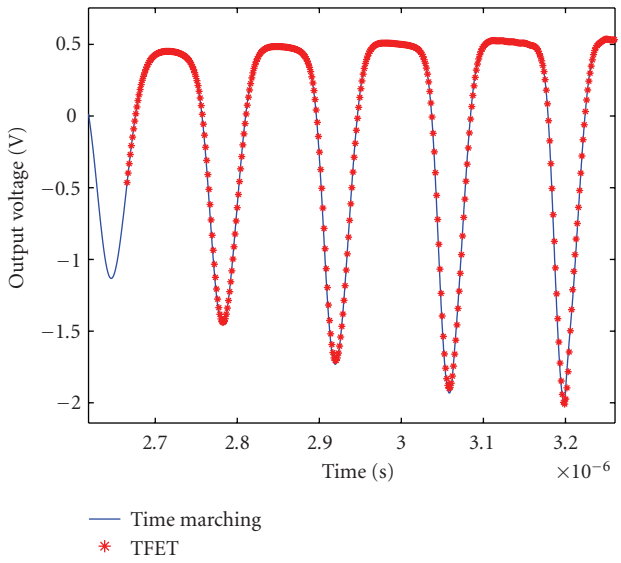


Figure 18: Time-marching and TFET results compared.

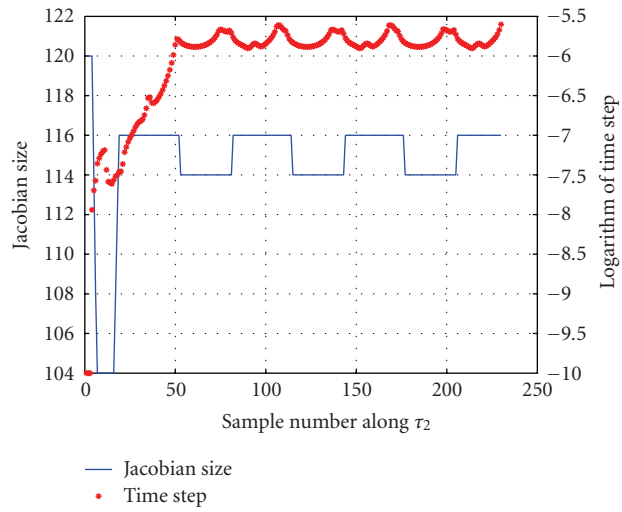


Figure 20: Size of the Jacobian matrix and the size of the time-step along τ_2 for each step in τ_2 for the VCO.

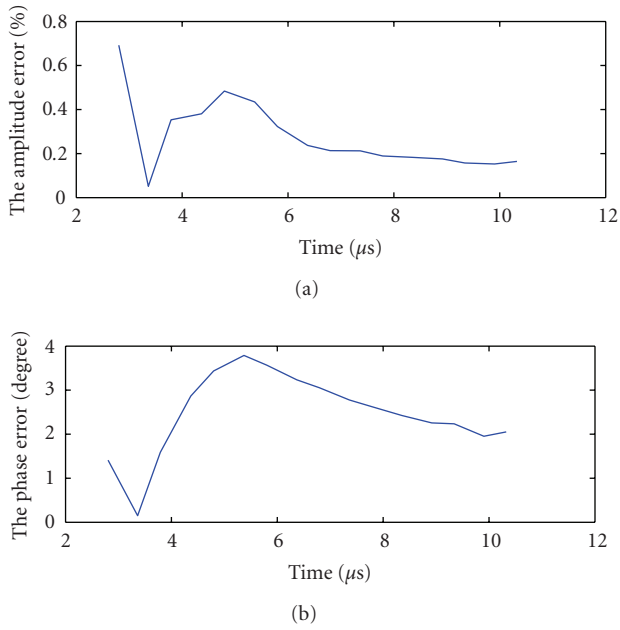


Figure 19: Comparison of ODE and warped MPDE in first harmonic.

and a voltage controlled Clapp-Gouriet oscillator were presented. In both cases there was good agreement between time-marching and TFET analyses, but it was noted that some phase error may occur if the local truncation error tolerance is not kept small enough. This may be an important consideration for the analysis of PLLs and should be further investigated. The simulations indicated that the adaptive HB technique significantly reduces the computational effort by reducing the size of the Jacobian matrix in the Newton method. It was shown that an accelerated TFET analysis can be effectively used to improve the convergence of the HB oscillator analysis.

ACKNOWLEDGMENT

This work was supported by Natural Sciences and Engineering Research Council of Canada (NSERC).

REFERENCES

[1] J. Roychowdhury, "Analyzing circuits with widely separated time scales using numerical PDE methods," *IEEE Transactions on Circuits and Systems I: Fundamental Theory and Applications*, vol. 48, no. 5, pp. 578–594, 2001.

[2] V. Rizzoli, A. Neri, F. Mastri, and A. Lipparini, "A Krylov-subspace technique for the simulation of integrated RF/microwave subsystems driven by digitally modulated carriers," *International Journal of RF and Microwave Computer-Aided Engineering*, vol. 9, no. 6, pp. 490–505, 1999.

[3] E. Ngoya and R. Larchevèque, "Envelop transient analysis: a new method for the transient and steady state analysis of microwave communication circuits and systems," in *Proceedings of IEEE MTT-S International Microwave Symposium Digest*, vol. 3, pp. 1365–1368, San Franscisco, Calif, USA, June 1996.

[4] K. S. Kundert, J. K. White, and A. Sangiovanni-Vincentelli, "An envelope-following method for the efficient transient simulation of switching power and filter circuits," in *Proceedings of IEEE International Conference on Computer-Aided Design (ICCAD '88)*, pp. 446–449, Santa Clara, Calif, USA, November 1988.

[5] A. Brambilla and P. Maffezzoni, "Envelope following method for the transient analysis of electrical circuits," *IEEE Transactions on Circuits and Systems I: Fundamental Theory and Applications*, vol. 47, no. 7, pp. 999–1008, 2000.

[6] N. B. Carvalho, J. C. Pedro, W. Jang, and M. B. Steer, "Nonlinear simulation of mixers for assessing system-level performance," *International Journal of RF and Microwave Computer-Aided Engineering*, vol. 15, no. 4, pp. 350–361, 2005.

[7] C. E. Christoffersen and J. Alexander, "An adaptive time step control algorithm for nonlinear time domain envelope transient," in *Proceedings of IEEE Canadian Conference on Electrical and Computer Engineering*, vol. 2, pp. 0883–0886, Niagara Falls, Ontario, Canada, May 2004.

[8] J. Roychowdhury, "Resolving fundamental issues of slowness in envelope simulation methods," *International Journal of RF and Microwave Computer-Aided Engineering*, vol. 15, no. 4, pp. 371–381, 2005.

[9] E. Dautbegović, M. Condon, and C. Brennan, "An efficient nonlinear circuit simulation technique," *IEEE Transactions on Microwave Theory and Techniques*, vol. 53, no. 2, pp. 548–555, 2005.

[10] S. Sancho, A. Suarez, and J. Chuan, "General envelope-transient formulation of phase-locked loops using three time scales," *IEEE Transactions on Microwave Theory and Techniques*, vol. 52, no. 4, pp. 1310–1320, 2004.

[11] M. B. Steer, C. Chang, and G. W. Rhyne, "Computer-aided analysis of nonlinear microwave circuits using frequency-domain nonlinear analysis techniques: the state of the art," *International Journal of Microwave and Millimeter-Wave Computer-Aided Engineering*, vol. 1, no. 2, pp. 181–200, 1991.

[12] V. Rizzoli, F. Mastri, F. Sgallari, and G. Spaletta, "Harmonic-balance simulation of strongly nonlinear very large-size microwave circuits by inexact Newton methods," in *Proceedings of IEEE MTT-S International Microwave Symposium Digest*, vol. 3, pp. 1357–1360, San Franscisco, Calif, USA, June 1996.

[13] H. Asai and H. Makino, "Frequency domain latency and relaxation-based harmonic analysis of nonlinear circuits," in *Proceedings of the 34th Midwest Symposium on Circuits and Systems*, vol. 1, pp. 202–205, Monterey, Calif, USA, May 1991.

[14] M. M. Gourary, S. G. Rusakov, S. L. Ulyanov, M. M. Zharov, K. K. Gullapalli, and B. J. Mulvaney, "A new technique to exploit frequency domain latency in harmonic balance simulators," in *Proceedings of the Conference on Asia South Pacific Design Automation*, vol. 1, pp. 65–68, Wanchai, Hong Kong, January 1999.

[15] L. (Lana) Zhu and C. E. Christoffersen, "Adaptive harmonic balance analysis of oscillators using multiple time scales," in *Proceedings of 3rd International IEEE Northeast Workshop on Circuits and Systems (NEWCAS '05)*, pp. 187–190, Quebec, Canada, June 2005.

[16] K. S. Kundert, J. K. White, and A. Sangiovanni-Vincentelli, *Steady-State Methods for Simulating Analog and Microwave Circuits*, Kluwer Academic, Boston, Mass, USA, 1990.

[17] N. Soveiko and M. S. Nakhla, "Steady-state analysis of multitone nonlinear circuits in wavelet domain," *IEEE Transactions on Microwave Theory and Techniques*, vol. 52, no. 3, pp. 785–797, 2004.

[18] A. Wenzler and E. Lueder, "Analysis of the periodic steady-state in nonlinear circuits using an adaptive function base," in *Proceedings of IEEE International Symposium on Circuits and Systems*, vol. 6, pp. 1–4, Orlando, Fla, USA, July 1999.

[19] J. Roychowdhury, "A multitime circuit formulation for closely spaced frequencies," *International Journal of RF and Microwave Computer-Aided Engineering*, vol. 15, no. 4, pp. 382–393, 2005.

[20] A. Brambilla and P. Maffezzoni, "Envelope-following method to compute steady-state solutions of electrical circuits," *IEEE Transactions on Circuits and Systems I: Fundamental Theory and Applications*, vol. 50, no. 3, pp. 407–417, 2003.

[21] A. Collado, F. Ramírez, A. Suarez, and J. P. Pascual, "Harmonic-balance analysis and synthesis of coupled-oscillator arrays," *IEEE Microwave and Wireless Components Letters*, vol. 14, no. 5, pp. 192–194, 2004.

[22] O. Narayan and J. Roychowdhury, "Analyzing oscillators using multitime PDEs," *IEEE Transactions on Circuits and Systems I: Fundamental Theory and Applications*, vol. 50, no. 7, pp. 894–903, 2003.

[23] M. Gourary, S. Ulyanov, M. Zharov, and S. Rusakov, "Simulation of high-Q oscillators," in *Proceedings of IEEE/ACM International Conference on Computer-Aided Design (ICCAD '98)*, pp. 162–169, San Jose, Calif, USA, November 1998.

[24] V. Rizzoli, A. Costanzo, and A. Neri, "Harmonic-balance analysis of microwave oscillators with automatic suppression of degenerate solution," *Electronics Letters*, vol. 28, no. 3, pp. 256–257, 1992.

[25] C.-R. Chang, M. B. Steer, S. Martin, and E. Reese Jr., "Computer-aided analysis of free-running microwave oscillators," *IEEE Transactions on Microwave Theory and Techniques*, vol. 39, no. 10, pp. 1735–1745, 1991.

[26] E. Ngoya, A. Suarez, R. Sommet, and R. Quéré, "Steady state analysis of free or forced oscillators by harmonic balance and stability investigation of periodic and quasi-periodic regimes," *International Journal of Microwave and Millimeter-Wave Computer-Aided Engineering*, vol. 5, no. 3, pp. 210–223, 1995.

[27] O. Narayan and J. Roychowdhury, "Analyzing oscillators using multitime PDEs," *IEEE Transactions on Circuits and Systems I: Fundamental Theory and Applications*, vol. 50, no. 7, pp. 894–903, 2003.

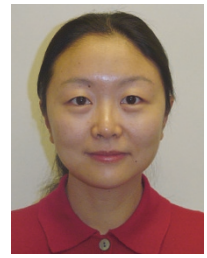
[28] H. G. Brachtendorf, G. Welsch, and R. Laur, "A time-frequency algorithm for the simulation of the initial transient response of oscillators," in *Proceedings of IEEE International Symposium on Circuits and Systems*, vol. 6, pp. 236–238, Monterey, Calif, USA, May-June 1998.

[29] L. (Lana) Zhu and C. E. Christoffersen, "Fast transient analysis of oscillators using multiple time scales with accurate initial conditions," in *Proceedings of IEEE Canadian Conference on Electrical and Computer Engineering*, pp. 718–721, Saskatoon, SK, Canada, May 2005.

[30] R. Larchevéque and E. Ngoya, "Compressed transient analysis speeds up the periodic steady state analysis of nonlinear microwave circuits," in *Proceedings of IEEE MTT-S International Microwave Symposium Digest*, vol. 3, pp. 1369–1372, San Franscisco, Calif, USA, June 1996.

[31] R. R. Spencer and M. S. Ghausi, *Introduction to Electronic Circuit Design*, Prentice Hall, Upper Saddle River, NJ, USA, 2003.

[32] J. A. Smith, *Modern Communication Circuits*, McGraw-Hill, New York, NY, USA, 2nd edition, 1997.



Lei (Lana) Zhu received her Bachelor degree of Electrical Engineering from Southeast University in China in August 1998. From 1994 to 2001, she was a Junior Design Engineer and later a Design Engineer in Electrical Engineering Design Department of Nanjing Power Supply Bureau, China. She received her M.S. degree from Lakehead University, Canada, in November 2005. Her research interest includes circuits analysis and simulation in multiple time axes, oscillator design, phase-locked loops, and general communication circuits. Currently she works at Manitoba Hydro, Canada.

Carlos E. Christoffersen received the Electronic Engineer degree at the National University of Rosario, Argentina, in 1993. From 1993 to 1995, he was a Research Fellow of the National Research Council of Argentina (CONICET). He received an M.S. degree and a Ph.D. degree in electrical engineering in 1998 and 2000, respectively, from North Carolina State University (NCSU). Currently he is an Assistant Professor in the Department of Electrical Engineering at Lakehead University, Thunder Bay, Canada. He is a Member of the IEEE. His current research interests include analogue and RF circuit computer-aided design including electromagnetic and thermal interactions.