# A Multicarrier Multiplexing Method for Very Wide Bandwidth Transmission 

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#### Abstract

The multicarrier orthogonal code division multiplexing (MC-OCDM) introduced here has been designed for very wide bandwidth (VWB) point-to-point and point-to-multipoint transmission. In order to meet VWB transmission requirements, the MC-OCDM design has two components, the basic and the composite. The basic MC-OCDM is a generalized form of the standard orthogonal frequency division multiplexing (OFDM). It has the property of distributing the power of each transmitted symbol into all subcarrier frequencies. Each subcarrier will then carry all transmitted symbols which are distinguished by orthogonal Hadamard sequences. The resulting system is shown to improve the performance of OFDM by introducing frequency and time diversity. As shown, by both analysis and simulation, the basic MC-OCDM combats the effects of narrowband interference (NBI). In particular, the simulation results show that the BER performance of the basic MC-OCDM in the presence of NBI is better than OFDM for both coded and uncoded systems. Furthermore, the composite MC-OCDM is a method of orthogonal frequency division multiplexing (OFDM) basic MC-OCDM channels. This allows us to multiplex more than one basic MC-OCDM channel into a VWB transmission system which can have the performance and spectral efficiency required in fixed wireless transmission environments.


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## 1. INTRODUCTION

Multicarrier (MC) transmission methods have been widely accepted for use in fixed and mobile wireless links. In particular, the multicarrier approach as realized by orthogonal frequency division multiplexing (OFDM) has been chosen for several new standards which include digital audio broadcasting (DAB), digital video broadcasting (DVB) [1], and wireless LANs such as 802.11a [2]. The DVB [3] is similar to DAB standard but is used for broadcasting digital television signals. It uses 8 MHz bandwidth and the OFDM signal is modulated up to 64 QAM points.

The OFDM transmission in a very wideband (VWB) channel, although it is resistant to multipath fading, is vulnerable to narrowband interference which often appears in wideband channels. In this paper, we propose an enhancement to OFDM which improves its performance and flexibility by introducing and exploiting frequency and time diversity. This enhancement is based on orthogonal code division multiplexing (OCDM). The resulting system formed by combining OCDM with the standard OFDM is
called multicarrier orthogonal code division multiplexing (MC-OCDM).

There are several related methods in the literature known as multicarrier CDMA or multicarrier DS-CDMA, which are proposed as multiple-access (multipoint-to-point) transmission [4-6]. These methods are the results of combining OFDM with CDMA. A multicarrier (MC) CDMA system may be synchronous [5], or asynchronous [6, 7] or it may be bandwidth expanding (spreading the spectrum) $[7,8]$ or nonbandwidth expanding (not spectrum spreading) [6]. Asynchronous access techniques do not require synchronization between transmitting users but they suffer from multiuser interference [9]. In all the above MC-CDMA methods, the spreading of each OFDM subcarrier (by orthogonal or PN codes) results in loosing the orthogonality between them. That is, although there are multiple subcarriers which may carry the same symbols, these subcarriers interfere with each other.

The MC-OCDM system presented here is novel and different from the above systems in more than one way. It is


Figure 1: (a) The basic MC-OCDM transmitter; (b) the basic MC-OCDM receiver.
assumed to be used for point-to-point and for point-tomultipoint transmission. In order to meet the required performance in VWB transmission, its design has two components: the basic and the composite. The basic MC-OCDM is a non-spectrum-spreading transmission method which has the property of distributing the power of each transmitted symbol into all subcarrier frequencies. Each subcarrier will then carry all transmitted symbols which are distinguished by orthogonal Hadamard sequences. Also, unlike MC-CDMA, all subcarriers are orthogonal to each other as in OFDM. The MC-OCDM provides frequency and time diversity by transmitting symbols in parallel both in the frequency and time domains. Unlike the standard OFDM in which each symbol is carried by only one subcarrier, the basic MC-OCDM may combat the effects of narrowband interference (NBI). The basic MC-OCDM is an original idea and has been patented under the title "interference suppressing OFDM method for wireless communications" [10]. The composite MC-OCDM is a method of multiplexing basic MC-OCDM channels into a VWB channel. This method is based on OFDM; that is, each basic MC-OCDM channel is orthogonally frequency division multiplexed into a composite VWB system. The choice of basic MC-OCDM bandwidth and the number of basic MC-OCDM subchannels are system parameters and their values are determined from the propagation characteristics of the wireless channel.

The article is organized as follows: in Sections 2 and 3, we present the descriptions of the transmitter and receiver of the basic MC-OCDM and the composite MC-OCDM, respectively, verification of its functional correctness and
establishment of orthogonality requirements in ideal channel conditions. Then in Section 4 we present the system's performance evaluation. This includes analysis and simulation of the effects of narrowband interference on the basic MCOCDM and comparisons with the standard OFDM. Then we provide an assessment of the composite system in terms of the performance, spectral efficiency multiplexing capability, and implementation for very wideband channel application.

## 2. THE BASIC MC-OCDM

### 2.1. The transmitter

The transmitter of the proposed basic MC-OCDM is illustrated in Figure 1(a). The input data stream $x(n)$ enters a serial-to-parallel (S/P) converter which provides $\widetilde{M}$ parallel data streams. At the output of the $\mathrm{S} / \mathrm{P}$ converter, the data signal $x_{q}$ ( $T$ seconds long) of parallel stream $q$ is spread by orthogonal binary Hadamard sequence $\mathbf{w}_{\mathbf{q}}=$ $\left[w_{q, 0}, w_{q, 1}, \ldots, w_{q, \widetilde{M}-1}\right]$, for $q=0, \ldots, \widetilde{M}-1$. In the spreading process the entire sequence of length $T$ has to "overlay" a single data symbol also of length $T$. Assuming that $x_{q}$ represents a complex-valued signaling point in a QAM constellation, that is, $x_{q}=\alpha_{q}+j \beta_{q}$, the spread signal then is

$$
\begin{equation*}
X_{q, k}=x_{q} w_{q, k}=\alpha_{q} w_{q, k}+j \beta_{q} w_{q, k} \tag{1}
\end{equation*}
$$

for $k=0, \ldots, \widetilde{M}-1$. The above process is called orthogonal code division multiplexing (OCDM) and provides a set of $\widetilde{M}$ parallel data streams which are separated from each other by orthogonal Hadamard codes.

On the next step, each of the parallel orthogonal streams enters a second S/P bit buffer and encoder which provides $M$ parallel substreams. The encoder creates $M=2 \widetilde{M}$ complex data points defined by

$$
b_{k}=\sum_{q=0}^{\widetilde{M}-1} y_{q k}= \begin{cases}\widetilde{M}-1 & \text { for } k=0,  \tag{2}\\ \sum_{q=0}^{\widetilde{M}-1} \alpha_{q} w_{q, 0} & \text { for } k=1,2, \ldots, \widetilde{M}-1, \\ \widetilde{M}-1 & \text { for } k=\widetilde{M} \\ \sum_{q=0} \beta_{q} w_{q, 0} & \text { for } k=\widetilde{M}+1, \ldots, M-1,\end{cases}
$$

where $(\cdot)^{*}$ denotes conjugation. In the above, both $y_{q, 0}$ and $y_{q, \widetilde{M}}$ are real valued.

Then, the $M$ parallel data points $b_{k}$ enter an inverse fast Fourier transform (IFFT) encoder the output of which is given by

$$
\begin{equation*}
B_{m}=\frac{1}{M} \sum_{k=0}^{M-1} b_{k} e^{j 2 \pi(k m / M)}=\frac{1}{M} \sum_{k=0}^{N-1} \sum_{q=0}^{\widetilde{M}-1} y_{q, k} e^{j 2 \pi(k m / M)} \tag{3}
\end{equation*}
$$

The Hermitian symmetry provided in (2) and shown in Figure $1(\mathrm{a})$ as an " $\mathrm{S} / \mathrm{P}$ and encoder" allows us to have real valued signal at the IFFT output. That is, the real part of the signal is transmitted by $\widetilde{M}$ subcarriers in one side of the spectrum and the imaginary part by another $\widetilde{M}$ subcarriers in the other side of the spectrum. The modulated signal then has one real (not quadrature) component with $M=2 \widetilde{M}$ subcarriers. The parallel IFFT outputs $B_{m}$ for $m=0,1, \ldots, M-1$, then enters a $\mathrm{P} / \mathrm{S}$ converter where a cyclic prefix or guard interval is added. The output of the P/S converter $s(m)$ is then converted to an analog signal $s(t)$ which is then up-converted to a carrier frequency and transmitted at the assigned frequency band.

Based on the above description, the $\widetilde{M}$ incoming data symbols $\left[x_{0}, x_{1}, \ldots, x_{\widetilde{M}-1}\right]$, to the input of the MC-OCDM encoder for the period of a frame, can be arranged as illustrated by the matrix $\mathscr{D}_{\widetilde{M}}$ below:

$$
\begin{gather*}
\mathcal{D}_{\widetilde{M}}=\left[\begin{array}{cccc}
x_{0} & x_{0} & \cdots & x_{0} \\
x_{1} & x_{1} & \cdots & x_{1} \\
\cdots & \cdot & \cdots & \cdots \\
x_{\widetilde{M}-1} & x_{\widetilde{M}-1} & \cdots & x_{\widetilde{M}-1}
\end{array}\right] \leftarrow  \tag{4}\\
\uparrow \\
\uparrow \\
\\
f_{0}
\end{gather*} f_{1} \quad \begin{array}{cc}
\mathbf{w}_{0} \\
\mathbf{w}_{1} \\
& \mathbf{w}_{\widetilde{M}-1}
\end{array}
$$

As we observe, every frequency bin or subcarrier $f_{i}, i=0, \ldots$, $\widetilde{M}-1$, carries all data bits $x_{0}, x_{1}, \ldots, x_{\widetilde{M}-1}$, which are distinguished from each other by the orthogonal Hadamard sequences $\mathbf{w}_{\mathbf{q}}=\left[w_{q, 0}, w_{q, 1}, \ldots, w_{q, \widetilde{M}-1}\right]$, for $q=0, \ldots, \widetilde{M}-1$.

This means that the power of each data bit is distributed or "spread" to all subcarriers as opposed to the orthogonal frequency division multiplexing (OFDM) in which each subcarrier carries only one symbol.

Let us now consider the special case where the orthogonal sequences are not Hadamard but are having $(0,1)$ entries as follows: $\mathbf{w}_{\mathbf{q}}=\left[w_{q, k}\right]$ where

$$
w_{q, k}= \begin{cases}1 & \text { for } q=k  \tag{5}\\ 0 & \text { for } q \neq k\end{cases}
$$

Then, as we may easily verify, the MC-OCDM becomes OFDM. Hence, the OFDM is a special case of the MCOCDM, corresponding to the matrix $\mathscr{D}_{\widetilde{M}}$ shown below:

$$
\begin{gather*}
\mathscr{D}_{\widetilde{M}}=\left[\begin{array}{cccc}
x_{0} & 0 & \cdots & 0 \\
0 & x_{1} & \cdots & 0 \\
\cdots & \cdot & \cdots & \cdot \\
0 & 0 & \cdots & x_{\widetilde{M}-1}
\end{array}\right] \leftarrow \begin{array}{cc}
\mathbf{w}_{0} \\
& \mathbf{w}_{1} \\
\uparrow & \uparrow \\
& \mathbf{w}_{\widetilde{M}-1} \\
f_{0} & f_{1}
\end{array} \quad \begin{array}{l}
\widetilde{M}-1
\end{array} \tag{6}
\end{gather*}
$$

### 2.2. The receiver

The receiver of the basic MC-OCDM is illustrated in Figure $1(\mathrm{~b})$. As shown, the received analog signal $r(t)$ is digitized by an A/D converter and then enters a S/P converter where also the cyclic prefix is removed. The $\mathrm{S} / \mathrm{P}$ converter output provides $M$ parallel data points $z_{m}$ for $m=0,1, \ldots, M-1$, which then enter a fast Fourier transform (FFT). The FFT output provides $M$ complex data signal points given by

$$
\begin{equation*}
\bar{Z}_{k}=\sum_{m=0}^{M-1} z_{m} e^{-j 2 \pi(k m / M)} \quad \text { for } k=0,1, \ldots, M-1 \tag{7}
\end{equation*}
$$

The above parallel data then enters a decoder/demapper which creates $\widetilde{M}=M / 2$ data points defined by

$$
Z_{k}= \begin{cases}\bar{Z}_{k} & \text { for } k=1,2, \ldots, \widetilde{M}-1  \tag{8}\\ \bar{Z}_{0}+j \bar{Z}_{\widetilde{M}} & \text { for } k=0\end{cases}
$$

Now, the $\widetilde{M}$ parallel $Z_{k}$ points enter a P/S converter the output of which is despread by $\widetilde{M}$ Hadamard sequences $\mathbf{w}_{\mathbf{q}}=$ $\left[w_{q, 0}, w_{q, 1}, \ldots, w_{q, \widetilde{M}-1}\right]$ in parallel for $q=0,1, \ldots, \widetilde{M}-1$, for recovering the data.

In order to verify the functional correctness of the MCOCDM process we assume that the channel is noiseless (the effects of channel noise and interference are examined in the performance section). The received signal is given by $r(t)=$ $\sum_{i} h_{i}(t) * s(t)$, where $h_{i}(t)$ is the channel impulse response at multipath $i$ and $(*)$ denotes convolution. Now, it can be shown that the post-FFT signal is $\bar{Z}_{k}=H_{k} b_{k}$, where $H_{k}$ is the channel transfer function at subcarrier $k$ and $b_{k}$ is given by (2). The post decoder/demapper signal then becomes

$$
\begin{equation*}
Z_{k}=H_{k} \sum_{q=0}^{\widetilde{M}-1} x_{q} w_{q, k} \quad \text { for } k=0,1, \ldots, \widetilde{M}-1 \tag{9}
\end{equation*}
$$



Figure 2: (a) The composite MC-OCDM; (b) the distribution of symbols into frequency bins.

After the $\mathrm{P} / \mathrm{S}$ converter the signal at the output of the de-spreader-1 is given by

$$
\begin{align*}
\sum_{k=0}^{\widetilde{M}-1} Z_{k} w_{1, k} & =\sum_{k=0}^{\widetilde{M}-1}\left[H_{k} \sum_{q=0}^{\widetilde{M}-1} x_{q} w_{q, k}\right] w_{1, k} \\
& =\sum_{q=0}^{\widetilde{M}-1} H x_{q} \sum_{k=0}^{\widetilde{M}-1} w_{q, k} w_{1, k}= \begin{cases}\widetilde{M} H x_{1} & \text { for } q=1, \\
0 & \text { for } q \neq 1 .\end{cases} \tag{10}
\end{align*}
$$

In the above result we have made the assumption that the channel magnitude is frequency flat, that is, $\left|H_{k}\right|=|H|$ for all $k$. We also assume that the channel phase rotation between subcarriers $e^{-j 2 \pi k / \widetilde{M}}$ is corrected for each subcarrier $k$.

## 3. THE COMPOSITE MC-OCDM

We may now extend the basic MC-OCDM into a composite MC-OCDM system which will have the capability of high transmission rates in VWB channels. The concept of the composite MC-OCDM is illustrated in Figure 2(a). As shown, the outputs of $L$ basic MC-OCDM encoders are multiplexed by an OFDM encoder into the composite system. The entire VWB channel will have a total of $N(N=L M)$
frequency bins which are grouped into $L$ groups called subchannels. Each subchannel will then carry $M$ data symbols per frame and the transmit power of each symbol will be distributed over all $M$ frequency bins as in the basic MCOCDM, see Figure 2(b). The different subchannels will carry different data symbols which will be orthogonal to each other as in an ordinary OFDM. The transmitter and receiver designs of the composite MC-OCDM are described below.

### 3.1. The transmitter

The composite MC-OCDM transmitter is shown in Figure 3. As shown, an input data stream of rate $R \mathrm{bps}$, enters a $\mathrm{S} / \mathrm{P}$ converter which provides $L$ parallel streams. Each parallel stream of rate $R / L$ enters again a second $\mathrm{S} / \mathrm{P}$ converter which provides $\widetilde{M}$ parallel streams each with rate $R / \tilde{N}$, where $\tilde{N}=$ $L \widetilde{M}$. At the output of the S/P converter, a data signal $x_{q}(T$ seconds long) of a parallel stream $q$ is spread by an orthogonal binary Hadamard sequence $\mathbf{w}_{\mathbf{q}}=\left[w_{q, 0}, w_{q, 1}, \ldots, w_{q, \widetilde{M}-1}\right]$, for $q=0, \ldots, \widetilde{M}-1$, (the entire sequence of length $T$ has to "overlay" a single data symbol also of length $T$ ). After the spreading operation the signal rate is $R / L$ bps. Assuming that $x_{q}^{(l)}$ represents a complex-valued signaling point in a QAM constellation, that is, $x_{q}^{(l)}=\alpha_{q}^{(l)}+j \beta_{q}^{(l)}$, the spread signal is

$\mathrm{P} / \mathrm{S}$ : parallel-to-serial conversion
S/P: serial-to-parallel conversion

Figure 3: The composite MC-OCDM transmitter.
given by

$$
\begin{equation*}
x_{q}^{(l)} w_{q, k}=\alpha_{q}^{(l)} w_{q, k}+j \beta_{q}^{(l)} w_{q, k} \tag{11}
\end{equation*}
$$

where $k=0, \ldots, \widetilde{M}-1$ and $l=1, \ldots, L$. Let us now define

$$
\begin{equation*}
b_{k}^{(l)}=\sum_{q=0}^{\widetilde{M}-1} x_{q}^{(l)} w_{q, k} \tag{12}
\end{equation*}
$$

where $k=0, \ldots, \widetilde{M}-1$ and $l=1, \ldots, L$. For any pair $(k, l)$, we then define

$$
a_{i}= \begin{cases}\mathfrak{R}\left\{b_{0}^{(1)}\right\}=\sum_{q=0}^{\widetilde{M}-1} \alpha_{q}^{(1)} w_{q, 0} & \text { for } i=0,  \tag{13}\\ b_{k}^{(l)} & \text { for } i=k L+l-1, \\ & \forall(k, l) \neq(0,1), \\ \mathfrak{I}\left\{b_{0}^{(1)}\right\}=\sum_{q=0}^{\widetilde{M}-1} \beta_{q}^{(1)} w_{q, 0} & \text { for } i=\widetilde{N} \\ \left\{b_{k}^{(l)}\right\}^{*} & \text { for } i=2 \widetilde{M} L-(k L+l-1), \\ & \forall(k, l) \neq(0,1) .\end{cases}
$$

In the above equation we have assumed that $\tilde{N}=L \widetilde{M}$ and $N=2 \tilde{N}$. This process takes place in the "encoder" shown
in Figure 3, provides $N$ parallel points $a_{i}$ to the input of the IFFT, the output of which is given by

$$
\begin{equation*}
s_{n}=\frac{1}{\sqrt{N}} \sum_{i=0}^{N-1} a_{i} e^{j 2 \pi(i n / N)} \quad \text { for } n=0,1, \ldots, N-1 \tag{14}
\end{equation*}
$$

As in the basic system the IFFT output of the composite one is then real valued. If we now assume $L=1$, then the resulting system having $\tilde{N}=\widetilde{M}$ is the basic MC-OCDM. In addition, if the spreading orthogonal Hadamard matrix $\mathbf{W}=\left[\mathbf{w}_{1}, \mathbf{w}_{2}, \ldots, \mathbf{W}_{\tilde{\mathbf{N}}}\right]^{T}$ is replaced with an identity matrix $\mathbf{W}=\mathbf{I}$, the resulting system is the ordinary OFDM. Also, if we take $\widetilde{M}=1$ and $\widetilde{N}=L$, the resulting system is again the ordinary OFDM.

### 3.2. The receiver

The composite MC-OCDM receiver design is shown in Figure 4. As shown, the received signal enters an OFDM receiver, which provides parallel outputs $Z_{k}^{(l)}$, for each frequency bin $k=0,1, \ldots, \widetilde{M}-1$ and each group $l=1,2, \ldots, L$ $(\tilde{N}=L \widetilde{M})$. The outputs of each group $l, Z_{k}^{(l)}$ then enter a $\mathrm{P} / \mathrm{S}$ converter the output of which is despread by the orthogonal sequences for recovering the data. The output of the


Figure 4: The composite MC-OCDM receiver.
despreader 0 of group $l=1$ is given by

$$
\begin{equation*}
Z_{0}^{(1)}=\sum_{k=0}^{\widetilde{M}-1} Z_{k}^{(1)} w_{0, k}=\sum_{k=0}^{\widetilde{M}-1}\left[b_{k}^{(1)} H_{k}^{(1)}+\mathrm{n}_{k}^{(1)}\right] w_{0, k} \tag{15}
\end{equation*}
$$

where $b_{k}^{(l)}=\sum_{q=0}^{\widetilde{M}-1} x_{q}^{(l)} w_{q, k}, H_{k}^{(1)}$ is the channel transfer function, and $\mathrm{n}_{k}^{(1)}$ is the noise in each bin $k$ of group 1. Assuming an ideal channel ( $H_{k}^{(1)}=1$ for all $k$ ), the useful part of the signal $\mathbb{Z}_{0}^{(1)}$ provides the data $x_{0}^{(1)}$ at the output of the despreader 0 , as shown below,

$$
\begin{align*}
\sum_{k=0}^{\widetilde{M}-1} b_{k}^{(1)} w_{0, k} & =\sum_{k=0}^{\widetilde{M}-1}\left[\sum_{q=0}^{\widetilde{M}-1} x_{q}^{(1)} w_{q, k}\right] w_{0, k} \\
& =\sum_{q=0}^{\widetilde{M}-1} x_{q}^{(1)} \sum_{k=0}^{\widetilde{M}-1} w_{q, k} w_{0, k}  \tag{16}\\
& = \begin{cases}\widetilde{M} x_{0}^{(1)} & \text { for } q=0, \\
0 & \text { for } q \neq 0 .\end{cases}
\end{align*}
$$

In a frequency selective channel the choice of the parameters $L$ and $\widetilde{M}$ will be made so that the channel is relatively flat in the bandwidth of $\widetilde{M}$ frequency bins.

## 4. PERFORMANCE EVALUATION

### 4.1. The signal model

Let us consider the basic MC-OCDM transmitter presented in Section 2.1. The signal $B_{l, m}$ is the IFFT of $b_{l, k}$, see (3), where $b_{l, k}$ is the $k$ th parallel IFFT input at the $l$ th frame,
see (2). After the P/S converter and the addition of $M_{g}$ guard samples, the output digital signal is $s_{l}(m)$ for $m=$ $-M_{g}, \ldots, M-1$; where there are $M_{s}=M+M_{g}$ samples per frame. The equivalent time intervals are $T_{s}=T+T_{g}, T_{g}$ is the guard time (or cyclic prefix), and the sampling time interval is $T_{M}=T / M$. This analysis is based on the assumption that the maximum channel dispersion $\tau_{\max }<T_{g}$.

Assuming that the channel remains unchanged for the duration of the frame, the post-FFT and decoder/demapper signal at the $l$ th frame and $k$ th subcarrier is given by

$$
\begin{gather*}
Z_{l, k}=a_{l, k} \cdot H_{l, k}+\mathrm{n}_{I ; l, k}+\mathrm{n}_{N ; l, k}, \\
\text { where } a_{l, k}=\sum_{q=0}^{\widetilde{M}-1} x_{q}^{(l)} w_{q, k} \tag{17}
\end{gather*}
$$

for $k=0,1, \ldots, \widetilde{M}-1 . H_{l, k}$ is the channel transfer function (CTF) during the $l$ th frame at subcarrier frequency $k$ and is considered to include both the response of the channel and the transmission filter. Also, $\mathrm{n}_{I ; k, l}$ and $\mathrm{n}_{k, l}$ are the interference and AWGN component, respectively. Next, the signal $Z_{l, k}$ enters a P/S converter, the output of which will be despread by each orthogonal sequence in parallel for recovering the corresponding data. Now, since the channel is a stationary process, we may focus our attention on a particular frame and drop the subscript $l$. The output of the despreader- 1 is then given by

$$
\begin{equation*}
Z_{1}=\sum_{k=0}^{\widetilde{M}-1} Z_{k} w_{1, k}=\sum_{k=0}^{\widetilde{M}-1} a_{k} H_{k} w_{1, k}+\sum_{k=0}^{\widetilde{M}-1}\left[\mathrm{n}_{I ; k}+\mathrm{n}_{N ; k}\right] w_{1, k} . \tag{18}
\end{equation*}
$$

### 4.2. The effects of narrowband interference and AWGN

Let us now assume that the interference noise $\mathrm{n}_{I}$ considered in the previous subsection, takes the form of narrowband interference (NBI). We also assume that no other interference is present except AWGN and that the channel multipath fading is frequency flat.

Based on the assumption of frequency-flat fading, $H_{k}$ has the same value for all subcarriers, that is, $H_{k}=H$ for $k=0,1,2, \ldots, \widetilde{M}-1$. As shown in (10), the first term of (18), representing the useful part of the signal at the output of despreader- 1 , is given by

$$
\sum_{k=0}^{\widetilde{M}-1} a_{k} H_{k} w_{1, k}= \begin{cases}\widetilde{M} H x_{1} & \text { for } q=1  \tag{19}\\ 0 & \text { for } q \neq 1\end{cases}
$$

Therefore,

$$
\begin{equation*}
z_{1}=\widetilde{M} H x_{1}+\sum_{k=0}^{\widetilde{M}-1} \mathrm{n}_{I ; k} w_{1, k}+\sum_{k=0}^{\widetilde{M}-1} \mathrm{n}_{N ; k} w_{1, k} . \tag{20}
\end{equation*}
$$

The useful power of the received signal then is $P_{U}=\widetilde{M}^{2}|H|^{2} x_{1}^{2}$.
The power of the NBI is given by

$$
\begin{equation*}
P_{I}=E\left\{\left(\sum_{k \in \mathcal{K}_{1}} \mathrm{n}_{I ; k} w_{1, k}\right)^{2}\right\}=\sum_{k \in \mathcal{X}_{1}} E\left\{\left|\mathrm{n}_{I ; k}\right|^{2}\right\} \tag{21}
\end{equation*}
$$

where $\mathcal{K}_{l}$ is the set of bins affected by NBI. The number of bins in the set $\mathcal{K}_{\perp}$ is assumed to be $K<\widetilde{M}$. In the above we have made the assumption that random variables $n_{I ; k}$ are mutually independent. If we also assume that random variables $\mathrm{n}_{I ; k}$ are identically distributed with variance $\sigma_{I ; k}^{2}=\sigma_{I}^{2}$ for all $k$, then

$$
\begin{equation*}
P_{I}=\sum_{k \in \mathcal{K}_{I}} E\left\{\left|\mathrm{n}_{I ; k}\right|^{2}\right\}=\sum_{k \in \mathcal{K}_{I}} \sigma_{I ; k}^{2}=K \sigma_{I}^{2} . \tag{22}
\end{equation*}
$$

The power of AWGN is

$$
\begin{equation*}
P_{N}=E\left\{\left[\sum_{k=0}^{\widetilde{M}-1} \mathrm{n}_{N ; k} w_{1, k}\right]^{2}\right\}=\sum_{k=0}^{\widetilde{M}-1} E\left\{\left|\mathrm{n}_{N ; k}\right|^{2}\right\}=\widetilde{M} \sigma_{N}^{2}, \tag{23}
\end{equation*}
$$

where $\sigma_{N}^{2}=E\left\{\left|\mathrm{n}_{N ; k}\right|^{2}\right\}$ for all $k$. The signal-to-interference-and-noise ratio (SINR) $\gamma_{1}$ of the MC-OCDM at the output of despreader-1 then is

$$
\begin{equation*}
\gamma_{1}=\frac{P_{U}}{P_{I}+P_{N}}=\frac{\widetilde{M}^{2}|H|^{2} x_{1}^{2}}{\sum_{k \in \mathcal{K}_{I}} \sigma_{I ; k}^{2}+\widetilde{M} N_{o}}=\frac{\widetilde{M}|H|^{2} x_{1}^{2}}{(K / \widetilde{M}) \sigma_{I}^{2}+\sigma_{N}^{2}} . \tag{24}
\end{equation*}
$$

The signal-to-interference-and-noise ratio of the OFDM $\gamma_{1}^{\prime}$ in frequency bin- 1 (assuming that bin- 1 is affected by NBI) is given by

$$
\begin{equation*}
\gamma_{1}^{\prime}=\frac{P_{U}^{\prime}}{P_{I}^{\prime}+P_{N}^{\prime}}=\frac{\widetilde{M}|H|^{2} x_{1}^{2}}{\sigma_{I ; 1}^{2}+\sigma_{N}^{2}} \tag{25}
\end{equation*}
$$

In the above we have assumed that $P_{U}=P_{U}^{\prime}=\widetilde{M}|H|^{2} x_{1}^{2}$. This means that the total power of symbol $x_{1}$ in the basic MC-OCDM (accross all frequency bins) must be equal to the power of $x_{1}$ in frequency bin-1 for OFDM.

Comparing $\gamma_{1}$ with $\gamma_{1}^{\prime}$ we observe that the basic MCOCDM has an advantage over OFDM in the presence of narrowband interference (NBI). As shown, the received signal power of symbol $x_{1}$, that is, $H^{2} x_{1}^{2}$ is spread to all $\widetilde{M}$ frequency bins while the narrowband interference power only exists in $K$ out of $\widetilde{M}$ frequency bins $(K<\widetilde{M})$. In OFDM on the other hand, the SINR at a frequency bin-1 will be much smaller if that bin is affected by NBI. The uncoded probability of error $P_{e}$ for the coherent basic MC-OCDM (antipodal) signal at the output of despreader-1 is given by

$$
\begin{equation*}
P_{e}=Q\left(\sqrt{2 \gamma_{1}}\right) \tag{26}
\end{equation*}
$$

where $Q(x)=(1 / \sqrt{2 \pi}) \int_{x}^{\infty} e^{-t^{2} / 2} d t$. Then, $P_{e}<P_{e}^{\prime}$ where $P_{e}^{\prime}$ is the corresponding OFDM probability of error of bin-1 $P_{e}^{\prime}$, because $\gamma_{1}>\gamma_{1}^{\prime}$.

In order to counteract the effects of NBI, OFDM systems utilize forward error correcting codes and interleaving across frequencies. In this case the analytic evaluation of the OFDM bit error probability is quite complicated because the random variables of interference are not identically distributed across subcarrier frequencies. The evaluation of the coded OFDM with NBI is achieved by simulation (presented in the next section) which indicates significant reduction in the bit error probability. However as shown, the error-rate increase due to NBI cannot be completely eliminated unless the coding rate is sufficiently low in which case the cost in terms of spectral efficiency loss is high. We also observe that in AWGN (i.e., without NBI) the bit error probability is the same for both MC-OCDM and OFDM systems.

### 4.3. Simulation results

In this section, we provide simulation results that compare the BER performance of a basic MC-OCDM system with OFDM in the presence of narrowband interference (NBI). Similar to the 802.11a model, we set the transmission bandwidth of the basic MC-OCDM $(L=1)$ to be equal to 20 MHz and consist of $N=64$ frequency bins. Then, the bandwidth of each subcarrier is given by $B_{s}=20 / 64=0.3125 \mathrm{MHz}$. Furthermore, according to 802.11a, the transmitter model consists of a random data generator, followed by rate $1 / 2$ convolutional encoder and puncturing (to achieve higher coding rates), interleaver (of length equal to one OFDM symbol), signal modulator (QPSK, or 16 QAM), MC-OCDM symbol modulator, cyclic copy, and windowing. The only addition to the standard OFDM system is the OCDM encoder which spreads the input symbols to all frequency bins (this corresponds to setting the parameter $M=N$, while for $M=1$ the MC-OCDM reduces to the standard OFDM). Similar to the 802.11a model, the number of input data bits per encoder frame corresponds to multiple MC-OCDM (or OFDM) symbols, depending on thesimulated data rate.


Figure 5: The uncoded BER versus $E_{b} / N_{0}$ for the basic MC-OCDM and OFDM with QPSK modulation in AWGN channel with NBI in one frequency bin.

Furthermore, in the basic MC-OCDM system no interleaving/deinterleaving is needed.

The channel model under consideration (for point-topoint or point-to-multipoint communication) consists of AWGN with the addition of NBI. The NBI is generated by a zero-mean Gaussian random process $j(t)$ with double-sided power spectral density $N_{o} / 2$ and bandwidth of $B_{j}=B_{s}$, which is centered at a subcarrier $k, k=0, \ldots, N-1$. In the simulation model, this is accomplished by passing the noise $j(t)$ through a "brick-wall" filter with bandwidth $B_{s}$ around subcarrier $k$ [7]. Then, the NBI power is given by $J=E\left[j^{2}(t)\right]=B_{s} N_{o} / 2$. The ratio $J / S$ of the interference power per frequency bin over the signal power per frequency bin can take different values. Furthermore, the channel may have one or more such narrowband interferers. At the receiving end, after the FFT decoder and OCDM decoder, the data are recovered using a soft Viterbi decoder. In conclusion, the main difference between the basic MC-OCDM and OFDM simulation models is the OCDM encoder/decoder that is used only in MC-OCDM.

Figure 5 shows the uncoded BER versus $E_{b} / N_{0}$ of the basic MC-OCDM $(M=N)$ and OFDM $(M=1)$ systems with QPSK modulation in AWGN channel with NBI in one frequency bin and $J / S=-\infty, 0,3$. We observe that while MCOCDM and OFDM have the same performance in an AWGN channel without NBI, the uncoded OFDM is extremely sensitive to NBI even when this is present in only one frequency bin.

In Figure 6 we show the BER versus $E_{b} / N_{0}$ of $3 / 4$ convolutionally coded basic MC-OCDM and OFDM systems with QPSK modulation in AWGN channel with NBI in one


Figure 6: The convolutionally 3/4 coded and soft Viterbi decoded BER versus $E_{b} / N_{0}$ for the basic MC-OCDM and OFDM with QPSK modulation in AWGN channel with NBI in one frequency bin.
frequency bin for various values of $J / S$. We observe that the MC-OCDM has significantly better performance than OFDM, although the coded OFDM system has much improved its performance as compared with the uncoded OFDM in the presence of NBI in one frequency bin.

Figure 7 shows the convolutionally $1 / 2$ coded and soft Viterbi decoded BER versus $E_{b} / N_{0}$ of the basic MC-OCDM and OFDM systems with 16 QAM modulation in AWGN channel with NBI in three consecutive frequency bins and $J / S=-\infty, 0,3 \mathrm{~dB}$. We again observe that the MC-OCDM has better performance than OFDM, although the performance of either system is not satisfactory when $J / S=3 \mathrm{~dB}$.

Figure 8 shows the convolutionally $1 / 2$ coded and soft Viterbi decoded BER versus $E_{b} / N_{0}$ of the basic MC-OCDM and OFDM with QPSK modulation in AWGN channel with NBI in 0,5 , and 10 frequency bins when $J / S=3 \mathrm{~dB}$. Once again we observe that MC-OCDM outperforms OFDM. We may then conclude that when the number of bins with interference is greater than a threshold, low rate coding with interleaving does not help the OFDM system.

Although fading was not considered in this work because of space limitations, we found that basic MC-OCDM and OFDM systems have identical BER performance in timeselective flat fading Rayleigh channels for all values of the product $f_{D} T$ (where $f_{D}$ is the Doppler frequency and $T$ is the symbol length) as well as in Rician flat fading channels. Furthermore, MC-OCDM in frequency selective channels where the channel transfer function fades for consecutive frequency bins across the total bandwidth outperforms OFDM as the spreading across frequencies offers increased protection capability, similar to the NBI case. Therefore, the


Figure 7: The convolutionally $1 / 2$ coded and soft Viterbi decoded BER versus $E_{b} / N_{0}$ for the basic MC-OCDM and OFDM with 16QAM modulation in AWGN channel with NBI in three frequency bins.


Figure 8: The convolutionally $1 / 2$ coded and soft Viterbi decoded BER versus $E_{b} / N_{0}$ for the basic MC-OCDM and OFDM with QPSK modulation in AWGN channel with NBI and $J / S=3 \mathrm{~dB}$.

MC-OCDM, the choice of the basic subchannel bandwidth is made so that it is wider than NBI, but narrower than the average width in which the magnitude of the channel transfer function is constant. In addition, the composite MC-OCDM has all the advantages of a VWB multicarrier transmission time-dispersive channels. It allows the support of high data rates while maintaining symbol durations longer than the channel's dispersion time.

## Spectral efficiency

The composite system has the same spectral efficiency as each basic subchannel because multiplexing is achieved by using OFDM. That is, no frequency guard bands or guardtime exist between the subchannels. In addition, each basic MC-OCDM subchannel has high spectral efficiency since it does not spread its bandwidth. Also, while all subcarriers in the basic MC-OCDM subchannel must have the same modulation, different subchannels may have different modulation load. Therefore, the composite MC-VWB system may use variable modulation loading so that the modulation load of each subchannel is adapted to its propagation conditions. This is another way of enhancing the spectral efficiency of the system.

## Multiplexing

The composite system in addition to broadcasting a single VWB channel also has the capability of multiplexing subchannels(up to $L$ different users) as in point-to-multipoint
transmissions. This property is useful in broadcasting applications such as video-on-demand.

## Implementation

The composite MC-OCDM can be implemented with $L$ standard IFFT devices (each for a basic MC-OCDM) instead of one high-speed IFFT. This approach allows us to implement the VWB channel by overcoming the hardware speed (MIPs) and complexity limitations of existing (available) hardware components. As an implementation example we may consider a MC-VWB system with total number of subcarrier $N=M L=516$. Let us consider the choice $M=32$ subcarriers per subchannel and $L=16$ subchannels. Assuming subcarrier spacing $10 \mathrm{MHz} / 32=0.3125 \mathrm{MHz}$, (the same as in 802.11a) and since the number of subcarriers per subchannel is $M=32$, the subchannel bandwidth is about 10 MHz wide. Then the MC-VWB system has a bandwidth of $16 \times 10=$ 160 MHz . Assuming each subchannel has QPSK modulation and $3 / 4$ channel coding rate, a data rate of $9 \mathrm{Mb} / \mathrm{s}$ can be provided. The resulting MC-VWB system data rate then is $144 \mathrm{Mb} / \mathrm{s}$.

## 5. CONCLUSION

In this article we have presented a novel MC-OCDM system appropriate for VWB point-to-point and point-tomultipoint transmission. its design has two components: the basic and the composite.

The basic MC-OCDM is a generalized form of the orthogonal frequency division multiplexing (OFDM). It has the property of distributing the power of each transmitted symbol to all subcarrier frequencies. Each subcarrier will then carry all transmitted symbols which are distinguished by orthogonal Hadamard sequences. The basic MCOCDM has shown to improve the performance of the standard OFDM by introducing frequency and time diversity. In particular MC-OCDM combats the effects of narrowband interference, while it maintains all the advantages of the standard OFDM in terms of having reliable high data rate transmission in time-dispersive wireless channels. The properties of the basic MC-OCDM have been established analytically and then verified by simulation. The system simulation is based on the 802.11a OFDM standard and is used to provide the coded and uncoded BER in different propagation environments. The above analysis and simulation led us to the following conclusion.

The basic MC-OCDM has better BER performance than OFDM in the presence of NBI for both the coded and uncoded systems. In the case of uncoded channel the symbols in the frequency bins that are corrupted by NBI are recoverable if MC-OCDM is used because every symbol is carried in all frequency bins, while this is not true in OFDM systems. In the case of coded channel it has been shown that the BER increase due to NBI is greater in OFDM than it is in MCOCDM. Therefore, the MC-OCDM may easily be protected
from NBI with "little" channel coding and thus can achieve higher spectral efficiency than OFDM.

The composite MC-OCDM synthesized a VWB transmission channel by multiplexing with OFDM basic MCOCDM subchannels. It is optimized by choosing the bandwidth of each subchannel to be wider than the NBI and narrower than the average width of a frequency selective fade so that the magnitude of the channel transfer function is approximately constant. The composite MC-VWB system may adapt the modulation load on each subchannel according to the propagation conditions in each of them. This way we can optimize performance of the entire width of the VWB channel. The composite MC-OCDM also provides multiplexing capability of multiple user subchannels in a spectrally efficient manner. That is, without frequency guard bands between subchannels. Finally, the composite MC-OCDM provides an approach for implementing the VWB system by using $L$ parallel low speed FFTs instead of one having high speed whose implementation may not be easy.

## REFERENCES

[1] ETSI ETS 300 744, "Digital video broadcasting; frame structure, channel coding and modulation for digital terrestrial television (DVB-T)," ETSI Tech. Rep., March 1997.
[2] IEEE 802.11a-1999, "Wireless LAN Medium Access Control and Physical Layer specifications-High Physical Layer in tha 5GHz Band".
[3] H. Sari, G. Karam, and I. Jeanclaude, "Transmission techniques for digital terrestrial TV broadcasting," IEEE Communications Magazine, vol. 33, no. 2, pp. 100-109, 1995.
[4] K. Fazel and L. Papke, "On the performance of convolution-ally-coded CDMA/OFDM for mobile communications," in Proceedings of 4th IEEE International Symposium on Personal, Indoor and Mobile Radio Communications (PIMRC '93), pp. 109-113, Yokohama, Japan, September 1993.
[5] V. M. Dasilva and E. S. Sousa, "Multicarrier orthogonal CDMA for quasi-synchronous communication systems," IEEE Journal on Selected Areas in Communications, vol. 12, no. 5, pp. 842-852, 1994.
[6] X. Gui and T. S. Ng, "Performance of asynchronous orthogonal multicarrier CDMA system in frequency selective fading channel," IEEE Transactions on Communications, vol. 47, no. 7, pp. 1084-1091, 1999.
[7] S. Kondo and L. Milstein, "Performance of multicarrier DS CDMA systems," IEEE Transactions on Communications, vol. 44, no. 2, pp. 238-246, 1996.
[8] D. A. Wiegandt, Z. Wu, and C. R. Nassar, "High-throughput, high-performance OFDM via pseudo-orthogonal carrier interferometry speading codes," IEEE Transactions on Communications, vol. 51, no. 7, pp. 1123-1134, 2003.
[9] X. Cai, S. Zhou, and G. B. Giannakis, "Group-orthogonal multicarrier CDMA," IEEE Transactions on Communications, vol. 52, no. 1, pp. 90-99, 2004.
[10] D. Gerakoulis, "Interference suppressing OFDM method for wireless communications," United States Patent no. 6,882,619, Granted April, 2005.

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