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Capacity Planning for Group-Mobility Users in OFDMA Wireless Networks

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Because of the random nature of user mobility, the channel gain of each user in a cellular network changes over time causing the signal-to-interference ratio (SNR) of the user to fluctuate continuously. Ongoing connections may experience outage events during periods of low SNR. As the outage ratio depends on the SNR statistics and the number of connections admitted in the system, admission capacity planning needs to take into account the SNR fluctuations. In this paper, we propose new methods for admission capacity planning in orthogonal frequency-division multiple-access (OFMDA) cellular networks which consider the randomness of the channel gain in formulating the outage ratio and the excess capacity ratio. Admission capacity planning is solved by three optimization problems that maximize the reduction of the outage ratio, the excess capacity ratio, and the convex combination of them. The simplicity of the problem formulations facilitates their solutions in real time. The proposed planning method provides an attractive means for dimensioning OFDMA cellular networks in which a large fraction of users experience group-mobility.

1. INTRODUCTION

Orthogonal frequency-division multiple-access (OFDMA) is one of the most promising solutions to provide a high-performance physical layer in emerging cellular networks. OFDMA is based on OFDM and inherits immunity to intersymbol interference and frequency selective fading. Recently, adaptive resource management for multiuser OFDMA systems has attracted enormous research interest [1–7].

In [1], the authors studied how to minimize the total transmission power while satisfying a minimum rate constraint for each user. The problem was formulated as an integer programming problem and a continuous-relaxation-based suboptimal solution method was studied. In [2], a class of computationally inexpensive methods for power allocation and subcarrier assignment were developed, and those are shown to achieve comparable performance, but do not require intensive computation.

Several studies have considered providing a *fair* opportunity for users to access a wireless system so that no user may dominate in resource occupancy while others starve. In [3], the authors proposed a fair-scheduling scheme to minimize the total transmit power by allocating subcarriers to the users and then to determine the number of bits transmitted on each subcarrier. Also, they developed suboptimal solution algorithms by using the linear programming technique and the Hungarian method. In [4], the authors formulated a combinatorial problem to jointly optimize the subcarrier and power allocation. In their formulation they considered a constraint to allocate resources to users according to the predetermined fractions with respect to the transmission opportunity. By using the constraint, the resources can be fairly allocated. A novel scheme to fairly allocate subcarriers, rate, and power for multiuser OFDMA system was proposed [6], where a new generalized proportional fairness criterion, based on Nash bargaining solutions (NBS) and coalitions, was used. The study in [6] is very different from the previous OFDMA scheduling studies in the sense that the resource allocation is performed with a game-theoretic decision rule. They proposed a very fast near-optimal algorithm using the Hungarian method. They showed by simulations that their fair-scheduling scheme provides a similar overall rate to that of the rate-maximizing scheme. In [7], they provided achievable rate formulations from the physical layer perspective and studied algorithms using Lagrangian multiplier theorem, and they showed that their algorithms can find the global optimum even though the problems have multiple local optima.

However, most previous studies on resource allocation in OFDMA systems did not consider the connection-level performance which is limited by the fluctuations in performance, for example, signal-to-interference ratio (SNR) in the lower layer. Because of the random user mobility, the average channel gain of a targeted group of users (referred simply as the average channel gain in the rest of the paper) in a cellular network changes over time causing the average SNR of the user group to continuously fluctuate. Since the maximum achievable transmit rate is bounded by the SNR, ongoing connections may experience outage events and, furthermore, the outage ratio increases for any given number of connections admitted in the system. Therefore, it is necessary to take the fluctuating nature of SNR into account when planning for the admission capacity. Several different optimization criteria have been used for admission capacity planning, such as the average call blocking probability, the average delay, and the utilization of bandwidth resources.

More specifically, we consider admission capacity planning for cellular networks in which a significant fraction of users experience "group-mobility," which is commonly observed in mass transportation systems (e.g., bus or train passengers). In general, the mobility patterns of users experiencing group-mobility are correlated causing their channel gains to be correlated as well. From the perspective of queuing theory, group-mobility users arrive at a network according to the "bulk arrival" process, which tends to degrade the teletraffic performance (for more details, refer to Section 3.2). In the case of a batch of users arriving at a new cell, for example, during a handover event involving the mobile platform, there are bulk arrivals of calls in the cell. During the cell dwell time of users within a mobility-group, new calls may arrive and ongoing calls may be completed. The system model based on batch arrivals therefore gives pessimistic results. However, as the cell size gets smaller, the number of handovers increases and the results based on batch arrivals become closer to the actual system performance.

Thus, on the one hand, evaluation of admission capacity without considering the degrading effect of group-mobility users may produce results that are too optimistic. On the other hand, it is clear that the proposed admission capacity planning based on group-mobility analysis yields a worst-case quality of service (QoS). However, from service providers' perspectives, to provide QoS has higher priority than to improve bandwidth utilization. For example, even though one handover call and one new call will pay the same cost per unit time, handover calls are usually given a higher priority than new calls from the QoS satisfaction perspective. This implies that service provider may prefer the degree of bandwidth wastage caused by the proposed pessimistic planning approach compared to the QoS degradation caused by a more optimistic planning approach. Therefore, it stands to reason that while admission capacity planning in the presence of group-mobility users gives pessimistic results when group-mobility patterns are absent, the possibility of adverse impact of group-mobility users must be properly taken into account. With the proposed method, by modifying the outage ratio and the excess capacity ratio, the admission capacity planning approach can also be applied to situations with individual mobility.

Recently, Niyato and Hossain [8] studied two call admission schemes in OFDMA networks. However, they did not consider the nonstationary nature of SNR in determining the threshold value for admission control, which is the major difference between their contributions and ours. In this paper, we propose new methods for admission capacity planning in OFMDA cellular networks, which take into consideration the random nature of the average channel gain. We derive the outage ratio and the excess capacity ratio, and formulate three optimization problems to maximize the reduction of the outage ratio, the excess capacity ratio, and the convex combination of them. The simplicity of the problem formulation enables the admission capacity planning problems to be solved in real time. Extensive simulation results show that (1) the outage ratio and the excess capacity ratio are small when the variance of the average channel gain is small; (2) the desired bit-error rate (BER) and the minimum required transmit rate per connection affect the optimal admission capacity but have little affect on the Pareto efficiency between the outage ratio and the excess capacity ratio; and (3) for relatively small (large) values of targeted outage ratio, the admission capacity increases (decreases) when the variance of the average channel gain is small. We believe that the proposed admission capacity planning method provides an attractive means for dimensioning of OFDMA cellular networks in which a large fraction of users experience group-mobility.

The remainder of this paper is organized as follows. Section 2 gives the motivations of this work. Section 3 describes the model considered in this paper. In Section 4, we derive the outage ratio and the excess capacity ratio. In Sections 5 to 7, we formulate three optimization problems and develop exact solution methods for maximizing the reduction in the outage ratio, the excess capacity ratio, and the convex combination of them. We present simulation results in Section 8 and discuss their implications. Section 9 concludes the paper.

2. MOTIVATIONS AND SCOPE OF THIS WORK

2.1. Motivations of this work

There are extensive studies on subcarrier and power allocations in OFDM (see [1–7] and the literature therein), where the authors assume that the SNR is not variable during the scheduling period. The results of these studies can be used in an adaptive manner in accordance with the frequent changes of SNR. Regardless of adaptations with respect to SNR variations, outage events of ongoing real-time connections are unavoidable in the cases where the instantaneous capacity with respect to the locations of users residing in a cell becomes lower than the minimum capacity required to serve those connections (see Figure 1). A simple solution to improve the outage ratio of ongoing connections is to apply a certain "bound" to the maximum number of connections. Because of simplicity of this type of solution, it is useful for

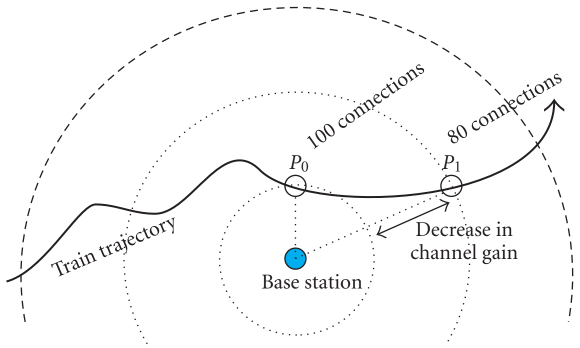

Figure 1: An example of group-mobility users on board a train. The maximum capacities are 100 connections at location P_0 and 80 connections at P_1. For a planned admission capacity $y = 100$, a small excess capacity exists and 20 connections are likely to be dropped. For a planned admission capacity $y = 80$, a large excess capacity exists and 0 connections are likely to be dropped.

practical applications. However, it is necessary to investigate how to find appropriate bounds for connection admission that take into account the particular characteristics of OFDM systems, which differentiates this problem from similar problems in the other wireless systems.

2.2. Scope of this work

The scope of this work is to find appropriate upper bounds of the number of ongoing connections. The objectives are to minimize the number of outage events while keeping capacity wastage below a specific limit, or to minimize capacity wastage while keeping the number of outage events below a given tolerance level. In this paper, we call these upper bounds the "admission capacity." We consider the case where the channel gain of user j using subcarrier i, denoted by G_{ij}, is a random variable that varies over time. In this case, the optimal subcarrier and power allocations will vary over time as they are completely dependent on the values of the random variables G_{ij}'s. We assume the perfect condition that optimum power and subcarrier allocations are made given the values of G_{ij}'s. This assumption is necessary and widely adopted in the literature to enable an analytical evaluation of the achievable system capacity. For example, in capacity planning of CDMA systems with time-division duplex (TDD), it is commonly assumed to have perfect power control and resource allocation [9, 10].

3. MODEL DESCRIPTION

3.1. System model

We consider an OFDMA cellular system. A cell has a total of C subcarriers and each user has a transmission power limit of $\bar{p}$. The achievable rate of user j using subcarrier i, C_{ij}, is given by

$$C_{ij} = W \log_2 \left(1 + a \cdot \frac{G_{ij} p_{ij}}{\sigma^2}\right), \tag{1}$$

where $a \approx -1.5/\log(5\,\text{BER})$ (BER denotes desired bit-error rate), G_{ij} denotes the channel gain of user j at subcarrier i, σ^2 is the thermal noise power, and p_{ij} denotes the power allocated to user j at subcarrier i [6]. Each connection has a minimum rate requirement ϕ, such that an outage event occurs if the assigned rate is smaller than the minimum required transmit rate ϕ.

Since the users are generally mobile, we consider that the channel gains G_{ij}'s are random variables. Thus, the optimal allocation of subcarrier and power is dependent upon the instantaneous values of the random variables. Thus, it is not possible to use a fixed allocation strategy.

In such situations, we propose an alternative to approximate the average rate per connection when y connections are ongoing as follows:

$$\begin{aligned} R(y) &\approx \frac{C}{y} W \log_2 \left(1 + a \cdot \frac{\bar{G} \cdot (y/C \cdot \bar{p})}{\sigma^2}\right) \\ &= \frac{C}{y} W \log_2 \left(1 + \frac{a\bar{p}y}{\sigma^2 C} \cdot \bar{G}\right) \\ &= \frac{C}{y} W \log_2 \left(1 + \rho(y) \cdot \bar{G}\right) \quad \left(\rho(y) = \frac{a\bar{p}y}{\sigma^2 C}\right), \end{aligned} \tag{2}$$

where C/y denotes the average number of subcarriers allocated to a connection, W is the bandwidth of a subcarrier, $\bar{G} = (1/yC)\sum_{i=1}^{C}\sum_{j=1}^{y} G_{ij}$, and $y/C \cdot \bar{p}$ is the average power allocated to a subcarrier. There are practical reasons to use $\bar{G}$ instead of the individual random variables G_{ij}'s. First, the variances of G_{ij}'s with respect to indices i and j are small in the case of group-mobility users because the users are located at the nearly same position with respect to the base station. Second, the mean value $\bar{G}$ is an unbiased estimator that provides sufficient statistical information on the targeted population. The probability density function (pdf) of random variable $\bar{G}$ is denoted by $f_G(\cdot)$. In the case of a system filled with individual mobility users, the approximation used in (2) may not be sufficiently accurate because the channel gains and allocated powers of individual mobility users are quite different, which is beyond the scope of this work. In the case of group-mobility users, however, because of the first reason, the approximation is much more accurate.

3.2. Connections of group-mobility users

Figure 1 gives an example of group-mobility users traveling onboard a train. The real-time traffic performance of group-mobility users is usually lower than that of individual mobility users. For example, consider two $M/M/m/m$ queue models with the same service rate: an $M/M/2c/2c$ queue with the arrival and departure rates λ and μ, respectively, where each arrival requires two channels and $M/M/2c/2c$ one with the arrival and departure rates 2λ and μ, respectively, where each arrival requires a single channel [11]. The former is the 2-user group-mobility example. It can be simply verified that the blocking probability in the former queue model is greater than that in the latter queue model. This is because group-mobility users move in bulk, requesting the respective minimum capacities almost at the same epoch, in the event of handovers in the case of a cellular network. Here, note that although each bulk arrival in the former queue model is a Poisson process, the arrival process of each user is not generally Poisson and, furthermore, it is not a stationary process. In this case, the blocking probability of a customer is usually greater even when the utilization of bandwidth resources is low.

The other property of group-mobility users is that they have an approximately equal SNR ceteris paribus. This also reduces the capacity that a base station can achieve, as it cannot take full advantage of *multiuser diversity*.

The reason that we take group-mobility users into account is to examine worst-case performance for admission control planning, whereas a great number of previous studies overestimated the performance by simplifying the arrival model into a Poisson arrival process [12].

4. OUTAGE RATIO AND EXCESS CAPACITY RATIO

In this section, we derive the outage ratio and the excess capacity ratio. The *outage ratio* is defined as the average fraction of the total number of connections suffering from outages, whereas the *excess capacity ratio* is defined as the average fraction of the achievable capacity that is not utilized for real-time traffic delivery, even though used for non-real-time traffic delivery, out of the total achievable capacity.

4.1. Outage ratio

Let random variable $K_D(y)$ denote the number of outages (or number of dropped connections) when y connections are ongoing. The probability that k users are dropped by outage is given by

$$\begin{aligned}\Pr\{K_D(y)=k\} &= \binom{y}{k}\cdot\{\Pr(R(y)<\phi)\}^k\cdot\{1-\Pr(R(y)<\phi)\}^{y-k}\\ &= \binom{y}{k}\cdot F_R(\phi)^k\cdot\{1-F_R(\phi)\}^{y-k}.\end{aligned} \tag{3}$$

The average number of connections experiencing outages is given by

$$\mathbb{E}\{K_D(y)\} = \sum_{k=1}^{y} k\cdot\Pr\{K_D(y)=k\} = yF_R(\phi). \tag{4}$$

By substituting G for R, we have

$$\mathbb{E}\{K_D(y)\} = yF_G(G_R(y)), \tag{5}$$

where $G_R(y)$ is the solution of (2) at $R=\phi$ with respect to G, that is,

$$G_R(y) = \frac{2^{y\phi/(CW)}-1}{\rho(y)}. \tag{6}$$

Thus, the outage ratio is expressed as

$$\begin{aligned}P_O(y) &= \frac{\mathbb{E}\{K_D(y)\}}{y}\\ &= F_G(G_R(y)).\end{aligned} \tag{7}$$

4.2. Excess capacity ratio

The average amount of excess capacity $S(y)$ is given by

$$\begin{aligned}S(y) &= \sum_{k=1}^{y}\int_{\phi}^{\infty}(r-\phi)\cdot f_R(r)dr\\ &= y\int_{\phi}^{\infty}(r-\phi)\cdot f_R(r)dr\\ &= y\int_{\phi}^{\infty} r\cdot f_R(r)dr - \phi y\int_{\phi}^{\infty} f_R(r)dr,\end{aligned} \tag{8}$$

where $f(x)=dF(x)/dx$. Substituting G for R, that is, $G_R(y)$ for $R(y)$, we have

$$f_R(r) = f_G(g)\cdot\left|\frac{dr}{dg}\right|^{-1}, \tag{9}$$

which gives

$$\begin{aligned}\int_{r=\phi}^{R^{\max}} r\cdot f_R(r)dr &= \frac{CW}{y}\int_{g=G_R(y)}^{G_R^{\max}}\log_2\{1+\rho(y)g\}\cdot f_G(g)\cdot\left|\frac{dr}{dg}\right|^{-1}\cdot\frac{dr}{dg}dg\\ &= \frac{CW}{y}\int_{G_R(y)}^{G_R^{\max}}\log_2\{1+\rho(y)g\}\cdot f_G(g)dg,\\ \int_{r=\phi}^{R^{\max}} f_R(r)dr &= \int_{g=G_R(y)}^{G_R^{\max}} f_G(g)dg,\end{aligned} \tag{10}$$

where $R^{\max}=\max R(y)$ and $G_R^{\max}=\max G_R(y)$. Thus, (8) is rewritten as

$$\begin{aligned}S(y) = CW&\int_{G_R(y)}^{G_R^{\max}}\log_2\{1+\rho(y)g\}\cdot f_G(g)dg\\ &-\phi y\{1-F_G(G_R(y))\}.\end{aligned} \tag{11}$$

When y ongoing connections have been admitted, the total amount of the achievable capacity is given by

$$\begin{aligned}S_T(y) &= \sum_{k=1}^{y}\int_{r=0}^{R^{\max}} r\cdot f_R(r)dr\\ &= y\int_{g=0}^{G_R^{\max}}\log_2\{1+\rho(y)g\}\cdot f_G(g)dg.\end{aligned} \tag{12}$$

Finally, the excess capacity ratio is given by

$$P_S(y) = \frac{S(y)}{S_T(y)}. \tag{13}$$

5. MINIMIZATION OF OUTAGE RATIO OF ONGOING CONNECTIONS

We can find the optimal y that minimizes the outage ratio of ongoing connections by solving the following simple problem (P1).

5.1. Problem formulation: outage ratio minimization

(P1)

$$\begin{aligned} &\text{minimize } P_O(y),\\ &\text{subject to } P_S(y) \le \gamma_S, \\ &y : \text{nonnegative integer.}\end{aligned} \tag{14}$$

The role of problem (P1) is to find y that minimizes the outage ratio of ongoing connections subject to the constraint that the excess capacity ratio is not greater than γ_S.

5.2. Solution method of (P1)

Proposition 1. *$P_O(y)$ is strictly increasing.*

Proof.

$$\frac{dP_O}{dy} = f_G(G_R(y)) \cdot \frac{dG_R(y)}{dy} > 0. \tag{15}$$

□

Proposition 2. *$P_S(y)$ is strictly decreasing.*

Proof. We have

$$\begin{aligned}\frac{dS(y)}{dy} &= -CW \log_2 \{1+\rho(y)G_R(y)\} \\ &\quad \cdot f_G(G_R(y)) \cdot \frac{dG_R(y)}{dy} \\ &\quad - \phi\{1 - F_G(G_R(y))\} + y\phi f_G(G_R(y)) \cdot \frac{dG_R(y)}{dy} \\ &= -y\phi f_G(G_R(y)) \cdot \frac{dG_R(y)}{dy} \\ &\quad - \phi\{1 - F_G(G_R(y))\} + y\phi f_G(G_R(y)) \cdot \frac{dG_R(y)}{dy} \\ &= -\phi\{1 - F_G(G_R(y))\} < 0,\end{aligned} \tag{16}$$

$$\begin{aligned}\frac{dS_T(y)}{dy} &= \int_0^{G_R^{\max}} \log_2 \{1+\rho(y)g\} \\ &\quad \cdot f_G(g)dg + y\frac{d}{dy}\int_0^{G_R^{\max}} \log_2 \{1+\rho(y)g\} \cdot f_G(g)dg \\ &= \int_0^{G_R^{\max}} \log_2 \{1+\rho(y)g\} \\ &\quad \cdot f_G(g)dg + y\int_0^{G_R^{\max}} \frac{(a\bar{p}/\sigma^2 C)}{\{1+\rho(y)g\}} \cdot f_G(g)dg > 0.\end{aligned} \tag{17}$$

The inequality (18) can also be demonstrated by the property of *multiuser diversity*, where the achievable capacity increases as the number of users increases [6].

From the above results, we have

$$\frac{dP_S}{dy} = \frac{(dS(y)/dy) \cdot S_T(y) - S(y) \cdot (dS_T(y)/dy)}{\{S_T(y)\}^2} < 0. \tag{18}$$

□

The feasible region of y in problem (P1) is given by

$$\mathcal{F}_1 = \{y : P_S(y) \le \gamma_S\} = \{y : y \ge P_S^{-1}(\gamma_S)\}. \tag{19}$$

This is supported by Proposition 2, namely, $P_S^{-1}(\cdot)$ exists and, furthermore,

$$\frac{dP_S^{-1}}{dy} = \frac{1}{dP_S/dy} < 0. \tag{20}$$

Thus, there exists a unique optimal solution of (P1), which is given by

$$y_O^* = \lceil P_S^{-1}(\gamma_S) \rceil, \tag{21}$$

where $\lceil x \rceil$ is the smallest integer not less than x.

6. MINIMIZATION OF EXCESS CAPACITY RATIO

Next, we consider the problem of minimizing the fraction of excess capacity. The amount of excess capacity represents capacity that is not used by any real-time traffic users and is therefore wasted. The problem is formulated by (P2) as follows.

6.1. Problem formulation: excess capacity ratio minimization

(P2)

$$\begin{aligned} &\text{minimize } P_S(y),\\ &\text{subject to } P_O(y) \le \gamma_O, \\ &y : \text{nonnegative integer.}\end{aligned} \tag{22}$$

Problem (P2) is subject to the constraint that the outage ratio is not greater than γ_O.

6.2. Solution method of (P2)

The feasible region of y in problem (P2) is given by

$$\begin{aligned}\mathcal{F}_2 &= \{y : P_O(y) \le \gamma_O\} \\ &= \{y : y \le P_O^{-1}(\gamma_O)\}.\end{aligned} \tag{23}$$

Similar to the case of (P1), this is supported by Proposition 1. Thus, there exists a unique optimal solution of (P2), which is given by

$$y_S^* = \lfloor P_O^{-1}(\gamma_O) \rfloor, \tag{24}$$

where $\lfloor x \rfloor$ is the largest integer not greater than x.

7. JOINT MINIMIZATION OF OUTAGE RATIO AND CAPACITY WASTAGE

7.1. Definition and formalism

(P3)

$$\begin{aligned}&\text{minimize } P_C(y:\alpha) \\ &\quad = \alpha P_O(y) + (1-\alpha)P_S(y), \quad y : \text{nonnegative integer.}\end{aligned} \tag{25}$$

Here, α is a constant between 0 and 1, which denotes the *relative* marginal utility[1] of the outage ratio with respect to $P_S(y)$ (see Figures 13–15). The objective function is a convex combination of outage ratio and capacity waste fraction. Note that the objective function is not always strictly convex. The necessary and sufficient condition for the objective function $(\alpha P_O(y) + (1-\alpha)P_S(y))$ to be strictly convex is that the second difference[2] is positive for all integers $y = 1, \ldots, C-1$. For the sake of tractability, we may consider as a sufficient condition that the second derivative of $\{\alpha P_O(y) + (1-\alpha)P_S(y)\}$ is positive if

$$\frac{df_G}{dy} > -f_G(G_R(y)) \cdot \left\{ \frac{d^2 G_R/dy^2}{dG_R/dy} - \left(\frac{1}{\alpha} - 1\right)\phi \right\} \qquad (26)$$

for $1 < y < C-1$. The nonconvexity of $P_C(y:\alpha)$ with respect to y can be observed in the examples shown in Figure 2.

7.2. *Is it useful?*

Even though applying (P1) and (P2) for admission capacity planning is useful under the condition that the required levels of $P_O(y)$ or $P_S(y)$, namely γ_O or γ_S, are given, these problems are not enough for us to plan the admission capacity in all cases. In some cases, the required level is not given and the only information available for planning is the relative marginal utility α. In such cases, the above problem (P3) is useful to determine the admission capacity (examples for this case can be found in Figures 13–15). Given that the relative marginal utility α is 0.5, the left point y^* (specified by $\alpha = 0.5$) is optimal. However, if the relative marginal utility decreases to 0.3, then the optimal point moves to the right one (specified by y^* at $\alpha = 0.3$), causing a balance with a decrease in P_S (denotes P_S gains more weight) and an increase in P_O (denotes P_O loses more weight). The solution methods used for solving (P1) and (P2) can be applied for (P3) after simple modifications. A simple and exact solution method is demonstrated in Figures 13–15 Section 8. Because there is a unique inflection point for $P_O(y)$ and $P_S(y)$ and the two functions, namely $P_O(y)$ and $-P_S(y)$, are strictly increasing, there are at most two local minima of function $P_C(y:\alpha) = \alpha P_O(y) + (1-\alpha)P_S(y)$.

Proposition 3. *The necessary condition for (local) optimality is*

$$\frac{dP_C}{dy} = \alpha\frac{dP_O}{dy} + (1-\alpha)\frac{dP_S}{dy} = 0. \qquad (27)$$

Alternatively, the necessary condition for (local) optimality can be expressed as

$$\frac{dP_O}{dP_S} = -\frac{1-\alpha}{\alpha}. \qquad (28)$$

[1] This denotes the marginal utility with respect to $P_S(y)$ instead of the marginal utility with respect to y.

[2] The first difference of a function is defined as $\Delta f(n) = f(n+1) - f(n)$ and the second difference is defined as $\Delta^2 f(n) = \Delta f(n+1) - \Delta f(n)$.

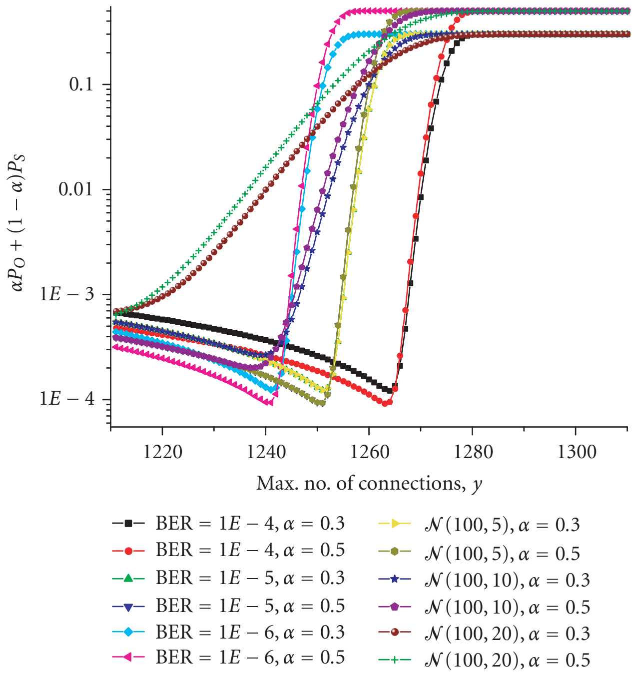

FIGURE 2: Nonconvexity of $P_C(y:\alpha)$ with respect to y ($P_C(y:\alpha) = \alpha P_O(y) + (1-\alpha)P_S(y)$).

8. EXPERIMENTAL RESULTS

We examine the three proposed methods for various probability density functions (pdf's) of the average channel gain $\bar{G}$ and for various values of BER, ϕ, σ^2, and $\bar{p}$. In our simulation setups the transmission power is $\bar{p} = 50$ mW, the thermal noise power is $\sigma^2 = 10^{-11}$ W, the number of subcarriers is $C = 128$ over a 3.2 MHz band, BER $= 10^{-5}$, and the minimum rate requirement is $\phi = 100$ kbps; all are used as default values. Table 1 shows the simulation parameters values.

Figures 3–7 show the admission capacity y versus the threshold value of excess capacity ratio. Note that in these figures, the actual shape of the curves are given by the step functions denoting $\lceil P_S^{-1}(\gamma_S)\rceil$. In Figure 3, the real shapes of the curves are shown whereas the curves are smooth in the other four figures; that is, in Figures 4–7, the curves denote $P_S^{-1}(\gamma_S)$ instead of $\lceil P_S^{-1}(\gamma_S)\rceil$.

In Figure 3, the admission capacities are shown with respect to desired bit-error rate (BER). As we can see through the achievable rate formula (1), the admission capacity decreases when BER decreases and when the targeted excess capacity ratio increases. In both cases, the admission capacity decreases approximately linearly with the decrease in BER. It is observed that the differences between admission capacities at different values of BER decrease when the targeted outage ratio γ_O increases.

Figure 4 shows the admission capacity versus the threshold value of excess capacity ratio with respect to transmit power. It is observed that the admission capacity increases as the transmit power $\bar{p}$ increases but with a decreasing rate, which we can conjecture from (1). In addition, it is observed

Table 1: Parameters used in experiments.

Item	Value	Description
$\bar{p}$	50	Avg. transmit power (mW)
σ^2	1e −11	Thermal noise level (W)
C	128	No. of subcarriers
BER	1e −5	Desired bit-error rate
W	25 000	Bandwidth of subcarrier (Hz)
ϕ	100	Min. required rate per connection (kbps)
$\bar{G}$	$\sim \mathcal{N}(100,5)$	—

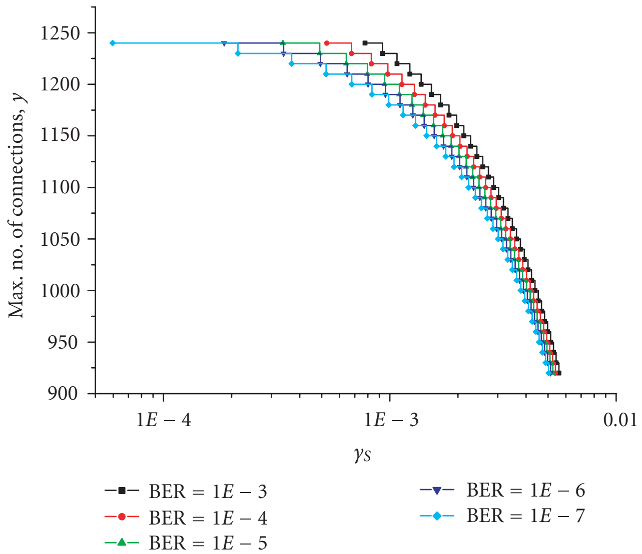

Figure 3: The maximum number of connections y versus γ_S with respect to BER ($\bar{p} = 50$ mW, $\sigma^2 = 10^{-11}$, $\phi = 100$ kbps, $\mathcal{N}(100, 5)$).

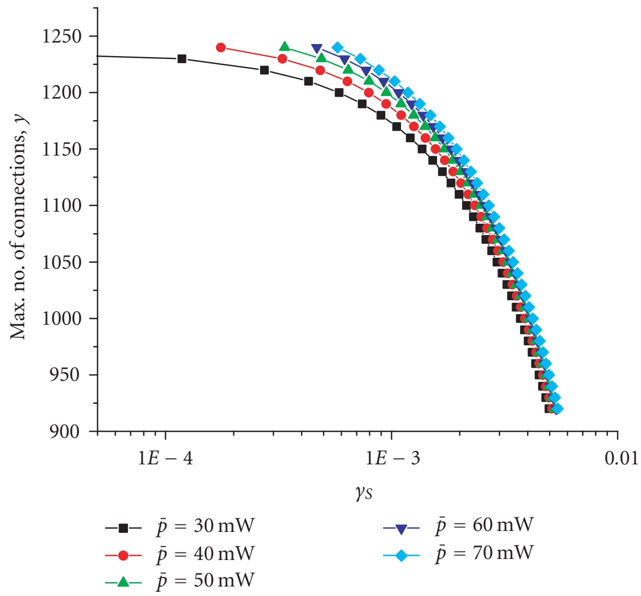

Figure 4: The maximum number of connections y versus γ_S with respect to $\bar{p}$ (BER = 10^{-5}, $\sigma^2 = 10^{-11}$, $\phi = 100$ kbps, $\mathcal{N}(100, 5)$).

that a ±10% increase in transmit power at 50 mW can increase approximately ±10% of admission capacity at any given threshold value of excess capacity ratio. Similarly, a ±20% increase in transmit power at 50 mW results in approximately ±20% increase in admission capacity.

Figure 5 shows the admission capacity versus the targeted excess capacity ratio with respect to the minimum required transmit rate per connection. It is observed that a ±1, 2% increase in ϕ results in an approximately equal decrease in admission capacity y^*. This is because the total capacities, $y^* \cdot \phi$, are approximately equal regardless of the value of ϕ. Figure 6 shows the admission capacity versus the targeted excess capacity ratio with respect to the thermal noise power. Similar patterns of admission capacity are observed.

Figure 7 shows the admission capacity versus the targeted excess capacity ratio with respect to the pdf of the random variable $\bar{G}$, that is, the average channel gain, where $\mathcal{N}(x, y)$ denote a normal distribution with mean x and variance y. Obviously, a large variance implies a high degree of variation. In this case, a dynamic planning strategy, such as admission planning with a dynamic value of admission threshold, is preferred compared to a static planning strategy, such as admission planning with a fixed value of admission threshold. This is because a static planning strategy does not adjust well to the high variations in the case of a large variance. This fact demonstrates that the admission capacity decreases as the variance of $\bar{G}$ increases, which is observed in the figure. However, it is observed that an 8-fold increase in the variance at 5 results in a 0.5% decrease in admission capacity. Thus, we can safely conclude that under the condition that $\bar{G}$ has a large variance the admission capacity decreases but the amount of decrease is slight.

Figures 8–12 show the maximum number of connections that can be accommodated, which is defined as the admission capacity and is denoted by y in this paper, versus the threshold value of outage ratio. In these figures, note that the actual shape of the curves are the step functions denoting $\lfloor P_O^{-1}(\gamma_O) \rfloor$. In Figure 8, the actual shapes of the curves are shown whereas the curves are smoothed in the other four

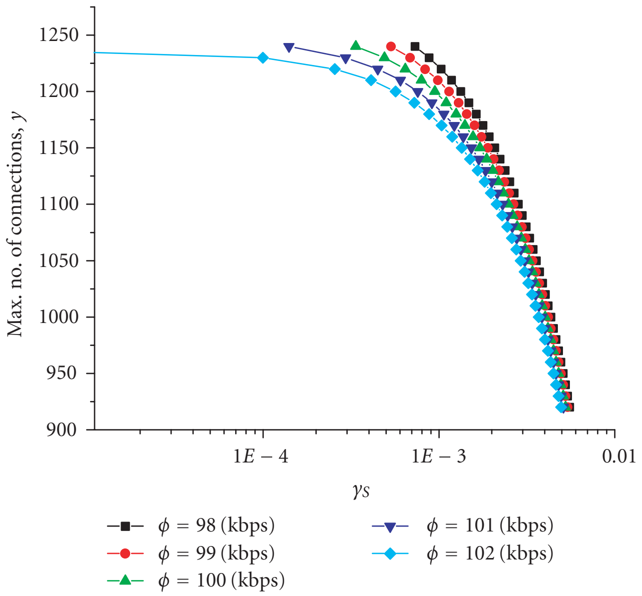

Figure 5: The maximum number of connections y versus γ_S with respect to ϕ(BER = 10^{-5}, $\bar{p}$ = 50 mW, $\sigma^2 = 10^{-11}$, $\mathcal{N}(100, 5)$).

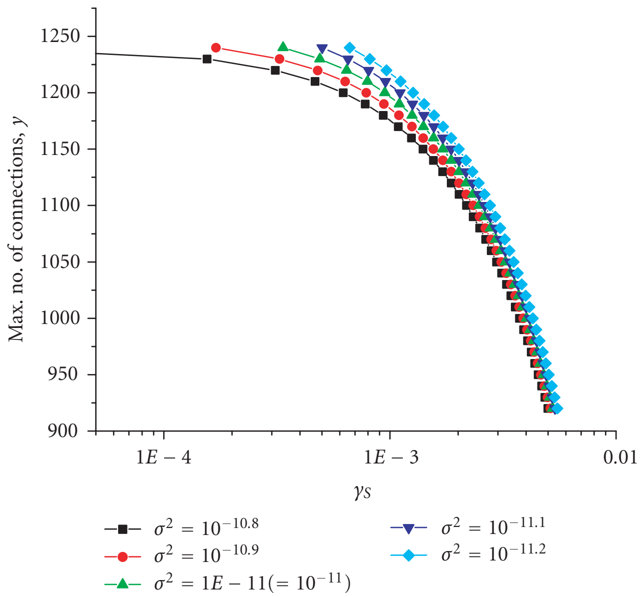

Figure 6: The maximum number of connections y versus γ_S with respect to σ^2 (BER = 10^{-5}, $\bar{p}$ = 50 mW, ϕ = 100 kbps, $\mathcal{N}(100, 5)$).

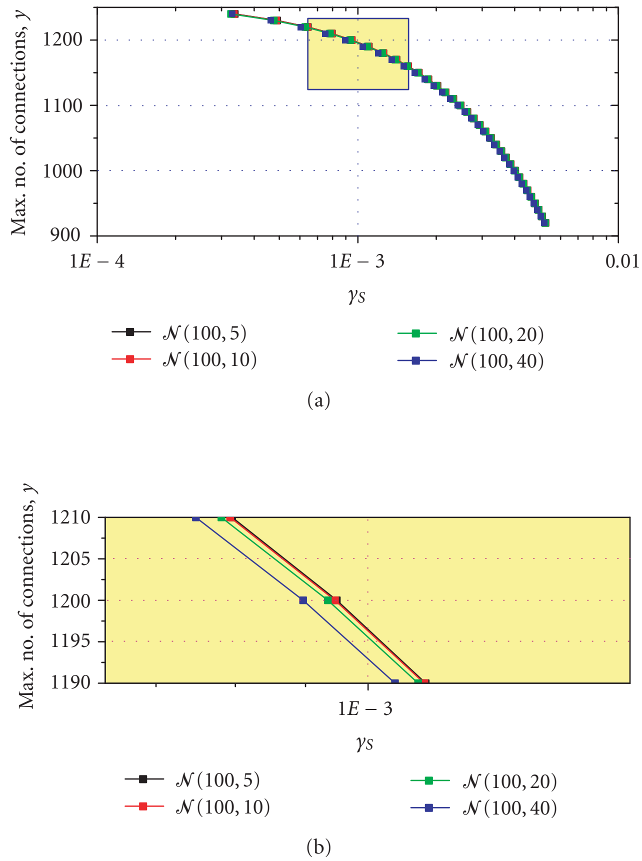

Figure 7: The maximum number of connections y versus γ_S with respect to the pdf of $\bar{G}$ (BER = 10^{-5}, $\bar{p}$ = 50 mW, $\sigma^2 = 10^{-11}$, ϕ = 100 kbps).

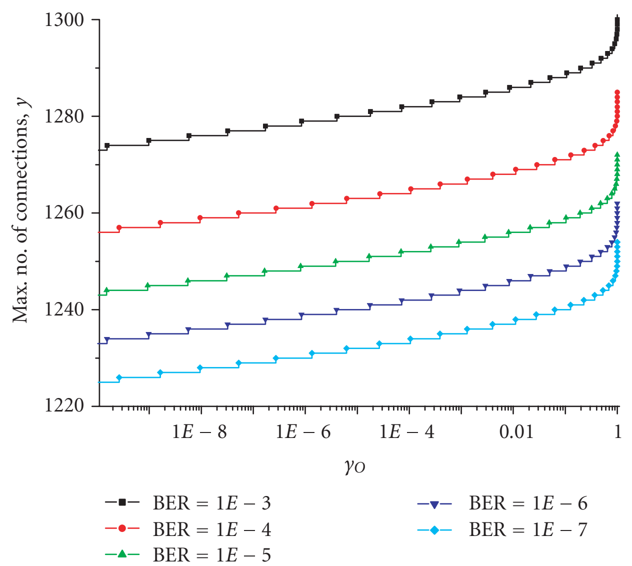

Figure 8: The maximum number of connections y versus γ_O with respect to BER ($\bar{p}$ = 50 mW, $\sigma^2 = 10^{-11}$, ϕ = 100 kbps, $\mathcal{N}(100, 5)$).

figures, that is, in Figures 9–12, the curves denote $P_O^{-1}(\gamma_O)$ instead of $\lfloor P_O^{-1}(\gamma_O) \rfloor$.

In Figure 8, the admission capacities are shown with respect to desired bit-error rate. It is observed that the differences between admission capacities with respect to different values of BER are nearly equivalent regardless of the targeted outage ratio γ_O. Obviously, the admission capacity increases when BER decreases and the targeted outage ratio increases.

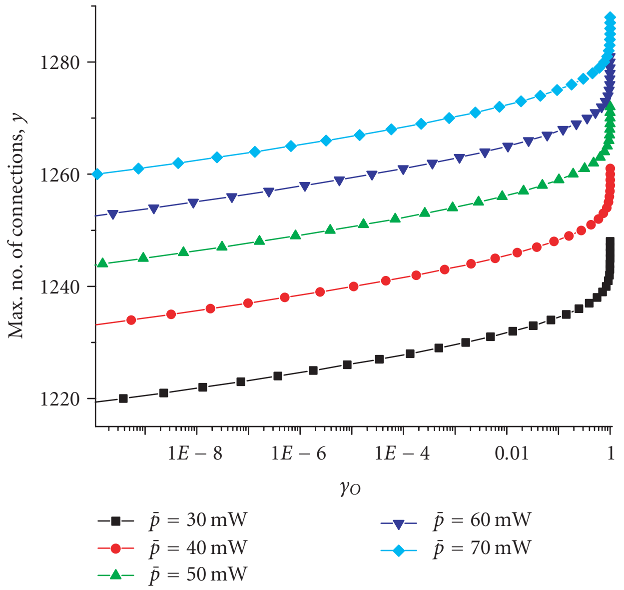

Figure 9: The maximum number of connections y versus γ_O with respect to $\bar{p}$ (BER $= 10^{-5}$, $\sigma^2 = 10^{-11}$, $\phi = 100$ kbps, $\mathcal{N}(100, 5)$).

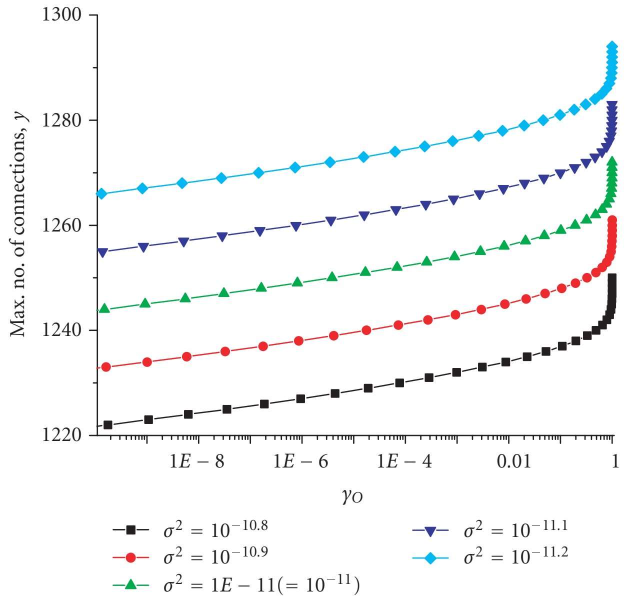

Figure 11: The maximum number of connections y versus γ_O with respect to σ^2 (BER $= 10^{-5}$, $\bar{p} = 50$ mW, $\phi = 100$ kbps, $\mathcal{N}(100, 5)$).

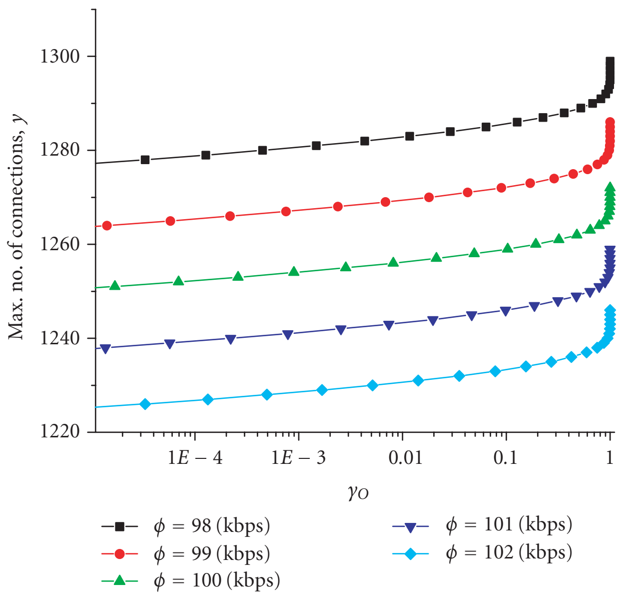

Figure 10: The maximum number of connections y versus γ_O with respect to ϕ (BER $= 10^{-5}$, $\bar{p} = 50$ mW, $\sigma^2 = 10^{-11}$, $\mathcal{N}(100, 5)$).

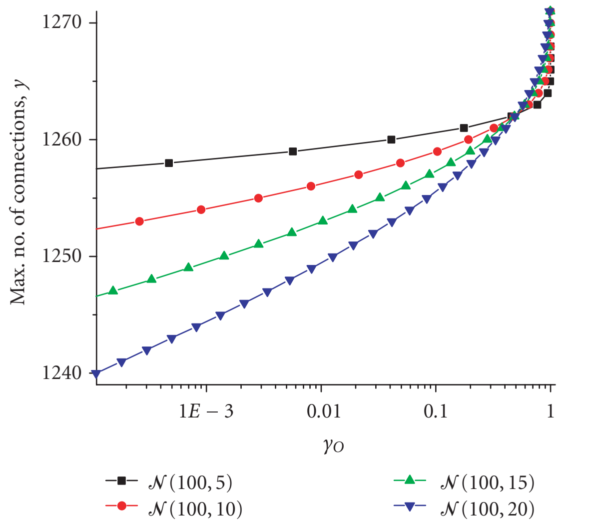

Figure 12: The maximum number of connections y versus γ_O with respect to the pdf of $\bar{G}$ (BER $= 10^{-5}$, $\bar{p} = 50$ mW, $\sigma^2 = 10^{-11}$, $\phi = 100$ kbps).

In both situations, the quality of service, such as link error quality and dropping probability, is relatively bad.

Figure 9 shows the admission capacity versus the targeted outage ratio with respect to the transmit power. It is observed that the admission capacity increases as the transmit power $\bar{p}$ increases. In addition, it is observed that the differences between admission capacities with respect to different values of $\bar{p}$ are nearly equivalent regardless of the targeted outage ratio γ_O. The rate of increase in admission capacity decreases as the transmit power increases, following the logarithmic scale.

Figure 10 shows the admission capacity versus the targeted outage ratio with respect to the minimum required transmit rate per connection. It is observed that a ±1, 2% of increase in ϕ results in an approximately equal amount of decrease in admission capacity y^*. This is because the total

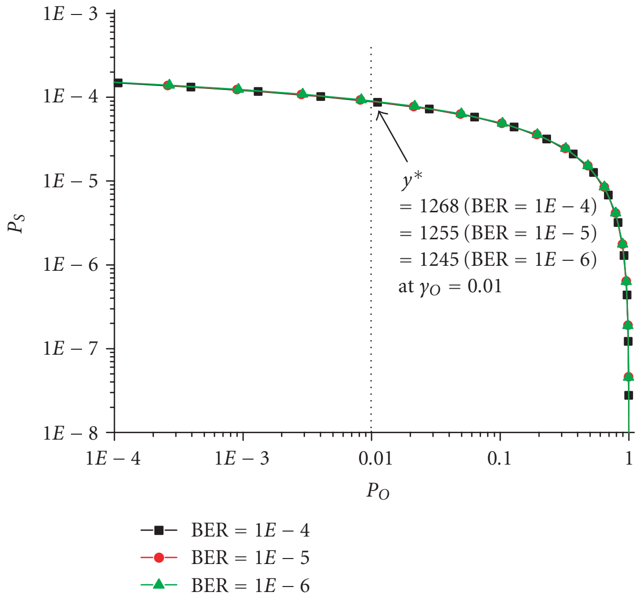

Figure 13: $P_O(y)$ versus $P_S(y)$ with respect to BER ($\bar{p}$ = 50 mW, ϕ = 100 kbps, $\sigma^2 = 10^{-11}$, $\mathcal{N}(100, 5)$). In the case that $\alpha = 0.5$, $y^* = 1263, 1251, 1241$ for BER = $10^{-4}, 10^{-5}, 10^{-6}$, respectively. In the case that $\gamma_O = 0.01$, $y^* = 1268, 1255, 1245$ for BER = $10^{-4}, 10^{-5}, 10^{-6}$, respectively.

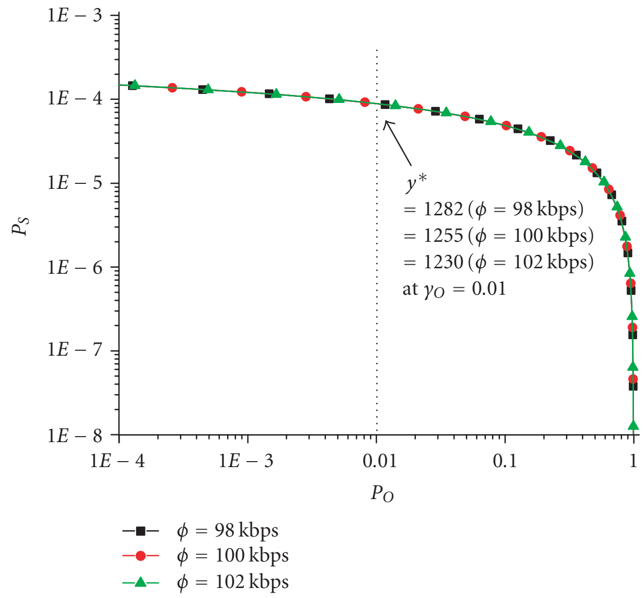

Figure 14: $P_O(y)$ versus $P_S(y)$ with respect to ϕ (BER = 10^{-5}, $\bar{p}$ = 50 mW, $\sigma^2 = 10^{-11}$, $\mathcal{N}(100, 5)$). In the case that $\alpha = 0.5$, $y^* = 1278, 1251, 1226$ for $\phi = 98(-2\%), 100, 102(+2\%)$ (kbps), respectively.

capacities, namely $y^* \cdot \phi$, are approximately equal regardless of the value of ϕ. Figure 11 shows the admission capacity versus the targeted outage ratio with respect to the thermal noise power. Similar patterns of admission capacity are observed.

Figure 12 shows the admission capacity versus the targeted outage ratio with respect to the variance of the random variable $\bar{G}$, that is, the average channel gain. When γ_O is less than about 0.46, the larger the variance of $\bar{G}$ is, the higher the rate of increase in the admission capacity is, and the admission capacity in the case of a small variance is greater than in the case of a large variance. However, when $\gamma_O > 0.46$, the admission capacity in the case of a large variance is greater than that in the case of a small variance.

Figure 13 shows the relation between excess capacity ratio P_S and outage ratio P_O with respect to the desired bit-error rate (BER). In Figures 8 and 3, it has been shown that BER affects the admission capacity in both cases of (P1) and (P2). However, the effect of BER on the relation between P_S and P_O is very small. This implies that the regions of Pareto efficiency between P_S and P_O are almost equivalent regardless of the desired bit-error rate. For the respective values BER = $1E-4, 1E-5, 1E-6$, the admission capacity y^* is equal to 1264, 1251, 1241 in the case of $\alpha = 0.3$, y^* is equal to 1263, 1251, 1241 in the case of $\alpha = 0.5$, and y^* is equal to 1263, 1250, 1240 in the case of $\alpha = 0.7$. This implies that the larger α is, the smaller is the admission capacity. A larger α should result in a smaller outage ratio.

Figure 14 shows the relation between excess capacity ratio P_S and outage ratio P_O with respect to the minimum required transmit rate ϕ. For the respective values ϕ = 98, 100, 102, the admission capacity y^* is equal to 1278, 1251, 1226 in the case of $\alpha = 0.3$; y^* is equal to 1277, 1251, 1225 in the case of $\alpha = 0.5$; and y^* is equal to 1277, 1251, 1225 in the case of $\alpha = 0.7$.

Figure 15 shows the relation between excess capacity ratio P_S and outage ratio P_O with respect to the pdf's of the average channel gain $\bar{G}$. For the respective pdf's $\mathcal{N}(100, 5)$, $\mathcal{N}(100, 10)$, $\mathcal{N}(100, 20)$, the admission capacity y^* is given by 1251, 1239, 1206 in the case of $\alpha = 0.3$; y^* is given by 1251, 1237, 1200 in the case of $\alpha = 0.5$; y^* is given by 1250, 1236, 1193 in the case of $\alpha = 0.7$. Unlike Figures 13 and 14, the regions of Pareto efficiency between P_S and P_O are quite different from each other with respect to the variance of the random variable $\bar{G}$. It is observed that the smaller the variance is, the better both P_S and P_O are.

9. CONCLUDING REMARKS

Because the *admission capacity*, which is defined as the upper bound of the number of connections that a base station can accommodate, fluctuates in accordance with the signal-to-noise ratio, a portion of ongoing connections may be dropped prior to their normal completion because of outage events. In this paper, we have developed three methods for admission capacity planning of an orthogonal frequency-division multiple-access system. Taking into account of the fluctuations of the average channel gains, we have derived outage ratio at the connection level, and the excess capacity ratio. Based on these metrics, we have formulated three problems to optimize admission capacity by maximizing

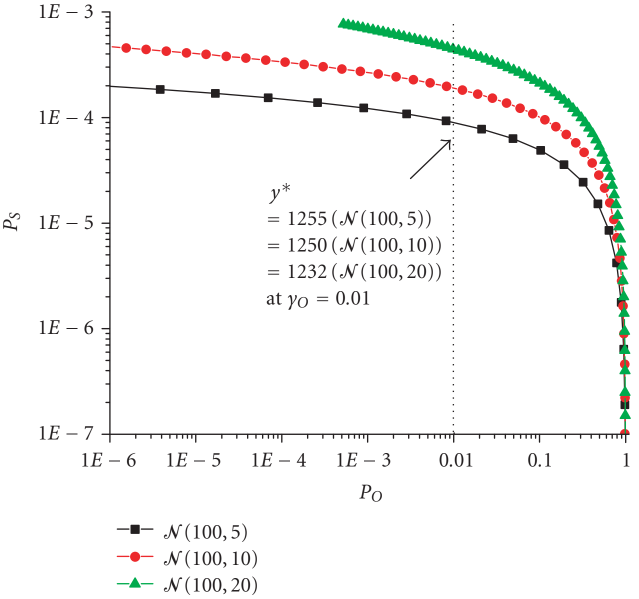

Figure 15: $P_O(y)$ versus $P_S(y)$ with respect to the pdf of $\bar{G}$ (BER = 10^{-5}, $\bar{p}$ = 50 mW, $\sigma^2 = 10^{-11}$, ϕ = 100 kbps). In the case that $\alpha = 0.5$, $y^* = 1251, 1238, 1201$ for $\mathcal{N}(100,5)$, $\mathcal{N}(100,10)$, $\mathcal{N}(100,20)$, respectively.

the reduction of the outage ratio, the excess capacity ratio, and the convex combination of them. Because of the simplicity of its formulation, each problem can be solved in real time. We believe that the proposed capacity planning method can be effectively applied in the design and dimensioning of OFDMA cellular networks, especially in situations where a significant fraction of the users experience group-mobility.

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