

# On-Off Frequency-Shift Keying for Wideband Fading Channels

Mustafa Cenk Gursoy,<sup>1,2</sup> H. Vincent Poor,<sup>1</sup> and Sergio Verdú<sup>1</sup>

<sup>1</sup>Department of Electrical Engineering, Princeton University, Princeton, NJ 08544, USA

<sup>2</sup>Department of Electrical Engineering, University of Nebraska-Lincoln, Lincoln, NE 68588, USA

Received 9 March 2005; Revised 20 August 2005; Accepted 15 September 2005

Recommended for Publication by Richard Kozick

$M$ -ary on-off frequency-shift keying (OOFSK) is a digital modulation format in which  $M$ -ary FSK signaling is overlaid on on/off keying. This paper investigates the potential of this modulation format in the context of wideband fading channels. First, it is assumed that the receiver uses energy detection for the reception of OOFSK signals. Capacity expressions are obtained for the cases in which the receiver has perfect and imperfect fading side information. Power efficiency is investigated when the transmitter is subject to a peak-to-average power ratio (PAR) limitation or a peak power limitation. It is shown that under a PAR limitation, it is extremely power inefficient to operate in the very-low-SNR regime. On the other hand, if there is only a peak power limitation, it is demonstrated that power efficiency improves as one operates with smaller SNR and vanishing duty factor. Also studied are the capacity improvements that accrue when the receiver can track phase shifts in the channel or if the received signal has a specular component. To take advantage of those features, the phase of the modulation is also allowed to carry information.

Copyright © 2006 M. C. Gursoy et al. This is an open access article distributed under the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

## 1. INTRODUCTION

A wide range of digital communication systems in wireless, deep space, and sensor networks operate in the low-power regime where power consumption rather than bandwidth is the limiting factor. For such systems, power-efficient transmission schemes are required for effective use of scarce energy resources. For example, in sensor networks [1], nodes that are densely deployed in a region may be equipped with only a limited power source and in some cases replenishment of these resources may not be possible. Therefore, energy-efficient operation is vital in these systems. Recently, there has also been much interest in ultra-wideband systems in which low-power pulses of very short duration are used for communication over short distances. These wideband pulses must satisfy strict peak power requirements in order not to interfere with existing systems.

The power efficiency of a communication system can be measured by the energy required for reliable communication of one bit. When communicating at rate  $R$  bps with power  $P$ , the transmitted energy per bit is  $E_b = P/R$ . Since the maximum rate is given by the channel capacity,  $C$ , the least amount of bit energy required for reliable communication is  $E_b = P/C$ . In [2], Shannon showed that the capacity of an ideal bandlimited additive white Gaussian noise channel is  $C = B \log_2(1 + P/BN_0)$  bps, where  $P$  is the received

power,  $B$  is the channel bandwidth, and  $N_0$  is the one-sided noise spectral level. As the bandwidth grows to infinity, the capacity monotonically increases to  $(P/N_0) \log_2 e$  bps, therefore decreasing the required received bit energy normalized to the noise power to

$$\frac{E_b^r}{N_0} = \frac{P/N_0}{C} \xrightarrow{B \rightarrow \infty} \log_e 2 = -1.59 \text{ dB}. \quad (1)$$

This minimum bit energy (1) can be approached by pulse-position modulation with vanishing duty cycle [3] or by  $M$ -ary orthogonal signaling as  $M$  becomes large [4]. In the presence of unknown fading, Jacobs [5] and Pierce [6] have noted that  $M$ -ary orthogonal signaling obtained by frequency-shift keying (FSK) modulation can still approach the limit in (1) for large values of  $M$ . Gallager [7, Section 8.6] also demonstrated that over fading channels  $M$ -ary orthogonal FSK signaling with vanishing duty cycle approaches the infinite bandwidth capacity of unfaded Gaussian channels as  $M \rightarrow \infty$ , thereby achieving (1). The result that the infinite bandwidth capacity of fading channels is the same as that of unfaded Gaussian channels is also noted by Kennedy [8]. Telatar and Tse [9] considered a more general fading channel model that consists of a finite number of time-varying paths and showed that the infinite bandwidth capacity of this channel is again approached by using peaky FSK signaling. Luo

and Médard [10] have shown that FSK with small duty cycle can achieve rates of the order of capacity in ultrawideband systems with limits on bandwidth and peak power. Reference [11] shows, in wider generality than was previously known, that the minimum received bit energy normalized to the noise level in a Gaussian channel is  $-1.59$  dB, regardless of the knowledge of the fading at the receiver and/or transmitter. It is also shown in [11] that if the receiver does not have perfect knowledge of the fading, flash signaling is required to achieve the minimum bit energy. The performance degradation in the wideband regime incurred by using signals with limited peakedness is discussed in [9, 12, 13]. The error performance of FSK signals used with a duty cycle is analyzed in [14, 15].

Besides approaching the minimum energy per bit, FSK modulation is particularly suitable for noncoherent communications. Butman et al. [16] studied the performance of  $M$ -ary FSK, which has unit peak-to-average power ratio, over noncoherent Gaussian channels by computing the capacity and computational cutoff rate. Stark [17] analyzed the capacity and cutoff rate of  $M$ -ary FSK signaling with both hard and soft decisions in the presence of Rician fading and noted that there exists an optimal code rate for which the required bit energy is minimized.

In this paper, we study the power efficiency of  $M$ -ary on/off FSK (OOFSK) signaling in which  $M$ -ary FSK signaling is overlaid on top of on/off keying, enabling us to introduce peakedness in both time and frequency. Our main focus will be on cases in which the peakedness of input signals is limited. The organization of the paper is as follows. Section 2 introduces the channel model. In Section 3, we find the capacity of  $M$ -ary orthogonal OOFSK signaling with energy detection at the receiver and investigate the power efficiency in two cases: limited peak-to-average power ratio and limited peak power. In Section 4, we consider joint frequency and phase modulation and analyze the capacity and power efficiency of  $M$ -ary OOFPSK signaling in which the phase of FSK signals also convey information. Finally, Section 5 includes our conclusions.

## 2. CHANNEL MODEL

In this section, we present the system model. We assume that  $M$ -ary orthogonal OOFSK signaling, in which FSK signaling is combined with on/off keying with a fixed duty factor,  $\nu \leq 1$ , is employed at the transmitter for communication over a fading channel. In this signaling scheme, over the time interval of  $[0, T]$ , the transmitter either sends no signal with probability  $1 - \nu$  or sends one of  $M$  orthogonal sinusoidal signals,

$$s_i(t) = \sqrt{\frac{P}{\nu}} e^{j(\omega_i t + \theta_i)}, \quad 0 \leq t \leq T, \quad 1 \leq i \leq M, \quad (2)$$

with probability  $\nu$ . To ensure orthogonality, adjacent frequency slots satisfy  $|\omega_{i+1} - \omega_i| = 2\pi/T$ . Choosing  $\nu = 1$ , we obtain ordinary FSK signaling. If the channel input is  $X = i$  for  $1 \leq i \leq M$ , the transmitter sends the sine wave  $s_i(t)$ , while no transmission is denoted by  $X = 0$ . Note that OOFSK

signaling has average power  $P$ , and peak power  $P/\nu$ . We assume that the transmitted signal undergoes stationary and ergodic fading and that the delay spread of the fading is much less than the symbol duration. Under these assumptions, the fading has a multiplicative effect on the transmitted signal and the received signal can be modeled as follows:

$$r(t) = h(t)s_{X_k}(t - (k-1)T) + n(t), \quad (3)$$

$$(k-1)T \leq t \leq kT, \quad \text{for } k = 1, 2, \dots,$$

where  $\{X_k\}_{k=1}^{\infty}$  is the input sequence with  $X_k \in \{0, 1, 2, \dots, M\}$ ,  $h(t)$  is a proper<sup>1</sup> complex stationary ergodic fading process with  $E\{h(t)\} = d$  and  $\text{var}(h(t)) = \gamma^2$ , and  $n(t)$  is a zero-mean circularly symmetric complex white Gaussian noise process with single-sided spectral density  $N_0$ . Note that  $s_0(t) = 0$ . If we further assume that the symbol duration  $T$  is less than the coherence time of the fading, then the fading stays constant over the symbol duration and the channel model now becomes

$$r(t) = h_k s_{X_k}(t - (k-1)T) + n(t), \quad (k-1)T \leq t \leq kT. \quad (4)$$

At the receiver, a bank of correlators is employed in each symbol interval to obtain the  $M$ -dimensional vector  $\mathbf{Y}_k = (Y_{k,1}, \dots, Y_{k,M})$ , where

$$Y_{k,i} = \frac{1}{\sqrt{N_0 T}} \int_{(k-1)T}^{kT} r(t) e^{-j\omega_i t} dt, \quad i = 1, 2, \dots, M. \quad (5)$$

It is easily seen that, given the symbol  $X_k = i$ , phase  $\theta_i$  and fading coefficient  $h_k$ ,  $Y_{k,j}$  is a proper complex Gaussian random variable with

$$E\{Y_{k,j} | X_k = i, \theta_i, h_k\} = \alpha h_k e^{j\theta_i} \delta_{ij}, \quad (6)$$

$$\text{var}(Y_{k,j} | X_k = i, \theta_i, h_k) = 1,$$

where  $\delta_{ij} = 1$  if  $i = j$  and is zero otherwise, and  $\alpha^2 = PT/\nu N_0 = \text{SNR}/\nu$  with SNR denoting the signal-to-noise ratio per symbol.

## 3. CAPACITY OF $M$ -ARY ORTHOGONAL OOFSK SIGNALING WITH ENERGY DETECTION

In this section, we analyze the capacity of  $M$ -ary orthogonal OOFSK signaling when in every symbol interval, the noncoherent receiver measures the energy at each of the  $M$  frequencies, that is, computes

$$R_{k,i} = |Y_{k,i}|^2 = \left| \frac{1}{\sqrt{N_0 T}} \int_{(k-1)T}^{kT} r(t) e^{-j\omega_i t} dt \right|^2, \quad (7)$$

$$1 \leq i \leq M, \quad \text{for } k = 1, 2, \dots,$$

and the decoder sees the vector  $\mathbf{R}_k = (R_{k,1}, \dots, R_{k,M})$ . With this structure, the receiver does not need to track phase

<sup>1</sup> See [18].

changes in the channel. We consider the cases where the receiver has either perfect or imperfect fading side information, while the transmitter has no knowledge of the fading coefficients. Besides providing the ultimate limits on the rate of communication, capacity results also offer insight into the power efficiency of OOFSK signaling by enabling us to obtain the energy required to send one bit of information reliably.

In the low-power regime, the spectral-efficiency/bit-energy tradeoff reflects the fundamental tradeoff between bandwidth and power. Assuming that the bandwidth of  $M$ -ary OOFSK modulation is  $M/T$ , where  $T$  is the symbol duration, the maximum achievable spectral efficiency is

$$C\left(\frac{E_b}{N_0}\right) = \frac{1}{M}C(\text{SNR}) \text{ bps/Hz}, \quad (8)$$

where  $C(\text{SNR})$  is the capacity in bits/symbol, and

$$\frac{E_b}{N_0} = \frac{\text{SNR}}{C(\text{SNR})} \quad (9)$$

is the bit energy normalized to the noise power. For average-power-limited channels, the bit energy required for reliable communications decreases monotonically with decreasing spectral efficiency, and the minimum bit energy is achieved at zero spectral efficiency, that is,  $E_b/N_{0\min} = \lim_{\text{SNR} \rightarrow 0} (\text{SNR}/C(\text{SNR})) = \log_e 2/\dot{C}(0)$ , where  $\dot{C}(0)$  is the first derivative of the capacity in nats. Hence, for fixed rate transmission, reduction in the required power comes only at the expense of increased bandwidth. Reference [11] analyzes the spectral-efficiency/bit-energy function in the low-power regime for a general class of average-power-limited fading channels and shows that the minimum bit energy is  $\log_e 2 = -1.59$  dB as long as the additive background noise is Gaussian. This minimum bit energy is achieved only in the asymptotic regime of infinite bandwidth. If one is willing to spend more power, then reliable communication over a finite bandwidth is possible. Hence, achieving the minimum bit energy is not a sufficient criterion for finite bandwidth analysis. The wideband slope [11], defined as the slope of the spectral efficiency curve  $C(E_b/N_0)$  in bps/Hz/3dB at zero spectral efficiency, is given by

$$\begin{aligned} S_0 &\stackrel{\text{def}}{=} \lim_{E_b/N_0 \downarrow E_b/N_0|_{C=0}} \frac{C(E_b/N_0)}{10 \log_{10}(E_b/N_0) - 10 \log_{10}(E_b/N_0)|_{C=0}} \\ &\quad \times 10 \log_{10} 2 \\ &= \frac{1}{M} \frac{2(\dot{C}(0))^2}{-\ddot{C}(0)}, \end{aligned} \quad (10)$$

where  $\dot{C}(0)$  and  $\ddot{C}(0)$  denote the first and second derivatives of the capacity in nats. Note that differing from the original definition in [11], normalization by  $M$  is introduced in (10) due to the scaling in (8). The wideband slope closely approximates the growth of the spectral-efficiency curve in the power-limited regime and hence is a useful tool providing insightful results when bandwidth is a resource to be conserved.

### 3.1. Perfect receiver side information

We first assume that the receiver has perfect knowledge of the magnitude of the fading,  $|h|$ . For this case, the capacity as a function of  $\text{SNR} = PT/N_0$  of  $M$ -ary OOFSK signaling with energy detection is given by the following proposition. Throughout the paper, we denote the probability density function and distribution function of a random variable  $Z$  by  $p_Z$  and  $F_Z$ , respectively, with arguments omitted in equations in order to avoid cumbersome expressions.

*Proposition 1.* Consider the fading channel model (4) and assume that the receiver knows the magnitude but not the phase of the fading coefficients  $\{h_k, k = 1, 2, \dots\}$ . Further assume that the transmitter has no fading side information. Then the capacity of  $M$ -ary orthogonal OOFSK signaling with a fixed duty factor  $\nu \leq 1$  with energy detection is

$$\begin{aligned} C_M^p(\text{SNR}) &= E_{|h|} \left\{ (1 - \nu) \int p_{\mathbf{R}|X=0} \log \frac{p_{\mathbf{R}|X=0}}{p_{\mathbf{R}|h}} d\mathbf{R} \right. \\ &\quad \left. + \nu \int p_{\mathbf{R}|X=1,|h|} \log \frac{p_{\mathbf{R}|X=1,|h|}}{p_{\mathbf{R}|h}} d\mathbf{R} \right\}, \end{aligned} \quad (11)$$

where

$$p_{\mathbf{R}|h} = (1 - \nu)p_{\mathbf{R}|X=0} + \frac{\nu}{M} \sum_{i=1}^M p_{\mathbf{R}|X=i,|h|}, \quad (12)$$

$$p_{\mathbf{R}|X=0} = e^{-\sum_{j=1}^M R_j}, \quad (13)$$

$$p_{\mathbf{R}|X=i,|h|} = e^{-\sum_{j=1}^M R_j} f(R_i, |h|, \text{SNR}), \quad 1 \leq i \leq M, \quad (14)$$

$$f(R_i, |h|, \text{SNR}) = \exp\left(-\frac{\text{SNR}}{\nu}|h|^2\right) I_0\left(2\sqrt{\frac{\text{SNR}}{\nu}}|h|^2 R_i\right). \quad (15)$$

For the proof, see Appendix A.

Formula (11) must be evaluated numerically, and computational complexity imposes a burden on numerical techniques for large  $M$ . Fortunately, a simpler expression is obtained in the limit  $M \rightarrow \infty$ .

*Proposition 2.* The capacity expression (11) for  $M$ -ary OOFSK signaling in the limit as  $M \uparrow \infty$  becomes

$$C_\infty^p(\text{SNR}) = D(p_{\mathbf{R}|\tilde{x},|h|} \| p_{\mathbf{R}|\tilde{x}=0,|h|} | F_{|h|} F_{\tilde{x}}), \quad (16)$$

where

$$R = |y|^2 = |h\tilde{x} + n|^2, \quad (17)$$

$\tilde{x}$  is a two-mass-point discrete random variable with the following mass-point locations and probabilities,

$$\tilde{x} = \begin{cases} 0, & \text{with probability } 1 - \nu, \\ \sqrt{\frac{\text{SNR}}{\nu}}, & \text{with probability } \nu, \end{cases} \quad (18)$$

and  $n$  is zero-mean circularly symmetric complex Gaussian random variable with  $E\{|n|^2\} = 1$ . Therefore,

$$p_{\mathbf{R}|\tilde{x},|h|} = e^{-R - \tilde{x}^2 |h|^2} I_0\left(2\sqrt{\tilde{x}^2 |h|^2 R}\right). \quad (19)$$

For the proof, see Appendix B.

### 3.2. Imperfect receiver side information

In this section, we assume that neither the receiver nor the transmitter has any side information about the fading. Unlike the previous section, here we consider a more special fading process: memoryless Rician fading where each of the i.i.d.  $h_k$ 's is a proper complex Gaussian random variable with  $E\{h_k\} = d$  and  $\text{var}(h_k) = \gamma^2$ . Note that the unknown Rician fading channel can also be regarded as an imperfectly known fading channel where the specular component is the channel estimate and the fading component is the Gaussian-distributed error in the estimate. As argued in [19], the Bayesian least-squares estimation over the Rayleigh channel leads to such a channel model. However, we want to emphasize that no explicit channel estimation method is considered in this section.

The following result gives the maximum rate at which reliable communication is possible with OOFSK signaling using energy detection over the memoryless Rician fading channel. As noted in Section 1, the capacity of the special case of  $M$ -ary FSK signaling ( $\nu = 1$ ) was previously obtained by Stark [17].

*Proposition 3.* Consider the fading channel (4), and assume that the fading process  $\{h_k\}$  is a sequence of i.i.d. proper complex Gaussian random variables with  $E\{h_k\} = d$  and  $\text{var}(h_k) = \gamma^2$ , which are not known at either the receiver or the transmitter. Further, assume that energy detection is performed at the receiver. Then the capacity of  $M$ -ary orthogonal OOFSK signaling with fixed duty factor  $\nu \leq 1$  is given by

$$C_M^{ip}(\text{SNR}) = (1 - \nu) \int p_{R|X=0} \log \frac{p_{R|X=0}}{p_R} d\mathbf{R} + \nu \int p_{R|X=1} \log \frac{p_{R|X=1}}{p_R} d\mathbf{R}, \quad (20)$$

where

$$p_{\mathbf{R}} = (1 - \nu)p_{R|X=0} + \frac{\nu}{M} \sum_{i=1}^M p_{R|X=i}, \quad (21)$$

$$p_{R|X=0} = e^{-\sum_{j=1}^M R_j}, \quad (22)$$

$$p_{R|X=i} = e^{-\sum_{j=1}^M R_j} f(R_i, \text{SNR}), \quad 1 \leq i \leq M, \quad (23)$$

$$f(R_i, \text{SNR}) = \frac{1}{\gamma^2 \text{SNR}/\nu + 1} \exp\left(\frac{\text{SNR}/\nu(\gamma^2 R_i - |d|^2)}{\gamma^2 \text{SNR}/\nu + 1}\right) \times I_0\left(\frac{2\sqrt{\text{SNR}/\nu}|d|\sqrt{R_i}}{\gamma^2 \text{SNR}/\nu + 1}\right). \quad (24)$$

*Proof.* With the memoryless assumption, the capacity of the  $M$ -ary OOFSK signaling can be formulated as the maximum mutual information between the channel input  $X_k$  and output vector  $\mathbf{R}_k$  for any  $k$ . Thus, considering a generic symbol

interval, and dropping the time index  $k$ , we have

$$C = \max_X I(X; \mathbf{R}) = \max_X (1 - \nu) \int p_{R|X=0} \log \frac{p_{R|X=0}}{p_R} d\mathbf{R} + \sum_{i=1}^M P(X=i) \int p_{R|X=i} \log \frac{p_{R|X=i}}{p_R} d\mathbf{R}. \quad (25)$$

Similarly as in the proof of Proposition 1, due to the symmetry of the channel, an input distribution equiprobable over nonzero input values, that is,  $P(X=i) = \nu/M$  for  $1 \leq i \leq M$ , where  $P(X=0) = 1 - \nu$  achieves the capacity, and we easily obtain (20) by noting that conditioned on  $X=i$ ,  $R_j = |Y_j|^2$  is a chi-square random variable with two degrees of freedom, or more generally,

$$p_{R_j|X=i} = \begin{cases} \frac{1}{\alpha^2 \gamma^2 + 1} \exp\left(-\frac{R_j + \alpha^2 |d|^2}{\alpha^2 \gamma^2 + 1}\right) I_0\left(\frac{2\sqrt{\alpha^2 |d|^2 R_j}}{\alpha^2 \gamma^2 + 1}\right), & j=i, \\ e^{-R_j}, & j \neq i, \end{cases} \quad (26)$$

where, as before,  $\alpha^2 = PT/\nu N_0$ . Note also that due to the orthogonality of signaling, the vector  $\mathbf{R}$  has independent components and we denote  $\text{SNR} = PT/N_0$ .  $\square$

Similarly to Proposition 2, we can find the infinite bandwidth capacity achieved as the number of orthogonal frequencies increases without bound. The proof is omitted as it follows along the same lines as in the proof of Proposition 2.

*Proposition 4.* The capacity expression (20) of  $M$ -ary OOFSK signaling in the limit as  $M \uparrow \infty$  becomes

$$C_\infty^{ip}(\text{SNR}) = D(p_{R|\bar{x}} || p_{R|\bar{x}=0} | F_{\bar{x}}), \quad (27)$$

where

$$R = |y|^2 = |h\bar{x} + n|^2, \quad (28)$$

$\bar{x}$  is a two-mass-point discrete random variable with mass-point locations and probabilities given in (18), and  $n$  is a zero-mean circularly symmetric complex Gaussian random variable with  $E\{|n|^2\} = 1$ . Therefore,

$$p_{R|\bar{x}} = \frac{1}{\gamma^2 \bar{x}^2 + 1} \exp\left(-\frac{R + \bar{x}^2 |d|^2}{\gamma^2 \bar{x}^2 + 1}\right) \times I_0\left(\frac{2\sqrt{\bar{x}^2 |d|^2 R}}{\gamma^2 \bar{x}^2 + 1}\right). \quad (29)$$

The following remarks are given for the asymptotic case in which  $M$  grows to infinity.

*Remark 1.* Assume that in the case of perfect receiver side information,  $\{h_k\}$  is a sequence of i.i.d. proper complex Gaussian random variables. Then the asymptotic loss in capacity incurred by not knowing the fading is

$$C_\infty^p(\text{SNR}) - C_\infty^{ip}(\text{SNR}) = D(p_{R|\bar{x}, |h|} || p_{R|\bar{x}=0, |h|} | p_{|h|} P_{\bar{x}}) - D(p_{R|\bar{x}} || p_{R|\bar{x}=0} | P_{\bar{x}}) = I(|h|; R | \bar{x}), \quad (30)$$

where  $R = |h\bar{x} + n|^2$ .



*Remark 2.* Consider the case of imperfect receiver side information, where

$$\begin{aligned} C_\infty^{ip} &= D(p_{R|\bar{x}} \| p_{R|\bar{x}=0} | P_{\bar{x}}) = (\gamma^2 + |d|^2) \text{SNR} \\ &\quad - \nu \log \left( \gamma^2 \frac{\text{SNR}}{\nu} + 1 \right) - \frac{2 \text{SNR} |d|^2}{\gamma^2 \text{SNR}/\nu + 1} \\ &\quad + \nu E_R \left\{ \log I_0 \left( \frac{2\sqrt{(\text{SNR}/\nu)|d|^2 R}}{\gamma^2 (\text{SNR}/\nu) + 1} \right) \right\} \end{aligned} \quad (31)$$

with  $\text{SNR} = PT/N_0$ . From (31), we can easily see that for fixed symbol interval  $T$ ,

$$\lim_{\nu \downarrow 0} \frac{1}{T} C_\infty^{ip}(\text{SNR}) = \frac{1}{T} (\gamma^2 + |d|^2) \text{SNR} = (\gamma^2 + |d|^2) \frac{P}{N_0} \text{ nats/s}, \quad (32)$$

and for fixed duty factor  $\nu$ ,

$$\lim_{T \uparrow \infty} \frac{1}{T} C_\infty^{ip}(\text{SNR}) = (\gamma^2 + |d|^2) \frac{P}{N_0} \text{ nats/s}. \quad (33)$$

Note that right-hand sides of (32) and (33) are equal to the infinite bandwidth capacity of the unfaded Gaussian channel with the same received signal power. Hence, these results agree with previous results [5–7], where it has been shown that the capacity of  $M$ -ary FSK signaling over noncoherent fading channels approaches the infinite bandwidth capacity of the unfaded Gaussian channel for large  $M$  and large symbol duration  $T$  or small duty factor  $\nu$ .

### 3.3. Limited peak-to-average power ratio

The peak-to-average power ratio (PAR) of OOFSK signaling is equal to the inverse of the duty factor,  $1/\nu$ . In this section, we examine the low-SNR behavior when we keep the duty factor fixed, while the average power  $P$  vanishes. We show that under this limited PAR condition, OOFSK communication with energy detection at low SNR values is extremely power inefficient even in the unfaded Gaussian channel.

*Proposition 5.* The first derivative of the capacity at zero SNR achieved by  $M$ -ary OOFSK signaling with a fixed duty factor  $\nu \leq 1$  over the unfaded Gaussian channel is zero, that is,  $\dot{C}_M^g(0) = 0$  and hence the bit energy required at zero spectral efficiency is infinite,

$$\left. \frac{E_b}{N_0} \right|_{C=0} = \lim_{\text{SNR} \rightarrow 0} \frac{\text{SNR}}{C_M^g(\text{SNR})} \log_e 2 = \frac{\log_e 2}{\dot{C}_M^g(0)} = \infty. \quad (34)$$

*Proof.* Since we consider the unfaded Gaussian channel, we set the fading variance  $\gamma^2 = 0$  in the capacity expression (20). Note that the only term in (20) that depends on the signal-to-noise ratio is  $f(R_i, \text{SNR}) = \exp(-|d|^2 \text{SNR}) I_0(2\sqrt{\text{SNR}|d|^2 R_i})$  in (24). Using the fact that  $\lim_{x \rightarrow 0} (I_1(a\sqrt{x})/\sqrt{x}) = a/2$  for  $a \geq 0$ , one can show that the derivative at  $\text{SNR} = 0$  is  $\dot{f}(R_i, 0) = |d|^2(-1 + R_i)$ . The result then follows by taking the derivative of the capacity (20) and evaluating it at  $\text{SNR} = 0$ .  $\square$

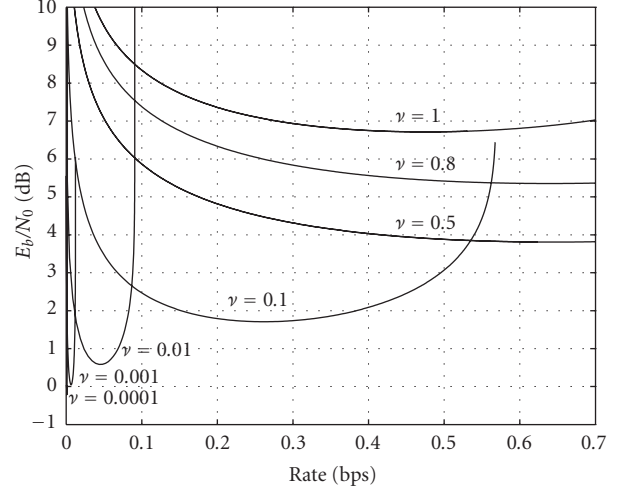


FIGURE 1:  $E_b/N_0$  (dB) versus rate (bps) for the unfaded Gaussian channel.  $M = 2$ .

Since the presence of fading that is unknown at the transmitter does not increase the capacity, from Proposition 5, we immediately conclude that  $\dot{C}(0) = 0$  for fading channels, regardless of receiver side information as long as  $\nu$  is fixed and hence the peak-to-average power ratio is limited. This result indicates that operating at very low SNR is power inefficient, and the minimum bit energy of  $M$ -ary OOFSK signaling is achieved at a nonzero spectral efficiency. Proposition 5 stems from the nonconcavity of the capacity-cost function under peak-to-average constraints (see [11]). The minimum energy per bit must be computed numerically.

Figure 1 plots bit-energy curves as a function of rate in (bps) achieved in the unfaded Gaussian channel by 2-OOFSK signaling for different values of fixed duty factor  $\nu$ . Notice that for all cases minimum bit-energy values are obtained at a nonzero rate and as the duty factor is decreased, the required minimum bit energy is also decreased. With  $\nu = 0.0001$ , the minimum bit energy is about  $-0.2$  dB. Note that this is a significant improvement over the case  $\nu = 1$ , where the minimum bit energy is about 6.7 dB. However, this gain is obtained at the cost of a considerable increase in the peak-to-average ratio. Figure 2 plots the bit-energy curves in the unknown Rician channel with Rician factor  $K = 0.5$ .

### 3.4. Limited peak power

In this section, we consider the case where the peak level of the transmitted signal is limited, while there is no constraint on the peak-to-average power ratio. Hence we fix the peak level to the maximum allowed level,  $A = P/\nu$ . Therefore, as  $P \rightarrow 0$ , the duty factor also has to vanish and hence the peak-to-average ratio increases without bound. In this case, the minimum bit energy is achieved at zero spectral efficiency, and the wideband slope provides a good characterization of the bandwidth/power tradeoff at low spectral-efficiency values.

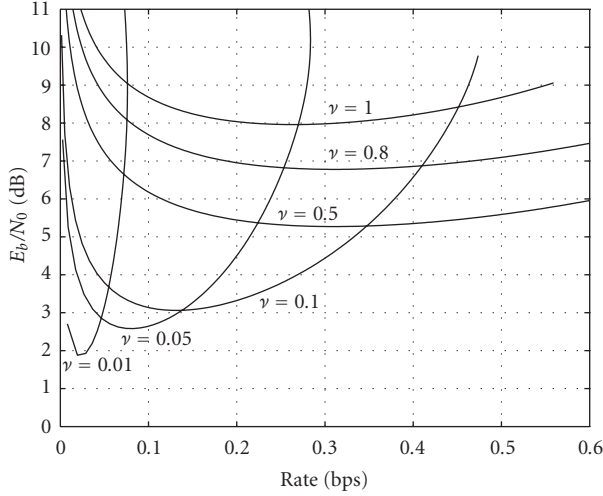


FIGURE 2:  $E_b/N_0$  (dB) versus rate (bps) for the unknown Rician channel with  $K = 0.5$ ,  $M = 2$ .

**Proposition 6.** Assume that the transmitter is limited in peak power,  $P/\nu \leq A$ , and the symbol duration  $T$  is fixed. Then

the capacity achieved by  $M$ -ary OOFSK signaling, with fixed peak power  $A$ , is a concave function of  $P$ . For the perfect receiver side information case, the minimum received bit energy and the wideband slope are

$$\begin{aligned} \frac{E_b^r}{N_{0 \min}} &= \frac{\log_e 2}{(E_{|h|} E_R \{ \log I_0(2\sqrt{\eta|h|^2 R}) \} / \eta(\gamma^2 + |d|^2)) - 1}, \\ S_0 &= \frac{2(E_h E_R \{ \log I_0(2\sqrt{\eta|h|^2 R}) \} - \eta(\gamma^2 + |d|^2))^2}{E_h \{ I_0(2\eta|h|^2) \} - 1}, \end{aligned} \quad (35)$$

respectively, where  $R$  is a noncentral chi-square random variable with

$$p_R = e^{-R - \eta|h|^2} I_0(2\sqrt{\eta|h|^2 R}) \quad (36)$$

and  $\eta = A(T/N_0)$  is the normalized peak power. For the imperfect receiver side information case, the minimum received bit energy and the wideband slope are

$$\frac{E_b^r}{N_{0 \min}} = \frac{\log_e 2}{1 - (1/(\gamma^2 + |d|^2))(2|d|^2/(\eta\gamma^2 + 1) + \log(\eta\gamma^2 + 1)/\eta - E\{\log I_0(2\sqrt{\eta|d|^2 R}/(\eta\gamma^2 + 1))\}/\eta)}, \quad (37)$$

$$S_0 = \begin{cases} \frac{2(\eta(\gamma^2 + |d|^2) - 2\eta|d|^2/(\eta\gamma^2 + 1) - \log(\eta\gamma^2 + 1) + E\{\log I_0(2\sqrt{\eta|d|^2 R}/(\eta\gamma^2 + 1))\})^2}{1/(1 - \eta^2\gamma^4) \exp(2\eta^2\gamma^2|d|^2/(1 - \eta^2\gamma^4)) I_0(2\eta|d|^2/(1 - \eta^2\gamma^4)) - 1}, & \eta\gamma^2 < 1, \\ 0, & \eta\gamma^2 \geq 1, \end{cases} \quad (38)$$

respectively, where  $R$  is a noncentral chi-square random variable with

$$p_R = \frac{1}{\eta\gamma^2 + 1} \exp\left(-\frac{R + \eta|d|^2}{\eta\gamma^2 + 1}\right) I_0\left(\frac{2\sqrt{\eta|d|^2 R}}{\eta\gamma^2 + 1}\right). \quad (39)$$

*Proof.* Since perfect and imperfect receiver side information cases are similar, for brevity we prove only the latter case. When we fix the peak power  $A = P/\nu$ , we have  $\nu = \text{SNR}/\eta$ , and the capacity becomes

$$\begin{aligned} C_M^{ip}(\text{SNR}) &= \left(1 - \frac{\text{SNR}}{\eta}\right) \int p_{R|X=0} \log \frac{p_{R|X=0}}{p_R} d\mathbf{R} \\ &+ \frac{\text{SNR}}{\eta} \int p_{R|X=1} \log \frac{p_{R|X=1}}{p_R} d\mathbf{R}. \end{aligned} \quad (40)$$

In the above capacity expression,  $p_R = (1 - \text{SNR}/\eta)p_{R|X=0} + (\text{SNR}/M\eta) \sum_{i=1}^M p_{R|X=i}$ , where  $p_{R|X=0}$  and  $p_{R|X=i}$  for  $1 \leq i \leq M$ , do not depend on SNR because the ratio  $\text{SNR}/\nu = \eta$  is a constant. Concavity of the capacity follows from the

concavity of  $-x \log x$  and the fact that  $p_R$  is a linear function of SNR. Since the capacity curve is concave, the minimum received bit energy is achieved at zero spectral efficiency,  $E_b^r/N_{0 \min} = E\{|h|^2\} \log_e 2 / \dot{C}(0)$ . The wideband slope is given by (10), and depends on both the first and second derivatives of the capacity. Hence the expressions in (37) and (38) are easily obtained by evaluating

$$\begin{aligned} \dot{C}_M^{ip}(0) &= \gamma^2 + |d|^2 - \frac{2|d|^2}{\eta\gamma^2 + 1} - \frac{\log(\eta\gamma^2 + 1)}{\eta} \\ &+ \frac{E\{\log I_0(2\sqrt{\eta|d|^2 R}/(\eta\gamma^2 + 1))\}}{\eta}, \end{aligned} \quad (41)$$

$$\begin{aligned} \ddot{C}_M^{ip}(0) &= \begin{cases} \frac{1}{\eta^2 M} \left(1 - \frac{1}{1 - \eta^2\gamma^4} \exp\left(\frac{2\eta^2\gamma^2|d|^2}{1 - \eta^2\gamma^4}\right) I_0\left(\frac{2\eta|d|^2}{1 - \eta^2\gamma^4}\right)\right), & \eta\gamma^2 < 1, \\ -\infty, & \eta\gamma^2 \geq 1. \end{cases} \end{aligned} \quad (42)$$

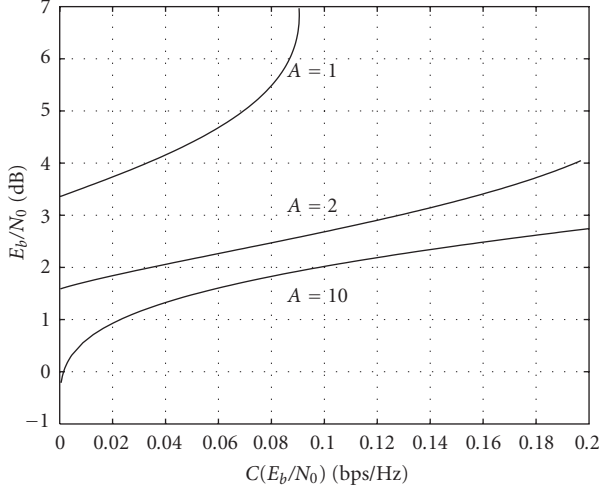


FIGURE 3:  $E_b/N_0$  (dB) versus spectral efficiency  $C(E_b/N_0)$  (bps/Hz) for the unfaded Gaussian channel.  $M = 2$ .

Similarly, for the perfect receiver side information case, we note that

$$\begin{aligned} \dot{C}_M^p(0) &= \frac{E_{|h|} E_R \{ \log I_0(2\sqrt{\eta|h|^2 R}) \}}{\eta} - (\gamma^2 + |d|^2), \\ \ddot{C}_M^p(0) &= \frac{1 - E_{|h|} \{ I_0(2\eta|h|^2) \}}{\eta^2 M}. \end{aligned} \quad (43)$$

□

In contrast to the limited PAR case, the minimum bit energy is achieved at zero spectral efficiency, and hence the power efficiency of the system improves if one operates at smaller SNR and vanishing duty factor. Note in this case that, although the average power  $P$  is decreasing, the energy of FSK signals,  $PT/\nu$ , is kept fixed, and the average power constraint is satisfied by sending these signals less frequently. In the imperfectly known channel, this type of peakedness introduced in time proves useful in avoiding adverse channel conditions. On the other hand, in the PAR limited case, the decreasing average power constraint is satisfied by decreasing the energy of FSK signals. Note that in the above result, for both perfect and imperfect side information cases, the minimum bit energy and the wideband slope do not depend on  $M$ . Therefore, on/off signaling with vanishing duty cycle is optimally power efficient at very low spectral-efficiency values, and there is no need for frequency modulation. Further note that in the imperfect receiver side information case, if  $\eta\gamma^2 \geq 1$ , then  $S_0 = 0$ , and hence approaching the minimum bit energy is extremely slow. If we relax the peak power limitation and let  $\eta \uparrow \infty$ , then it is easily seen that even in the imperfect receiver side information case,  $E_b^*/N_{0\min} \rightarrow \log_e 2 = -1.59$  dB. Indeed, [11] shows in a more general setting that flash signaling with increasingly high peak power is required to achieve the minimum bit energy of  $-1.59$  dB if the fading is not perfectly known at the receiver.

Figure 3 plots the bit-energy curves achieved by 2-OOFPSK signaling in the unfaded Gaussian channel for

different peak power values  $A$ . Notice that for all cases the minimum bit energy is achieved in the limit as the spectral efficiency goes to zero and this energy monotonically decreases to  $-1.59$  dB as  $A \rightarrow \infty$ .

#### 4. CAPACITY OF $M$ -ARY OOFPSK SIGNALING

In this section, we consider joint frequency and phase modulation to improve the power efficiency of communication with OOFPSK signaling. Combining phase and frequency modulation techniques has been proposed in the literature (see, e.g., [20–23]). As we have seen in the previous section, if the receiver employs energy detection and the peak-to-average power ratio is limited, then operating at very low SNR is extremely power inefficient. The peak-to-average power ratio constraint puts a restriction on the energy concentration in a fraction of time. Hence, for low average power values, the power of FSK signals is also low, and depending solely on energy detection leads to severe degradation in the performance. On the other hand, if the receiver can track phase shifts in the channel or if the received signal has a specular component as in the Rician channel, then the performance is improved at low spectral-efficiency values if information is conveyed in not only the amplitude but also the phase of each orthogonal frequency. Hence we propose employing phase modulation in OOFPSK signaling. Therefore, in this section, we assume that the phase  $\theta_i$  of the FSK signal,

$$s_{i,\theta_i}(t) = \sqrt{\frac{P}{\nu}} e^{j(w_i t + \theta_i)}, \quad 0 \leq t \leq T, \quad (44)$$

is a random variable carrying information. Henceforth this new signaling scheme is referred to as OOFPSK signaling. The channel input can now be represented by the pair  $(X, \theta)$ . If  $X = i$  for  $1 \leq i \leq M$ , and  $\theta = \theta_i$ , the transmitter sends the sine wave  $s_{i,\theta_i}(t)$ , while no transmission is denoted by  $X = 0$ , and hence  $s_0(t) = 0$ . As another difference from Section 3, the decoder directly uses the matched filtered output vector  $\mathbf{Y} = (Y_1, \dots, Y_M)$  instead of the energy measurements in each frequency component.

##### 4.1. Perfect receiver side information

We first consider the case where the receiver has perfect knowledge of the instantaneous realization of fading coefficients  $\{h_k\}$ , and obtain the capacity results both for fixed  $M$  and as  $M$  goes to infinity.

*Proposition 7.* Consider the fading channel model (4) and assume that the receiver perfectly knows the instantaneous values of the fading,  $h_k$ ,  $k = 1, 2, \dots$ , while the transmitter has no fading side information. Then the capacity of  $M$ -ary orthogonal OOFPSK signaling, with a fixed duty factor  $\nu \leq 1$ , is

$$\begin{aligned} C_M^p(\text{SNR}) &= -M - E_{|h|} \left\{ (1 - \nu) \int p_{\mathbf{R}|X=0} \log p_{\mathbf{R}||h|} d\mathbf{R} \right. \\ &\quad \left. + \nu \int p_{\mathbf{R}|X=1,|h|} \log p_{\mathbf{R}||h|} d\mathbf{R} \right\}, \end{aligned} \quad (45)$$

where  $p_{\mathbf{R}}|_{|h|}$ ,  $p_{\mathbf{R}}|_{X=0}$ ,  $p_{\mathbf{R}}|_{X=i,|h|}$ , and  $f(R_i, |h|, \text{SNR})$  for  $1 \leq i \leq M$  are defined in (12), (13), (14), and (15), respectively.

For the Proof, see Appendix C.

*Proposition 8.* The capacity expression (45) of  $M$ -ary OOF-PSK signaling in the limit as  $M \uparrow \infty$  becomes

$$\begin{aligned} C_{\infty}^p(\text{SNR}) &= D(P_{y|\tilde{x},h} || P_{y|\tilde{x}=0,h} | F_{\tilde{x}} F_h) \\ &= E\{|h|^2\} \text{SNR} \\ &= (\gamma^2 + |d|^2) \text{SNR}, \end{aligned} \quad (46)$$

where  $y = h\tilde{x} + n$ ,  $\tilde{x}$  is a two-mass-point discrete random variable with mass-point locations and probabilities given in (18), and  $n$  is zero-mean circularly symmetric Gaussian random variable with  $E\{|n|^2\} = 1$ .

Note that  $1/TC_{\infty}^p(\text{SNR}) = (\gamma^2 + |d|^2)P/N_0$  nats/s is equal to the infinite bandwidth capacity of the unfaded Gaussian channel with the same received power. Hence, in the perfect side information case, ordinary FSK signaling with duty factor  $\nu = 1$  is enough to achieve this capacity.

#### 4.2. Imperfect receiver side information

Similarly as in Section 3.2, we now assume that neither the receiver nor the transmitter has any fading side information and consider a more special fading process: memoryless Rician fading where each of the i.i.d.  $h_k$ 's is a proper complex Gaussian random variable with  $E\{h_k\} = d$  and  $\text{var}(h_k) = \gamma^2$ . The capacity of OOFPSK signaling is given by the following result.

*Proposition 9.* Consider the fading channel (4) and assume that the fading process  $\{h_k\}$  is a sequence of i.i.d. proper complex Gaussian random variables with  $E\{h_k\} = d$  and  $\text{var}(h_k) = \gamma^2$ , which are not known at either the receiver or the transmitter. Then the capacity of  $M$ -ary orthogonal OOFPSK signaling, with a duty factor  $\nu \leq 1$ , is given by

$$\begin{aligned} C_M^{ip}(\text{SNR}) &= -M - \nu \log\left(\gamma^2 \frac{\text{SNR}}{\nu} + 1\right) \\ &\quad - (1 - \nu) \int p_{\mathbf{R}}|_{X=0} \log p_{\mathbf{R}} \, d\mathbf{R} \\ &\quad - \nu \int p_{\mathbf{R}}|_{X=1} \log p_{\mathbf{R}} \, d\mathbf{R}, \end{aligned} \quad (47)$$

where  $p_{\mathbf{R}}$ ,  $p_{\mathbf{R}}|_{X=0}$ ,  $p_{\mathbf{R}}|_{X=i}$ , and  $f(R_i, \text{SNR})$  for  $1 \leq i \leq M$  are defined in (21), (22), (23), and (24), respectively.

*Proof.* The proof is almost identical to that of Proposition 7. Due to the symmetry of the channel, capacity is achieved by equiprobable FSK signals with uniform phases. Note that in this case,

$$\begin{aligned} C_M^{ip}(\text{SNR}) &= (1 - \nu) \int p_{\mathbf{Y}|X=0,\theta} \log \frac{p_{\mathbf{Y}|X=0,\theta}}{p_{\mathbf{Y}}} \, d\mathbf{Y} \frac{1}{2\pi} \, d\theta \\ &\quad + \nu \int p_{\mathbf{Y}|X=1,\theta} \log \frac{p_{\mathbf{Y}|X=1,\theta}}{p_{\mathbf{Y}}} \, d\mathbf{Y} \frac{1}{2\pi} \, d\theta, \end{aligned} \quad (48)$$

where

$$\begin{aligned} p_{\mathbf{Y}|X=i,\theta} &= \begin{cases} \frac{1}{\pi^{M-1}} e^{-\sum_{j \neq i} |Y_j|^2} \frac{1}{\pi(\gamma^2 \alpha^2 + 1)} e^{-|Y_i - \alpha d e^{j\theta}|^2 / (\gamma^2 \alpha^2 + 1)}, & 1 \leq i \leq M, \\ \frac{1}{\pi^M} e^{-\sum_{j=1}^M |Y_j|^2}, & i = 0. \end{cases} \end{aligned} \quad (49)$$

The capacity expression in (47) is then obtained by first integrating with respect to  $\theta$ , and then making a change of variables,  $R_j = |Y_j|^2$ .  $\square$

*Proposition 10.* The capacity expression (47) of  $M$ -ary OOF-PSK signaling in the limit as  $M \uparrow \infty$  becomes

$$\begin{aligned} C_{\infty}^{ip}(\text{SNR}) &= D(P_{y|\tilde{x}} || P_{y|\tilde{x}=0} | F_{\tilde{x}}) \\ &= (\gamma^2 + |d|^2) \text{SNR} - \nu \log\left(\gamma^2 \frac{\text{SNR}}{\nu} + 1\right), \end{aligned} \quad (50)$$

where  $y = h\tilde{x} + n$ ,  $h$  is a proper Gaussian random variable with  $E\{h\} = d$  and  $\text{var}(h) = \gamma^2$ ,  $\tilde{x}$  is a two-mass-point discrete random variable with mass-point locations and probabilities given in (18), and  $n$  is a zero-mean circularly symmetric complex Gaussian random variable with  $E\{|n|^2\} = 1$ .

Similarly as before, the remarks below are given for the asymptotic case in which  $M \rightarrow \infty$ .

*Remark 3.* Assume that in the case of perfect receiver side information,  $\{h_k\}$  is a sequence of i.i.d. proper complex Gaussian random variables. Then the asymptotic loss in capacity incurred by not knowing the fading is

$$\begin{aligned} C_{\infty}^p(\text{SNR}) - C_{\infty}^{ip}(\text{SNR}) &= D(p_{y|\tilde{x},h} || p_{y|\tilde{x}=0,h} | F_h F_{\tilde{x}}) \\ &\quad - D(p_{y|\tilde{x}} || p_{y|\tilde{x}=0} | F_{\tilde{x}}) \\ &= I(h; y | \tilde{x}). \end{aligned} \quad (51)$$

*Remark 4.* Consider the case of imperfect receiver side information. For unit duty factor  $\nu = 1$ , the capacity expression (50) is a special case of the result by Viterbi [24]. From (50), we can also see that for fixed symbol interval  $T$ ,

$$\lim_{\nu \downarrow 0} \frac{1}{T} C_{\infty}^{ip}(\text{SNR}) = \frac{1}{T} (\gamma^2 + |d|^2) \text{SNR} = (\gamma^2 + |d|^2) \frac{P}{N_0} \text{ nats/s}, \quad (52)$$

and for fixed duty factor  $\nu$ ,

$$\lim_{T \uparrow \infty} \frac{1}{T} C_{\infty}^{ip}(\text{SNR}) = (\gamma^2 + |d|^2) \frac{P}{N_0} \text{ nats/s}. \quad (53)$$

Note that right-hand sides of (52) and (53) are equal to the infinite bandwidth capacity of the unfaded Gaussian channel with the same received signal power.

#### 4.3. Limited peak-to-average power ratio

As in Section 3.3, we first consider the case where the transmitter peak-to-average power ratio is limited and hence the



duty factor  $\nu$  is kept fixed, while the average power varies. The power efficiency in the low-power regime is characterized by the following result.

*Proposition 11.* Assume that the transmitter is constrained to have limited peak-to-average power ratio and the PAR of  $M$ -ary OOFPSK signaling,  $1/\nu$ , is kept fixed at its maximum level. Then, for the perfect receiver side information case, the minimum received bit energy and the wideband slope are

$$\frac{E_b^r}{N_{0 \min}} = \log_e 2, \quad S_0 = \frac{2(E\{|h|^2\})^2}{E\{|h|^4\}} = \frac{2}{\kappa(|h|)}, \quad (54)$$

respectively, where  $\kappa(|h|)$  is the kurtosis of the fading magnitude. For the imperfect receiver side information case, the received bit energy required at zero spectral efficiency and the wideband slope are

$$\frac{E_b^r}{N_0} \Big|_{C=0} = \left(1 + \frac{1}{K}\right) \log_e 2, \quad S_0 = \frac{2K^2}{(1+K)^2 - M/\nu}, \quad (55)$$

respectively, where  $K = |d|^2/\gamma^2$  is the Rician factor.

*Proof.* For brevity, we show the result only for the imperfect receiver side information case. Note that in the capacity expression (47), the only term that depends on SNR is  $f(R_i, \text{SNR})$ . Using

$$\begin{aligned} \lim_{x \rightarrow 0} \frac{I_1(a\sqrt{x})}{\sqrt{x}} &= \frac{a}{2}, \\ \lim_{x \rightarrow 0} \frac{I_0(a\sqrt{x})}{x} - \frac{2I_1(a\sqrt{x})}{ax^{3/2}} &= \frac{a^2}{8}, \end{aligned} \quad (56)$$

one can easily show that the first and second derivatives with respect to SNR of  $f(R_i, \text{SNR})$  at zero SNR are

$$\begin{aligned} \dot{f}(R_i, 0) &= \frac{1}{\nu} (\gamma^2 + |d|^2) (-1 + R_i), \\ \ddot{f}(R_i, 0) &= \frac{1}{\nu^2} (|d|^4 + 2\gamma^4 + 4\gamma^2 |d|^2) \left(1 - 2R_i + \frac{R_i^2}{2}\right), \end{aligned} \quad (57)$$

respectively. Then, differentiating the capacity (47) with respect to SNR, we have

$$\dot{C}_M^i(0) = |d|^2, \quad \ddot{C}_M^i(0) = -\frac{(\gamma^2 + |d|^2)^2}{M} + \frac{\gamma^4}{\nu}. \quad (58)$$

The received bit energy required at zero spectral efficiency is obtained from the formula

$$\frac{E_b^r}{N_0} \Big|_{C=0} = \frac{(\gamma^2 + |d|^2) \log_e 2}{\dot{C}(0)}, \quad (59)$$

and the wideband slope is found by inserting the derivative expressions in (58) into (10). Similarly, for the perfect receiver side information case, we have

$$\dot{C}_M^p(0) = E\{|h|^2\} = (\gamma^2 + |d|^2), \quad \ddot{C}_M^p(0) = -\frac{E\{|h|^4\}}{M}. \quad (60)$$

□

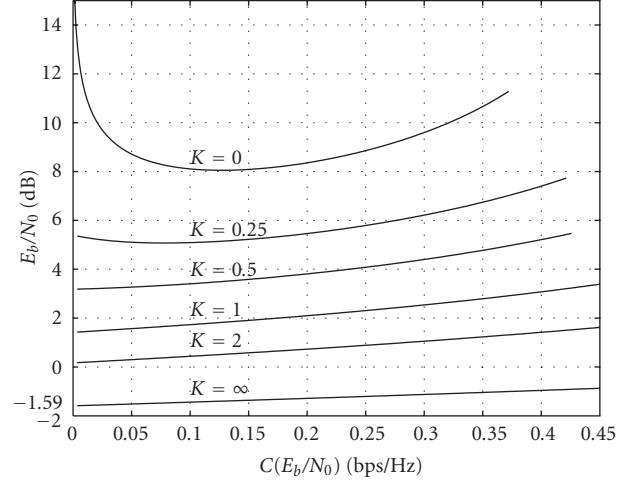


FIGURE 4:  $E_b/N_0$  (dB) versus spectral efficiency  $C(E_b/N_0)$  (bps/Hz) for the unknown Rayleigh channel ( $K = 0$ ), unknown Rician channels ( $K = 0.25, 0.5, 1, 2$ ), and the unfaded Gaussian channel ( $K = \infty$ ) when  $M = 2$  and  $\nu = 1$ .

Notice that in the perfect side information case, the minimum bit energy is  $-1.59$  dB, and the wideband slope does not depend on  $M$  and  $\nu$ . In fact, Verdú has obtained the same bit energy and wideband slope expression in [11] for discrete-time fading channels when the receiver knows the fading coefficients, and proved that QPSK modulation is optimally efficient achieving these values. More interesting is the imperfect receiver side information case, where the minimum bit energy is not necessarily achieved at zero spectral efficiency. Note that unlike the bit-energy expression in (55), the wideband slope is a function of  $M$  and  $\nu$ , and is negative if  $M/\nu > (1+K)^2$  in which case the minimum bit energy is achieved at a nonzero spectral efficiency.

Figure 4 plots the bit-energy curves as a function of spectral efficiency in bps/Hz for 2-FPSK signaling ( $\nu = 1$ ). Note that for  $K = 0.25$ , the wideband slope is negative, and hence the minimum bit energy is achieved at a nonzero spectral efficiency. On the other hand, for  $K = 0.5, 1, 2$ , the wideband slope is positive, and hence higher power efficiency is achieved as one operates at lower spectral efficiency. Similar observations are noted from Figure 5, where bit-energy curves are plotted for 3-FPSK signaling. Figure 6 plots the bit-energy curves for 2-OOFPSK signaling with different duty cycle parameters over the unknown Rician channel with  $K = 1$ . We observe that the required minimum bit energy is decreasing with decreasing duty cycle. For instance, when  $\nu = 0.01$ , the minimum bit energy of  $\sim 0.46$  dB is achieved at the cost of a peak-to-average ratio of 100. Note also that since the received bit energy at zero spectral efficiency (55) depends only on the Rician factor  $K$ , all the curves in Figure 6 meet at the same point on the  $y$ -axis.

#### 4.4. Limited peak power

Here we assume that the transmitter is limited in its peak power, while there is no bound on the peak-to-average power

ratio. We consider the power efficiency of  $M$ -ary OOFPSK signaling when the peak power is kept fixed at the maximum allowed level,  $A = P/\nu$ . Note that as the average power  $P \rightarrow 0$ , the duty factor  $\nu$  also must vanish, thereby increasing the peak-to-average power ratio without bound. For this case, we have the following result.

*Proposition 12.* Assume that the transmitter is limited in peak power,  $P/\nu \leq A$ , and the symbol duration  $T$  is fixed. Then the capacity achieved by  $M$ -ary OOFPSK signaling with fixed peak power  $A$  is a concave function of the SNR. For the case

of perfect receiver side information, the minimum received bit energy and the wideband slope are

$$\frac{E_b^r}{N_{0 \min}} = \log_e 2, \quad S_0 = \frac{2\eta^2 (E\{|h|^2\})^2}{E\{I_0(2\eta|h|^2)\} - 1}, \quad (61)$$

respectively, where  $\eta = A(T/N_0)$  is the normalized peak power. For the case of imperfect receiver side information, the minimum received bit energy and the wideband slope are

$$\frac{E_b^r}{N_{0 \min}} = \frac{\log_e 2}{1 - \log(\gamma^2 \eta + 1)/(\gamma^2 + |d|^2)\eta},$$

$$S_0 = \begin{cases} \frac{2(\eta(\gamma^2 + |d|^2) - \log(\eta\gamma^2 + 1))^2}{1/(1 - \eta^2\gamma^4) \exp(2\eta^2\gamma^2|d|^2/(1 - \eta^2\gamma^4)) I_0(2\eta|d|^2/(1 - \eta^2\gamma^4)) - 1}, & \eta\gamma^2 < 1, \\ 0, & \eta\gamma^2 \geq 1, \end{cases} \quad (62)$$

respectively.

*Proof.* As before, we consider only the imperfect receiver side information case. When we fix the peak power  $A = P/\nu$ , we have  $\nu = \text{SNR}/\eta$ , and the capacity becomes

$$C_M^{ip}(\text{SNR}) = -M - \frac{\text{SNR}}{\eta} \log(\gamma^2 \eta + 1) - \left(1 - \frac{\text{SNR}}{\eta}\right) \int p_{\mathbf{R}|X=0} \log p_{\mathbf{R}} d\mathbf{R} - \frac{\text{SNR}}{\eta} \int p_{\mathbf{R}|X=1} \log p_{\mathbf{R}} d\mathbf{R}. \quad (63)$$

In the above capacity expression,

$$p_{\mathbf{R}} = \left(1 - \frac{\text{SNR}}{\eta}\right) p_{\mathbf{R}|X=0} + \frac{\text{SNR}}{M\eta} \sum_{i=1}^M p_{\mathbf{R}|X=i}, \quad (64)$$

where  $p_{\mathbf{R}|X=0}$  and  $p_{\mathbf{R}|X=i}$  for  $1 \leq i \leq M$  do not depend on SNR because the ratio  $\text{SNR}/\nu = \eta$  is a constant. Concavity of the capacity follows from the concavity of  $-x \log x$  and the fact that  $p_{\mathbf{R}}$  is a linear function of SNR. Due to concavity of the capacity curve, the minimum bit energy is achieved at zero spectral efficiency. Differentiating the capacity with respect to SNR, we get

$$\dot{C}_M^{ip}(0) = \gamma^2 + |d|^2 - \frac{\log(\gamma^2 \eta + 1)}{\eta}, \quad (65)$$

and  $\dot{C}_M^{ip}(0)$  having the same expression as in (42). Then, (62) is easily obtained using the aforementioned formulas for the minimum bit energy and the wideband slope. Similarly, we note for the perfect side information case that

$$\dot{C}_M^p(0) = E\{|h|^2\} = \gamma^2 + |d|^2, \quad \dot{C}_M^p(0) = \frac{1 - E\{I_0(2\eta|h|^2)\}}{\eta^2 M}. \quad (66)$$

□

Note that the results in (61) and (62) do not depend on  $M$ , and hence they can be achieved by pure on/off keying. Further, note that  $(I_0(2\eta|h|^2) - 1)/\eta^2 > |h|^4$  for  $\eta > 0$ . Therefore, when the fading is perfectly known, the strategy of fixing the peak power and letting  $\nu \downarrow 0$  results in a wideband slope smaller than that of fixed duty factor and hence should not be preferred. In the imperfect receiver side information case, if the peak power limitation is relaxed, that is,  $\eta \uparrow \infty$ , the minimum bit energy approaches  $-1.59$  dB.

Figure 7 plots the bit-energy curves as a function of spectral efficiency for the unknown Rayleigh channel ( $K = 0$ ), unknown Rician channels ( $K = 0.25, 0.5, 1, 2$ ), and the unfaded Gaussian channel ( $K = \infty$ ) when the normalized peak power limit is  $\eta = 1$ . We observe that for all cases the required bit energy decreases with decreasing spectral efficiency, and therefore the minimum bit energy is achieved at zero spectral efficiency. Finally, Figures 8 and 9 plot the minimum bit-energy and wideband slope values, respectively, as functions of the normalized peak power limit  $\eta$  in the unknown Rician channel with  $K = 1$ . The curves are plotted for the case in which no phase modulation is used, and the receiver employs energy detection (Section 3), and also for the scenario in which phase modulation is employed.

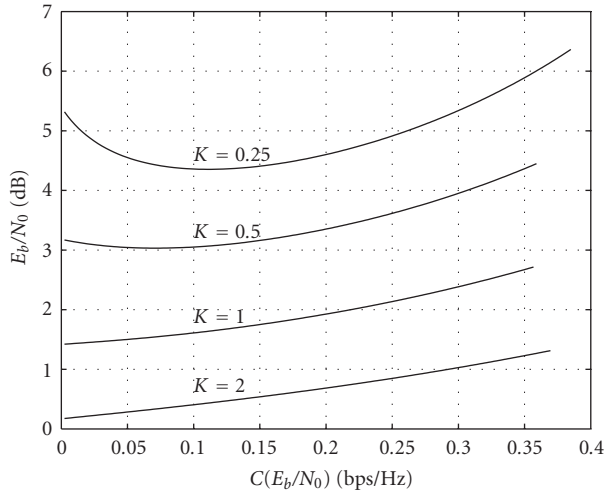


FIGURE 5:  $E_b/N_0$  (dB) versus spectral efficiency  $C(E_b/N_0)$  (bps/Hz) for unknown Rician channels ( $K = 0.25, 0.5, 1, 2$ ) when  $M = 3$  and  $\nu = 1$ .

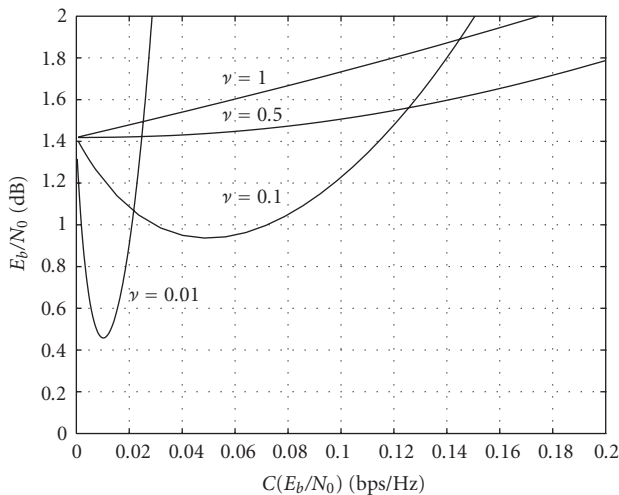


FIGURE 6:  $E_b/N_0$  (dB) versus spectral efficiency  $C(E_b/N_0)$  (bps/Hz) for the unknown Rician channel with  $K = 1$  for  $\nu = 1, 0.5, 0.1, 0.01$  when  $M = 2$ .

## 5. CONCLUSION

We have considered transmission of information over wide-band fading channels using  $M$ -ary orthogonal on/off FSK (OOFSK) signaling, in which  $M$ -ary FSK signaling is overlaid on top of on/off keying. We have first assumed that the receiver uses energy detection for the reception of OOFSK signals. We have obtained capacity expressions when the receiver has perfect and imperfect fading side information both for fixed  $M$  and as  $M$  goes to infinity. We have investigated power efficiency when the transmitter is subject to a peak-to-average power ratio (PAR) limitation or a peak power limitation. It is shown that under a PAR limitation, no matter how large the transmitted energy per information bit is, reliable communication is impossible for small enough spectral

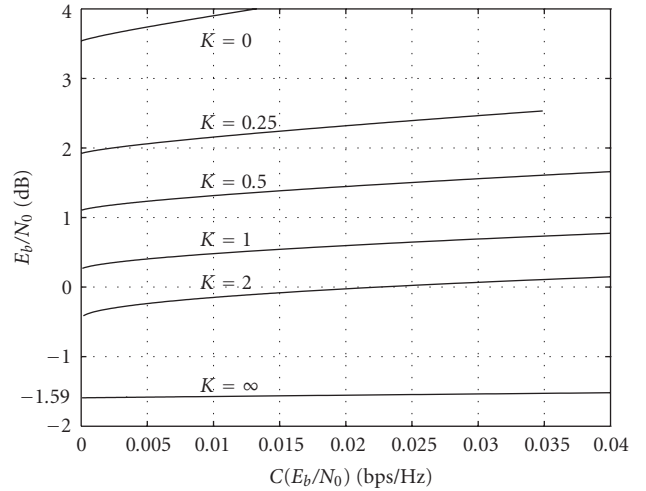


FIGURE 7:  $E_b/N_0$  (dB) versus spectral efficiency  $C(E_b/N_0)$  (bps/Hz) for the unknown Rayleigh channel ( $K = 0$ ), unknown Rician channels ( $K = 0.25, 0.5, 1, 2$ ), and the unfaded Gaussian channel ( $K = \infty$ ) when  $M = 2$  and fixed peak limit  $\eta = 1$ .

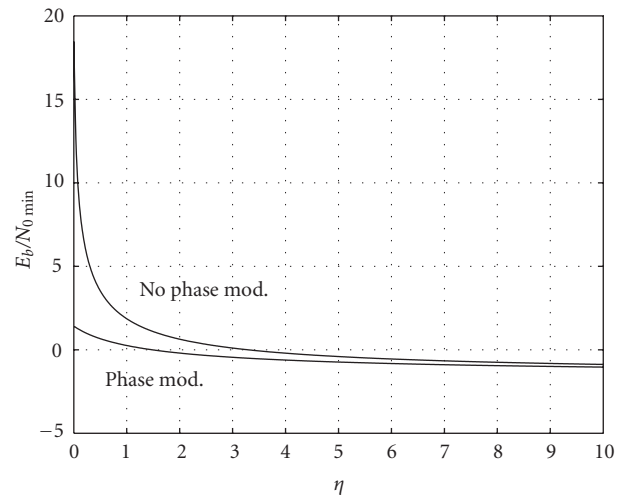


FIGURE 8:  $E_b/N_{0\min}$  versus normalized peak power limit  $\eta$  in the unknown Rician channel with  $K = 1$ .

efficiency even in the unfaded Gaussian channel, and hence it is extremely power inefficient to operate in the very low SNR regime. On the other hand, if there is only a peak power limitation, we have demonstrated that power efficiency improves as one operates with smaller SNR and vanishing duty factor. We note that, in this case, on/off keying (OOK) is an optimally efficient signaling in the low-power regime achieving the minimum bit energy and the wideband slope in both perfect and imperfect channel side information cases, while combined OOK and FSK signaling is required to improve energy efficiency when a constraint is imposed on the PAR.

We have also considered joint frequency-phase modulation schemes where the phase of the FSK signals are also used to convey information. Similarly, we have analyzed the capacity and power efficiency of these schemes. Assuming

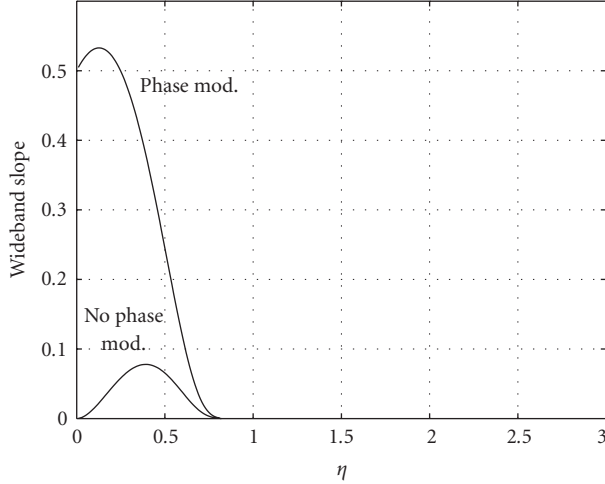


FIGURE 9: Wideband slope  $S_0$  versus normalized peak power limit  $\eta$  in the unknown Rician channel with  $K = 1$ .

perfect channel knowledge at the receiver, we have obtained the minimum bit-energy and wideband slope expressions. In this case, it is shown that FSK signaling is not required for optimum power efficiency in the low-power regime as pure phase modulation in the PAR limited case and OOK in the peak power limited case achieve both the minimum bit energy and the optimal wideband slope. For the case in which the receiver has imperfect channel side information and the input is subject to PAR constraints, we have shown that if  $M/\nu > (1 + K)^2$ , then the wideband slope is negative, and hence the minimum bit energy is achieved at a nonzero spectral efficiency,  $C^* > 0$ . It is concluded that, in these cases, operating in the region, where  $C < C^*$ , should be avoided. We also note that, in general, the combined OOK and FSK signaling performs better and indeed if the number of orthogonal frequencies, that is,  $M$ , is increased, then a smaller minimum bit-energy value is achieved. Furthermore, for the case in which only the peak power is limited with no constraints on the peak-to-average ratio, we have investigated the spectral-efficiency/bit-energy tradeoff in the low-power regime by obtaining both the minimum bit energy (attained at zero spectral efficiency) and the wideband slope which can be achieved by pure OOK signaling.

## APPENDIX

### A. PROOF OF PROPOSITION 1

Since the fading coefficients form a stationary ergodic process, the capacity of OOFSK signaling can be formulated as follows:

$$C(\text{SNR}) = \lim_{n \rightarrow \infty} \max_{X^n} \frac{1}{n} I(X^n; \mathbf{R}^n | |h|^n), \quad (\text{A.1})$$

where  $X^n = (X_1, \dots, X_n)$ ,  $\mathbf{R}^n = (\mathbf{R}_1, \dots, \mathbf{R}_n)$ , and  $|h|^n = (|h_1|, \dots, |h_n|)$ . As the additive Gaussian noisesamples are

independent for each symbol interval, the conditional output density satisfies

$$p_{\mathbf{R}^n | X^n, |h|^n} = \prod_{k=1}^n p_{\mathbf{R}_k | X_k, |h_k|}, \quad (\text{A.2})$$

where

$$p_{\mathbf{R}_k | X_k = i, |h_k|} = \begin{cases} e^{-\sum_{j=1}^M R_{kj}} e^{-\alpha^2 |h_k|^2} I_0(2\sqrt{R_{ki}\alpha^2 |h_k|^2}), & 1 \leq i \leq M, \\ e^{-\sum_{j=1}^M R_{kj}}, & i = 0, \end{cases} \quad (\text{A.3})$$

with  $\alpha^2 = PT/\nu N_0 = \text{SNR}/\nu$ . From the above fact, one can easily show that

$$\begin{aligned} I(X^n; \mathbf{R}^n | |h|^n) &= \sum_{k=1}^n I(X_k; \mathbf{R}_k | |h_k|) \\ &\quad - D\left(p_{\mathbf{R}^n | |h|^n} \left\| \prod_{k=1}^n p_{\mathbf{R}_k | |h_k|} \right. \middle| F_{|h|^n}\right) \\ &\leq \sum_{k=1}^n I(X_k; \mathbf{R}_k | |h_k|), \end{aligned} \quad (\text{A.4})$$

where  $D(\cdot || \cdot F_{|h|^n})$  denotes the conditional divergence. The above upper bound is achieved if the input vector  $X^n = (X_1, \dots, X_n)$  has independent components. Due to the symmetry of the channel, an input distribution equiprobable over nonzero input values, that is,  $P(X_k = i) = \nu/M$  for  $1 \leq i \leq M$ , where  $P(X_k = 0) = 1 - \nu$ , maximizes  $I(X_k; \mathbf{R}_k | |h_k|)$  for each  $k$ . To see this, note that since the mutual information is a concave function of the input vector, a sufficient and necessary condition for an input vector to be optimal is

$$\frac{\partial}{\partial P_i} \left[ I(X_k; \mathbf{R}_k | |h_k|) - \lambda \left( \sum_{j=1}^M P_j - \nu \right) \right] = 0, \quad 1 \leq i \leq M, \quad (\text{A.5})$$

where  $\lambda$  is a Lagrange multiplier for the equality constraint  $\sum_{j=1}^M P_j = \nu$ , and  $P_j$  denotes  $P(X_k = j)$  for  $1 \leq j \leq M$ . Note that the duty factor is fixed, and hence  $P(X = 0) = 1 - \nu$  is a predetermined constant. Evaluating the derivatives, the above condition can be reduced to

$$E_{|h_k|} \left\{ \int p_{\mathbf{R}_k | X_k = i, |h_k|} \log \frac{p_{\mathbf{R}_k | X_k = i, |h_k|}}{p_{\mathbf{R}_k | |h_k|}} d\mathbf{R}_k \right\} - 1 = \lambda, \quad (\text{A.6})$$

$$1 \leq i \leq M,$$

and due to the symmetry of the channel, letting  $P_i = P(X_k = i) = \nu/M$  for  $1 \leq i \leq M$  satisfies the condition. Therefore, an

i.i.d. input sequence with the above distribution achieves the capacity. The capacity expression in (11) is easily obtained by evaluating the mutual information achieved by the optimal input, considering a generic symbol interval, and dropping the time index  $k$ .

## B. PROOF OF PROPOSITION 2

The method of proof follows primarily from [25], where martingale theory is used to establish a similar result for  $M$ -ary FSK signaling over the noncoherent Gaussian channel. The capacity expression in (11) can be rewritten as

$$C_M^p(\text{SNR}) = \nu E_{|h|} \left\{ \int e^{-R-(\text{SNR}/\nu)|h|^2} I_0 \left( 2\sqrt{\frac{\text{SNR}}{\nu}} |h|^2 R \right) \log \frac{e^{-R-(\text{SNR}/\nu)|h|^2} I_0(2\sqrt{(\text{SNR}/\nu)} |h|^2 R)}{e^{-R}} dR \right\} - E_{|h|} \left\{ \int e^{-\sum_{i=1}^M R_i} \frac{S_M(\mathbf{R})}{M} \log \frac{S_M(\mathbf{R})}{M} d\mathbf{R} \right\}, \quad (\text{B.1})$$

where the first term on the right-hand side can be recognized as the conditional divergence  $D(p_{R|\bar{x},|h|} \| p_{R|\bar{x}=0,|h|} | F_{|h|} F_{\bar{x}})$ , and

$$S_M(\mathbf{R}) = \sum_{i=1}^M (\nu f(R_i, |h|, \text{SNR}) + (1 - \nu)) \quad (\text{B.2})$$

is a sum of i.i.d. random variables. The following result is noted in [25].

**Lemma 1.** *Let  $X_1, X_2, \dots$  be identically distributed random variables having finite mean. Let  $S_n = X_1 + \dots + X_n$ , and  $\beta_n = \beta(S_n, S_{n+1}, \dots)$ , the Borel field generated by  $S_n, S_{n+1}, \dots$ . Then  $\{\dots, S_n/n, S_{n-1}/n-1, \dots, S_1/1\}$  is a martingale with respect to  $\{\dots, \beta_n, \beta_{n-1}, \dots, \beta_1\}$ . Moreover, if  $g$  is a function which is convex and continuous on a convex set containing the range of  $X_1$ , and if  $E\{|g(X_1)|\} < \infty$ , then  $\{g(S_n/n)\}_\infty$  is a submartingale.*

From Lemma 1, we conclude that

$$\chi_M = g\left(\frac{S_M(\mathbf{R})}{M}\right) = \frac{S_M(\mathbf{R})}{M} \log \frac{S_M(\mathbf{R})}{M} \quad (\text{B.3})$$

is a submartingale, and hence from the martingale convergence theorem [26],  $\chi_M$  converges to a limit  $\chi_\infty$  almost surely and in mean. Therefore,  $\lim_{M \rightarrow \infty} E\{\chi_M\} = E\{\lim_{M \rightarrow \infty} \chi_M\} = E\{\chi_\infty\}$ . Note also that from the strong law of large numbers and continuity of the function  $g(x) = x \log x$ ,

$$\begin{aligned} \lim_{M \rightarrow \infty} \chi_M &= \lim_{M \rightarrow \infty} g\left(\frac{S_M(\mathbf{R})}{M}\right) = g\left(\lim_{M \rightarrow \infty} \frac{S_M(\mathbf{R})}{M}\right) \\ &= g\left(E_R\{\nu f(R, |h|, \text{SNR}) + (1 - \nu)\}\right) \\ &= g\left(\int e^{-R} (\nu f(R, |h|, \text{SNR}) + (1 - \nu)) dR\right) \\ &= g(1) = 0. \end{aligned} \quad (\text{B.4})$$

Hence, we conclude that

$$\lim_{M \rightarrow \infty} E_R \left\{ \left( \frac{S_M(\mathbf{R})}{M} \right) \log \left( \frac{S_M(\mathbf{R})}{M} \right) \right\} = 0. \quad (\text{B.5})$$

The first term on the right-hand side of (B.1) does not depend on  $M$ , and the second term can be expressed as  $E_{|h|} E_R \{(S_M(\mathbf{R})/M) \log(S_M(\mathbf{R})/M)\}$ . The proof is completed by showing that

$$\begin{aligned} \lim_{M \rightarrow \infty} E_{|h|} E_R \left\{ \frac{S_M(\mathbf{R})}{M} \log \frac{S_M(\mathbf{R})}{M} \right\} \\ = E_{|h|} \left\{ \lim_{M \rightarrow \infty} E_R \left\{ \frac{S_M(\mathbf{R})}{M} \log \frac{S_M(\mathbf{R})}{M} \right\} \right\} = 0, \end{aligned} \quad (\text{B.6})$$

where the interchange of limit and expectation needs to be justified by invoking the Dominated Convergence Theorem. Note that since  $\{(S_M(\mathbf{R})/M) \log(S_M(\mathbf{R})/M)\}$  is a submartingale,

$$0 \leq E_R \left\{ \frac{S_M(\mathbf{R})}{M} \log \frac{S_M(\mathbf{R})}{M} \right\} \leq E_R \{S_1(R) \log S_1(R)\} < \infty. \quad (\text{B.7})$$

By noting that  $f(R, |h|, \text{SNR})$  is an exponentially decreasing function of  $|h|$ , it can be easily shown that

$$\int E_R \{S_1(R) \log S_1(R)\} dF_{|h|} < \infty \quad (\text{B.8})$$

for any distribution function  $F_{|h|}$  with  $E\{|h|^2\} < \infty$ . Therefore, the Dominated Convergence Theorem applies using the integrable upper bound  $E_R \{S_1(R) \log S_1(R)\}$ .

## C. PROOF OF PROPOSITION 7

Similarly to the proof of Proposition 1, an i.i.d. input sequence achieves the capacity and due to the symmetry of the channel, equiprobable FSK signals each having uniformly distributed phases are optimal. Now, the maximum input-output mutual information is

$$\begin{aligned} I(X, \theta; \mathbf{Y} | h) &= E_h \left\{ (1 - \nu) \int p_{\mathbf{Y}|X=0, \theta} \log \frac{p_{\mathbf{Y}|X=0, \theta}}{p_{\mathbf{Y}||h|}} d\mathbf{Y} \frac{1}{2\pi} d\theta \right. \\ &\quad \left. + \nu \int p_{\mathbf{Y}|X=1, \theta, |h|} \log \frac{p_{\mathbf{Y}|X=1, \theta, |h|}}{p_{\mathbf{Y}||h|}} d\mathbf{Y} \frac{1}{2\pi} d\theta \right\}, \end{aligned} \quad (\text{C.1})$$



where

$$p_{Y|X=i,\theta,h} = \begin{cases} \frac{1}{\pi^{M-1}} e^{-\sum_{j \neq i} |Y_j|^2} \left(\frac{1}{\pi}\right) e^{-|Y_i - \alpha h e^{j\theta}|^2}, & 1 \leq i \leq M, \\ \left(\frac{1}{\pi^M}\right) e^{-\sum_{j=1}^M |Y_j|^2}, & i = 0. \end{cases} \quad (C.2)$$

In the above formulation,  $\alpha^2 = PT/\nu N_0 = \text{SNR}/\nu$ . It can be easily seen that

$$\int p_{Y|X=i,\theta} \log p_{Y|X=i,\theta} d\mathbf{Y} \frac{1}{2\pi} d\theta = -\log(\pi e)^M, \quad 0 \leq i \leq M. \quad (C.3)$$

The capacity expression in (45) is then obtained by first integrating

$$\int p_{Y|X=0,\theta} \log p_{Y|X=0,\theta} d\mathbf{Y} \frac{1}{2\pi} d\theta, \quad \int p_{Y|X=1,\theta} \log p_{Y|X=1,\theta} d\mathbf{Y} \frac{1}{2\pi} d\theta, \quad (C.4)$$

with respect to  $\theta$  and then making a change of variables,  $R_j = |Y_j|^2$ .

## ACKNOWLEDGMENTS

This paper was prepared through collaborative participation in the Communications and Networks Consortium sponsored by the US Army Research Laboratory under the Collaborative Technology Alliance Program, Cooperative Agreement DAAD19-01-2-0011. The material in this paper was presented in part at the IEEE Vehicular Technology Conference (VTC), Los Angeles, Calif, September 26–29 2004, and the IEEE Global Telecommunications Conference (GlobeCom), Dallas, Tex, 29 November–3 December 2004.

## DISCLAIMER

The US Government is authorized to reproduce and distribute reprints for Government purposes notwithstanding any copyright notation thereon.

## REFERENCES

- [1] I. F. Akyildiz, W. Su, Y. Sankarasubramaniam, and E. Cayirci, "A survey on sensor networks," *IEEE Communications Magazine*, vol. 40, no. 8, pp. 102–114, 2002.
- [2] C. E. Shannon, "A mathematical theory of communication," *Bell System Technical Journal*, vol. 27, pp. 379–423, 623–656, 1948.
- [3] M. J. E. Golay, "Note on the theoretical efficiency of information reception with PPM," *Proceedings of the IRE*, vol. 37, pp. 1031, 1949.
- [4] G. L. Turin, "The asymptotic behavior of ideal  $M$ -ary systems," *Proceedings of the IRE*, vol. 47, pp. 93–94, 1959.
- [5] I. Jacobs, "The asymptotic behavior of incoherent  $M$ -ary communications systems," *Proceedings of the IEEE*, vol. 51, no. 1, pp. 251–252, 1963.
- [6] J. R. Pierce, "Ultimate performance of  $M$ -ary transmissions on fading channels," *IEEE Transactions on Information Theory*, vol. 12, no. 1, pp. 2–5, 1966.
- [7] R. G. Gallager, *Information Theory and Reliable Communication*, John Wiley & Sons, New York, NY, USA, 1968.
- [8] R. S. Kennedy, *Fading Dispersive Communication Channels*, John Wiley & Sons, New York, NY, USA, 1969.
- [9] I. E. Telatar and D. N. C. Tse, "Capacity and mutual information of wideband multipath fading channels," *IEEE Transactions on Information Theory*, vol. 46, no. 4, pp. 1384–1400, 2000.
- [10] C. Luo and M. Médard, "Frequency-shift keying for ultrawideband—achieving rates of the order of capacity," in *Proc. 40th Annual Allerton Conference on Communication, Control, and Computing*, Monticello, Ill, USA, October 2002.
- [11] S. Verdú, "Spectral efficiency in the wideband regime," *IEEE Transactions on Information Theory*, vol. 48, no. 6, pp. 1319–1343, 2002.
- [12] M. Médard and R. G. Gallager, "Bandwidth scaling for fading multipath channels," *IEEE Transactions on Information Theory*, vol. 48, no. 4, pp. 840–852, 2002.
- [13] M. C. Gursoy, H. V. Poor, and S. Verdú, "The noncoherent Rician fading channel—Part II: Spectral efficiency in the low-power regime," *IEEE Transactions on Wireless Communications*, vol. 4, no. 5, pp. 2207–2221, 2005.
- [14] D. S. Lun, M. Médard, and I. C. Abou-Faycal, "On the performance of peaky capacity-achieving signaling on multipath fading channels," *IEEE Transactions on Communications*, vol. 52, no. 6, pp. 931–938, 2004.
- [15] C. Luo, M. Médard, and L. Zheng, "Error exponents for multi-tone frequency shift keying on wideband Rayleigh fading channels," in *Proc. IEEE Global Telecommunications Conference (GLOBECOM '03)*, vol. 2, pp. 779–783, San Francisco, Calif, USA, December 2003.
- [16] S. A. Butman, I. Bar-David, B. K. Levitt, R. F. Lyon, and M. J. Klass, "Design criteria for noncoherent Gaussian channels with MFSK signaling and coding," *IEEE Transactions on Communications*, vol. 24, no. 10, pp. 1078–1088, 1976.
- [17] W. E. Stark, "Capacity and cutoff rate of noncoherent FSK with nonselective Rician fading," *IEEE Transactions on Communications*, vol. 33, no. 11, pp. 1153–1159, 1985.
- [18] F. D. Neeser and J. L. Massey, "Proper complex random processes with applications to information theory," *IEEE Transactions on Information Theory*, vol. 39, no. 4, pp. 1293–1302, 1993.
- [19] I. C. Abou-Faycal, M. Médard, and U. Madhow, "Binary adaptive coded pilot symbol assisted modulation over Rayleigh fading channels without feedback," *IEEE Transactions on Communications*, vol. 53, no. 6, pp. 1036–1046, 2005.
- [20] R. Padovani and J. K. Wolf, "Coded phase/frequency modulation," *IEEE Transactions on Communications*, vol. 34, no. 5, pp. 446–453, 1986.
- [21] I. Ghareeb and A. Yongacoglu, "Performance of joint frequency phase modulation over Rayleigh fading channels," *IEE Proceedings. I, Communications, Speech and Vision*, vol. 141, no. 4, pp. 241–247, 1994.
- [22] R. A. Khalona, G. E. Atkin, and J. L. LoCicero, "On the performance of a hybrid frequency and phase shift keying modulation technique," *IEEE Transactions on Communications*, vol. 41, no. 5, pp. 655–659, 1993.
- [23] F.-C. Hung, C.-D. Chung, and Y.-L. Chao, "Coherent frequency/phase modulation scheme," *IEE Proceedings. I, Communications, Speech and Vision*, vol. 149, no. 1, pp. 36–44, 2002.

- [24] A. J. Viterbi, "Performance of an  $M$ -ary orthogonal communication system using stationary stochastic signals," *IEEE Transactions on Information Theory*, vol. 13, no. 3, pp. 414–422, 1967.
- [25] S. A. Butman and M. J. Klass, "Capacity of noncoherent channels," Tech. Rep. 32-1526, Jet Propulsion Laboratory, Pasadena, Calif, USA, vol. 18, pp. 85–93, September 1973.
- [26] G. R. Grimmett and D. R. Stirzaker, *Probability and Random Processes*, Oxford University Press, New York, NY, USA, 1998.

**Mustafa Cenk Gursoy** received the B.S. degree in electrical and electronic engineering from Bogazici University, Turkey, in 1999, and the Ph.D. degree in electrical engineering from Princeton University, Princeton, NJ, USA, in 2004. In the summer of 2000, he worked at Lucent Technologies, Holmdel, NJ, where he conducted performance analysis of DSL modems. Since September 2004, he has been an Assistant Professor in the Department of Electrical Engineering at the University of Nebraska-Lincoln. His research interests are in the general areas of wireless communications, information theory, communication networks, and signal processing. Dr. Gursoy is a recipient of Gordon Wu Graduate Fellowship from Princeton University.



**H. Vincent Poor** received the Ph.D. degree in electrical engineering and computer science from Princeton University in 1977. From 1977 until 1990, he was on the faculty of the University of Illinois at Urbana-Champaign. Since 1990, he has been on the faculty at Princeton, where he is the Michael Henry Strater University Professor of Electrical Engineering. He has also held visiting appointments at a number of universities, including recently Imperial College, Stanford, and Harvard. Dr. Poor's research interests are in the areas of advanced signal processing, wireless networks, and related fields. Among his publications in these areas is the recent book *Wireless Networks: Multiuser Detection in Cross-Layer Design* (Springer, 2005). Dr. Poor is a Member of the National Academy of Engineering and the American Academy of Arts & Sciences, and is a Fellow of the IEEE, the Institute of Mathematical Statistics, the Optical Society of America, and other organizations. He is a past President of the IEEE Information Theory Society, and is the current Editor-in-Chief of the *IEEE Transactions on Information Theory*. Recent recognition of his work includes the Joint Paper Award of the IEEE Communications and Information Theory Societies (2001), the NSF Director's Award for Distinguished Teaching Scholars (2002), a Guggenheim Fellowship (2002-2003), and the IEEE Education Medal (2005).



**Sergio Verdú** received the Telecommunications Engineering degree from the Polytechnic University of Catalonia, Barcelona, Spain, in 1980, and the Ph.D. degree in electrical engineering from the University of Illinois at Urbana-Champaign in 1984. He is a Professor of electrical engineering at Princeton University, Princeton, NJ. Professor Verdú is a recipient of several paper awards, including the IEEE Donald Fink Paper Award, a Golden Jubilee Paper Award from the Information



Theory Society, the 1998 Information Theory Society Paper Award, and the 2002 Leonard G. Abraham Prize Award from the IEEE Communications Society. He also received a Millennium Medal from the IEEE and the 2000 Frederick E. Terman Award from the American Society for Engineering Education. In 2005, he received a Doctorate Honoris Causa from the Polytechnic University of Catalonia. Professor Verdú served as an Associate Editor for Shannon Theory of the *IEEE Transactions on Information Theory*. In 1997, he served as President of the IEEE Information Theory Society. He is currently Editor-in-Chief of Foundations and Trends in Communications and Information Theory. He authored the text *Multiuser Detection* published by Cambridge University Press in 1998, and served as Guest Editor of the 1998 Special Commemorative Issue of the IEEE Transactions on Information Theory, reprinted as the IEEE Press volume "*Information Theory: Fifty years of discovery.*"