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Slotted Aloha with multi-AP diversity and APS transmit beamforming

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Abstract

Slotted Aloha is an effective random access protocol and can also be an important element of more advanced media access protocols. This paper investigates slotted Aloha in a radio environment with multiple access points. Specifically, we examine the impact of multi-access-point (multi-AP) diversity on the performance of slotted Aloha. The paper considers both omni-directional (OM) and beamforming (BF) antennas at transmission nodes. This leads to the investigation and comparison of four different network scenarios, i.e., OM with multi-AP diversity, OM without multi-AP diversity, BF with multi-AP diversity and BF without multi-AP diversity. Performance evaluations and comparisons are presented in terms of throughput and average packet delay.

Keywords: Slotted Aloha, Multi-access-point diversity, Beamforming, Capture effect, Rayleigh fading, Throughput, Average packet delay

I. Introduction

Slotted Aloha has been extensively used in wireless environments [1-4], in which the power levels of received packets can be different due to independent fading. It is possible that the strongest packet captures the receiver even when there is a packet collision [5], which could increase throughput. This phenomenon is referred to as the capture effect. A lot of research have been conducted for the investigations of the capture effect under various fading channels, including Rayleigh, Rician and Nakagami [6-8].

Besides the capture effect, beamforming (BF) techniques can also potentially increase throughput since they are able to reduce collisions in slotted Aloha as compared to omni-directional (OM) antennas. The applications of BF at both receiving and transmitting sides have been investigated. It is shown that a single-beam adaptive array at the receiver improves the performance of a slotted Aloha network by creating a strong capture effect [9] and a multiple receiving beam adaptive array can successfully receive two or more overlapping packets at the same time [10]. Slotted Aloha using transmit BF at mobile entities in mobile ad hoc networks has also been studied [11].

Notice that there can be two types of interference in slotted Aloha in a cellular environment, multiple access interference and cochannel interference. For a given user, multiple access interference is due to users within the same cell and cochannel interference is due to users in cochannel cells. The performance of slotted Aloha in Nakagami fading channels considering both synchronized and asynchronous cochannel cells is analyzed in [12], highlighting the differences between these two types of interference. While all cochannel interfering packets are discarded in [12], a model, in which multiple base stations are able to accept a packet from the same user as long as it captures the receivers, is studied in [13] through simulations. Clearly, such a scheme potentially improves the throughput of slotted Aloha as compared to the approach in [12].

The model in [13] is a type of multi-access-point (multi-AP) diversity, a concept also addressed in [14] which considers downlinks in cellular communications. It is pointed out that a user can simultaneously receive pilot channels from multiple base stations, which introduces multi-AP diversity due to independent channel variations between the user and the base stations [14]. Therefore, a user could choose one base station among a set of base stations as its server according to channel conditions. Similarly, a multi-AP

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architecture has been proposed for wireless local area networks, in which one user can associate with more than one access point [15].

This paper investigates slotted Aloha with multi-AP diversity and it differs from previous research in the following aspects. Firstly, we develop analytical models and derive closed-form solutions for the throughput and average packet delay. Secondly, we investigate the joint use of transmit BF and multi-AP diversity. We thus specifically study four network scenarios, i.e., OM with multi-AP diversity, OM without multi-AP diversity, BF with multi-AP diversity and BF without multi-AP diversity, to exam and compare various technical options.

The rest of this paper is organized as follows. Section 2 gives the system model of slotted Aloha with multi-AP diversity, including two cases in which OM and directional antennas are applied, respectively. Sections 3 and 4 analyze these two cases and derive the capture probabilities, throughput and average packet delay. In Section 5, numerical results are presented and, finally, Section 6 draws conclusions.

II. System Model

A. Network model

We consider a network with two access points (AP) A and B (two servers) (Figure 1) placed to cover a given area. Around AP A, there are a set of N_A users (*User Set A*), and around AP B, there are a set of N_B users (*User Set B*). A user $u_i (1 \leq i \leq N_A)$ in *User Set A* transmits its packet to AP A and/or AP B depending on its antenna structures (OM or BF). Similarly, a user $v_j (1 \leq j \leq N_B)$ in *User Set B* transmits its packet to AP B and/or AP A.

We apply a traffic and retransmission model as in [16]. If no packet retransmission is needed, each user generates a new packet with a probability σ and no packet with a probability $1 - \sigma$ during each time slot. Once a user generates a packet, it transmits the packet immediately. If the packet transmission fails, it will be retransmitted in each of the following slots with a probability σ until it is successfully transmitted. When a user needs to perform packet retransmissions, it does not generate any new packet.

B. Signal capture model

A transmission collision in fading channels does not always result in transmission failures of all packets due to the capture effect, in which a packet may capture a receiver if its power level is higher than the sum of powers of all interfering packets [17,18]. The capture probability, P_{cap}, can thus be calculated by

$$P_{cap}(I, J) = \Pr \left(\frac{x}{\sum_{i=1}^{I} y_i + \sum_{j=1}^{J} z_j} > R \right) \qquad (1)$$

for $R \geq 1$, $I \geq 0$, $J \geq 0$, where x is the power of the desired packet; R is a capture ratio; I and J are the total numbers of interfering packets from the same user set as the desired packet and from the other user set, respectively. y_i and z_j indicate the powers of interfering packets from the two user sets. In a Rayleigh fading channel, x, y_i, z_j follow exponential distributions [17,19]. There are two scenarios in determining the mean powers of x, y_i, and z_j. When the desired packet is transmitted from *User Set A* (or B) to AP A (or B), the mean powers are assumed to be X, Y and Z. When the desired packet is transmitted from *User Set A* (or B) to AP B (or A), the mean powers are assumed to be $\mathcal{X}$, $\mathcal{Y}$ and Z. Notice that the mean powers X, Y and Z relate to packet transmissions (desired or interfering) from *User Set A* (or B) to AP A (or B). The mean powers $\mathcal{X}$, $\mathcal{Y}$ and $\mathcal{Z}$ relate to packet transmissions (desired or interfering) from *User Set A* (or B) to AP B (or A). Figure 2 illustrates the packet transmissions and the notations of signal and interference powers and their mean powers. We assume that the mean powers satisfy
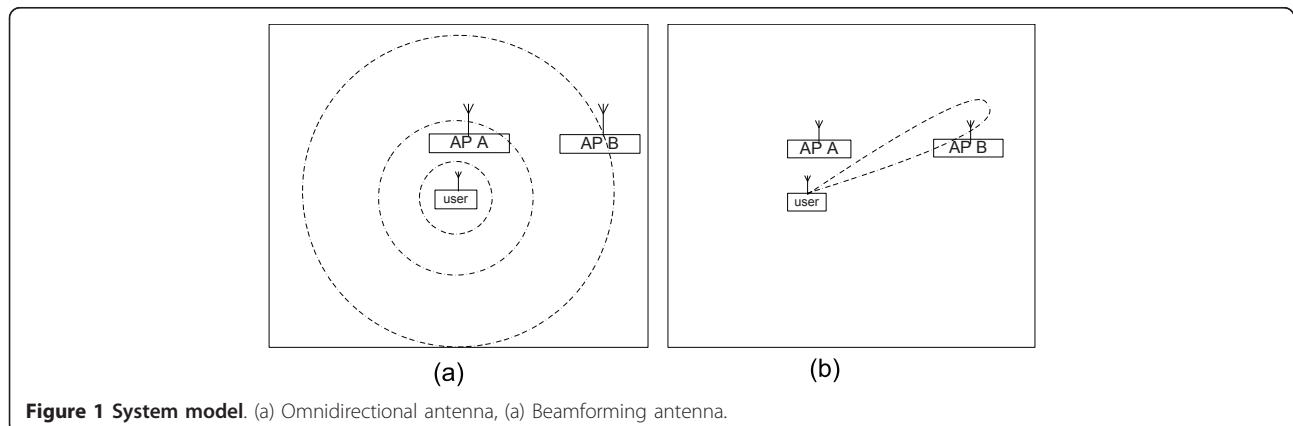
$$X = Y = Z \qquad (2)$$

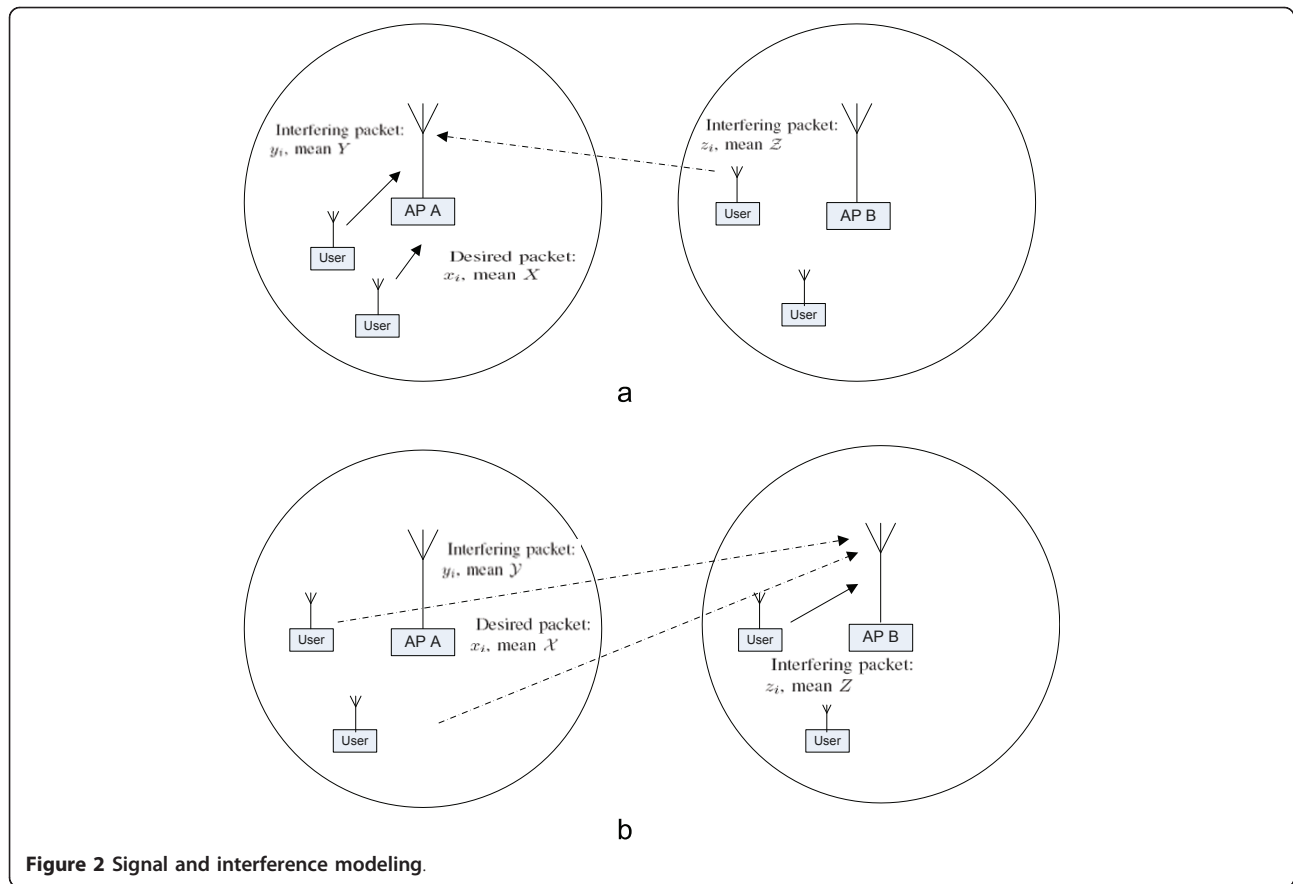**Figure 1 System model**. (a) Omnidirectional antenna, (a) Beamforming antenna.

Figure 2 Signal and interference modeling.

and

$$\mathcal{X} = \mathcal{Y} = \mathcal{Z} \qquad (3)$$

We further define

$$\gamma = \frac{\mathcal{X}}{X} = \frac{\mathcal{Y}}{Y} = \frac{\mathcal{Z}}{Z} \qquad (4)$$

Notice that the signal and capture model consider a Rayleigh fading channel environment. There are several capture models which have been investigated in literatures [17-20]. This paper only considers one model as defined in Equation 1. Near-far effects [19,20] due to user spatial distributions are not considered in this model and the combined effect of Rayleigh fading and user spatial distributions will be investigated in our future research.

C. Multi-AP diversity
Multi-AP diversity, in which one user can be associated with more than one access point (e.g., base stations in cellular networks or hot spots in wireless local area networks), is investigated in [14,15]. In the network model we defined above, each user could potentially transmit a packet through two independent channels to two APs. Therefore, there is multi-AP diversity in the system to

potentially provide diversity gains. The following explains how the diversity is exploited when OM or BF antennas are applied at the transmit side.

D. OM versus BF antennas
When users employ OM transmit antennas, any packet transmitted by any user can potentially reach both APs (see Figure 1a). Therefore, a packet has to compete with other packets from all users (*User Set A* and *User Set B*) in order to capture a receiver. If transmit BF is used, each user can choose one AP as its server where its packet will have stronger power as compared to that at the other AP. Such an AP selection task can be accomplished based on feedback information or pilot signals. The user steers its beam towards only the chosen AP. Therefore, under the BF antenna mode, any packet can only reach one AP (see Figure 1b). And this leads to potentially less interference.

III. Slotted Aloha with Multi-AP Diversity and OM Antenna
A. Capture probability
Considering the transmission of a desired packet from *User Set A* to AP *A*, following the definition in Section

2.A, we find its capture probability as follows,

$$P_{\text{cap}S_A \to A}(I, J) = \Pr\left(\frac{x}{\sum_{i=1}^{I} y_i + \sum_{j=1}^{J} z_j} > R\right)$$

$$= \int_0^\infty \cdots \int_0^\infty \int_{R(\sum_{i=1}^{I} y_i + \sum_{j=1}^{J} z_j)}^{\infty} \frac{1}{X} e^{-\frac{x}{X}} dx \prod_{i=1}^{I} \frac{1}{Y} e^{-\frac{y_i}{Y}} \prod_{j=1}^{J} \frac{1}{Z} e^{-\frac{z_j}{Z}} dy_1 \cdots dy_I dz_1 \cdots dz_J \quad (5)$$

$$= \left(\frac{X}{RY+X}\right)^I \left(\frac{X}{RZ+X}\right)^J$$

Following (2)-(4), (5) can be rewritten as

$$P_{\text{cap}S_A \to A}(I, J) = \left(\frac{1}{R+1}\right)^I \left(\frac{1}{R\gamma+1}\right)^J \quad (6)$$

Similarly, considering other transmission scenarios, we are able to obtain the following capture probabilities (from *User Set A* to AP *B*, from *User Set B* to AP *B*, and from *User Set B* to AP *A*).

$$P_{\text{cap}S_A \to B}(I, J) = \left(\frac{1}{R+1}\right)^I \left(\frac{\gamma}{R+\gamma}\right)^J \quad (7)$$

$$P_{\text{cap}S_B \to B}(I, J) = \left(\frac{1}{R+1}\right)^I \left(\frac{1}{R\gamma+1}\right)^J \quad (8)$$

$$P_{\text{cap}S_B \to A}(I, J) = \left(\frac{1}{R+1}\right)^I \left(\frac{\gamma}{R+\gamma}\right)^J \quad (9)$$

B. Throughput

We consider the throughput per AP, S, which is defined as the total number of packets successfully received by the two APs during one time slot and divided by two. The following defines several events during a period of one time slot.

E: AP *A* successfully receives one packet and AP *B* successfully receives one packet and the packets are different.
F : AP *A* and AP *B* both successfully receive the same packet.
G: Only AP *A* successfully receives a packet.
H : Only AP *B* successfully receives a packet.
$T_{i, j}$: There are i users in *User Set A* and j users in *User Set B* attempting to transmit. If one packet is received successfully at both APs, it is only counted as one. The throughput is thus calculated as

$$S = 0.5 \times [2\Pr(E) + \Pr(F) + \Pr(G) + \Pr(H)]$$

$$= 0.5 \times \left\{\sum_{i=0}^{N_A} \sum_{j=0}^{N_B} \binom{N_A}{i} \sigma^i (1-\sigma)^{N_A-i} \binom{N_B}{j} \sigma^j (1-\sigma)^{N_B-j}\right. \quad (10)$$

$$\left.\times [2\Pr(E|T_{i,j}) + \Pr(F|T_{i,j}) + \Pr(G|T_{i,j}) + \Pr(H|T_{i,j})]\right\}$$

in which

$$\Pr(E|T_{i,j}) = \Pr(\text{AP } A \text{ successfully receives a packet}|T_{i,j})$$
$$\times \Pr(\text{AP } B \text{ successfully receives a packet}|T_{i,j})$$
$$- (\Pr(\text{A user in } User Set A \text{ successfully transmits a packet to AP } A \text{ and AP } B|T_{i,j}) \quad (11)$$
$$+ \Pr(\text{A user in } User Set B \text{ successfully transmits a packet to AP } A \text{ and AP } B|T_{i,j}))$$

where

$$\Pr(\text{AP } A \text{ successfully receives a packet}|T_{i,j})$$
$$= iP_{\text{cap}S_A \to A}(i-1, j) + jP_{\text{cap}S_B \to A}(j-1, i) \quad (12)$$

$$\Pr(\text{AP } B \text{ successfully receives a packet}|T_{i,j})$$
$$= iP_{\text{cap}S_A \to B}(i-1, j) + jP_{\text{cap}S_B \to B}(j-1, i) \quad (13)$$

$$\Pr(\text{A user in } User Set A \text{ successfully transmits a packet to AP } A \text{ and AP } B|T_{i,j})$$
$$= iP_{\text{cap}S_A \to A}(i-1, j)P_{\text{cap}S_A \to B}(i-1, j) \quad (14)$$

$$\Pr(\text{A user in } User Set B \text{ successfully transmits a packet to AP } A \text{ and AP } B|T_{i,j})$$
$$= jP_{\text{cap}S_B \to A}(j-1, i)P_{\text{cap}S_B \to B}(j-1, i) \quad (15)$$

Combining (6)-(9) and (11)-(15) we obtain

$$\Pr(E|T_{i,j}) = \left[i\left(\frac{1}{R+1}\right)^{i-1}\left(\frac{1}{R\gamma+1}\right)^j + j\left(\frac{1}{R+1}\right)^{j-1}\left(\frac{\gamma}{R+\gamma}\right)^i\right]$$

$$\times \left[i\left(\frac{1}{R+1}\right)^{i-1}\left(\frac{\gamma}{R+\gamma}\right)^j + j\left(\frac{1}{R+1}\right)^{j-1}\left(\frac{1}{R\gamma+1}\right)^i\right]$$

$$- \left[i\left(\frac{1}{R+1}\right)^{i-1}\left(\frac{1}{R\gamma+1}\right)^j\left(\frac{1}{R+1}\right)^{i-1}\left(\frac{\gamma}{R+\gamma}\right)^j\right. \quad (16)$$

$$\left.+ j\left(\frac{1}{R+1}\right)^{j-1}\left(\frac{\gamma}{R+\gamma}\right)^i\left(\frac{1}{R+1}\right)^{j-1}\left(\frac{1}{R\gamma+1}\right)^i\right]$$

Considering $\Pr(F|T_{i, j})$ in (10), we have

$$\Pr(F|T_{i,j})$$
$$= \Pr(\text{A user in } User Set A \text{ successfully transmits a packet to AP } A \text{ and AP } B|T_{ij}) \quad (17)$$
$$+ \Pr(\text{A user in } User Set B \text{ successfully transmits a packet to AP } A \text{ and AP } B|T_{ij})$$

After combining (6)-(9), (14), (15) and (17), we obtain

$$\Pr(F|T_{i,j}) = \left[i\left(\frac{1}{R+1}\right)^{i-1}\left(\frac{1}{R\gamma+1}\right)^j\left(\frac{1}{R+1}\right)^{i-1}\left(\frac{\gamma}{R+\gamma}\right)^j\right.$$

$$\left.+ j\left(\frac{1}{R+1}\right)^{j-1}\left(\frac{\gamma}{R+\gamma}\right)^i\left(\frac{1}{R+1}\right)^{j-1}\left(\frac{1}{R\gamma+1}\right)^i\right] \quad (18)$$

We also have

$$\Pr(G|T_{i,j}) = \Pr(\text{AP } A \text{ successfully receives a packet}|T_{i,j})$$
$$\times (1 - \Pr(\text{AP } B \text{ successfully receives a packet}|T_{i,j})) \quad (19)$$

After combining (6)-(9), (12), (13) and (19), we obtain

$$\Pr(G|T_{i,j}) = \left[i\left(\frac{1}{R+1}\right)^{i-1}\left(\frac{1}{R\gamma+1}\right)^j + j\left(\frac{1}{R+1}\right)^{j-1}\left(\frac{\gamma}{R+\gamma}\right)^i\right]$$

$$\times \left[1 - i\left(\frac{1}{R+1}\right)^{i-1}\left(\frac{\gamma}{R+\gamma}\right)^j - j\left(\frac{1}{R+1}\right)^{j-1}\left(\frac{1}{R\gamma+1}\right)^i\right] \quad (20)$$

Similarly, we are able to obtain

$$\Pr(H|T_{i,j}) = \left[1 - i\left(\frac{1}{R+1}\right)^{i-1}\left(\frac{1}{R\gamma+1}\right)^j - j\left(\frac{1}{R+1}\right)^{j-1}\left(\frac{\gamma}{R+\gamma}\right)^i\right]$$

$$\times \left[i\left(\frac{1}{R+1}\right)^{i-1}\left(\frac{\gamma}{R+\gamma}\right)^j + j\left(\frac{1}{R+1}\right)^{j-1}\left(\frac{1}{R\gamma+1}\right)^i\right] \quad (21)$$

Finally, the average throughput per access point, S, can be obtained by inserting (16), (18), (20) and (21) into (10).

C. Delay

One method to quantify the delay characteristics is to examine the average number of transmission attempts for each successful transmission, which is defined as A_{avg}. We define p as the probability of a successful reception of a packet when it is transmitted. We have

$$A_{\text{avg}} = \frac{1}{p} \tag{22}$$

Let the probability that a user successfully transmits a packet after it is generated is p_A or p_B when this packet is in *User Set A* or *User Set B*. We have

$$p = p_A \frac{N_A}{N_A + N_B} + p_B \frac{N_B}{N_A + N_B} \tag{23}$$

in which

$$
\begin{aligned}
p_A = \sum_{i=0}^{N_A-1} \sum_{j=0}^{N_B} \binom{N_A-1}{i} \sigma^i (1-\sigma)^{N_A-1-i} \binom{N_B}{j} \sigma^j (1-\sigma)^{N_B-j} \\
[\Pr(\text{The concerned packet is successfully transmitted to both AP A and AP B}|T_{i+1,j}) \\
+ \Pr(\text{The concerned packet is successfully transmitted to AP A only }|T_{i+1,j}) \\
+ \Pr(\text{The concerned packet is successfully transmitted to AP B only }|T_{i+1,j})] \\
= \sum_{i=0}^{N_A-1} \sum_{j=0}^{N_B} \binom{N_A-1}{i} \sigma^i (1-\sigma)^{N_A-1-i} \binom{N_B}{j} \sigma^j (1-\sigma)^{N_B-j} \\
\times \{P_{\text{capS}_A \to A}(i,j) P_{\text{capS}_A \to B}(i,j) + P_{\text{capS}_A \to A}(i,j) \left[1 - P_{\text{capS}_A \to B}(i,j)\right] \\
+ P_{\text{capS}_A \to B}(i,j) \left[1 - P_{\text{capS}_A \to A}(i,j)\right]\}
\end{aligned}
\tag{24}
$$

Inserting (6) and (7) into (24), we obtain

$$
\begin{aligned}
p_A = \sum_{i=0}^{N_A-1} \sum_{j=0}^{N_B} \binom{N_A-1}{i} \sigma^i (1-\sigma)^{N_A-1-i} \binom{N_B}{j} \sigma^j (1-\sigma)^{N_B-j} \\
\times \left\{ \left(\frac{1}{R+1}\right)^i \left(\frac{1}{R\gamma+1}\right)^j \left(\frac{1}{R+1}\right)^i \left(\frac{\gamma}{R+\gamma}\right)^j \right. \\
+ \left(\frac{1}{R+1}\right)^i \left(\frac{1}{R\gamma+1}\right)^j \left[1 - \left(\frac{1}{R+1}\right)^i \left(\frac{\gamma}{R+\gamma}\right)^j\right] \\
\left. + \left(\frac{1}{R+1}\right)^i \left(\frac{\gamma}{R+\gamma}\right)^j \left[1 - \left(\frac{1}{R+1}\right)^i \left(\frac{1}{R\gamma+1}\right)^j\right] \right\}
\end{aligned}
\tag{25}
$$

Similarly, we are able to find

$$
\begin{aligned}
p_B = \sum_{i=0}^{N_B-1} \sum_{j=0}^{N_A} \binom{N_B-1}{i} \sigma^i (1-\sigma)^{N_B-1-i} \binom{N_A}{j} \sigma^j (1-\sigma)^{N_A-j} \\
\times \left\{ \left(\frac{1}{R+1}\right)^i \left(\frac{1}{R\gamma+1}\right)^j \left(\frac{1}{R+1}\right)^i \left(\frac{\gamma}{R+\gamma}\right)^j \right. \\
+ \left(\frac{1}{R+1}\right)^i \left(\frac{1}{R\gamma+1}\right)^j \left[1 - \left(\frac{1}{R+1}\right)^i \left(\frac{\gamma}{R+\gamma}\right)^j\right] \\
\left. + \left(\frac{1}{R+1}\right)^i \left(\frac{\gamma}{R+\gamma}\right)^j \left[1 - \left(\frac{1}{R+1}\right)^i \left(\frac{1}{R\gamma+1}\right)^j\right] \right\}
\end{aligned}
\tag{26}
$$

Combining (22), (23), (25) and (26), the average number of transmission attempts is obtained.

D. Special case comparison: no multi-AP diversity

The following gives the performance results of slotted Aloha without multi-AP diversity in an OM transmit scenario. Following [12] and based on the derivations in Section 3.B, we are able to obtain the throughput as

$$
\begin{aligned}
S = 0.5 \times \left[\sum_{i=0}^{N_A} \sum_{j=0}^{N_B} \binom{N_A}{i} \sigma^i (1-\sigma)^{N_A-i} \binom{N_B}{j} \sigma^j (1-\sigma)^{N_B-j} i \left(\frac{1}{R+1}\right)^{i-1} \left(\frac{1}{R\gamma+1}\right)^j \right. \\
\left. + \sum_{i=0}^{N_B} \sum_{j=0}^{N_A} \binom{N_B}{i} \sigma^i (1-\sigma)^{N_B-i} \binom{N_A}{j} \sigma^j (1-\sigma)^{N_A-j} i \left(\frac{1}{R+1}\right)^{i-1} \left(\frac{1}{R\gamma+1}\right)^j \right]
\end{aligned}
\tag{27}
$$

The average number of transmission attempts expressed in (22) and (23) still applies with p_A and p_B as follows,

$$
p_A = \sum_{i=0}^{N_A-1} \sum_{j=0}^{N_B} \binom{N_A-1}{i} \sigma^i (1-\sigma)^{N_A-1-i} \binom{N_B}{j} \sigma^j (1-\sigma)^{N_B-j} \left(\frac{1}{R+1}\right)^i \left(\frac{1}{R\gamma+1}\right)^j \tag{28}
$$

$$
p_B = \sum_{i=0}^{N_B-1} \sum_{j=0}^{N_A} \binom{N_B-1}{i} \sigma^i (1-\sigma)^{N_B-1-i} \binom{N_A}{j} \sigma^j (1-\sigma)^{N_A-j} \left(\frac{1}{R+1}\right)^i \left(\frac{1}{R\gamma+1}\right)^j \tag{29}
$$

IV. Slotted Aloha with Multi-AP Diversity and BF Antenna

A. Capture probability

In order to investigate the capture effect in this multi-AP diversity and BF scenario, we define a function

$$f(I, J, \Gamma) = \Pr\left(\frac{x}{\sum_{i=1}^{I} y_i + \sum_{j=1}^{J} z_j} > R, x > \tilde{x}, \, y_i > \tilde{y}_i, \, z_j > \tilde{z}_j \right) \tag{30}$$

where x, y_i, and z_j are the received power of the desired packet, the received power of interfering packets from the same user set as the desired packet, and the received power of interfering packets from the different user set as the desired packet, and respectively, for a target AP; $\tilde{x}$ is the received power of the desired packet if the desired packet is received at the AP other than the target AP. $\tilde{y}_i$ and $\tilde{z}_j$ are similarly defined. We let

$$\frac{E[\tilde{x}]}{E[x]} = \frac{E[\tilde{y}_i]}{E[y_i]} = \frac{E[z_j]}{E[\tilde{z}_j]} = \Gamma \tag{31}$$

For examples, $f(m-1, n, \gamma)$ denotes the probability that for a given AP (say AP A), m transmitting users of user set A and n transmitting users of user set B choose AP A and one of the m users successfully captures AP A; $f(m-1, n, \frac{1}{\gamma})$ denotes the probability that for a given AP (say AP A), m transmitting users of user set B and n transmitting users of user set A choose AP A and one of the m users successfully captures AP A. The following equation derives this function.

$$
\begin{aligned}
& f(I, J, \Gamma) \\
&= \prod_{i=1}^{I} \int_{\tilde{y}_i}^{\infty} \frac{1}{\mu} e^{-\frac{y_i}{\mu}} dy_i \prod_{j=1}^{J} \int_{\tilde{z}_j}^{\infty} \frac{1}{\nu} e^{-\frac{z_j}{\nu}} dz_j \prod_{i=1}^{I} \int_{0}^{\infty} \frac{1}{\nu} e^{-\frac{\tilde{y}_i}{\nu}} d\tilde{y}_i \prod_{j=1}^{J} \int_{0}^{\infty} \frac{1}{Z_2} e^{-\frac{\tilde{z}_j}{Z_2}} d\tilde{z}_j \\
& \times \left[\int_{R(\sum_{i=1}^{I} y_i + \sum_{j=1}^{J} z_j)}^{\infty} \frac{1}{\mu} e^{-\frac{x}{\mu}} dx \int_{0}^{R(\sum_{i=1}^{I} y_i + \sum_{j=1}^{J} z_j)} \frac{1}{\nu} e^{-\frac{\tilde{x}}{\nu}} d\tilde{x} + \int_{\tilde{x}}^{\infty} \frac{1}{\mu} e^{-\frac{x}{\mu}} dx \right. \\
& \left. \int_{R(\sum_{i=1}^{I} x_i + \sum_{j=1}^{J} y_j)}^{\infty} \frac{1}{\nu} e^{-\frac{\tilde{x}}{\nu}} d\tilde{x} \right] \\
&= \frac{1}{[(1+R)(1+\Gamma+R\Gamma)]^I} \frac{1}{[(1+R\Gamma)(1+\frac{1}{\Gamma}+R)]^J} - \frac{1}{\frac{1}{\Gamma}+1} \\
& \times \frac{1}{\{[1+R(1+\frac{1}{\Gamma})][1+\Gamma+R(\Gamma+1)]\}^I} \frac{1}{\{[1+R(\Gamma+1)][1+\frac{1}{\Gamma}+R(1+\frac{1}{\Gamma})]\}^J}
\end{aligned}
\tag{32}
$$

B. Throughput

To calculate the average throughput per access point in the BF cases, we can still use the modeling approach based on the event T_{ij} as defined in Section 3.B. Furthermore, a new event $Q_{m,\,n}$ is defined below.

$Q_{m,\,n}$: m transmitting users in *User Set A* choose AP A and n transmitting users in *User Set B* choose AP A as their server.

The throughput of AP A, S_a, can be calculated as follows.

$$
\begin{aligned}
S_a &= \sum_{i=0}^{N_A}\sum_{j=0}^{N_B}\binom{N_A}{i}\sigma^i(1-\sigma)^{N_A-i}\binom{N_B}{j}\sigma^j(1-\sigma)^{N_B-j}\Pr(\text{AP } A \text{ successfully}\\
&\quad \text{receives a packet}|T_{i,j})\\
&= \sum_{i=0}^{N_A}\sum_{j=0}^{N_B}\binom{N_A}{i}\sigma^i(1-\sigma)^{N_A-i}\binom{N_B}{j}\sigma^j(1-\sigma)^{N_B-j}\sum_{m=0}^{i}\sum_{n=0}^{j}\Pr(Q_{m,n}|T_{i,j})\\
&\quad \times \Pr(\text{AP } A \text{ successfully receives a packet}|T_{i,j}Q_{m,n})
\end{aligned}
\tag{33}
$$

Expanding the conditional probability $\Pr(Q_{m,\,n}|T_{i,\,j})$, the throughput of AP A is expressed as

$$
\begin{aligned}
S_a &= \sum_{i=0}^{N_A}\sum_{j=0}^{N_B}\binom{N_A}{i}\sigma^i(1-\sigma)^{N_A-i}\binom{N_B}{j}\sigma^j(1-\sigma)^{N_B-j}\sum_{m=0}^{i}\sum_{n=0}^{j}\binom{i}{m}\binom{j}{n}\\
&\quad \times (\Pr(\text{a transmitting user in } User\ Set\ A \text{ chooses AP } A))^m\\
&\quad \times (\Pr(\text{a transmitting user in } User\ Set\ B \text{ chooses AP } A))^n\\
&\quad \times (1-\Pr(\text{a transmitting user in } User\ Set\ A \text{ chooses AP } A))^{i-m}\\
&\quad \times (1-\Pr(\text{a transmitting user in } User\ Set\ B \text{ chooses AP } A))^{j-n}\\
&\quad \times \Pr(\text{AP } A \text{ successfully receives one packet}|T_{i,j}Q_{m,n})
\end{aligned}
\tag{34}
$$

Notice

$$
\begin{aligned}
&\Pr(\text{AP } A \text{ successfully receives one packet}|T_{i,j}Q_{m,n})\\
&= \Pr(\text{AP } A \text{ successfully receives one packet from } User\ Set\ A|T_{i,j}Q_{m,n})\\
&\quad + \Pr(\text{AP } A \text{ successfully receives one packet from } User\ Set\ B|T_{i,j}Q_{m,n})
\end{aligned}
\tag{35}
$$

and

$$
\begin{aligned}
&(\Pr(\text{a transmitting users in } User\ Set\ A \text{ chooses AP } A))^m\\
&\quad \times (\Pr(\text{a transmitting users in } User\ Set\ B \text{ chooses AP } A))^n\\
&\quad \times \Pr(\text{AP } A \text{ successfully receives one packet from } User\ Set\ A|T_{i,j}Q_{m,n})\\
&= m\Pr\left(\frac{x}{\sum_{i=1}^{m-1}y_i+\sum_{j=1}^{n}z_j}>R|x>\tilde{x},\ y_i>\tilde{y}_i,\ z_j>\tilde{z}_j\right)\Pr(x>\tilde{x},y_i>\tilde{y}_i,\ z_j>\tilde{z}_j)\\
&= mf(m-1,\ n,\ \gamma)
\end{aligned}
\tag{36}
$$

Similarly, we are able to obtain

$$
\begin{aligned}
&(\Pr(\text{a transmitting users in } User\ Set\ A \text{ chooses AP } A))^m\\
&\quad \times (\Pr(\text{a transmitting users in } User\ Set\ B \text{ chooses AP } A))^n\\
&\quad \times \Pr(\text{AP } A \text{ successfully receives one packet from } User\ Set\ B|T_{i,j}Q_{m,n})\\
&= nf\left(n-1,\ m,\ \frac{1}{\gamma}\right)
\end{aligned}
\tag{37}
$$

Inserting (34)-(36) into (33), we obtain

$$
\begin{aligned}
S_a &= \sum_{i=0}^{N_A}\sum_{j=0}^{N_B}\binom{N_A}{i}\sigma^i(1-\sigma)^{N_A-i}\binom{N_B}{j}\sigma^j(1-\sigma)^{N_B-j}\\
&\quad \times \sum_{m=0}^{i}\sum_{n=0}^{j}\binom{i}{m}\binom{j}{n}\left(mf(m-1,\ n,\ \gamma)+nf\left(n-1,\ m,\ \frac{1}{\gamma}\right)\right)\\
&\quad \times (1-\Pr(\text{a transmitting users in } User\ Set\ A \text{ chooses AP } A))^{i-m}\\
&\quad \times (1-\Pr(\text{a transmitting users in } User\ Set\ B \text{ chooses AP } A))^{j-n}
\end{aligned}
\tag{38}
$$

Following the derivations in (5), we get

$$
\Pr(\text{a transmitting user in } User\ Set\ A \text{ chooses AP } A) = \frac{X}{\mathcal{X}+X}
\tag{39}
$$

and

$$
\Pr(\text{a transmitting user in } User\ Set\ B \text{ chooses AP } A) = \frac{\mathcal{X}}{X+\mathcal{X}}
\tag{40}
$$

Using (2)-(4) and inserting (31), (38), (39) into (37), we obtain

$$
\begin{aligned}
S_a &= \sum_{i=0}^{N_A}\sum_{j=0}^{N_B}\binom{N_A}{i}\sigma^i(1-\sigma)^{N_A-i}\binom{N_B}{j}\sigma^j(1-\sigma)^{N_B-j}\\
&\quad \times \sum_{m=0}^{i}\sum_{n=0}^{j}\binom{i}{m}\binom{j}{n}\Bigg\{\frac{m}{[(1+R)(1+\gamma+R\gamma)]^{m-1}[(1+R\gamma)(1+\frac{1}{\gamma}+R)]^n}\\
&\quad -\frac{m\gamma}{(1+\gamma)[(R+1)(1+R(1+\frac{1}{\gamma}))(1+\gamma)]^{m-1}[(R+1)(1+R(1+\gamma))(1+\frac{1}{\gamma})]^n}\\
&\quad +\frac{n}{[(1+R)(1+\frac{1}{\gamma}+\frac{R}{\gamma})]^{n-1}[(1+\frac{R}{\gamma})(1+\gamma+R)]^m}\\
&\quad -\frac{n}{(1+\gamma)[(R+1)(1+R(1+\gamma))(1+\frac{1}{\gamma})]^{n-1}[(R+1)(1+R(1+\frac{1}{\gamma}))(1+\gamma)]^m}\Bigg\}\\
&\quad \times \left(\frac{\gamma}{\gamma+1}\right)^{i-m}\left(\frac{1}{1+\gamma}\right)^{j-n}
\end{aligned}
\tag{41}
$$

Following a similar derivation process as (32)-(40), we obtain the throughput of access point B, S_b, as

$$
\begin{aligned}
S_b &= \sum_{i=0}^{N_B}\sum_{j=0}^{N_A}\binom{N_B}{i}\sigma^i(1-\sigma)^{N_B-i}\binom{N_A}{j}\sigma^j(1-\sigma)^{N_A-j}\\
&\quad \times \sum_{m=0}^{i}\sum_{n=0}^{j}\binom{i}{m}\binom{j}{n}\Bigg\{\frac{m}{[(1+R)(1+\gamma+R\gamma)]^{m-1}[(1+R\gamma)(1+\frac{1}{\gamma}+R)]^n}\\
&\quad -\frac{m\gamma}{(1+\gamma)[(R+1)(1+R(1+\frac{1}{\gamma}))(1+\gamma)]^{m-1}[(R+1)(1+R(1+\gamma))(1+\frac{1}{\gamma})]^n}\\
&\quad +\frac{n}{[(1+R)(1+\frac{1}{\gamma}+\frac{R}{\gamma})]^{n-1}[(1+\frac{R}{\gamma})(1+\gamma+R)]^m}\\
&\quad -\frac{n}{(1+\gamma)[(R+1)(1+R(1+\gamma))(1+\frac{1}{\gamma})]^{n-1}[(R+1)(1+R(1+\frac{1}{\gamma}))(1+\gamma)]^m}\Bigg\}\\
&\quad \times \left(\frac{\gamma}{\gamma+1}\right)^{i-m}\left(\frac{1}{1+\gamma}\right)^{j-n}
\end{aligned}
\tag{42}
$$

The average throughput per AP, S, is thus $\frac{S_a+S_b}{2}$.

C. Delay

The derivation of the delay in the BF case is similar to that in the OM case. We use the parameters p, p_A, p_B defined in Section 3.C and event $T_{i,\,j}$ defined in Section 3.B. The user transmitting a concerned packet is referred to as a concerned user and all other users are called non-concerned users. Furthermore, a new event $J_{m,\,n}$ is defined below.

$J_{m,\,n}$: Excluding the concerned user, m transmitting users in *User Set A* choose AP A and n transmitting users in *User Set B* choose AP A as their server.

We have

$$
\begin{aligned}
p_A &= \sum_{i=0}^{N_A-1}\sum_{j=0}^{N_B}\binom{N_A-1}{i}\sigma^i(1-\sigma)^{N_A-1-i}\binom{N_B}{j}\sigma^j(1-\sigma)^{N_B-j}\\
&\quad \times \Pr(\text{AP } A \text{ or AP } B \text{ successfully receives the concerned packet}|T_{i+1,j})\\
&= \sum_{i=0}^{N_A-1}\sum_{j=0}^{N_B}\binom{N_A-1}{i}\sigma^i(1-\sigma)^{N_A-1-i}\binom{N_B}{j}\sigma^j(1-\sigma)^{N_B-j}\sum_{m=0}^{i}\sum_{n=0}^{j}\Pr(J_{m,n}|T_{i+1,j})\\
&\quad \times \Pr(\text{AP } A \text{ or AP } B \text{ successfully receives the concerned packet}|T_{i+1,j}J_{m,n})
\end{aligned}
\tag{43}
$$

Expanding $\Pr(J_{m,\,n}|T_{i+1,\,j})$ and $\Pr$(AP A or AP B successfully receives the concerned packet $|T_{i+1,j}J_{m,\,n})$, we have

$$
p_A = \sum_{i=0}^{N_A-1}\sum_{j=0}^{N_B}\binom{N_A}{i}\sigma^i(1-\sigma)^{N_A-i-1}\binom{N_B}{j}\sigma^j(1-\sigma)^{N_B-j}\sum_{m=0}^{i}\sum_{n=0}^{j}\binom{i}{m}\binom{j}{n}
$$
$$
\times\ (\Pr(\text{a non - concerned transmitting user in } User\ Set\ A \text{ chooses AP } A))^m
$$
$$
\times\ (\Pr(\text{a non - concerned transmitting user in } User\ Set\ B \text{ chooses AP } A))^n
$$
$$
\times\ (1-\Pr(\text{a non - concerned transmitting user in } User\ Set\ A \text{ chooses AP } A))^{i-m}
$$
$$
\times\ (1-\Pr(\text{a non - concerned transmitting user in } User\ Set\ B \text{ chooses AP } A))^{j-n}
$$
$$
\times\ [\Pr(\text{AP } A \text{ successfully receives the concerned packet}|T_{i+1,j}J_{m,n})
$$
$$
+\Pr(\text{AP } B \text{ successfully receives the concerned packet}|T_{i+1,j}J_{m,n})] \tag{44}
$$

Notice that

$$
(\Pr(\text{a non - concerned transmitting user in } User\ Set\ A \text{ chooses AP } A))^m
$$
$$
\times\ (\Pr(\text{a non - concerned transmitting user in } User\ Set\ B \text{ chooses AP } A))^n
$$
$$
\times\ \Pr(\text{AP } A \text{ successfully receives the concerned packet}|T_{i+1,j}J_{m,n}))
$$
$$
= \Pr\left(\frac{x}{\sum_{i=1}^{m}y_i+\sum_{j=1}^{n}z_j}>R,\ x>\bar{x}|y_i>\bar{y}_i,\ z_j>\bar{z}_j\right)\Pr(y_i>\bar{y}_i,\ z_j>\bar{z}_j) \tag{45}
$$
$$
= f(m,\ n,\ \gamma)
$$

Similarly, we have

$$
(1-\Pr(\text{a non - concerned transmitting user in } User\ Set\ A \text{ chooses AP } A))^{i-m}
$$
$$
\times\ (1-\Pr(\text{a non - concerned transmitting user in } User\ Set\ B \text{ chooses AP } A))^{j-n}
$$
$$
+\Pr(\text{AP } B \text{ successfully receives the concerned packet}|T_{i+1,j}J_{m,n})) \tag{46}
$$
$$
= f\left(i-m,\ j-n,\ \frac{1}{\gamma}\right)
$$

Inserting (38), (39), (44), (45) into (43) and using (2)-(4) and the function defined in (31), (43) can be rewritten as

$$
p_A = \sum_{i=0}^{N_A-1}\sum_{j=0}^{N_B}\binom{N_A-1}{i}\sigma^i(1-\sigma)^{N_A-1-i}\binom{N_B}{j}\sigma^j(1-\sigma)^{N_B-j}\sum_{m=0}^{i}\sum_{n=0}^{j}\binom{i}{m}\binom{j}{n}
$$
$$
\times\left\{\left(\frac{\gamma}{\gamma+1}\right)^{i-m}\left(\frac{1}{1+\gamma}\right)^{j-n}\left\{\frac{1}{[(1+R)(1+\gamma+R\gamma)]^m[(1+R\gamma)(1+\frac{1}{\gamma}+R)]^n}\right.\right.
$$
$$
\left.-\frac{\gamma}{(1+\gamma)[(R+1)(1+R(1+\frac{1}{\gamma}))(1+\gamma)]^m[(R+1)(1+R(1+\gamma))(1+\frac{1}{\gamma})]^n}\right\} \tag{47}
$$
$$
+\left(\frac{1}{\gamma+1}\right)^m\left(\frac{\gamma}{1+\gamma}\right)^n\left\{\frac{1}{[(1+R)(1+\frac{1}{\gamma}+\frac{R}{\gamma})]^{i-m}[(1+\frac{R}{\gamma})(1+\gamma+R)]^{j-n}}\right.
$$
$$
\left.\left.-\frac{1}{(1+\gamma)[(R+1)(1+R(1+\gamma))(1+\frac{1}{\gamma})]^{i-m}[(R+1)(1+R(1+\frac{1}{\gamma}))(1+\gamma)]^{j-n}}\right\}\right\}
$$

The probability p_B can be similarly found as

$$
p_B = \sum_{i=0}^{N_B-1}\sum_{j=0}^{N_A}\binom{N_B-1}{i}\sigma^i(1-\sigma)^{N_B-1-i}\binom{N_A}{j}\sigma^j(1-\sigma)^{N_A-j}\sum_{m=0}^{i}\sum_{n=0}^{j}\binom{i}{m}\binom{j}{n}
$$
$$
\times\left\{\left(\frac{\gamma}{\gamma+1}\right)^{i-m}\left(\frac{1}{1+\gamma}\right)^{j-n}\left\{\frac{1}{[(1+R)(1+\gamma+R\gamma)]^m[(1+R\gamma)(1+\frac{1}{\gamma}+R)]^n}\right.\right.
$$
$$
\left.-\frac{\gamma}{(1+\gamma)[(R+1)(1+R(1+\frac{1}{\gamma}))(1+\gamma)]^m[(R+1)(1+R(1+\gamma))(1+\frac{1}{\gamma})]^n}\right\} \tag{48}
$$
$$
+\left(\frac{1}{\gamma+1}\right)^m\left(\frac{\gamma}{1+\gamma}\right)^n\left\{\frac{1}{[(1+R)(1+\frac{1}{\gamma}+\frac{R}{\gamma})]^{i-m}[(1+\frac{R}{\gamma})(1+\gamma+R)]^{j-n}}\right.
$$
$$
\left.\left.-\frac{1}{(1+\gamma)[(R+1)(1+R(1+\gamma))(1+\frac{1}{\gamma})]^{i-m}[(R+1)(1+R(1+\frac{1}{\gamma}))(1+\gamma)]^{j-n}}\right\}\right\}
$$

Applying p_A and p_B into (22) and (23), the average number of transmission attempts is obtained.

D. Special case comparison: no Multi-AP diversity
The following presents the throughput and delay expressions considering BF but without multi-AP diversity. Following [19], we are able to obtain the throughput as

$$
S = 0.5\times\left[\sum_{i=0}^{N_A}\binom{N_A}{i}\sigma^i(1-\sigma)^{N_A-i}\frac{i}{(R+1)^{i-1}}+\sum_{j=0}^{N_B}\binom{N_B}{j}\sigma^j(1-\sigma)^{N_B-j}\frac{j}{(R+1)^{j-1}}\right] \tag{49}
$$

The delay expression follows (22) and (23), with the probabilities p_A and p_B given as

$$
p_A = \sum_{i=0}^{N_A-1}\binom{N_A-1}{i}\sigma^i(1-\sigma)^{N_A-1-i}\frac{1}{(R+1)^i} \tag{50}
$$

$$
p_B = \sum_{j=0}^{N_B}\binom{N_B-1}{i}\sigma^j(1-\sigma)^{N_B-1-j}\frac{1}{(R+1)^j} \tag{51}
$$

V. Numerical Results: Theoretical and Simulation
Numerical results presented in this section are mostly based on theoretical formulas. For the comparison purpose, a number of simulation results are also presented. All simulation results are obtained by running MATLAB programs for 500000 time slots. Rayleigh fading and independent transmission links are assumed in generating signal strength values. For packet arrivals, a Poisson distribution is used in determining the number of packets generated in each time slot. Signaling is not implemented in the simulation, assuming that all acknowledgments are received successfully.

Figure 3 compares the throughput of slotted Aloha when BF with multi-AP diversity and OM with multi-AP diversity are used. Both analytical and simulation results are presented. System parameters considered include $N_A = N_B = 25$, $\gamma = 0.1$, and $R = 3$ dB. Numerical results illustrate that the analytical evaluation and simulation results match very well. The scenario with BF clearly outperforms the OM case under high traffic load
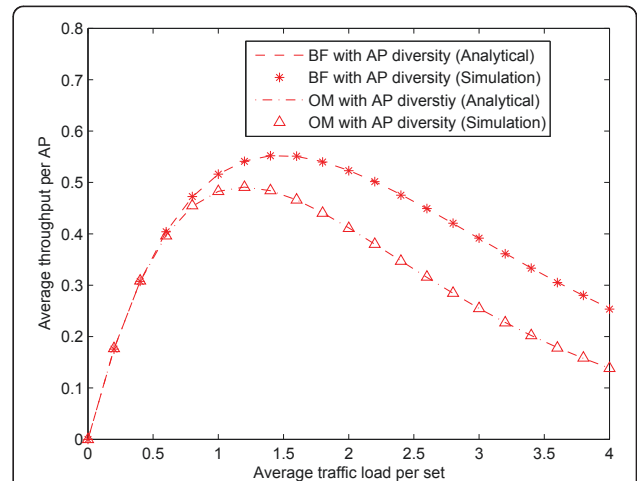

Figure 3 Throughput comparison: OM versus BF, with AP diversity; analytical versus simulation results, $N_A = N_B = 25$, $\gamma = 0.1$, $R = 3$ dB.

conditions with an approximately 12% improvement in peak throughput.

Figure 4 considers the OM case and examines the impact of the capture ratio, R. System parameters N_A and N_B are assumed to be 25 and γ is assumed to be 0.1. It is seen that a lower capture ratio leads to higher throughput. The OM case with AP diversity consistently outperforms that without AP diversity, especially when the capture ratio is small.
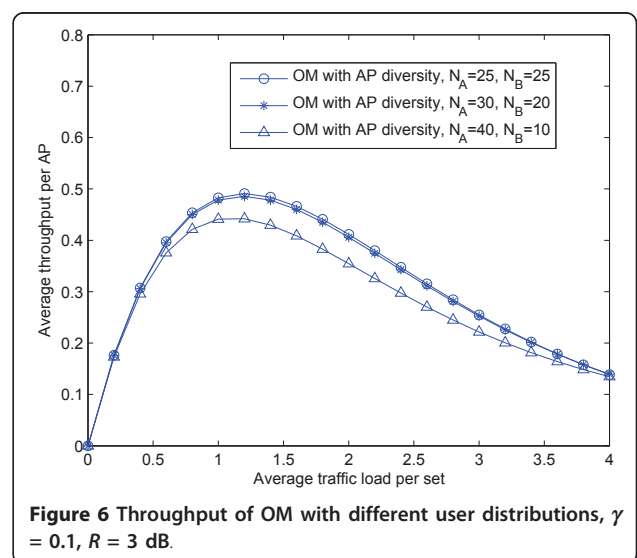
Figure 5 considers the OM case and examines the impact of γ values (see (4)). System parameters N_A and N_B are assumed to be 25 and R is assumed to be 3 dB. The throughput decreases as γ increases (due to more interference between the two APs). It is also noted that the throughput gain due to multi-AP diversity is more significant when γ is larger.

Figure 6 examines the impact of user distributions (N_A versus N_B) in the OM case with multi-AP diversity. The system parameter γ is assumed to be 0.1 and R is assumed to be 3 dB. The scenario with even user distributions ($N_A = 25$ and $N_B = 25$) outperforms other scenarios with uneven distributions. When the user distributions become very uneven (e.g., $N_A = 40$ and $N_B = 10$), throughput is noticeably lower due to the potential of a higher collision probability at the heavy-load AP ($N_A = 40$).
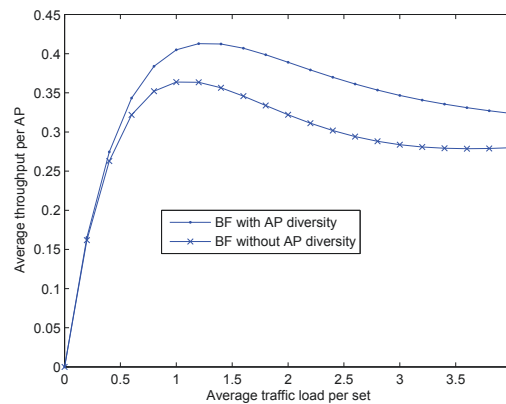
Figure 7a, b, c considers the BF scenario and examines the impact of multi-AP diversity. System parameter γ is assumed to be 0.1 and R is assumed to be 3 dB. The figures show that the advantage, if any, of multi-AP diversity in the BF case depends on the user distributions between the two user sets. When the distributions are extremely uneven (e.g., $N_A = 45$ and $N_B = 5$), the multi-

Figure 5 Throughput of OM with different γ values, N_A = N_B = 25, R = 3 dB.
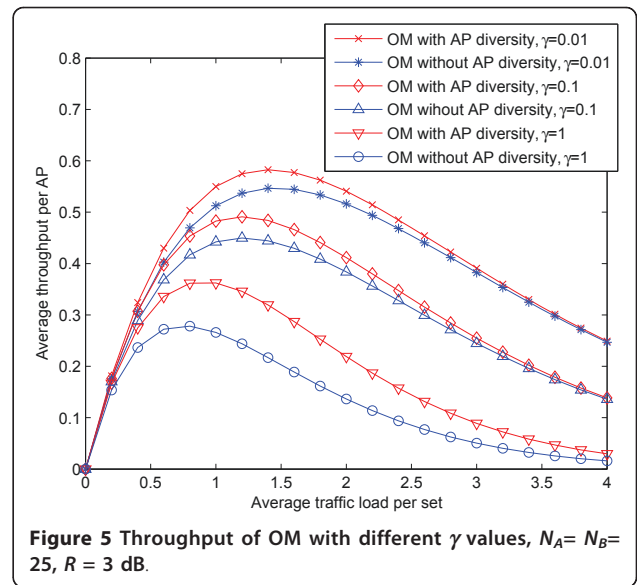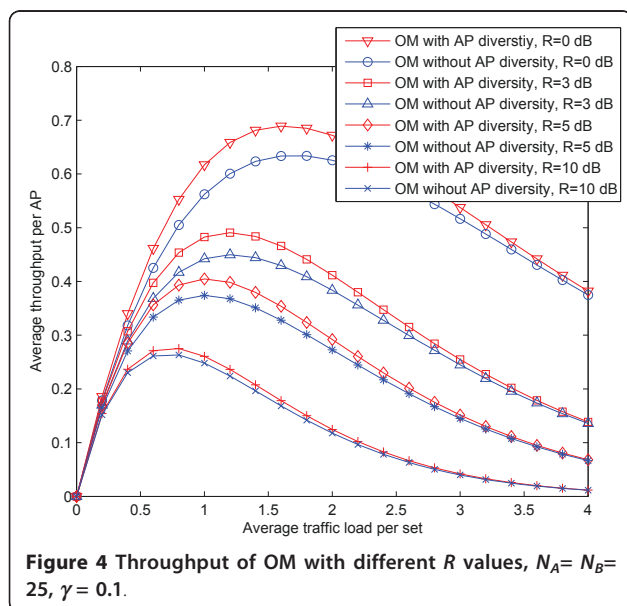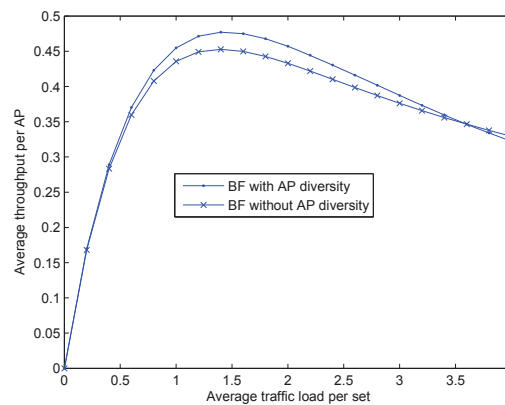
AP diversity clearly shows its advantage. When the distributions become less uneven (e.g., $N_A = 40$ and $N_B = 10$), the advantage of multi-AP diversity is seen for a wide traffic load range, but not for extremely high traffic load conditions. When the user distributions become even (e.g., $N_A = 25$ and $N_B = 25$), the advantage of multi-AP diversity disappears. These observations are due to a traffic redistribution characteristics of AP diversity. When the user distribution is uneven, with AP diversity, some users could effectively migrate from the AP with a heavy load to the AP with a light load, which may lead to an overall performance improvement. However, when the user distribution is even, AP diversity may cause a situation where one AP gets overly loaded, which brings down overall throughput.

Figure 4 Throughput of OM with different R values, N_A = N_B = 25, γ = 0.1.

Figure 6 Throughput of OM with different user distributions, γ = 0.1, R = 3 dB.

$[N_A = 45, \; N_B = 5.]$

$[N_A = 40, \; N_B = 10.]$

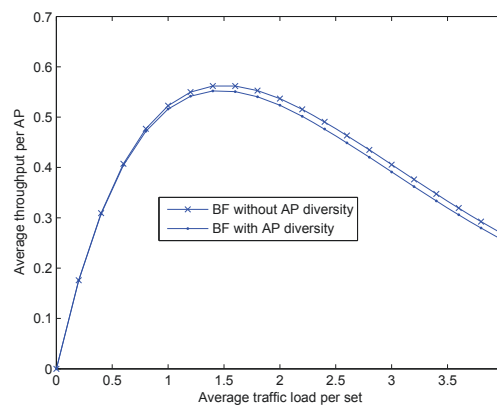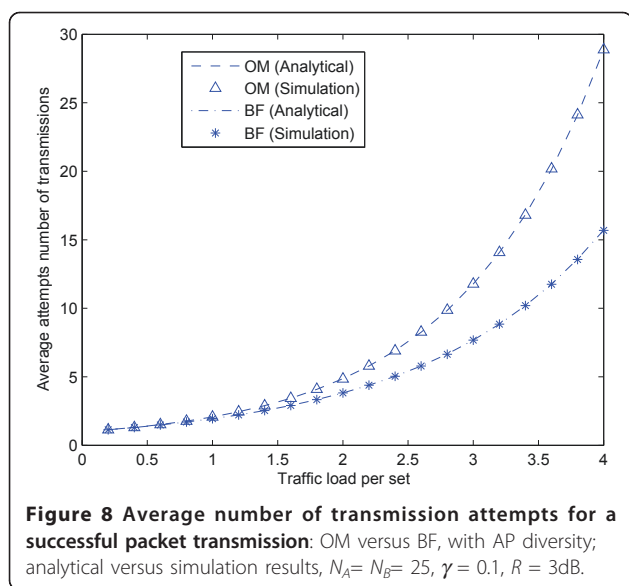
$[N_A = 25, \; N_B = 25.]$

Figure 7 Throughput of BF with different user distributions, $\gamma = 0.1$ and $R = 3$ dB.

One method to study the delay performance is to examine the average number of transmission attempts for each successful packet transmission. In Figure 8, OM with multi-AP diversity and BF with multi-AP diversity are compared in terms of the average number of transmission attempts for each successful transmission. System parameters considered include $N_A = 25$, $N_B = 25$, $\gamma = 0.1$, and $R = 3$ dB. Both analytical and simulation results are presented in Figure 8 and the analytical evaluation and simulation match very well. Figure

8, which illustrates that BF with multi-AP diversity outperforms OM with multi-AP diversity in the delay performance.

VI. Conclusions

This paper investigates the impact of multi-AP diversity and BF in slotted Aloha. A total of four network scenarios are examined, i.e., OM with multi-AP diversity, OM without multi-AP diversity, BF with multi-AP diversity and BF without multi-AP diversity. Performance

Figure 8 Average number of transmission attempts for a successful packet transmission: OM versus BF, with AP diversity; analytical versus simulation results, $N_A = N_B = 25$, $\gamma = 0.1$, $R = 3$dB.

evaluations conclude that, for OM systems, a configuration with multi-AP diversity always outperforms that without multi-AP diversity (Figures 4 and 5). For BF systems, multi-AP diversity provides performance advantages only under conditions with extremely uneven user distributions (Figure 7). Considering multi-AP diversity, BF systems outperform OM systems in terms of throughput and delay (Figures 3 and 8).

VII. Competing interests
The authors declare that they have no competing interests.

Abbreviations
AP: access points; BF: beamforming; multi-AP: multi-access-point; OM: omni-directional.

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