Wang et al. EURASIP Journal on Wireless Communications and Networking 2012, 2012:101
http://jwcn.eurasipjournals.com/content/2012/1/101

EURASIP Journal on Wireless Communications and Networking
a SpringerOpen Journal

RESEARCH Open Access

Transmission strategies with interference coordination in spectrum sharing cognitive radio networks under outage probability constraint

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Abstract

In this article, we consider a cognitive radio (CR) communication system based on spectrum sharing schemes, where we have a secondary user (SU) link with multiple transmitting antennas and a single receiving antenna, coexisting with a primary user (PU) link with a single receiving antenna. At the SU transmitter (SU-Tx), the channel state information (CSI) of the SU link is assumed to be perfectly known; while the interference channel from the SU-Tx to the PU receiver (PU-Rx) is not perfectly known due to less cooperation between the SU and the PU. As such, the SU-Tx is only assumed to know that the interference channel gain can randomly take values from a finite set with certain probabilities. Considering a SU transmit power constraint, our design objective is to determine the transmit covariance matrix that maximizes the SU rate, while we protect the PU by enforcing both a PU average interference constraint and a PU outage probability constraint. This problem is first formulated as a non-convex optimization problem with a non-explicit probabilistic constraint, which is then approximated as a mixed binary integer programming (MBIP) problem and solved with the branch and bound (BB) algorithm. The complexity of the BB algorithm is analyzed and numerical results are presented to validate the effectiveness of the proposed algorithm. A key result proved in the article is that the rank of the optimal transmit covariance matrix is one, i.e., CR beamforming is optimal under PU outage constraints. Finally, a heuristic algorithm is proposed to provide a suboptimal solution to our MBIP problem by efficiently (in polynomial time) solving a particularly-constructed convex problem.

Keywords: cognitive radio (CR), spectrum sharing, convex optimization, branch and bound, probabilistic constraint, integer programming

1 Introduction

The evolution from static spectrum allocation policies to dynamic ones can significantly increase the utilization efficiency of the radio spectrum. One promising platform to support such transitions is the cognitive radio (CR) system that was invented for opportunistic spectrum sharing with existing primary links, where CRs dynamically adapt their transmission patterns to access under-utilized frequency segments while regulating the interference to PUs [1,2]. As such, the key design challenge is how to maximize the SU rate while maintaining an acceptable level of interference to PUs.

Recently, there has been much research devoted to this interesting problem. Gastpar [3] studied the channel capacity of a single secondary transmission when the interference power received at the PU-Rx is limited below a given threshold, which is the so-called interference temperature constraint. Along a similar line, Xing et al. [4] studied the problem of maximizing the sum utility over multiple SUs under the interference temperature constraints. In more recent research, the role of multi-antennas has been investigated under CR network settings. Scutari et al. [5,6] proposed an approach to share resource among SUs from the game theoretic point of view. Larsson and Jorswieck [7] studied the beamforming vectors in MISO interference channel also from a game theoretic perspective. Zhang and Liang [8] studied the channel capacity of secondary multiple-input multiple-output (MIMO) and multiple-input single-output

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(MISO) channels when the channel state information (CSI) between the SU-Tx and PU was perfectly known at the SU-Tx. In the MISO case, under an average secondary transmit power constraint and an interference temperature constraint at each PU-Rx, beamforming was proved to be the optimal strategy. In the MISO case where only one PU was present with one receiving antenna, a closed-form solution was derived. In [9-11], the authors considered a similar MISO scenario where, instead of complete CSI between the SU-Tx and PU, only partial CSI was known. In [9], channel capacity was studied with only the mean of the channel between the SU-Tx and PU-Rx was known at the SU-Tx, where beamforming was proved to be optimal. Such study was extended in [10,11] to consider both the mean and covariance feedbacks at the SU-Tx, where two algorithms were presented to solve for the optimal solution: one based on a second-order cone programming approach; and the other based on a geometric interpretation. Kang et al. [12] studied ergodic and outage capacities of SU under the constraint on the PU transmission outage probability assuming that all the instantaneous channel power gains in the PU-SU network were available to the SU at each fading state. Phan et al. [13] proposed and designed optimal multicast beamforming with inaccurate channel state information at the secondary transmitters in a quality-of-service (QoS) aware cognitive multicast network. The robust beamforming optimization problems with outage probability constraints had been investigated by Vorobyov et al. [14] and Chalise et al. [15]. And in cognitive radio networks the robust optimization problems had been studied as well. Zheng et al. [16] maximized the minimum of the SUs' signal-to-interference-plus-noise ratio (SINR) to obtain the optimal robust beamforming vectors under the constraints of the overall SU transmit power and the received interference power at the PUs. Gharavol et al. [17,18] designed a transmit power minimization problem of the SU-Tx while simultaneously achieving a lower bound on the received SINR for the SUs and imposing an upper limit on the interference power at the PUs with only imperfect CSI. Sun et al. [19] also investigated the robust problem in cognitive radio networks to minimize the total power consumption of SU-Tx under the QoS constraint at SU-Rx and the interference constraint at PU-Rx. Xiong et al. [20] studied a max-min SINR problem of the SUs with controlling the interference leakage to PUs using the probability based approach.

In this article, we model a practical scenario where we only know the imperfect CSI of the SU-Tx to PU-Rx channel and formulate the problem under a PU outage probability constraint in addition to the transmit power constraint and the average interference power constraint. In our study, we define the outage probability to be the probability of interference power at the PU-Rx exceeding a given threshold, in a manner similar to the outage constraint considered in [12,14,15]. The main motivation for this formulation is to allow some interference from the SU-Tx to the PU-Rx as long as the resulting outage probability is kept small. Our aim in this article is to investigate the SU system performance with this more practical regulation over the SU interference to the PU-Rx. The main contribution of this article is summarized as follows. We formulate the transmit covariance matrix design problem for a single secondary link under an average interference power constraint and an outage probability constraint to protect a given PU-Rx. The two constraints are both motivated by the interference temperature concept. The PUs can be protected from the received interference under both constraints. The average interference power constraint is used to guarantee a long-term QoS of PU, for example, delay-insensitive services of PU. The outage probability constraint is considered to ensure the instantaneous reliable transmission with a prescribed outage probability. Due to the introduction of the outage probability constraint, this resulting design problem is non-convex with non-explicit constraints. To solve this problem, we reformulate it into an MBIP problem with a deterministic constraint on the outage upper bound. Then we use a BB algorithm to compute the numerical results, which is highly efficient in solving the MBIP problem compared with exhaustive searching for the original non-convex problem. A key result proved in this article is that the rank of the optimal transmit covariance matrix is one, i.e., CR beamforming is optimal under the PU outage constraint. Finally, a heuristic algorithm is proposed to provide a suboptimal solution to our MBIP problem by efficiently (in polynomial time) solving a particularly-constructed convex problem.

The rest of the article is organized as follows. In Section 2, we discuss the system and signal models. In Section 3, the MBIP transformation is discussed along with the BB algorithm and the complexity analysis, and we show that the rank of the optimal transmit covariance matrix is always one. In addition, we also propose a simple algorithm as an alternative to the BB algorithm for finding a good suboptimal solution to the MBIP problem. In Section 4, the numerical results are presented. Section 5 draws the conclusions. *Notations*: $\mathbf{x}^{\dagger}$ denotes the conjugate transpose, $\mathrm{tr}(\cdot)$ denotes the trace operator, $rank(\cdot)$ denotes the rank of a matrix, $E[\cdot]$ denotes the statistical expectation, and $C^{M \times N}$ denotes the space of $M \times N$ matrices with complex entries. Boldface upper and lower case letters are used to denote matrices and vectors, respectively, with "~" standing for "distributed as".

Re(·) and Im(·) represent the real and imaginary parts of the operand, respectively. The log(·) functions are over base 2.

2 System and signal model

We consider a spectrum sharing scenario in which CR users coexist with primary users in the same frequency band. For the purpose of exposition, we consider in this article a simple CR system, where one SU link and one PU link share the same spectrum simultaneously. Here the SU-Tx is equipped with M_t transmitting antennas, and both the secondary and primary receivers are each equipped with a single antenna, as illustrated in Figure 1. We assume that the SU-Tx knows the MISO channel $\mathbf{h} \in C^{M_t \times 1}$ from the SU-Tx to the SU-Rx, which is randomly distributed according to $\mathbf{h} \sim \mathcal{CN}(0, \mathbf{I})$. The MISO interference channel from the SU-Tx to the PU-Tx, denoted as $\mathbf{g} \in C^{M_t \times 1}$, is not perfectly known to the SU-Tx due to less cooperation between the SU and the PU. In this article, we assume that the interference channel gain $\mathbf{g}$ is randomly selected from a finite set $\mathbf{G} = \{\mathbf{g}_1, \mathbf{g}_2, \ldots, \mathbf{g}_\mathbf{N}\}$ with a corresponding probability set $\{p_1, p_2, \ldots, p_N\}$. We model the interference channel with finite number of discrete states from a practical perspective. In practical scenario, due to the limited capacity of feedback links, the transmitter can only obtain the quantized values of the channel states, i.e., the imperfect channel state information. Therefore, the interference channel model with finite number of channel realizations is an approximation of the continuous channel model. And the accuracy of this approximation can be improved with the increasing value of N. The similar channel model has been adopted in [21,22].

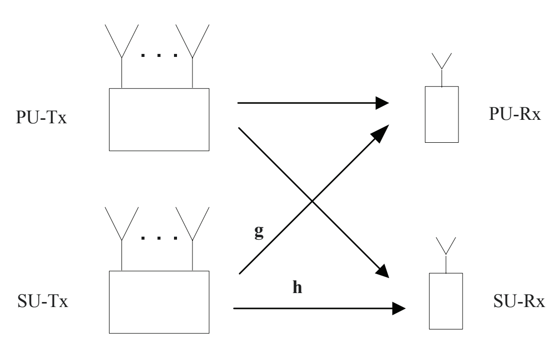

Figure 1 System model of the SU coexisting with the PU. A simple CR system is considered shown in Figure 1, where one SU link, $\mathbf{g}$, and one PU link, $\mathbf{h}$, share the same spectrum simultaneously. Both SU-Tx and PU-Tx are equipped with M_t transmitting antennas. Both SU-Rx and PU-Rx are each equipped with a single antenna.

Furthermore, since the SU-Rx cannot differentiate the interference from the PU-Tx from the background noise, it is assumed that the interference from PU-Tx to SU-Rx can be considered in the Gaussian noise at SU-Rx. Under these assumptions, the SU-Tx adapts the transmission rate, power, and spatial spectrum to maximize its own transmission rate, while maintaining the interference to the PU-Rx below a certain level. Such an interference regulation is achieved by enforcing a set of constraints over the SU transmit covariance matrix, which will be discussed later in details.

The signal model for the system under consideration is given as

$$y = \mathbf{h}^\dagger \mathbf{x} + w, \tag{1}$$

where y and $\mathbf{x} \in C^{M_t \times 1}$ are the received and transmitted signals at the SU-Rx and SU-Tx, respectively, and w is the additive Gaussian noise with $w \sim \mathcal{CN}(0, 1)$. The transmit covariance matrix is denoted by $\mathbf{K_x} = E[\mathbf{x}\mathbf{x}^\dagger] \succcurlyeq 0$.

Our goal in this article is to balance the maximum transmit rate of the SU and the interference from the SU-Tx to the PU-Rx by adjusting the spatial spectrum of the SU signals. As such, we need to design the optimal transmit covariance matrix, $\mathbf{K_x}$, to maximize the SU rate with some tolerable interference to the PU-Rx. In particular, we cast this problem as follows:

$$\begin{aligned}
(\text{P1}) : \underset{\mathbf{K_x}}{\text{maximize}} : & \ \mathbf{h}^\dagger \mathbf{K_x} \mathbf{h} && (2)\\
\text{subject to} : & \ \text{tr}(\mathbf{K_x}) \le P_{\text{tr1}} && (3)\\
& \ E[\mathbf{g}^\dagger \mathbf{K_x} \mathbf{g}] \le P_{\text{tr2}} && (4)\\
& \ \Pr\{\mathbf{g}^\dagger \mathbf{K_x} \mathbf{g} \ge r\} \le p_{\text{th}} && (5)\\
& \ \mathbf{K_x} \succcurlyeq 0, && (6)
\end{aligned}$$

where the objective is equivalent to maximizing the achievable rate $\log(1 + \mathbf{h}^\dagger \mathbf{K_x} \mathbf{h})$, P_{tr1} is the SU transmit power limit, P_{tr2} is the average interference power limit, r is the instantaneous interference power tolerance at the PU-Rx, and p_{th} is the PU outage probability limit. The objective function is the SU transmission rate, and the four constraints are the average transmit power, the average interference power, the PU outage probability constraints, and the positive semi-definite constraint, respectively.

Due to the probabilistic constraint in (5), problem (P1) is generally hard to solve directly. For a probabilistic constraint where the random vector has a continuous distribution, checking the feasibility of each feasible point requires a complex multi-dimensional integration. Even when the random vector has a discrete distribution, the feasible set defined by the probabilistic constraint is generally non-convex and it cannot be described by explicit functions [23]. Fortunately, as shown in [24,25], the above probabilistically constrained problem can be solved as an integer programming (IP) problem with deterministic constraints.

For our problem, under the assumption that the SU-Tx knows that the interference channel $\mathbf{g}$ has finite number of discrete states, we take the approach in [24,25] to first approximate (P1) as an MBIP problem with deterministic constraints, and then deploy a BB algorithm [26-28] to seek the solution. The details will be discussed in the following sections, together with complexity analysis and simulation results. We will also propose a heuristic algorithm to efficiently solve a sub-optimal solution for the MBIP problem.

3 Optimization algorithm

3.1 MBIP transformation

In this section, we first discuss a deterministic transformation of the probabilistic constraint in (P1). As assumed, the random variable $\mathbf{g}$ takes values from a finite set $\mathbf{G} = \{\mathbf{g}_1, \mathbf{g}_2, \ldots, \mathbf{g_N}\}$ with a corresponding probability set $\{p_1, p_2, \ldots, p_N\}$. We refer to each probable value $\mathbf{g_n}$ as one scenario. The probabilistic constraint can then be interpreted as that the sum probability over all possible interference-violating scenarios must be bounded by p_{th}. Therefore, we can reformulate the probabilistic constraint in (P1) as shown in the following problem:

$$
\begin{aligned}
\text{(P2)} : \underset{\mathbf{K_x}, b_n}{\text{maximize}} : \ & \mathbf{h}^\dagger \mathbf{K_x} \mathbf{h} && (7)\\
\text{subject to} : \ & \text{tr}(\mathbf{K_x}) \le P_{\text{tr}1} && (8)\\
& E[\mathbf{g}^\dagger \mathbf{K_x} \mathbf{g}] \le P_{\text{tr}2} && (9)\\
& \mathbf{g_n^\dagger K_x g_n} - M b_n \le r, n = 1, \ldots, N. && (10)\\
& \sum_{n=1}^{N} b_n p_n \le p_{\text{th}}, b_n \in \{0, 1\} && (11)\\
& \mathbf{K_x} \succcurlyeq 0. && (12)
\end{aligned}
$$

The two newly added constraints (10) and (11) are deterministic and only involving explicit functions, which can be easily handled by numerical algorithms. The design variables here are now both the matrix $\mathbf{K_x}$ and the binary variables b_n, $n = 1, 2, \ldots, N$, where the binary variables are used to indicate whether the interference outage check needs to be performed: if $b_n = 0$, it means no outage is possible under the scenario $\mathbf{g_n}$ given the constraint (10), such that p_n needs not to be included in the left-hand sum of (11); if $b_n = 1$, there may or may not be an outage if the slack constant M is chosen large enough, which leads to the fact that (11) is enforcing an outage probability upper-bound to be less than p_{th} since p_n is now always counted in the left-hand sum of (11). The positive slack constant, M, is chosen to be of a large value since it is used to deactivate the outage check in (10) when $b_n = 1$. Given the fact that $\sum_{n=1}^{N} b_n p_n$ incurs an outage probability upper-bound, (P2) is actually a stricter version of (P1) with tighter constraints. As a result, the optimal objective value of (P2) will be slightly less than that of (P1). However, as we show later that the resulting performance is still much better than reference approaches.

We now discuss how to determine the value for M, which needs to guarantee the satisfaction of the inequality (10) when $b_n = 1$. It means that the value of M is chosen large enough to deactivate the constraint (10) when $b_n = 1$, i.e., the corresponding scenario in which the SU-Tx may cause harm to PU doesn't need to enforce the interference constraint (10) when we solve the problem (P2). For sufficiency, we could find an M that is larger than the maximum value of $\mathbf{g_n^\dagger K_x g_n}$ over $n = 1, \ldots, N$. One way to achieve that is as follows:

$$
\begin{aligned}
\mathbf{g_n^\dagger K_x g_n} &= \text{tr}(\mathbf{g_n^\dagger K_x g_n}) && (13)\\
&= \text{tr}(\mathbf{K_x g_n g_n^\dagger}) && (14)\\
&\le \text{tr}(\mathbf{K_x}) \text{tr}(\mathbf{g_n g_n^\dagger}) && (15)\\
&\le P_{\text{tr}1} \text{tr}(\mathbf{g_n g_n^\dagger}). && (16)
\end{aligned}
$$

Given above inequality, one way to choose M is to get different values for each scenario n. Instead of that, we conveniently choose one value of M for all scenarios and take $M = \max_n P_{\text{tr}1} \text{tr}(\mathbf{g_n g_n^\dagger})$ since the only purpose of M is to deactivate the constraints. With the value of M available, we next solve the MBIP problem (P2), for which a direct approach is through exhaustive search over the binary variables b_n's, where for each feasible choice of b_n's we solve the following convex semi-definite programming (SDP) problem:

$$
\begin{aligned}
\underset{\mathbf{K_x}}{\text{maximize}} : \ & \text{tr}(\mathbf{K_x h h^\dagger}) && (17)\\
\text{subject to} : \ & \text{tr}(\mathbf{K_x}) \le P_{\text{tr}1} && (18)\\
& E[\text{tr}(\mathbf{K_x g g^\dagger})] \le P_{\text{tr}2} && (19)\\
& \text{tr}(\mathbf{K_x g_n g_n^\dagger}) \le M b_n + r, \quad n = 1, \ldots, N. && (20)\\
& \mathbf{K_x} \succcurlyeq 0. && (21)
\end{aligned}
$$

Unfortunately, such an exhaustive search in general incurs exponential total complexity. So instead, we discuss a BB approach to search over the binary variables more efficiently in the average sense.

3.2 Branch and bound algorithm

As mentioned before, one way to solve an MBIP problem is through exhaustive search, where the feasible space grows exponentially with the number of binary variables, which leads to the NP-hardness of most binary optimization problems. Fortunately, BB algorithms [26-28] can often be used in solving discrete and combinatorial optimization problems to reduce the average complexity, when the problem has a finite but very large solution set with certain structures.

We first give a brief overview of the BB algorithm, followed by specific implementations for solving the MBIP problem (P2). Two components are usually required for an effective implementation of a BB algorithm. The first is a *branching procedure* and the second is a *bounding function*. Given a set S, the branching procedure returns non-overlapping subsets S_1, S_2, . . . , whose union is the set S. The bounding function then computes the upper and/or lower bounds of the optimal value given each subset S_i. The upper and lower bounds are then used to determine one of the following two outcomes: split the subset S_i into more subsets for further bounding, or discard the subset S_i from the searching space, which is also referred to as pruning and is the reason why the BB algorithm is more efficient than exhaustive search.

It is clear that problem (P2) can be cast as a SDP problem over the design variable $\mathbf{K_x}$ when the binary variables are relaxed to be within [0,1]. With this property, we next implement the BB algorithm to jointly search over $\mathbf{K_x}$ and the binary variable b_n's. Due to the recursive nature of the BB algorithm, it traverses a binary search tree (BST), as shown in Figure 2. Each *node* in the BST represents a particular case when the relaxed SDP problem from (P2) is solved at a partial or complete binary solution. In particular, the *root node* corresponds to the case where all b_n's are relaxed to be within [0,1]; and a *leaf node* is a node at the bottom of the BST, which denotes the case with a complete binary solution, where the resulting objective value of (P2) is called an *incumbent* if it is the best objective value found so far across all the known leaf nodes. The *depth* of a node, D, is defined to be the number of determined binary variables in the partial binary solution at this node. As D increases from $D = j$ to $D = j + 1$, one additional binary variable b_n is being determined. Specifically, at one particular node let us assume that b_1, b_2, . . . , b_{n-1} have been determined. We then create two child nodes corresponding to two sub-problems in the relaxed SDP form of (P2) with $b_n = 0$ and $b_n = 1$, respectively, while keeping b_1, b_2, . . . , b_{n-1} unchanged and rounding all undetermined binary variables, b_{n+1}, b_{n+2}, . . . , b_n, to be ones. For a given sub-problem, if the achieved optimal objective value is lower than the current incumbent, the corresponding child node (as well as all of its descendants) is discarded, i.e., pruned from the searching space. Otherwise, the corresponding child node is kept in the BST, and the searching continues to b_{n+1} until we reach the leaf node with a complete binary solution.

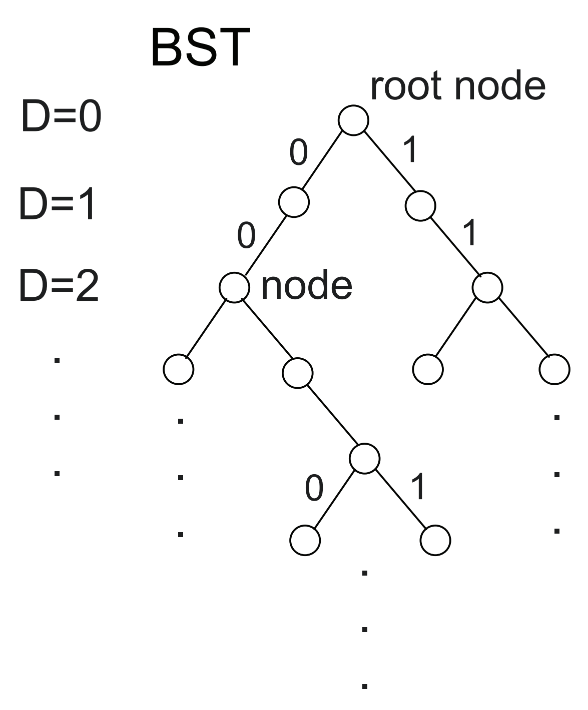

Figure 2 Binary Search Tree (BST). A binary search tree (BST) shown in Figure 2 is illustrated the recursive process of the BB algorithm. Each *node* in the BST represents a particular case when the relaxed SDP problem from (P2) is solved at a partial or complete binary solution. The *depth* of a node, D, is defined to be the number of determined binary variables in the partial binary solution at this node. At one particular node two child nodes will be created corresponding to two sub-problems in the relaxed SDP form of (P2) with $b_n = 0$ and $b_n = 1$, respectively.

Following the above procedure, the BB algorithm traverses through the BST by solving one relaxed SDP for an optimal $\mathbf{K_x}$ at each node. The algorithm is terminated when the entire BST has been either pruned or processed. All computations in our algorithm are performed using the matlab-based software package CVX [29,30] which deploys SeDuMi [31] as its back-end solver for SDP problems.

3.3 Complexity analysis

In this section, we discuss the complexity of the proposed algorithm. The efficiency of the algorithm depends critically on the branching and bounding procedure, where the entire searching space is branched into non-overlapping subsets, and the bounding procedure then calculates bounds for each subset with decisions made on whether to continue branching or to discard the entire subset. The pruning process, which allows the algorithm to only traverse a fraction of the entire BST, is the key to decrease the overall searching complexity. In our implementation, at the root node, there are no determined binary variables, i.e., all binary variables are relaxed. At each child node, one additional binary variable is determined. During each iteration, one node is chosen and the bound is calculated after solving the relaxed SDP. If the bound is lower than the incumbent, then it means that no child nodes branched from this node will yield a solution better than the incumbent; the node is therefore pruned. If the node at depth j is pruned, we can calculate how many potential child nodes of this branch are pruned, which indicates how much searching complexity is reduced. For simulations, we set $M_t = 2$. We assume that each element of $\mathbf{g_n}$ is generated by quantizing a random variable distributed as $\mathcal{CN}(0, 0.1)$ into four levels, and the corresponding p_n is determined by integrating the probability density function over the associated quantization levels. The secondary transmit power ranges from 0 dB to 10 dB. Accordingly, the MBIP problem has 16 binary design variables, such that if exhaustive search is deployed, there will be a total of $2^{16} = 65536$ sub-problems need to be solved. With our approach, Figure 3 shows the update progress of the incumbent, and Figure 4 shows the progress of pruned nodes in percentages at each iteration, where we only need to solve 273 sub-problems in the simulation.

Remark: The number of sub-problems solved in our BB algorithm varies over different channel realizations. Typically, we observe that less than 700 sub-problems in total are solved with our BB algorithm across a large number of channel realizations.

3.4 Rank-one property of the optimal $\mathbf{K_x}$

Note that Zhang and Liang [8] studied a similar problem without the PU outage constraint, where they proved that the optimal $\mathbf{K_x}$ must be a rank-one matrix. To prove the rank-one property of the optimal matrix $\mathbf{K_x}$ in our case, we focus on the following equivalent problem to the relaxed SDP problem at each given set of b_n's:

$$(\text{P3}): \underset{\mathbf{K_x}}{\text{maximize}}: \log(1 + \mathbf{h}^{\dagger}\mathbf{K_x}\mathbf{h}) \tag{22}$$
$$\text{subject to}: \text{tr}(\mathbf{K_x}) \le P_{\text{tr1}} \tag{23}$$
$$\text{tr}(\mathbf{K_x}E[\mathbf{g}\mathbf{g}^{\dagger}]) \le P_{\text{tr2}} \tag{24}$$
$$\text{tr}(\mathbf{K_x}\mathbf{g_n}\mathbf{g_n}^{\dagger}) \le r, \ \forall n \in T_1 \tag{25}$$
$$\text{tr}(\mathbf{K_x}\mathbf{g_m}\mathbf{g_m}^{\dagger}) \le r + M, \forall m \in T_2 \tag{26}$$
$$\mathbf{K_x} \succcurlyeq 0, \tag{27}$$

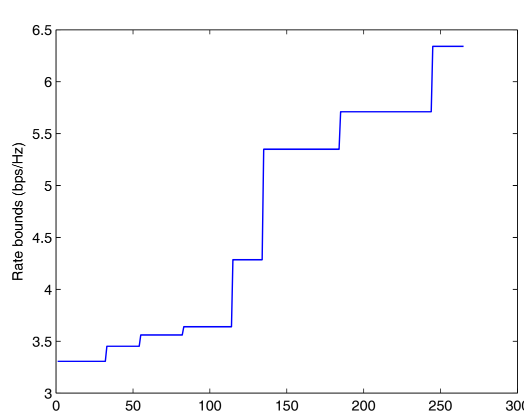

Figure 3 Bounding progress of BB versus the number of iterations. Figure 3 shows the update progress of the incumbent.

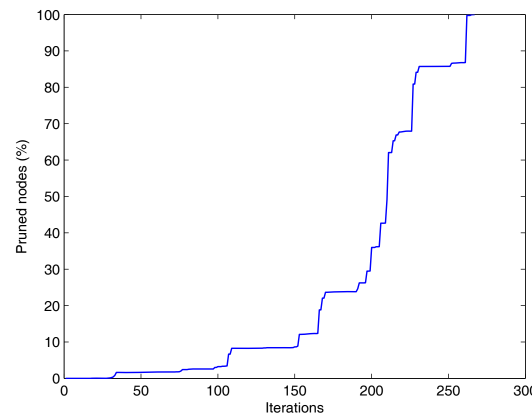

Figure 4 Pruned nodes of BB versus the number of iterations. Figure 4 shows the progress of pruned nodes in percentages at each iteration.

where the replacement of $\mathbf{h}^{\dagger}\mathbf{k_x}\mathbf{h}$ by $\log(1 + \mathbf{h}^{\dagger}\mathbf{k_x}\mathbf{h})$ in the objective function is for the convenience of applying the Karush-Kuhn-Tucker (KKT) optimality conditions [30], the set T_1 contains all the indices with $b_n = 0$, and the set T_2 contains all the indices with $b_n = 1$.

The Lagrange dual function of (P3) can thus be written as

$$g(\nu, \theta, \mu_n, \lambda_m) = \sup_{\mathbf{K_x} \succcurlyeq 0} \quad \log(1 + \mathbf{h}^{\dagger}\mathbf{K_x}\mathbf{h}) + \text{tr}\Bigg[\mathbf{K_x}\Bigg(\nu\mathbf{I} + \theta E[\mathbf{g}\mathbf{g}^{\dagger}] + \sum_{n \in T_1} \mu_n \mathbf{g_n}\mathbf{g_n}^{\dagger} + \sum_{m \in T_2} \lambda_m \mathbf{g_m}\mathbf{g_m}^{\dagger}\Bigg)\Bigg], \tag{28}$$

where ν, θ, μ_n, and λ_m are the dual variables associated with the constraints (23)-(26), respectively. We then define matrix $\mathbf{A}$ as

$$\mathbf{A} = \nu\mathbf{I} + \theta E[\mathbf{g}\mathbf{g}^{\dagger}] + \sum_{n \in T_1} \mu_n \mathbf{g_n}\mathbf{g_n}^{\dagger} + \sum_{m \in T_2} \lambda_m \mathbf{g_m}\mathbf{g_m}^{\dagger}, \tag{29}$$

and we show that $\mathbf{A}$ must have a full rank of M_t in order for the dual function to have a bounded value. First, it is clear that in the case of either $\nu > 0$ or $\theta > 0$, $\mathbf{A}$ must have full rank. When both $\nu = 0$ and $\theta = 0$, we prove that $\mathbf{A}$ also needs to have full rank by a contradiction approach discussed in [32]. Let us assume $\mathbf{A}$ is rank deficient; then it is possible to have a $\mathbf{K_x} = t\mathbf{v_j}\mathbf{v_j}^{\dagger}$, where $\mathbf{v_j}$ is an eigenvector of $\mathbf{A}$ corresponding to a zero eigenvalue and t is some scaling coefficient. As such, the trace term on the right-hand side of (28) goes to zero. Since $\mathbf{h}$ is drawn from a continuous distribution, the probability that $\mathbf{h}$ is orthogonal to $\mathbf{v_j}$ is zero. It thus follows that the supremum in (28) would be unbounded by choosing an appropriate polarity of t and scaling up the magnitude of t to infinity. As such, we conclude that $\mathbf{A}$ must have full rank, which allows us to define a new design variable $\overline{\mathbf{K}}_\mathbf{x} = \mathbf{A}^{\frac{1}{2}}\mathbf{K}_\mathbf{x}\mathbf{A}^{\frac{1}{2}}$ and rewrite the Lagrange dual function as the optimal value of the following problem:

$$\underset{\overline{\mathbf{K}}_\mathbf{x}}{\text{maximize}} : \log(1 + \mathbf{h}^\dagger\mathbf{A}^{-\frac{1}{2}}\overline{\mathbf{K}}_\mathbf{x}\mathbf{A}^{-\frac{1}{2}}\mathbf{h}) + \text{tr}(\overline{\mathbf{K}}_\mathbf{x}) \tag{30}$$

$$\text{subject to: } \overline{\mathbf{K}}_\mathbf{x} \succcurlyeq 0. \tag{31}$$

This problem is convex and has strictly feasible points; thus the optimal $\overline{\mathbf{K}}_\mathbf{x}$ must satisfy the KKT conditions [30] as follows,

$$\frac{1}{\ln 2}(\mathbf{A}^{-\frac{1}{2}})^\dagger\mathbf{h}(1 + \mathbf{h}^\dagger\mathbf{A}^{-\frac{1}{2}}\overline{\mathbf{K}}_\mathbf{x}\mathbf{A}^{-\frac{1}{2}}\mathbf{h})^{-1}\mathbf{h}^\dagger(\mathbf{A}^{-\frac{1}{2}})^\dagger + \mathbf{\Phi} = -\mathbf{I} \tag{32}$$

$$\text{tr}(\mathbf{\Phi}\overline{\mathbf{K}}_\mathbf{x}) = 0, \tag{33}$$

where $\Phi \succcurlyeq 0$ is the dual variable associated with the constraint (31). Here, we see that since the right-hand side of (32) is a matrix of full rank M_t, and the first term on the left-hand side has unit rank, the matrix Φ must be of a rank greater than or equal to M_t - 1. Given $\overline{\mathbf{K}}_\mathbf{x} \succcurlyeq 0$ and $\mathbf{\Phi} \succcurlyeq 0$, together with (33), we conclude that the rank of the nontrivial optimal $\overline{\mathbf{K}}_\mathbf{x}$, and also the optimal $\mathbf{K}_\mathbf{x}$, is one. Since the above result holds for all of the feasible dual variables, when the ν, θ, μ_n, and λ_m are taking the optimal values in the dual problem, the resulting optimal solution of $\mathbf{K}_\mathbf{x}$ from the optimal $\overline{\mathbf{K}}_\mathbf{x}$ in (30), (31) is also the optimal solution for the original problem in (22)-(27), which is of rank one. As such, beamforming is optimal for the CR transmitter even under the PU outage probability constraint, where the optimal beamformer can be directly obtained as the eigenvector of the rank-one optimal $\mathbf{K}_\mathbf{x}$.

3.5 An efficient heuristic algorithm

As an alternative to the high-complexity BB-based solution, we propose a heuristic but efficient algorithm for finding a suboptimal solution of the MBIP problem (P2). By observing the objective function and the constraint (10) in (P2), we see that we will severely limit the SU received signal power when we limit the interference to the PU via (10) in the case of a $\mathbf{g_n}$ that is highly correlated with $\mathbf{h}$. To prevent this, we could manually set such a case as an outage scenario with $b_n = 1$ as long as the outage probability constraint is still satisfied. By doing so, the corresponding constraint $\mathbf{g_n}^\dagger\mathbf{K}_\mathbf{x}\mathbf{g_n} \le r + M$ becomes inactive with a large M, such that no power restriction is enforced over the correlated direction of $\mathbf{h}$. With the above approach applied to a part of $\mathbf{g_n}$'s correlated with $\mathbf{h}$, we could achieve a good balance between maximizing the SU rate and protecting the PU, where the philosophy is that since certain PU outage is allowed, we should greedily utilize such an outage allowance to cover certain $\mathbf{g_n}$'s that are aligned in a similar direction to $\mathbf{h}$. Note that the comment on the rank of $\mathbf{K}_\mathbf{x}$ given in the last subsection is also applicable to this heuristic algorithm and the beamforming is the optimal transmission strategy. Specifically, we use the angles between $\mathbf{g_n}$'s and $\mathbf{h}$, defined by $\cos(\theta_n) = \frac{\bar{\mathbf{g}}_\mathbf{n}^\dagger \cdot \bar{\mathbf{h}}}{||\mathbf{g_n}||\,||\mathbf{h}||}$, with $\bar{\mathbf{g}}_\mathbf{n} = [\text{Re}(\mathbf{g_n})\text{Im}(\mathbf{g_n})]^\dagger$ and $\overline{\mathbf{h}} = [\text{Re}\,(\mathbf{h})\text{Im}\,(\mathbf{h})]^\dagger$, as a measure for the correlation of directions. The smaller angle means that the direction of $\mathbf{g_n}$ is closer to $\mathbf{h}$. The proposed algorithm first sorts the set of $\{\mathbf{g_1}, \mathbf{g_2}, \ldots, \mathbf{g_N}\}$ in descending order of $|\cos(\theta_n)|$ and forms a new set $\{\tilde{\mathbf{g}}_\mathbf{1}, \tilde{\mathbf{g}}_\mathbf{2}, \ldots, \tilde{\mathbf{g}}_\mathbf{N}\}$ with a corresponding probability set $\{\tilde{p}_1, \tilde{p}_2, \ldots, \tilde{p}_N\}$. The $\tilde{b}_n$ values are all initialized to zeros. Starting with $\tilde{\mathbf{g}}_\mathbf{1}$, which has the smallest angle between $\mathbf{h}$, named the highest correlation to $\mathbf{h}$ relative to other $\tilde{\mathbf{g}}_\mathbf{n}$'s in this article, we add this scenario to the outage probability by setting the corresponding $\tilde{b}_1$ to one, as long as doing so does not violate the sum probability constraint $\sum_{n=1}^{N} \tilde{b}_n\tilde{p}_n \le p_{\text{th}}$, otherwise $\tilde{b}_1$ is set to zero. This process continues sequentially for the set of $\{\tilde{\mathbf{g}}_\mathbf{1}, \tilde{\mathbf{g}}_\mathbf{2}, \ldots, \tilde{\mathbf{g}}_\mathbf{N}\}$, which results in a pre-determined set of $\tilde{b}_n$'s that satisfies the sum outage constraint. And the convex SDP problem in the form of (P3) with the pre-determined binary variables can be solved to get the optimal covariance matrix $\mathbf{K}_\mathbf{x}$. Although the optimality (with respect to the solution of (P2)) of the SU rate obtained by solving the above resulting problem is not guaranteed, the heuristic algorithm does offer an efficient solution to an otherwise complex problem by solving only one SDP problem. Numerical results in the following section show the encouraging performance of this heuristic algorithm in comparison with the BB and reference algorithms.

4 Numerical results and comparison

In this section, numerical results are presented to show the performance of the CR system under consideration with our optimal solution. The simulation setup of $\mathbf{g_n}$ is the same as that for generating Figures 3 and 4.

Figure 5 illustrates the maximum achievable transmit rate for the SU using the BB algorithm in comparison with the heuristic algorithm and a reference algorithm

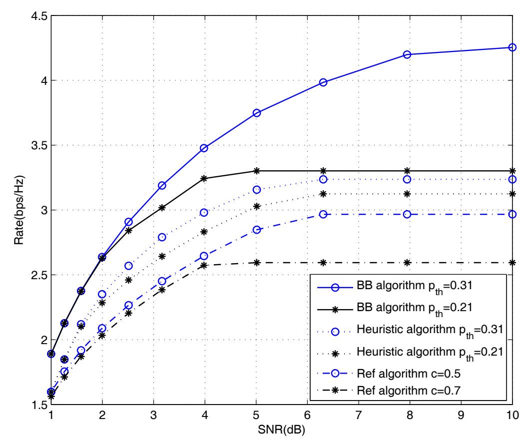

Figure 5 Comparison of the achievable transmit rates with the BB algorithm, the heuristic algorithm, and the reference algorithm. Figure 5 illustrates the maximum achievable transmit rate for the SU using the BB algorithm in comparison with the heuristic algorithm and a reference algorithm from [9-11]. The outage probability limits, p_{th}, are set as 0.21 and 0.31, respectively. The parameter values for *c* in the reference algorithm are set to 0.7 and 0.5, which leads to outage probabilities of 0.21 and 0.31, respectively.

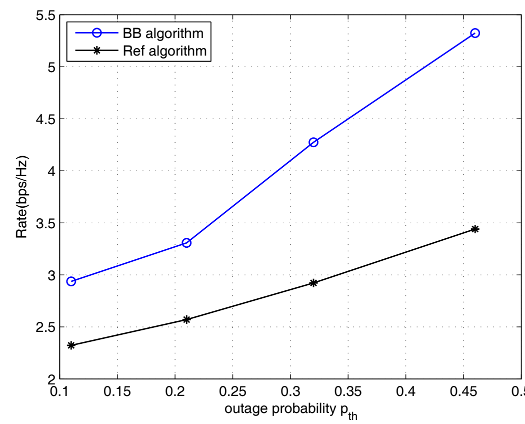

Figure 6 Comparison of the achievable transmit rates with the BB algorithm and the reference algorithm under different outage probability limits. Figure 6 illustrates the maximum achievable transmit rate for the SU obtained by the BB algorithm and the reference algorithm under different values of the PU outage probability limit and the corresponding values of *c*. In the simulation, the simulation setup is kept while the transmit power limit is 8 dB.

from [9-11].The reference algorithm adopts the concept that the interference power to the PU should be less than a given threshold with certain probability. The parameter c in the reference algorithm determines the probability that the PU interference power constraint is violated. Therefore, the second set of constraints of the robust design problem (P1) in [10,11] has the similar meaning with the PU outage probability constraint of our problem (P1). In the simulation, we assume that the average interference power is limited to 2, the transmit power limit is ranging from 1 dB to 10 dB, and the outage probability limits are set as 0.21 and 0.31, respectively. The parameter values for c in the reference algorithm are set to 0.7 and 0.5, which leads to outage probabilities of 0.21 and 0.31, respectively. From Figure 5, we see that the transmit rate with p_{th} = 0.31 is always greater than or equal to the rate with p_{th} = 0.21, which is as expected. Moreover, we note that the maximum achievable transmit rate with the BB approach is always higher than the rate of the heuristic approach, which is higher than the reference algorithm. In Figure 6, we compare the two results of the maximum achievable transmit rate obtained by our algorithm and the reference algorithm under different values of the PU outage probability limit and the corresponding values of c in the reference algorithm. In the simulation, we assume that the simulation setup is kept while the transmit power limit is 8 dB. From Figure 6, we can see that the transmit rate with our algorithm is always higher than the reference algorithm under different values of outage probability.

5 Conclusions

In this article, we consider a secondary communication link sharing the same spectrum with a primary link in a CR network. Multiple transmitting antennas are exploited at the SU-Tx to achieve balance between the SU transmit rate maximization and the interference regulation at the PU-Rx. We introduce the PU outage probability constraint in our formulation to model a more practical scenario, where the problem is formulated as a non-convex optimization problem with a probabilistic constraint, in addition to the SU transmit power constraint and the PU average interference constraint. To make the non-convex problem solvable, a deterministic transformation is used to approximate the original problem as an MBIP problem. An efficient BB algorithm and a heuristic algorithm are proposed to solve the MBIP problem, with simulation results to illustrate the superior performance of our algorithms over an existing reference algorithm. A key result proved in the article is that the rank of the optimal transmit covariance matrix is one, i.e., CR beamforming is optimal under the PU outage constraint. To deal with the complexity issue of the BB algorithm, a heuristic algorithm is also proposed to provide a suboptimal solution for our MBIP problem by efficiently solving a particularly-constructed convex SDP problem.

Acknowledgements

This study was supported by the National Science & Technology Specific Program of China (No. 2010ZX03005-001-03), the National High Tech. Research & Development of China (863 Program, No. 2009AA02Z412), the International Science and Technology Cooperation Project (No. 2010DFA11590) and the Program for New Century Excellent Talents in University (NCET).

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Competing interests

The authors declare that they have no competing interests.

Received: 28 June 2011 Accepted: 12 March 2012
Published: 12 March 2012

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doi:10.1186/1687-1499-2012-101
Cite this article as: Wang *et al.*: **Transmission strategies with interference coordination in spectrum sharing cognitive radio networks under outage probability constraint.** *EURASIP Journal on Wireless Communications and Networking* 2012 **2012**:101.