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Localized geometric topologies with bounded node degree for three-dimensional wireless sensor networks

Fan Li[1], Zeming Chen[1] and Yu Wang[2*]

Abstract

Three-dimensional (3D) wireless sensor networks have attracted a lot of attention due to their great potential usages in both commercial and civilian applications, such as environmental data collection, pollution monitoring, space exploration, disaster prevention, and tactical surveillance. Topology control in 3D sensor networks has been studied recently, and different 3D geometric topologies were proposed to be the underlying network topologies to achieve the sparseness of the communication networks. However, most of these proposed 3D topologies cannot bound the maximum node degree, i.e., some nodes may need to maintain a large number of neighbors in the constructed topologies, which is not energy efficient and may lead to large contention. In this article, we extend several existing 3D geometric topologies to a set of new 3D topologies with bounded node degree. We provide both theoretical analysis and simulation evaluation on their power efficiency and node degree distributions. Our simulation results over random 3D sensor networks confirm nice performances of these proposed 3D topologies.

1 Introduction

Due to its wide-range potential applications (such as environmental data collection, pollution monitoring, space exploration, disaster prevention, and tactical surveillance), 3D wireless sensor network has recently emerged as a premier research topic. Most current researches in 3D sensor networks primarily focus on coverage [1-4], connectivity [4,5], and routing issues [6-12]. In this article, we focus on how to efficiently control the 3D network topology to maintain both network connectivity and energy efficiency of routes in 3D wireless sensor networks.

Topology control [13-15] has been well-studied for 2D wireless ad hoc and sensor networks in the past decade [16-26]. Topology control methods allow each sensor node to *locally* adjust its transmission range and select certain neighbors for communication, while maintaining a structure that can support energy efficient routing and improve the overall network performance. Given the dynamic nature of sensor networks, the topology should

be locally and self-adaptively maintained without affecting the whole network and the communication cost during maintaining should not be too high. There exist several topology control techniques such as localized geometrical structures [17-25], dynamic cluster techniques [27-30] and power assignment protocols [31-37]. In this article, we focus on geometrical structures based methods.

Though many 2D geometrical structures have been proposed, surprisingly, there is not much study of 3D geometrical methods for topology control in 3D sensor networks, except for [38-44]. Bahramgiri et al. [38], Ghosh et al. [42], and Poduri et al. [43] proposed methods based on generalized cone-based topology control algorithm [17,20] to preserve connectivity in 3D sensor networks, however, all of their 3D structures cannot bound the node degree, i.e., some nodes may have large unbounded number of neighbors. In addition, their construction methods are complex. Wang et al. [39-41] and Kim et al. [44] then proposed a set of 3D structures based on Yao structures [18,19] in 3D space. These 3D Yao structures can be constructed locally and efficiently, and they do have bounded node out-degree. However, none of these existing 3D structures can bound the

* Correspondence: yu.wang@uncc.edu
[2]Department of Computer Science, University of North Carolina at Charlotte, Charlotte, NC 28223, USA
Full list of author information is available at the end of the article

node in-degree. Possible unbounded in-degree at some nodes will often cause large overhead or contention at those nodes which may make them exhausted earlier than other nodes. Faced up with this challenge, in this article we study how to efficiently construct 3D topology with bounded node degree (both out-degree and in-degree) to maintain connectivity, conserve energy, and enable energy efficient routing. In particular, we propose two general frameworks: *3D Symmetric Yao Graph* and *3D Yao and Reverse Yao Graph*, which are based on any existing 3D Yao structures. We then theoretically prove that the new 3D structures built by our proposed method can guarantee connectivity of the network, have a constant-bounded node degree, and support energy efficient routes. We believe that these structures are the first set of 3D structures that can achieve all of these properties.

The rest of this article is organized as follows. Section 2 introduces the network model and desired properties of our 3D topology control problem. Section 3 provides a brief review of related study on existing 3D topology control methods. Section 4 presents the algorithms to build 3D Yao-based topologies with bounded degree and the theoretical proofs of their nice properties, such as connectivity, degree bound, and power stretch factor. Section 5 presents the simulation results on random 3D networks and Section 6 concludes the article.

2 Network models and assumptions

A 3D wireless network consists of a set V of n wireless nodes distributed in a 3D plane $\mathbb{R}^3$. Each node has the same *maximum transmission range R*. These wireless nodes define a *unit ball graph* (UBG), or called *unit sphere graph*, in which there is an edge uv between two nodes u and v iff (if and only if) the Euclidean distance $||uv||$ between u and v in $\mathbb{R}^3$ is at most R. In other words, two nodes can always receive the signal from each other directly if the distance between them is not more than R. If there exists a link uv in UBG, v is a neighbor of u. All neighbors of u form its one-hop neighborhood, denoted as $N_{UBG}(u)$ or $N(u)$. The size of $N_{UBG}(u)$ is the node degree of u. We assume that all wireless nodes have distinctive identities and each node knows its position information either through a low-power GPS receiver or some other ways (such as 3D localization methods in [45-47]). By one-hop broadcasting, each node u can gather the location information of all nodes within its transmission range. As in the most common power-attenuation model, the power to support a link uv is assumed to be $||uv||^\beta$, where β is a real constant between 2 and 5 depending on the wireless transmission environment.

Topology control protocols aim to maintain a structure H from the original communication graph UBG that can preserve connectivity, optimize network throughput with power-efficient routing and conserve energy. The constructed topology H could be a directed or undirected graph. In the literature, the following desirable features of the structure are well-regarded and preferred in wireless sensor networks:

(1) **Connectivity:** To guarantee communications among all sensor nodes, the constructed topology H needs to be *connected*, i.e., there exists a path between any pair of nodes in the topology. This is the most fundamental requirement of topology control. Here, we always assume that the original communication graph UBG is a connected graph.

(2) **Bounded node degree:** It is also desirable that node degree in the constructed topology H is small and upper-bounded by a constant. If H is a directed graph, both in-degree and out-degree should be bounded. A small node degree reduces the MAC-level contention and interference, and may help to mitigate the well-known hidden and exposed terminal problems. In addition, if a graph has a bounded node degree, it is also a sparse graph, i.e., the total number of links is linear with the total number of nodes in the graph. A sparse graph conserves more energy in term of maintaining the constructed network topology.

(3) **Power spanner:** A good network topology should be *energy efficient*, i.e., the total power consumption of the least energy cost path between any two nodes in final topology should not exceed a constant factor of the power consumption of the least energy cost path in original network [18]. Given a path $v_1v_2\cdots v_h$ connecting two nodes v_1 and v_h, the energy cost of this path is $\sum_{j=1}^{h-1} \|v_jv_{j+1}\|^\beta$. The path with the least energy cost is called the shortest path in a graph. A subgraph H is called a *power spanner* of a graph G if there is a positive real constant ρ such that for any two nodes, the power consumption of the shortest path in H is at most ρ times of the power consumption of the shortest path in G. The constant ρ is called the *power stretch factor*. A power spanner of the communication graph (e.g., UBG) is usually energy efficient for routing.

(4) **Localized construction:** Due to limited resources and high mobility of wireless nodes, it is preferred that the topology can be constructed locally and in a self-organizing fashion. Here, a topology is *localized*, i.e., can be constructed locally, if every node u can decide all edges incident on itself in the topology by only using the information of nodes within a constant hops of u. Actually, all construction algorithms of our topologies presented here only use 1-hop neighbor information.

3 Related works

In this section, we briefly review the current techniques of topology control in both 2D and 3D wireless networks.

3.1 Topology control in 2D networks

With the objective of achieving energy efficiency and maintaining network connectivity, several localized geometrical structures have been proposed for topology control in 2D wireless networks, such as *local minimum spanning tree* (LMST) [21,22], *relative neighborhood graph* (RNG) [48,49], *Gabriel graph* (GG) [48,50], *Yao graph* (YG) [18,19], *cone-based topology control* (CBTC) [17,20], *Delaunay-based graph* [23-25], and different combinations of these graphs [51-53]. By constructing such sparse topology structures, transmission power of nodes can be minimized. As a result, the number of links in the constructed topology is significantly reduced compared with that of the original communication graph which contains all links supported by the maximum transmission power. Among these 2D structures, some are planar structures (such as LMST, RNG, GG, and Delaunay-based graphs), some are power spanners (such as GG, Yao graph, CBTC, and Delaunay-based graphs), and some are with bounded node degree (such as Yao graph).

Besides these localized geometrical structures, there are also other various techniques proposed by researchers for topology control in 2D sensor networks, such as how to construct a virtual backbone for routing [27-30] and how to minimize the total transmission power while maintaining connectivity or other properties [31,32,36,37]. Since this article only focuses on localized geometrical structures with bounded node degree, we refer readers to some nice surveys [13-15] on topology control for more details.

3.2 Topology control in 3D networks

Although geometric topology control protocols have been well studied in 2D networks, the design of 3D topology control is surprisingly more difficult than the design in 2D. Current 2D methods cannot be directly applied in 3D networks. Wang et al. [39,40] proved that there is no embedding method mapping a 3D network into a 2D plane so that the relative scale of all edge length is preserved and all 2D geometric topology control protocols can still be applied for power efficiency. Thus, any simple mapping method from 3D to 2D does not work. On the other hand, many properties of 3D networks require additional computational complexity. Until recently, little research has been done on topology control for 3D wireless networks.

To solely achieve the connectivity of a 3D network, *local minimum spanning tree* (LMST) [21,22] may be the best choice, since it is very sparse (with a bounded node degree) and can be easily constructed even for 3D networks and preserve connectivity. However, LMST may have very large power stretch factor. Similarly, *relative neighborhood graph* (RNG) and *Gabriel graph* (GG)

can be easily extended to 3D, as shown in [39,40]. However, both of them do not have bounded node degree.

Bahramgiri et al. [38] generalized CBTC algorithm from 2D to 3D to preserve connectivity. Basically, each node u increases its transmission power until there is no empty 3D-cone with angle degree α, i.e., there exists at least a node in each 3D-cone of degree α centered at u, if $\alpha \leq \frac{2\pi}{3}$. This algorithm can be extended to ensure k-connectivity with $\alpha \leq \frac{2\pi}{3k}$. Even though this approach can guarantee connectivity, the gap detection algorithm applied to check the existence of the empty 3D-cone of degree α is very complicated. The time complexity of the gap detection algorithm at a node u is $O(d^3 \log d)$, where d is the node degree of u. Moreover, their method cannot bound node degree, as shown by [54].

Wang et al. [39,40] proposed a set of 3D Yao-based topologies (FiYG and FlYG), which can be constructed locally and efficiently. They proved some nice properties of these 3D Yao-based structures, e.g., bounded node out-degree and constant power stretch factor. Later, Wang et al. [41] also extended all of their 3D structures to support fault tolerance.

Ghosh et al. [42] also presented two CBTC-based approaches for 3D wireless networks. Though the first approach, a heuristic based on 2D orthographic projections, can provide excellent performance in practice, it cannot guarantee connectivity for sure. In the second approach, a spherical Delaunay triangulation (SDT) is built to determine the existence of empty 3D cones. Although the second approach can guarantee connectivity of the network, the expense to construct the SDT is very high. Similarly, Poduri et al. [43] also used the spherical Delaunay triangulation to find the largest empty 3D cone in order to apply a CBTC-based topology control. The expense of SDT construction makes it inefficient in practice.

Recently, Kim et al. [44] proposed another localized Yao-based structure with Platonic solid (PYG). The basic idea is the same as the 3D Yao structures in [39,40] except for the partition method of 3D cones. To construct PYG, each node divides the 3D sphere neighborhood into k equal cones by using regular k-polyhedron and selects the neighbor that requires the lowest transmit power in each cone. The authors also consider the interference effects to adaptively choose the minimum transmit power and adjust the topology. However, such modification will break the connectivity and power spanner guarantees of 3D Yao structures.

Overall, all of these existing 3D structures cannot achieve bounded node degree. Notice that even though 3D Yao structures (including FiYG, FlYG, and PYG) can bound the node out-degree, their node in-degree could be as large as $O(n)$ where n is the number of nodes in

the networks. Faced up with this challenge, in this article we study how to efficiently construct 3D topology with bounded node degree to maintain connectivity, conserve energy and enable energy efficient routing.

## 4. Degree-bounded 3D topologies: 3D symmetric Yao graph and 3D Yao & reverse Yao graph

In this section, we propose two general frameworks to build degree-bounded 3D topologies for wireless sensor networks. The proposed frameworks basically apply the existing 3D Yao structures via two techniques developed in [19,23,55] for 2D topology control protocols. In addition, we provide detailed analysis on the degree bound and power stretch factor of the proposed new 3D topologies.

4.1 Review of basic 3D Yao structures

Our general frameworks to building degree-bounded 3D topologies are based on any existing 3D Yao structure (such as FiYG [39,40], FlYG [39,40] and PYG [44]). These 3D Yao structures use certain types of 3D cones to partition the transmission range of a node (which is a sphere), and inside each 3D cone the node only keeps a link to the nearest neighbor. Since the number of such 3D cones is bounded by a constant k, all of them can bound the node out-degree by k. Here, k is a constant depending on which method and parameter are used. Basically, these structures can be categorized into two sets: fixed partition and flexible partition.

3D Yao structures based on fixed partition

In fixed partition, 3D cones from one node do not intersect with each other and the partition method is the same for all nodes. In [39,40], Wang et al. first proposed two methods to divides the transmission range of a node into certain number of 3D cones. Figure 1a,b illustrates these two methods, which divide the transmission ball into 32 and 56 cones, respectively. For each cone, node u will choose the shortest edge $uv \in UBG$ among all edges emanated from u, if there is any, and add a directed link $\overrightarrow{uv}$. Ties are broken arbitrarily or by ID. The resulting directed graph are denoted by $FiY\ G_{32}$ and $FiY\ G_{56}$, respectively. Notice that these cones in $FiY\ G_{32}$ and $FiY\ G_{56}$ are different and do not intersect with each other. In [44], Kim et al. then proposed another type of fixed partition method which divides the unit ball into k equal cones by using a regular k-polyhedron and selects the nearest neighbor in each cone. The resulting directed graph is denoted by $PY\ G_k$. Possible polyhedrons include tetrahedron, cube, octahedron, dodecahedron and icosahedron for $k = 4$, 6, 8, 12, 20 respectively. Figure 1c,d illustrates partition examples with a octahedron $k = 8$ and a dodecahedron $k = 12$. Notice that the cones in this method are with same shape/size and do not intersect with each other. All above methods based on fixed partition can be performed locally using 1-hop neighbor information and with $O(d)$ time, where d is the number of 1-hop neighbors.

(a) FiYG$_{32}$

(b) FiYG$_{56}$

(b) PYG$_8$

(c) PYG$_{12}$
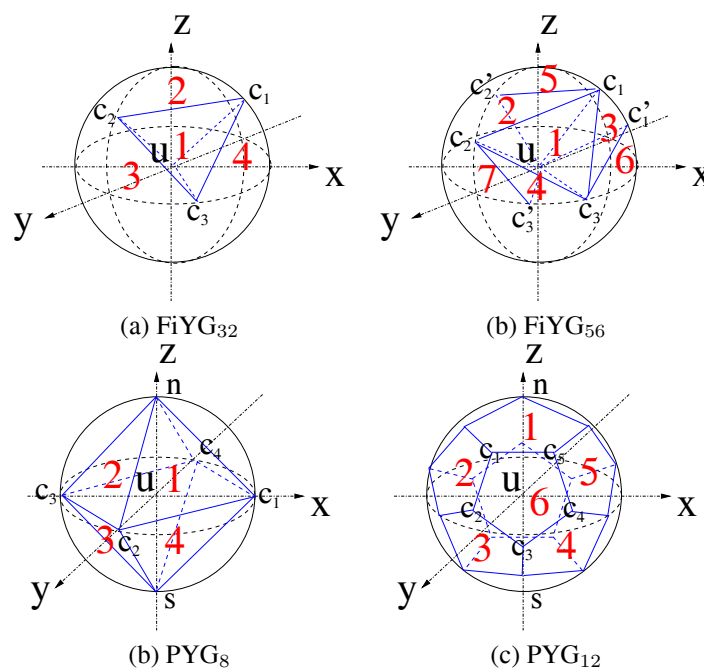
Figure 1 Definitions of 3D Yao Structures with fixed partitions. (**a**) and (**b**) show partitions of 1/8 of the ball in FiYG; (**c**) and (**d**) show partitions using a octahedron or a dodecahedron for PYG.

3D Yao structures based on flexible partition

In flexible partition, identical 3D cones with a top angle 2θ are used to partition the transmission ball and where to define these cones depends on the locations of neighbors around node u. In [39,40], Wang et al. proposed three different methods to perform the partition. However, they showed that the first method does not bound the node out-degree. Here, we just review their third method. Initially, all neighbors v_i of node u are *unprocessed* and ordered by the distance to u. The algorithm processes link uv from the shortest link and follows an ascending order. When it processes uv_i, it defines the 3D cone C_{uv_i} which uses uv_i as its axis (as shown in Figure 2a), adds the link uv_i, and marks all other links in C_{uv_i} as *processed*. We denote the final structure as FlY G_θ or FlYG when value of θ is clear. Algorithm 1 illustrates the detailed algorithm. The time complexity of this algorithm is $O(d \log d)$ due to the sorting. Notice that the 3D cones in this method are in the same size/shape and can intersect with each other (as in Figure 2b). By using a volume argument[a], Wang et al. showed that the node out-degree of FlY G is bounded by

$$k = \left\lceil \frac{2}{1-\cos\left(\frac{\theta}{2}\right)} \right\rceil.$$ If $\theta = \pi/4$, the degree bound is

$\frac{2}{1-\cos\left(\frac{\theta}{2}\right)} \approx 26$, and if $\theta = \pi/6$, the degree bound is 58.

Algorithm 1. Construct 3D Yao Structure *FlY G* for Node *u*

Input: all neighbors $N_{UBG}(u)$ of node u in UBG.

Output: neighbors $N_{FlY\ G}(u)$ of u in the constructed FlYG.

1: Sort all neighbors $v_i \in N_{UBG}(u)$ by its length such that $||uv_i|| \le ||uv_{i+1}||$, where $i = 1$ to $|N_{UBG}(u)|$.

2: Set PROCESSED(v_i) = 0 for all neighbor $v_i \in N_{UBG}(u)$.

3: **for** $i = 1$ to $|N_{UBG}(u)|$ **do**

4: **if** PROCESSED(v_i) = 0 **then**

5: As shown in Figure 2a, let C_{uv_i} be the cone using uv_i as the axis and 2θ as the top angle.

6: Keep v_i as a neighbor of u in FlYG, i.e., add v_i in $N_{FlYG}(u)$.

7: Set PROCESSED(w) = 1 for every other neighbor w inside C_{uv_i}.

8: **end if**

9: **end for**

10: **return** $N_{FlY\ G}(u)$

4.2 General frameworks to build 3D symmetric Yao graph and 3D Yao & reverse Yao graph

Bounded out-degree from 3D Yao structures gives us advantages when apply several routing algorithms on these structures. However, possible unbounded in-degree at some nodes will often cause large overhead or contention at those nodes which may make them exhausted earlier than other nodes. Therefore it is often imperative to construct a sparse network topology such that both the in-degree and the out-degree are bounded by a constant while it is still power spanner. Hereafter, we define a general function 3D-YAO-Structure() which can generate the neighbor set of 3D Yao structure at node u given the current neighbor set of u. The 3D-YAO-Structure() function can be any generation methods of existing Yao-based 3D structures. We use YG to denote the generated 3D Yao structure.

The first set of 3D topologies is 3D symmetric Yao graph (SYG), an undirected graph, which guarantees that the node degree is at most k. It first applies the 3D Yao structure to select the closest node in each 3D cone. An link uv is selected to graph SYG if and only if both u and v are selected to be kept by each other in YG, i.e., $v \in N_{Y\ G}(u)$ and $u \in N_{Y\ G}(v)$. See Figure 3a,b for illustrations. Algorithm 2 shows the framework. It is clear that only one-hop information is used and total $O(n)$ of messages are used. Thus, the SYG can be built locally and efficiently. Notice that similar idea has been used in 2D networks [23,55,56].
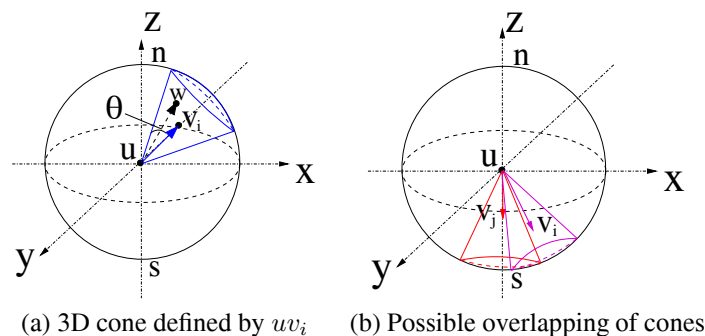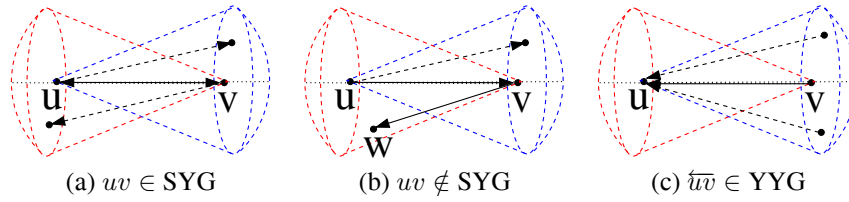

(a) 3D cone defined by uv_i (b) Possible overlapping of cones

Figure 2 Definitions of 3D Yao structures with flexible partitions.

Figure 3 Illustrations of 3D Yao Structures with bounded degree. (**a**) and (**b**) show 3D symmetric Yao graph; (**c**) shows 3D Yao and reverse Yao graph.

Algorithm 2. Building 3D symmetric Yao graph at node u

Input: all neighbors $N_{UBG}(u)$ of node u in UBG.

Output: neighbors $N_{SY\ G}(u)$ of u in the constructed SYG.

1: $N_{YG}(u)$ = 3D-YAO-Structure($N_{UBG}(u)$).

2: Broadcast $N_{Y\ G}(u)$ to all neighbors $N_{UBG}(u)$.

3: **for all** node $v \in N_{Y\ G}(u)$ do

4: **if** $u \in N_{Y\ G}(v)$ **then**

5: Keep v as a neighbor of u in SYG, i.e., add v in $N_{SY\ G}(u)$.

6: **end if**

7: **end for**

8: **return** $N_{SY\ G}(u)$

Algorithm 3. Building 3D Yao and reverse Yao graph at node u

Input: all neighbors $N_{UBG}(u)$ of node u in UBG.

Output: neighbors $N_{YY\ G}(u)$ of u in the constructed YYG.

1: $N_{Y\ G}(u)$ = 3D-YAO-Structure($N_{UBG}(u)$).

2: Broadcast $N_{Y\ G}(u)$ to all neighbors $N_{UBG}(u)$.

3: Let $N_{Y\ G}^{in}(u)$ be the set of u's incoming neighbors, i. e., all node v satisfying $u \in N_{Y\ G}(v)$.

4: $N_{YYG}^{in}(u) = \text{3D - YAO - Structure}\left(N_{Y\ G}^{in}(u)\right)$.

5: Broadcast $N_{YYG}^{in}(u)$ to all neighbors $N_{U\ BG}(u)$.

6: **for all** node $v \in N_{YG}(u)$ **do**

7: **if** $u \in N_{YYG}^{in}(v)$ **then**

8: Keep v as a neighbor of u in YYG, i.e., add v in $N_{Y\ Y\ G}(u)$.

9: **end if**

10: **end for**

11: **return** $N_{Y\ Y\ G}(u)$

The second set of 3D topologies is 3D Yao and reverse Yao graph (YYG), a directed graph, which guarantees that both node in-degree and node out-degree are at most k. The basic idea is to apply reverse 3D Yao structure on YG to bound the node in-degree. Node u chooses a node v from each 3D cone, if there is any, so the incoming link $\overleftarrow{uv}$ in YG has the smallest length among all incoming links from YG in that cone as shown in Figure 3c. Similar idea has been used for 2D

networks by [16,19,57]. Algorithm 3 shows the detailed algorithm. 3D YYG can be built locally and efficiently with only 1-hop neighbor information and linear number of messages.

4.3 Performance analysis of 3D SYG and 3D YYG

We are now ready for providing some analysis on the 3D structures built by our general frameworks. We will use two basic properties of the underlying 3D Yao structures: (1) the out-degree of 3D YG is bounded by k; and (2) if a link $uv \in UBG$ is not kept in 3D YG, there must exist a shorter link uw kept in 3D YG and $\angle vuw < \theta$. Here θ is the largest angle possible in a 3D cone in FiYG or the half of the top angle of the 3D cone in FlYG. For simplification, we assume the maximum transmission range $R = 1$.

Theorem 1 *Both 3D SYG and 3D YYG are strongly connected if the original 3D UBG is connected and the angle parameter θ in 3D YG is bounded by $\pi/3$.*

PROOF. We first prove the connectivity of 3D SYG, which is equivalent to prove that there is a directed path from u to v in SYG for any two nodes u and v with $||uv|| \le 1$. We prove this claim by an induction over the distance $||uv||$ between nodes u and v. First, note that the edge between the closest pair of nodes must be kept in SYG. Assume that the claim is true for all links less than $||uv||$. Now we consider nodes u and v. If uv is kept in SYG, the claim is true. If uv is not in SYG, there must a node w inside one of 3D cones at u or v who causes the deletion of uv. Assume w and v are in the same cone of u and $||uw|| < ||uv||$. Because the angle $\angle wuv$ is less than $\theta \le \frac{\pi}{3}$, we have $||vw|| < ||uv||$. By induction there is a path from u to w and a path from w to v in SYG. Therefore a path from u to v exists in SYG. This finishes the proof for SYG.

We next prove that SYG is a subgraph of YYG, which then can imply the connectivity of 3D YYG automatically. Assume that there exists a link uv in SYG but not in YYG. From the definition of SYG, we know link uv is selected by both u and v in 3D YG. Then if we apply reverse Yao structure on incoming neighbor of 3D YG (as Line 4 in Algorithm 3), uv will also be selected by node v. Thus, uv must be in YYG. This is a

contradiction. Therefore, YYG is a supergraph of SYG and fully connected. □

The above theorem shows that our proposed algorithms can guarantee the connectivity of the final structure given that the underlying UBG is connected. In other words, our algorithms only remove links whose deletions will not affect the connectivity. Therefore, if the underlying UBG is already very sparse, our algorithms will not remove many links or even any links. But if the UBG is denser, more links will be removed by our proposed algorithms. This has been confirmed by our simulations (presented in Section 5). Notice that how to guarantee the connectivity of the UBG (where every node uses the maximum transmission power) is beyond the scope of this article. However, there are several existing studies [31,33-35,54] on how to set up the maximum transmission power to guarantee the network connectivity.

Theorem 2 *The node degree of 3D SYG is bounded by k while both node out-degree and in-degree of 3D YYG is bounded by k, where k is the degree bound of underlying 3D YG.*

PROOF. This theorem is straightforward from the construction methods of SYG and YYG. Both methods first apply 3D YG. Since each node has at most k 3D cones during this construction, the out-degree is bounded by k. For 3D SYG, a link is kept only if both endpoints keep it in 3D YG. Thus, the node degree of SYG is obviously bounded by k. For 3D YYG, the second round of 3D YG is applied to incoming links, thus the node in-degree is also bounded by k. Notice that in 3D YYG, the out-degree and in-degree neighbors of a node may be different set of nodes. □

Theorem 3 *The 3D SYG is not a power spanner of UBG, while 3D YYG is a power spanner of UBG when β ≥ 3 and θ < π /3.*

PROOF. The first half of this theorem can be directly obtained from a result by Grunewald et al. [16]. They basically show how to construct a counter example of a 2D network in which SYG is not a power spanner. Since the 2D network is a special case of 3D networks, the same counter example works for 3D networks.

The proof of power spanner property of YYG is much challenging, even in 2D. Jia et al. [58] first proved that 2D YYG is a power spanner when $\theta \le \pi / 60$ (i.e., $k \ge 120$). It seems that their proof might be extended to 3D case, however, the node degree bound will be huge (larger than $\frac{4\pi/3}{2\pi(1-\cos(\theta/2))/3} \ge 5836$). Thus it is not very useful in practice. Schindelhauer et al. [59] then proved that 2D YYG is a power spanner with power spanning ratio $(8c + 1)^2 \frac{(2c)^\beta}{1-2^{(2-\beta)}}$ for $\beta > 2$ when $k > 6$. Here $c = \frac{1}{1-2\sin(\pi/k)}$. They proved this by first proving that 2D YYG is a weak c-spanner. In a weak c-spanner, between any two nodes there exists a path which remains within a disk or sphere of radius c-times the Euclidean distance between them. Their proof of weak spanner property of YYG can also be extended to 3D YYG with $\theta < \pi / 3$. However, to further extend it to 3D power spanner, it requires $\beta \ge 3$. More specifically, 3D YYG is a power spanner with power spanning ratio $(8c + 1)^3 \frac{(2c)^\beta}{1-2^{(3-\beta)}}$ for $\beta > 3$ or $O(c^{12})$ for $\beta = 3$ when $\theta < \pi/3$. Therefore, we can claim that 3D YYG is a power spanner for $\beta \ge 3$ and $\theta < \pi / 3$. When $2 \le \beta < 3$, the power spanner property is still open. □

Table 1 summarizes the properties of all proposed and existing 3D topologies. Notice that the time complexity of 3D Yao-based structures is $O(d)$ for fixed partitions and $O(d \log d)$ for flexible partitions. Here d is the number of 1-hop neighbors. In addition, the power stretch factor of 3D YYG is $O(1)$ only for $\beta \ge 3$, but still open for $2 \le \beta < 3$.

5 Simulation results

In order to evaluate the performance of our new 3D topologies with bounded degree, we conduct simulations by generating random 3D sensor networks. In our experiments, we randomly generate a set V of n wireless nodes and the UBG, then test the connectivity of UBG. If it is connected, we construct different localized 3D topologies proposed in this article and some existing ones, and measure the node degree as well as power efficiency of these topologies. We repeat the experiment

Table 1 Property summary for existing and proposed 3D topologies

3D structure	connectivity	out-degree	in-degree	power stretch factor	message	time
3D LMST	yes	$O(1)$	$O(1)$	$O(n)$	$O(n)$	$O(d^2)$
3D RNG	yes	$O(n)$	$O(n)$	$O(n)$	$O(n)$	$O(d)$
3D GG	yes	$O(n)$	$O(n)$	1	$O(n)$	$O(d)$
3D YG	yes	$O(1)$	$O(n)$	$O(1)$	$O(n)$	$O(d)$ or $O(d \log d)$
3D SYG	yes	$O(1)$	$O(1)$	$O(n)$	$O(n)$	$O(d)$ or $O(d \log d)$
3D YYG	yes	$O(1)$	$O(1)$	$O(1)$	$O(n)$	$O(d)$ or $O(d \log d)$

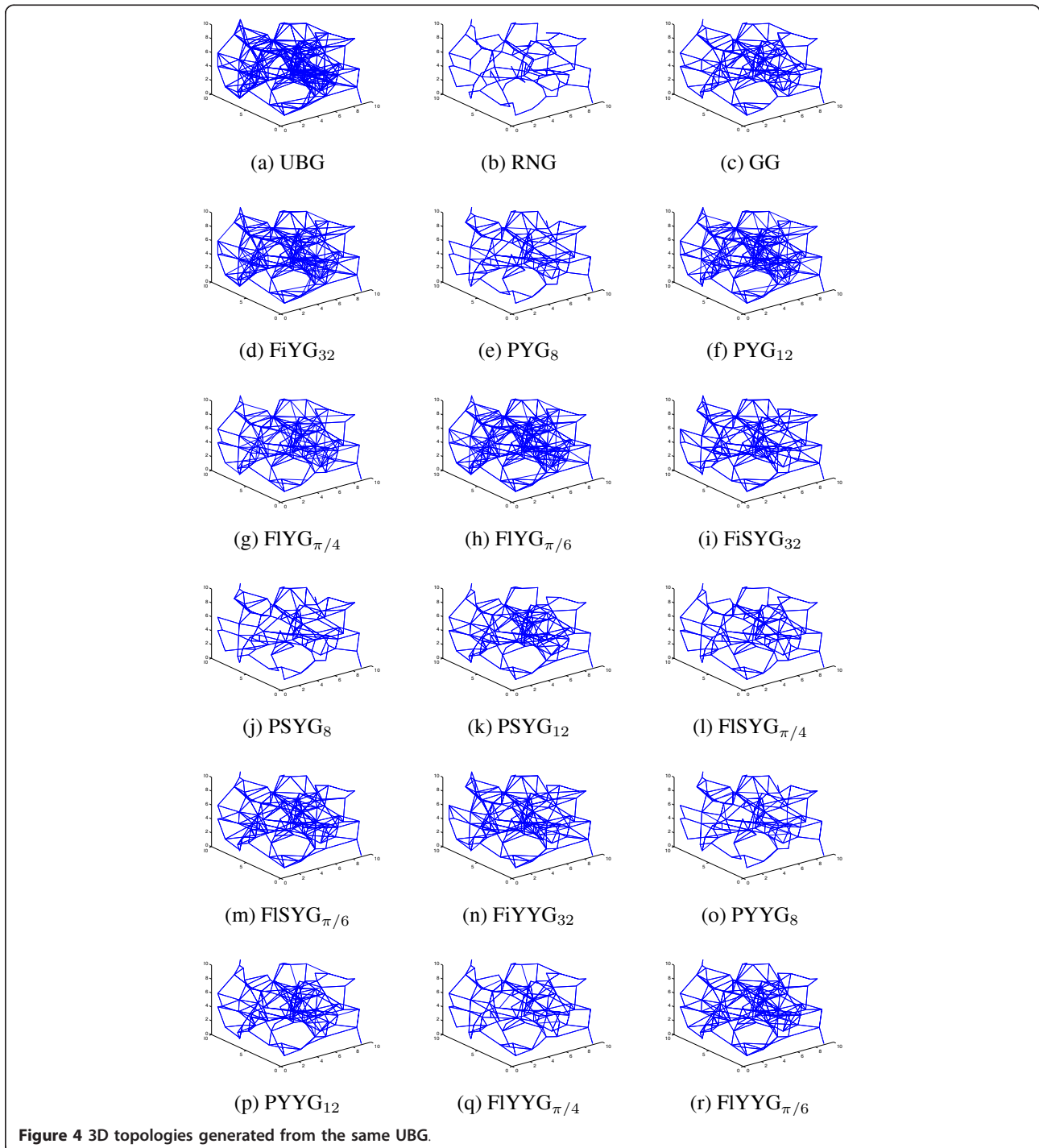for multiple times and report the average or maximum values of these metrics in all of the simulations.

We evaluate the following localized 3D topologies:

• Sparse topologies: RNG and GG;

• 3D Yao structures: $FiYG_{32}$, PYG_8, PYG_{12}, $FlYG_{\pi/4}$, and $FlYG_{\pi/6}$;

• 3D symmetric Yao structures: $FiSYG_{32}$, $PSYG_8$, $PSYG_{12}$, $FlSYG_{\pi/4}$, and $FlSYG_{\pi/6}$;

• 3D Yao & reverse Yao Structures: $FiYYG_{32}$, $PYYG_8$, $PYYG_{12}$, $FlYTG_{\pi/4}$, and $FlYYG_{\pi/6}$;

Here, *SYG and *YYG are the symmetric Yao structure and Yao & Reverse Yao structure based on the corresponding underlying 3D Yao structures, respectively.

Figure 4 shows a set of topologies generated for a UBG with 100 wireless nodes. In the experimental results presented here, we generate n random wireless

(a) UBG

(b) RNG

(c) GG

(d) $FiYG_{32}$

(e) PYG_8

(f) PYG_{12}

(g) $FlYG_{\pi/4}$

(h) $FlYG_{\pi/6}$

(i) $FiSYG_{32}$

(j) $PSYG_8$

(k) $PSYG_{12}$

(l) $FlSYG_{\pi/4}$

(m) $FlSYG_{\pi/6}$

(n) $FiYYG_{32}$

(o) $PYYG_8$

(p) $PYYG_{12}$

(q) $FlYYG_{\pi/4}$

(r) $FlYYG_{\pi/6}$

Figure 4 3D topologies generated from the same UBG.

nodes in a $10 \times 10 \times 10$ cube; the maximum transmission range R is set to $\sqrt{10}$ and the power constant $\beta = 2$, thus the maximum transmission power $P_{\max} = R^2 = 10$. It is clear that RNG is the sparsest structure,

however, we know that it is not a power spanner of UBG. For all Yao structures, the more 3D cones defined (larger k or smaller θ) the denser the topology is. For the structures based on the same 3D Yao construction
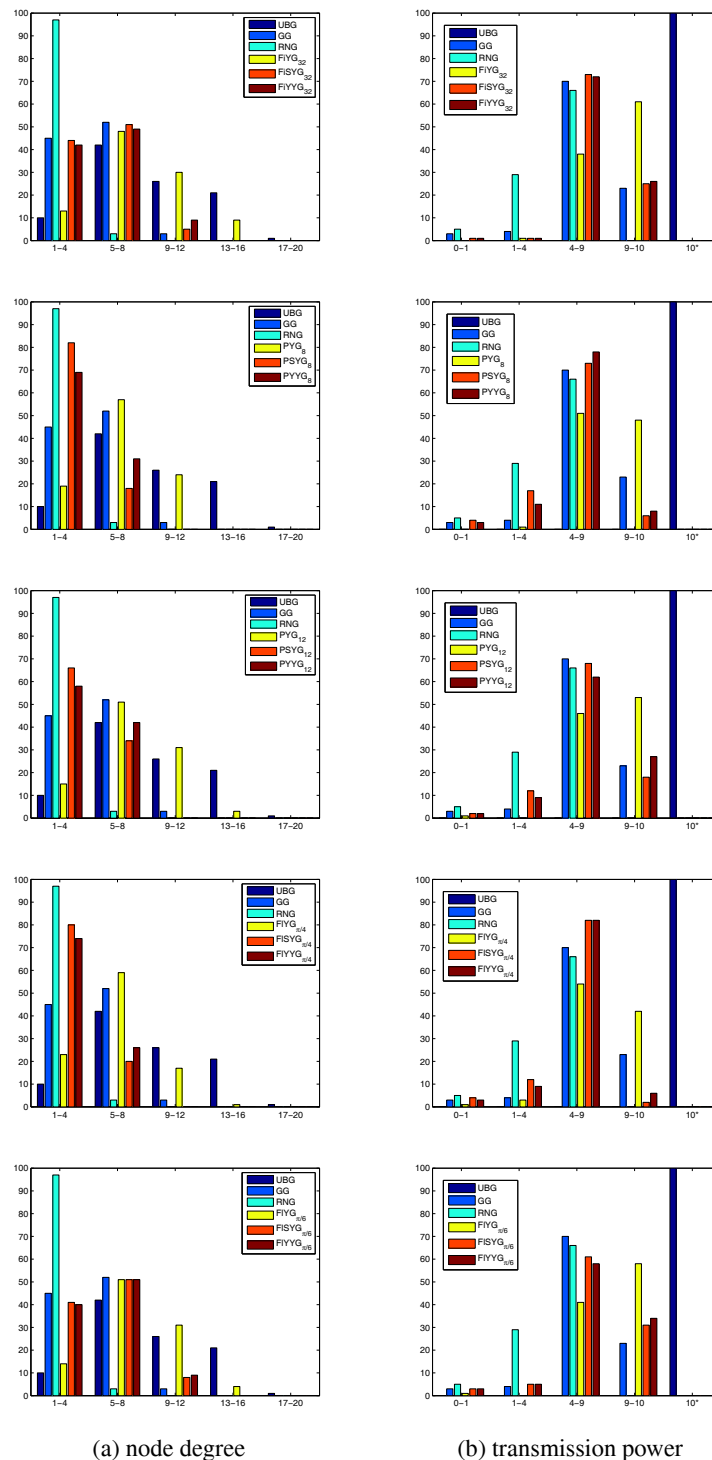
Figure 5 Degree distributions and transmission power distributions. (**a**) shows the distributions of node degrees of the UBG and other topologies; (**b**) shows the distributions of final assigned transmission power levels of nodes.

method, symmetric Yao structure is sparser than Yao & reverse Yao structure.

For the same instance, we plotted the node degrees of different topologies in Figure 5a. It is clear that UBG has more nodes with high node degree. While RNG, GG and all Yao structures can drastically reduce node degrees. Figure 5b shows the assigned minimal power levels for all the nodes. Here we assume that each node
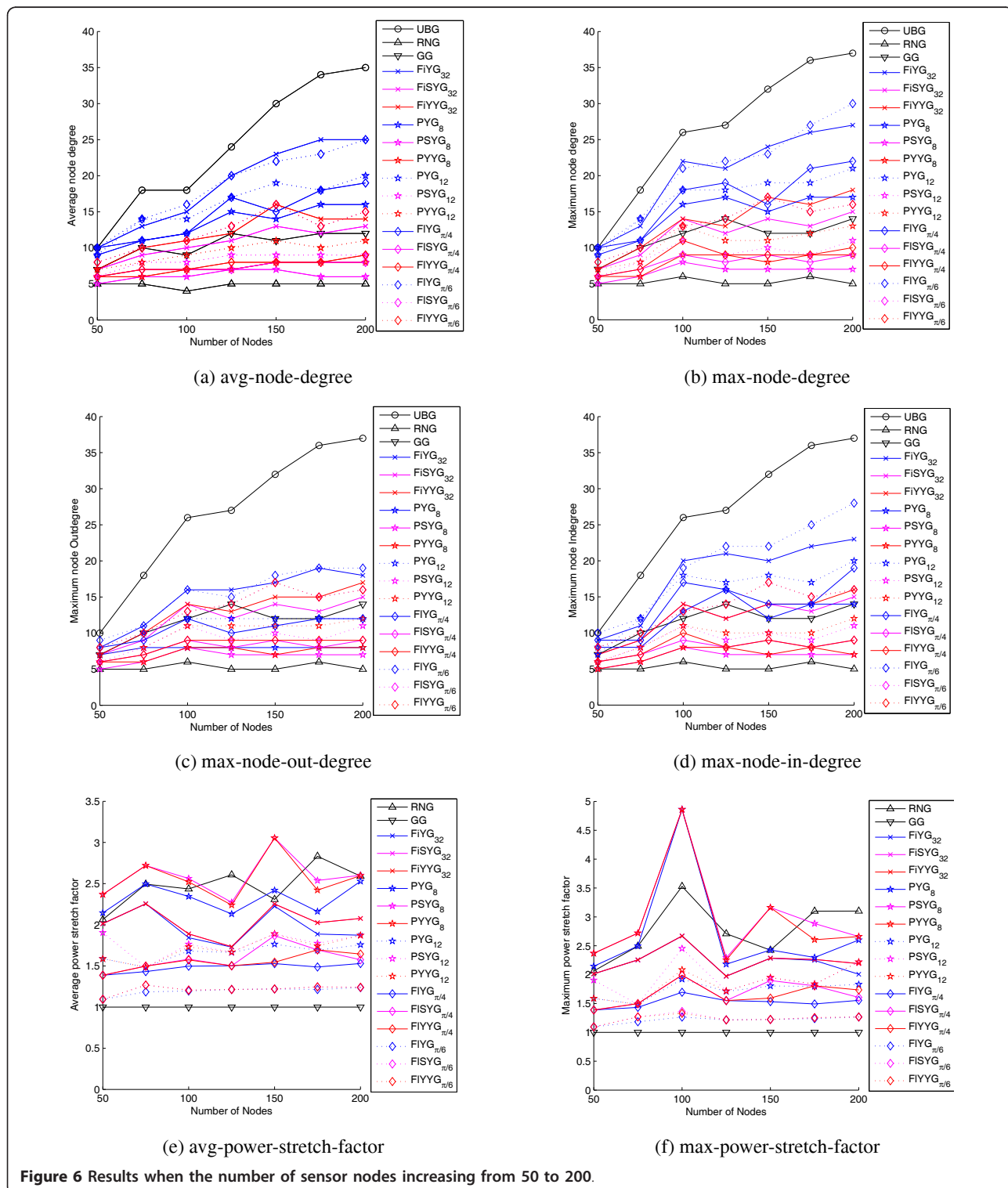

Figure 6 Results when the number of sensor nodes increasing from 50 to 200.

can shrink its power level to support the longest link in the generated topology. Clearly, no node needs to transmit at its maximum power level anymore in these sparse topologies. RNG and GG use the smallest power level, since they are the sparsest graphs. All Yao structures can also save a lot of energy compared with UBG. Clearly, more 3D cones defined in Yao structure, less energy it saves and larger node degree it has.
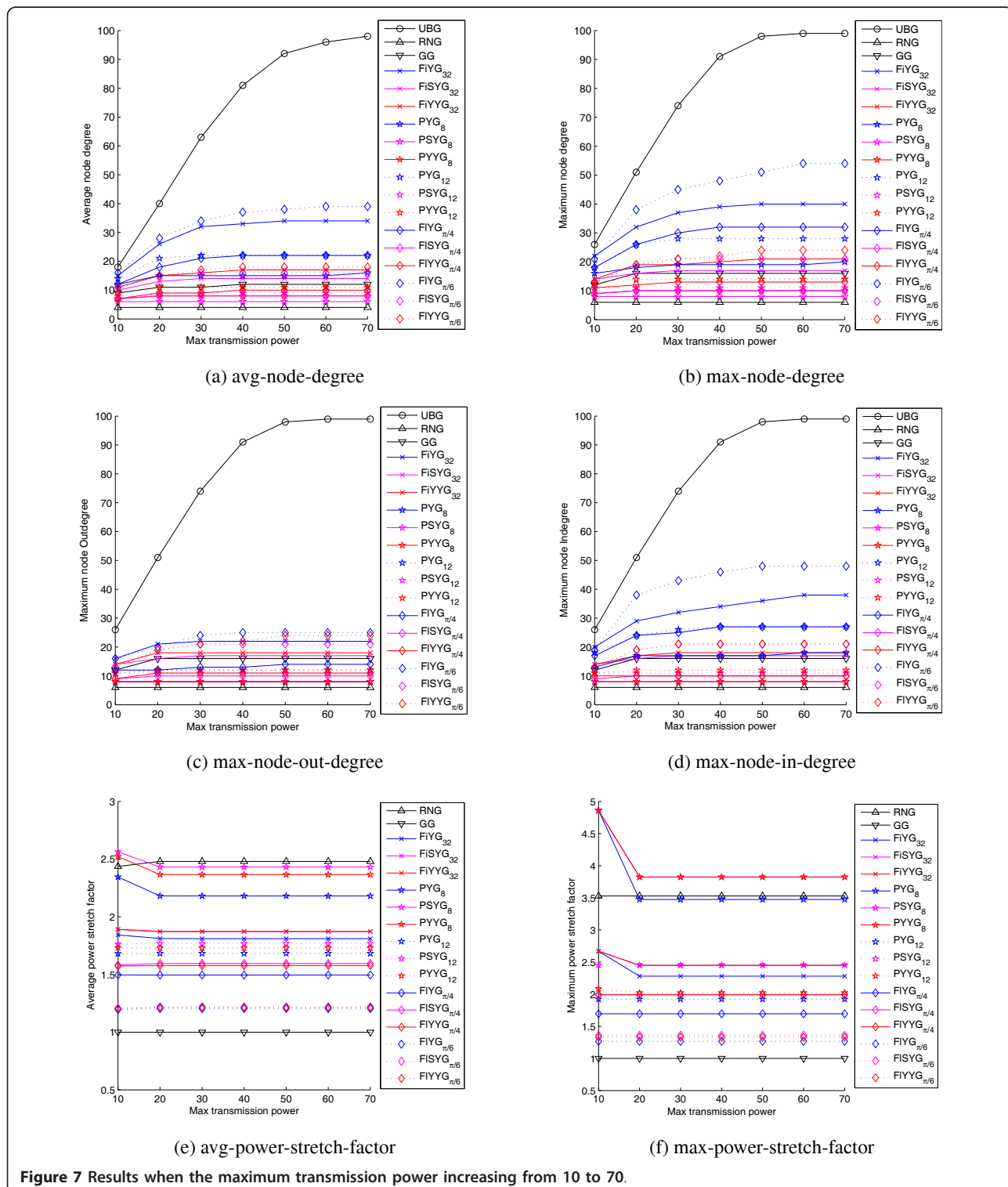

(a) avg-node-degree

(b) max-node-degree

(c) max-node-out-degree

(d) max-node-in-degree

(e) avg-power-stretch-factor

(f) max-power-stretch-factor

Figure 7 Results when the maximum transmission power increasing from 10 to 70.

We vary the number of nodes n in the network from 50 to 200, where 100 vertex sets are generated for each case. The average and the maximum are computed over all these 100 vertex sets. All experimental results are plotted in Figure 6.

The node degree of wireless networks should not be too large to avoid interference, collision, and overhead. It should not be too small either: a low node degree usually implies that the network has a low fault tolerance and tends to increase the overall network power consumption as longer paths may have to be taken. Figure 6a shows all localized 3D topologies have lower average degrees compared with UBG and keep small degrees when the degree of UBG becomes denser and denser as the number of nodes in the network increases. Clearly, Yao structures with larger number of 3D cones lead to denser structures, while *RNG* is much sparser than most of Yao-based structures. Notice that the structures sparser than *GG* are Symmetric Yao structures and Yao & Reverse Yao structures of PYG_{12}, PYG_8 and $FlYG_{\pi/4}$ since these structures have smallest 3D cones. These can also be verified by Figure 4 and 6b (the maximum node degree). Figure 6c shows that the maximum node out-degree of these localized 3D-topologies are small. The out-degrees of all Yao structures are smaller than the theoretical bounds. Figure 6d also gives the maximum node in-degree of these topologies which are a little bit larger than their out-degrees. It is clear that both symmetric Yao structures and Yao & reverse Yao structures have smaller in-degrees than 3D Yao structures. Remember that theoretically they can bound the node in-degree. The results also showed that RNG and GG do not have large degrees and 3D Yao structures do not have large in-degree in this experiment, and the reasons are that the nodes are distributed randomly in the area. In real life, the network may not be distributed randomly, so it is possible that RNG and GG have large degrees and 3D Yao structures have large in-degrees. Such examples and simulation results can be found in [19].

Besides connectivity, the most important design metric of wireless networks is perhaps power efficiency, because it directly affects both nodes and network lifetime. Figure 6e,f show all proposed structures have small power stretch factors even when the network is very dense. Notice that Yao structures based on PYG_8 and RNG have a little bit higher stretch factor than GG and other Yao-based structures, however, their maximum power stretch factors are still smaller than 5. The reason again is that the nodes are distributed randomly in the area and the number of nodes is not too large. As we expected, GG has a power stretch factor of one and all power stretch factors of 3D Yao structures are smaller than their theoretical bounds if they have ones.

Among all 3D Yao structures, $FlYG_{\pi/6}$ and $FlYYG_{\pi/6}$ have the smallest stretch factor, since they are the densest 3D Yao structures.

We also conduct simulations on random networks with fixed 100 nodes and various maximum transmission power $P_{\max}$ is from 10 to 70. Simulation results are plotted in Figure 7. Basic conclusions from this set of simulations are coherent with previous simulation set.

6 Conclusion

Topology control for 3D wireless sensor networks has been well studied recently and different 3D geometric topologies were proposed to achieve the sparseness and power efficiency. However, most of them cannot bound the node degree. Even though some of 3D structures based on Yao graph can bound the node out-degree, they may still lead to large in-degree at some nodes. Therefore, in this article, we propose two general frameworks to build degree-bounded 3D topologies for wireless sensor networks. These frameworks can use all existing Yao-based 3D structures as the underlying methods, and only use local information with linear number of messages. We show some of them can also guarantee the constant power stretch factor in 3D. Our simulation results confirm the good performance of our new 3D topologies.

Endnote

[a]As shown in Figure 2b, the angle between two neighbors in $FlYG_\theta$ must be larger than θ, i.e., $\angle v_i u v_j < \theta$.

Author details
[1]School of Computer Science, Beijing Institute of Technology, Beijing, 100081, China [2]Department of Computer Science, University of North Carolina at Charlotte, Charlotte, NC 28223, USA

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