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Amplify-forward relaying for multiple-antenna multiple relay networks under individual power constraint at each relay

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Abstract

This article considers the design of an optimal beamforming weight matrix of multiple-antenna multiple-relay networks. It is assumed that each relay utilizes the amplify and forward strategy, i.e., it multiplies the received signal vector by a matrix, dubbed the relay weight matrix, and forwards the resulting vector to the destination. Furthermore, we assume that the source and the destination have the same number of antennas and that each transmit antenna is virtually paired to a different destination antenna. The relay weight matrices are concurrently designed to optimize the mean square error (MSE) criterion at the destination, assuming each relay node is subject to a power constraint. Accordingly, it is demonstrated that this problem can be cast as a convex optimization problem in which the individual power constraints are tackled by employing the method of Lagrange multipliers in two stages. First, the relay gain matrix is computed analytically in terms of Lagrange dual variables, thereby converting the original problem into a scalar optimization problem. Then, these scalar variables are computed numerically. The proposed scheme is evaluated through simulation with various numbers of relays and antennas to obtain MSE and bit error rate (BER) metrics and it is shown that the resulting MSE and BER achieved through using the proposed method outperforms that of MMSE-MMSE method introduced by Oyman et.al., which is regarded as the best known method for the underlying problem.

Keywords: co-operative communication, multiple-antenna multiple-relay networks, convex optimization, amplify and forward relaying

1. Introduction

It is well established that in most cases relaying techniques provide considerable advantages over direct transmission, provided that the source and relay cooperate efficiently. The choice of relay function is especially important as it directly affects the potential capacity benefits of node cooperation [1-5]. In this regard, two relaying methods, amplify-forward (AF) [6,7] and estimate-forward [8,9], are extensively addressed in the literature. As the names imply, the former just amplifies the received signal but the latter estimates the signal with errors and then forwards it to the destination.

It has been shown that increasing the number of relays has the advantage of increasing the diversity gain and flexibility of the network; however, it renders some new issues to arise [10]. For instance, the relaying algorithm and power allocation across relays should be addressed is such cases. Relay selection [11,12] and power allocation [13,14] are two well-known methods when dealing with the power management issues.

The capacity and reliability of the relay channel can be further improved by using multiple antennas at each node. The use of relays together with using multiple antennas has made it a versatile technique to be used in emerging wireless technologies [15-20]. Relaying strategies for the multi-antenna multiple-relay (MAMR) networks is more challenging than single-antenna networks, since in addition to scaling and phase operations, matrix operations should also employed at the relays.

AF MIMO relay systems have drawn considerable attention in the literature due to their simplicity and ease of implementation. In this regard, a plethora of

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works are devoted to finding a proper relaying strategy for AF MAMR networks. In [21], the idea of linear distributed multi-antenna relay beamforming is introduced where each relay performs a linear reception and transmission in addition to output power normalization. In this article, K single antenna transmitted independent data streams to their respected single antenna receivers. The linear operations suggested in this article are matched filter, zero forcing, and minimum mean square error (MMSE). They are briefly called MF-MF, ZF-ZF, and MMSE-MMSE schemes, respectively. In [22], a method based on QR decomposition is suggested which works better than the ZF-ZF scheme. Combinations of various schemes are also considered in [22]. For example in ZF-QR scheme, relays perform ZF algorithm in reception and QR algorithm (channel triangulation) in transmission.

In [23], the so-called incremental cooperative beamforming is introduced and it is shown that it can achieve the network capacity in the asymptotic case of large K with a gap no more than $O(1/\log(K))$. However, this method is not suited when few relays are incorporated since this method only works properly when the number of relays tends to infinity.

In [24], a wireless sensor network that is composed of some multi-antenna sensors aimed to transmit a noisy measurement vector parameter to the fusion centre is formulated as a MAMR network. Moreover, it is assumed that the second hop associated with the resulting MAMR network has a diagonal channel matrix and the destination noise is small enough to be ignored. The current manuscript is actually an extension of [24] since neither the channel matrices need to be diagonal nor the destination noise is restricted to be zero.

In [25], it is shown that an MAMR network with single-antenna source and destination can be transformed to a single-antenna multiple relay (SAMR) network by performing maximal ratio combining at reception and transmission for each relay nodes. This enables the network beamforming introduced in [14] to be readily employed.

In [26], by using ZF-ZF scheme, an MAMR network with M single-antenna source-destination pairs is transformed to M SAMR networks to which network-beamforming proposed in [14] is applied.

In [27], the relay gain matrices are obtained by maximizing the MSE at destination restricting the received power at the destination. In [28], a linear relaying scheme for an MAMR network fulfilling the target SNRs on different independent substreams transmitted from each source antennas is proposed and the power-efficient relaying strategy is derived in closed form. In [29], a nearly optimal relaying scheme is proposed to maximize the mutual information between the source and the destination under total relay power constraint.

In this article, the problem of MAMR network with multiple antennas at source and destination with individual relays power constraints is formulated as a convex optimization problem. The optimum relay gain matrices are obtained by solving the optimization problem using Lagrange dual variables method. This relays gain matrices are obtained in terms of K scalar variables where K is the number of relays. Then those variables are computed numerically. As noted before, the articles that investigate this configuration either suggest the relay gain matrix heuristically or concern another constraint such as a limited power constraint at the destination, the destination quality of service or the sum power of relays. In our opinion, the limited power for each relay is a more realistic assumption, because each relay in the network has its own power supply and unused power for each relay cannot be used by other relays. In the same manner as [26-29], complete CSI is considered to be available for optimum relay design. The optimization can be performed at the destination, and then the processing results are fed back to the relays. Although the closed form formula is not obtained but a parametric relation form of the relay gain matrices are derived. These parameters can be calculated either numerically or heuristically. A simpler form of the relay gain matrices is derived for the two relay case. The initial works on this issue are first addressed in [30] while the optimal solution is not fully treated there.

2. System model

Figure 1 illustrates a typical MAMR relay network system in which there are M single-antenna sources, trying to send independent data streams through K multi-antenna relays to their affiliated single-antenna

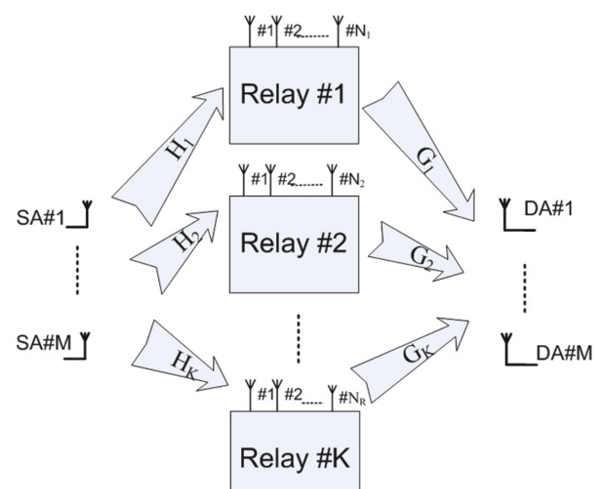

Figure. 1

Figure 1 A typical MAMR relay system model.

destinations. In fact, the aim is to send independent data streams from each source antenna to the corresponding single-antenna destination. Thus, each single-antenna destination can merely apply a simple scaling to its received signal and the integral part of the interference cancellation process must be performed at multi-antenna relays.

It is assumed that the ith relay has N_i antennas. Hence, the transmission occurs in two hops. During the first hop, the transmitter broadcasts the desired signal to the relays. Then, throughout the second hop, each relay applies a weight matrix to the received signal vector and retransmits it to the destination.

We consider $\mathbf{x}$ as an $M \times 1$ vector whose elements are independent zero mean Gaussian random variables with covariance matrix $\mathbf{E}(\mathbf{x}\mathbf{x}^{\mathrm{H}}) = \mathbf{P}_{\mathrm{s}}\mathbf{I}_{\mathrm{M}}$ Thus, the received signal vector at the ith relay can be represented as

$$\mathbf{y}_i = \mathbf{H}_i\mathbf{x} + \mathbf{n}_i, \tag{1}$$

where $\mathbf{n}_i$ is a $N_i \times 1$ Gaussian noise vector, representing the input noise vector at the ith relay with the covariance matrix $\mathrm{E}\left(\mathbf{n}_i\mathbf{n}_i^{\mathrm{H}}\right) = P_{\mathrm{n}_i}\mathbf{I}_{N_i}$ where $\mathbf{I}_{N_i}$ denotes the identity matrix and $\mathrm{P}_{\mathrm{n}_i}$ is the noise power associated with each entry of $\mathbf{n}_i$. $\mathbf{H}_i$ is a known $N_i \times M$ matrix with complex elements, representing the channel gain matrix between the transmitter and the ith relay. Moreover, $(.)^{\mathrm{H}}$ is Hermitian operation. Assuming the ith relay multiplies its received signal by a weight matrix $\mathbf{W}_i$ and forwards the resulting vector, $\mathbf{x}_i$, to the destination, thus

$$\mathbf{x}_i = \mathbf{W}_i\mathbf{y}_i = \mathbf{W}_i\left(\mathbf{H}_i\mathbf{x} + \mathbf{n}_i\right) = \mathbf{W}_i\mathbf{H}_i\mathbf{x} + \mathbf{W}_i\mathbf{n}_i. \tag{2}$$

$$\mathrm{P}_i^{\mathrm{out}} = \mathrm{E}\left(|\mathbf{x}_i|^2\right) = \mathrm{E}\left(|\mathbf{W}_i\mathbf{H}_i\mathbf{x} + \mathbf{W}_i\mathbf{n}_i|^2\right) \le \mathrm{P}_{\mathrm{r}_i}, \tag{3}$$

where $\mathrm{P}_i^{\mathrm{out}}$ is the average transmit power which is assumed to be lower than $\mathrm{P}_{\mathrm{r}_i}$, considering $\|.\|$ is *frobenius norm.* Thus, referring to Figure 1, it follows

$$\mathbf{y} = \sum_{i=1}^{K}\mathbf{G}_i\mathbf{x}_i + \mathbf{n} = \sum_{i=1}^{K}\mathbf{G}_i\mathbf{W}_i\mathbf{y}_i + \mathbf{n} = \sum_{i=1}^{K}\mathbf{G}_i\mathbf{W}_i\mathbf{H}_i\mathbf{x} + \sum_{i=1}^{K}\mathbf{G}_i\mathbf{W}_i\mathbf{n}_i + \mathbf{n} \tag{4}$$

where $\mathbf{G}_i$ is the $M \times N_i$ channel gain matrix between the ith relay and the destination whose entries are complex and assumed to be known completely at the destination. Also, $\mathbf{n}$ is an $M \times 1$ zero-mean noise vector whose entries are of power $P_{\mathrm{n_d}}$ Finally, $\mathbf{n}_i$ for $i = 1,2,\ldots,$ K and $\mathbf{n}$ are assumed to be statistically independent.

Furthermore, as it is noted earlier, a scalar operation is merely done at each destination. In other words, the weight matrices $\mathbf{W}_i$ for $i = 1,2,\ldots, K$ are computed so that the received vector $\mathbf{y}$ is a scaled unbiased estimation of the transmitted vector $\mathbf{x}$. Note that when sources and destinations are equipped with multiple antennas, joint precoder and reception matrices must be concurrently designed along with the relay matrices. However, this is a completely different problem which is out of the scope of the current work. It should be emphasised that since there is a correspondence between each source and its affiliated destination, the number of sources and destinations remains the same.

3. Optimization problem

In this section, we aim at addressing the problem formulation using the MSE criterion, assuming each relay is subject to an individual power constraint. In what follows, we first formalize and then present the proposed approach to get the optimal solution. Referring to (3) and (4), the optimization problem can be represented as

$$\begin{cases} \min\limits_{\mathbf{W}_1,\ldots,\mathbf{W}_K} \xi = \mathrm{E}_{\mathbf{x},\mathbf{n}_1,\ldots,\mathbf{n}_K,\mathbf{n}}\left\{|y - \eta\mathbf{x}|^2\right\} \\ \text{w.r.t} \quad \mathrm{P}_{\mathrm{s}}\|\mathbf{W}_i\mathbf{H}_i\|^2 + \mathrm{P}_{\mathrm{n}_i}\|\mathbf{W}_i\|^2 < \mathrm{P}_{\mathrm{r}_i} \qquad i = 1\ldots K. \end{cases} \tag{5}$$

where η is a positive constant value which affects the signal power and consequently the resulting SNR at the destination. The choice of η would ensure a certain target SNR at the destination as follows [31]:

$$\eta = \sqrt{\gamma_t\frac{P_n}{P_s}} \tag{6}$$

where γ_t is the target SNR. Although increasing η can increase the SNR, there is a threshold beyond which the choice of η cannot improve the SNR and merely increases the noise power [27]. Finding the best value for η is a difficult task when relying upon analytical methods; one can think of numerical methods to tackle a relation close to optimal solution. Section 5 aims at addressing this issue. In what follows we assume η is a known parameter. Thus, from (5) the objective function can be expanded as

$$\xi = \mathrm{E}_{\mathbf{x},\mathbf{n}_1,\ldots,\mathbf{n}_K,\mathbf{n}}\left\{\left|\sum_{i=1}^{K}\mathbf{G}_i\mathbf{W}_i\mathbf{H}_i\mathbf{x} + \sum_{i=1}^{K}\mathbf{G}_i\mathbf{W}_i\mathbf{n}_i + \mathbf{n} - \eta\mathbf{x}\right|^2\right\} \tag{7}$$

$$\begin{aligned} &= \mathrm{P}_{\mathrm{s}}\left\|\sum_{i=1}^{K}\mathbf{G}_i\mathbf{W}_i\mathbf{H}_i\right\|^2 + \sum_{i=1}^{K}\mathrm{P}_{\mathrm{n}_i}\|\mathbf{G}_i\mathbf{W}_i\|^2 \\ &\quad - 2\eta\sum_{i=1}^{K}\mathrm{P}_{\mathrm{s}}\mathrm{Re}\left\{\mathrm{tr}\left(\mathbf{G}_i\mathbf{W}_i\mathbf{H}_i\right)\right\} + M\mathrm{P}_{\mathrm{n}} + M\eta^2\mathrm{P}_{\mathrm{s}} \end{aligned} \tag{8}$$

Discarding the constant terms in (8), the original problem can be rewritten as

$$\begin{cases} \min .\mathrm{P_s}\left\|\sum_{i=1}^{K}\mathbf{G}_i\mathbf{W}_i\mathbf{H}_i\right\|^2 \\ \quad +\sum_{i=1}^{K}\mathrm{P}_{\mathrm{n}_i}\|\mathbf{G}_i\mathbf{W}_i\|^2 - 2\eta\mathrm{P_s}\sum_{i=1}^{K}\mathrm{Re}\left\{\mathrm{tr}\left(\mathbf{G}_i\mathbf{W}_i\mathbf{H}_i\right)\right\} \\ \mathrm{wrt}\quad \mathrm{P_s}\|\mathbf{W}_i\mathbf{H}_i\|^2 + \mathrm{P}_{\mathrm{n}_i}\|\mathbf{W}_i\|^2 - \mathrm{P}_{\mathrm{r}_i} < 0 \quad i = 1\ldots K \end{cases} \tag{9}$$

Without loss of generality, $\mathrm{P_s}$ can be set equal to one. The Lagrangian [32] associated with (9) can then be written as

$$\begin{aligned} &\mathrm{L}\left(\mathbf{W}_1,\mathbf{W}_2,\ldots,\mathbf{W}_K,\lambda_1,\lambda_2,\ldots,\lambda_K\right) = \\ &\left\|\sum_{i=1}^{K}\mathbf{G}_i\mathbf{W}_i\mathbf{H}_i\right\|^2 + \sum_{i=1}^{K}\mathrm{P}_{\mathrm{n}_i}\|\mathbf{G}_i\mathbf{W}_i\|^2 - 2\eta\sum_{i=1}^{K}\mathrm{Re}\left\{\mathrm{tr}\left(\mathbf{G}_i\mathbf{W}_i\mathbf{H}_i\right)\right\} \\ &+\sum_{i=1}^{K}\lambda_i\left(\|\mathbf{W}_i\mathbf{H}_i\|^2 + \mathrm{P}_{\mathrm{n}_i}\|\mathbf{W}_i\|^2 - \mathrm{P}_{\mathrm{r}_i}\right) \end{aligned} \tag{10}$$

where λ_i for $i = 1,\ldots,K$ are the corresponding Lagrange multipliers. The Lagrangian can be expressed as

$$\begin{aligned} &\mathrm{L}\left(\mathbf{W}_1,\mathbf{W}_2,\ldots,\mathbf{W}_K,\lambda_1,\lambda_2,\ldots,\lambda_K\right) = \\ &\left|\sum_{i=1}^{K}\mathrm{vec}\left(\mathbf{G}_i\mathbf{W}_i\mathbf{H}_i\right)\right|^2 + \sum_{i=1}^{K}\mathrm{P}_{\mathrm{n}_i}|\mathrm{vec}\left(\mathbf{G}_i\mathbf{W}_i\right)|^2 \\ &-2\eta\sum_{i=1}^{K}\mathrm{Re}\left\{\mathrm{vec}(\mathbf{I})^{\mathrm{T}}\left(\mathbf{H}_i{}^T\otimes\mathbf{G}_i\right)\mathrm{vec}\left(\mathbf{W}_i\right)\right\} \\ &+\sum_{i=1}^{K}\lambda_i|\mathrm{vec}\left(\mathbf{W}_i\mathbf{H}_i\right)|^2 + \sum_{i=1}^{K}\mathrm{P}_{\mathrm{n}_i}\lambda_i|\mathrm{vec}\left(\mathbf{W}_i\right)|^2 - \sum_{i=1}^{K}\lambda_i\mathrm{P}_{\mathrm{r}_i} \end{aligned} \tag{11}$$

where the fact $tr(\mathbf{AXB}) = \mathrm{vec}\ (\mathbf{I})^{\mathrm{T}}\ (\mathbf{B^T}\otimes\mathbf{A})\ \mathrm{vec}\ (\mathbf{X})$ from [33] is used in the third term in (11) and the fact that $\|\mathbf{A}\| = |\mathrm{vec}\,(\mathbf{A})|$ from [33] is used in the remaining terms.

Furthermore, using the fact that $\mathrm{vec}(\mathbf{AXB}) = (\mathbf{B^T}\otimes\mathbf{A})\ \mathrm{vec}(\mathbf{X})$ from [33], the Lagrangian can then be rewritten as

$$\begin{aligned} \mathrm{L} =& \\ &\left|\sum_{i=1}^{K}\left(\mathbf{H}_i{}^T\otimes\mathbf{G}_i\right)\mathrm{vec}\left(\mathbf{W}_i\right)\right|^2 \\ &+\sum_{i=1}^{K}\mathrm{P}_{\mathrm{n}_i}|\left(\mathbf{I}\otimes\mathbf{G}_i\right)\mathrm{vec}\left(\mathbf{W}_i\right)|^2 - 2\eta\sum_{i=1}^{K}\mathrm{Re}\left\{\mathrm{vec}(\mathbf{I})^{\mathrm{T}}\left(\mathbf{H}_i{}^T\otimes\mathbf{G}_i\right)\mathrm{vec}\left(\mathbf{W}_i\right)\right\} \\ &+\sum_{i=1}^{K}\lambda_i\left|\left(\mathbf{H}_i{}^T\otimes\mathbf{I}\right)\mathrm{vec}\left(\mathbf{W}_i\right)\right|^2 + \sum_{i=1}^{K}\lambda_i\mathrm{P}_{\mathrm{n}_i}|\mathrm{vec}\left(\mathbf{W}_i\right)|^2 - \sum_{i=1}^{K}\lambda_i\mathrm{P}_{\mathrm{r}_i}.(12) \end{aligned} \tag{12}$$

To simplify (12), the following matrix and vectors are defined:

$$\begin{aligned} &\mathbf{T_i} = \left(\mathbf{H}_i{}^T\otimes\mathbf{G}_i\right), \\ &\bar{\mathbf{G}}_i = \left(\mathbf{I}\otimes\mathbf{G}_i\right), \\ &\mathbf{f}_i^T = \mathrm{vec}(\mathbf{I})^{\mathrm{T}}\left(\mathbf{H}_i{}^T\otimes\mathbf{G}_i\right) = \mathrm{vec}(\mathbf{I})^{\mathrm{T}}\mathbf{T_i}, \\ &\bar{\mathbf{H}}_i = \left(\mathbf{H}_i{}^T\otimes\mathbf{I}\right), \\ &\mathbf{w}_i = \mathrm{vec}\left(\mathbf{W}_i\right) \end{aligned} \tag{13}$$

We can reformulate the Lagrangian (12) as

$$\begin{aligned} \mathrm{L} =& \left|\sum_{i=1}^{K}\mathbf{T}_i\mathbf{w}_i\right|^2 + \sum_{i=1}^{K}\mathrm{P}_{\mathrm{n}_i}\left|\bar{\mathbf{G}}_i\mathbf{w}_i\right|^2 - 2\eta\sum_{i=1}^{K}\mathrm{Re}\left\{\mathbf{f}_i^T\mathbf{w}_i\right\} + \sum_{i=1}^{K}\lambda_i\left|\bar{\mathbf{H}}_i\mathbf{w}_i\right|^2 \\ &+\sum_{i=1}^{K}\mathrm{P}_{\mathrm{n}_i}|\lambda_i\mathbf{w}_i|^2 - \sum_{i=1}^{K}\mathrm{P}_{\mathrm{r}_i}\lambda_i. \end{aligned} \tag{14}$$

To obtain the optimum $\mathbf{w}_p$ s, the differentiation of the Lagrangian with respect to $\mathbf{w}_p$ ($p = 1,2\ldots, K$) has to be set to zero:

$$\begin{aligned} \frac{\partial \mathrm{L}}{\partial \mathbf{w}_p} =& 2\left(\mathbf{T}_p{}^H\mathbf{T}_p + \lambda_p\bar{\mathbf{H}}_p^H\bar{\mathbf{H}}_p + \mathrm{P}_{\mathrm{n}_p}\bar{\mathbf{G}}_p^H\bar{\mathbf{G}}_p + \mathrm{P}_{\mathrm{n}_p}\lambda_p\mathbf{I}\right)\mathbf{w}_p \\ &+ 2\sum_{\substack{i=1\\ i\neq p}}^{\mathrm{K}}\left(\mathbf{T}_p{}^H\mathbf{T}_i\mathbf{w}_i\right) - 2\eta^{-1}\mathbf{f}_p{}^*.p = 1, 2\ldots, \mathrm{K} \end{aligned} \tag{15}$$

Setting the derivation to zero, it can be concluded that

$$\begin{aligned} &\frac{\partial \mathrm{L}}{\partial \mathbf{w}_i} = 0\ i = 1\ldots K \\ &\Rightarrow \begin{cases} \left(\mathbf{T}_1{}^H\mathbf{T}_1 + \lambda_1\bar{\mathbf{H}}_1^H\bar{\mathbf{H}}_1 + \mathrm{P}_{\mathrm{n}_1}\bar{\mathbf{G}}_1^H\bar{\mathbf{G}}_1 + \mathrm{P}_{\mathrm{n}_1}\lambda_1\mathbf{I}\right)\mathbf{w}_{1o} \\ \quad +\sum_{i=2}^{\mathrm{K}}\left(\mathbf{T}_1{}^H\mathbf{T}_i\mathbf{w}_{io}\right) = \eta\mathbf{f}_1{}^* \\ \quad\vdots \\ \left(\mathbf{T}_p{}^H\mathbf{T}_p + \lambda_p\bar{\mathbf{H}}_p^H\bar{\mathbf{H}}_p + \mathrm{P}_{\mathrm{n}_p}\bar{\mathbf{G}}_p^H\bar{\mathbf{G}}_p + \mathrm{P}_{\mathrm{n}_p}\lambda_p\mathbf{I}\right)\mathbf{w}_{po} \\ \quad +\sum_{\substack{i=1\\ i\neq p}}^{\mathrm{K}}\left(\mathbf{T}_p{}^H\mathbf{T}_i\mathbf{w}_{io}\right) = \eta\mathbf{f}_p{}^* \\ \quad\vdots \\ \left(\mathbf{T}_K{}^H\mathbf{T}_K + \lambda_K\bar{\mathbf{H}}_K^H\bar{\mathbf{H}}_K + \mathrm{P}_{\mathrm{n}_K}\bar{\mathbf{G}}_K^H\bar{\mathbf{G}}_K + \mathrm{P}_{\mathrm{n}_K}\lambda_K\mathbf{I}\right)\mathbf{w}_{Ko} \\ \quad +\sum_{i=1}^{\mathrm{K}-1}\left(\mathbf{T}_K{}^H\mathbf{T}_i\mathbf{w}_{io}\right) = \eta\mathbf{f}_K{}^* \end{cases} \end{aligned} \tag{16}$$

If the following parameters are defined as follows:

$$\mathbf{w_o} = \left[\mathbf{w}_{1o}{}^T\ \ldots\ \mathbf{w}_{Ko}{}^T\right]^T\ \mathbf{f} = \left[\mathbf{f}_1{}^H\ \ldots\ \mathbf{f}_K{}^H\right]^T, \tag{17}$$

And also the sub-matrices$(\mathbf{A})_{pp}$ and $(\mathbf{A})_{pi}$ for $p,i = 1,\ldots, K$ define as

$$\begin{aligned} &(\mathbf{A})_{pp} \triangleq \left(\mathbf{T}_p{}^H\mathbf{T}_p + \lambda_p\overline{\mathbf{H}}_p{}^H\mathbf{H}_p + \mathrm{P}_{\mathrm{n}_p}\overline{\mathbf{G}}_p{}^H\overline{\mathbf{G}}_p + \mathrm{P}_{\mathrm{n}_p}\lambda_p\mathbf{I}\right), \\ &(\mathbf{A})_{pi} \triangleq \left(\mathbf{T}_p{}^H\mathbf{T}_i\right)\ i\neq p. \end{aligned} \tag{18}$$

Hence the matrix $\mathbf{A}$ is defined as

$$\mathbf{A} \triangleq \begin{bmatrix} (\mathbf{A})_{11} & & (\mathbf{A})_{1\mathrm{K}} \\ & \ddots & \\ (\mathbf{A})_{\mathrm{K}1} & & (\mathbf{A})_{\mathrm{KK}} \end{bmatrix} \tag{19}$$

Therefore, the relation (16) can be represented simply as

$$\sum_{i=1}^{K} (\mathbf{A})_{pi}\mathbf{w}_{io} = \eta \mathbf{f}_p^{*} \quad p = 1 \ldots K \tag{20}$$

or

$$\mathbf{A}\mathbf{w}_o = \eta \mathbf{f}. \Rightarrow \mathbf{w}_o = \eta \mathbf{A}^{-1}\mathbf{f}. \tag{21}$$

Then, by substituting (21) into (14) one can readily arrive at Lagrange dual problem [32], considering λ_i for $i = 1,...,K$ are non-negative values. Thus, maximizing the obtained dual object function yields the optimal values for the corresponding Lagrange coefficients. However, this dual problem is too complicated to differentiate, thus does not lead to an analytical solution. Hence, a numerical method, called the active set method [34], is employed to compute the Lagrange multipliers.

It is worth mentioning that the dual problem involves just K scalar variables, however, the primary problem contains K unknown matrices each of size $N_i \times N_i$. Thus, relying on dual problem, results in a simplification which can be effectively addressed through using the aforementioned numerical method.

Inserting the obtained $\mathbf{w}$ from (21) to (14), the Lagrange dual problem can be written as

$$\left\{ \begin{array}{l} \max . \left| \sum\limits_{i=1}^{K} \mathbf{T}_i \mathbf{w}_i \right|^2 + P_n \sum\limits_{i=1}^{K} \left| \bar{\mathbf{G}}_i \mathbf{w}_i \right|^2 \\ \quad -2\eta \sum\limits_{i=1}^{K} \mathrm{Re}\left\{ \mathbf{f}_i^T \mathbf{w}_i \right\} + \sum\limits_{i=1}^{K} \lambda_i \left| \bar{\mathbf{H}}_i \mathbf{w}_i \right|^2 \\ \quad + P_n \sum\limits_{i=1}^{K} \lambda_i |\mathbf{w}_i|^2 - \sum\limits_{i=1}^{K} \mathrm{P}_{\mathrm{r}_i} \lambda_i \\ \lambda_i > 0 \quad i = 1, \ldots, K \end{array} \right\} \tag{22}$$

From KKT condition [32], if λ_i is found to be non-zero, the ith relay has to transmit with its full power. In the same approach as [35] in which the precoder is designed for a MIMO transmitter using the Lagrangian method, Lagrange multipliers are found by solving a set of nonlinear equations. In these equations, the multiplications of Lagrange multipliers with their corresponding inequality constraints have to be set to zero concurrently.

$$\lambda_i \left(\|\mathbf{W}_i \mathbf{H}_i\|^2 + \mathrm{P}_{\mathrm{n}_i} \|\mathbf{W}_i\|^2 - \mathrm{P}_{\mathrm{r}_i} \right) = 0 \qquad i = 1, 2, \ldots, K. \tag{23}$$

In [32], this is not solved but a value is suggested heuristically for the λ_i and the output is then normalized to the transmitter output power. Here, such values are determined numerically.

4. Discussion on the parameter η

Increasing the parameter η in (5) not only improves the received signal power, but it also renders the noise power to be increased, thereby the received signal-to-interference and noise ratio (SINR) may not be improved as η exceeds a certain threshold. Note that the optimal value of η cannot be derived analytically. This motivated us to rely upon some numerical methods to indicate how η may affect both the received SINR and bit error rate (BER) which are served as performance functions in the current study. Specifically, two different approaches are exploited in our numerical study. In the first part of our study, the resulting received SINR against η for many realizations of channel matrices and for various values of transmitted SNR is computed under different network's configurations. Note that in this case, the transmitted SNR is defined as $\mathrm{TSNR} = P_s/P_n$ and consequently the received SINR is computed as

$$\mathrm{SINR} = \frac{P_s \left\| \mathrm{diag}\left(\sum_{i=1}^{K} \mathbf{G}_i \mathbf{W}_i \mathbf{H}_i \right) \right\|^2}{P_s \left\| \left(\sum_{i=1}^{K} \mathbf{G}_i \mathbf{W}_i \mathbf{H}_i \right) - \mathrm{diag}\left(\sum_{i=1}^{K} \mathbf{G}_i \mathbf{W}_i \mathbf{H}_i \right) \right\|^2 + \sum_{i=1}^{K} P_i \|\mathbf{G}_i \mathbf{W}_i\|^2 + MP_n} \tag{24}$$

where "diag(**A**)" represents a diagonal matrix with the same diagonal entries as matrix **A**. Figures 2 and 3 represent the sensitivity of the received SINR against η for 2 and 4 relay networks, respectively. The simulation is performed for different channel realizations considering the transmitted SNR (TSNR) is set to 12 dB.

It can be observed that for each channel realization, there is an optimum value for η that is dependent upon the instantaneous first and second hop channel matrices. Thus, the optimum value of η is a random variable for each network configuration as well as TSNR. The

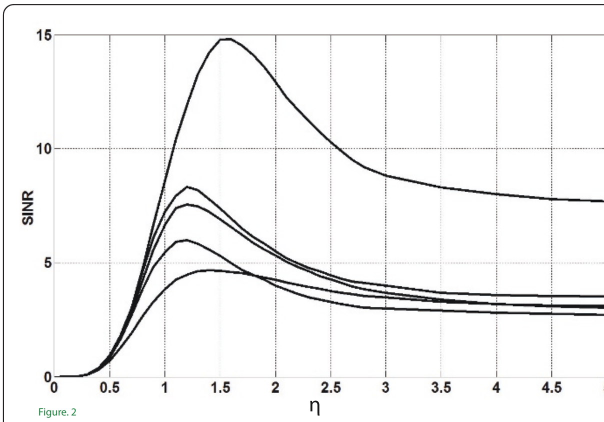

Figure 2 SINR at destination for different channel realizations considering a two relay network each with two antennas.

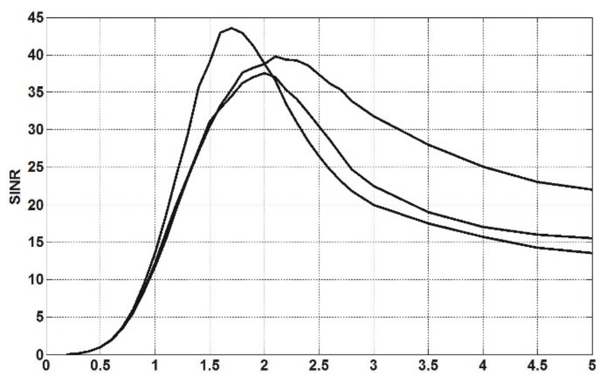

Figure. 3

Figure 3 **Received SINR for different channel realizations, considering four relays each with four antennas**.

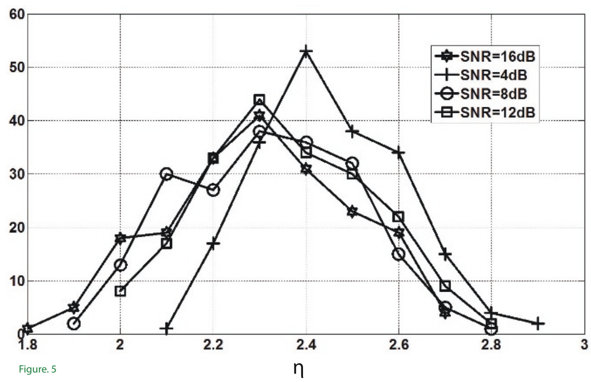

Figure. 5

Figure 5 **The estimated histogram for the optimum value of η considering four-relay network each having four antennas and four source-destination pairs**.

probability density function (PDF) of η can be estimated through using Mont Carlo simulation method. In Figures 4 and 5, the estimated PDF for the optimum η is depicted for a network of two antennas, two relays and four antennas, four relays, respectively. It can also be observed that the PDFs are very thin, i.e., a low variance value. Thus, we can select the mean value of the optimum value of η for simulation purposes. So, for each network configuration, the best value of η can be determined for the performance evaluation.

Furthermore, for different configurations of the relay network, the BER at the destination is computed against η for various values of SNRs. It can be seen that at the beginning, increasing η results in decreasing BER. However, as it increases beyond a certain value, the BER increases. Accordingly, Figure 6 depicts the BER against η for a network with two relays each having two antennas. It can be seen that the selected η from this diagram is in agreement with the value that is obtained from Figure 4.

Also, Figures 7 and 8 are provided for various network configurations with different number of relays and antennas.

Referring to the results, it can be observed that at each SNR point, there is an η in which the resulting BER is minimized. Moreover, results show that there is a close agreement between the optimum value of η from BER curves to that obtained from the estimated PDF for η. The obtained values are employed later in the simulation results provided in Section 6.

5. The proposed algorithm implementation procedure

The material proposed in the previous sections can be summarized for system implementation as follows.

Channel estimation has to be performed primarily. The channel estimation for AF relaying is considered in related literatures [36,37]. It is assumed that the estimation and

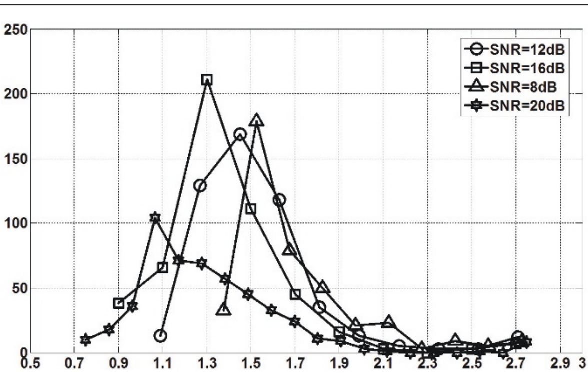

Figure. 4

Figure 4 **The estimated histogram for the optimum value of η considering two-relay network each having two antennas and two source-destination pairs**.

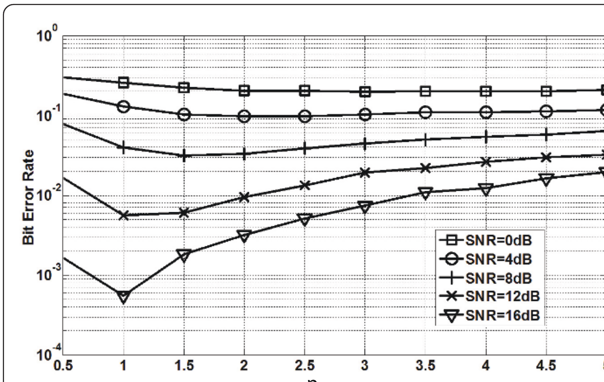

Figure. 6

Figure 6 **BER at destination with two relays each having two antennas for various values of η and SNRs**.

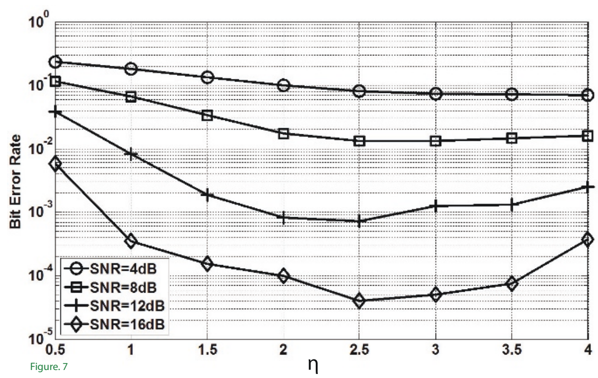

Figure 7 BER at destination with four relays each having two antennas for various values of η and SNRs.

transmission of channel matrices are error free. Assuming a slow fading channel, the first and second hop channels can be modeled as block fading channels and it can be assumed that it does not change during the block. The block can be a fraction of coherent time of the channel.

Knowing the TSNR, the best value for η can be determined by the methods introduced in Section 4. Furthermore, **A** and **f** are computed from (19) and (17), respectively. Then, **w** can be computed from Equation (21) (**w** is a function of λ_i's). Inserting **w** to (22), an object function with K scalar unknown variables is obtained. This function has to be maximized with respect to the set of non-negative λ_i's. Then using the active set method that does not need the closed loop form of the gradient is used to find the optimum values of λ_i's. The stopping criterion is are the difference between the primary object function and the dual object function or

$$\epsilon = \sum_{i=1}^{K} \lambda_i \left(\left| \bar{\mathbf{H}}_i \mathbf{w}_{oi} \right|^2 + P_n |\mathbf{w}_{oi}|^2 - P_{r_i} \right) \tag{25}$$

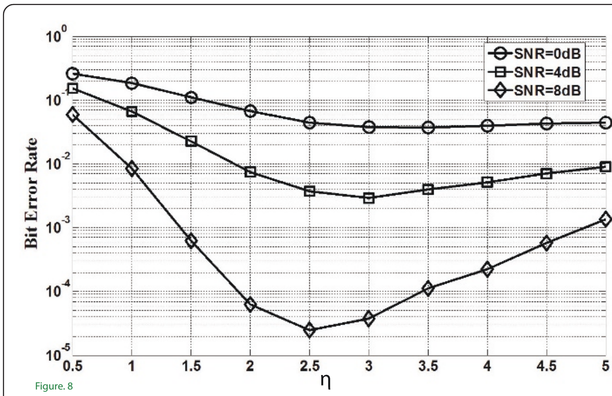

Figure 8 BER at destination with four relays each have four antennas for various values of η and SNRs.

Thus, the algorithm at the boundary of each block is as follows.

Initialization: set λ_i to an arbitrary start value for $i = 1,...,K$,

iterate: compute **A**, **f** *and* **w**

$$Compute : \epsilon = \sum_{i=1}^{K} \lambda_i \left(\left| \bar{\mathbf{H}}_i \mathbf{w}_{oi} \right|^2 + P_n |\mathbf{w}_{oi}|^2 - P_{r_i} \right) \tag{26}$$

If $\lfloor < \lfloor_0$ *end,*

else modify λ_i *for* $i = 1,...,K$, *goto iterate,*

where $\lfloor_0$ is a predetermined constant value that can be chosen arbitrary according to specific design accuracy.

Modification of λ_i in the last line of the algorithm is performed based on the Active Set method [34]. In this method, during each step the gradient of the cost function is estimated using three points in the space. The MATLAB function "fmincon" can be used to implement this method.

6. Simulation results

To confirm the superiority of the proposed schemes over MMSE-MMSE and ZF-ZF method, their average BER and MSE are compared by varying the number of relay nodes, K, and the number of relay antennas N. It is also assumed that the input noise power at the destination and the relays are the same. The channel matrices are generated independently during subsequent iterations. It is further assumed that the first and the second hop channels for all relays are known perfectly. Networks with various numbers of nodes and antennas are simulated and the average BER and the MSE parameter are used as the performance metrics and they are compared with MMSE-MMSE and ZF-ZF methods. Independent un-coded QPSK modulated symbol streams are transmitted from each of the source antennas.

The average BER and MSE versus SNR_t for $N = M = 3$ for a two relay network are shown in Figures 9 and

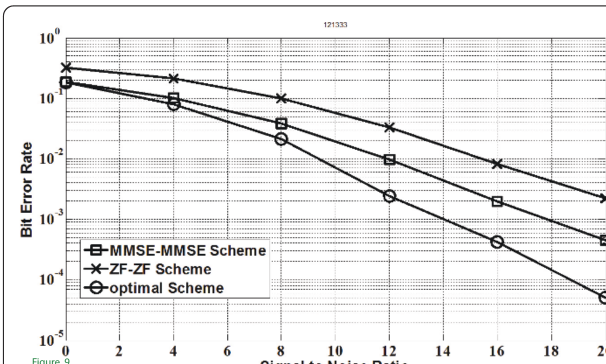

Figure 9 BER at destination with two relays each having three antennas.

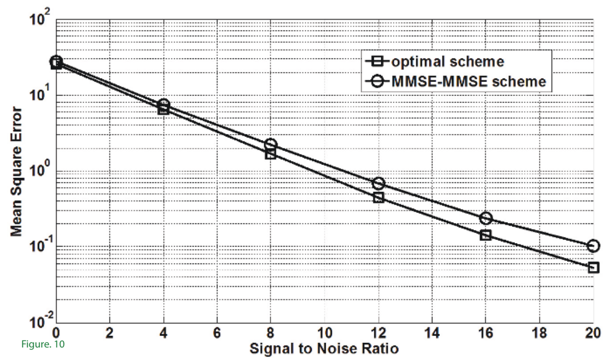

Figure 10 MSE at destination with two relays each having three antennas.

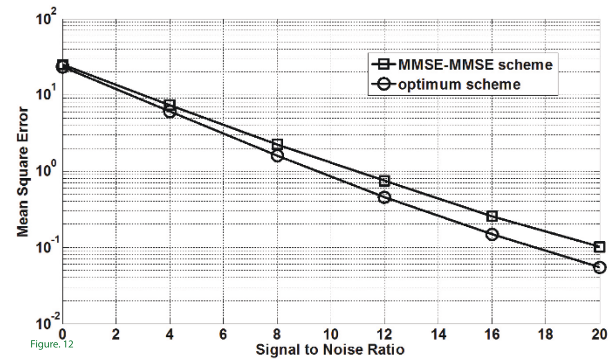

Figure 12 MSE at destination with two relays each have four antennas.

10, respectively. From these figures, it is found that the proposed scheme outperforms ZF-ZF and MMSE-MMSE schemes in all the examined cases.

For the second network configuration, it is assumed that $N = M = 4$, and the number of relays is 2. The average BER and MSE for three mentioned methods are depicted in Figures 11 and 12, respectively. It can be easily observed that the proposed optimum scheme outperforms both MMSE-MMSE and ZF-ZF methods.

Finally, networks with 4 and 6 relays are simulated. In the former setup each relay has four antennas and in the later case three antennas. For this case, The BER and MSE versus SNR are depicted in Figures 13, 14, 15, and 16.

In these cases too, the simulation results reveal that the optimum scheme outperforms the other two methods. Furthermore, the complexity observed by the proposed optimum method although seems to be a bit higher than MMSE-MMSE scheme, but provides a solution that would reduce the power consumption by approximately 3 dB.

7. Conclusion

A relay network with multiple relay each having multiple antennas is considered. The relay matrices are found by solving an optimization problem. In this problem, the MSE at the destination is minimized and the individual relay output power considered as constraint. The Lagrange dual problem is then obtained to compute the Lagrange dual variables numerically. Solving Lagrange dual problem (22) is simpler than the primary problem (9). This is because, solving Lagrange dual problem requires the calculation of K scalar unknown variables but in primary problem case, K unknown $N \times N$ matrices needs to be computed. So, the dimension of the problem decreases $N \times N$ times.

Two numerical methods based upon SINR and BER are introduced to obtain the optimum value of η that is employed for the actual simulation of the proposed optimum scheme.

The system with the proposed optimum, MMSE-MMSE and ZF-ZF schemes, is simulated and the average BER as well as MSE at destination are obtained. The

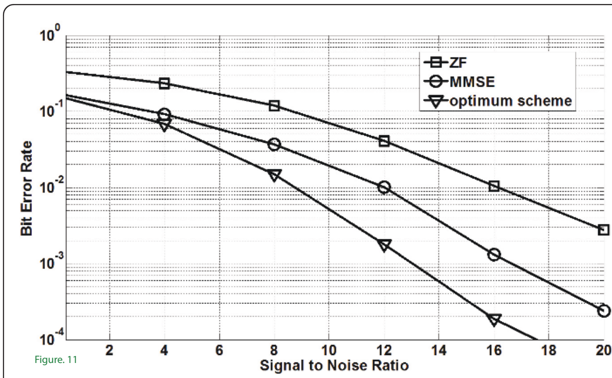

Figure 11 BER at destination with two relays each have four antennas.

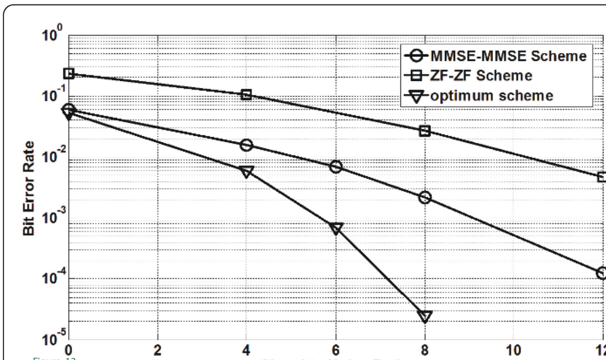

Figure 13 BER at destination with four relays each having four antennas.

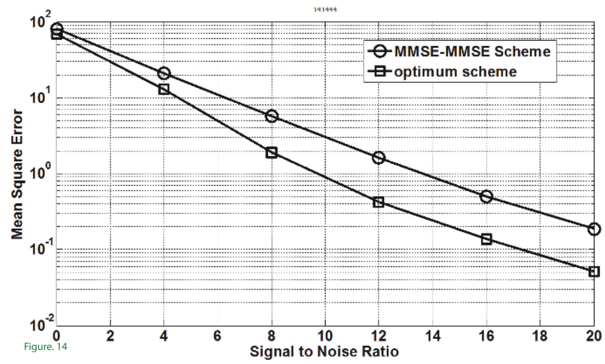

Figure 14 MSE at destination with four relays each having four antennas.

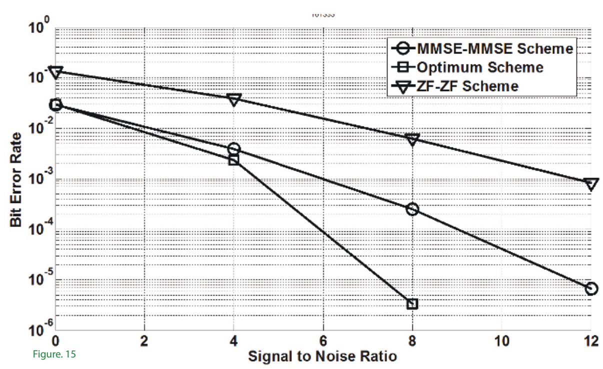

Figure 15 BER at destination with six relays each having three antennas.

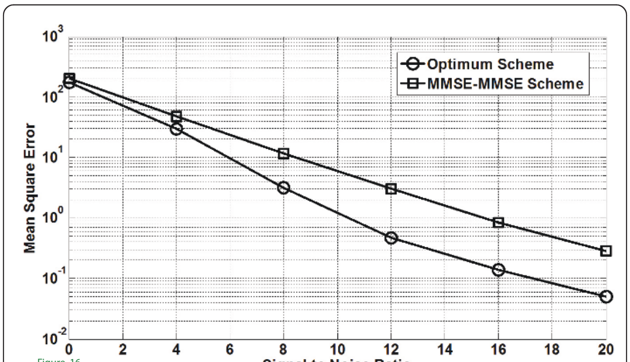

Figure 16 MSE at destination with six relays each having three antennas.

results show that the proposed optimum scheme outperforms MSE-MSE and ZF-ZF schemes by a good margin. Indeed, analytical computation of Lagrange dual variables and considering normalization parameter η as the optimization problem variable can be considered for future investigations.

Appendix

Two relays network case

For two relay network further simplification can be performed. Rewriting (16)

$$\begin{cases} \left(\mathbf{T}_1{}^H\mathbf{T}_1 + \lambda_1\bar{\mathbf{H}}_1^H\bar{\mathbf{H}}_1 + P_n\bar{\mathbf{G}}_1^H\bar{\mathbf{G}}_1 + P_n\lambda_1\mathbf{I}\right)\mathbf{w}_1 + \mathbf{T}_1{}^H\mathbf{T}_2\mathbf{w}_2 = \eta\mathbf{f}_1{}^* \\ \left(\mathbf{T}_2{}^H\mathbf{T}_2 + \lambda_2\bar{\mathbf{H}}_2^H\bar{\mathbf{H}}_2 + P_n\bar{\mathbf{G}}_2^H\bar{\mathbf{G}}_2 + P_n\lambda_2\mathbf{I}\right)\mathbf{w}_2 + \mathbf{T}_2{}^H\mathbf{T}_1\mathbf{w}_1 = \eta\mathbf{f}_2{}^* \end{cases} \tag{27}$$

and removing $\mathbf{W}_2$ in that will lead to

$$\Rightarrow \mathbf{w}_1 = \\ \begin{bmatrix} \left(\mathbf{T}_1{}^{\mathbf{H}}\mathbf{T}_2\right)^{-1}\left(\mathbf{T}_1{}^H\mathbf{T}_1 + \lambda_1\bar{\mathbf{H}}_1^H\bar{\mathbf{H}}_1 + P_n\bar{\mathbf{G}}_1^H\bar{\mathbf{G}}_1 + P_n\lambda_1\mathbf{I}\right) \\ \quad -\left(\mathbf{T}_2{}^{\mathbf{H}}\mathbf{T}_2 + \lambda_2\bar{\mathbf{H}}_2^H\bar{\mathbf{H}}_2 + P_n\bar{\mathbf{G}}_2^H\bar{\mathbf{G}}_2 + P_n\lambda_2\mathbf{I}\right)^{-1}P_r\mathbf{T}_2{}^{\mathbf{H}}\mathbf{T}_1 \end{bmatrix}^{-1} \\ \begin{bmatrix} \left(\mathbf{T}_1{}^{\mathbf{H}}\mathbf{T}_2\right)^{-1}\eta\mathbf{f}_1{}^* \\ \quad -\left(\mathbf{T}_2{}^{\mathbf{H}}\mathbf{T}_2 + \lambda_2\bar{\mathbf{H}}_2^H\bar{\mathbf{H}}_2 + P_n\bar{\mathbf{G}}_2^H\bar{\mathbf{G}}_2 + P_n\lambda_2\mathbf{I}\right)^{-1}\eta\mathbf{f}_2{}^* \end{bmatrix}. \tag{28}$$

Recalling defined parameters from (13) and some manipulation we can derive

$$\mathbf{w}_1 = \\ \eta\left(\mathbf{H}_1{}^* \otimes \mathbf{G}_1{}^H\right) \\ \left[\left(\boldsymbol{H}_2{}^T \otimes \boldsymbol{H}_2\right)\left(\boldsymbol{H}_1{}^T \otimes \boldsymbol{H}_1\right) - \left(\left(\mathbf{H}_2{}^H\mathbf{H}_2\right)^T \otimes \mathbf{G}_2\mathbf{G}_2{}^H\right)\left(\left(\mathbf{H}_1{}^H\mathbf{H}_1\right)^T \otimes \mathbf{G}_1\mathbf{G}_1{}^H\right)\right]^{-1} \\ \operatorname{vec}\left(\boldsymbol{H}_2\boldsymbol{H}_2 - \mathbf{G}_2\mathbf{G}_2{}^H\mathbf{H}_2{}^H\mathbf{H}_2\right) \tag{29}$$

$$\mathbf{w}_2 = \eta\left(\mathbf{H}_2{}^* \otimes \mathbf{G}_2{}^H\right) \\ \left[\left(\boldsymbol{H}_1{}^T \otimes \boldsymbol{H}_1\right)\left(\boldsymbol{H}_2{}^T \otimes \boldsymbol{H}_2\right) - \left(\left(\mathbf{H}_1{}^H\mathbf{H}_1\right)^T \otimes \mathbf{G}_1\mathbf{G}_1{}^H\right)\left(\left(\mathbf{H}_2{}^H\mathbf{H}_2\right)^T \otimes \mathbf{G}_2\mathbf{G}_2{}^H\right)\right]^{-1} \\ \operatorname{vec}\left(\boldsymbol{H}_1\boldsymbol{H}_1 - \mathbf{G}_1\mathbf{G}_1{}^H\mathbf{H}_1{}^H\mathbf{H}_1\right). \tag{30}$$

Where

$$\begin{aligned} \boldsymbol{H}_2 &= \left(\mathbf{H}_2{}^H\mathbf{H}_2 + P_n\mathbf{I}\right), \\ \boldsymbol{H}_2 &= \left(\mathbf{G}_2\mathbf{G}_2{}^H + \lambda_2\mathbf{I}\right), \\ \boldsymbol{H}_1 &= \left(\mathbf{H}_1{}^H\mathbf{H}_1 + P_n\mathbf{I}\right), \\ \boldsymbol{H}_1 &= \left(\mathbf{G}_1\mathbf{G}_1{}^H + \lambda_1\mathbf{I}\right), \end{aligned} \tag{31}$$

Competing interests

The authors declare that they have no competing interests.

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