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Low complexity soft demapping for non-binary LDPC codes

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Abstract

This article focuses on non-binary wireless transmission, where “non-binary” refers to the use of non-binary Low Density Parity Check (LDPC) codes for Forward Error Correction. The complexity of the non-binary soft demapper is addressed in particular when one non-binary Galois Field (GF) symbol spreads across multiple Quadrature Amplitude Modulation (QAM) symbols and Space-Time Block Code (STBC) codewords. A strategy is devised to guarantee an efficient mapping at the transmitter, together with an algorithm at the receiver for low complexity soft Maximum Likelihood demapping. The proposed solution targets a trade-off between performance and complexity, and removes any restriction on the setting of the GF order, QAM constellation order, and STBC scheme. This makes the non-binary LDPC codes even more appealing for potential use in practical wireless communication systems.

Keywords: non-binary, LDPC codes, mapping, soft values, maximum likelihood, MIMO

1. Introduction

Non-binary channel codes (i.e., defined over high-order Galois Field (GF) $q > 2$) have been researched in the literature to achieve higher error protection than conventional binary codes for transmission over different noisy channels [1-3]. More recently, the European FP7 DAVINCI project [4] has explored the design of innovative non-binary Low Density Parity Check (LDPC) codes with tailored link level technologies over wireless fading channels, whilst aiming at small added complexity to conventional binary receivers.

The DAVINCI project considers LDPC codes defined over a GF of order $q = 64$ (denoted as GF(64)). The proposed non-binary LDPC codes were shown to outperform their binary counterparts, e.g., binary LDPC and (duo-) binary Turbo Codes, with higher gains for higher constellation orders and higher coding rates [5]. Moreover, these non-binary codes were shown to boost the system spectral efficiency when combined with high-order Quadrature Amplitude Modulation (QAM) constellations and MIMO spatial multiplexing [6]. This boosting effect comes from the inherently higher capacity of the single-input single-output (SISO) equivalent

channel as seen by the non-binary code with high-order constellations and multiple antennas [6,7].

Complexity-wise, for high GF order, e.g., $q = 64$, some relatively low complexity LDPC decoding algorithms have been proposed in [8]. Now, if we consider mapping the encoded symbols onto QAM constellation symbols and Space-Time Block Code (STBC) codewords, the complexity of the soft demapper at the receiver turns out to represent a real challenge, especially when one GF symbol spreads across multiple QAM constellation symbols and STBC codewords. This can be seen for example in the simple case of GF order $q = 64$ with 16QAM constellation in SISO (single antenna) transmission, where two GF64 coded symbols (total of $2 \times 6 = 12$ bits) jointly map onto three 16QAM symbols (total of $3 \times 4 = 12$ bits). Thus, one of the three 16QAM symbols has to contain coded bits from two GF symbols. This spreading of the GF coded symbols across more than one QAM symbol drastically increases the complexity of the soft demapper, the latter already being more complex than in the binary case ($q = 2$). This complexity issue may become even more problematic in the mapping of GF coded symbols to STBC codewords. This is particularly true when one GF coded symbol does not fit into exactly one STBC codeword. In order to avoid such complexity, most of the recent studies have been restricted to the configurations where each GF

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symbol can be individually processed in its mapping onto QAM symbols and STBC codewords [5,6]. This led in certain cases to non-practical assumptions, such as 3×3 antenna configuration for GF64 with Quadrature Phase Shift Keying (QPSK) and MIMO spatial multiplexing [6].

This article tackles the challenging complexity of the non-binary soft demapper when the GF symbol spreads across multiple QAM symbols and STBC codewords. The mappings at the transmitter and the soft demapping at the receiver are both considered with the aim to achieve the best trade-off between performance and complexity. A strategy is devised to guarantee an efficient mapping at the transmitter, together with an algorithm for low complexity soft demapping at the receiver. The proposed algorithm borrows a key finding in [8] which is by feeding the non-binary LDPC decoder with only a limited number of the highest A Posteriori Probability (APP) values for each GF symbol (with this limited number being much less than the GF order q) one can still achieve very close to the optimal performance whilst reducing significantly the non-binary LDPC decoding complexity and memory requirements.

The rest of the article is structured as follows. Section 2 describes the system model, and Section 3 follows with the problem statement. Section 4 presents the mapping and demapping solutions proposed. Section 5 shows numerical results to illustrate the performance and complexity of the proposed solutions. Finally, Section 6 draws our conclusions and suggests some perspectives for future work.

2. System model: non-binary wireless transmission

The key notations used throughout this article are listed in Table 1.

A block diagram of the non-binary transmission chain is depicted in Figure 1. In the following, we first introduce the non-binary LDPC codec used, and then describe the functioning of the non-binary wireless transmission chain illustrated in Figure 1.

2.1 Non-binary LDPC codes

The non-binary LDPC codes used are taken directly from DAVINCI project [4]. These codes have been designed with a very sparse parity check matrix. The non-zero elements of the matrix are defined over a GF of order $q = 64$, denoted by Ω . The variable node degree is fixed to $d_v = 2$ (optimal when $q \rightarrow \infty$ and codeword length $\rightarrow \infty$), whereas the check node degree d_c is variable and adapted to the coding rate (i.e., $d_c = \{4, 6, 8, 12\}$ for rate = $\{1/2, 2/3, 3/4, 5/6\}$, respectively). The DAVINCI codes are obtained as regular LDPC codes over the GF Ω following the optimization process described in [9]. At the receiver side, we use a reduced complexity non-binary decoder based on the Extended Min-Sum algorithm proposed in [8] for practical hardware implementation of the DAVINCI codes. This low complexity decoder takes only the q_m ($q_m < q$) highest APP values out of the $q - 1$ values available at the output of the soft demapper. This truncation of the APP values at the input of the decoder reduces significantly the decoder complexity at the cost of slight performance degradation.

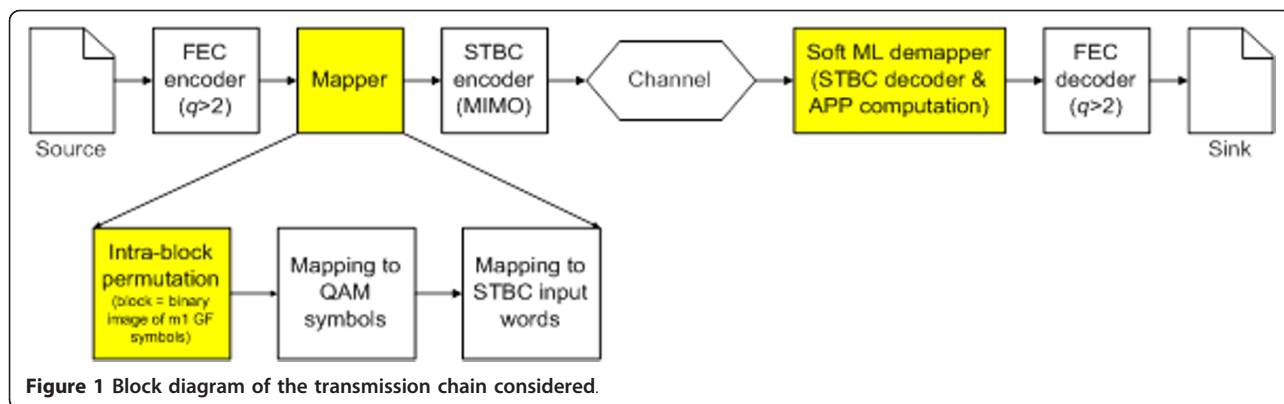
2.2 Non-binary wireless transmission chain

2.2.1 Transmitter operations

As illustrated in Figure 1, the binary information message issued from the source is first converted into non-binary GF(q) message by simply mapping groups of $\log_2(q)$ bits onto their corresponding GF(q) images. The

Table 1 Key notations used throughout the article

q	GF order (default value $q = 64$)
Ω	Alphabet of q GF symbols
M	QAM constellation order (e.g., QPSK $\rightarrow M = 4$; 16QAM $\rightarrow M = 16$)
\mathbf{A}	Alphabet of M QAM constellation symbols
q_m	Number of APP values per GF symbol fed to the decoder ($< q$)
n_t	Number of transmitter antennas
n_r	Number of receiver antennas
Q	Number of QAM symbols mapped onto one STBC codeword
T	STBC block length (as expressed in MIMO channel uses)
m_1	Minimum integer number of GF(q) symbols which map onto m_2 M -QAM symbols and m_3 STBC codewords
n_1	Number of GF(q) symbols multiplexing within n_2 ($\leq m_2$) M -QAM symbols and n_3 ($\leq m_3$) STBC codewords, ($n_1 \leq m_1$)
m_2	Minimum integer number of M -QAM symbols which map to m_1 GF(q) symbols and m_3 STBC codewords
n_2	Number of M -QAM symbols carrying one GF(q) symbol, ($n_2 \leq m_2$)
m_3	Minimum integer number of STBC codewords which map to m_1 GF(q) symbols and m_2 M -QAM symbols
n_3	Number of STBC codewords carrying one GF(q) symbol, ($n_3 \leq m_3$)



binary images are obtained from the primitive polynomial below used in the DAVINCI project to optimize the DAVINCI codes [4,9]:

$$P(x) = x^6 + x + 1 \quad (1)$$

Blocks of K GF(q) symbols are then passed to the non-binary LDPC encoder which generates the non-binary codeword of length N GF(q) symbols. In order then to map the non-binary Forward Error Correction (FEC) codeword onto the M -QAM constellation symbols, each of the GF(q) symbols in the FEC codeword is first converted back to its binary image of $\log_2(q)$ bits (using the same primitive polynomial in (1)). The resulting binary stream is then passed to the Mapping module, which is in charge of mapping the GF(q) symbols onto the M -QAM constellation symbols and STBC codewords. As highlighted in Figure 1, the mapping function features a novel module referred to as *intra-block permutation* which permutes/re-arranges the bits (per block of m_1 GF symbols) in the binary stream in accordance with three design rules devised in this article to achieve the trade-off between performance and complexity. Next to the intra-block permutation, each group of $\log_2(M)$ adjacent bits of the permuted output stream is mapped onto one QAM constellation symbol. A conventional gray-mapping is used to produce the stream of complex-valued QAM symbols. The QAM symbols are then directly sent for transmission over the wireless multi-path fading channel in the context of a single antenna transmission.

In the context of multi-antenna transmission, the stream of QAM symbols undergoes a further step of spatial encoding represented by the STBC encoder depicted in Figure 1. The QAM symbols are arranged in groups of Q symbols, and each group is encoded by the STBC encoder resulting into a STBC codeword \mathbf{V}_j of $n_t \times T$ complex symbols, with n_t being the number of transmitter antennas and T the STBC block length. The spatial rate is then given by $R_S = Q/T$. The output

stream of the STBC encoder is then transmitted across the multiple antennas through the multi-path fading channel.

2.2.2 Receiver operations

At the receiver side, with single antenna (i.e., single output), the complex-valued received signal can be modeled as:

$$y = hx + v \quad (2)$$

where h is the channel fading, x the transmitted symbol per channel use, and v the background noise assumed to be complex-valued Gaussian distributed with zero mean and single-sided variance N_0 .

The received symbols are de-interleaved at the QAM level, and next fed into the soft demapper, which computes the APP values of all GF(q) symbols in the codeword. The computation of the APP values in the non-binary case is much heavier than in the case of binary transmission for two reasons: first, each GF(q) coded symbol calls for the computation of $(q - 1)$ APP values, and second the computation of each APP value turns out to be particularly complicated whenever one GF(q) symbol is spread across different QAM symbols (see Section 3 for more details). The APP values are fed into the non-binary FEC decoder, and the decoded GF symbols are finally converted into bits to represent the received binary message.

The received signal model is slightly more complicated with $n_t \times n_r$ MIMO transmission.

The received STBC codeword \mathbf{W}_j of size $n_r \times T$ is a linear transformation of the transmitted STBC codeword \mathbf{V}_j plus additive Gaussian noise \mathbf{v}_j , as shown below:

$$\mathbf{W}_j = \mathbf{H}_j \mathbf{V}_j + \mathbf{v}_j \quad (3)$$

where \mathbf{H}_j is an $n_r \times n_t$ complex matrix representing the MIMO frequency-flat channel coefficients for the j th STBC codeword \mathbf{V}_j .

The received STBC codewords are fed into the so-called Soft Maximum Likelihood (ML) demapper which

combines the STBC ML detection and the APP computation. It is noteworthy here that sub-optimal linear equalizers may be considered for the STBC detection, which will then be followed in a second step by the APP computation for non-binary LDPC decoder. Such linear approach was compared to the soft ML demapper in [4,10], where the latter was shown to significantly outperform the former. It is not in the scope of this article to reproduce such comparison, but rather focus on the complexity reduction of the soft ML demapper with the aim to make it practical for wireless communication systems.

3. Problem statement: mapping and Soft ML demapping

Figure 2 illustrates the different stages for mapping the $GF(q)$ non-binary symbols onto the M -QAM constellation symbols, and later on onto the STBC codewords should MIMO transmission be considered.

3.1 Mapping of $GF(q)$ symbols onto M -QAM symbols

As suggested in [6], in order to ensure a bijective mapping between $GF(q)$ and M -QAM symbols, we have to map a vector of m_1 $GF(q)$ symbols onto a vector of m_2 M -QAM symbols, such that both vectors are of the same length as expressed in (coded) bits:

$$m_1 \times \log_2(q) = m_2 \times \log_2(M) \quad (4)$$

In [6], in order to minimize the complexity of the mapper and demapper, the parameters m_1 and m_2 are

set to their minimum possible integer values in accordance with Equation (4). Assuming $q = 64$, Table 2 lists the values of m_1 and m_2 to be used with QPSK, 16QAM, and 64QAM constellations.

For example for QPSK, one $GF(64)$ symbol maps onto three QPSK symbols. For 16QAM however, two $GF(64)$ symbols will map onto three 16QAM symbols, and consequently two $GF(64)$ symbols will be spread onto at least one 16QAM symbol. For 64QAM, the mapping is obviously one-to-one since both $GF(64)$ and 64QAM symbols are represented by the same number of bits ($= \log_2(64) = 6$).

The mapping between $GF(q)$ symbols and M -QAM symbols can be formulated as:

$$\mathbf{d}^{(n)} = [d_0^{(n)} \dots d_{m_1-1}^{(n)}] \xrightarrow{\text{Mapping } \mu(\cdot)} \mathbf{x}^{(n)} = [x_0^{(n)} \dots x_{m_2-1}^{(n)}] \quad (5)$$

$$\mathbf{d} = \left[\mathbf{d}^{(0)} \dots \mathbf{d}^{\left(\frac{N}{m_1}-1\right)} \right]; \quad \mathbf{x} = \left[\mathbf{x}^{(0)} \dots \mathbf{x}^{\left(\frac{N}{m_2}-1\right)} \right]$$

In the sequel, we consider only one vector $\mathbf{d}^{(n)}$ and corresponding vector $\mathbf{x}^{(n)}$, and omit the superscript index n for the sake of simplicity. Thus, \mathbf{d} and \mathbf{x} refer now to two vectors of lengths m_1 and m_2 , respectively, associated by the mapping function $\mu(\cdot)$. As illustrated in Figures 1 and 2, the mapping function $\mu(\cdot)$ features a novel component introduced in this article which is the so-called “intra-block permutation”. In this component, the bits in the binary image of the vector of m_1 $GF(q)$ symbols are permuted (re-arranged) in accordance with the design rules proposed in Section 3.1. This is with

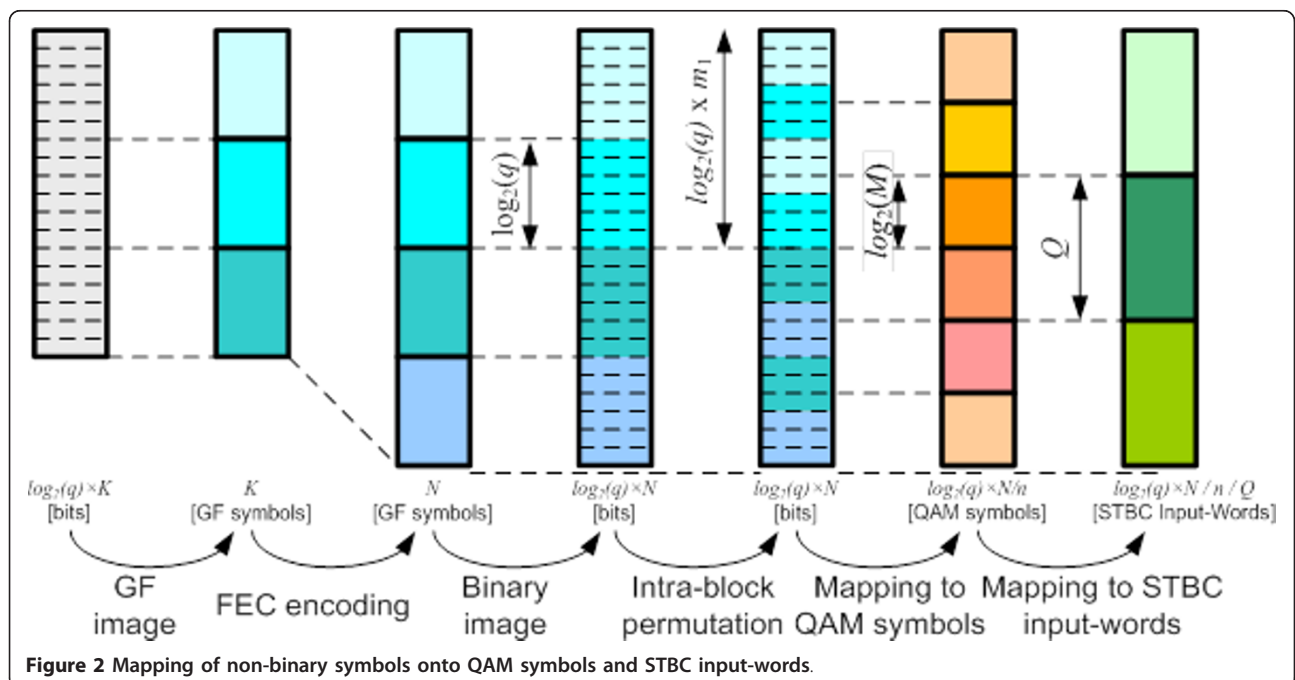


Figure 2 Mapping of non-binary symbols onto QAM symbols and STBC input-words.

Table 2 Values of m_1 and m_2 for GF(64) to QAM mapping

Constellation	QPSK	16QAM	64QAM
(m_1, m_2)	(1, 3)	(2, 3)	(1, 1)

the aim to achieve the best trade-off between performance and complexity.

At the receiver side, the soft demapper computes $q - 1$ APP values for each GF(q) coded symbol. For a memoryless SISO channel and assuming all GF(q) coded symbols are equiprobable, we can write the k th ($k = 0 \dots q - 1$) APP value for the i th ($i = 0 \dots m_1 - 1$) GF(q) coded symbol in the vector \mathbf{d} of m_1 GF(q) symbols mapped onto the vector \mathbf{x} of m_2 QAM symbols as [6]:

$$\lambda_{i,k} = \ln \frac{\sum_{\mathbf{d} \in \Delta_i^k} \exp \left(-\frac{1}{N_0} \sum_{j=0}^{m_2-1} |y_j - h_j \mu_j(\mathbf{d})|^2 \right)}{\sum_{\mathbf{d} \in \Delta_i^0} \exp \left(-\frac{1}{N_0} \sum_{j=0}^{m_2-1} |y_j - h_j \mu_j(\mathbf{d})|^2 \right)} \quad (6)$$

where $y_j = h_j x_j + v_j$ is the j th received symbol given in (2), h_j is the corresponding equivalent channel coefficient (that we assume perfectly known), $x_j = \mu_j(\mathbf{d})$ the j th entry in the vector \mathbf{x} of m_2 QAM symbols mapping onto the vector \mathbf{d} of m_1 GF(q) symbols, and v_j the noise term. The set Δ_i^k includes all configurations of vector \mathbf{d} with i th component $d_i = \alpha_k$, where α_k denotes the k th entry in the GF Ω . The cardinality of Δ_i^k is clearly equal to q^{m_1-1} .

The computational complexity of the soft demapper is of a magnitude order $O((q - 1) \times q^{m_1-1} \times m_2)$ per GF symbol. This reflects an exponential growth with the GF order q whenever the minimum number m_1 of GF(q) symbols that are spread onto M -QAM symbols is strictly greater than 1 ($m_1 > 1$). This is the case with 16QAM as given in Table 2.

3.2 Mapping of GF(q) symbols onto STBC codewords

With MIMO transmission, a number Q of M -QAM constellation symbols (equal to one input word to the STBC encoder which outputs once corresponding STBC codeword) are encoded together and transmitted across n_t transmitter antennas during T transmission intervals (also known as MIMO channels uses). Thus, similarly to (4), we have to map here the original vector of m_1 GF(q) symbols (mapping onto the vector of m_2 M -QAM symbols) onto a third vector of m_3 STBC codewords, such that all three vectors are of the same length in bits:

$$m_1 \times \log_2(q) = m_2 \times \log_2(M) = m_3 \times Q \times \log_2(M) \quad (7)$$

The parameters m_1 , m_2 , and m_3 should be set to their minimum possible integer values in accordance with

Equation (7). Assuming $q = 64$, STBC codeword size $Q = 2$, Table 3 gives the values of m_1 , m_2 , and m_3 , with QPSK, 16QAM, and 64QAM constellations.

Compared to Table 2, the values of m_1 and m_2 in Table 3 are simply doubled as a result of MIMO transmission with $Q = 2$. For a memoryless MIMO channel, we can write the k th ($k = 0 \dots q - 1$) APP value (assuming the to use Log-MAP) for the i th ($i = 0 \dots m_1 - 1$) GF(q) coded symbol in the vector \mathbf{d} of m_1 GF(q) symbols mapped onto the vector of m_2 QAM symbols (equivalent to m_3 STBC codewords) as

$$\lambda_{i,k} = \ln \frac{\sum_{\mathbf{d} \in \Delta_i^k} \exp \left(-\frac{1}{N_0} \sum_{j=0}^{n_3-1} \|\mathbf{W}_j - \mathbf{H}_j SM(\mu(\mathbf{d}))\|_F^2 \right)}{\sum_{\mathbf{d} \in \Delta_i^0} \exp \left(-\frac{1}{N_0} \sum_{j=0}^{n_3-1} \|\mathbf{W}_j - \mathbf{H}_j SM(\mu(\mathbf{d}))\|_F^2 \right)} \quad (8)$$

where $SM(\cdot)$ denotes the MIMO encoder operation, which encodes the stream of QAM symbols into STBC codewords, and $\|\cdot\|_F$ is the Frobenius norm. The parameter n_3 is defined in Table 1 as the number of STBC codewords carrying one GF(q) symbol ($n_3 \leq m_3$). The value of n_3 may then vary from one GF symbol to another in the vector of m_1 GF symbols and therefore depends on the index i of the GF(q) symbol. The vector \mathbf{d} may thus have different bit lengths depending on whether n_3 is equal to or greater than 1. Furthermore, in order to minimize the computational weight of the APP extraction, we can exploit the max-Log-MAP, so that (8) becomes the difference between the maximum sum at the numerator and the maximum sum at the denominator.

Taking into account the inherent matrix multiplication required to compute the distance between STBC codewords \mathbf{V}_j and \mathbf{W}_j (cf. (3)), the computation complexity of the soft demapper becomes of the magnitude order $O((q - 1) \times q^{m_1-1} \times m_3 \times n_r \times Q \times T)$. When $m_1 > 1$ different GF(q) are spread into different STBC codewords, and this occurs here for any constellation, unlike the SISO case where it only occurs for 16QAM constellation (cf. Table 2). This emphasizes how problematic the complexity of the soft demapper may become with MIMO transmission, even with simple practical configurations (e.g., $Q = 2$). The main problem tackled in this article is the reduction of the complexity of the soft demapper when one GF(q) symbol spreads across different QAM symbols and STBC codewords (i.e., $m_1 > 1$), without sacrificing the error protection performance.

Table 3 Values of m_1 , m_2 , and m_3 for GF(64) to MIMO ($Q = 2$) mapping

Constellation	QPSK	16QAM	64QAM
(m_1, m_2, m_3)	(2, 6, 3)	(4, 6, 3)	(2, 2, 1)

4. Novel mapping strategy and low complexity soft demapping

Our solution to the problem stated above consists of a mapping strategy at the transmitter side together with an algorithm for low complexity soft demapping when one GF(q) symbol spreads across different QAM symbols and STBC codewords (i.e. $m_1 > 1$).

4.1 Mapping strategy at the transmitter

Three rules are introduced hereafter with the aim to achieve the best trade-off between error protection performance and soft demapper complexity.

First rule: The I or Q component of an M-QAM symbol should carry (in part or in full) the binary image of only one GF(q) symbol

This rule naturally applies to the particular case of $m_1 = 1$, and can always be met whenever the number of bits per GF(q) symbol $\log_2(q)$ is an integer multiple of the number of bits per I or Q component $\log_2(M)/2$. Otherwise, the rule requires mapping as many I and Q components as possible to binary parts issued from the binary image of only one single GF symbol. This ensures better performance compared to all other schemes not obeying to this rule, as will be proven in Section 5.

Assuming SISO 16QAM with $m_1 = 2$ and $m_2 = 3$, Table 4 gives four possible patterns to map the two GF(64) symbols **a** and **b** with binary images respectively, $a_0a_1a_2a_3a_4a_5$ and $b_0b_1b_2b_3b_4b_5$, onto the three 16QAM symbols with I and Q components, I_0Q_0 , I_1Q_1 , and I_2Q_2 . Amongst the four patterns shown in Table 4, only **P1** and **P3** obey the first rule.

Second rule: Map as many I/Q components as possible issued from the same GF(q) symbol onto the same STBC codeword

This will ensure a minimum number ($n_3 \leq m_3$) of STBC codewords to be considered by the soft demapper for the computation of the APP values of each GF(q) symbol, and so will contribute to the reduction of the complexity of soft ML demapping as proposed in Section 4.2, but to the detriment of limiting the maximum channel selectivity that can be achieved within one GF(q) symbol. This is because ideally by letting each I or Q component issued from one GF(q) symbol map onto different STBC codewords, we create higher chance for

these parts of the same GF(q) symbol to experience uncorrelated channel fading. This rule clearly restricts the freedom to let the GF(q) symbol enjoy higher channel selectivity, but fortunately has the advantage of reducing drastically the complexity of the soft ML demapper. This is where the complexity of the soft ML demapper is traded off with the error protection performance of the GF(q) symbols.

Third rule: Under the constraint of the second rule, map the I/Q components issued from one GF(q) symbol onto the transmission units ideally of independent channel fading within the STBC codeword carrying this GF(q) symbol

This rule obviously targets the maximum achievable channel selectivity (i.e., number of independent channel fading) within each GF(q) symbol under the constraint of the second rule. The higher the channel selectivity within one GF(q) symbol is (i.e., the number of independent channel fading affecting the different parts of the GF(q) symbol), the better the error protection performance is expected to be. The margin for this rule to achieve higher channel selectivity order is clearly bound by the second rule.

For example, in the case of MIMO uncoded spatial multiplexing ($Q = 2$) and 16QAM where $m_1 = 4$, $m_2 = 6$, and $m_3 = 3$ (cf. Table 3), we give in Table 5 three possible patterns for mapping the four GF(64) symbols, **a**, **b**, **c**, and **d**, of binary images, respectively, $a_0a_1a_2a_3a_4a_5$, $b_0b_1b_2b_3b_4b_5$, $c_0c_1c_2c_3c_4c_5$, $d_0d_1d_2d_3d_4d_5$, onto the six 16-QAM symbols representing three STBC codewords. Each STBC codeword carries $Q = 2$ 16QAM symbols concurrently transmitted over 2 antennas.

All three patterns in Table 5 follow the first rule by not mixing bits from different GF symbols into the same I or Q component.

Patterns **P1** and **P3** further obey the second rule by mapping as many I/Q components from the same GF symbol as possible into the same STBC codeword, whilst Pattern **P2** does not. For patterns **P1** and **P3**, GF(64) symbols **a** and **d** are carried within one single STBC codeword, and GF(64) symbols **b** and **c** are mapped onto two STBC codewords. However, for

Table 4 Example of four patterns for mapping GF(64) symbols to 16-QAM symbols

Number	Mapping pattern ($m_1 = 2, m_2 = 3$)					
	I0	Q0	I1	Q1	I2	Q2
P1	a_0a_1	a_2a_3	a_4a_5	b_0b_1	b_2b_3	b_4b_5
P2	a_0b_0	a_1b_1	a_2b_2	a_3b_3	a_4b_4	a_5b_5
P3	a_0a_1	b_0b_1	a_2a_3	b_2b_3	a_4a_5	b_4b_5
P4	a_0b_0	b_1a_1	a_2b_2	b_3a_3	a_4b_4	b_5a_5

Table 5 Example of three patterns for mapping GF(64) symbols to STBC codewords

Number	Antenna number	Mapping pattern ($m_1 = 4, m_2 = 6, m_3 = 3$)					
		I0	Q0	I1	Q1	I2	Q2
P1	A#1	a_0a_1	a_2a_3	b_2b_3	b_4b_5	c_4c_5	d_0d_1
	A#2	a_4a_5	b_0b_1	c_0c_1	c_2c_3	d_2d_3	d_4d_5
P2	A#1	a_0a_1	b_0b_1	a_2a_3	b_2b_3	a_4a_5	b_4b_5
	A#2	c_0c_1	d_0d_1	c_2c_3	d_2d_3	c_4c_5	d_4d_5
P3	A#1	a_0a_1	a_2a_3	b_2b_3	c_0c_1	c_4c_5	d_0d_1
	A#2	a_4a_5	b_0b_1	b_4b_5	c_2c_3	d_2d_3	d_4d_5

pattern **P2**, each GF(64) symbol is spread out over all of the $m_3 = 3$ STBC codewords. In terms of complexity of the soft demapper, patterns **P1** and **P3** will enable reduced complexity, whereas the complexity with pattern **P2** will be drastically higher, as shown later in Section 4.2.

Now with regard to the third rule, the channel selectivity order (i.e., maximum number of independent channel fading) for pattern **P1** is equal to 2 for all GF symbols, **a**, **b**, **c**, and **d**. This is clear since any of these GF symbols is mapped onto exactly two QAM symbols within only one single STBC codeword, with the first QAM symbol transmitted on the first antenna port and the second QAM symbol transmitted on the second antenna port. For pattern **P2** however, the channel selectivity order is higher and equal to 3, since any GF symbol is mapped onto exactly three QAM symbols transmitted within three different STBC codewords, hence ideally subject to three independent channel fading. The last pattern **P3** has its channel selectivity order equal to 2 for the edge symbols **a** and **d** (since carried in two QAM symbols within one single STBC codeword), whereas it is equal to 3 for the middle symbols **b** and **c** (since these are carried in two QAM symbols within two STBC codewords). Amongst all three patterns, only **P1** and **P3** respect the second rule, but only **P3** which further respects the third rule as it attempts to achieve the highest possible channel selectivity order under the constraint of the second rule.

In summary, by obeying all the three rules introduced above, we aim to obtain mapping patterns which ensure the best trade-off between performance and complexity. This will be further detailed and proven in Sections 4.2 and 5.

4.2 Low complexity soft demapping at the receiver

As highlighted in Equation (6), the soft demapper at the receiver requires two major steps for the computation of the APP values of the GF(q) coded symbols: (i) Euclidean distances computation, and (ii) Marginalization across all possible combinations. The Euclidean distances computation is typically required for ML hard detector. In our case, since soft values are required, the MIMO ML detection and non-binary soft demapping are combined together into one single function, referred to as soft ML demapping.

First step: Computation of the Euclidean distances

In the decoding of STBC, each received STBC codeword \mathbf{W}_j is processed individually in order to obtain its distance to all possible transmitted STBC codewords \mathbf{V}_j . In our non-binary case ($q > 2$), one GF(q) coded symbol may span more than one STBC codeword. Thus, for the computation of the APP values of one GF(q) symbol, there is a need to store the Euclidean distances of all of

the STBC codewords which carry the binary image of the given GF(q) symbol. Thanks to our second rule in the design of the mapper at the transmitter (which limits the number of STBC codewords carrying the binary image of one GF(q) symbol to the minimum possible), only the Euclidean distances of $n_3 \leq m_3$ STBC codewords are needed. This clearly reduces the memory requirements at the receiver.

Second step: Marginalization across all possible combinations

The marginalization takes the form of a summation in the general case (i.e., log-Map) reflected in Equation (6). Should the Max-log approximation be used, it takes instead the form of a comparison. The marginalization (or summation) involves the Euclidean distances of $n_3 \leq m_3$ STBC codewords and the binary sub-parts of the $n_1 - 1$ ($n_1 \leq m_1$) GF(q) symbols multiplexing with the binary image of the desired GF(q) symbol in their mapping to the $n_2 \leq m_2$ M-QAM symbols and $n_3 \leq m_3$ STBC codewords.

For the sake of simplicity, let us consider first the case where $n_3 = 1$, i.e., the desired GF(q) symbol is mapped onto a single STBC codeword. This is the case of SISO transmission but also applies for instance to MIMO transmission for the edge GF(q) symbols **a** and **d** in patterns **P1** and **P3** in Table 5. Let us focus first on the simple case of SISO transmission with 16QAM as in Table 4 with the straightforward mapping **P1** = $[a_0a_1a_2a_3]$; $[a_4a_5b_0b_1]$; $[b_2b_3b_4b_5]$; for $m_1 = 2$ and $m_2 = 3$. In order to compute the APP values for the first GF(64) symbol **a**, the Euclidean distances involving the first $n_2 = 2 \leq 3$ QAM symbols are required. For the second GF(64) symbol **b**, those involving the second and the third QAM symbols are required. For the computation of the APP values of **a**, a marginalization is required across all of the possible combinations of the sub-part b_0b_1 from GF(64) symbol **b** due to their mix with the sub-part a_4a_5 in the second QAM symbol (i.e., $[a_4a_5b_0b_1]$). The number of all possible combinations is clearly equal to $2^2 = 4$. The number of operations per received GF symbol is $(q - 1) \times 2^2 \times 3$, a factor $2^2/q^{m_1-1} = 4/64 = 1/16$ smaller than the value $O((q - 1) \times q^{m_1-1} \times m_2)$ indicated earlier in Section 3.1. Thanks to the specific mapping where the two edge 16QAM symbols carry information from only one single GF(64) symbol. Similar marginalization is required in the second case of MIMO transmission for the edge GF(q) symbols **a** and **d** in patterns **P1** and **P3** in Table 5.

Consider now the more general case of $n_3 > 1$, for example in the case of MIMO transmission for the middle GF(64) symbols **b** and **c** in patterns **P1** and **P3** in Table 5 the GF(64) symbol **b** is mapped onto the first ($= [a_0a_1a_2a_3a_4a_5b_0b_1]$) and second ($= [b_2b_3b_4b_5c_0c_1c_2c_3]$) STBC codewords. The marginalization here is required

across all the possible combinations of $a_0a_1a_2a_3a_4a_5$ due to the mix with the sub-part b_0b_1 in the first STBC codeword, and also across all of the possible combinations of $c_0c_1c_2c_3$ due to the mix with the sub-part $b_2b_3b_4b_5$ in the second STBC codeword. This adds up to the total of $2^6 \times 2^4 = 1024$ combinations per APP value. Demapping complexity clearly depends on the mapping pattern used. Table 6 gives an example of the number of distances required for marginalization of each APP value for the two mapping patterns **P2** and **P3** from Table 5.

Table 6 reflects the huge complexity incurred with mapping pattern P2 (although as said earlier, this pattern achieves the maximum transmit diversity order 3 for all the GF(64) symbols). This confirms the tremendous complexity advantage of the mapping patterns respecting the second rule devised previously. Yet, whilst only 4 combinations are required for the edge symbols **a** and **d**, 1024 combinations are required for the symbols in the middle **b** and **c**, which is still relatively a high number.

Still, 1024 is a relatively large number causing excessive complexity. To further reduce the number of combinations to a relatively low level, we propose the following algorithm which exploits the correlation existing between GF(q) symbols produced by the code. The algorithm introduces a threshold parameter called N_m . The algorithm proceeds with the following steps:

- **Step 1:** Set the value of N_m . For example, N_m is set to the value 8.
- **Step 2:** For any GF(q) symbol entailing a number N_e of combinations required for marginalization lower than the threshold N_m , obtain the corresponding APP values using an exhaustive search over all N_e required combinations.
 - *Example:* This applies to the edge symbols **a** and **d** in **P1** and **P3** in Table 5, where the number of combinations required is $N_e = 4 < N_m = 8$.
- **Step 3:** For GF(q) symbols that multiplex **only** with symbols falling under step 2, compute the APPs by limiting the combinations associated with the GF(q) symbol from step 2 only to the ones yielding the N_m largest APPs for this symbol.

- *Example:* Assume we are transmitting three consecutive GF(256)*symbols α , β , and γ mapped onto two consecutive 64-QAM STBC codewords. Then symbols α and γ fall under Step 2, while the APPs for β have to be computed as above. Assuming $N_m = 16$, then the marginalization over α and β will be carried out by considering only $N_m \cdot N_m = 256$ terms instead of the $256 \cdot 256 = 65536$ terms in the exhaustive search.

**NB: Switching to GF(256) in this example is simply because no such case occurs with our default GF(64).*

- **Step 4:** For each remaining GF(q) symbol, not falling under steps 2 and 3, proceed with the following sub-steps:

- **Step 4.1:** Limit the combinations associated with the multiplexing GF(q) symbol from step 2 to the ones yielding the N_m maximum APP values for this multiplexing symbol.

- **Step 4.2:** Complete the marginalization of the APPs with respect to the adjacent GF(q) symbol whose APPs are still unavailable with an iterative procedure for a number r of iterations and depending on a parameter N_q . At i th iteration, the marginalization runs across the N_q combinations of the interleaved symbol with the highest N_q APP values. Such combinations are those computed in the previous $i - 1$ iteration of the algorithm. At the initialization stage, the N_q combinations are chosen randomly.

- *Example:* Step 4.1 applies to the middle GF(q) symbol **b** in **P1** and **P3** in Table 5, where marginalization is required across the interleaved edge GF(q) symbol **a**. The APP values of the edge symbol **a** are obtained from step 2. Thus, instead of searching over all the $2^6 = 64$ possible values of symbol **a**, we only limit the search to the $N_m = 8$ values of symbol **a** yielding the highest APP values (thus 8 highest likelihood values). Step 4.2 applies to the middle GF(q) symbol **b** in **P1** and **P3** in Table 5, where marginalization is required across the interleaved other middle GF(q) symbol **c**, whose APP values are not available from step 2. We start considering $N_q = 8$ randomly selected APP values for symbol **c** (out of the $2^4 = 16$ values theoretically needed) to obtain the (marginalized) APP values of symbol **b**. Then, we compute the APP values for symbols **b** and **c**, with the marginalization limited to the N_q random values of each. We refine then the choice of the N_q combinations used for marginalization to the ones yielding the highest N_q APP values for symbols **b** and **c**. This is repeated for r iterations.

Table 6 Example of number of combinations to be considered for APP marginalization

Number of combinations	P2	P3
GF(64) symbol a	$q^{m-1} = 262144$	$2^2 = 4$
GF(64) symbol b		$2^6 \times 2^4 = 1024$
GF(64) symbol c		$2^4 \times 2^6 = 1024$
GF(64) symbol d		$2^2 = 4$

The above algorithm may better be illustrated with the graph depicted in Figure 3, where the nodes represent the computation of the APP values for each GF symbol, and the arrows indicate the propagation of the most likely combinations of one GF symbol at a given node to the GF symbol at an adjacent node. This for the purpose of reduced marginalization according the different steps described in the above algorithm. The edge symbols **a** and **d** fall under step 2 and will therefore get their APP values available simply from step 2. The middle symbols **b** and **c** make use of N_m most likely combinations yielding the highest APP values for the edge GF symbols **a** and **d**, respectively. The propagation of these combinations is illustrated in Figure 3 by the arrows coming into nodes **b** (from node **a**) and **c** (from node **d**). Since symbols **b** and **c** multiplex together, then an iterative process as described in step 4.2 is followed by reusing the N_q most likely combinations of one symbol for marginalization to obtain the APP values of the adjacent symbol. This exchange of N_q combinations between GF symbols **b** and **c** is illustrated in the graph by the arrows connecting nodes **b** to **c**. Although in the

example the same value (equal to 8) is set for both numbers N_m and N_q , this does not reflect the general case where these two variables can be set with different values.

The introduction of the variables N_m and N_q taking values much lower than the total number q of the APP values is mainly inspired from the work originally done by the authors in [8,11] in their contribution to the FP7 DAVINCI project [4]. The authors of [8,11] have conducted a thorough analysis of the behavior of the non-binary LDPC decoder specifically in term of the APP distribution of the GF symbols at the input of the LDPC decoder. The main motivation in [8] is the proposal of sub-optimal LDPC decoding algorithm of reduced complexity and less memory requirements for real implementation. A key result found in [8,11] is that by feeding the non-binary LDPC decoder with only a limited number N_m of the highest APP values for each GF symbol (with N_m much less than the GF order q) achieves very close to the optimal performance whilst reducing significantly the non-binary LDPC decoding complexity and APP memory requirements. This finding

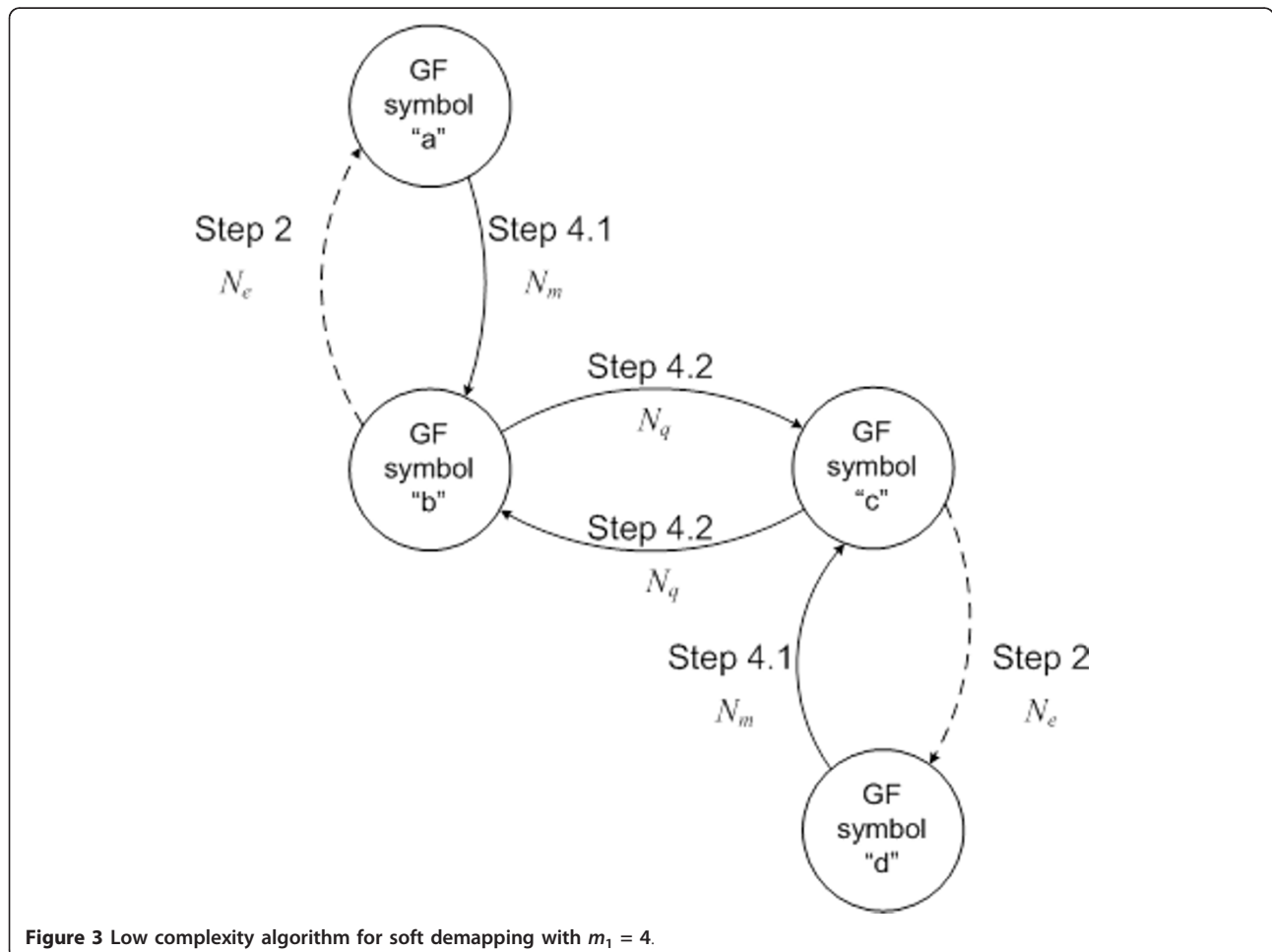


Figure 3 Low complexity algorithm for soft demapping with $m_1 = 4$.

is borrowed in our algorithm above to reduce the complexity challenge of the soft ML demapper.

Table 7 reports the number of combinations to be explored for extracting the q -ary APP values of each GF(64) symbol first without the low complexity algorithm and then for two settings of the low complexity algorithm. When the low complexity algorithm is selected, GF(64) symbols **b** and **c** require first a number of operations for sorting the APP values of GF(64) symbols **a** and **d**, respectively. The implemented algorithm, which is based on Merge and Sort approach, has a complexity of $n \cdot \log(n)$ (with n being the length of the vector to sort). The proposed algorithm reduces the number of combinations used for marginalization to obtain the APP values of the middle symbols **b** and **c** by a factor of approximately 7.5 without iterations, and a factor of approximately 10 with 3 iterations. The impact of the proposed algorithm on the error performance is assessed in Section 5.

5. Numerical results

Table 8 summarizes the simulation set up that was used to derive performance results. Both a SISO and a MIMO scenario are considered as representative of next generation cellular communication systems.

Figure 4 depicts the Frame Error Rate (FER) results obtained in the SISO scenario using two different patterns to map the GF(64) symbols onto QAM constellation symbols. The first mapping is an arbitrary mapping which does not respect the first rule devised in our solution, whereas the second mapping referred to as optimum mapping does.

As illustrated in Figure 4, for QPSK and 64QAM, where $m_1 = 1$ (cf. Table 2), there is no significant difference between the arbitrary and the proposed mapping patterns, since inherently here only one GF(64) symbol maps onto three QPSK symbols or one 64QAM symbol. However, for 16QAM, where $m_1 = 2$ and $m_2 = 3$ (cf. Table 2), two GF(64) symbols are mapped onto the same mapping onto one 16QAM symbol, and here the results show clear SNR gain of 0.5 dB for the mapping respecting the first design rule as compared to a pattern not respecting this rule, hence validating the merits of this rule. It is noteworthy here that at this stage, there is no issue of trade-off between performance and

complexity (this will come later when considering the second and third design rules proposed).

We then move to the MIMO context in order to validate the second and third rules introduced in our mapping strategy, which aim to achieve a trade-off between performance and complexity. First we analyze the complexity in terms of number of operations of the APP extraction. Specifically, with the term *operation* we refer to a summation or a comparison of real-valued numbers, so either the summation or comparison operation has the same computational weight. We consider first the case of 16QAM with the three patterns given in Table 5, where patterns P1 and P3 respect the second rule, but not pattern P2. Figure 5 depicts the number of operations required for marginalization in the computation of the APP values of the $m_1 = 4$ GF(64) symbols which map together onto $m_2 = 6$ QAM symbols and $m_3 = 3$ STBC codewords. Four curves show the number of operations (in logarithm scale) as a function of the threshold N_m introduced in the proposed algorithm. The curves are as follows:

- The first curve in black circular marker gives the number of operations when an exhaustive search with pattern P2 is performed.
- The second curve in red circular marker gives the number of operations when an exhaustive search with pattern P1 or P3 is performed.
- The third curve in blue downwards triangular marker shows the number of operations using the proposed algorithm without the iterative step 4 (i.e., simply replace sub-step 4.2 by an exhaustive search).
- The fourth curve in green with diamond markers considers the iterative step 4 of the proposed algorithm with $r = 3$ iterations and $N_q = 10$.

From Figure 5, we can first clearly appreciate the huge reduction in complexity (cf. gap between first curve using P2, and the other curves using P1 and P3). This clearly validates the merit of our second rule from the complexity perspective, where patterns P1 and P3 respect this second rule, but not pattern P2. Moreover, from Figure 5, we can also clearly appreciate the significant reduction in complexity (cf. gap between second curve, and third and fourth curves) brought by the use

Table 7 Example of reduction of the number combinations for P1 and P3 from Table 6

Number of combinations	Without the algorithm	Prop. Algorithm with $r = 0, N_m = 8$	Prop. algorithm with $r = 3, N_m = 8, N_q = 8$
GF(64) symbol a	$64 \times 2^2 = 256$	$64 \times 2^2 = 256$	$64 \times 2^2 = 256$
GF(64) symbol b	$64 \times 2^6 \times 2^4 = 65536$	$64 \times 6 + 64 \times N_m \times 2^4 = 8576$	$64 \times 3 \times N_m \times N_q + 2 \times 64 \times 6 = 13056$
GF(64) symbol c	$64 \times 2^6 \times 2^4 = 65536$	$64 \times 6 + 64 \times N_m \times 2^4 = 8576$	
GF(64) symbol d	$64 \times 2^2 = 256$	$64 \times 2^2 = 256$	$64 \times 2^2 = 256$
Block of m_1 GF symbols	$64 \times 2 \times (2^{10} + 2^2) = 131584$	$2 \times 64 \times (2^2 + 6 + N_m \times 2^4) = 17664$	$2 \times 64 \times (2^2 + 6) + 64 \times 3 \times N_m \times N_q = 13568$

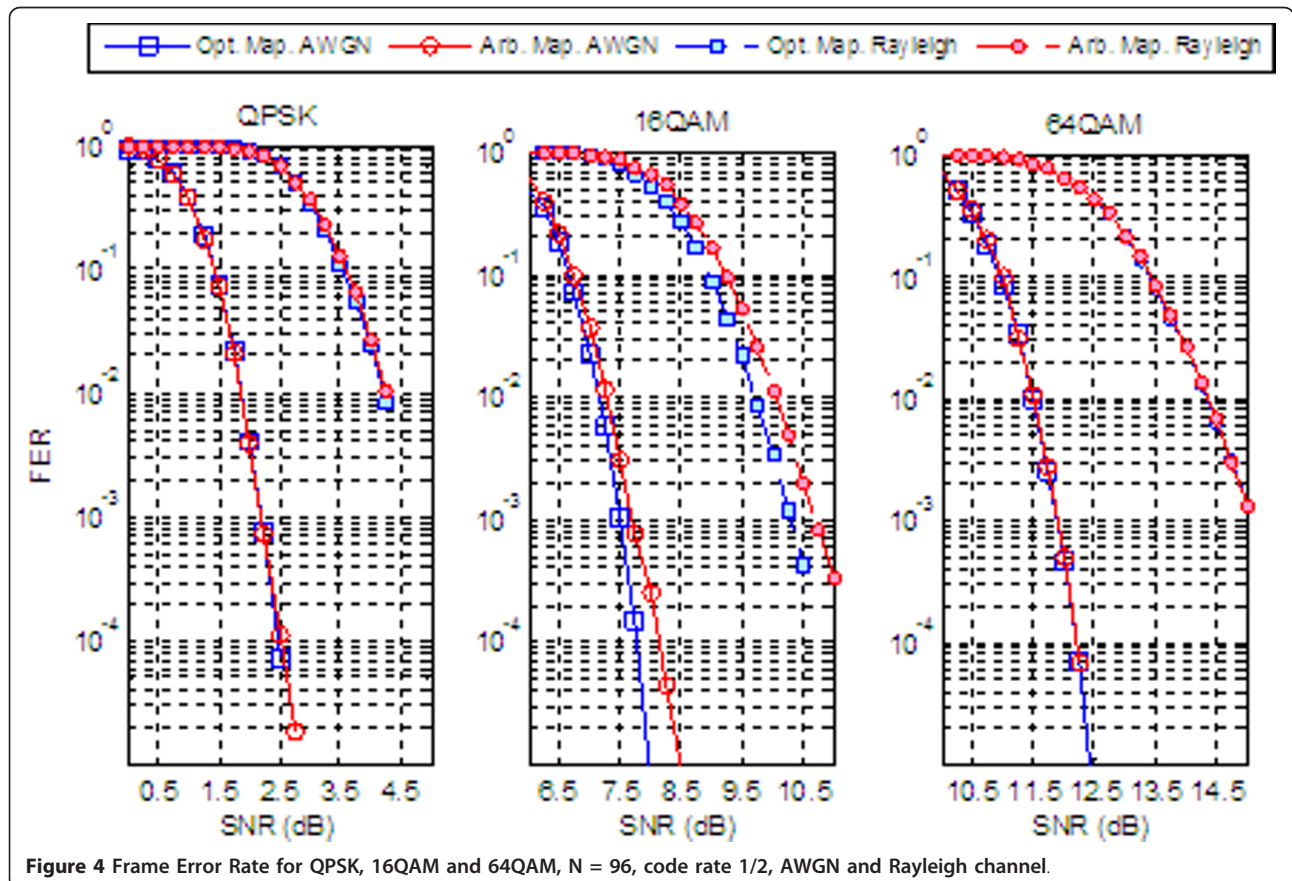
Table 8 Simulations set up

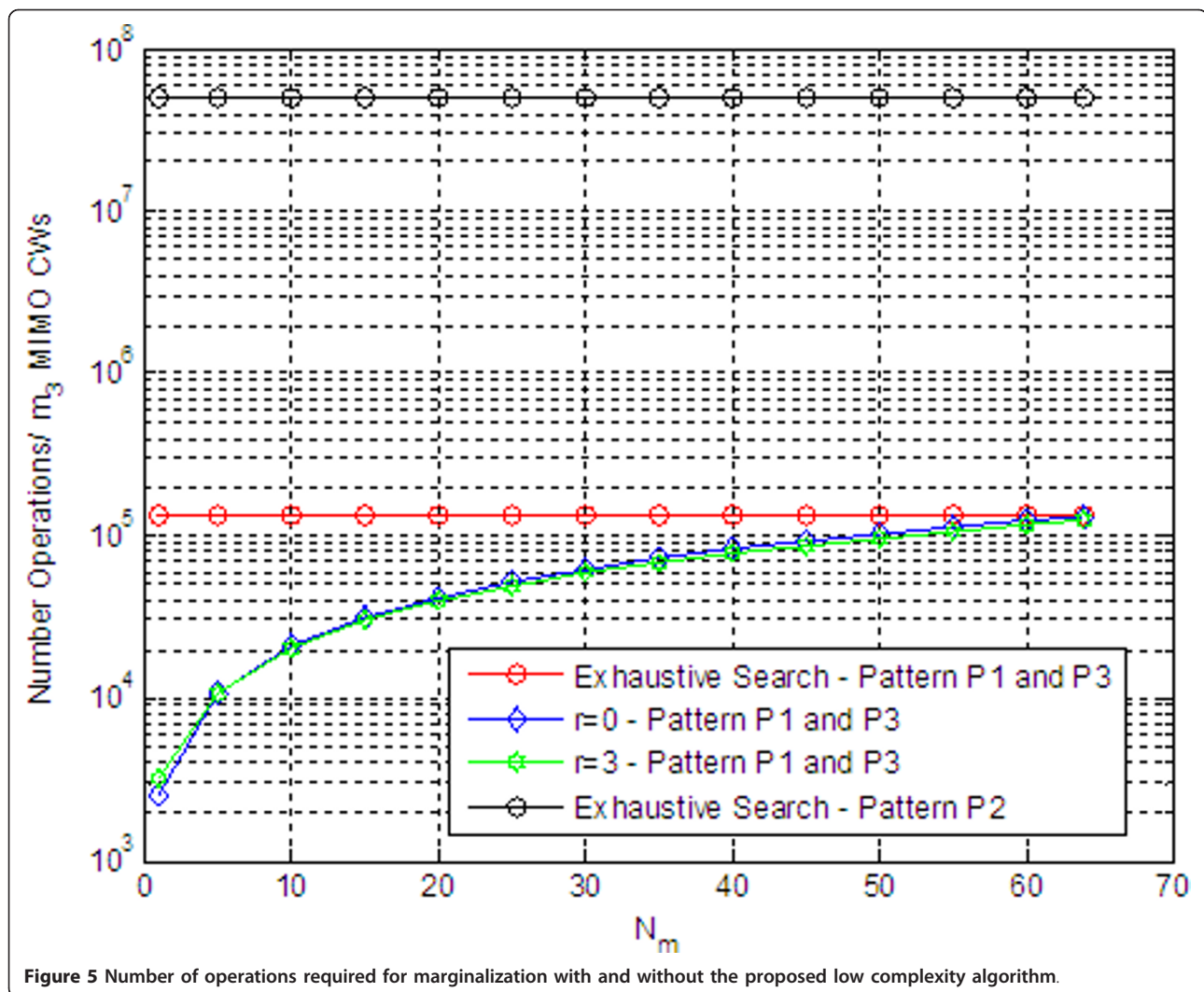
Modules	Set up
FEC encoder	DAVINCI NB-LDPC codes GF order = 64 Codeword length = 96 symbols = 576 bits
FEC decoder	Extended Min-Sum algorithm Number of soft values per symbol fed to the decoder = $q_m = 16$ (highest values) Maximum number of decoding iterations = 30
Constellation	QPSK, 16QAM, 64QAM
MIMO encoder	2×2 antennas configuration Uncoded Spatial multiplexing STBC codeword length $Q = 2$
Channel model	AWGN and Rayleigh channels
Soft demapper	Soft ML demapping Proposed low complexity algorithm with $N_m = 8$, $N_q = 4$ to 16, number of demapping iterations $r = 0$ to 3

of the proposed algorithm (with and without iterations) as compared to the exhaustive search. The reduction in complexity clearly decreases when increasing the threshold N_m . For a typical value of $N_m = 8$, we can appreciate nearly one decay (i.e., a factor of 10) complexity reduction; thanks to the proposed algorithm.

The second aspect to be assessed here is the impact of the proposed mapping strategy and demapping

algorithm on the error protection performance. This is illustrated in Figure 6, for patterns P1 and P3 with different configurations. It is worth noting here that the pattern P2 could not be evaluated since its breach of the second rule makes it non practical for computer simulations. Our reference curves are the ones in red solid line which perform an exhaustive search (i.e., do not implement the proposed algorithm). In this figure, square



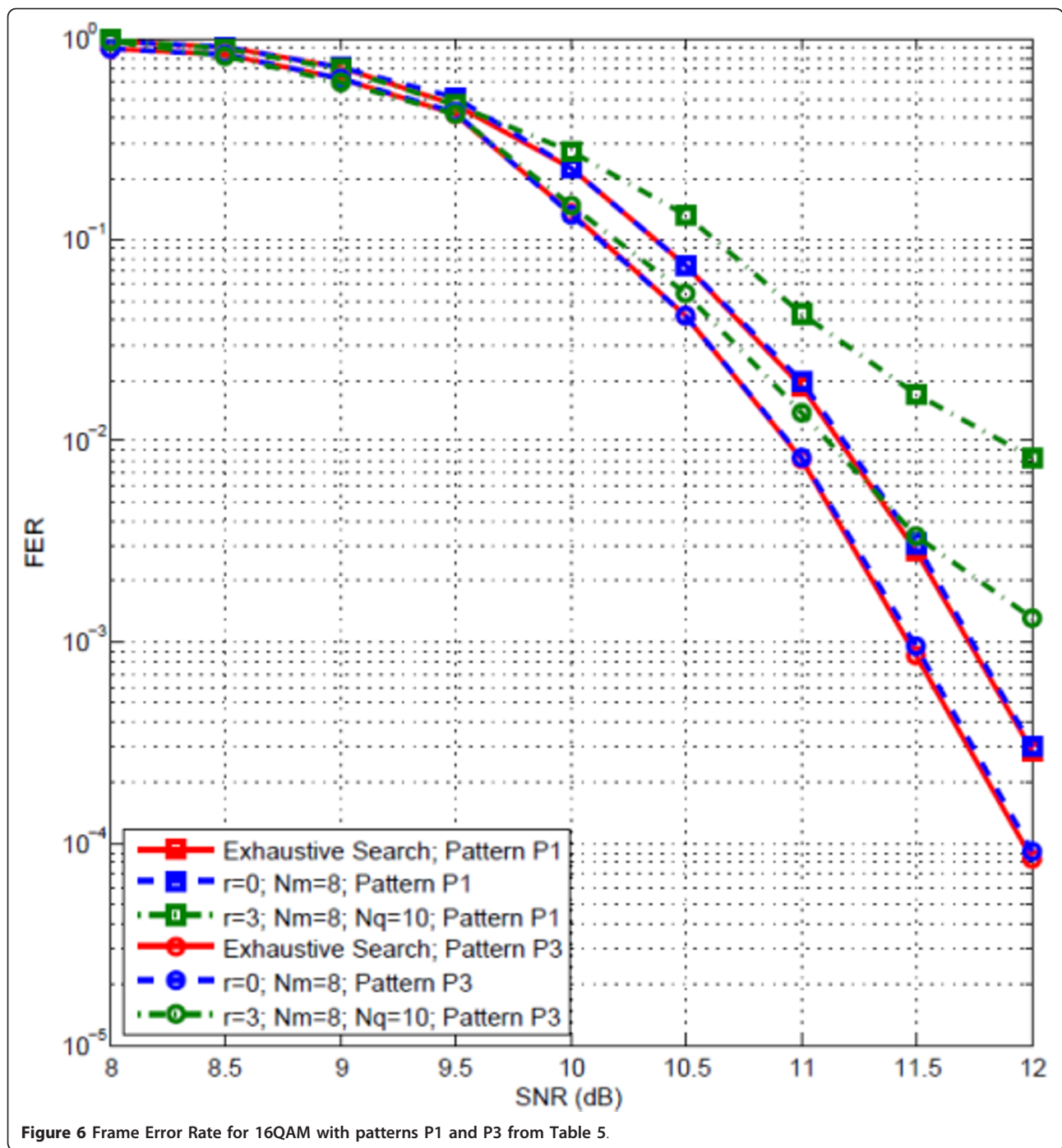


marker is used for mapping pattern P1, and circular marker for mapping pattern P3.

From Figure 6, we first compare the performance gap between patterns P1 and P3 with the exhaustive search used in both. This is in order to appreciate the trade-off in performance due to the second rule and the merits of the third rule. The performance gap between P1 and P3 is almost 0.25 dB, when P1 has a constant channel selectivity order equal to 2 and P3 has an average channel selectivity order equal to 2.5 (it is equal to 2 at the edge symbols and 3 at the middle symbols). As mentioned previously, both patterns P1 and P3 respect the second rule, but only P3 respects the third rule. Hence, from this comparison, the merit of the third rule is clearly appreciated (approximately 0.25 dB SNR gain) at the same level of complexity. The same performance gap is expected between patterns P2 and P3 (although as said before simulations with pattern P2 are not feasible since it breaches the second rule). This expectation

is motivated by the fact that the gap in channel selectivity order between P2 and P3 is equal to 0.5, which is the same gap between P3 and P1 (PS: the average channel selectivity order is equal to 3, 2.5, and 2, respectively, for patterns P2, P3, and P1). Hence, the penalty in performance of the second design rule is expected to be around 0.25 dB, compared to a pattern P3 respecting the second and third design rules, and 0.5 dB compared to a pattern P1 respecting the second rule but not the third rule.

Now let us compare the performance of both patterns P1 and P3 when using the proposed soft demapping algorithm. From Figure 6, for both patterns P1 and P3, we do not notice any appreciable degradation when using the proposed algorithm with threshold $N_m = 8$, and without using the iterative process, compared to when using the exhaustive search. This is an important result as it shows the potential of the proposed algorithm to reduce the complexity by tenfold without



practical degradation in the FER performance. Further reduction of the complexity by means of the iterative process for example, does degrade the FER performance. The degradation of the iterative process in the waterfall region at target FER of 10^{-2} appears tolerable (up to 0.5 dB), whilst the degradation in the error floor region appears significant. This reflects the trade-off someone can obtain between FER performance and further reduction of the complexity with the iterative process.

Further analysis was carried out for the case of MIMO 64QAM. In such a case, we consider two different mapping patterns as illustrated in Table 9.

Both P1 and P2 respect the first and second rule, but only P2 respects the third rule. With 64QAM, the proposed algorithm must necessarily use the iterative process, since there are no edge symbols falling under step 2 of the proposed algorithm. Similarly to the 16QAM case, the sorting of the APP values should be taken into

Table 9 Used patterns for mapping GF(64) symbols to STBC codewords (64QAM)

Number	Antenna number	Mapping pattern ($m_1 = 2, m_2 = 2, m_3 = 1$)	
		I0	Q0
P1	A#1	$a_0a_1a_2$	$a_3a_4a_5$
	A#2	$b_0b_1b_2$	$b_3b_4b_5$
P2	A#1	$a_0a_1a_2$	$b_0b_1b_2$
	A#2	$a_3a_4a_5$	$b_3b_4b_5$

account in the computation of the complexity. Table 10 shows a complexity reduction of 35% with respect to the exhaustive search when N_q is equal to 24 and 44% when N_q is equal to 20.

Figure 7 also shows the FER performance results for both patterns with and without the proposed algorithm in the configurations given in Table 10. A circle marker is used for the curves with pattern P1 and square marker for the curves with pattern P2.

Table 10 Example of reduction of the number combinations for 64QAM

Number of combinations	Without the algorithm	Prop. algorithm with $r = 3, N_q = 20$	Prop. algorithm with $r = 3, N_q = 24$
GF(64) symbol a	$64 \times 2^6 = 4096$	$64 \times 3 \times N_q + 2 \times 64 \times 6 = 4608$	$64 \times 3 \times N_q + 2 \times 64 \times 6 = 5376$
GF(64) symbol b	$64 \times 2^6 = 4096$		
Block of m_1 GF symbols	$2 \times 64 \times (2^6) = 8192$	$64 \times 3 \times N_q + 2 \times 64 \times 6 = 4608$	$64 \times 3 \times N_q + 2 \times 64 \times 6 = 5376$

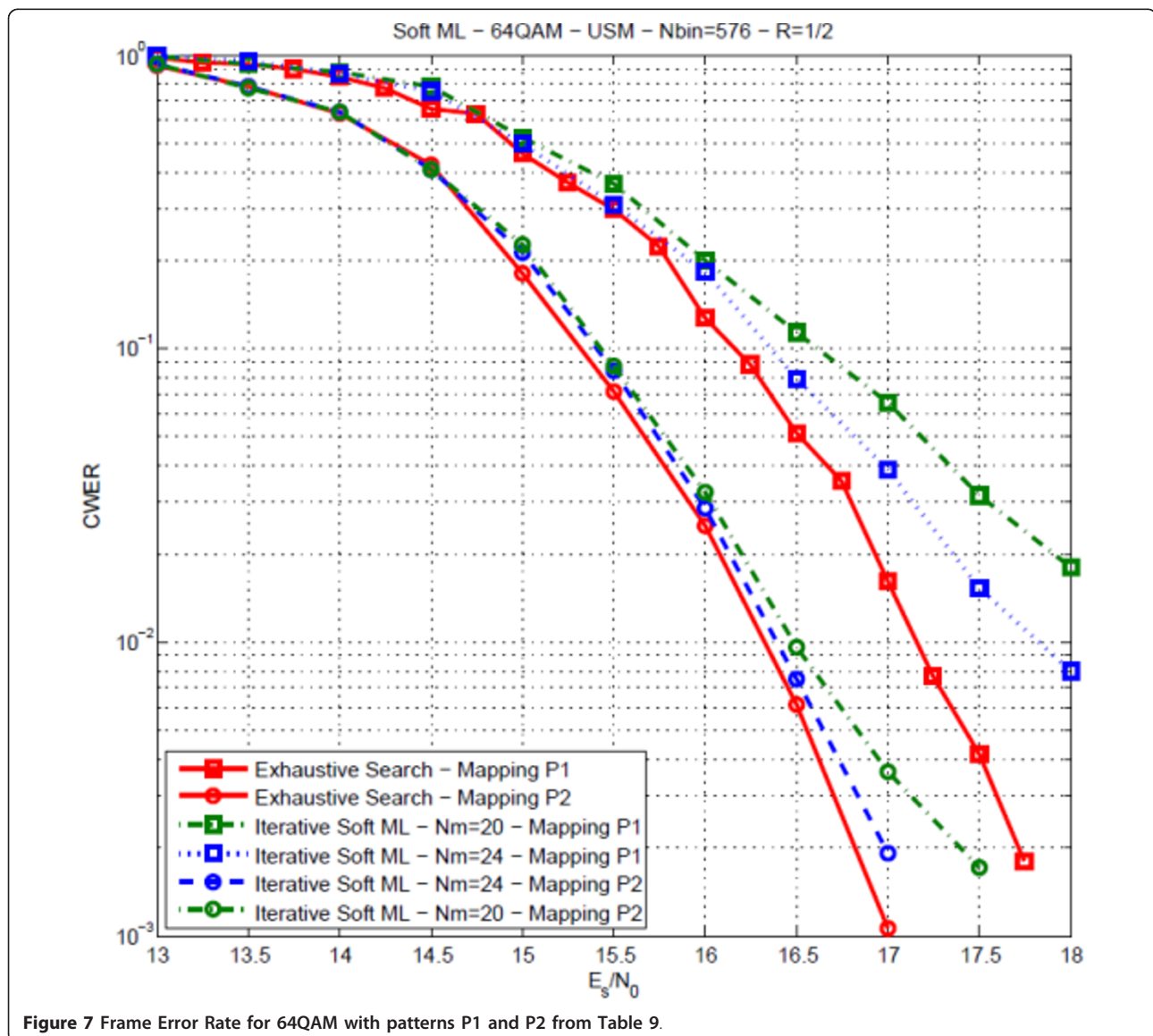


Figure 7 Frame Error Rate for 64QAM with patterns P1 and P2 from Table 9.

From Figure 7, we can first appreciate a gain of nearly 0.8 dB for pattern P2 as compared to P1. This confirms further the potential of the third rule in achieving much higher diversity. Second, with pattern P2, we can clearly appreciate a slight degradation in performance nearly 0.2 dB when using the proposed low complexity iterative demapping algorithm with $N_q = 24$ (35% complexity reduction). The degradation becomes higher 0.5 dB for $N_q = 20$ (44% complexity reduction). So clearly, there is a trade-off between the tolerable FER performance degradation and the target complexity reduction, and the proposed mapping strategy and low complexity demapping algorithm provide the tools to achieve the trade-off desired.

6. Conclusions

In this article, we have addressed the particular complexity challenge of the soft ML demapping faced with non-binary LDPC codes when one $GF(q)$ symbol spreads across multiple QAM symbols and STBC codewords. A solution is proposed combining a mapping strategy based on three design rules at the transmitter, and a low complexity soft ML demapping algorithm at the receiver.

At the transmitter side, the mapping strategy introduced three design rules to achieve the best trade-off between performance and complexity. In the first rule, the I or Q component of an M -QAM symbol should carry (in part or in full) the binary image of only one $GF(q)$ symbol. This rule was shown to bring an SNR performance gain of approximately 0.5 dB compared to mapping patterns not respecting this rule. In the second rule, the I/Q components issued from one $GF(q)$ symbol are carried into the minimum possible number of STBC codewords. This second rule clearly restricts the freedom to let the $GF(q)$ symbol enjoy higher channel selectivity, but fortunately has the advantage of reducing drastically the complexity of the soft ML demapper. In the third rule, the I/Q components issued from one $GF(q)$ symbol are mapped onto the transmission units which ideally can experience independent channel fading within the STBC codeword carrying this $GF(q)$ symbol. This third rule aims at exploiting the last degree of freedom left by the binding second rule to achieve high channel selectivity within the $GF(q)$ symbol. With mapping patterns respecting the second rule, it was shown that a tenfold complexity reduction can be achieved compared to patterns not respecting this second rule. The trade-off in performance was shown to be small, 0.25 and 0.5 dB performance degradation for the patterns respecting the second rule with and without the third rule, respectively.

At the receiver side, an algorithm was proposed to reduce the complexity of the soft ML demapper. The

algorithm exploits the correlation existing between $GF(q)$ symbols but also any knowledge available on the APP values of the $GF(q)$ symbols in the vector of m_1 $GF(q)$ symbols, which map together onto the vector of m_2 M -QAM symbols and further on onto the vector of m_3 STBC codewords. The algorithm also considers only a limited number of potential combinations for each $GF(q)$ symbol, those associated with this same limited number of highest APP values for this symbol. This latter consideration has been inspired from the original work done by the authors of [8,11] to reduce the complexity of the non-binary LDPC decoder. Our proposed algorithm was shown to further reduce the complexity of the soft ML demapper by up to 85%.

The proposed solution mitigates the complexity challenge at the receiver faced with non-binary LDPC codes when one $GF(q)$ symbol spreads across multiple QAM constellation symbols and STBC codewords, at the expense of a slight performance degradation but not sacrificing the performance merits of non-binary LDPC codes. This removes any restriction on the size of the GF order, QAM constellation order, and MIMO scheme, whilst preserving the merits of non-binary LDPC codes at very reasonable receiver complexity. Future work will be focused on the assessment of the MIMO schemes which are best suited for combination with $GF(64)$ non-binary LDPC codes, with the ultimate goal of proposing non-binary LDPC codes for beyond 4 G wireless communication systems.

Acknowledgements

This study was supported by INFSCO-ICT-216203 DAVINCI "Design And Versatile Implementation of Non-binary wireless Communications based on Innovative LDPC Codes" <http://www.ict-davinci-codes.eu> funded by the European Commission under the Seventh Framework Programme (FP7) [4].

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Competing interests

The authors declare that they have no competing interests.

Received: 27 April 2011 Accepted: 20 February 2012

Published: 20 February 2012

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doi:10.1186/1687-1499-2012-55

Cite this article as: Mourad et al.: Low complexity soft demapping for non-binary LDPC codes. *EURASIP Journal on Wireless Communications and Networking* 2012 **2012**:55.

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