Noori and Ardakani *EURASIP Journal on Wireless Communications and Networking* 2013, **2013**:11
http://jwcn.eurasipjournals.com/content/2013/1/11

EURASIP Journal on
Wireless Communications and Networking
a SpringerOpen Journal

RESEARCH Open Access

On the achievable rates of symmetric Gaussian multi-way relay channels

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Abstract

Considering a symmetric Gaussian multi-way relay channel (MWRC) with K users, this work compares two transmission strategies, namely one-way relaying (OWR) and multi-way relaying (MWR), in terms of their achievable rates. While in OWR, only one user acts as data source at each time and transmits in the uplink channel access, users can make simultaneous transmissions in MWR. First, we prove that for MWR, lattice-based relaying ensures a gap less than $\frac{1}{2(K-1)}$ bit from the capacity upper bound while MWR based on decode-and-forward (DF) or amplify-and-forward (AF) is unable to guarantees this rate gap. For DF and AF, we identify situations where they also have a rate gap less than $\frac{1}{2(K-1)}$ bit. Later, we show that although MWR has higher relaying complexity, surprisingly, it can be outperformed by OWR depending on K and the system SNR. Summarily speaking, for large K and small users' transmit power, OWR usually provides higher rates than MWR.

Keywords: Multi-way relay channel, Multi-way relaying, One-way relaying, Capacity gap, Achievable rate

1 Introduction

As an extension of two-way relay channels (TWRCs) [1-3], multi-way relay channels (MWRCs) have been introduced by Gunduz et al. [4] to improve the spectral efficiency in multicast communication networks [5,6]. In an MWRC, several users want to (fully) share their information with the help of one or more relays. Some practical examples of MWRCs are conference calls in a cellular network, file sharing between several wireless devices, and device-to-device communications.

Different relaying schemes are applicable to MWRCs. One approach is to divide the data transmission time into several one-way relaying (OWR) phases. Conventional relaying strategies, i.e. amplify-and-forward (AF) and decode-and-forward (DF), can be accommodated by OWR. A more recent approach is to employ multi-way relaying (MWR) where several users are allowed to simultaneously transmit to the relay. For MWR, several schemes have been proposed including AF, DF, and compress-and-forward (CF)[7]. Further, an MWR approach based on lattice codes has been proposed [8-10] which is called functional-decode-forward (FDF). In the following, we generally use OWR and MWR to refer to the discussed relaying schemes for MWRCs. Note that MWR generally has a higher relaying complexity than OWR.

There exist several works studying the performance of MWRCs in terms of their achievable rate. In [4], it is shown that MWR with CF can achieve to within $\frac{1}{2(K-1)}$ bit of the common rate capacity where K is the number of users. Also, for TWRCs with FDF, it is shown that the capacity gap is less than $\frac{1}{2}$ bit [11] while FDF achieves the capacity for binary MWRCs [9]. Ong et al. [10] show that under some conditions, FDF achieves the common rate capacity of Gaussian MWRCs. Furthermore, they briefly discuss the capacity gap of FDF when all users and the relay have equal power.

In this work, a detailed performance comparison between MWR and OWR is provided. More specifically, we focus on the common rate of these relaying schemes over symmetric Gaussian MWRCs. The Gaussian symmetric model can be practically associated with situations where dynamic power adjustment mechanism at the users is applied to compensate for the slow fading effect. For instance, in a cellular CDMA system, dynamic power adjustment is used to equalize the received power of users at the base-station (relay) resulting in a higher achievable rates in the system [12].

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For MWR, we prove that similar to CF, FDF assures a gap less than $\frac{1}{2(K-1)}$ bit with the common rate capacity of symmetric Gaussian MWRCs. For AF and DF, we first show that they may have a larger than $\frac{1}{2(K-1)}$ bit capacity gap and then we find the SNR regions where a gap smaller than $\frac{1}{2(K-1)}$ bit is guaranteed. In the next step, we study the achievable rate of MWRCs using OWR. For this purpose, we consider OWR with AF and DF and show that for the considered MWRC setup, DF always outperforms AF when OWR is used. Then, the achievable rate of MWR with CF and FDF (guaranteeing a less than $\frac{1}{2(K-1)}$-bit gap) is compared with the rate of DF OWR. Surprisingly, in spite of a higher relaying complexity, MWR is not always superior to OWR and we find the SNR regions where OWR indeed outperforms MWR. According to our study, by decreasing SNR or increasing K, we may see OWR surpassing MWR.

The article is organized as follows: Section 2 provides the system model and some definitions. The capacity gap analysis for MWR is discussed in Sections 3 and 4 focuses on the rate study for OWR. Rate comparison between MWR and OWR is presented in Sections 5 and 6 concludes the article. Further, all proofs are provided in Appendix.

2 Preliminaries

Consider an MWRC where $K \geq 2$ users want to share their data without having direct user-to-user links. It means that each user aims to receive all other users data as well as to transmit its data to all other users. We name users by $u_1, u_2, \ldots, u_K$ and their data by $X_1, X_2, \ldots, X_K$. Each user has a limited average power P, thus, for all i, $E[X_i^2] \leq P$. To enable data communication between users, a relay, $\mathcal{R}$, with average transmit power P_r is employed.

Data communication consists of uplink and downlink phases. In the uplink phase, users transmit their data to $\mathcal{R}$ while in the downlink phase $\mathcal{R}$ broadcasts its message. We assume that the received signals at the relay and users are contaminated by a zero-mean Gaussian noise with unit variance. Due to considering AWGN channel, we refer to this MWRC by *Gaussian MWRC*.

In this article, we consider the common rate capacity of Gaussian MWRCs. The common rate capacity is the maximum data rate at which all users can reliably transmit and receive data. In other words, if we denote the achievable data rates at all user by a K-tuple $(R_1, R_2, \ldots, R_K)$, where R_i is achievable at u_i, then

$$R^c = \sup\{R : (R, R, \ldots, R) \text{is achievable.}\} \quad (1)$$

For more details on common rate definition and its applications in MWRCs, the reader is referred to [4,10]. Note that for a general Gaussian MWRC, the common rate capacity is yet to be known. Thus, in the following, we use the capacity upper bound for our capacity gap analysis instead of the capacity itself. For this purpose, we borrow the following lemma from [4].

Lemma 1. *An upper bound on the common rate capacity of a symmetric Gaussian MWRC is*

$$R^c_{\mathrm{UB}} = \min\left\{\frac{\log\left(1+(K-1)P\right)}{2(K-1)}, \frac{\log\left(1+P_r\right)}{2(K-1)}\right\} \quad (2)$$

Please notice that in this article, $\log(\cdot)$ represents the logarithm in base 2.

3 Rate analysis for MWR

Here, we focus on the achievable rate of MWR and study the capacity gap for FDF, DF and AF. We prove that similar to CF, FDF guarantees a capacity gap less than $\frac{1}{2(K-1)}$ bit.

3.1 Capacity gap of FDF

As suggested in [10], for MWR with FDF, the uplink transmission is divided into $K-1$ multiple-access (MAC) slots. In each MAC slot, a pair of users transmit their data to the relay. Each user encodes its data using nested lattice codes [13]. This enables $\mathcal{R}$ to directly decode the modulo-sum of the users data, instead of decoding their data separately, after receiving the superimposed users' signals. Then, $\mathcal{R}$ encodes the sum value and broadcasts it to the users. This pair-wise transmission continues for $K-1$ times until u_{K-1} and u_K transmit their data. Now, in addition to own data, each user has received $K-1$ independent linear combinations of other users data. By forming a system of linear equations, consisting of these $K-1$ independent equations, each user can find the data of any other user. For more information on this pairwise transmission strategy, the interested reader is referred to [9-11].

The achievable rate of lattice-based relaying was first studied in [8] for TWRC. Later, the following lemma was proposed [10] for the achievable common rate of FDF.

Lemma 2. *The maximum achievable rate of FDF over a symmetric Gaussian MWRC is*

$$R^c_{\mathrm{FDF}} = \min\left\{\frac{\log\left(\frac{1}{2}+\frac{KP}{2}\right)}{2(K-1)}, \frac{\log\left(1+P_r\right)}{2(K-1)}\right\}. \quad (3)$$

Proof. Please see [10]. □

The following theorem states the performance of FDF in comparison with the capacity upper bound.

Theorem 1. *The gap between the achievable rate of FDF and the capacity of a K-user symmetric Gaussian MWRC is less than* $\frac{1}{2(K-1)}$ *bit.*

Proof. See Appendix. □

For numerical illustrations, the achievable rate of FDF and the capacity upper bound for several cases are depicted in Figures 1, 2 and 3. In Figure 1, users' SNR effect on the capacity gap is studied while the effect of the relay SNR and K are presented in Figures 2 and 3, respectively. As seen, the achievable rate of FDF always sits above the $\frac{1}{2(K-1)}$-bit gap. Further, when downlink limits the rate, FDF achieves the capacity.

3.2 Capacity gap of DF

For DF MWR, all users share the same uplink transmission time and simultaneously send their data to the relay. Then, $\mathcal{R}$ decodes the data of all users and broadcasts them over the downlink as described in [4].

Lemma 3. *The maximum achievable common rate of DF MWR is*

$$R^c_{\mathrm{DF}} = \min\left\{\frac{\log(1+KP)}{2K}, \frac{\log(1+P_r)}{2(K-1)}\right\}. \tag{4}$$

Proof. See [4]. □

Our analysis reveals that depending on SNR and K, DF may not be able to guarantee a $\frac{1}{2(K-1)}$-bit gap to R^c_{UB}. The following theorem summarizes the result.

Theorem 2. *The gap between R^c_{DF} and R^c_{UB} is less than $\frac{1}{2(K-1)}$ bit if either $P_r < \min\{2(1+KP)^{\frac{K-1}{K}} - 1, (K-1)P\}$ or $(K-1)P < P_r$ and $(K-1)P < 2(1+KP)^{\frac{K-1}{K}} - 1$.*

Proof. Please see Appendix. □

As the numerical results in Figures 1, 2 and 3 indicate, in some SNR regions and depending on the number of users, the capacity gap might be larger than $\frac{1}{2(K-1)}$ bit for DF.

3.3 Capacity gap of AF

When AF is used for MWR, similar to DF, all users simultaneously transmit their data to the relay. Unlike DF, however, $\mathcal{R}$ only amplifies the received signal, while meeting the relay power constraint, and transmits it back to the users [4]. Then, each user cancels out its own signal from the broadcast signal and decodes the other users data. In this case, it is easy to prove the following lemma for the achievable rate of AF.

Lemma 4. *In a K-user symmetric Gaussian MWRC,*

$$R^c_{\mathrm{AF}} = \frac{1}{2(K-1)}\log\left(1+\frac{(K-1)PP_r}{1+KP+P_r}\right) \tag{5}$$

is the maximum common rate that AF can achieve.

Now, the following theorem is presented on the capacity gap of AF.

Theorem 3. *The gap between R^c_{AF} and R^c_{UB} is less than $\frac{1}{2(K-1)}$ if $P_r \le (K-1)P$ and $P_r^2 - (K-2)PP_r < KP$ or $(K-1)P < P_r$ and $K(K-1)P^2 - P - 1 < P_r + (K-1)PP_r$.*

Proof. Please see Appendix. □

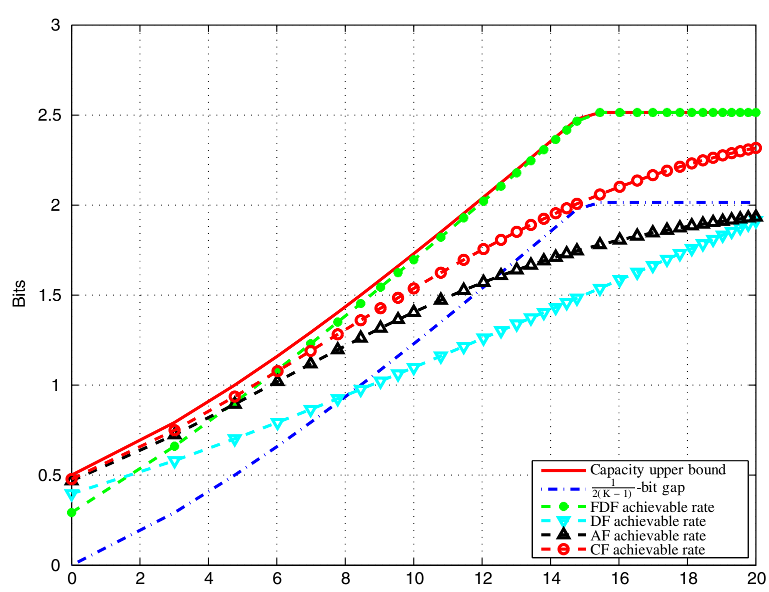

Figure 1 Achievable rates of relaying schemes when $K = 3$ and $P_r = 15$ dB.

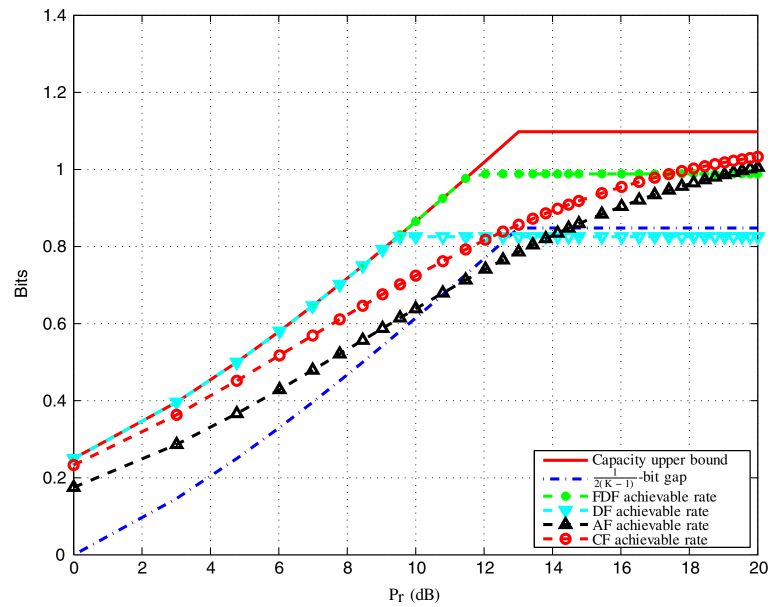

Figure 2 Achievable rates of relaying schemes when $K = 3$ and $P = 10$ dB.

Depending on the SNR and K, the achievable rate of AF may fall under the $\frac{1}{2(K-1)}$-bit gap from the capacity upper bound (Figures 1, 2 and 3).

4 Rate analysis for OWR

In this section, we study the achievable rate of OWR. In a MWRC with OWR, transmission time in both uplink and downlink phases is divided into K slots. In each slot, one user serves as the source and the rest are the data destinations. First, the source user transmits in the uplink slot and then $\mathcal{R}$ broadcasts the data back to the users in the downlink slot. Since each user transmits in only one uplink slot and stays silent in the rest, it can upscale its power to KP during its transmission turn without violating the power constraint.

When DF is employed for OWR, $\mathcal{R}$ first decodes the received data from the source in the uplink and then broadcasts it to the users. Then, destination users decode

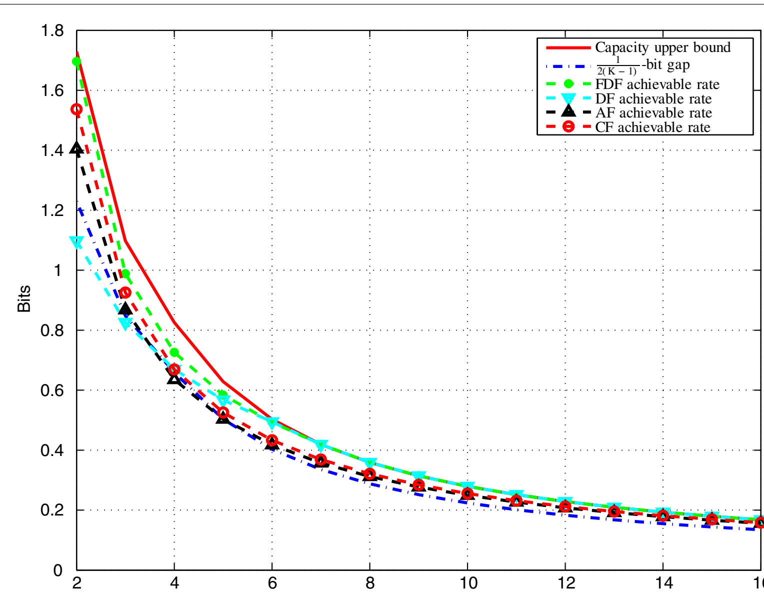

Figure 3 Achievable rates of relaying schemes when $P = 10$ dB and $P_r = 15$ dB.

the received signal from the relay. It is easy to show that the achievable rate of DF OWR is

$$R_{\mathrm{DF_O}} = \min\left\{\frac{\log(1+KP)}{2K}, \frac{\log(1+P_r)}{2K}\right\} \tag{6}$$

For AF, $\mathcal{R}$ amplifies and forwards the received signal in the uplink without further processing. The decoding is done at the destination users. The achievable rate of this scheme is

$$R_{\mathrm{AF_O}} = \frac{1}{2K}\log\left(1+\frac{KPP_r}{1+KP+P_r}\right) \tag{7}$$

It can be shown that OW (with DF or AF) does not guarantee a $\frac{1}{2(K-1)}$-bit gap.

Now, we like to compare the performance of AF and DF for OW. Using the achievable rates in (6) and (7), we can derive the following theorem.

Theorem 4. *In a symmetric Gaussian MWRC with OWR, DF always outperforms AF in terms of the achievable rate.*

Proof. See Appendix. □

5 Comparison between the rate of OWR and MWR

In this section, we compare the performance of OWR and MWR. For OWR, we consider DF which has the superior performance over AF. Also, FDF and CF are considered for MWR since they provide a guaranteed rate performance (capacity gap).

5.1 Comparison of DF OWR and FDF MWR

First, assume $P_r < \frac{KP}{2}$. Thus,

$$R_{\mathrm{FDF}} = \frac{\log(1+P_r)}{2(K-1)}, \quad R_{\mathrm{DF_O}} = \frac{\log(1+P_r)}{2K}. \tag{8}$$

In this region, it is clear that MWR outperforms OWR due to its smaller pre-log factor. However, increasing K decreases the gap between MWR and OWR. Consider the second SNR region where $\frac{KP}{2} \le P_r < KP$ and

$$R_{\mathrm{FDF}} = \frac{\log(1+\frac{KP}{2})}{2(K-1)}, \quad R_{\mathrm{DF_O}} = \frac{\log(1+P_r)}{2K}. \tag{9}$$

In this SNR region, FDF MWR surpasses DF OWR if

$$P_r < \left(1+\frac{KP}{2}\right)^{\frac{K}{K-1}} - 1 \tag{10}$$

Since the right hand side of (10) is an increasing function of P, it can be concluded that for a fixed P_r, decreasing P reduces the chance of holding the inequality (10). It means that when the relay's received SNR decreases, OWR may start performing better than MWR.

Now, we consider a third region where $KP \le P_r$. Here,

$$R_{\mathrm{FDF}} = \frac{\log(1+\frac{KP}{2})}{2(K-1)}, \quad R_{\mathrm{DF_O}} = \frac{\log(1+KP)}{2K}. \tag{11}$$

Thus, MWR performs better if

$$(1+KP)^{\frac{1}{K}} < \left(1+\frac{KP}{2}\right)^{\frac{1}{K-1}}. \tag{12}$$

From (12) and noticing that $(1+x)^{\frac{1}{x}}$ is a decreasing function and $\lim_{x\to 0}(1+x)^{\frac{1}{x}} = e^x$, it can be concluded that decreasing P or increasing K (without violating $KP \le P_r$) is in favor of OWR. Numerical results for the comparison between the achievable rate of DF OWR and FDF MWR are presented in Figure 4 and 5. As seen, when $K = 2$, for small P (low receive SNR at the relay), OWR performs close to MWR and even outperforms FDF. Increasing SNR causes the gap between OWR and MWR to largen. By setting $K = 8$, we see that for a significant SNR region OWR surpasses FDF.

5.2 Comparison of DF OWR and CF MWR

To compare the performance of DF OWR and CF MWR, we use two SNR regions. First, assume $P_r < KP$. Thus,

$$R_{\mathrm{CF}} = \frac{1}{2(K-1)}\log\left(1+\frac{(K-1)PP_r}{1+(K-1)P+P_r}\right),$$
$$R_{\mathrm{DF_O}} = \frac{\log(1+P_r)}{2K}. \tag{13}$$

From (13), we can conclude that MWR outperforms OWR in this SNR region when

$$P_r < \left(1+\frac{(K-1)PP_r}{1+(K-1)P+P_r}\right)^{\frac{K}{K-1}} - 1. \tag{14}$$

In (14), if $P_r \ge 1$, using the derivative of the right hand side of (14), it can be shown that when P decreases, MWR may lose its advantage over OWR. Now, we consider the second SNR region where $KP \le P_r$. Thus

$$R_{\mathrm{CF}} = \frac{1}{2(K-1)}\log\left(1+\frac{(K-1)PP_r}{1+(K-1)P+P_r}\right),$$
$$R_{\mathrm{DF_O}} = \frac{\log(1+KP)}{2K}. \tag{15}$$

MWR with CF performs better than DF OWR if

$$\frac{(1+(K-1)P)((1+KP)^{\frac{K-1}{K}}-1)}{(K-1)P+1-(1+KP)^{\frac{K-1}{K}}} < P_r. \tag{16}$$

It can be concluded that for low SNRs, (16) does not hold and OWR outperforms MWR. Further, the left side

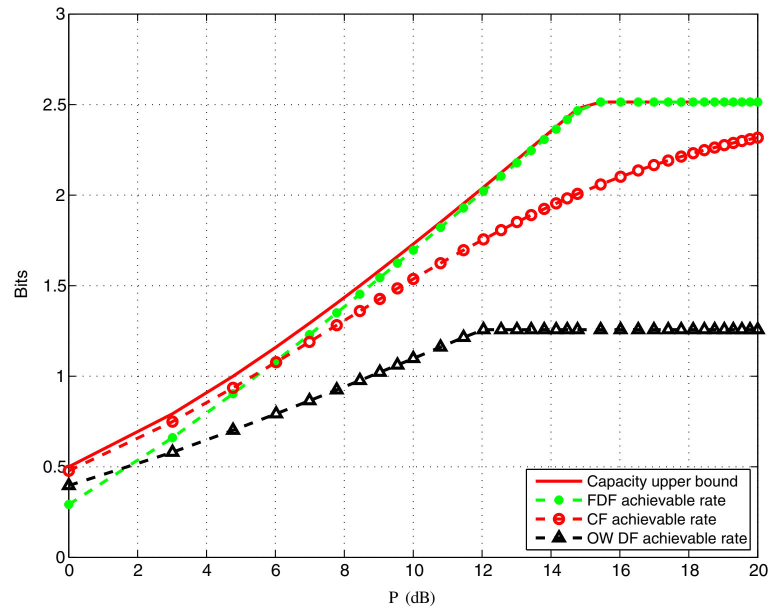

Figure 4 Comparison between the achievable rates of OWR and MWR when $P_r = 15$ dB and $K = 2$.

of (16) is an increasing function of K. Thus, by increasing K, we may start seeing higher rates from OWR than MWR. Figures 4 and 5 depict the comparison between the achievable rate of DF OWR and CF MWR.

6 Conclusion

In this article, we compared the performance of OWR and MWR in a symmetric Gaussian MWRC where several users want to share their data through a relay. To this end, we first proved that FDF always have a capacity gap less than $\frac{1}{2(K-1)}$ bit while depending on the users' and relay SNR, AF and DF may have a capacity gap larger than $\frac{1}{2(K-1)}$. Furthermore, for OWR, we showed that DF is always superior to AF. By comparing the achievable rate of DF OWR with CF and FDF MWR, we concluded that MWR is likely to outperform OWR in high SNR regions

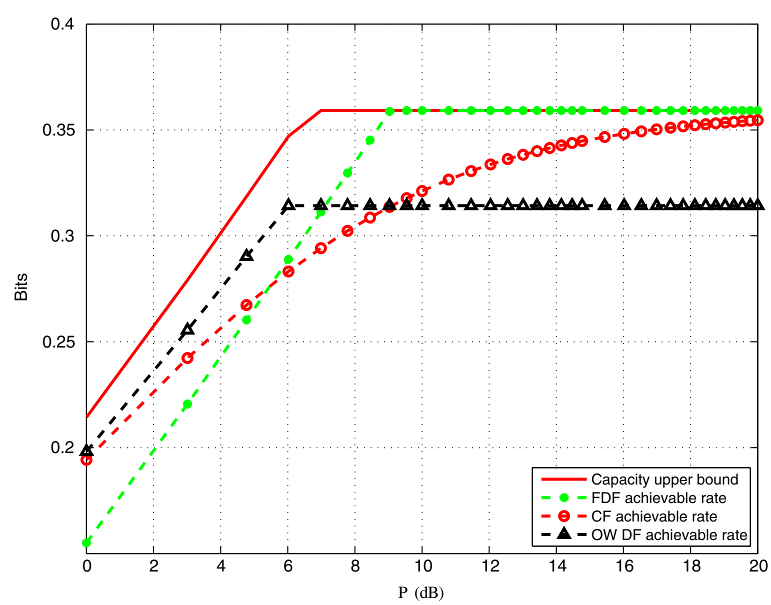

Figure 5 Comparison between the achievable rates of OWR and MWR when $P_r = 15$ dB and $K = 8$.

or for small K. Conducting a similar study for asymmetric Gaussian channels is considered as a direction for future research.

Appendix

Before presenting proofs, we state the following propositions based on Lemma 1, 2 and 3.

Proposition 1. *If $P_r \leq (K-1)P$, i.e. downlink is the rate bottleneck, we have*

$$R^c_{\mathrm{UB}} = \frac{\log(1+P_r)}{2(K-1)}. \tag{17}$$

Otherwise

$$R^c_{\mathrm{UB}} = \frac{\log(1+(K-1)P)}{2(K-1)}. \tag{18}$$

Proposition 2. *In a Gaussian MWRC with FDF MWR, if $P_r \leq \frac{K}{2}P - \frac{1}{2}$, then downlink is the bottleneck resulting in*

$$R^c_{\mathrm{FDF}} = \frac{\log(1+P_r)}{2(K-1)}. \tag{19}$$

If $\frac{K}{2}P - \frac{1}{2} \leq P_r$

$$R^c_{\mathrm{FDF}} = \frac{\log\left(\frac{1}{2}+\frac{KP}{2}\right)}{2(K-1)}. \tag{20}$$

Proposition 3. *When $P_r \leq (1+KP)^{\frac{K-1}{K}} - 1$, downlink constrains the rate of DF MWR and*

$$R^c_{\mathrm{DF}} = \frac{\log(1+P_r)}{2(K-1)}. \tag{21}$$

Further, when $(1+KP)^{\frac{K-1}{K}} - 1 < P_r$, uplink is the rate bottleneck and

$$R^c_{\mathrm{DF}} = \frac{\log(1+KP)}{2K}. \tag{22}$$

Proof of Theorem 1

We start the proof by partitioning the range of P_r and P using Proposition 1 and 2. Then, the achievable rate of FDF and the rate upper bound are compared in each region in order to complete the proof. The partitions specify which constraints in (2) and (3) are active. Since $K \geq 2$, we have $\frac{K}{2}P - \frac{1}{2} < (K-1)P$. To this end, the regions of interest are specified as $P_r \leq \frac{K}{2}P - \frac{1}{2}$, $\frac{K}{2}P - \frac{1}{2} < P_r \leq (K-1)P$, and $(K-1)P < P_r$. These partitions are denoted by A_1^{FDF}, A_2^{FDF} and A_3^{FDF}, respectively.

Capacity gap on A_1^{FDF}: The achievable rate of FDF as well as the upper bound is determined by downlink on this region. Using Proposition 1 and 2, we conclude that $R^c_{\mathrm{FDF}} = R^c_{\mathrm{UB}}$. In other words, FDF achieves the capacity upper bound and the gap between R^c_{UB} and R^c_{FDF}, $G_U = R^c_{\mathrm{UB}} - R^c_{\mathrm{FDF}}$, is 0.

Capacity Gap on A_2^{FDF}: For this region, the achievable rate of FDF is bounded by uplink while the rate upper bound is forced by downlink. Thus,

$$\begin{aligned} G_U &= \frac{1}{2(K-1)}\left[\log(1+P_r) - \log\left(\frac{1}{2}+\frac{K}{2}P\right)\right] \\ &= \frac{1}{2(K-1)}\log\left(\frac{1+P_r}{\frac{1}{2}+\frac{K}{2}P}\right). \end{aligned} \tag{23}$$

Since $\log(\cdot)$ is an increasing function, the maximum of G_U happens when P_r has its maximum value on A_2. Since $P_r < (K-1)P$, it is easy to show that

$$\frac{1+P_r}{\frac{1}{2}+\frac{K}{2}P} < 2. \tag{24}$$

As a consequence, $G_U < \frac{1}{2(K-1)}$.

Capacity Gap on A_3^{FDF}: Both R^c_{FDF} and R^c_{UB} are limited by the uplink in this case. Thus, using Proposition 1 and 2

$$G_U = \frac{1}{2(K-1)}\log\left(\frac{1+(K-1)P}{\frac{1}{2}+\frac{K}{2}P}\right). \tag{25}$$

Now, it is inferred from (25) that $G < \frac{1}{2(K-1)}$.

Proof of Theorem 2

Similar to FDF, we partition the SNR region and study the capacity gap for DF over different partitions. First, we point out that $(1+KP)^{\frac{K-1}{K}} < (K-1)P$. To this end, we define three SNR regions namely A_1^{DF}, A_2^{DF}, and A_3^{DF} denoting $P_r \leq (1+KP)^{\frac{K-1}{K}} - 1$, $(1+KP)^{\frac{K-1}{K}} - 1 < P_r \leq (K-1)P$, and $(K-1)P < P_r$, respectively.

Capacity gap on A_1^{DF}: R^c_{UB} and R^c_{DF} are limited by downlink. Using propositions 1 and 3, it is concluded that $G_U = R^c_{\mathrm{UB}} - R^c_{\mathrm{DF}} = 0$.

Capacity gap on A_2^{DF}: For this partition

$$\begin{aligned} G_U &= \frac{\log(1+P_r)}{2(K-1)} - \frac{\log(1+KP)}{2K} \\ &= \frac{1}{2(K-1)}\log\left(\frac{1+P_r}{(1+KP)^{\frac{K-1}{K}}}\right). \end{aligned} \tag{26}$$

Now, the capacity gap is less than $\frac{1}{2(K-1)}$ bit if

$$P_r < 2(1+KP)^{\frac{K-1}{K}} - 1. \tag{27}$$

Considering that $(1+KP)^{\frac{K-1}{K}} \leq P_r < (K-1)P$, it is easy to show that (27) does not necessarily hold for all values of P_r and P in this region.

Capacity gap on A_3^{DF}: Here, uplink is the rate bottleneck for the upper bound as well as DF. Thus,

$$G_U = \frac{1}{2(K-1)}\log\left(\frac{1+(K-1)P}{(1+KP)^{\frac{K-1}{K}}}\right) \tag{28}$$

and $G_U < \frac{1}{2(K-1)}$ if $(K-1)P < 2(1+KP)^{\frac{K-1}{K}} - 1$ which does not necessarily hold for all P and P_r values within A_3^{DF}.

Proof of Theorem 3

We again define SNR regions, called A_1^{AF} and A_2^{AF} based on Proposition 1. The first region is where $P_r \le (K-1)P$ and the second region includes $(K-1)P < P_r$.

Capacity Gap on A_1^{AF}: In this region, we have

$$\begin{aligned} G_U^{\mathrm{AF}} &= R_{\mathrm{UB}}^c - R_{\mathrm{AF}}^c \\ &= \frac{1}{2(K-1)} \log\left(\frac{(1+P_r)(1+KP+P_r)}{1+KP+P_r+(K-1)PP_r}\right) \end{aligned} \tag{29}$$

Now, from (29), one can show that $G_U^{\mathrm{AF}} < \frac{1}{2(K-1)}$ if $P_r^2 - (K-2)PP_r < KP$.

Capacity Gap on A_2^{AF}: On this partition,

$$G_U^{\mathrm{AF}} = \frac{1}{2(K-1)} \log\left(\frac{(1+(K-1)P)(1+KP+P_r)}{1+KP+P_r+(K-1)PP_r}\right) \tag{30}$$

Using (30), it is easy to conclude that if $K(K-1)P^2 - (K-1)PP_r < 1+P_r+P$ then AF has a capacity gap smaller than $\frac{1}{2(K-1)}$.

Proof of Theorem 4

First assume $P_r < KP$. Since

$$\frac{KPP_r}{1+KP+P_r} < P_r \tag{31}$$

holds, then $R_{\mathrm{DF_O}} > R_{\mathrm{AF_O}}$. For $KP \le P_r$,

$$\frac{KPP_r}{1+KP+P_r} < KP \tag{32}$$

is always correct. As a consequence, for this SNR region $R_{\mathrm{DF_O}} > R_{\mathrm{AF_O}}$ still holds.

Competing interests

The authors declare that they have no competing interests.

Acknowledgements

The authors would like to thank the Natural Sciences and Engineering Research Council of Canada (NSERC) and the Alberta Innovates Technology Futures (AITF) for supporting our research.

Received: 28 September 2012 Accepted: 3 December 2012
Published: 22 January 2013

References

1. Y Wu, PA Chou, S Kung, Information exchange in wireless networks with network coding and physical-layer broadcast. Tech. rep., Microsoft Research (2004)
2. B Rankov, A Wittneben, in *Asilomar Conference on Signals, Systems, and Computers*. Spectral efficient signaling for half-duplex relay channels (Pacific Grove, USA, 2005), pp. 1066–1071
3. P Popovski, H Yomo, in *IEEE Intl. Conf. on Communications (ICC)*. The anti-packets can increase the achievable throughput of a wireless multi-hop network (Istanbul, Turkey, 2006), pp. 3885–3890
4. D Gunduz, A Yener, A Goldsmith, H Poor, in *IEEE Intl. Symp. on Inf. Theory (ISIT)* . The multi-way relay channel (IEEE, Seoul, South Korea, 2009), pp. 339–343
5. A Amah, A Klein, Regenerative multi-group multi-way relaying. IEEE Trans. Veh. Technol. **60**(7), 3017–3029 (2011)
6. C Hausl, O Iscan, F Rossetto, Resource allocation for asymmetric multi-way relay communication over orthogonal channels. EURASIP J. Wirel. Commun. Netw. **2012**(20), 1–12 (2012)
7. T Cover, A Gamal, Capacity theorems for the relay channel. IEEE Trans. Inf. Theory. **25**(5), 572–584 (1979)
8. W Nam, SY Chung, Y Lee, in *IEEE International Zurich Seminar on Communications*. Capacity bounds for two-way relay channels (Zurich, Switzerland, 2008), pp. 144–147
9. L Ong, S Johnson, C Kellett, An optimal coding strategy for the binary multi-way relay channel. IEEE Commun. Lett. **14**(4), 330–332 (2010)
10. L Ong, C Kellett, S Johnson, in *IEEE Intl. Symp. on Inf. Theory (ISIT)*. Capacity theorems for the AWGN multi-way relay channel (USA, Austin, 2010), pp. 664–668
11. W Nam, SY Chung, Y Lee, Capacity of the Gaussian two-way relay channel to within 1/2 bit. IEEE Trans. Inf. Theory. **56**(11), 5488–5494 (2010)
12. S Ariyavisitakul, LF Chang, Signal and interference statistics of a CDMA system with feedback power control. IEEE Trans. Commun. **41**(11), 1626–1634 (1993)
13. U Erez, R Zamir, Achieving 1/2 log (1+SNR) on the AWGN channel with lattice encoding and decoding. IEEE Trans. Inf. Theory. **50**(10), 2293–2314 (2004)

doi:10.1186/1687-1499-2013-11
Cite this article as: Noori and Ardakani: **On the achievable rates of symmetric Gaussian multi-way relay channels.** *EURASIP Journal on Wireless Communications and Networking* 2013 **2013**:11.