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On performance of cooperative network based on OFDM combined with TDM using MMSE-FDC in the presence of nonlinear HPA

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Abstract

In this paper, we analyze the impact of nonlinear high-power amplifier (HPA) on the performance of cooperative network based on orthogonal frequency division multiplexing combined with time-division multiplexing (OFDM/TDM) using minimum mean-square-error frequency-domain combining (MMSE-FDC) in a frequency-selective fading channel. We design a novel MMSE-FDC weights while taking into account the nonlinearity of HPA at source and relay. Closed-form symbol error rate and outage probability expressions are derived while approximating the residual inter-slot interference after the MMSE-FDC as a random Gaussian variable. We discuss and address the nonlinear OFDM/TDM system design issues in cooperative network using the obtained simulation and theoretical results. We show that the OFDM/TDM with MMSE-FDC can be used to reduce the impact of nonlinear HPA on overall performance of cooperative network in comparison to OFDM while providing the target quality-of-service for reduced required signal-to-noise ratio. This is because OFDM/TDM with MMSE-FDC achieves frequency diversity in addition to cooperative diversity, while reduced peak-to-average power ratio makes it more robust on nonlinear degradation due to HPA saturation in comparison to conventional OFDM.

1 Introduction

Multi-antenna techniques are promising candidates to achieve broadband data services in a limited available bandwidth. However, their application often encounters practical implementation problem if a large number of antennas is to be deployed. In order to overcome this problem, a new mode of transmit diversity, called cooperative diversity, was proposed based on user cooperation [1,2], where the antennas of the sender and the partners together form a multiple transmit antenna situation. A variety of algorithms have been developed to obtain cooperative diversity gain [3-9]. Cooperative network based on orthogonal frequency division multiplexing (OFDM) physical-layer access is an attractive solution to achieve cooperative diversity while overcoming the channel frequency selectivity. However, high peak-to-average power ratio (PAPR) of OFDM causes performance degradation due to the nonlinearity of high-power amplifiers (HPA) at the transmitters.

The importance of design choices for different relaying protocols and the implementation complexity as a result of a particular protocol has been discussed with regards to the achieved average system capacity in [10]. In [11], a trade-off between half-duplex (HD) and full-duplex (FD) mode in relay link was studied and a new scheme named hybrid HD/FD based on opportunistic switching between the two modes was presented. Cooperative dual-hop network based on OFDM using amplify-and-forward (AF) relaying with average power scaling (APS) and instantaneous power scaling (IPS) at HPA is analyzed in [12,13], and it was shown that AF-IPS outperforms AF-APS. The authors in [14] showed that in dual-hop cooperative network based on OFDM with AF protocol relay's HPA cause only minimal loss in performance, while the authors in [15] showed that the choice of the HPA input back-off is a trade-off between good performance at low signal-to-noise ratio (SNR) or low bit error rate (BER) at high SNR. Methods for compensating the effect of nonlinear HPA in cooperative network based on OFDM using AF and

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decode-and-forward protocol were presented in [16,17], showing that the performance considerably approaches to the case with linear HPA. At receiver, a maximum ratio combiner is used in [16], while the authors in [17] used maximum a posteriori probability expectation maximization. Cooperative network using single-carrier frequency-domain equalization (SC-FDE) was presented in [18], while [19] gives a performance comparison of cooperative network based on SC-FDE and OFDM.

The OFDM combined with time-division multiplexing (OFDM/TDM) using minimum mean-square-error frequency-domain equalization (MMSE-FDE) can be used to reduce the PAPR of conventional OFDM [20]. Thus, the cooperative network using OFDM/TDM access based on MMSE frequency-domain combining (MMSE-FDC) in a frequency-selective fading channel may be an attractive solution to achieve both cooperative and frequency diversity gains with a lower PAPR in comparison with cooperative OFDM [21]. Still, the PAPR is not completely eliminated and thus, the nonlinear HPA may considerably degrade the performance of cooperative network based on OFDM/TDM with MMSE-FDC. It is worth mentioning that SC-FDE outperforms OFDM/TDM with MMSE-FDE in terms of BER performance but suffers from inferior channel capacity [22]. However, the benefit of OFDM/TDM over SC-FDE is related to its multicarrier properties which can be exploited for bit, subcarrier, and power allocation.

Unlike [21], in this work, we present extensive analysis of cooperative network based on OFDM/TDM with MMSE-FDC in frequency-selective fading channel in respect to impact of nonlinear HPA. We resolve to AF protocol at relay with time-domain amplification and equal time sharing for uplink/downlink due to its simple implementation while achieving satisfactory system performance. In addition, we adopt HD at relay (on the cost of reduced spectrum efficiency) since there is no self-interference at relay which is present in FD operation, and consequently, algorithms for mitigating this interference are not needed. We design a MMSE-FDC weights while taking into account nonlinearity of HPA at source and relay. Derivation of the closed-form symbol error rate (SER) and outage probability expressions is done while approximating the residual inter-slot interference (ISI) after MMSE-FDC as a zero-mean Gaussian variable. In addition, we present a closed-form average sum-rate expression for high SNR region. Theoretical average SER has been consisted with the simulation result validating the presented analysis. We discuss and address the nonlinear OFDM/TDM design issue and show that cooperative network based on OFDM/TDM with MMSE-FDC can be used to improve the robustness against nonlinear degradation due to HPA saturation in comparison to conventional OFDM. Simultaneously, the target quality-of-service (QoS) is achieved for reduced required SNR. The remainder of this paper is organized as follows: section 2 presents the network model; performance analysis is provided in section 3, while numerical results and discussions are presented in section 4; and section 5 concludes the paper.

2 Network model overview

In this section, we give an overview of the cooperative network based on OFDM/TDM with MMSE-FDC in the presence of nonlinear HPA. We present the transmit-and-receive signal representation, and then derive the decision variable. In this paper, T_c - spaced discrete time signal representation is used, where T_c represents a sampling interval of fast Fourier transform (FFT).

2.1 Transmit signal representation

The data-modulated symbols are divided into OFDM/TDM frames denoted as $\{d(i);\ i = 0 \sim N_c - 1\}$ with $\mathbb{E}[\,|d(i)|^2\,] = 1$ (where $\mathbb{E}[\,\cdot]$ denotes the ensemble average operation), and then each frame is decomposed into K blocks, each of which has $N_m\ (= N_c/K)$ data-modulated symbols [20]. The kth $(k = 0, ..., K-1)$ block of the OFDM/TDM frame is denoted as $\{d\,(k,i)\,;\ i = 0 \sim N_m - 1\}$, where $d\,(k,i) = d\,(kN_m + i)$. Then, an N_m- point inverse FFT (IFFT) is applied to each of the K blocks to generate a sequence of K OFDM signals with N_m subcarriers. Unlike conventional OFDM, the guard interval (GI) with cycle prefix (CP) is inserted over K slots which constitute the OFDM/TDM frame [20]. The resulting OFDM/TDM signal can be expressed using the equivalent low-pass representation as $s(t) = \sqrt{P}\sum_{i=0}^{N_m-1} d(\lfloor t/N_m \rfloor, i)\exp[\,j2\pi t(i/N_m)]$, for $t = 0 \sim N_c - 1$, where P denotes the power coefficient given by $P = 2E_s/T_cN_c$ with E_s being the data-modulated symbol energy and T_c the sampling period. We note here that OFDM/TDM signal with $K = 1$ (i.e., $N_m = N_c$) represents the conventional OFDM system with N_c subcarriers, while for $K = N_c$ (i.e., $N_m = 1$), it becomes the SC system.

We assume a soft envelope limiter model for HPA [23], where the output of HPA represents a linearly amplified input signal for the input amplitude below the HPA saturation level φ_s, while for the input signal amplitudes beyond φ_s, the output signal amplitude is limited to φ_s. Due to the nonlinear input-output characteristic of HPA, the output signal can be approximated as a sum of attenuated input replica and nonlinear noise as [24] $\hat{s}(t) = \alpha_{\mathrm{s(or\,r)}}s(t) + c_{\mathrm{s(or\,r)}}(t)$, where $\alpha_{\mathrm{s(or\,r)}}$, $s(t)$, and $c_{\mathrm{s(or\,r)}}(t)$, respectively, denote the attenuation constant, the transmitted signal, and the nonlinear noise due to HPA at source (s) or relay (r). Finally, the signal is multiplied with power coefficient as $\tilde{s}(t) = \sqrt{P}\hat{s}(t)$ and transmitted over a frequency-selective fading channel.

We assume frequency-selective fading channel having a discrete-time channel impulse response given by $h_{xy}(\tau) = \sum_{l=0}^{L-1} h_{l,xy}\delta(\tau - \tau_l)$, where L, $h_{l,xy}$, τ_l, and $\delta(\tau)$, respectively, denote the number of paths, the path gain for xy link, and the delay of the lth path and the delta function. $xy \in \{\mathrm{sd}, \mathrm{sr}, \mathrm{rd}\}$ where sd, sr and rd, respectively, denote source-to-destination, source-to-relay, and relay-to-destination links. We assume Jake's isotropic scattering model where incoming rays constituting each propagation path arrive at a user with uniformly distributed angles [25]. Thus, the normalized autocorrelation function of a Rayleigh faded channel with motion at a constant velocity is given by $R(\varepsilon) = \mathbb{E}[h_{l,xy}h^*_{l,xy+\varepsilon}] = J_0(2\pi f_D\varepsilon)$ at delay ε when the maximum Doppler shift is f_D, and where $J_0(\varrho) = (1/\pi)\int_0^\pi \exp(j\varrho\cos(\phi)d\phi)$ is the 0th order Bessel function of the first kind.

The signal is transmitted in two orthogonal time stages. In the first time stage, the signal from source is received at destination and relay, while in the second time stage, the signal from relay is forwarded towards destination.

2.2 Received signal representation

2.2.1 Stage I

During the first stage, after GI removal and N_c-point FFT, signal received at destination is given by

$$\begin{aligned} R_{d,1}(n) &= \sqrt{P}\alpha_s H_{\mathrm{sd}}(n)\Psi_1(n,i)d(k,i) \\ &\quad + \sqrt{P}C_s(n)H_{\mathrm{sd}}(n) + N_{d,1}(n) \end{aligned} \tag{1}$$

for $n = 0 \sim N_c - 1$, where $\Psi_1(n,i)$ denotes the frequency-domain filter defined as

$$\Psi_1(n,i) = \begin{cases} 1, & \text{if } n = Ki \\ \frac{1}{N_c}\frac{\sin(\pi N_m \frac{n-Ki}{N_c})}{\sin(\pi\frac{n-Ki}{N_c})}\exp\{j\pi\,[(2k+1)\,N_m - 1]\,\frac{n-Ki}{N_c}\} & \text{otherwise.} \end{cases}$$

In Equation 1, α_s, $C_s(n)$, $d(k,i)$, $N_{d,1}(n)$ and $H_{\mathrm{sd}}(n)$, respectively, denote the attenuation constant, nonlinear noise due to the source terminals' HPA, data-modulated symbol, the additive white Gaussian noise (AWGN) having the variance $2N_0/T_cN_c$, with N_0 being the single-sided power spectrum density, and the channel gain between source and destination defined as $H_{\mathrm{sd}}(n) = \sum_{t=0}^{N_c-1} h_{\mathrm{sd}}(t)\exp(-j2\pi nt/N_c)$, for $n = 0 \sim N_{\mathrm{c}} - 1$. We note here that the ISI during the first stage is embedded in $s(t) * h_{\mathrm{sd}}(t)$, and it is defined after the data-demodulation in section 2.4.

During the first stage, the signal received at relay is given by

$$r_r(t) = \sqrt{P}[\,\alpha_s s(t) + c_s(t)] * h_{\mathrm{sr}}(t) + n_r(t) \tag{2}$$

for $n = 0 \sim N_c - 1$, where $*$, $s(t)$, $h_{\mathrm{sr}}(t)$, and $n_r(t)$, respectively, denote the convolution operator, the OFDM/TDM transmitted signal, the channel impulse response between source, and relay and the AWGN at relay.

2.2.2 Stage II

At relay, the AF-IPS protocol is implemented where the received signal is normalized with coefficient $\beta = \sqrt{P/\mathbb{E}[\,|r_r(t)|^2]}$, and transmitted over the channel during the second time stage. The nonlinear output of HPA at relay can be represented as $\tilde{r}_r(t) = \beta[\,\alpha_r r_r(t) + c_r(t)]$, where α_r and $c_r(t)$, respectively, denote the attenuation constant and nonlinear noise corresponding to relay's HPA.

During the second stage, after GI removal and N_c-point FFT, signal received at destination is given by

$$\begin{aligned} R_{d,2}(n) &= \tilde{R}_r(n)H_{\mathrm{rd}}(n) + N_{d,2}(n) \\ &= \sqrt{P}\alpha_s\alpha_r\beta S(n)H_{\mathrm{sr}}(n)H_{\mathrm{rd}}(n) \\ &\quad + \sqrt{P}\alpha_r\beta C_s(n)H_{\mathrm{sr}}(n)H_{\mathrm{rd}}(n) + \sqrt{P}\beta C_r(n)H_{\mathrm{rd}}(n) \\ &\quad + N_r(n)H_{\mathrm{rd}}(n) + N_{d,2}(n), \end{aligned} \tag{3}$$

where $\tilde{R}_r(n)$, $H_{\mathrm{rd}}(n)$, and $N_{d,2}(n)$, respectively, denote the frequency-domain representation of relay's HPA output, the channel gain between relay and destination, and the AWGN at relay during the second stage. We underline that the ISI during the second stage can be clearly defined only after the data-demodulation in section 2.4.

2.3 MMSE-FDC

To combat the negative effect of frequency-selective fading channel and nonlinear HPA, at destination, we apply MMSE-FDC as

$$\hat{R}(n) = \sum_{j=1}^{2} R_{d,j}(n)w_j(n), \tag{4}$$

where $w_j(n)$ denotes the equalization weight for the jth stage. Below, we present the equalization weight based on MMSE criteria to capture the effect of nonlinear HPA. The equalization weight at the jth ($j = 1, 2$) stage is chosen to minimize the mean square error (MSE) $e_j(n) = R_{d,j}(n)w_j(n) - S(n)$ as $\mathrm{MSE}_j = \mathbb{E}\left[\left|e_j(n)\right|^2\right]$. We take into account the following assumptions: (1) the transmitted signal is not correlated with nonlinear noise due to HPA and AWGN at source and relay, (2) the nonlinear noise due to HPA at source/relay is not correlated with AWGN at source/relay, (3) the nonlinear noise due to HPA at source and relay are not correlated, and (4) the AWGN at source and relay are not correlated.

First, we derive equalization weights for the first stage corresponding to the source-destination link. By using Equation 1, the MSE for the first stage can be represented as (see Appendix)

$$\begin{aligned}\mathrm{MSE}_1 &= \left(\frac{2E_s}{T_cN_c}\right)\alpha_s^2\mathbb{E}[\,|S(n)|^2]\,|H_{\mathrm{sd}}(n)|^2|w_1(n)|^2 + \left(\frac{2E_s}{T_c}\right)\mathbb{E}[\,|C_s(n)|^2]\,|H_{\mathrm{sd}}(n)|^2|w_1(n)|^2\\ &- \left(\frac{4E_s}{T_cN_c}\right)\alpha_s\mathrm{Re}\{H_{\mathrm{sd}}^*(n)w_1(n)\} + \left(\frac{2N_0}{T_cN_c}\right)|w_1(n)|^2 + \frac{2E_s}{T_cN_c},\end{aligned} \tag{5}$$

where $(\cdot)^*$ denotes the complex conjugate operation. Now, by solving $\partial\mathbb{E}[\,|e_1(n)|^2]\,/\partial w_1(n) = 0$, while taking that $\mathbb{E}[\,|S(n)|^2] = 1$ and $\mathbb{E}[\,|C_{\mathrm{s(or\,r)}}(n)|^2] = \frac{1}{N_c}\left(1-\exp(-\varphi_{s(\mathrm{or}\,r)}^2) - \alpha_{\mathrm{s(or\,r)}}^2\right)$ [23] for $\varphi_{\mathrm{s(or\,r)}} < 7$ dB, the MMSE equalization weight for the first stage is given by

$$w_1(n) = \frac{\alpha_s H_{\mathrm{sd}}^*(n)}{(1-\exp(-\varphi_s^2))|H_{\mathrm{sd}}(n)|^2 + (E_s/N_0)^{-1}}. \tag{6}$$

The attenuation constant can be well approximated as [26] $\alpha_{\mathrm{s(or\,r)}} = 1-\exp(-\varphi_{\mathrm{s(or\,r)}}^2) + \frac{\sqrt{\pi}}{2}\varphi_{\mathrm{s(or\,r)}}\mathrm{erfc}(\varphi_{\mathrm{s(or\,r)}})$ and $\mathrm{erfc}(x) = (2/\sqrt{\pi})\int_x^\infty \exp(-t^2)dt$ is the complementary error function.

By using Equation 3 and the abovementioned assumptions, the MSE for the second stage can be represented as (see Appendix)

$$\begin{aligned}\mathrm{MSE}_2 &= \left(\frac{2E_s}{T_cN_c}\right)\alpha_s^2\alpha_r^2\beta^2\mathbb{E}[\,|S(n)|^2]\,|H_{\mathrm{srd}}(n)|^2|w_2(n)|^2 + \left(\frac{2E_s}{T_c}\right)\alpha_r^2\beta^2\mathbb{E}[\,|C_s(n)|^2]\,|H_{\mathrm{srd}}(n)|^2|w_2(n)|^2\\ &+ \left(\frac{2E_s}{T_c}\right)\alpha_r^2\beta^2\mathbb{E}[\,|C_r(n)|^2]\,|H_{\mathrm{rd}}(n)|^2|w_2(n)|^2 - \left(\frac{4E_s}{T_cN_c}\right)\alpha_s^2\alpha_r^2\mathrm{Re}\{H_{\mathrm{srd}}^*(n)w_2(n)\}\\ &+ \left(\frac{2N_0}{T_cN_c}\right)|H_{sr}(n)|^2|w_2(n)|^2 + \left(\frac{2N_0}{T_cN_c}\right)|w_2(n)|^2 + \frac{2E_s}{T_cN_c},\end{aligned} \tag{7}$$

where $H_{\mathrm{srd}}(n) = H_{\mathrm{sr}}(n)H_{\mathrm{rd}}(n)$ denotes the cascade channel gain between source to relay and relay to destination links. We note here that before transmission, the GI with CP is inserted, and thus the product of channel gains $H_{\mathrm{sr}}(n)$ and $H_{\mathrm{rd}}(n)$ is described by FFT of the circular convolution of their corresponding impulse responses $h_{\mathrm{sr}}(t)$ and $h_{\mathrm{rd}}(t)$. Now, by solving $\partial\mathbb{E}[\,|e_2(n)|^2]\,/\partial w_2(n) = 0$ and taking that $\beta^2 \approx 1$, the MMSE equalization weight for the second stage is given by

$$w_2(n) = \frac{\alpha_s\alpha_r H_{\mathrm{sr}}^*(n)H_{\mathrm{rd}}^*(n)}{\alpha_r^2(1-\exp(-\varphi_s^2))|H_{\mathrm{srd}}(n)|^2 + (1-\exp(-\varphi_r^2)-\alpha_r^2)|H_{\mathrm{rd}}(n)|^2 + \left(\frac{E_s}{N_0}\right)^{-1}(|H_{\mathrm{sr}}(n)|^2+1)}. \tag{8}$$

We observe from Equations 6 and 8 that unlike previous works [19-21], the negative effect of HPA saturation at both source and relay has been taken into account when calculating the equalization weights.

2.4 Data demodulation

Here, we derive the decision variable for cooperative OFDM/TDM using MMSE-FDC in the presence of HPA. After MMSE-FDC given by Equation 4, the time-domain signal is obtained by applying the N_c-point IFFT as

$$\begin{aligned}\hat{r}(t) &= \sqrt{P}\alpha_s\left(\frac{1}{N_c}\sum_{n=0}^{N_c-1}\hat{H}_{\mathrm{sd}}(n) + \alpha_r\beta\frac{1}{N_c}\sum_{n=0}^{N_c-1}\hat{H}_{\mathrm{srd}}(n)\right)s(t)\\ &+ \sqrt{P}\alpha_s\frac{1}{N_c}\sum_{n=0}^{N_c-1}\left\{\hat{H}_{\mathrm{sd}}(n)\sum_{t'=0,t'\neq t}^{N_c-1}s(t')\exp\left(-j2\pi n\frac{t'-t}{N_c}\right)\right\}\\ &+ \sqrt{P}\alpha_s\alpha_r\beta\frac{1}{N_c}\sum_{n=0}^{N_c-1}\left\{\hat{H}_{\mathrm{srd}}(n)\sum_{t'=0,t'\neq t}^{N_c-1}s(t')\exp\left(-j2\pi n\frac{t'-t}{N_c}\right)\right\}\\ &+ \sqrt{P}\left(\frac{1}{N_c}\sum_{n=0}^{N_c-1}\hat{H}_{\mathrm{sd}}(n) + \alpha_r\beta\frac{1}{N_c}\sum_{n=0}^{N_c-1}\hat{H}_{\mathrm{srd}}(n)\right)c_s(t) + \sqrt{P}\beta\left(\frac{1}{N_c}\sum_{n=0}^{N_c-1}\hat{H}_{\mathrm{rd}}(n)\right)c_r(t)\\ &+ \hat{n}_{d,1}(t) + \left(\frac{1}{N_c}\sum_{n=0}^{N_c-1}\hat{H}_{\mathrm{rd}}(n)\right)\hat{n}_r(t) + \hat{n}_{d,2}(t)\end{aligned} \tag{9}$$

for $t = 0 \sim N_c - 1$, where $\hat{H}_{\mathrm{sd}}(n) = H_{\mathrm{sd}}(n)w_1(n)$, $\hat{H}_{\mathrm{srd}}(n) = H_{\mathrm{sr}}(n)H_{\mathrm{rd}}(n)w_2(n)$, $\hat{n}_{d,1}(t) = \mathrm{IFFT}\{\hat{N}_{d,1}(n)\}$, $\hat{n}_r(t) = \mathrm{IFFT}\{\hat{N}_r(n)\}$, $\hat{n}_{d,2}(n) = \mathrm{IFFT}\{\hat{N}_{d,2}(n)\}$ with $\hat{N}_{d,1}(n) = N_{d,1}(n)w_1(n)$, $\hat{N}_{d,2}(n) = N_{d,2}(n)w_2(n)$ and $\hat{N}_r(n) = N_r(n)w_2(n)$. Then, the time-domain equalized signal $\hat{r}(t)$ is fed to N_m-point FFT to obtain the decision variable as

$$
\begin{aligned}
\hat{d}(k,i) &= \frac{1}{N_m}\sum_{t=kN_m}^{(k+1)N_m} \hat{r}(t)\exp\left(-j2\pi i\frac{t}{N_m}\right) \\
&= \sqrt{P}\alpha_s\left(\frac{1}{K}\sum_{n=0}^{N_c-1}\hat{H}_{\mathrm{sd}}(n) + \alpha_r\beta\frac{1}{K}\sum_{n=0}^{N_c-1}\hat{H}_{\mathrm{srd}}(n)\right)d(k,i)\Psi^2(n,i) \\
&\quad + \sqrt{P}\alpha_s\frac{1}{N_c}\sum_{n=0}^{N_c-1}\hat{H}_{\mathrm{sd}}(n)\sum_{\substack{k'=0\\k'\neq k}}^{K-1}\sum_{\substack{i'=0\\i'\neq i}}^{N_m-1}\sum_{t=k'N_m}^{(k'+1)N_m}\exp\left(j2\pi t\frac{n}{N_c}\right)\exp\left(-j2\pi t\frac{i'}{N_m}\right)\Psi(n,i)d(k',i') \\
&\quad + \sqrt{P}\alpha_s\alpha_r\beta\frac{1}{N_c}\sum_{n=0}^{N_c-1}\hat{H}_{\mathrm{srd}}(n)\sum_{\substack{k'=0\\k'\neq k}}^{K-1}\sum_{\substack{i'=0\\i'\neq i}}^{N_m-1}\sum_{t=k'N_m}^{(k'+1)N_m}\exp\left(j2\pi t\frac{n}{N_c}\right)\exp\left(-j2\pi t\frac{i'}{N_m}\right)\Psi(n,i)d(k',i') \\
&\quad + \frac{1}{N_c}\sum_{n=0}^{N_c-1}\left(\sqrt{P}C_s(n)\hat{H}_{\mathrm{sd}}(n) + \hat{N}_{d,1}(n)\right)\Psi(n,i) \\
&\quad + \frac{1}{N_c}\sum_{n=0}^{N_c-1}\left(\sqrt{P}\alpha_r\beta\hat{H}_{\mathrm{srd}}(n)C_s(n) + \sqrt{P}\beta\hat{H}_{\mathrm{rd}}(n)C_r(n) + \hat{N}_r(n)\hat{H}_{\mathrm{rd}}(n) + \hat{N}_{d,2}(n)\right)\Psi(n,i), \qquad (10)
\end{aligned}
$$

for $i = 0 \sim N_m - 1$ and $k = 0 \backsim K - 1$, where $\Psi(n,i)$ denotes the frequency-domain filter which is equivalent to successive N_c-point IFFT and N_m-point FFT operations in Figure 1. The coefficients of $\Psi(\cdot)$ are defined as $\Psi(n,i) = K\Psi_1(n,i)$, where $\Psi_1(n,i)$ is given by Equation 2.

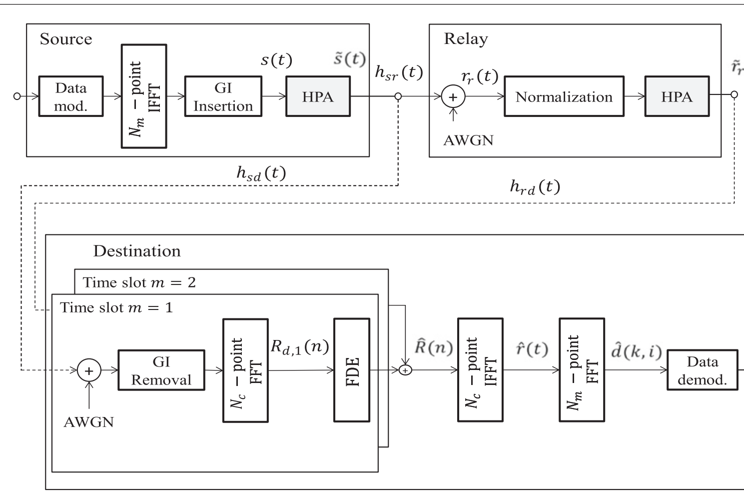

Figure 1 System model block scheme.

After some manipulation, the kth decision variable in the ith slot is obtained as

$$\hat{d}(k,i) = \sqrt{P}\frac{1}{K}\sum_{n=0}^{N_c-1}\left[\alpha_s\hat{H}_{\mathrm{sd}}(n) + \alpha_s\alpha_r\beta\hat{H}_{\mathrm{srd}}(n)\right]\Psi^2(n,i)d(k,i) + \sum_{j=1}^{2}\left[\sqrt{P}\mu_{j,i}(k,i) + \sqrt{P}\mu_{j,hpa}(k,i) + \mu_{j,n}(k,i)\right] \quad (11)$$

for $n = 0 \backsim N_c - 1$ and $i = 0 \sim N_m - 1$. The first term in Equation 11 is the transmitted data-modulated sequence, while $\mu_{j,i}(k,i)$, $\mu_{j,hpa}(k,i)$, and $\mu_{j,n}(k,i)$, denote the residual ISI after the MMSE-FDC, nonlinear noise due to HPA saturation at the source terminal and relay and the AWGN at the jth stage. $\mu_{j,i}(k,i)$, $\mu_{j,hpa}(k,i)$, and $\mu_{j,n}(k,i)$ are given by

$$\begin{cases} \mu_{1,i}(k,i) = \frac{1}{K}\sum_{n=0}^{N_c-1}\alpha_s\hat{H}_{\mathrm{sd}}(n)\sum_{k'=0,k'\neq k}^{K-1}\sum_{i'=0,i'\neq i}^{N_m-1}\Psi(n,i)\Psi(n,i')d(k',i') \\ \mu_{2,i}(k,i) = \frac{1}{K}\sum_{n=0}^{N_c-1}\alpha_s\alpha_r\beta\hat{H}_{\mathrm{srd}}(n)\sum_{k'=0,k'\neq k}^{K-1}\sum_{i'=0,i'\neq i}^{N_m-1}\Psi(n,i)\Psi(n,i')d(k',i') \\ \mu_{1,hpa}(k,i) = \frac{1}{N_c}\sum_{n=0}^{N_c-1}C_s(n)\hat{H}_{\mathrm{sd}}(n)\Psi(n,i) \\ \mu_{2,hpa}(k,i) = \frac{1}{N_c}\sum_{n=0}^{N_c-1}\left[\alpha_r\beta C_s(n)\hat{H}_{\mathrm{srd}}(n) + \beta C_r(n)\hat{H}_{\mathrm{rd}}(n)\right]\Psi(n,i) \\ \mu_{1,n}(k,i) = \frac{1}{N_c}\sum_{n=0}^{N_c-1}\hat{N}_{d,1}(n)\Psi(n,i) \\ \mu_{2,n}(k,i) = \frac{1}{N_c}\sum_{n=0}^{N_c-1}\left[\hat{H}_{\mathrm{rd}}\hat{N}_r(n) + \hat{N}_{d,2}(n)\right]\Psi(n,i). \end{cases} \quad (12)$$

3 Performance analysis

Here, we first derive the instantaneous SINR expression and afterwards a closed-form SER and outage probability for cooperative network based OFDM/TDM with MMSE-FDC and nonlinear HPA is designed. Finally, the average sum rate is presented.

3.1 SINR

In this subsection, we derive the instantaneous SINR expression $\gamma\,[P,\varphi_s,\varphi_r]$ while taking into account nonlinear HPA at source and relay.

We assume that the residual ISI after MMSE-FDC, the nonlinear noise and AWGN given by Equation 12 are complex-valued random Gaussian variable. Thus, the average values of the transmitted data-modulated symbol, the residual ISI after MMSE-FDC, and AWGN at the first and second stages are given by

$$\begin{cases} \sigma_{1,sig}^2 = \left|\frac{1}{K}\alpha_s\sum_{n=0}^{N_c-1}\hat{H}_{\mathrm{sd}}(n)\Psi^2(n,i)\right|^2 \\ \sigma_{1,i}^2 = \mathbb{E}[\,|\mu_{1,i}(k,i)|^2] = \frac{1}{K}\sum_{n=0}^{N_c-1}\alpha_s^2\left|\hat{H}_{\mathrm{sd}}(n)\right|^2\Psi^2(n,i)\sum_{i'=0}^{N_m-1}\Psi^2(n,i') - \left|\frac{1}{K}\sum_{n=0}^{N_c-1}\alpha_s\hat{H}_{\mathrm{sd}}(n)\Psi^2(n,i)\right|^2 \\ \sigma_{1,hpa}^2 = \mathbb{E}[\,|\mu_{1,hpa}(k,i)|^2] = \frac{1}{N_c}\sum_{n=0}^{N_c-1}\left[(1-\exp(-\varphi_s^2)-\alpha_s^2)|\hat{H}_{\mathrm{sd}}(n)|^2\Psi^2(n,i)\right] \\ \sigma_{1,n}^2 = \mathbb{E}[\,|\mu_{1,n}(k,i)|^2] = \sum_{n=0}^{N_c-1}|w_1(n)|^2\,\Psi^2(n,i) \\ \sigma_{2,sig}^2 = \left|\frac{1}{K}\alpha_s\alpha_r\beta\sum_{n=0}^{N_c-1}\hat{H}_{\mathrm{srd}}(n)\Psi^2(n,i)\right|^2 \\ \sigma_{2,i}^2 = \mathbb{E}[\,|\mu_{2,i}(k,i)|^2] = \frac{1}{K}\sum_{n=0}^{N_c-1}(\alpha_s\alpha_r\beta)^2\left|\hat{H}_{\mathrm{srd}}(n)\right|^2\Psi^2(n,i)\sum_{i'=0}^{N_m-1}\Psi^2(n,i') \\ \qquad - \left|\frac{1}{K}\sum_{n=0}^{N_c-1}\alpha_s\alpha_r\beta\hat{H}_{srd}(n)\Psi^2(n,i)\right|^2 \\ \sigma_{2,hpa}^2 = \mathbb{E}[\,|\mu_{2,hpa}(k,i)|^2] = \frac{1}{N_c}\sum_{n=0}^{N_c-1}\begin{bmatrix}\alpha_r\beta(1-\exp(-\varphi_s^2)-\alpha_s^2)|\hat{H}_{\mathrm{srd}}(n)|^2 \\ +\beta(1-\exp(-\varphi_r^2)-\alpha_r^2)|\hat{H}_{\mathrm{rd}}(n)|^2\end{bmatrix}\Psi^2(n,i) \\ \sigma_{2,n}^2 = \mathbb{E}[\,|\mu_{2,n}(k,i)|^2] = \sum_{n=0}^{N_c-1}|w_2(n)|^2\,\Psi^2(n,i)\left(1+|H_{\mathrm{rd}}(n)|^2\right). \end{cases} \quad (13)$$

Now, using Equations 11 and 13, we obtain the instantaneous SINR as

$$\gamma\,[P,\varphi_s,\varphi_r] = \frac{\sum_{j=1}^{2}P\sigma_{j,sig}^2}{\sum_{j=1}^{2}\left(P\sigma_{j,i}^2 + P\sigma_{j,hpa}^2 + \sigma_{j,n}^2\right)}, \quad (14)$$

Next, we present two special cases of OFDM/TDM: the conventional OFDM when $K = 1$ and SC-FDE when $K = N_c$.

3.1.1 Special case of conventional OFDM ($K = 1$)

In the case of conventional OFDM, the expression for instantaneous SINR can be obtained by substituting Equation 13 into Equation 14 for $N_m = N_c$ and $\Psi(n,i) = \delta(i-n)$, while the ISI is omitted since GI is inserted between two consecutive OFDM data symbols.

3.1.2 Special case of SC-FDE ($K = N_c$)

In the case of SC-FDE, the instantaneous SINR is obtained by substituting Equation 13 into Equation 14 for $N_m = 1$ and $\Psi(n,i) = 1$, while nonlinear noise due to HPA can be neglected. This is because SC-FDE with quadrature phase shift keying (QPSK) data modulation has a low PAPR, and consequently, nonlinear HPA has no impact on its performance.

3.2 Closed-form SER

To evaluate the impact of nonlinear HPA on QoS degradation of cooperative network based on OFDM/TDM with MMSE-FDC, we design the closed-form SER expression. The SER for M-PSK modulation is given by [27]

$$P_s = \frac{1}{\pi}\int_0^{\pi-\frac{\pi}{M}} \mathcal{M}_\gamma\left(\frac{\sin^2(\pi/M)}{\sin^2\theta}\right)d\theta, \tag{15}$$

where $\mathcal{M}_\gamma(\cdot)$ denotes the moment-generating function (MGF) as a function of the random variable γ which is the instantaneous SINR of OFDM/TDM with MMSE-FDC. The MGF can be calculated as a Laplace transform of the probability density function (PDF), while the PDF is obtained as a derivation of the corresponding SINR's cumulative density function (CDF).

The SINR given by Equation 14 can be rewritten as $\gamma = X/Y$, where X and Y, respectively, denote the useful signal component and the composite noise (i.e., sum of the nonlinear noise due to HPA saturation, the residual ISI after the MMSE-FDE and AWGN). The random variables X and Y represent a sum of many random variables including $\{\hat{H}_{\text{sr}}(n);\, n = 0 \sim N_c - 1\}$ and $\{\hat{H}_{\text{srd}}(n);\, n = 0 \sim N_c - 1\}$. According to the central limit theorem, each of those variables X_i (Y_i) for $i = 1, 2$, can be approximated as exponentially distributed (chi-square distribution with 2 degrees of freedom) random variable (as we assume mobile terminals, i.e., Rayleigh fading channels [10,11]). As a consequence, X and Y may follow a chi-square distribution with 4 degrees of freedom if both variables have the same average. However, the averages may not be the same since the path losses between the source-destination and the source-relay-destination are not the same. Moreover, it was also shown in [28] that in the case when there is a low number of exponentially distributed random variables in the sum, it is reasonable to approximate the resulting variable as exponentially distributed random variable [28]. Thus, to facilitate the analysis, we approximate X and Y as exponential distributed random variables. The random variables X and Y represent a combination of the same equalized channel gains, but one must know that besides those channel gains, each term contains also independent variables (i.e., $C_s(n)$, $C_r(n)$, $N_{d,1}(n)$, $N_r(n)$, $N_{d,2}(n)$) which are not correlated to the useful signal. Therefore, the nonlinear noise due to HPA, AWGN term, and the residual ISI after the MMSE-FDE are not correlated with the useful signal [22,24]. Moreover, the Rayleigh fading channel coefficients $H_{\text{sr}}(n)$, $H_{\text{sd}}(n)$, and $H_{\text{rd}}(n)$ are assumed to be independent random variables. By taking into account the above mentioned, it is reasonable to assume as well that X and Y are independent random variables.

Now, the CDF of the instantaneous SINR is obtained as

$$F_\gamma(\gamma) = \Pr\left[\frac{X}{Y} \le \gamma\right] = \frac{1}{\overline{X}\overline{Y}}\int_0^\infty \int_0^{\gamma Y} \exp\left(-\frac{X}{\overline{X}}\right)\exp\left(-\frac{Y}{\overline{Y}}\right)dXdY, \tag{16}$$

where $\overline{X}$ and $\overline{Y}$, respectively, denote the average values of the random variables X and Y. After some manipulations, CDF of the instantaneous SINR is given by

$$F_\gamma(\gamma) = 1 - \frac{\overline{\gamma}}{\gamma + \overline{\gamma}}, \tag{17}$$

where $\overline{\gamma} = \overline{X}/\overline{Y}$ denotes the average SINR. Now, the PDF of the SINR is calculated as a derivation of the CDF and it is given by

$$p_\gamma(\gamma) = \frac{\overline{\gamma}}{(\gamma + \overline{\gamma})^2}. \tag{18}$$

By using $\mathcal{L}\{1/(\gamma + \overline{\gamma})\} = -\exp(\overline{\gamma}s)\text{Ei}(-\overline{\gamma}s)$, with $\text{Ei(x)} = \int_x^\infty \exp(-t)/t dt$ being the exponential integral and the property $\mathcal{L}\{dx(t)/dt\} = sX(s)$, the MGF of γ is obtained as

$$\mathcal{M}_\gamma(s) = \overline{\gamma}s\exp(\overline{\gamma}s)\text{Ei}(-\overline{\gamma}s). \tag{19}$$

By using Chebyshev approximation of the exponential integral [28], the MGF of the SINR can be represented as

$$\mathcal{M}_\gamma(s) = \sum_{j=1}^{n} \frac{b_{j-1}}{a_{j-1} - s\overline{\gamma}} - a_0, \tag{20}$$

where coefficients (a_j, b_j) are given in [29] and $n \leq 9$. Finally, by substituting Equation 20 into Equation 15, closed-form SER for cooperative OFDM/TDM with MMSE-FDC taking into account the nonlinear HPA is given by

$$P_s = \frac{1}{\pi}\left\{\sum_{j=1}^{n}\frac{b_{j-1}}{a_j}\left[\arctan\left(\pi - \frac{\pi}{M}\right)\left(1 - \frac{a_j}{\overline{\gamma}\sin^2(\pi/M)}\right) - \arctan\left(\pi - \frac{\pi}{M}\right)\right] - a_0\right\}. \tag{21}$$

We observe that HPA saturation level and OFDM/TDM parameter K affects the SER. In particular, lower saturation level (greater parameter K) will cause higher nonlinear noise (lower residual ISI after MMSE-FDC), and the average SINR will decrease (increase).

3.3 Outage probability

Outage probability is an important performance metric which reflects that the cooperative network will not be able to support the target QoS (i.e., SER). The outage probability P_{out} at any given average received SINR is defined as the probability that the instantaneous SINR γ at destination is lower than the given threshold γ_{th}, and it is given by $P_{\text{out}} = P\{\gamma < \gamma_{\text{th}}\} = F_\gamma(\gamma_{\text{th}})$, where $F_\gamma(\gamma_{\text{th}})$ denotes the cumulative density function (i.e., CDF) of the random variable γ. By using Equation 17, the outage probability of cooperative network based on OFDM/TDM with MMSE-FDC in the presence of nonlinear HPA is given by

$$F_\gamma(\gamma_{\text{th}}) = 1 - \frac{\overline{\gamma}}{\gamma_{\text{th}} + \overline{\gamma}}, \tag{22}$$

where γ_{th} denotes the SINR threshold. We observe that the outage probability of cooperative network is a function of the OFDM/TDM design parameter K and HPA saturation level. As the parameter K increases, so does the residual ISI after the MMSE-FDC and consequently the outage probability as well. However, if the HPA saturation level increases, corresponding total noise due to HPA saturation level will decrease as will the outage probability. Thus, to achieve the target QoS, these parameters have to be designed properly.

3.4 Average sum-rate

Here, we give a brief overview of the average sum rate of cooperative network based on OFDM/TDM access with MMSE-FDC in the presence of nonlinear HPA at source and relay. We provide an analytical description of the impact of nonlinear degradation on average sum rate performance of cooperative network.

Average mutual information of cooperative network based on AF protocol is given by [30]

$$I = \frac{1}{2}\log_2(1+\gamma), \tag{23}$$

where γ is given by Equation 14. Now, the average sum rate is obtained as

$$R = \frac{1}{2}\mathbb{E}_\gamma\left\{\log_2(1+\gamma)\right\}. \tag{24}$$

By using Equation 18, the average sum rate can be rewritten as

$$R = \frac{\overline{\gamma}}{2\ln 2}\int_0^\infty \frac{\ln(1+\gamma)}{(\gamma+\overline{\gamma})^2}d\gamma. \tag{25}$$

Now, by using $\int_0^\infty \ln x/(x+a)^2 dx = \ln a/a$, the the average sum rate in the high SNR region in closed-form can be expressed as

$$R = \frac{1}{2\ln 2}\ln(\overline{\gamma}). \tag{26}$$

We note here that the average sum rate upper bound which can be easily derived from Equation 24 converges to Equation 26 in the high SNR region.

4 Simulation results and discussion

Simulation conditions are shown in Table 1. In our computer simulation, we assume an OFDM/TDM frame size of $N_c = 256$ samples with the GI length of $N_g = 32$ samples, single relay ($M = 1$), and ideal coherent QPSK data modulation/demodulation. The propagation channel is an $L = 16$ -path block Rayleigh fading channel having uniform power-delay profile, where the path

Table 1 Numerical parameters

	Parameter	Value
	Data modulation	QPSK
	IFFT/FFT size	$N_m = 256/K$
Transmitter	No. of	$K = 1$, 16
	slots	AND 256
	GI	$N_g = 32$
Channel	L	16-path frequency-selective
		block Rayleigh fading
	FDE	MMSE
Receiver	No. of FFT	$N_c = 256$
	points	$N_m = 256/K$
	Channel estimation	Ideal

gains $\{h_{l,xy};\ l = 0 \sim L-1\}$ remain constant over one OFDM/TDM frame length and vary frame-by-frame. The path gains are zero-mean independent complex variables with $\mathbb{E}\left[\left|h_{l,xy}\right|^2\right] = 1/L$, where all paths in any channel are independent with each other (we assume the same number of paths in each link). Without loss of generality, we assume $\tau_0 = 0 < \tau_1 < \ldots < \tau_{L-1}$ and that the lth path time delay is $\tau_l = l\triangle$, where $\triangle (= 1)$ denotes the time delay separation between adjacent paths (i.e., $(l-1)$th and lth path). Thus, the maximum time delay of the channel is less than the GI length (i.e., $L < N_g$). We assume no knowledge of the channel state information at the transmitters, the perfect channel state information at destination and perfect synchronization. The assumption on synchronization is most critical since synchronization becomes increasingly challenging in larger networks. We note that the analysis is done for general case in which HPA saturation level at source (φ_s) differs from the one at the relay's HPA (φ_r). However, in simulation results, the HPA at source and relay have the same saturation levels (i.e., $\varphi_s = \varphi_r = \varphi$).

4.1 SER

In this subsection, we investigate the SER performance of cooperative network based on OFDM/TDM with MMSE-FDC in the case of ideal and nonlinear HPA.

4.1.1 Ideal HPA

The performance of cooperative network based on OFDM/TDM with MMSE-FDC is largely dependent on channel frequency selectivity; thus in this section, we investigate the effect of different propagation scenarios. The channel frequency selectivity is a function of the number of paths L; as L decreases, the channel becomes less frequency selective, and when L =1 it becomes a frequency-nonselective channel (i.e., single-path channel). Figure 2 illustrates the average SER as a function of the average SNR ($P|H_{xy}|^2/\sigma_n^2$) with the number of paths L as a parameter. It can be seen that as L decreases (i.e., the channel becomes less frequency-selective) the performance of OFDM/TDM based cooperative network degrades since the channel is becoming less frequency-selective. This is because as L decreases the channel becomes less frequency-selective and lower frequency diversity gain is obtained through MMSE-FDC. In the case of L = 1, the performance of cooperative OFDM/TDM and cooperative OFDM is the same since the channel becomes frequency nonselective.

4.1.2 Nonlinear HPA

Here, we investigate the impact of nonlinear HPA on SER performance of cooperative network based on OFDM/TDM with MMSE-FDC. Figure 3 shows the average SER for K=16 as a function of average SNR at transmitter and relay, with HPA saturation level as a parameter. By using presented analysis, we plot theoretical results and simulated results for the comparison, and a fair agreement is observed.

Then, we investigate the impact of nonlinear degradation due to HPA saturation on cooperative network based on different access schemes. Figure 4 shows the average

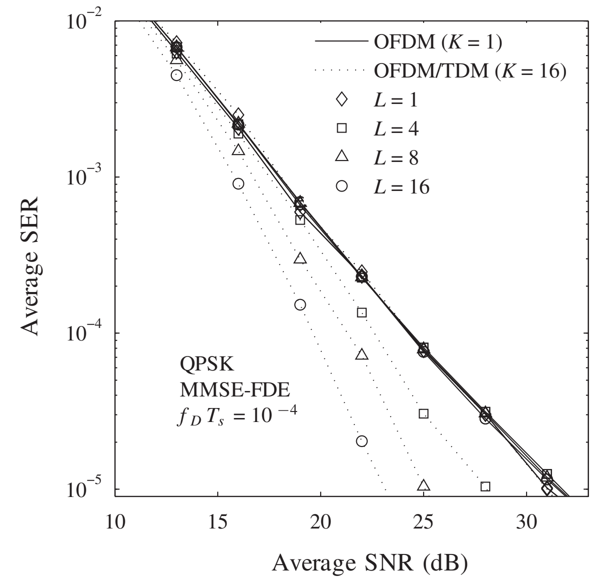

Figure 2 Simulated average SER vs. SNR.

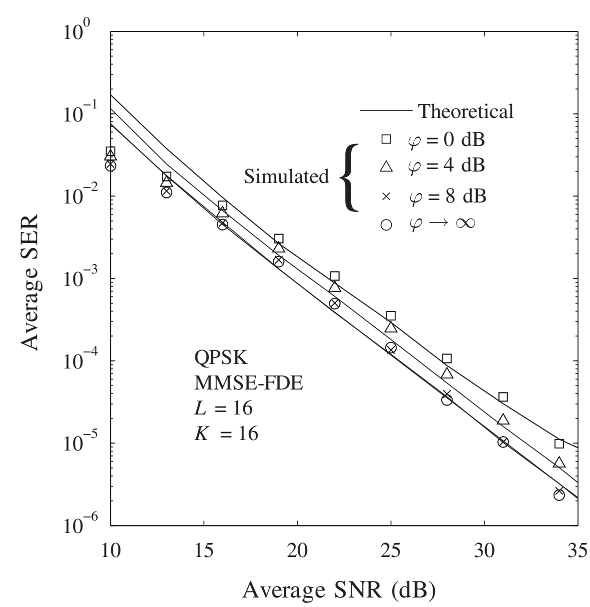

Figure 3 Average SER vs. SNR.

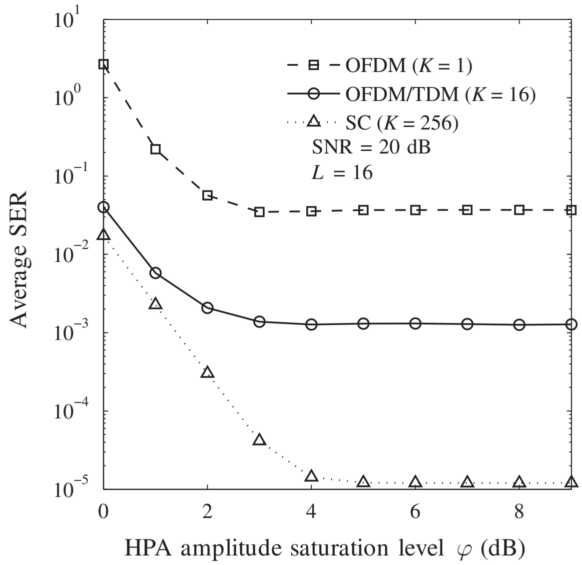

Figure 4 Theoretical average SER vs. φ**.**

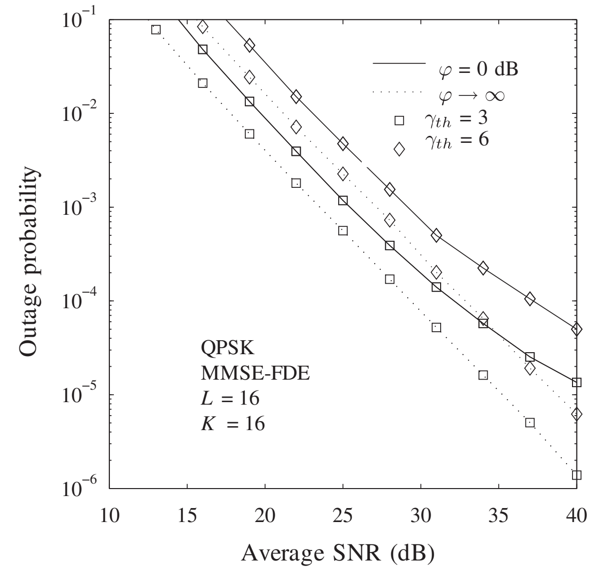

Figure 5 Theoretical outage probability vs. SNR.

SER performance of cooperative network based on OFDM ($K = 1$), OFDM/TDM ($K = 16$), and SC ($K = 256$), as a function of HPA saturation level φ, for SNR $= 20$ dB. We can see that the SER performance significantly improves for φ up to 4 dB irrespective of an access scheme, while for φ greater than 4 dB, the performance remains the same. This is because the corresponding noise due to HPA saturation level decreases (i.e., becomes negligible) and consequently does not affect the performance. Moreover, above $\varphi \geqslant 4$ dB floors the corresponding SER performance values are in order of 100.

4.2 Outage probability

In this subsection, we investigate the impact of nonlinear degradation due to HPA saturation on the outage probability of cooperative OFDM/TDM. Figure 5 shows the outage probability of cooperative network based on OFDM/TDM ($K = 16$) with γ_{th} as a parameter. We plot the case of an ideal HPA ($\varphi \to \infty$) and worse HPA saturation level ($\varphi = 0$ dB), for the comparison. We observe that the nonlinear HPA has almost the same impact on outage probability as increasing the required γ_{th} from 3 dB to 6 dB.

Next, we investigate the impact of SINR threshold γ_{th} on the outage probability of cooperative network in the presence of nonlinear HPA. Figure 6 shows the outage probability of cooperative network based on OFDM/TDM (i.e., $K = 16$) as a function of SINR threshold γ_{th} with (without) nonlinear degradation corresponding to φ=0 dB ($\varphi \to \infty$) with average SNR as a parameter. We observe that the impact of nonlinear HPA on outage probability degradation increases as the average SNR. Thus, for achieving the target QoS, one must take care to properly design the HPA since the performance improvement in high SNR region becomes more sensitive to nonlinear HPA.

4.3 Average sum rate

Here, we evaluate the average sum-rate performance of cooperative OFDM/TDM in the presence of nonlinear HPA. Figure 7 shows the average sum rate as a function of average SNR for different cases depending on which

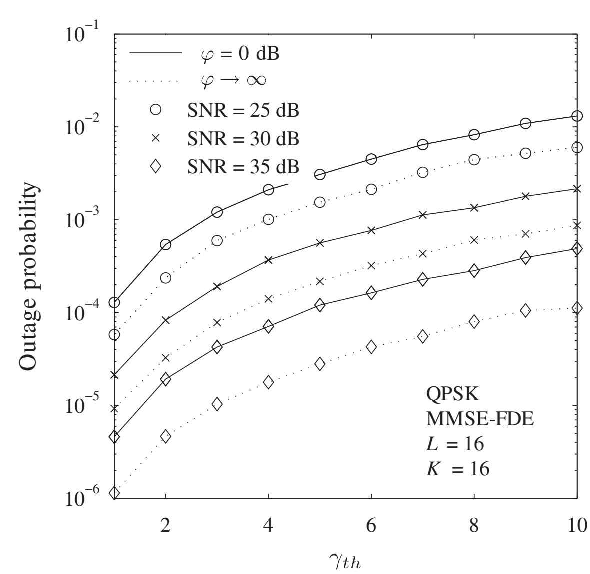

Figure 6 Theoretical outage probability vs. γ_{th}**.**

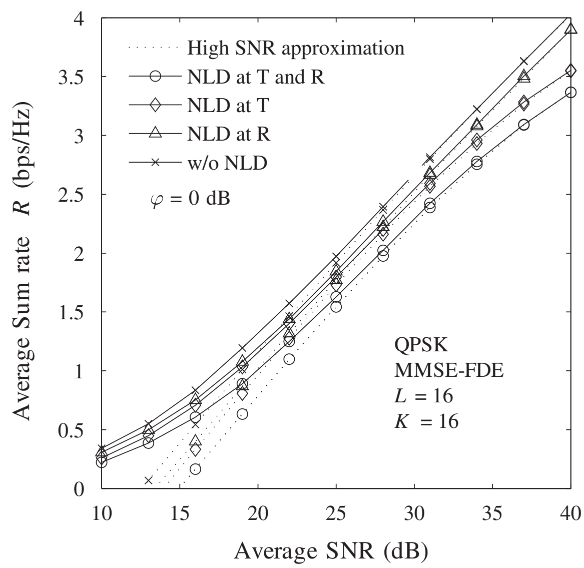

Figure 7 Theoretical average Sum rate vs. SNR.

terminal (i.e., transmitter and relay) is nonlinear HPA present. We observe that the average sum-rate performance is more sensitive to the nonlinear HPA introduced at source. This is because the signal affected by the nonlinear HPA is transmitted in the second stage as well (over the relay), and consequently, the worse performance is obtained. We note here that the average sum-rate upper bound also performs well in the low SNR region.

5 Conclusion

In this paper, we investigate the impact of nonlinear HPA using AF-IPS at relay on the performance of cooperative OFDM/TDM with MMSE-FDC in the frequency-selective fading channel. We design the MMSE-FDC weights while taking into account the nonlinear HPA at source and relay. In addition, we design a closed-form SER and outage probability expressions while approximating the residual ISI after the MMSE-FDC and noise due to HPA saturation as Gaussian random variables. It has been showed that by appropriate design, OFDM/TDM with MMSE-FDC can be used to reduce the impact of nonlinear HPA in cooperative network in comparison to OFDM access while achieving the target QoS for reduced required SNR. This is due to the reduced PAPR of OFDM/TDM and frequency diversity obtained through MMSE-FDC. Furthermore, we show that the nonlinear HPA at relay has limited impact on overall performance of cooperative OFDM/TDM.

Appendix

Here, we present a derivation of the MSE expressions for both stages.

The MSE term for the first stage is given by

$$\mathrm{MSE}_1 = \mathbb{E}[\, |R_{d,1}(n) w_1(n) - S(n)|^2]\,,$$

where $R_{d,1}(n)$, $w_1(n)$, and $S(n)$, respectively, denote the received signal at destination during the first stage, the equalization weight for the first stage, and the transmitted signal. We note here that the ISI term is embedded into $s(t) * h_{\mathrm{sd}}(t)$, and can be clearly defined only after the data demodulation in section 2.4. Therefore, the received signal at destination during the first stage after the N_{c}-point FFT can be represented as

$$R_{d,1}(n) = \sqrt{P}\alpha_s S(n) H_{\mathrm{sd}}(n) + \sqrt{P} C_s(n) H_{\mathrm{sd}}(n) + N_{d,1}(n).$$

Now, the MSE term can be rewritten as

$$\begin{aligned}
\mathrm{MSE}_1 =\, & \mathbb{E}[\, |\sqrt{P}\alpha_s S(n) H_{\mathrm{sd}}(n) w_1(n) \\
& + \sqrt{P} C_s(n) H_{\mathrm{sd}}(n) w_1(n) \\
& + N_{d,1}(n) w_1(n) - S(n)|^2] \\
=\, & \frac{2E_s}{T_c N_c}\alpha_s^2 \mathbb{E}[\, |S(n)|^2]\, |H_{\mathrm{sd}}(n)|^2 |w_1(n)|^2 \\
& + \frac{4E_s}{T_c N_c}\alpha_s \mathbb{E}[\, S(n) C_s(n)]\, \mathrm{Re}\{H_{\mathrm{sd}}^*(n)\} |w_1(n)|^2 \\
& + 2\sqrt{\frac{2E_s}{T_c N_c}}\alpha_s \mathbb{E}[\, S(n) N_{d,1}(n)]\, |H_{\mathrm{sd}}(n)|^2 |w_1(n)|^2 \\
& - \frac{4E_s}{T_c N_c}\alpha_s \mathbb{E}[\, |S(n)|^2]\, \mathrm{Re}\{H_{\mathrm{sd}}^*(n) w_1(n)\} \\
& + \frac{2E_s}{T_c}\mathbb{E}[\, |C_s(n)|^2]\, |H_{\mathrm{sd}}(n)|^2 |w_1(n)|^2 \\
& + 2\sqrt{\frac{2E_s}{T_c N_c}}\alpha_s \mathbb{E}[\, C_s(n) N_{d,1}(n)]\, \mathrm{Re}\{H_{\mathrm{sd}}^*(n)\} |w_1(n)|^2 \\
& - \frac{4E_s}{T_c N_c}\alpha_s \mathbb{E}[\, S(n) C_s(n)]\, \mathrm{Re}\{H_{\mathrm{sd}}^*(n) w_1(n)\} \\
& + \mathbb{E}[\, |N_{d,1}(n)|^2]\, |w_1(n)|^2 \\
& - 2\sqrt{\frac{2E_s}{T_c N_c}}\mathbb{E}[\, S(n) N_{d,1}(n)]\, \mathrm{Re}\{w_1(n)\} + \frac{2E_s}{T_c N_c}.
\end{aligned}$$

By taking into account that; (1) the transmitted signal $S(n)$ is not correlated with nonlinear noise due to HPA at source (i.e., $C_s(n)$) and AWGN at destination (i.e., $N_{d,2}(n)$) and (2) the nonlinear noise due to HPA at source (i.e., $C_s(n)$) is not correlated with AWGN at destination (i.e., $N_{d,2}(n)$), we come to Equation 5 in the main text.

The MSE term for the second stage is given by

$$\mathrm{MSE}_2 = \mathbb{E}[\, |R_{d,2}(n) w_2(n) - S(n)|^2]\,,$$

where $R_{d,2}(n)$ and $w_2(n)$, respectively, denote the received signal at destination during the second stage and the equalization weight for the second stage. Now, using Equation 3, the MSE term for the second stage can be represented as

$$
\begin{aligned}
MSE_2 &= \frac{2E_s}{T_cN_c}\alpha_s^2\alpha_r^2\beta^2\mathbb{E}[\,|S(n)|^2]\,|H_{\rm srd}(n)|^2|w_2(n)|^2 + \frac{4E_s}{T_cN_c}\alpha_s\alpha_r^2\beta^2\mathbb{E}[\,S(n)C_s(n)]\,|H_{\rm srd}(n)|^2|w_2(n)|^2\\
&+ \frac{4E_s}{T_cN_c}\alpha_s\alpha_r\beta^2\mathbb{E}[\,S(n)C_r(n)]\,\mathrm{Re}\{H^*_{\rm srd}(n)H_{\rm rd}(n)\}|w_2(n)|^2\\
&+ 2\sqrt{\frac{2E_s}{T_cN_c}}\alpha_s\alpha_r\beta\mathbb{E}[\,S(n)N_r(n)]\,\mathrm{Re}\{H^*_{\rm srd}(n)H_{\rm rd}(n)\}|w_2(n)|^2\\
&+ 2\sqrt{\frac{2E_s}{T_cN_c}}\alpha_s\alpha_r\beta\mathbb{E}[\,S(n)N_{d,2}(n)]\,\mathrm{Re}\{H^*_{\rm srd}(n)\}|w_2(n)|^2 - \frac{4E_s}{T_cN_c}\alpha_s\alpha_r\beta\mathbb{E}[\,|S(n)|^2]\,\mathrm{Re}\{H^*_{\rm srd}(n)w_2(n)\}\\
&+ \frac{2E_s}{T_c}\alpha_r^2\beta^2\mathbb{E}[\,|C_s(n)|^2]\,|H_{\rm srd}(n)|^2|w_2(n)|^2 + 2\sqrt{\frac{2E_s}{T_c}}\alpha_r\beta\mathbb{E}[\,C_s(n)C_r(n)]\,\mathrm{Re}\{H^*_{\rm srd}(n)H_{\rm rd}(n)\}|w_2(n)|^2\\
&+ 2\sqrt{\frac{2E_s}{T_c}}\alpha_r\beta\mathbb{E}[\,C_s(n)N_r(n)]\,\mathrm{Re}\{H^*_{\rm srd}(n)H_{\rm rd}(n)\}|w_2(n)|^2 + 2\sqrt{\frac{2E_s}{T_c}}\alpha_r\beta\mathbb{E}[\,C_s(n)N_{d,2}(n)]\\
&\times \mathrm{Re}\{H^*_{\rm srd}(n)\}|w_2(n)|^2 - \frac{4E_s}{T_c}\alpha_r\beta\mathbb{E}[\,C_s(n)S(n)]\,\mathrm{Re}\{H^*_{\rm srd}(n)w_2(n)\} + \frac{2E_s}{T_c}\beta^2\mathbb{E}[\,|C_r(n)|^2]\,|H_{\rm rd}(n)|^2|w_2(n)|^2\\
&+ 2\sqrt{\frac{2E_s}{T_cN_c}}\beta\mathbb{E}[\,C_r(n)N_r(n)]\,|H_{\rm rd}(n)|^2|w_2(n)|^2 + 2\sqrt{\frac{2E_s}{T_cN_c}}\beta\mathbb{E}[\,C_r(n)N_{d,2}(n)]\,\mathrm{Re}\{H^*_{\rm rd}(n)\}|w_2(n)|^2\\
&+ \mathbb{E}[\,|N_r(n)|^2]\,|H_{\rm rd}(n)|^2|w_2(n)|^2 + \mathbb{E}[\,N_r(n)N_{d,2}(n)]\,\mathrm{Re}\{H^*_{\rm rd}(n)\}|w_2(n)|^2\\
&+ 2\sqrt{\frac{2E_s}{T_cN_c}}\mathbb{E}[\,N_r(n)S(n)]\,\mathrm{Re}\{H^*_{\rm rd}(n)w_2(n)\} + \mathbb{E}[\,|N_{d,2}(n)|^2]\,|w_2(n)|^2\\
&+ 2\sqrt{\frac{2E_s}{T_cN_c}}\mathbb{E}[\,S(n)N_{d,2}(n)]\,\mathrm{Re}\{w_2(n)\} + \frac{2E_s}{T_cN_c}.
\end{aligned}
$$

Now, by taking into account that (1) the transmitted signal $S(n)$ in not correlated with nonlinear noise due to HPA at source/relay (i.e., $C_s(n)$ and $C_r(n)$) and AWGN at relay/destination (i.e., $N_r(n)$ and $N_{d,2}(n)$), (2) the non-linear noise due to HPA at source/relay (i.e., $C_s(n)$ and $C_r(n)$) is not correlated with AWGN at relay/destination (i.e., $N_r(n)$ and $N_{d,2}(n)$), (3) the nonlinear noises due to HPA at source and at destination are not correlated ($C_s(n)$ and $C_r(n)$), and (4) the AWGN at relay and destination are not correlated ($N_r(n)$ and $N_{d,2}(n)$), we come to Equation 7 in the main text.

Competing interests
The authors declare that they have no competing interests.

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