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Impact of partial relay selection on the capacity of communications systems with outdated CSI and adaptive transmission techniques

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Abstract

The impact of outdated channel state information (CSI) on the capacity of amplify-and-forward (AF) partial relay selection systems is studied in this article. The closed-form expressions for the distribution of received signal-to-noise ratio (SNR) in a multi-relay cooperative communications system is first derived, with independent and identically distributed (i.i.d.) Rayleigh fading channels being assumed in each wireless link. After that, the theoretical closed-form expressions for both outage probability and channel capacity of partial relay selection are derived, with four classical adaptive transmission techniques, including the constant power with optimal rate adaption (ORA), the optimal power and rate adaption (OPRA), the channel inversion with fixed rate (CIFR) and truncated channel inversion with fixed rate (TIFR), being considered. Numerical analysis proves that the channel capacity of partial relay selection is impacted considerably by some critical parameters, including the number of relays, the channel correlation coefficient and the end-to-end SNR, etc. It's also exhibited in the numerical results that among the four adaptive transmission techniques, the diversity order of OPRA is larger than that of TIFR, and the OPRA outperforms TIFR with about 0.15 bits/s/Hz in terms of average channel capacity.

Keywords: Cooperative networks, AF partial relaying, Outdated channel state information, Adaptive transmission techniques, Average channel capacity

1 Introduction

Cooperative communications technique, where distributed multiple nodes form a virtual multiple-antenna array, has the potential of offering a number of significant performance benefits, such as an expanded wireless coverage, a system-wide power saving, a throughput improvement, and a better immunity against severe channel fading [1-3]. Relaying protocols are also expected to be adopted as standard in future mobile broadband communications systems such as beyond-third-generation (B3G) and fourth-generation (4G) systems [4-7].

Spatial diversity gain can be improved considerably by employing cooperative relays (without loss of generality, R is used to represent relay nodes), which forward the received message from the source node (i.e., S) to the destination (i.e., D) using the source-to-relay-to-destination ($S \rightarrow R \rightarrow D$) link. The type of relaying modes can be largely classified as amplify-and-forward (AF) and decode-and-forward (DF), where in the former, relay directly retransmits a linearly amplified version of the received signal to the destination without any modification to the signal, but in the latter, relay demodulates and possibly re-modulates the received information before forward it [8,9]. Besides AF and DF, some other well-known relaying mode, such as compress-and-forward (CF), has also been studied considerably to obtain a performance gain by performing signal compression at the relays [10].

As compared to the single-relay systems, a higher spatial diversity gain is announced in a multiple-relay system due to more redundant signal transmission paths being created in it. However, the advantage of multi-relay systems in terms of spatial diversity gain mainly comes from

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the redundant relays, which usually require an orthogonal channel (in either the time or frequency domain) being allocated to each individual to effectively avoid the inter-relay interference [11,12]. This kind of orthogonal-allocation will inevitably consume more bandwidth to convey the same message.

Relay selection is proven to be one of the most efficient methods to improving the spectral efficiency and maximizing the spatial diversity gain at the same time in multiple-relay systems [13-18]. By selecting the optimal relay from multiple candidates, only one relay is active in each time so as to improve the spectral efficiency considerably, with full spatial diversity gain being obtained. Relay selection techniques in cooperative communications systems can be classified as two modes, i.e., the opportunistic relay selection [19,20] and the partial relay selection [21], where in the former, the signal-to-noise ratio (SNR) of both the source-to-relay ($S \rightarrow R$) and relay-to-destination ($R \rightarrow D$) links must be considered by the central unit, but in the latter, the SNR of only the $S \rightarrow R$ link or $R \rightarrow D$ link is necessarily considered.

Currently, most of the studies about relay selection have been focused on the scenarios of a perfect channel state information (CSI) being available [22,23]. However, in practice, an outdated CSI is usually obtained, especially in a high-mobility scenario with a high Doppler Shift and a long feedback delay. In this case, performing relay selection by considering an outdated CSI feedback would be a suitable method to optimize the performance of the cooperative systems at a reasonable CSI feedback cost [24,25]. In [21], a closed-form expression on the capacity of partial relay selection under outdated CSI was studied, with non-adaptive fixed-rate transmission being considered. The ergodic channel capacity of reactive DF relaying system for non-adaptive fixed-rate transmission is studied in [20,26]. Besides, an analytical expression of channel capacity in an AF opportunistic relaying system with adaptive transmission has also been derived in [19]. However, an assumption of non-adaptive fixed-rate transmission has been considered in all the studies aforementioned, and partial relay selection with an outdated CSI feedback and adaptive transmission in cooperative networks is still not considerably studied in prior studies.

In this article, channel capacity for AF partial relay selection with adaptive transmission techniques under an outdated CSI will be studied, with four classic adaptive transmission techniques, including the constant power with optimal rate adaption (ORA), the optimal power and rate adaption (OPRA), the channel inversion with fixed rate (CIFR) and truncated channel inversion with fixed rate (TIFR), being considered. The main contributions of this article as compared to the existing works are exhibited as follows: (1) the $S \rightarrow D$ channel is employed in the proposed cooperative transmission to improve the spatial diversity gain, with variant figures of merit, such as channel capacity, outage probability, being analyzed; (2) in consideration of the fact that an outdated CSI may be obtained due to Doppler Shift or feedback delay, relay selection with an outdated CSI is studied; (3) considering the CSI feedback load reduction in relay selection, partial relay selection instead of full opportunistic relaying is studied in this article, with a closed-form expression of the channel capacity for each adaptive transmission technique being derived. The advantages of partial relay selection in the presence of imperfect CSI feedback have also been proven in this article. It's proven in the numerical results that the diversity order of OPRA is larger than that of TIFR, and the OPRA outperforms TIFR with about 0.15 bits/s/Hz in terms of average channel capacity.

The remainder of this article is organized as follows. Section 2 introduces the channel model of partial relay selection with outdated CSI. The closed-form expressions for average channel capacity of partial relay selection under four adaptive transmission techniques with outdated CSI are derived in Section 3. Section 4 gives out the numerical results. Finally, Section 5 concludes this article.

Notation. $\mathbb{E}\{x\}$ denotes the mean of x. γ_{SR_i} represents the SNR of $S \rightarrow R_i$ link. $\bar{\gamma}_{SR_i}$ and $\hat{\gamma}_{SR_i}$ stand for the mean and estimation of γ_{SR_i}, respectively. $f_X(\cdot)$ and $F_X(\cdot)$ denote the probability density function (PDF) and cumulative distribution function (CDF) of random variable (RV) X, respectively. $\mathcal{M}_X(s)$ represents the moment generating function (MGF) of RV X.

2 System model

In this section, a cooperative network comprising a source terminal (S), N half-duplex AF fixed-gain relays (as denoted by the set of $\Omega = \{i = 1, 2, \ldots, N\}$) and a destination terminal (D), is considered. Without loss of generality, the wireless channels of all $S \rightarrow R_i$ links and $R_i \rightarrow D$ links are assumed to be independent and identically distributed (i.i.d.) Rayleigh distributed RVs, and the channel gain between terminals S and R_i is denoted by h_{SR_i}. The average SNR of each $S \rightarrow R_i$ link can be represented as $\bar{\gamma}_{SR}$. Similarly, the average SNR of all $R_i \rightarrow D$ links and $S \rightarrow D$ link can be denoted by $\bar{\gamma}_{RD}$ and $\bar{\gamma}_{SD}$, respectively.

The optimal relay is selected as

$$k = \arg\max_i(\hat{\gamma}_{SR_i}), \tag{1}$$

where $i \in \{1, 2, \ldots, N\}$. Note that the selected optimal relay at time t may no necessarily be the best choice for the time $t + \tau$ due to the existence of feedback delay τ. In this case, the estimated SNR is outdated, and based on it, the system performance will be degraded consequently.

The effect of outdated CSI can be reduced to some extent by taking into consideration of the correlation between the actual CSI (i.e., h_{SR_k}) and its outdated estimation (i.e., $\hat{h}_{SR_k}$), as given by $\rho_{SR_k} = J_0(2\pi\tau f_D)$, which leads to

$$\hat{h}_{SR_k} = \rho_{SR_k} h_{SR_k} + \sqrt{1-\rho_{SR_k}^2}\, w_{SR_k}, \tag{2}$$

where the Jakes' autocorrelation mode is assumed, w_{SR_k} denotes a circularly symmetric complex Gaussian RV, $J_0(.)$ stands for the zeroth Bessel function of the first kind, and f_D represents the maximum Doppler frequency on the $S \to R_k$ link.

After maximal ratio combining (MRC) at the destination[a], we can obtain the effective SNR for partial relay selection with outdated CSI as γ_{total}. The PDF of γ_{total} can be given as (please see Appendix 1 for detail)

$$\begin{aligned} f_{\gamma_{\text{total}}}(\gamma) = & N \sum_{m=0}^{N-1} \frac{(-1)^m \binom{N-1}{m}}{m+1} \\ & \cdot \frac{\mathcal{A}_m}{\mathcal{A}_m \bar{\gamma}_{SD} - 1}\left(e^{-\gamma/\bar{\gamma}_{SD}} - e^{-\mathcal{A}_m \gamma}\right), \end{aligned} \tag{3}$$

where $\mathcal{A}_m = \frac{m+1}{(m(1-\rho_{SR_k}^2)+1)\bar{\gamma}_{SR}} + \frac{1}{\bar{\gamma}_{RD}}$.

The outage probability is then given by

$$P_{\text{out}} = N \sum_{m=0}^{N-1} \frac{(-1)^m \binom{N-1}{m}}{m+1} \cdot \mathcal{A}_m \cdot \mathcal{D}(N, \gamma_0, \bar{\gamma}_{SD}, \mathcal{A}_m), \tag{4}$$

where

$$\begin{aligned} \mathcal{D}(N, \gamma_0, \bar{\gamma}_{SD}, \mathcal{A}_m) = & \frac{1}{\mathcal{A}_m} + \frac{1/\bar{\gamma}_{SD} e^{-\mathcal{A}_m \gamma_0}}{\mathcal{A}_m(\mathcal{A}_m - 1/\bar{\gamma}_{SD})} \\ & - \frac{e^{-\gamma_0/\bar{\gamma}_{SD}}}{\mathcal{A}_m - 1/\bar{\gamma}_{SD}} \end{aligned} \tag{5}$$

and $\gamma_0 = 2^{2R_{\text{sbol}}} - 1$ represents the SNR threshold, with R_{sbol} denoting the symbols rate. Note that the exponent becomes R_{sbol} if full-duplex AF relays are considered [27].

3 Channel capacity analysis

In consideration of the CSI feedback reduction in practical systems, partial relay selection with an outdated CSI will be studied in this section Channel capacity of partial relay selection with four classical adaptive transmission techniques, including ORA, OPRA, CIFR, and TIFR [28], will be analyzed. Approximate closed-form expressions of the average channel capacity will also be derived.

3.1 Constant power with optimal rate adaption (ORA)

The channel capacity for the partial relay selection should be derived based on the PDF of effective SNR γ_{total}, as given by (3). Like in [28], the channel capacity of the partial relay selection with ORA can be derived as

$$\mathbb{E}\{C_{\text{ORA}}\} = \frac{1}{2}\int_0^\infty \log_2(1+\gamma) f_{\gamma_{\text{total}}}(\gamma) d\gamma, \tag{6}$$

where the factor 1/2 comes from the fact that the proposed cooperative communications is performed within two orthogonal channels or time-slots.

Note that a closed-form representation of (6) is not straightforward to derive. By substituting (3) into (6), it yields

$$\begin{aligned} \mathbb{E}\{C_{\text{ORA}}\} = & N \sum_{m=0}^{N-1} \frac{(-1)^m \binom{N-1}{m}}{m+1} \cdot \frac{\mathcal{A}_m}{2\ln 2\,(\mathcal{A}_m \bar{\gamma}_{SD} - 1)} \quad , \\ & \times \left(\bar{\gamma}_{SD} e^{1/\bar{\gamma}_{SD}} \mathcal{G}_1\left(\frac{1}{\bar{\gamma}_{SD}}\right) - \frac{e^{\mathcal{A}_m} \mathcal{G}_1(\mathcal{A}_m)}{\mathcal{A}_m}\right) \text{bit/s/Hz} \end{aligned} \tag{7}$$

where

$$\begin{aligned} \mathcal{G}_1(x) &= \frac{x}{e^x}\int_0^\infty \ln(1+t) e^{-xt} dt \\ &= \int_x^\infty \frac{e^{-t}}{t} dt \\ &= -Ei(-x), \qquad x > 0 \end{aligned} \tag{8}$$

denotes the exponential integral function of first order ([29], Equation (8.211.1)), and the first step in (8) is derived from [28].

Evidently, a finite series representation may provide a proper way to give out a closed-form analysis of the channel capacity.

3.2 Optimal power and rate adaptation (OPRA)

In OPRA, both transmission power and rate are adaptively optimized according to the CSI, with a given average transmit power constraint P being satisfied, i.e.,

$$\begin{aligned} & \mathbb{E}(P_{\text{OPRA}}/P) = 1 \\ & \quad \text{subject to} \\ & \quad \begin{cases} P_{\text{OPRA}}/P = 0 \text{ if } \gamma < \gamma_0, \\ P_{\text{OPRA}}/P = (1/\gamma_0 - 1/\gamma) \text{ if } \gamma \geq \gamma_0, \end{cases} \end{aligned} \tag{9}$$

where γ_0 denotes the optimal cut-off SNR threshold below which no data can be transmitted, and P_{OPRA} denotes the power of OPRA. From [28], the constraint condition $\mathbb{E}(P_{\text{OPRA}}/P) = 1$ is equivalent to

$$\int_{\gamma_0}^\infty \left(\frac{1}{\gamma_0} - \frac{1}{\gamma}\right) f_{\gamma_{\text{total}}}(\gamma) d\gamma = 1. \tag{10}$$

Before analyze the channel capacity of OPRA, let's first define a new parameter, as referred to Y_{γ_0}, by substituting (3) into (10):

$$Y_{\gamma_0} = N \sum_{m=0}^{N-1} \frac{(-1)^m \binom{N-1}{m}}{m+1} \cdot \frac{\mathcal{A}_m}{\mathcal{A}_m \bar{r}_{SD} - 1} \cdot \lambda(m, \gamma_0) - 1, \tag{11}$$

where

$$\begin{aligned}\lambda(m,\gamma_0) = &\left[\mathcal{G}_0\left(\frac{\gamma_0}{\bar{\gamma}_{SD}}\right) - \mathcal{G}_0\left(\mathcal{A}_m\gamma_0\right)\right] \\ &- \left[\mathcal{G}_1\left(\frac{\gamma_0}{\bar{\gamma}_{SD}}\right) - \mathcal{G}_1\left(\mathcal{A}_m\gamma_0\right)\right],\end{aligned} \tag{12}$$

and

$$\mathcal{G}_0(x) = \frac{1}{x}e^{-x}. \tag{13}$$

It's proven by Appendix 2 that Y_{γ_0} is a monotonically decreasing function of γ_0. Since the curve crosses the x-coordinate, it ensures that a unique cut-off SNR level can always be found. In this case, the instantaneous transmission rate can be defined as $R_{\mathrm{OPRA}} = 1/2\log_2[1 + (P_{\mathrm{OPRA}}/P)\gamma]$.

Like in [28], the average capacity of OPRA is given by

$$\mathbb{E}\{C_{\mathrm{OPRA}}\} = \frac{1}{2}\int_{\gamma_0}^{\infty}\log_2\left(\frac{\gamma}{\gamma_0}\right)f_{\gamma_{\mathrm{total}}}(\gamma)d\gamma. \tag{14}$$

The closed-form representation of (14) can also be derived by using a finite series representation. Like in ORA, we also substitute (3) into (14), which yields

$$\begin{aligned}\mathbb{E}\{C_{\mathrm{OPRA}}\} &= N\sum_{m=0}^{N-1}\frac{(-1)^m\binom{N-1}{m}}{m+1}\cdot\frac{\mathcal{A}_m}{2\ln 2\left(\mathcal{A}_m\bar{\gamma}_{SD}-1\right)} \\ &\times\left(\bar{\gamma}_{SD}\mathcal{G}_1\left(\frac{\gamma_0}{\bar{\gamma}_{SD}}\right) - \frac{\mathcal{G}_1\left(\gamma_0\mathcal{A}_m\right)}{\mathcal{A}_m}\right)\ \text{bit/s/Hz}\end{aligned}. \tag{15}$$

Since both transmission power and rate are adaptively optimized in OPRA according to the CSI, the channel capacity, i.e., $\mathbb{E}\{C_{\mathrm{OPRA}}\}$, can always be optimized by adaptively modulating each sub-channel based on (9). It is also shown in (15) that a water filling principle can be employed to optimize the OPRA channel capacity: we can allocate lower rate and power levels for unfavorable channel conditions and higher rates and power levels for a good-quality channel.

3.3 Channel inversion with fixed rate (CIFR)

In this scheme, the source terminal adjusts its transmission power so as to maintain a constant SNR (γ_C) level at the destination, and γ_C is maintained subject to the average power constraint $\bar{P}$ with $\mathbb{E}\left\{P(\gamma)/\bar{P}\right\} = 1$. A fixed-rate modulation with fixed-code design is also assumed in CIFR. Therefore, power allocation in CIFR must follow the constrain of

$$P(\gamma)/\bar{P} = \gamma_C/\gamma, \tag{16}$$

which yields

$$\begin{aligned}\mathbb{E}\{\gamma_C\} &= \mathbb{E}\{P(\gamma)/\bar{P}\}\left[\mathbb{E}\{1/\gamma\}\right]^{-1} \\ &= \left[\mathbb{E}\{1/\gamma\}\right]^{-1} \\ &= \left[\int_0^{\infty}\frac{1}{\gamma}f_{\gamma_{\mathrm{total}}}(\gamma)d\gamma\right]^{-1}.\end{aligned} \tag{17}$$

The channel capacity of CIFR can be defined as

$$\mathbb{E}\{C_{\mathrm{CIFR}}\} = 1/2\log_2(1+\gamma_C), \tag{18}$$

and by substituting (3) and (17) into (18), it yields [30]

$$\begin{aligned}\mathbb{E}\{C_{\mathrm{CIFR}}\} &= \frac{1}{2}\log_2\left(1+\left[\int_0^{\infty}\frac{1}{\gamma}f_{\gamma_{\mathrm{total}}}(\gamma)d\gamma\right]^{-1}\right) \\ &= \frac{1}{2}\log_2\left(1+\left[N\sum_{m=0}^{N-1}\frac{(-1)^m\binom{N-1}{m}}{m+1}\cdot\frac{\ln(\mathcal{A}_m\bar{\gamma}_{SD})}{\bar{\gamma}_{SD}-\frac{1}{\mathcal{A}_m}}\right]^{-1}\right).\end{aligned} \tag{19}$$

Different from OPRA, adaptive modulation and power allocation have not been considered in CIFR, and $\mathbb{E}\{C_{\mathrm{CIFR}}\}$ is expected to suffer a large capacity penalty due to a large amount of the transmitted power being required to compensate for the deep-fading channels.

3.4 Truncated channel inversion with fixed rate (TIFR)

Although adaptive transmission with the least-complex technique can be obtained in CIFR, it suffers a large capacity degradation in a deep-fading channel. In order to combat the disadvantage aforementioned, the TIFR technique, which performs the channel inversion when the end-to-end SNR γ exceeds the cut-off SNR level γ_0, is introduced. The average capacity of TIFR can then be derived as [28]

$$\mathbb{E}\{C_{\mathrm{TIFR}}\} = \frac{1}{2}\log_2\left(1+\left[\int_{\gamma_0}^{\infty}\frac{1}{\gamma}f_{\gamma_{\mathrm{total}}}(\gamma)d\gamma\right]^{-1}\right)(1-P_{\mathrm{out}}). \tag{20}$$

By substituting (3) into (20), it yields

$$\begin{aligned}\mathbb{E}\{C_{\mathrm{TIFR}}\} &= \frac{1}{2}\log_2\left(1+\left[\theta(N,\gamma_0,\bar{\gamma}_{SD},\mathcal{A}_m)\right]^{-1}\right) \\ &\times\left(1-N\sum_{m=0}^{N-1}\frac{(-1)^m\binom{N-1}{m}}{m+1}\right. \\ &\left.\cdot\mathcal{A}_m\mathcal{D}\left(N,\gamma_0,\bar{\gamma}_{SD},\mathcal{A}_m\right)\right),\end{aligned} \tag{21}$$

where

$$\begin{aligned}\theta(N,\gamma_0,\bar{\gamma}_{SD},\mathcal{A}_m) = &N\sum_{m=0}^{N-1}\frac{(-1)^m\binom{N-1}{m}}{m+1}\cdot\frac{\mathcal{A}_m}{\mathcal{A}_m\bar{\gamma}_{SD}-1} \\ &\times\left(\mathcal{G}_1\left(\frac{\gamma_0}{\bar{\gamma}_{SD}}\right) - \mathcal{G}_1\left(\mathcal{A}_m\gamma_0\right)\right).\end{aligned} \tag{22}$$

The optimal cut-off SNR level, i.e., γ_0^*, can always be obtained in CIFR to maximize the channel capacity $\mathbb{E}\{C_{\mathrm{TIFR}}\}$. Since CIFR only *selectively* performs channel inversion, a cost-effective capacity optimization can be obtained in this scheme.

From the analysis aforementioned, it has been emphasized that the finite series representation plays a critical role in simplifying the analysis of channel capacity for partial relay selection. Also the closed-form representation for the capacity can be obtained, with the exponential integral function of first order, i.e., $\mathcal{G}_1(x) = -Ei(-x)$, being resolved by using some specific method (e.g., numerical analysis using MATLAB).

4 Numerical results

In this section, a normalized system bandwidth is assumed. Without loss of generality, the wireless channel fading is also assumed as follows, i.e., $\bar{\gamma}_{SD} = \bar{\gamma}_{SR} = \bar{\gamma}_{RD} = \bar{\gamma}$. Monte Carlo simulations are used in this article to verify the proposed analysis, with the detailed simulation being performed as follows. For each adaptive transmission scheme with a given parameter (e.g., N, SNR), we first generate 5000 i.i.d. wireless channel gains for each link. Without loss of generality, Rayleigh fading is assumed in each link. Additive noise is then generated by using MATLAB, with a given SNR level being satisfied. In each iteration, Equations of (7), (15), (19), and (21) will be used to calculate the instantaneous channel capacity for the scheme of ORA, OPRA, CIFR, and TIFR, respectively. After that, the 5000 results will be averaged to obtain the final channel capacity for each scheme in a given parameter condition.

As depicted in Figure 1, the outage probabilities of the two scenarios, including that of relay selection with full CSI and that of relay selection with partial CSI, are compared. In this simulation, the number of relaying nodes is assumed to be 3. Without loss of generality, we also assume that the distance between the source and each relay is not far away and channel quality of $S \to R$ links is relatively constant but that of $R \to D$ links varies relatively faster. Therefore, the value of the channel correlation coefficient ρ_{R_kD}, which is related to the maximum Doppler Shift and the feedback delay of $R \to D$ links, falls within the range of [0, 1]. However, ρ_{SR_k} (i.e., the

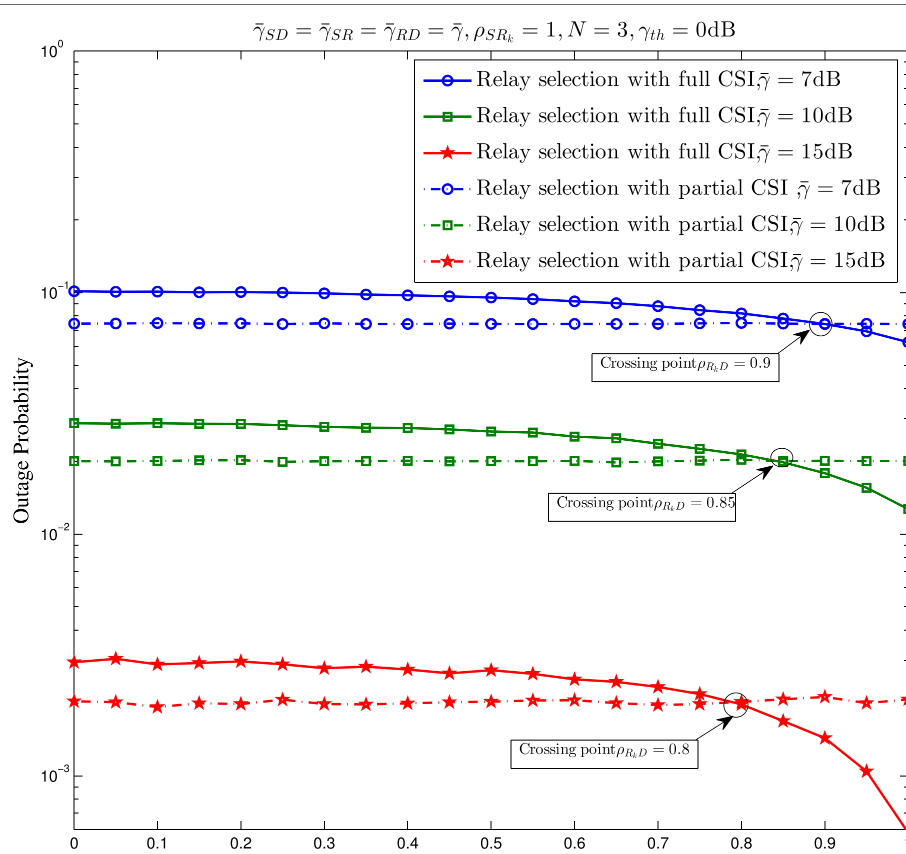

Figure 1 Outage probability of the relay selection with full CSI and relay selection with partial CSI versus the channel correlation coefficient of $R \to D$ link (ρ_{R_kD}) for the i.i.d. links.

channel correlation coefficient for the $S \rightarrow R$ links), on the other hand, always equals 1. Note that there always exists errors in channel estimation by using outdated CSI due to the channel variant characteristics of $R \rightarrow D$ links, and those errors will consequently deteriorate the relay selection if full CSI is employed. In consideration of this degradation, the performance of relay selection can be improved by considering only partial perfect CSI (e.g., only the CSI of $S \rightarrow R$ or $R \rightarrow D$ link is considered to alleviate the degradation due to estimation errors). Therefore, the outage performance of relay selection with partial relay selection may outperform that with full CSI, especially in the presence of a relatively small channel correlation coefficient. For each SNR, it is shown in this figure that we can always find a ρ_{R_kD} threshold, within which scope the partial relay selection outperforms the full selection. Furthermore, it is also observed that this threshold is a monotonically decreasing function of SNR, and a higher superiority over the full selection can be observed in the partial relay selection in the low SNR conditions.

Figure 2 illustrates the optimum cut-off SNR of both the OPRA and TIFR schemes versus the number of relays, with $\rho_{SR_k} = 0, 0.1, 0.5, 0.9, 1$ and $\bar{\gamma} = 10\,\text{dB}$ being assumed. It's interesting to observe that the optimal cut-off SNR for both OPRA and TIFR schemes changes slowly as N increases, although both curves are monotonically increasing functions of N. When the number of relays increases, the opportunity of obtaining an optimum relay is improved consequently, and the optimum cut-off SNR is increased as a result. However, the performance of both techniques is impacted greatly by ρ_{SR_k} and $\bar{\gamma}$. Data transmission in high SNR zone can be exploited to improve the average channel capacity. It's observed that the optimum cut-off SNR of OPRA is lower than 0 dB, but that for TIFR falls in the interval (0 dB, ∞). It's also shown that a larger ρ_{SR_k} implies a higher optimum cut-off SNR value in each scheme.

Figure 3 illustrates the optimum cut-off SNR of both the OPRA and TIFR schemes versus the correlation coefficient ρ_{SR_k}. It's observed that the optimum cut-off SNR is a monotonically increasing function of either ρ_{SR_k} or $\bar{\gamma}$, and the upper bound of the optimal cut-off SNR, i.e., 0 db, is approached in OPRA as $\bar{\gamma}$ increases. The optimum cut-off SNR of TIFR is always 2.9 dB higher than that of OPRA, regardless of what the parameters (including N,

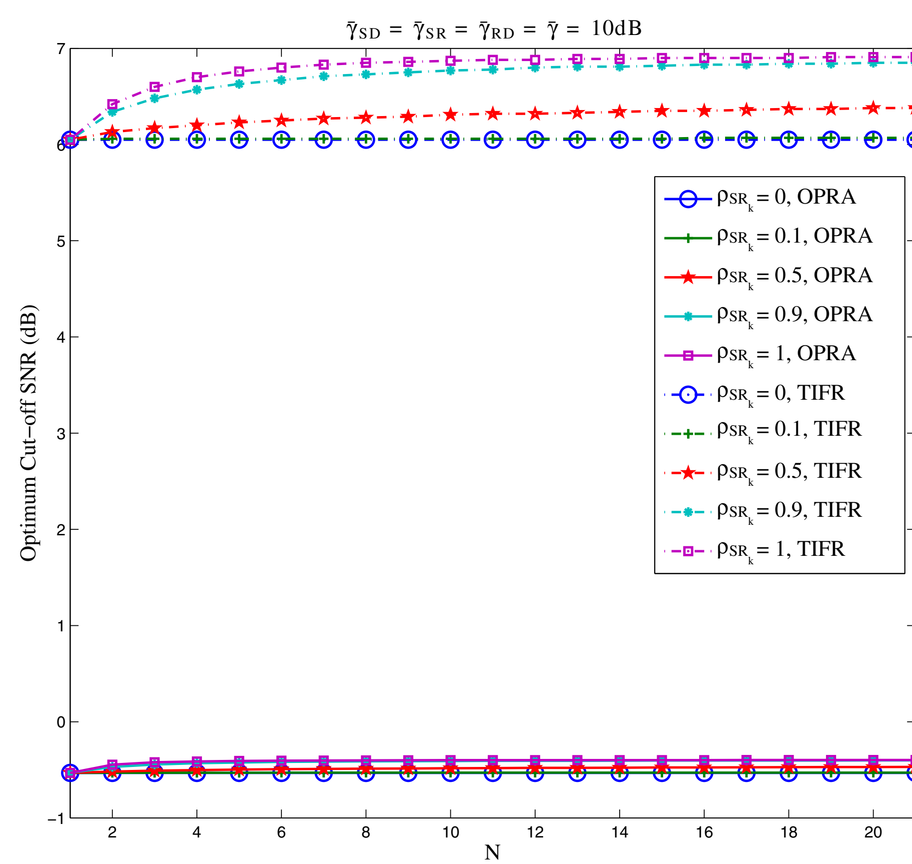

Figure 2 Optimum Cut-off SNR of the OPRA and TIFR schemes versus the number of relays *N* for the i.i.d. links.

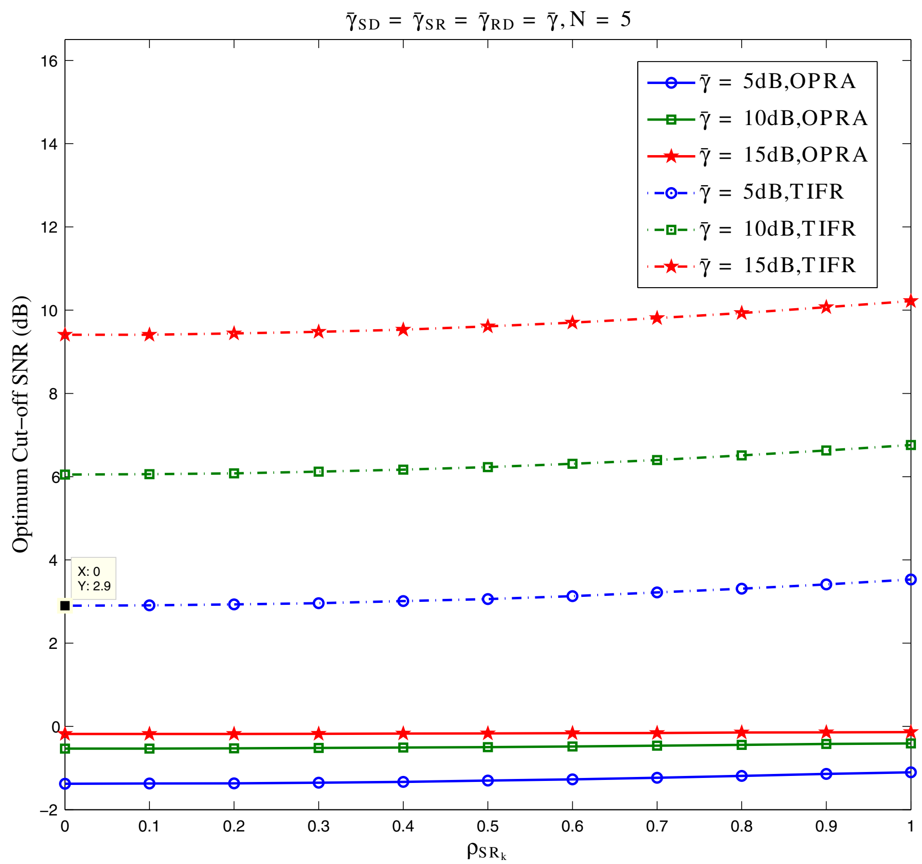

Figure 3 Optimum Cut-off SNR of the OPRA scheme as a function of ρ_{SR_k} and $\bar{\gamma}$ for the i.i.d. links.

ρ_{SR_k} and $\bar{\gamma}$) might be. In order to keep a constant SNR at receiver side in TIFR scheme, the optimum cut-off SNR in TIFR is more prone to be impacted by the channel correlation coefficient than the OPRA scheme.

The average channel capacity per unit bandwidth versus the number of relays N is shown in Figure 4. It's observed that in each technique, a larger number of relays implies a higher average capacity being obtained. The capacity of OPRA and ORA schemes is larger than that of CIFR or TIFR scheme, provided that the same correlation coefficient ρ_{SR_k} being considered. It's also observed that the gap between ORA and OPRA (or between TIFR and CIFR) tends to decrease as ρ_{SR_k} increases. In each scheme, when $\rho_{SR_k} = 0$ (i.e., most of the CSI feedback is imperfect), employing multiple relays cannot improve the spatial diversity gain efficiently. When $\rho_{SR_k} > 0$, on the other hand, both the spatial diversity gain and channel capacity in the cooperative systems can be improved by utilizing more relays. Since the CIFR scheme requires a relatively high transmit power so as to keep a constant SNR at the receiver side, the channel capacity will be sacrificed to some extent (that's why the capacity of CIFR scheme is always lower than the other schemes). TIFR can be regarded as an improvement of CIFR, with CIFR being performed only if the SNR is higher than the cut-off threshold. As compared to the conventional CIFR scheme, a performance improvement of about 0.1 bits/s/Hz can be obtained in the TIFR scheme, and this performance advantage becomes 0.4 bits/s/Hz in ORA scheme. The OPRA scheme, which employs water-filling mechanism in ORA, can further increase the channel capacity of about 0.005 bits/s/Hz as compared to conventional ORA.

The gap between ORA and OPRA (or between TIFR and CIFR) also tends to decreases as $\bar{\gamma}$ increases, as shown in Figure 5. Among all the techniques, the ORA scheme achieves almost the same average capacity as that obtained in the OPRA scheme. However, as compared to the CIFR scheme, a significant performance improvement (about 1 dB to 1.8 dB) is obtained in the TIFR technique. Note that signal transmitted through any wireless link can always be received successfully by the receiver for a high SNR, and in this case, only a limited benefit in terms of channel capacity can be brought by adaptive power allocation.

Figure 6 illustrates the outage probability as a monotonically decreasing function of $\bar{\gamma}$, regardless of whether

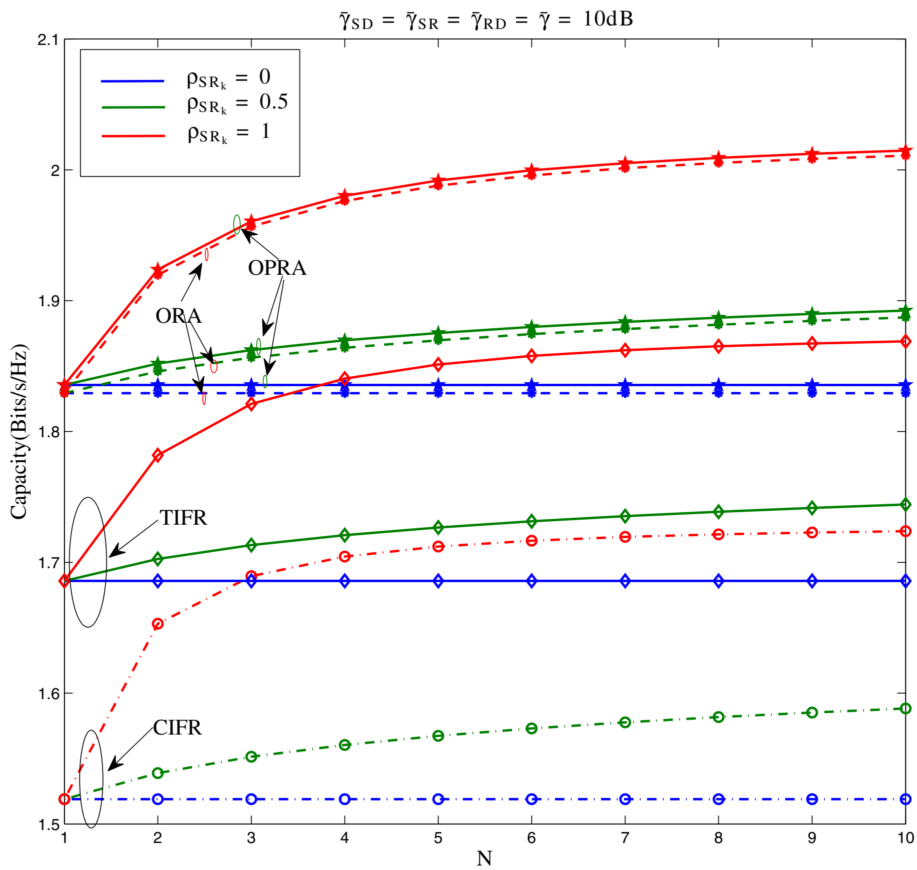

Figure 4 Average channel capacity per unit bandwidth as a function of *N* and ρ_{SR_k} for the i.i.d. links.

$\rho_{SR_k} = 0$ or $\rho_{SR_k} = 1$ (i.e., the spatial diversity gain is proven to be independent of ρ_{SR_k}). It's also shown that the slopes of outage probability for OPRA are steeper than that for TIFR. For the same parameters N and ρ_{SR_k}, both the OPRA and TIFR schemes can obtain their own optimum cut-off SNR, and the outage probability of OPRA is smaller than that of TIFR due to a lower optimum cut-off SNR threshold being utilized in the former. Evidently, OPRA always outperforms the TIFR in terms of channel capacity.

5 Conclusions

The average channel capacity of relay selection in multi-relay cooperative communications systems with outdated CSI was studied, with four classical adaptive transmission techniques over Rayleigh fading links being considered. The approximated closed-form expressions for some important parameters, including PDF, CDF, outage probability, and the average capacity, were also derived. The validity of proposed theoretical approximation on the critical figures of merit, including the optimum cut-off SNR, the outage probability and the capacity, was proven via numerical evaluations, and the theoretical analysis was consistent with the simulation results. It was also shown in the numerical results that some other parameters, such as the number of relays, the channel coefficient, etc, impact the capacity considerably. Numerical results proved that the OPRA technique outperforms the TIFR technique in terms of both the diversity gain and the average channel capacity in the optimum relay selection.

Appendix 1

The probability density function

For each i.i.d. Rayleigh-fading $S \to R_n, n = 1, \ldots, N$ link, the PDF of its estimated SNR, i.e., $f_{\hat{\gamma}_{SR_n}}(\cdot)$, is identical to $f_{\hat{\gamma}_{SR}}(\cdot)$, which is obtain as

$$f_{\hat{\gamma}_{SR}}(y) = \frac{1}{\bar{\gamma}_{SR}} e^{-\frac{y}{\bar{\gamma}_{SR}}}. \tag{23}$$

Likewise, the CDF of the estimated SNR is

$$\begin{aligned} F_{\hat{\gamma}_{SR}}(y) &= \int_0^y f_{\hat{\gamma}_{SR}}(x)dx \\ &= 1 - e^{-\frac{y}{\bar{\gamma}_{SR}}}. \end{aligned} \tag{24}$$

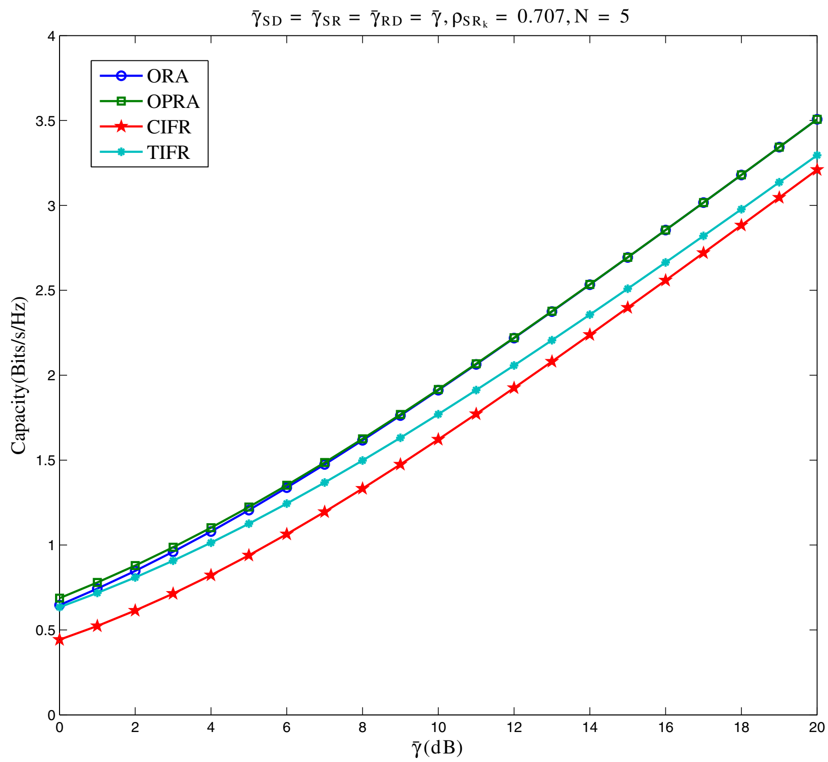

Figure 5 Average channel capacity per unit bandwidth versus the average SNR $\bar{\gamma}$.

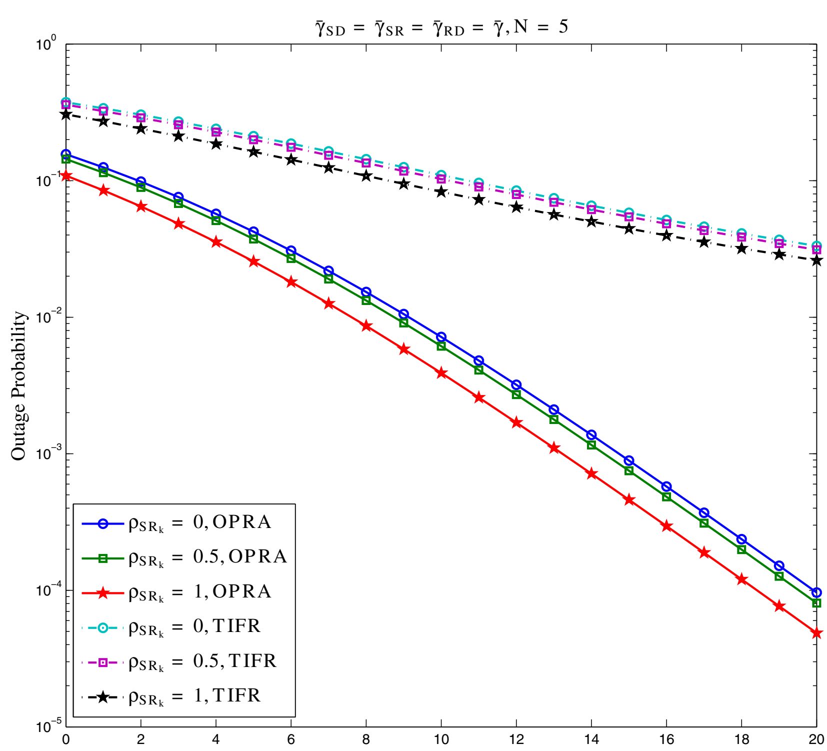

Figure 6 Outage probability of the partial relay selection scheme as a function of $\bar{\gamma}$ and ρ_{SR_k}.

From (1), we have ([24], Equation (31))

$$f_{\hat{\gamma}_{SR_k}}(y) = N[F_{\hat{\gamma}_{SR}}(y)]^{N-1} f_{\hat{\gamma}_{SR}}(y). \tag{25}$$

Based on (2), the conditional PDF of γ_{SR_k} is given by

$$f_{\gamma_{SR_k}|\hat{\gamma}_{SR_k}}(\gamma|y) = \frac{e^{-\frac{\gamma+\rho^2_{SR_k}y}{(1-\rho^2_{SR_k})\bar{\gamma}_{SR}}}}{(1-\rho^2_{SR_k})\bar{\gamma}_{SR}} I_0\left(\frac{2\sqrt{\rho^2_{SR_k}\gamma y}}{(1-\rho^2_{SR_k})\bar{\gamma}_{SR}}\right), \tag{26}$$

with $I_0(\cdot)$ denoting the zeroth order modified Bessel function of the first kind.

Note that

$$f_{\gamma_{SR_k}}(\gamma) = \int_0^\infty f_{\gamma_{SR_k}|\hat{\gamma}_{SR_k}}(\gamma|y) f_{\hat{\gamma}_{SR_k}}(y)dy, \tag{27}$$

by substituting (25) and (26) into (27), $f_{\gamma_{SR_k}}(\gamma)$ can be further represented as

$$f_{\gamma_{SR_k}}(\gamma) = \int_0^\infty \frac{e^{-\frac{\gamma+\rho^2_{SR_k}y}{(1-\rho^2_{SR_k})\bar{\gamma}_{SR}}}}{(1-\rho^2_{SR_k})\bar{\gamma}_{SR}} I_0 \times \left(\frac{2\sqrt{\rho^2_{SR_k}\gamma y}}{(1-\rho^2_{SR_k})\bar{\gamma}_{SR}}\right) N[F_{\hat{\gamma}_{SR}}(y)]^{N-1} f_{\hat{\gamma}_{SR}}(y)dy, \tag{28}$$

where

$$I_0\left(\frac{2\sqrt{\rho^2_{SR_k}\gamma y}}{(1-\rho^2_{SR_k})\bar{\gamma}_{SR}}\right) = \sum_{k=0}^{\infty} \frac{\left\{\frac{1}{4}\left[\frac{2\sqrt{\rho^2_{SR_k}\gamma y}}{\left(1-\rho^2_{SR_k}\right)\bar{\gamma}_{SR}}\right]^2\right\}^k}{k!\,\Gamma(k+1)} = \sum_{k=0}^{\infty} \frac{1}{(k!)^2}\left\{\frac{\rho^2_{SR_k}\gamma y}{\left[\left(1-\rho^2_{SR_k}\right)\bar{\gamma}_{SR}\right]^2}\right\}^k. \tag{29}$$

$[F_{\hat{\gamma}_{SR}}(y)]^{N-1}$ can be derived by using binomial expansion as

$$\left[F_{\hat{\gamma}_{SR}}(y)\right]^{N-1} = \sum_{m=0}^{N-1}\binom{N-1}{m}(-1)^m e^{-\frac{my}{\bar{\gamma}_{SR}}}, \tag{30}$$

where $\binom{N-1}{m} = \frac{(N-1)!}{m!(N-1-m)!}$. Substituting (23), (29), and (30) into (28) we obtain

$$\begin{aligned}
f_{\gamma_{SR_k}}(\gamma) &= \int_0^\infty \frac{e^{-\frac{\gamma+\rho^2_{SR_k}y}{(1-\rho^2_{SR_k})\bar{\gamma}_{SR}}}}{(1-\rho^2_{SR_k})\bar{\gamma}_{SR}} \sum_{k=0}^{\infty}\frac{1}{(k!)^2}\left\{\frac{\rho^2_{SR_k}\gamma y}{\left[\left(1-\rho^2_{SR_k}\right)\bar{\gamma}_{SR}\right]^2}\right\}^k \\
&\quad \times N\sum_{m=0}^{N-1}\binom{N-1}{m}(-1)^m e^{-\frac{my}{\bar{\gamma}_{SR}}}\frac{1}{\bar{\gamma}_{SR}}e^{-\frac{y}{\bar{\gamma}_{SR}}}dy \\
&= \frac{e^{-\frac{\gamma}{(1-\rho^2_{SR_k})\bar{\gamma}_{SR}}}}{(1-\rho^2_{SR_k})\bar{\gamma}_{SR}} \sum_{k=0}^{\infty}\frac{1}{(k!)^2}\left\{\frac{\rho^2_{SR_k}\gamma}{\left[\left(1-\rho^2_{SR_k}\right)\bar{\gamma}_{SR}\right]^2}\right\}^k \\
&\quad \times N\sum_{m=0}^{N-1}\binom{N-1}{m}(-1)^m \frac{1}{\bar{\gamma}_{SR}} \\
&\quad \times \int_0^\infty e^{-\frac{\rho^2_{SR_k}y}{(1-\rho^2_{SR_k})\bar{\gamma}_{SR}}} y^k e^{-\frac{my}{\bar{\gamma}_{SR}}} e^{-\frac{y}{\bar{\gamma}_{SR}}}dy \\
&= N\sum_{m=0}^{N-1}\binom{N-1}{m}(-1)^m \frac{1}{\bar{\gamma}_{SR}} \frac{e^{-\frac{\gamma}{(1-\rho^2_{SR_k})\bar{\gamma}_{SR}}}}{(1-\rho^2_{SR_k})\bar{\gamma}_{SR}} \sum_{k=0}^{\infty}\frac{1}{(k!)^2} \\
&\quad \times \left\{\frac{\rho^2_{SR_k}\gamma}{\left[\left(1-\rho^2_{SR_k}\right)\bar{\gamma}_{SR}\right]^2}\right\}^k \\
&\quad \times \int_0^\infty e^{-\left(\frac{\rho^2_{SR_k}}{(1-\rho^2_{SR_k})\bar{\gamma}_{SR}}+\frac{m+1}{\bar{\gamma}_{SR}}\right)y} y^k dy \\
&= N\sum_{m=0}^{N-1}\binom{N-1}{m}(-1)^m \frac{1}{\bar{\gamma}_{SR}} \frac{e^{-\frac{\gamma}{(1-\rho^2_{SR_k})\bar{\gamma}_{SR}}}}{(1-\rho^2_{SR_k})\bar{\gamma}_{SR}} \sum_{k=0}^{\infty}\frac{1}{(k!)^2} \\
&\quad \times \left\{\frac{\rho^2_{SR_k}\gamma}{\left[\left(1-\rho^2_{SR_k}\right)\bar{\gamma}_{SR}\right]^2}\right\}^k \int_0^\infty e^{-ty}y^k dy,
\end{aligned} \tag{31}$$

where $t = \left(\frac{\rho^2_{SR_k}}{(1-\rho^2_{SR_k})\bar{\gamma}_{SR}}+\frac{m+1}{\bar{\gamma}_{SR}}\right) = \frac{m(1-\rho^2_{SR_k})+1}{(1-\rho^2_{SR_k})\bar{\gamma}_{SR}}$, and $\int_0^\infty e^{-ty}y^k dy = \frac{k!}{t^{k+1}} = \frac{k!\left[(1-\rho^2_{SR_k})\bar{\gamma}_{SR}\right]^{k+1}}{\left[m(1-\rho^2_{SR_k})+1\right]^{k+1}}$. Therefore, (31) can be further simplify as

$$\begin{aligned}
f_{\gamma_{SR_k}}(\gamma) &= N\sum_{m=0}^{N-1}\binom{N-1}{m}(-1)^m \frac{1}{\bar{\gamma}_{SR}} \frac{e^{-\frac{\gamma}{(1-\rho^2_{SR_k})\bar{\gamma}_{SR}}}}{(1-\rho^2_{SR_k})\bar{\gamma}_{SR}} \\
&\quad \times \sum_{k=0}^{\infty}\frac{1}{k!}\left\{\frac{\rho^2_{SR_k}\gamma}{\left[\left(1-\rho^2_{SR_k}\right)\bar{\gamma}_{SR}\right]^2}\frac{(1-\rho^2_{SR_k})\bar{\gamma}_{SR}}{m(1-\rho^2_{SR_k})+1}\right\}^k \\
&\quad \times \frac{(1-\rho^2_{SR_k})\bar{\gamma}_{SR}}{m(1-\rho^2_{SR_k})+1}
\end{aligned}$$

$$
\begin{aligned}
&= N\sum_{m=0}^{N-1}\binom{N-1}{m}(-1)^m \frac{1}{\bar{\gamma}_{SR}} \frac{1}{m(1-\rho_{SR_k}^2)+1} e^{-\frac{\gamma}{(1-\rho_{SR_k}^2)\bar{\gamma}_{SR}}} \\
&\quad \times \sum_{k=0}^{\infty}\frac{1}{k!}\left\{\frac{\rho_{SR_k}^2\gamma}{\left(1-\rho_{SR_k}^2\right)\bar{\gamma}_{SR}(m(1-\rho_{SR_k}^2)+1)}\right\}^k \\
&= N\sum_{m=0}^{N-1}\binom{N-1}{m}(-1)^m \frac{1}{\bar{\gamma}_{SR}} \frac{1}{m(1-\rho_{SR_k}^2)+1} \\
&\quad \times e^{-\frac{\gamma}{(1-\rho_{SR_k}^2)\bar{\gamma}_{SR}}} e^{\frac{\rho_{SR_k}^2\gamma}{\left(1-\rho_{SR_k}^2\right)\bar{\gamma}_{SR}\left(m\left(1-\rho_{SR_k}^2\right)+1\right)}} \\
&= N\sum_{m=0}^{N-1}\frac{(-1)^m\binom{N-1}{m}e^{-\frac{(m+1)}{(m(1-\rho_{SR_k}^2)+1)\bar{\gamma}_{SR}}\gamma}}{(m(1-\rho_{SR_k}^2)+1)\bar{\gamma}_{SR}}.
\end{aligned} \tag{32}
$$

The CDF of γ_{SR_k} is then given by

$$
\begin{aligned}
F_{\gamma_{SR_k}}(\gamma) &= \int_0^{\gamma} f_{\gamma_{SR_k}}(\gamma)d\gamma \\
&= N\sum_{m=0}^{N-1}\frac{(-1)^m\binom{N-1}{m}}{(m+1)}\left(1-e^{-\frac{(m+1)}{(m(1-\rho_1^2)+1)\bar{\gamma}_{SR}}\gamma}\right).
\end{aligned} \tag{33}
$$

Also note that the CDF of SNR for the $R_k \rightarrow D$ link is derived as $f_{\gamma_{R_kD}}(x) = \frac{1}{\bar{\gamma}_{RD}}e^{-\frac{x}{\bar{\gamma}_{RD}}}$, and the equivalent SNR of $S \rightarrow R_k \rightarrow D$ link can be denoted by $\gamma_k = \frac{\gamma_{SR_k}\gamma_{R_kD}}{1+\gamma_{SR_k}+\gamma_{R_kD}}$, the CDF of γ_k can therefore be derived as ([24], Equation (17))

$$
\begin{aligned}
F_{\gamma_k}(\gamma) &= P_r[\gamma_k < \gamma] \\
&= \int_0^{\infty} P_r\left[\frac{\gamma_{SR_k}x}{1+\gamma_{SR_k}+x} < \gamma\right] f_{\gamma_{R_kD}}(x)dx \\
&= \int_0^{\gamma} P_r\left[\frac{\gamma_{SR_k}x}{1+\gamma_{SR_k}+x} < \gamma\right] f_{\gamma_{R_kD}}(x)dx \\
&\quad + \int_{\gamma}^{\infty} P_r\left[\frac{\gamma_{SR_k}x}{1+\gamma_{SR_k}+x} < \gamma\right] f_{\gamma_{R_kD}}(x)dx \\
&= \int_0^{\gamma} f_{\gamma_{R_kD}}(x)dx + \int_{\gamma}^{\infty} P_r\left[\gamma_{SR_k} < \frac{(1+x)\gamma}{x-\gamma}\right] \\
&\quad \times f_{\gamma_{R_kD}}(x)dx \\
&= \int_0^{\gamma}\frac{1}{\bar{\gamma}_{RD}}e^{-\frac{x}{\bar{\gamma}_{RD}}}dx + \int_{\gamma}^{\infty} F_{\gamma_{SR_k}}\left(\frac{(1+x)\gamma}{x-\gamma}\right) \\
&\quad \times f_{\gamma_{R_kD}}(x)dx \\
&= 1-e^{-\frac{\gamma}{\bar{\gamma}_{RD}}} + N\sum_{m=0}^{N-1}\frac{(-1)^m\binom{N-1}{m}}{(m+1)\bar{\gamma}_{RD}} \\
&\quad \times \int_{\gamma}^{\infty}\left(1-e^{-\frac{(m+1)}{(m(1-\rho_1^2)+1)\bar{\gamma}_{SR}}\frac{\gamma x+\gamma}{x-\gamma}}\right)e^{-\frac{x}{\bar{\gamma}_{RD}}}dx \\
&= 1-e^{-\frac{\gamma}{\bar{\gamma}_{RD}}} + N\sum_{m=0}^{N-1}\frac{(-1)^m\binom{N-1}{m}}{(m+1)\bar{\gamma}_{RD}} \\
&\quad \times \left(\int_{\gamma}^{\infty} e^{-\frac{x}{\bar{\gamma}_{RD}}}dx \right. \\
&\quad \left. - \int_{\gamma}^{\infty} e^{-\frac{(m+1)}{(m(1-\rho_1^2)+1)\bar{\gamma}_{SR}}\frac{\gamma x+\gamma}{x-\gamma}} e^{-\frac{x}{\bar{\gamma}_{RD}}}dx\right) \\
&= 1-N\sum_{m=0}^{N-1}\frac{(-1)^m\binom{N-1}{m}}{(m+1)\bar{\gamma}_{RD}} \\
&\quad \int_{\gamma}^{\infty} e^{-\frac{(m+1)}{(m(1-\rho_1^2)+1)\bar{\gamma}_{SR}}\frac{\gamma x+\gamma}{x-\gamma}} e^{-\frac{x}{\bar{\gamma}_{RD}}}dx \\
&= 1-N\sum_{m=0}^{N-1}\frac{(-1)^m\binom{N-1}{m}}{(m+1)\bar{\gamma}_{RD}} e^{-\left(\frac{m+1}{(m(1-\rho_1^2)+1)\bar{\gamma}_{SR}}+\frac{1}{\bar{\gamma}_{RD}}\right)x} \\
&\quad \times \int_0^{\infty} e^{-\frac{\eta}{4x}-\xi x}dx,
\end{aligned} \tag{34}
$$

where $P_r\{\cdot\}$ stands for the probability distribution, $\eta = \frac{4(m+1)(\gamma^2+\gamma)}{[m(1-\rho_1^2)+1]\bar{\gamma}_{SR}}$, and $\xi = \frac{1}{\bar{\gamma}_{RD}}$. Solving (34) using ([29], Equation (3.324.1)), we obtain

$$
\begin{aligned}
F_{\gamma_k}(\gamma) &= 1-N\sum_{m=0}^{N-1}\frac{(-1)^m\binom{N-1}{m}}{(m+1)\bar{\gamma}_{RD}} e^{-\left(\frac{m+1}{(m(1-\rho_1^2)+1)\bar{\gamma}_{SR}}+\frac{1}{\bar{\gamma}_{RD}}\right)x} \\
&\quad \times \sqrt{\frac{\eta}{\xi}}K_1(\sqrt{\eta\xi}) \\
&= 1-N\sum_{m=0}^{N-1}\frac{(-1)^m\binom{N-1}{m}}{(m+1)\bar{\gamma}_{RD}} e^{-\mathcal{A}_m x} \\
&\quad \times \sqrt{\frac{\eta}{\xi}}K_1(\sqrt{\eta\xi}),
\end{aligned} \tag{35}
$$

where $K_1(x) \approx \frac{1}{x}$ ([30], Equation (9.6.9)).

Furthermore, the CDF of γ_k for partial relay selection scheme is given by

$$
F_{\gamma_k}(\gamma) \approx 1-N\sum_{n=0}^{N-1}\frac{(-1)^m\binom{N-1}{m}}{(m+1)}e^{-\mathcal{A}_m\gamma}, \tag{36}
$$

which leads to

$$
f_{\gamma_k}(\gamma) = N\sum_{m=0}^{N-1}\frac{(-1)^m\binom{N-1}{m}}{m+1}\cdot\mathcal{A}_m e^{-\mathcal{A}_m\gamma}. \tag{37}
$$

By performing Laplace transform on (37), it yields

$$
\mathcal{M}_{\gamma_k}(s) = N\sum_{m=0}^{N-1}\frac{(-1)^m\binom{N-1}{m}}{m+1}\cdot\frac{\mathcal{A}_m}{s+\mathcal{A}_m}. \tag{38}
$$

Moreover,

$$\begin{aligned}\mathcal{M}_{\gamma_{SD}}(s) &= \int_0^{\infty} e^{-s\gamma} f_{\gamma_{SD}}(\gamma) d\gamma \\ &= \frac{1/\bar{\gamma}_{SD}}{s + 1/\bar{\gamma}_{SD}}\end{aligned}, \tag{39}$$

where $f_{\gamma_{SD}}(\gamma) = \frac{1}{\bar{\gamma}_{SD}} e^{-\frac{\gamma}{\bar{\gamma}_{SD}}}$.

The MGF of $\gamma_{\text{total}} = \gamma_{SD} + \gamma_k$ is given by

$$\mathcal{M}_{\gamma_{\text{total}}}(s) = \mathcal{M}_{\gamma_{SD}}(s)\mathcal{M}_{\gamma_k}(s), \tag{40}$$

and by substituting (38) and (39) into (40), we obtain

$$\mathcal{M}_{\gamma_{\text{total}}}(s) = N \sum_{m=0}^{N-1} \frac{(-1)^m \binom{N-1}{m}}{m+1} \cdot \frac{\mathcal{A}_m}{s + \mathcal{A}_m} \frac{1/\bar{\gamma}_{SD}}{s + 1/\bar{\gamma}_{SD}}. \tag{41}$$

After performing an inverse Laplace transform on (41), the PDF of γ_{total} can be derived as

$$\begin{aligned} f_{\gamma_{\text{total}}}(\gamma) =& N \sum_{m=0}^{N-1} \frac{(-1)^m \binom{N-1}{m}}{m+1} \\ & \cdot \frac{\mathcal{A}_m}{\mathcal{A}_m \bar{\gamma}_{SD} - 1} \left(e^{-\gamma/\bar{\gamma}_{SD}} - e^{-\mathcal{A}_m \gamma} \right). \end{aligned} \tag{42}$$

Appendix 2

Proof of monotonically decreasing property of Y_{γ_0}

By takeing partial derivative of Y_{γ_0} with respect to γ_0, it leads to

$$\begin{aligned} \frac{dY_{\gamma_0}}{d\gamma_0} &= N \sum_{m=0}^{N-1} \frac{(-1)^m \binom{N-1}{m}}{m+1} \cdot \frac{\mathcal{A}_m}{\mathcal{A}_m \bar{\gamma}_{SD} - 1} \\ & \times \left\{ \frac{e^{-\mathcal{A}_m \gamma_0}}{(\mathcal{A}_m \gamma_0)^2} - \left(\frac{\bar{\gamma}_{SD}}{\gamma_0} \right)^2 e^{-\gamma_0/\bar{\gamma}_{SD}} \right\}. \end{aligned} \tag{43}$$

Now let's prove the monotonically decreasing property of Y_{γ_0} in terms of γ_0.

1. *If* $\frac{1}{(\mathcal{A}_m \gamma_0)^2} e^{-\mathcal{A}_m \gamma_0} - \left(\frac{\bar{\gamma}_{SD}}{\gamma_0} \right)^2 e^{-\gamma_0/\bar{\gamma}_{SD}} > 0$, it yields $e^{-(\mathcal{A}_m - 1/\bar{\gamma}_{SD})\gamma_0} > (\mathcal{A}_m \bar{\gamma}_{SD})^2$, which leads to

$$\begin{cases} \mathcal{A}_m - 1/\bar{\gamma}_{SD} > 0 \ \& \ e^{-(\mathcal{A}_m - 1/\bar{\gamma}_{SD})\gamma_0} < 1 \\ \text{if } \mathcal{A}_m \bar{\gamma}_{SD} > 1 \ (\textbf{contrary to assumption}), \\ \mathcal{A}_m - 1/\bar{\gamma}_{SD} \le 0 \ \& \ e^{-(\mathcal{A}_m - 1/\bar{\gamma}_{SD})\gamma_0} \ge 1 \\ \text{if } \mathcal{A}_m \bar{\gamma}_{SD} \le 1 \ (\textbf{consistent with assumption}); \end{cases} \tag{44}$$

2. *If* $\frac{1}{(\mathcal{A}_m \gamma_0)^2} e^{-\mathcal{A}_m \gamma_0} - \left(\frac{\bar{\gamma}_{SD}}{\gamma_0} \right)^2 e^{-\gamma_0/\bar{\gamma}_{SD}} \le 0$, it yields $e^{-(\mathcal{A}_m - 1/\bar{\gamma}_{SD})\gamma_0} \le (\mathcal{A}_m \bar{\gamma}_{SD})^2$, which leads to

$$\begin{cases} \mathcal{A}_m - 1/\bar{\gamma}_{SD} \ge 0 \ \& \ e^{-(\mathcal{A}_m - 1/\bar{\gamma}_{SD})\gamma_0} \le 1 \\ \text{if } \mathcal{A}_m \bar{\gamma}_{SD} \ge 1 \ (\textbf{consistent with assumption}), \\ \mathcal{A}_m - 1/\bar{\gamma}_{SD} < 0 \ \& \ e^{-(\mathcal{A}_m - 1/\bar{\gamma}_{SD})\gamma_0} > 1 \\ \text{if } \mathcal{A}_m \bar{\gamma}_{SD} < 1 \ (\textbf{contrary to assumption}). \end{cases} \tag{45}$$

Evidently, Y_{γ_0} is a monotonically decreasing function of γ_0, and $\lim_{\gamma_0 \to 0^+} (Y_{\gamma_0}) = +\infty$, $\lim_{\gamma_0 \to +\infty} (Y_{\gamma_0}) = -1$.

Endnote

[a]Note that the other combining methods, such as Equal Gain Combining (EGC), can also be used in the proposed relay selection method.

Competing interests

The authors declare that they have no competing interests.

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