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Combining cooperative diversity and network coding in uplink multi-source multi-relay networks

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Abstract

In this paper, a new scheme of combining cooperative diversity with network coding is proposed for wireless uplink multi-source multi-relay networks. The existing network-coded cooperative scheme always conducts network coding operation at relays in moderate-to-high signal-to-noise ratio region. Distinct from it, the proposed scheme determines either a direct cooperative mode or a network-coded cooperative mode at relays according to the channel qualities of the broadcast phase. Compared with the existing network-coded cooperative scheme, the proposed scheme achieves a performance gain in terms of both diversity order and system ergodic capacity without extra bandwidth resource consumption. Both theoretical analysis and simulations verify the validity and superiority of the proposed cooperative scheme.

1 Introduction

Cooperative transmission has been considered a bandwidth-efficient and low-cost method to combat the channel fading in wireless communication scenarios [1]. Moreover, cooperative transmission can achieve cooperative diversity gain through the way that some idle users or relay nodes help the data transmission of other users in the wireless communication system.

There have existed tremendous literatures on the study of cooperative schemes for multi-relay networks to achieve a desirable cooperative diversity gain [2-4]. However, most of the existing cooperative schemes suffer the loss of ergodic capacity. On the other hand, network coding [5,6] has emerged as a promising technique that is well known for its capability to increase system throughput. Network coding technique encourages a relay to forward the mixture of its observations to the destination. Because of the broadcast nature of radio signals, it is natural to combine network coding method with cooperative diversity in wireless cooperative networks for better spectral efficiency.

There have existed numerous literatures about the cooperative transmission based on network coding techniques. The combination of network coding and cooperative diversity has been studied by [7], which demonstrates that a full cooperative diversity can be achieved by cooperative scheme of [7], based on network coding with less loss of ergodic capacity. The diversity network codes over finite fields has been designed for multiple-user cooperative communication networks by [8], which can achieve higher diversity order than the schemes without network coding or with binary network coding. Deterministic and random network coding schemes have been designed for a cooperative communication setup with multiple sources and destinations [9]. It shows that the schemes in [9] perform better than conventional cooperation in terms of both the diversity-multiplexing tradeoff (DMT) and the diversity order. A generalized dynamic network codes was proposed by [10] for a network consisting of M users sending independent information to a common base station, which offers a much better tradeoff between the rate and diversity order compared to the dynamic network codes [8]. A binary network coding and space-time network coding are applied to a multi-source multi-relay cooperative wireless network in [11] and [12].

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However, in [7], a network-coded operation is always needed in moderate-to-high signal-to-noise ratio (SNR) regime regardless of the channel qualities of its user broadcast phase. Actually, the broadcast phase can be seen as a multiple-access channel that consisted of two users and one destination in [7]. According to the multiple-access channel capacity region [13], if all of the uplink channels are good enough, the destination can decode each user's information correctly with only one time slot. Otherwise, only the user whose channel quality cannot support its current transmission rate is helped by the relay nodes to communicate with the destination. That is, one extra network-coded operation of both users' messages is not always necessary by the cooperative scheme of [7] in moderate-to-high SNR regime.

Motivated by the above observation, a new cooperative scheme is proposed in this paper. A formal proof that the network-coded cooperative scheme can achieve the largest ergodic throughput with two sources involved in the cooperative transmission has been provided [7]. So, in this paper, we consider a multi-source multi-relay network, where all L relays and $M - 2$ idle users help arbitrary two-user out of all the M users that communicate with the destination during a cooperative process. Further, in the next cooperation, another two-user can be serviced by the L relays and the $M - 2$ idle users. The idle users are hereinafter referred to as relay. It can be seen that in fact there are $L + M - 2$ relays in a cooperative process. In addition, two serviced users can be selected in the following way. Firstly, M users are grouped in pair randomly, and the round robin scheduling is then applied among the groups. In this way, the M users can communicate with the destination with the help of relays.

The proposed scheme consists of broadcast phase and opportunistic network coding cooperation phase in the data transmission process, where two users transmit their signals simultaneously at the broadcast phase and then a direct cooperative mode or a network-coded cooperative mode at relays will be determined according to the channel qualities of the broadcast phase. Compared with the network-coded cooperative scheme, the proposed scheme achieves a performance gain in terms of both diversity order and system ergodic capacity without extra bandwidth resource consumption. Furthermore, some system overheads, which are used by network coding operation, can also be reduced.

The rest of this paper is organized as follows. Section 2 describes a uplink multi-relay system model and the key motivation of this paper. The proposed opportunistic network-coded cooperative scheme is given in Section 3. Section 4 analyzes the performance of the proposed scheme in terms of diversity gain. Numerical simulations and comparisons are shown in Section 5. Finally, the conclusions are provided in Section 6.

1.1 Notations

Superscripts $[\cdot,\cdot]^T$ and $[\cdot,\cdot]^H$ represent the transpose and Hermitian of its arguments, respectively.

2 System model and motivation

2.1 System model

Consider a wireless uplink multi-relay cooperative network which consists of M users, L relays, and one destination. M users transmit individual information to the destination with the help of L relays. Each of the nodes is equipped with a single antenna and has unit transmit power. We assume that all the channels in the network are flat Rayleigh faded with additive white Gaussian noise (AWGN), and all channel fading coefficients are zero-mean independent circularly symmetric complex Gaussian (ZMCSCG) random variables with variance σ^2. The half-duplex mode is employed in this paper, i.e., each node cannot transmit and receive at the same time, and time synchronization is assumed among all nodes including the users, relays, and destination. Note that according to Theorem 1 in [7], the largest ergodic sum rate can be achieved, where there are only two users participating in the cooperation. So, it only focuses on the cooperative scenario, where only two users out of the M users are involved into the communication during one cooperative process. In this paper, we consider an uplink cooperative network, where there are M users participating in the transmission to the destination with the help of L relays. For fair comparison, during one cooperative process, only two users are serviced by L relays and $M - 2$ idle users as shown in Figure 1. During the next cooperative process, another two-user can be serviced by the L relays and $M - 2$ idle users. Therefore, only one best relay is needed in the proposed network-coded cooperation phase according to [7].

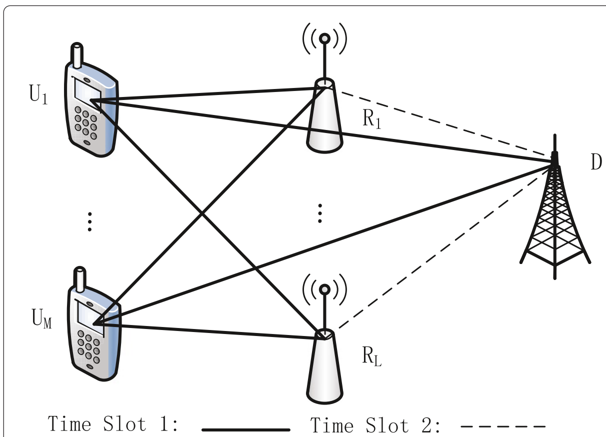

Figure 1 System model.

2.2 Motivation of our work

In [7], the authors combine the network coding and cooperative diversity in wireless uplink transmission network as shown in Figure 1. It has been proved that in high-SNR region, there always existed at least one qualified relay node which can decode both users' messages correctly. Then, a cooperative scheme which incorporates the network coding into the cooperative network is proposed. The proposed scheme by [7] consists of broadcast phase and cooperative phase. In the broadcast phase, two users transmit their messages simultaneously, and all the L relay nodes, the $M-2$ idle users, and the destination listen. In the cooperative phase, a best relay out of the N qualified relay nodes is selected to retransmit the network symbols of its received users' information.

2.2.1 Multi-access channel capacity

In the cooperative scheme proposed by [7], the two users transmit their messages simultaneously in the broadcast phase. This broadcast process is actually a multi-access channel, where two users communicate with a common destination by sharing the same bandwidth resource. Therefore, the channel capacity region can be written as

$$\mathcal{R}_1 \leq \log(1+\rho|h_{1D}|^2) \tag{1}$$

$$\mathcal{R}_2 \leq \log(1+\rho|h_{2D}|^2) \tag{2}$$

$$\mathcal{R}_1+\mathcal{R}_2 \leq \log(1+\rho\sum_{m=1}^{2}|h_{mD}|^2). \tag{3}$$

2.2.2 Motivation

Outage event is defined as one that the current channel condition cannot support its user's transmit rate, i.e., $\mathcal{C}_m < \mathcal{R}_m \cup \mathcal{C}_{\text{sumrate}} < \sum_{m=1}^{2} \mathcal{R}_m$, where $\mathcal{C}_m = \log(1+\rho|h_{mD}|^2)$, $m=1,2$ and $\mathcal{C}_{\text{sumrate}} = \log(1+\rho\sum_{m=1}^{2}|h_{mD}|^2)$. According to the above analysis of channel capacity region, we can deduce that no outage happens to both users at the same time when the channel condition is good enough. The deduction motivates us that the network coding operation is not always necessary in high-SNR region, and the relay only needs to retransmit the outage user's message. In addition, if no outage happens in the broadcast phase, one transmission time slot can just satisfy the communication requirement. In this paper, an opportunistic network coding cooperative scheme is proposed by taking full account of the channel qualities of the broadcast phase. The detail of the proposed cooperative scheme is described in the following section.

3 The proposed transmission scheme description

There consists of initialization phase and cooperation mode selection phase in the proposed scheme to finish the uplink communication. In the initialization phase, the destination estimates the channel state information (CSI) from two users. Each relay estimates the CSI between two users and itself. In the cooperative phase, two users communicate with the destination according to the scheduling strategy decided by the destination in the initialization phase. Then, the best relay node retransmits the outage user's information. The scheduling of the best relay can be accomplished in a distributed way by adjusting each relay's backoff time that is inversely proportional to the quality of its relay's destination channel $h_{r_lD}|^2$ [14]. The detailed transmission process and signal format are described as follows.

3.1 Initialization

Before data transmission, each user broadcasts the training information in turns. After that, the destination D, the $M-2$ idle users, and the L relays use such training information to accomplish their incoming CSI estimation from two users, respectively. Based on the local CSIs from user U_1 and U_2, each relay can determine whether or not it can successfully decode both users' information based on the criterion of the capacity region for multiple-access channels [13]:

$$\sum_{m\in\mathcal{S}} \mathcal{R}_m \leq \log(1+\rho\sum_{m\in\mathcal{S}}|h_{mr_l}|^2),\ \ \forall\mathcal{S}\subseteq\{1,2\}, \quad l=1,\cdots,L+M-2 \tag{4}$$

where $\mathcal{R}_m$ denotes the targeted data rate for the mth user and ρ is the transmit SNR. As long as one relay's incoming channel $h_{mr_l}, m=1,2$, can support all of the inequalities in (4), the relay will be marked as a qualified relay that is able to decode both users' information correctly. Then the qualified relays can be used to participate into the cooperative process to help two users' communication with destination. Denote the total number of the qualified relays as N.

3.2 Opportunistic network-coded cooperative phase

In the second phase, the destination decides which kind of communication mode to be used according to the CSIs coming from the two users. The detailed process is shown as follows:

(1) If each user's transmit rate satisfies

$$\begin{aligned}\mathcal{R}_1 &\leq \mathcal{C}_1\\ \mathcal{R}_2 &\leq \mathcal{C}_2\\ \mathcal{R}_1+\mathcal{R}_2 &\leq \mathcal{C}_{\text{sumrate}},\end{aligned} \tag{5}$$

i.e., no outage happens under current channel condition, two users broadcast their messages simultaneously to the destination. Also, the received signals at the destination can be formulated as

$$y_{D1} = h_{1D}s_1 + h_{2D}s_2 + n_1,$$

where s_m is the mth users' transmitted signal, and n_1 denotes the AWGN with zero mean and variance N_0. Then, successive decoding- and interference cancelation (SIC)-based approaches [13] can be applied by the destination to decode the two users' signals correctly. We can see that only one time slot is needed for two users' communication, and the system sum rate capacity is equal to

$$\mathcal{C}_{\text{sumrate}} = \log(1 + \rho \sum_{m=1}^{2} |h_{mD}|^2). \tag{6}$$

(2) If each user's transmit rate satisfies

$$\begin{aligned} \mathcal{R}_1 &> \mathcal{C}_1 \\ \mathcal{R}_2 &\leq \mathcal{C}_2 \\ \mathcal{R}_1 + \mathcal{R}_2 &\leq \mathcal{C}_{\text{sumrate}}, \end{aligned} \tag{7}$$

i.e., user U_1 outages under the current channel condition, the following transmission will be carried out.
In the first time slot, two users broadcast their messages simultaneously, and the destination and $L+M-2$ relays listen. The received signals at the destination $L+M-2$ relays can be formulated as

$$y_{D1} = \sum_{m=1}^{2} h_{mD} s_m + n_1, \tag{8}$$

$$y_{r_l} = \sum_{m=1}^{2} h_{mr_l} s_m + n_{r_l}, \ \ \forall l \subseteq \{1, \cdots, L+M-2\}, \tag{9}$$

where n_{r_l} denotes the AWGN at the receiver of the lth relay.
In the subsequent time slot, the destination selects the best relay R out of the N qualified relays, which satisfies $R = \arg \max_{n=1,\cdots,N} |h_{R_n D}|^2$, to retransmit the outage user's signal s_1 to the destination. If no qualified relay exists, the user will retransmit its original signal s_1 again. The received signal at the destination in the retransmission phase can be expressed as

$$y_{D2} = h'_D s_1 + n_2,$$

where $h'_D = h_{RD}$ or h'_{1D}, h_{RD} denotes the channel fading coefficient from the best relay R to the destination, h'_{1D} denotes the channel fading coefficient from outage user U_1 to the destination in the retransmitting time slot, and n_2 is the AWGN at the destination in the retransmitting time slot.

(3) Outage only occurs to user U_2 when the transmit rates satisfy

$$\begin{aligned} \mathcal{R}_1 &\leq \mathcal{C}_1 \\ \mathcal{R}_2 &> \mathcal{C}_2 \\ \mathcal{R}_1 + \mathcal{R}_2 &\leq \mathcal{C}_{\text{sumrate}}. \end{aligned} \tag{10}$$

Then, a similar transmission process with case (2) can be employed as mentioned above. In the retransmitting time slot, the received signal at the destination can be expressed as

$$y_{D2} = h'_D s_2 + n_2,$$

where $h'_D = h_{RD}$ or h'_{2D}, h_{RD} denotes the channel fading coefficient from the best relay to the destination, and h'_{2D} denotes the channel fading coefficient from the outage user U_2 to the destination in the retransmitting time slot.
Note that due to $\mathcal{C}_{\text{sumrate}} \leq \mathcal{C}_1 + \mathcal{C}_2$, it is impossible that both users U_1 and U_2 occur to outage when their transmission rates satisfy $\mathcal{R}_1 + \mathcal{R}_2 \leq \mathcal{C}_{\text{sumrate}}$.
To sum up, if user U_m, $m = 1, 2$, occurs to outage, the received signal after two time slots is written as

$$y_{D1} = h_{1D} s_1 + h_{2D} s_2 + n_1, \tag{11}$$

$$y_{D2} = h'_D s_m + n_2. \tag{12}$$

Here, we define $\mathrm{y} = [y_{D1}, y_{D2}]^T$, $\mathrm{s} = [s_1, s_2]^T$, and $\mathrm{n} = [n_1, n_2]^T$. Then Equations (11) and (12) can be rewritten as

$$\mathrm{y} = \begin{bmatrix} h_{1D} & h_{2D} \\ h'_D & 0 \end{bmatrix} \mathrm{s} + \mathrm{n} := \mathrm{H}_1 \mathrm{s} + \mathrm{n}, \ U_1 \text{ outages},$$

$$\mathrm{y} = \begin{bmatrix} h_{1D} & h_{2D} \\ 0 & h'_D \end{bmatrix} \mathrm{s} + \mathrm{n} := \mathrm{H}_2 \mathrm{s} + \mathrm{n}, \ U_2 \text{ outages},$$

which can be seen as a virtual multiple-input multiple-out (MIMO) model. Based on the MIMO channel capacity formula $\log\det(\mathbf{I}_2 + \rho \mathbf{H}\mathbf{H}^H)$, the system sum rate capacity in the case of (2) or (3) can be calculated as

$$\mathcal{C}_{CO} = \log \left\{ \left(1 + \rho \sum_{m=1}^{2} |h_{mD}|^2\right) (1 + \rho |h_{RD}|^2) - \rho^2 |h_{mD}|^2 |h_{RD}|^2 \right\}, m = 1 \text{ or } 2. \tag{13}$$

(4) If the total system capacity cannot support the current transmission rates, i.e., $\mathcal{R}_1 + \mathcal{R}_2 > \mathcal{C}_{\text{sumrate}}$, the cooperative scheme in [7] will be employed, where a round robin scheduling is applied to two users while no qualified relay exists; otherwise, a network coding-based approach is applied for user scheduling. The round robin scheduling is that two

users broadcast their individual messages in turns without the help of the relays, and after two time slots, the received signals at the destination can be formulated as

$$y_{D1} = h_{1D}s_1 + n_1,$$
$$y_{D2} = h_{2D}s_2 + n_2.$$

If there is a qualified relay, a network coding-based approach is proposed to assist user communication with two time slots. In the first time slot, two users broadcast their messages simultaneously, and the destination and $L+M-2$ relays listen. The received signals at the destination and $L+M-2$ relays have the same format as Equations (8) and (9). In the second time slot, the best qualified relay R forwards the mixture signals of its observations s_1 and s_2 in moderate-to-high SNR region. The received signal at the destination is expressed as

$$y_{D2} = h_{RD}(\gamma_1\hat{s}_1 + \gamma_2\hat{s}_2) + n_3,$$

where $\gamma_m, m = 1, 2$, is the network coding coefficient, and n_3 denotes the AWGN at the receiver. Note that for fair comparison, we employ the same network coding method as that of [7], where the network coding vector Γ_R is generated as a unit vector which satisfies [7]

$$\begin{cases} \Gamma_R \perp \Gamma_0, & R \in \{1, \cdots, N\} \\ \Gamma_i^H \Gamma_i = 1, & i = 0 \text{ or } R \end{cases} \tag{14}$$

where $\perp$ denotes orthogonality, $\Gamma_R = [\gamma_1, \gamma_2]^T$, and $\Gamma_0 = [h_{1D}, h_{2D}]^T / \sqrt{\sum_{m=1}^{2} |h_{mD}|^2}$. Because of the orthogonality feature of the network coding vector, the system sum rate capacity in the network-coded cooperative mode can be obtained directly from [7]

$$\mathcal{C}_{\text{NCCO}} = \log\left\{\left(1 + \rho\sum_{m=1}^{2}|h_{mD}|^2\right)\left(1 + \rho|h_{RD}|^2\right)\right\}. \tag{15}$$

4 Diversity order analysis of the proposed scheme

Diversity gain is one of the most important performance measure for a cooperative communication network. In this section, a brief analysis on diversity gain of the proposed scheme is given. The analysis result shows that the proposed scheme yields a diversity order of $L+M$, while the existing network-coded cooperative scheme proposed in [7] can only achieve a diversity order of $L+1$.

For a wireless communication system, the diversity gain is defined as [15]

$$d = -\lim_{\rho\to\infty} \frac{\log P_{\text{out}}}{\log \rho}, \tag{16}$$

where P_{out} is the system outage probability. Equation (16) can also be written as $P_{\text{out}} \doteq \rho^{-d}$ in an exponential equality [15].

The transmission diagram of the proposed opportunistic network-coded cooperative scheme can be summarized as a form of flow diagram in high-SNR region shown in Figure 2. Note that in terms of the definition of the diversity order, it only relates to the outage probability in the high-SNR region. So, only the transmission diagram in high-SNR region is shown here, and the transmission process in low-SNR region can refer to the description in Section 3.

According to the law of total probability, the system outage probability of the proposed scheme in high-SNR region can be formulated as

$$\begin{aligned} P_{\text{out}} &= Pr(\mathcal{C}_{\text{sumrate}} < 2\mathcal{R})Pr(\mathcal{O}_1|\mathcal{C}_{\text{sumrate}} < 2\mathcal{R}) \\ &\quad + Pr(\mathcal{C}_{\text{sumrate}} \ge 2\mathcal{R})Pr(\mathcal{O}_2|\mathcal{C}_{\text{sumrate}} \ge 2\mathcal{R}), \end{aligned} \tag{17}$$

where $\mathcal{O}_1$ and $\mathcal{O}_2$ represent the outage events under the condition $\mathcal{C}_{\text{sumrate}} < 2\mathcal{R}$ and $\mathcal{C}_{\text{sumrate}} \ge 2\mathcal{R}$, respectively.

For the case of $\mathcal{C}_{\text{sumrate}} < 2\mathcal{R}$, the existing network-coded cooperative mode is employed. According to [7], we can have

$$\begin{aligned} Pr(\mathcal{C}_{\text{sumrate}} < 2\mathcal{R}) &= Pr(\log\left(1 + \rho\sum_{m=1}^{2}|h_{mD}|^2\right) < 2\mathcal{R}) \\ &= (2^{2\mathcal{R}} - 1)^2/2\rho^2. \end{aligned} \tag{18}$$

The outage happens if the system cannot support the current transmission rates of both two users. The outage probability can be approximated as

$$Pr(\mathcal{O}_1|\mathcal{C}_{\text{sumrate}} < 2\mathcal{R}) \propto \rho^{-(L+M-1)},\ \rho \to \infty. \tag{19}$$

For the case of $\mathcal{C}_{\text{sumrate}} \ge 2\mathcal{R}$, we have

$$\begin{aligned} Pr(\mathcal{O}_2|\mathcal{C}_{\text{sumrate}} \ge 2\mathcal{R}) &= Pr(\mathcal{O}_2|\mathcal{C}_{\text{sumrate}}, \mathcal{C}_1 \text{ or } \mathcal{C}_2) \\ &\quad \times \{Pr(\mathcal{C}_1 \ge \mathcal{R}, \mathcal{C}_2 < \mathcal{R}) \\ &\quad + Pr(\mathcal{C}_1 < \mathcal{R}, \mathcal{C}_2 \ge \mathcal{R})\}. \end{aligned} \tag{20}$$

Since all the channels are independent, identical distribution and obey the same Rayleigh fading, we have

$$\begin{aligned} &Pr(\mathcal{C}_1 \ge \mathcal{R}, \mathcal{C}_2 < \mathcal{R}) + Pr(\mathcal{C}_1 < \mathcal{R}, \mathcal{C}_2 \ge \mathcal{R}) \\ &= 2Pr(\mathcal{C}_1 \ge \mathcal{R})Pr(\mathcal{C}_2 < \mathcal{R}) \\ &= 2(1 - Pr(\mathcal{C}_2 < \mathcal{R}))Pr(\mathcal{C}_2 < \mathcal{R}), \end{aligned} \tag{21}$$

where the first equality follows from the assumption of channel independence, and the last equality follows from

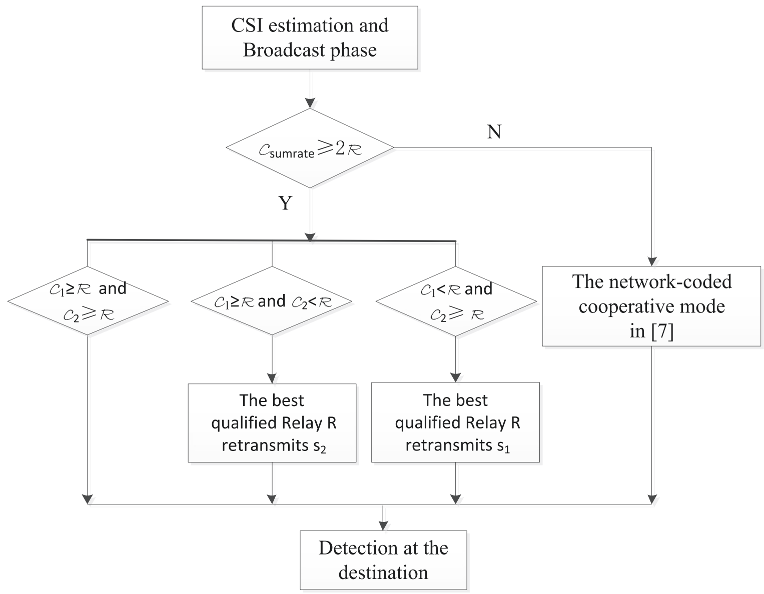

Figure 2 Diagram of the proposed opportunistic network-coded cooperative scheme.

the fact that all channels obey the same distribution. Furthermore, there is

$$\begin{aligned} Pr(\mathcal{C}_2 < \mathcal{R}) &= Pr(\log(1+\rho|h_{2D}|^2) < \mathcal{R}) \\ &= Pr(|h_{2D}|^2 < (2^{\mathcal{R}}-1)/\rho), \end{aligned} \tag{22}$$

where $|h_{2D}|^2$ is an exponential random variable with parameter 1 for Rayleigh fading channel of zero mean and variance one. Therefore, Equation (22) is the probability density function of random variable $|h_{2D}|^2$ and can be re-expressed as

$$Pr(\mathcal{C}_2 < \mathcal{R}) = 1 - \exp(-(2^{\mathcal{R}}-1)/\rho). \tag{23}$$

Similarly, the first factor $\Pr(\mathcal{O}_2|\mathcal{C}_{\text{sumrate}}, \mathcal{C}_1 \text{ or } \mathcal{C}_2)$ in Equation (20) is given by a theorem as follows.

Theorem 1. *Under the condition of $\mathcal{C}_{sumrate} \geq 2\mathcal{R}$, the outage probability can be expressed as*

$$Pr(\mathcal{O}_2|\mathcal{C}_{sumrate}, \mathcal{C}_1 \text{ or } \mathcal{C}_2) \propto \rho^{-(L+M-1)},\ \rho \to \infty.$$

Proof. Please refer to the Appendix section. □

According to the theorem above and Equation (19), we can have $Pr(\mathcal{O}_m|\cdot) \propto \rho^{-(L+M-1)}, m = 1 \text{ or } 2$ when $\rho \to \infty$. Then, by combining Equations (17) to (23), the outage probability of the proposed scheme is

$$P_{\text{out}} = \frac{(2^{2\mathcal{R}}-1)^2}{2\rho^2} Pr(\mathcal{O}_1|\mathcal{C}_{\text{sumrate}} < 2\mathcal{R}) + \left(1 - \frac{(2^{2\mathcal{R}}-1)^2}{2\rho^2}\right) \times \left(2e^{-\frac{2^{\mathcal{R}}-1}{\rho}}\left(1-e^{-\frac{2^{\mathcal{R}}-1}{\rho}}\right)\right) Pr(\mathcal{O}_2|\mathcal{C}_{\text{sumrate}}, \mathcal{C}_1 \text{ or } \mathcal{C}_2).$$

When SNR ρ tends to be infinite, and the targeted data rate R is set to a fixed system parameter, with the aid of the approximations that $\exp(t) \approx 1 - t$ when $t \to 0$, we have

$$P_{\text{out}} \propto \rho^{-(L+M)}.$$

Therefore, the diversity order of the proposed scheme is $L + M$ and the one more order diversity gain than the scheme in [7]. This is due to the user selection process according to the channel qualities of direct links.

5 Simulation results

In this section, we provide numerical simulations to verify the validity of the proposed scheme under different transmission rates. There is a same system setup as the that of the existing network-coded cooperative scheme. All of the addressed channels are assumed i.i.d. Rayleigh fading of zero mean and variance σ^2=1 with AWGN. Each node has a unit transmit power. The SNR in the simulations is defined as the ratio of transmit power for per bit to average noise power. The proposed opportunistic network-coded cooperative scheme is compared with the existing network-coded cooperative scheme proposed by [7] in terms of outage performance and system ergodic capacity.

Figure 3 shows the outage probabilities of the different transmission schemes with different numbers of relays L and numbers of users M. The targeted transmission data is set as $R = 1.5$ bits/Hz/s for each user. The comparison with the existing network-coded cooperative scheme [7] shows a significant performance gain achieved with the fixed L and M. Furthermore, Figure 3 also shows

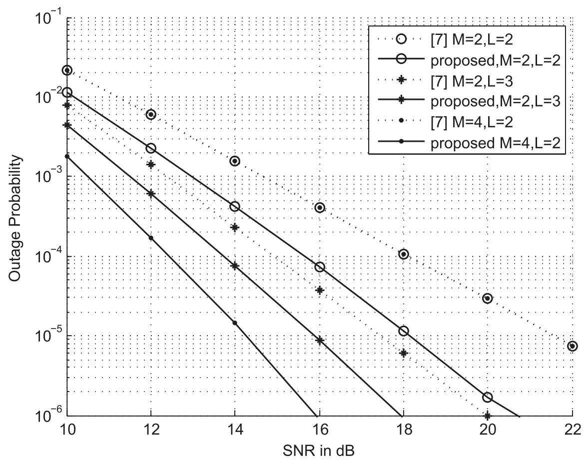

Figure 3 Outage probability versus SNR with different numbers of *L* and *M*. The data rate *R* is set to 1.5 bits/Hz/s.

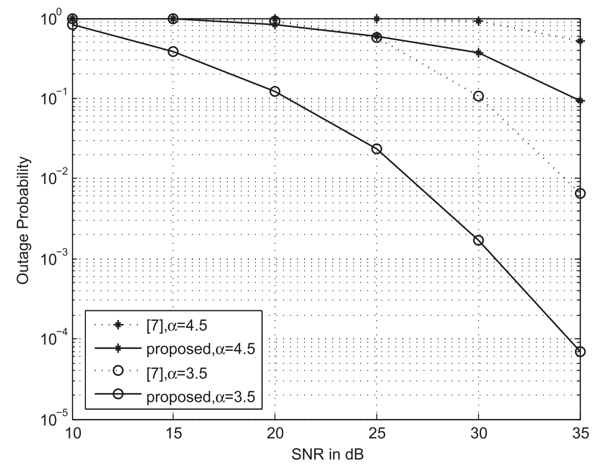

Figure 5 Outage probability versus SNR with $L = 2$ and $M = 2$.

the fact that the system outage performance can be continuously improved by deploying more relays and users. The proposed scheme achieves a total diversity order of $L + M$ instead of $L + 1$, thus provides a better outage performance than [7] in the moderate-to-high SNR regime. These results confirm our theoretical analysis on the diversity order in Section 4.

Figure 4 represents the system ergodic capacity of the different transmission schemes with different numbers of relays L and numbers of users M. It can be seen that the proposed scheme achieves a larger ergodic capacity than [7]. Moreover, the gap of the ergodic capacity becomes big as the number of users M increases.

The performance comparison between the proposed scheme and the scheme of [7] is also provided in the large-scale fading environment as shown in Figures 5 and 6. Assume that the destination is located at the center of the cell, with users and relays generating as a uniform distribution. The path loss (in dB) at distance d from each node is $L(d) = L(d_0) + 10\alpha \log_{10} \frac{d}{d_0}$, where $d_0 = 10m$ is used as a reference point in the measurements ($L(d_0) = 0$ dB), and α is set to 3.5 and 4.5 according to the recommended channel model in [16]. Shadow fading for each user is modeled as an independent lognormal random variable with standard deviation $\sigma = 10$ dB.

The outage performance is illustrated in Figure 5 for different α. It is shown that the proposed scheme still outperforms [7] in the large-scale fading environment. In addition, in Figure 6, the comparison of the system ergodic capacity between the two schemes for different α shows that the proposed scheme achieves better ergodic capacity than the scheme of [7]. Moreover, the outage performance gap and the ergodic capacity gap between the proposed scheme and [7] become big as α decreases,

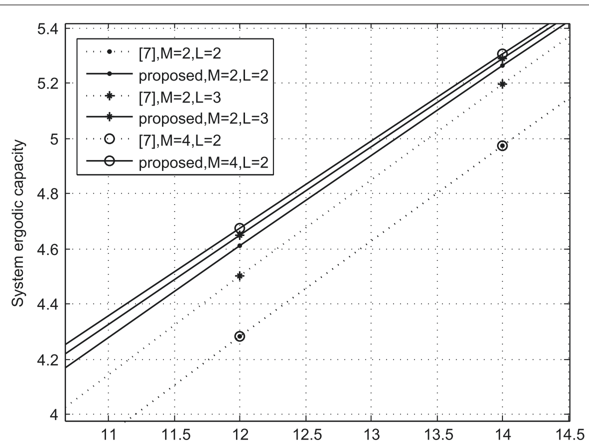

Figure 4 System ergodic capacity versus SNR with different numbers of *L* and *M*.

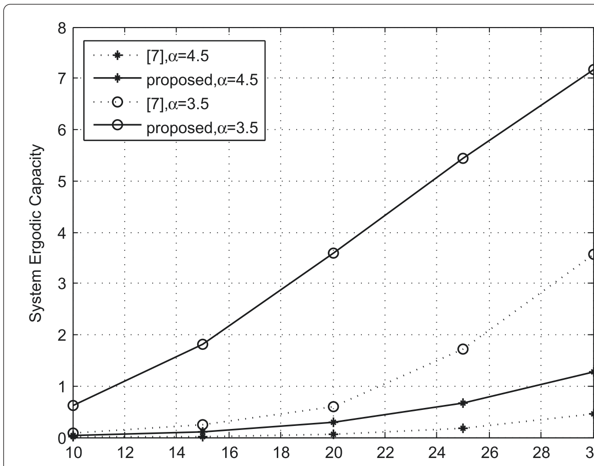

Figure 6 System ergodic capacity versus SNR with $L = 2$ and $M = 2$.

which means that the better the channel condition, the better the performance of the proposed scheme.

6 Conclusions

In this paper, a scheme of combining cooperative diversity and network coding is proposed for a wireless uplink M-source L-relay networks. Compared with the existing network-coded cooperative scheme [7] of diversity order $L+1$, the proposed scheme yields a diversity order of $L+M$. The more order diversity gain is caused by the user selection process according to the channel qualities of direct links and the idle users which served as relays. Furthermore, a significant performance gain in terms of both outage performance and system ergodic capacity can be achieved by the proposed scheme without extra bandwidth resource consumption. Both theoretical analysis and simulations confirm the effectiveness of the proposed scheme.

Appendix

Proof. The outage event in this paper is defined that the destination cannot decode both users' messages correctly. Here, $\mathcal{I}$ denotes the mutual information of the two nodes including users, relays, and destination. Therefore, for a given transmission rate $\mathcal{R}_m$ bit/s/Hz, the outage event $\mathcal{O}$ is defined as

$$\mathcal{O} = \underset{\mathcal{A}}{\cup}\mathcal{O}_{\mathcal{A}}, \tag{24}$$

where $\mathcal{A}$ takes over all the possible subsets of set $\{1,2\}$; furthermore, $\mathcal{O}_{\mathcal{A}}$ is defined as

$$\mathcal{O}_{\mathcal{A}} := \underset{N}{\cup}\mathcal{O}_{N,\mathcal{A}}\ \forall N \in \{1,\cdots,L+M-2\}, \tag{25}$$

$$\mathcal{O}_{N,\mathcal{A}} := \left\{\mathcal{I}(s_{\mathcal{A}};\mathbf{Y}|s_{\mathcal{A}^c},\mathbf{H},\mathrm{N}) < \sum_{m\in\mathcal{A}}\mathcal{R}_m\right\}. \tag{26}$$

For ease of theoretical analysis, a symmetric system is considered, where two users have the same data transmit rate $\mathcal{R}$. Therefore, the system outage probability in case (2) is equal to the outage probability in case (3). We only take case (2), for example, to analyze its system outage probability in this proof. Then, Equation (26) is written as $\mathcal{O}_{N,\mathcal{A}} := \{\mathcal{I}(s_{\mathcal{A}};\mathbf{Y}|s_{\mathcal{A}^c},\mathbf{H},\mathrm{N}) < |\mathcal{A}|\mathcal{R}$, and $|\mathcal{A}|$ is the cardinality of subset $\mathcal{A}$.

For the case of $\mathcal{C}_{\text{sumrate}} \geq 2\mathcal{R}$, the outage probability P_{out2} in case (2) can be written as

$$\begin{aligned}P_{\text{out2}} &= Pr(\mathcal{O}_2|\mathcal{C}_{\text{sumrate}},\mathcal{C}_1 \text{ or } \mathcal{C}_2) = P(\underset{\mathcal{A}}{\cup}\mathcal{O}_{2\mathcal{A}})\\ &= P(\underset{\mathcal{A}}{\cup}\underset{N}{\cup}\mathcal{O}_{N,2\mathcal{A}})\mathcal{A}\in\{1,2\},\ \forall N\in\{1,\cdots,L+M-2\}.\end{aligned} \tag{27}$$

Hence, when only user U_2 outage occurs, the mutual information is expressed as

$$\begin{aligned}I_{N,2\mathcal{A}_1} &= \log\{1+\rho|h_{1D}|^2\}\\ I_{N,2\mathcal{A}_2} &= \log\{1+\rho(|h_{2D}|^2+|h_{RD}|^2)\}\\ I_{N,\mathcal{A}_3} &= \log\left\{\left(1+\rho\sum_{m=1}^{2}|h_{mD}|^2\right)\left(1+\rho|h_{RD}|^2\right)\right.\\ &\quad\left. -\ \rho^2|h_{2D}|^2|h_{RD}|^2\right\}\\ &= \log\{(1+\rho|h_{1D}|^2)(1+\rho|h_{RD}|^2)+\rho|h_{2D}|^2\},\end{aligned} \tag{28}$$

where $\mathcal{A}_1 = \{1\}$, $\mathcal{A}_2 = \{2\}$, and $\mathcal{A}_3 = \{1,2\}$. Then, the outage probability $P(\mathcal{O}_{2\mathcal{A}_2})$ of user U_2 for the event $\mathcal{O}_{2\mathcal{A}_2}$ is given by

$$P(\mathcal{O}_{2\mathcal{A}_2}) = \sum_{N=0}^{L}P(\mathcal{O}_{N,2\mathcal{A}_2})P(\mathrm{N}=N). \tag{29}$$

In Equation (29), $P(\mathcal{O}_{N,2\mathcal{A}_2})$ is the probability for the case of $N \geq 0$, and $P(\mathrm{N}=N)$ is the probability that there exists N qualified relays out of $L+M-2$ candidates. In addition, [7] has given the expression of the probability $P(\mathrm{N}=N)$, which equals to $(\frac{2^{\mathcal{R}}-1}{\rho})^{L+M-2-N}\sum_{l=N}^{L+M-2}\frac{L+M-2!}{(L+M-2-l)!(l-N)!N!}$ approximately when $\rho\to\infty$.

Based on the definition of outage probability, $P(\mathcal{O}_{N,2\mathcal{A}_2})$ is written as

$$\begin{aligned}P(\mathcal{O}_{N,2\mathcal{A}_2}) &= Pr(\mathcal{I}_{N,2\mathcal{A}_2} < \mathcal{R})\\ &= Pr(\log\{1+\rho(|h_{2D}|^2+|h_{RD}|^2)\} < \mathcal{R}).\end{aligned} \tag{30}$$

By defining $u = |h_{2D}|^2$ and $v = |h_{RD}|^2$, the cumulative distribution function (CDF) of variable u and v can be obtained. In this paper, assuming that all channels are Rayleigh fading with zero mean and variance $\sigma^2 = 1$, then the CDF of the variable u is

$$F_U(u) = Pr(|h_{2D}|^2 \leq u) = 1-\exp(-u), \tag{31}$$

and the CDF of the variable v is

$$\begin{aligned}F_V(v) &= Pr(|h_{RD}|^2 \leq v) = Pr\left(\max_{n=1,\cdots,N}(|h_{R_nD}|^2) \leq v\right)\\ &= \prod_{n=1}^{N}Pr(|h_{R_nD}|^2 \leq v) = [\,1-\exp(-v)]^N,\end{aligned} \tag{32}$$

where the third equality follows the fact that all channels are of mutual independence. By combining Equations (30) to (32), the outage probability $P(\mathcal{O}_{N,2\mathcal{A}_2})$ is rewritten as

$$P(\mathcal{O}_{N,2\mathcal{A}_2}) = Pr(\log(1+\rho(u+v)) < \mathcal{R}) \overset{\rho\to\infty}{\approx} \int_0^{\frac{2^{\mathcal{R}}-1}{\rho}} \left(\frac{2^{\mathcal{R}}-1}{\rho} - v\right) N v^{N-1} dv = \frac{1}{N+1}\left(\frac{2^{R}-1}{\rho}\right)^{N+1}, \tag{33}$$

where $\overset{\rho\to\infty}{\approx}$ means approximate equality when ρ tends to be infinite. Then, the probability of event $\mathcal{O}_{2\mathcal{A}_2}$ of Equation (29) is

$$P(\mathcal{O}_{2\mathcal{A}_2}) = \left(\frac{2^{\mathcal{R}}-1}{\rho}\right)^{L+M-1} \sum_{N=0}^{L+M-2} \sum_{l=N}^{L+M-2} \frac{L+M-2!}{(L+M-2-l)!\,(l-N)!\,(N+1)!}. \tag{34}$$

Due to the symmetry, the probability of $P(\mathcal{O}_{2\mathcal{A}_1})$ is the same as that of $P(\mathcal{O}_{2\mathcal{A}_2})$. Similarly, for the case of $\mathcal{A}_3 = \{1,2\}$, the probability of outage event $\mathcal{O}_{3\mathcal{A}_3}$ is expressed as

$$P(\mathcal{O}_{3\mathcal{A}_3}) = \sum_{N=0}^{L} P(\mathcal{O}_{N,3\mathcal{A}_3}) P(\mathrm{N}=N), \tag{35}$$

and $P(\mathcal{O}_{N,3\mathcal{A}_3}) = Pr(\mathcal{I}_{N,3\mathcal{A}_3} < 2\mathcal{R}) = Pr(\log\{xy+\rho z\} \leq 2\mathcal{R})$ where $x = (1+\rho|h_{1D}|^2)$, $y = (1+\rho|h_{RD}|^2)$, and $z = |h_{2D}|^2$. By using the CDF of variables $|h_{2D}|^2$ in Equation (31) and $|h_{RD}|^2$ in Equation (32), the CDF of x and y can be obtained as follows:

$$F_X(x) = 1 - \exp\left(-\frac{x-1}{\rho}\right), \tag{36}$$

$$F_Y(y) = \left(1 - \exp\left(-\frac{y-1}{\rho}\right)\right)^N. \tag{37}$$

Thus,

$$P(\mathcal{O}_{N,3\mathcal{A}_3}) = \int_0^{\frac{\alpha-1}{\rho}} \int_1^{\alpha} F_Y\left(\frac{\alpha-\rho z}{x}\right) f_X(x) dx f_Z(z) dz \overset{\rho\to\infty}{\approx} \int_0^{\frac{\alpha-1}{\rho}} \int_1^{\alpha} \frac{1}{\rho^{N+1}} \left(\frac{\alpha-\rho z - x}{x}\right)^N dx dz = \frac{1}{\rho^{N+2}} C_R, \tag{38}$$

where $C_{\mathcal{R}} = \int_1^{2^{2\mathcal{R}}} \frac{1}{x^N} \frac{(2^{2\mathcal{R}}-x)^{N+1}-(1-x)^{N+1}}{N+1} dx$ which is only related with the transmit rate $\mathcal{R}$. So, the outage probability of $P(\mathcal{O}_{3\mathcal{A}_3})$ of Equation (35) is rewritten as

$$P(\mathcal{O}_{3\mathcal{A}_3}) = \left(\frac{1}{\rho}\right)^{L+M} \sum_{N=0}^{L+M-2} \sum_{l=N}^{L+M-2} C_{\mathcal{R}} (2^{\mathcal{R}}-1)^{L+M-2-N} \times \frac{L+M-2!}{(L+M-2-l)!\,(l-N)!\,N!}. \tag{39}$$

According to Equation (14) in [7], the outage probability P_{out2} under the condition of $\mathcal{C}_{\text{sumrate}} \geq 2\mathcal{R}$ is bounded as

$$P(\mathcal{O}_{2\mathcal{A}_2}) \leq P_{\text{out2}} = P(\underset{\mathcal{A}}{\cup}\mathcal{O}_{2\mathcal{A}}) \leq \sum_{\mathcal{A}} P(\mathcal{O}_{2\mathcal{A}}). \tag{40}$$

By using (34) and (39), the upper bound of P_{out2} is calculated as

$$\sum_{\mathcal{A}} P(\mathcal{O}_{2\mathcal{A}}) = P(\mathcal{O}_{2\mathcal{A}_2}) + P(\mathcal{O}_{2\mathcal{A}_3}) = \left(\frac{2^{\mathcal{R}}-1}{\rho}\right)^{L+M-1} \sum_{N=0}^{L+M-2} \sum_{l=N}^{L+M-2} \frac{L+M-2!}{(L+M-2-l)!\,(l-N)!\,(N+1)!} + \left(\frac{1}{\rho}\right)^{L+M} \sum_{N=0}^{L+M-2} \sum_{l=N}^{L+M-2} C_{\mathcal{R}} (2^{\mathcal{R}}-1)^{L+M-2-N} \times \frac{L+M-2!}{(L+M-2-l)!\,(l-N)!\,N!}, \tag{41}$$

and the lower bound of P_{out2} is expressed as

$$P(\mathcal{O}_{2\mathcal{A}_2}) = \left(\frac{2^{\mathcal{R}}-1}{\rho}\right)^{L+M-1} \sum_{N=0}^{L+M-2} \sum_{l=N}^{L+M-2} \frac{L+M-2!}{(L+M-2-l)!\,(l-N)!\,(N+1)!}. \tag{42}$$

Therefore, the diversity order under the condition of $\mathcal{C}_{\text{sumrate}} \geq 2\mathcal{R}$ is $L+M-1$, that is

$$P_{\text{out2}} = Pr(\mathcal{O}_2 | \mathcal{C}_{\text{sumrate}}, \mathcal{C}_1 \text{ or } \mathcal{C}_2) \propto \rho^{-(L+M-1)},\ \rho \to \infty. \tag{43}$$

□

Competing interests
The authors declare that they have no competing interests.

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