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Two-way amplify-and-forward relaying with carrier offsets in the absence of CSI: differential modulation-based schemes

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Abstract

In this paper, differential modulation (DM) schemes, including single differential and double differential, are proposed for amplify-and-forward two-way relaying (TWR) networks with unknown channel state information (CSI) and carrier frequency offsets caused by wireless terminals in high-speed vehicles and trains. Most existing work in TWR assumes perfect channel knowledge at all nodes and no carrier offsets. However, accurate CSI can be difficult to obtain for fast varying channels, while increases computational complexity in channel estimation and commonly existing carrier offsets can greatly degrade the system performance. Therefore, we propose the two schemes to remove the effect of unknown frequency offsets for TWR networks, when neither the sources nor the relay has any knowledge of CSI. Simulation results show that the proposed differential modulation schemes are both effective in overcoming the impact of carrier offsets with linear computational complexity in the presence of high mobility.

Keywords: Bidirectional relay communication; Amplify-and-forward; Differential; Carrier offsets

Introduction

Two-way relaying (TWR) has attracted much interest recently [1-7], where two source terminals communicate with each other through an intermediate relay. Both amplify-and-forward (AF) and decode-and-forward (DF) relaying schemes under one-way relaying have been extended to TWR [3,4]. In the DF protocol, the relay first decodes the information transmitted from both sources in the multiple-access (MA) phase, performs binary network coding to the decoded signal, then broadcasts the network-coded signal back to the sources in the broadcast (BC) phase. If the relay cannot decode the information correctly, erroneous relaying will cause significant performance degradation. For the AF-based TWR, the relay amplifies the superimposed signal received from the two sources and then broadcasts it back in the BC phase. AF-based TWR is particularly useful in wireless networks, since the wireless channel acts as a natural implementation of network coding by summing the wireless signals over the air. Therefore, we will focus on the AF-based TWR in this paper.

There has been some work investigating TWR using AF [4-6], referred to as analog network coding (ANC). However, most of the existing work assumes that perfect channel state information (CSI) is known at all transmission links. Although in some scenarios, the CSI is likely to be acquired through the use of pilot signals, it may be very difficult to obtain accurate CSI when the channel coefficients vary fast. Moreover, conventional estimation methods do not work for AF-based TWR, although they are effective for DF-based TWR. For example, channel estimation for TWR was studied in [8,9] for frequency-flat and frequency-selective environments, respectively. These studies showed that AF TWRN systems require very different estimation techniques from conventional point-to-point systems. Therefore, differential modulation for TWR without the knowledge of CSI is worth being exploited. Differential receivers for TWR were designed in [7,10,11]. However, perfect synchronization was assumed in [7,10], while imperfect synchronization scenario caused by different propagation delay from both sources to the relay due to the distributed nature of all nodes was investigated in [11]. To the best of our knowledge, no work has

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been reported in TWR with unknown carrier frequency offsets when CSI is not available at all nodes.

In wireless mobile communications, however, Doppler shift is common and inevitable, especially in the high-speed mobile environment. For example,it is anticipated that the third-generation European cellular standards will operate on trains moving as fast as 500 km/h. If the carrier frequency is 2 GHz, the induced Doppler shift may be up to 880 Hz. One technique to mitigate frequency offset is to estimate it at the receiver using a frequency acquisition and tracing circuit and then compensate it with single-differential modulation, resulting in increased computational complexity in the relay and reduced data rate [12,13]. Another approach is double-differential modulation [14-16], which can effectively handle frequency offsets in the presence of channel fading. A multiple symbol double-differential detection based on least squares criteria was proposed in [16], where the system performance was proved to be insensitive to different carrier offsets. However, all the above methods [12-16] are carried out on point-to-point communication links and cannot be directly applied to TWR with unknown carrier offsets, since the signal received at the relay is a mixture of both source signals, and CSI is not available at all nodes.

Therefore, we investigate both single-differential detection (SD) and double-differential detection (DD) for TWR using AF with unknown carrier offsets in this paper. For SD, a carrier offset estimation and compensation scheme with reduced computational complexity is employed. To further improve the performance of using DD, a fast algorithm of multiple-symbol-based signal detection is proposed. Simulation results show that the proposed SD and DD schemes are both effective in removing the carrier offsets, and the computational complexity remains linear.

Notation

Boldface lower-case letters denote vectors, $(\cdot)^*$ stands for complex conjugate, $(\cdot)^T$ represents transpose, $(\cdot)^H$ represents conjugate transpose, $\mathbb{E}\{.\}$ is used for expectation, $\|\cdot\|$ denotes the Euclidean vector norm, $\mathrm{CN}(0, N_0)$ denotes the set of Gaussian distributed complex numbers with the standard variance of N_0 (i.e., $0.5N_0$ per dimension), and $\mathrm{R}e\{\cdot\}$ denotes real part.

Single-differential modulation for bidirectional relay networks under carrier offsets

We consider a network with three nodes including two source nodes, denoted by S_1 and S_2, and one relay node R. A half-duplex system is assumed and all nodes are equipped with one antenna. Information is exchanged between S_1 and S_2 with the help of R, which is completed in two phases. In the first phase, the MA phase, both source nodes send the differentially encoded signals to the relay, and in the second phase, the BC phase, the relay broadcasts the superimposed signals back to both source nodes.

Let $z_i(k) \in \Omega, i \in \{1, 2\}$ denote the symbol to be transmitted by source node S_i at discrete symbol time k, where Ω represents a unity power M-PSK constellation set. As single-differential modulation is used, the signal $z_i(k)$ sent by source S_i is given as

$$s_i(k) = s_i(k-1)\, z_i(k), z_i(k) \in \Omega \tag{1}$$

In the MA phase, two terminals simultaneously transmit the differentially encoded information to the relay. For simplicity, we assume that the fading coefficients and the carrier offsets keep constant over the frame of length L and change independently from one frame to another [7,10]. The received signal at the relay at time k is then

$$y_{\mathrm{r}}(k) = \sqrt{P}_1 h_1 s_1(k)\, e^{j\omega_1 k} + \sqrt{P}_2 h_2 s_2(k)\, e^{j\omega_2 k} + n_{\mathrm{r}}(k), \tag{2}$$

where $h_i, i \in \{1, 2\}$ denotes the complex channel gain with zero mean and unit variance between S_i and R; $\omega_i = 2\pi f_{\mathrm{d}}^i T$, T is the symbol interval and f_{d}^i is the Doppler shift introduced between S_i and R. $n_{\mathrm{r}}(k)$ stands for a zero mean complex Gaussian random variable with variance σ_n^2, and P_i denotes the transmit power at source S_i.

In the BC phase, the relay R amplifies y_{r} by a factor α and then broadcasts its conjugate, denoted by $y_{\mathrm{r}}^*(k)$ back to both S_1 and S_2 with transmit power P_{r}. The corresponding signal received by S_1 at time k, denoted by $y_1(k)$, can then be written as

$$\begin{aligned}\tilde{y}_1(k) &= \alpha\sqrt{P_{\mathrm{r}}} h_1 y_{\mathrm{r}}^*(k)\, e^{j\omega_1 k} + n_1(k) \\ &= \alpha\sqrt{P_1 P_{\mathrm{r}}}\, |h_1|^2 s_1^*(k) + \alpha\sqrt{P_2 P_{\mathrm{r}}} h_1 h_2^* s_2^*(k)\, e^{j(\omega_1-\omega_2)k} \\ &\quad + \alpha\sqrt{P_{\mathrm{r}}} h_1 n_{\mathrm{r}}^*(k)\, e^{j\omega_1 k} + n_1(k)\end{aligned} \tag{3}$$

For the decoding simplicity at S_1, we can obtain the conjugate of (3) as

$$y_1(k) = \tilde{y}_1^*(k) = \mu s_1(k) + \nu s_2(k)\, e^{j(\omega_2-\omega_1)k} + \bar{n}_1(k), \tag{4}$$

where $\alpha = \left(P_1|h_1|^2 + P_2|h_2|^2 + N_0\right)^{-\frac{1}{2}}$, $\mu = \alpha\sqrt{P_1 P_{\mathrm{r}}}\, |h_1|^2$, $\nu = \alpha\sqrt{P_2 P_{\mathrm{r}}} h_1^* h_2$ and the equivalent noise $\bar{n}_1(k) = \alpha\sqrt{P_{\mathrm{r}}} n_{\mathrm{r}}(k)\, h_1^* e^{-j\omega_1 k} + n_1^*(k)$.

Similarly, the received signal at S_2 can be expressed as

$$\tilde{y}_2(k) = \alpha\sqrt{P_{\mathrm{r}}} h_2 y_{\mathrm{r}}^*(k)\, e^{j\omega_2 k} + n_2(k) \tag{5}$$

Given that S_1 and S_2 are mathematically symmetrical, as shown in (3) and (5), for simplicity, we only discuss the signal detection at S_1 in the following.

Since the relay has no knowledge of CSI, we cannot obtain the amplification factor α directly. We may rewrite the received signals at the relay in a vector format as

$$\mathbf{Y_r} = \sqrt{P_1} h_1 \mathbf{S}_1 + \sqrt{P_2} h_2 \mathbf{S}_2 + \mathbf{n}_\mathrm{r}, \tag{6}$$

where $\mathbf{Y}_\mathrm{r} = \left[y_\mathrm{r}(1), \cdots, y_\mathrm{r}(L)\right]^T$, $\mathbf{n}_\mathrm{r} = [n_\mathrm{r}(1), \cdots, n_\mathrm{r}(L)]^T$, $\mathbf{S}_\mathrm{i} = \left[s_i(1) e^{j\omega_i}, \cdots, s_i(L) e^{j\omega_i L}\right]^T$, $i \in \{1, 2\}$. We therefore have

$$\mathbf{Y}_\mathrm{r}^H \mathbf{Y}_\mathrm{r} = P_1 |h_1|^2 \mathbf{S}_1^H \mathbf{S}_1 + P_2 |h_2|^2 \mathbf{S}_2^H \mathbf{S}_2 + \mathbf{n}_\mathrm{r}^H \mathbf{n}_\mathrm{r} + \mathrm{Re}\left\{\sqrt{P_1}\sqrt{P_2} h_1^* h_2 \mathbf{S}_1^H \mathbf{S}_2 + \sqrt{P_1} h_1^* \mathbf{S}_1^H \mathbf{n}_\mathrm{r} + \sqrt{P_1} h_2^* \mathbf{S}_2^H \mathbf{n}_\mathrm{r}\right\}, \tag{7}$$

where $\mathbb{E}\left\{\mathbf{S}_1^H \mathbf{S}_1\right\} = \mathbb{E}\left\{\mathbf{S}_2^H \mathbf{S}_2\right\} = L, \mathbb{E}\left\{\mathbf{n}_\mathrm{r}^H \mathbf{n}_\mathrm{r}\right\} = LN_0$, $\mathbb{E}\left\{\mathbf{S}_1^H \mathbf{n}_\mathrm{r}\right\} = \mathbb{E}\left\{\mathbf{S}_2^H \mathbf{n}_\mathrm{r}\right\} = 0$ and $\mathbb{E}\left\{\mathbf{S}_1^H \mathbf{S}_2\right\} = \mathbb{E}\left\{s_1^*(1) s_2(1) e^{j(\omega_2-\omega_1)} + \cdots + s_1^*(L) s_2(L) e^{j(\omega_2-\omega_1)L}\right\} = e^{j(\omega_2-\omega_1)} \mathbb{E}\left\{s_1^*(1) s_2(1)\right\} + \cdots + e^{j(\omega_2-\omega_1)L} \mathbb{E}\left\{s_1^*(L) s_2(L)\right\} \approx 0$. α can thus be approximated at high signal-to-noise-ratio (SNR) as

$$\alpha = \sqrt{\frac{\mathbb{E}\left\{\mathbf{Y}_\mathrm{r}^H \mathbf{Y}_\mathrm{r}\right\}}{L}} \approx \sqrt{\frac{||\mathbf{Y}_\mathrm{r}||^2}{L}} \tag{8}$$

Similar to (6), the received signals at source S_1 can also be rewritten in the vector format as

$$\mathbf{Y}_1 = \mu \mathbf{S}_1 + \nu \mathbf{S}_2 + \bar{n}_1 \tag{9}$$

where $\mathbf{Y}_1 = \left[y_\mathrm{r}(1), \cdots, y_\mathrm{r}(L)\right]^T$, $\mathbf{S}_1 = [s_1(1), \cdots, s_1(L)]^T$, $\mathbf{S}_2 = \left[s_2(1) e^{j(\omega_2-\omega_1)}, \cdots, s_2(L) e^{j(\omega_2-\omega_1)L}\right]^T$ and $\bar{\mathbf{n}}_1 = [\bar{n}_1(1), \cdots, \bar{n}_1(L)]^T$.

It is shown in (3) that the signal received at source S_1 is a complex superimposed signal; therefore, the application of conventional single- or double-differential detection on point-to-point communication link to TWR is not straightforward. It is difficult to decode the expected information $z_2(k)$ if we cannot subtract the self-information $s_1(k)$ from $y_1(k)$ when μ is unknown, due to the lack of CSI at S_1. Therefore, we propose a three-step approach in the single-differential detection for TWR with carrier offsets: step 1, the self-information of $\mu s_1(k)$ is subtracted from $y_1(k)$, the most important step in the whole detection procedure. Step 2, the carrier frequency offset is estimated and compensated. Step 3, signal $z_2(k)$ differentially decoded using the single-symbol single-differential detector.

Step 1: self-information subtraction

Since terminal S_1 knows its own transmitted signal, μ needs to be estimated before we can subtract the contribution of $\mu s_1(k)$ from $y_1(k)$. We thereby propose a simple estimation method as follows

$$\mathbf{Y}_1 \mathbf{S}_1^H = \mu \mathbf{S}_1 \mathbf{S}_1^H + \nu \mathbf{S}_2 \mathbf{S}_1^H + \bar{n}_1 \mathbf{S}_1^H = L\mu + \nu \mathbf{S}_2 \mathbf{S}_1^H + \bar{n}_1 \mathbf{S}_1^H \tag{10}$$

By taking the expectation of $\mathbf{Y}_1 \mathbf{S}_1^H$, given that $s_1(k)$ and $s_2(k)$ are independent and have the same distribution, we can approximately obtain

$$\mathbb{E}\left\{\mathbf{Y}_1 \mathbf{S}_1^H\right\} \approx L\mu \tag{11}$$

$$\mu \approx \frac{\mathbb{E}\left\{\mathbf{Y}_1 \mathbf{S}_1^H\right\}}{L} \tag{12}$$

After obtaining the estimation of μ, we can easily subtract the self-information of $s_1(k)$ as

$$\bar{y}_1(k) \triangleq y_1(k) - \mu s_1(k) = \nu s_2(k) e^{j(\omega_2-\omega_1)k} + \bar{n}_1(k) \tag{13}$$

Step 2: carrier offset estimation

A frequency offset estimation method was introduced in [13], which is effective in removing the impact of carrier frequency offsets, independent of data symbols and channel gains. However, training symbols are required to be transmitted at the beginning of each frame to solve the ambiguous estimation problem. In this case, two training symbols are enough to provide a good estimation of the carrier offsets. Then, the signals received at S_1 can be rewritten as

$$\bar{\mathbf{Y}}_1 = \left[\bar{y}_1(-P), \cdots, \bar{y}_1(-1), \bar{y}_1(0), \bar{y}_1(1), \cdots, \bar{y}_1(N)\right]^T \tag{14}$$

$$\mathbf{S}_2 = [s_2(-P), \cdots, s_2(-1), s_2(0), s_2(1), \cdots, s_2(N)]^T, \tag{15}$$

where P is the number of training symbols. Define the training symbols as $s_i(-P) = 1$ and $P = 2$, we have

$$\begin{aligned} \bar{y}_1(-2) &= \nu e^{j\omega(-2)} + \bar{n}_1(-2) \\ \bar{y}_1(-1) &= \nu e^{j\omega(-1)} + \bar{n}_1(-1) \\ \bar{y}_1(0) &= \nu e^{j\omega 0} + \bar{n}_1(0) \end{aligned} \tag{16}$$

Since ν is also a complex value, the following transformation is made

$$\begin{aligned} \hat{y}_1(-1) &= \bar{y}_1(-1) \bar{y}_1^*(-2) = |\nu|^2 e^{j\omega} + \hat{n}_1(-1) \\ \hat{y}_1(0) &= \bar{y}_1(0) \bar{y}_1^*(-1) = |\nu|^2 e^{j\omega} + \hat{n}_1(0), \end{aligned} \tag{17}$$

where $\omega \triangleq \omega_2 - \omega_1$. Then, the estimation of ω can be obtained as $\hat{\omega} = \arg\left\{\sum_{l=-1}^{0} y(l)\right\} \in (-\pi, \pi]$.

Step 3: single-symbol single-differential detection

With the estimation of the carrier frequency offset $\hat{\omega}$, the frequency offset effect is compensated, and the received data after compensation can be expressed as

$$\hat{y}_1(k) = \bar{y}_1(k) e^{-j\hat{\omega}k} = \nu s_2(k) + \bar{n}_1(k) e^{-j\hat{\omega}k} \tag{18}$$

Consider the generalized likelihood ratio test (GLRT) detection of the multiple symbols $\{s_2(k-n)\}_{n=0}^{N}$. Define

$$\hat{\mathbf{Y}}_1 = \left[\hat{y}_1(0), \hat{y}_1(1), \cdots, \hat{y}_1(N)\right]^T$$
$$\mathbf{S}_2 = \left[s_2(0), s_2(1), \cdots, s_2(N)\right]^T \quad (19)$$

The GLRT algorithm for detection of $\{s_2(k-n)\}_{n=0}^{N}$ can be obtained by minimizing the following metric:

$$\min_{\{s_2(t)\}_{t=1}^{N-1}, \nu} \left\| \hat{\mathbf{Y}}_1 - \nu \mathbf{S}_2 \right\|^2 \quad (20)$$

Performing the minimization of the metric over ν results in the following decision algorithm:

$$\max_{\{s_2(k-n)\}_{n=0}^{N}} \left| \sum_{n=0}^{N} \hat{y}_1(k-n)\, s_2^*(k-n) \right|^2 \quad (21)$$

Let $\{\hat{s}_2(k-n)\}_{k=0}^{N}$ be the detection results during the observation length of N for the signal transmitted by S_2. Then, by differential decoding, we can recover the $z_2(k-n)$ as

$$\hat{z}_2(k-n) = \hat{s}_2(k-n)\, \hat{s}_2^*(k-n-1), \quad n = 0, 1, \cdots, N-1 \quad (22)$$

Double-differential modulation bidirectional relay networks under carrier offsets

In this section, we investigate the double-differential modulation for TWR. Similar to the single-differential modulation, the signal $s_i(k)$ sent by source S_i is given as

$$s_i(k) = s_i(k-1)\, p_i(k), \quad p_i(k) \in \Omega$$
$$p_i(k) = p_i(k-1)\, z_i(k), \quad z_i(k) \in \Omega \quad (23)$$

Same as single-differential modulation, the signals received at terminal S_1 can be transformed as

$$y_1(k) = \tilde{y}_1^*(k) = \mu s_1(k) + \nu s_2(k)\, e^{j(\omega_2-\omega_1)k} + \bar{n}_1(k) \quad (24)$$

The DD in TWR is divided into two steps. Step 1 is self-information elimination, similar to the first step of the single-differential detection method described in section 'Single-differential modulation for bidirectional relay networks under carrier offsets'. Step 2 is the double-differential demodulation. The attractive feature of double-differential modulation is its insensitivity to unknown frequency offset, so the frequency offset is not necessarily acquired and tracked in step 2. For the second step of DD detection, conventional double-differential detector, including symbol-by-symbol and multiple-symbol detection can be applied, once the self-information $\mu s_1(k)$ is subtracted from the received signal $y_1(k)$. Since the processing of step 1 has been introduced in the above section in detail, we in the next focus on step 2.

Symbol-by-symbol double-differential detection

From (13), the self-information of $\mu s_1(k)$ can be subtracted at S1 without the need of any CSI; therefore, (13) is equivalent to the DD detection on a direct transmission link [14]. A symbol-by-symbol double-differential detector is then developed to recover the desired information, as in the following:

$$\tilde{z}_2(k) = \underset{z_2(k)}{\operatorname{argmax}} \left\{ \bar{y}_1(k)\bar{y}_1^*(k-1) \left(\bar{y}_1(k-1)\bar{y}_1^*(k-2)\right)^* z_2^*(k) \right\} \quad (25)$$

Multiple-symbol double-differential detection

Even though double-differential modulation can eliminate the degradation due to frequency offset, it needs higher SNR power ratio than that of coherent detection, to achieve the same average bit error rate (BER) performance. An attractive approach to mitigate this SNR loss is called multiple-symbol double-differential detection [15,16].

In the absence of noise, we can obtain

$$\hat{y}_1(k) = \bar{y}_1(k)\,\bar{y}_1^*(k-1)$$
$$= |\nu|^2 e^{j(\omega_2-\omega_1)} p_2(k-1)\, z_2(k)$$
$$= \hat{h} p_2(k-1)\, z_2(k) \quad (26)$$

which is equivalent to single-differential detection, and when iterated, it becomes

$$\hat{y}_1(k-n) = \hat{h} p_2(k-N+2) \prod_{m=0}^{N-n-3} z_2(k-n-m),$$
$$n = 0, 1, 2, \cdots, N-2 \quad (27)$$

Here, N denotes the symbol length in the observation. Next, the minimum least-square (LS) criterion [16] is applied. By performing the minimization of the metric over $\hat{h}p_2^*(k-N+2)$, the following decision can be obtained:

$$\max_{z_2(k), z_2(k-1), \cdots, z_2(k-N+3)} \left| \sum_{n=0}^{N-2} y_1(k-n) \ldots \prod_{m=0}^{N-n-3} z_2^*(k-n-m) \right|^2 \quad (28)$$

However, (28) has a computational complexity of $\frac{(MN-3M-N+2)M^{N-1}}{(M-1)^2}$, which is prohibitively high. Then, a fast algorithm is introduced in the following with a complexity on the order of $N\log_2 N$ independent of the constellation size based on the principle in [17].

(26) can be rewritten as

$$\hat{y}_1(k) = \bar{y}_1(k)\,\bar{y}_1^*(k-1) = |\nu|^2 e^{j(\omega_2-\omega_1)} p_2(k) \quad (29)$$

With the theorem [17] that the vector $\mathbf{Z}_2$ maximizes $\mathrm{p}\left(\hat{y}_1|\mathbf{Z}_2\right)$ if and only if the vector $\mathbf{P}_2$ maximizes $\mathrm{p}\left(\hat{y}_1|\mathbf{P}_2\right)$, $\hat{\mathbf{P}}_2$ which maximizes the following is then selected:

$$\left|\sum_{\mathrm{k}=1}^{\mathrm{N}} \hat{y}_1(k)\,\hat{p}_2(k)\right|^2, \tag{30}$$

where $\mathbf{Z}_2 = [z_2(k+1), \cdots, z_2(k+N)]^T, \mathbf{P}_2 = \left[p_2(k+1), \cdots, p_2(k+N)\right]^T, k \geq 0$.

If $\hat{\mathbf{P}}_2 = \mathbf{P}_2$, following the corollary [17], for any k, l, with $1 \leq k,\ 1 \leq N$, we have

$$\left|\arg\left(\hat{y}_1(k)\,\hat{p}_2(k)\right) - \arg\left(\hat{y}_1(l)\,\hat{p}_2(l)\right)\right| < \frac{2\pi}{M} \tag{31}$$

For any $k, k = 1, \cdots, N$ and any $\hat{p}_2(k)$, $\hat{y}_1(k)\,\hat{p}_2(k)$ is termed as a re-modulation of $\hat{y}_1(k)$. Therefore, it is sufficient to consider only those sets of re-modulations of $\hat{y}_1(k), k = 1, \cdots, N$, which contain the re-modulations within $\frac{2\pi}{M}$. Let $\bar{\mathbf{P}}_2$ be the unique $\mathbf{P}_2$, which satisfies $\arg\left(\hat{y}_1(k)\,\bar{p}_2(k)\right) \in \left(0, \frac{2\pi}{M}\right]$. For simplicity, we define $d_k = \hat{y}_1(k)\,\bar{p}_2(k)$ and then list the $\arg\{d_k\}$ ordering from the largest to the smallest. Define the function $k(i)$ where the value of $k(i)$ denotes the subscript k of $d_{k(i)}$ and i represents the ith position in the list. To get all the possible re-modulations of $\hat{y}_1(k), k = 1, \cdots, N$, let the list going clockwise around the circle at the interval of $\frac{2\pi}{M}$. Let $q_i = d_{k(i)}$, $i = 1, \cdots, N$ for $m = 1, \cdots, M-1$ and then for $mN < i \leq (m+1)N, q_i = e^{j\frac{2\pi}{M}} q_{i-mN}$.

To maximize (30), it is sufficient to obtain the starting position as

$$\hat{i} = \underset{i\in\{1,\cdots,MN\}}{\operatorname{argmax}} \left|\sum_{i=n}^{(n+N-1)modMN} q_i\right|^2 \tag{32}$$

Note that the magnitudes in (32) are periodic in N, resulting in M-fold ambiguity in (30), which will not affect differential decoding. Thus, only the following is required to be obtained:

$$\hat{i} = \underset{i\in\{1,\cdots,N\}}{\operatorname{argmax}} \left|\sum_{i=n}^{(n+N-1)modMN} q_i\right|^2, \tag{33}$$

and hence, the algorithm has the complexity on the order of $N\log_2 N$. Then, vector $\hat{\mathbf{P}}_2$ can be obtained, where

$$\begin{aligned} \hat{p}_{2,k(i)} &= \bar{p}_{2,k(i)}, \hat{i} \leq i \leq N \\ \hat{p}_{2,k(i-N)} &= \bar{p}_{2,k(i-N)} + \frac{2\pi}{M}, N \leq i \leq \hat{i} + N - 1 \end{aligned} \tag{34}$$

By reordering the elements $\hat{p}_{2,k(i)}, i = 1, \cdots, N$ in the order of the subscript value (i), we can get the vector $\hat{p}_2(k), k = 1, \cdots, N$. For differential decoding, $z_2(k)$ can be recovered as

$$z_2(k) = \hat{p}_2(k+1)\,\hat{p}_2^*(k) \tag{35}$$

Simulation results

In this section, we present some simulation results for the proposed SD and DD schemes for TWR using AF with different Doppler shifts corresponding to different relative velocities between the relay R and the terminal $S_i, i \in \{1, 2\}$. We choose the carrier frequency 2 GHz and the symbol interval $T = 100us$. Three different normalized Doppler frequencies have been selected, $f_d T = 0.12$, $f_d T = 0.24$ and $f_d T = 0.36$, corresponding to a mobile terminal moving at speeds of 100, 200, and 300 Km/h, respectively. We also plot the performance of the analog network coding scheme with differential modulation (ANC-DM) [7] with no frequency offset for comparison. For simplicity, it is assumed that $P_1 = P_2 = P_r = 1$, both source nodes and the relay have the same noise variance N_0, and the variance of complex channel coefficient is set to 1 for all links. All simulations are performed with BPSK modulation and the length of the frame is set to 100.

The BER performance of estimating μ as described in (11), and (12) is presented in Figure 1a,b with random Doppler shift. For comparison, we also included the Genie-aided result by assuming that μ is perfectly known by the source such that traditional differential decoding without carrier offsets can be performed both for SD and DD. It is shown that there is almost no performance loss using the estimation method with the existence of carrier offsets, which clearly justifies the robustness of the proposed schemes.

In Figure 2, the BER of the proposed SD for TWR is compared with that of the ANC-DM [7] with different Doppler shifts. It can be observed that the proposed SD scheme based on Doppler shift estimation and compensation nearly has the same performance under different Doppler shifts. It is about 3 dB inferior to ANC-DM [7] without Doppler shift. However, ANC-DM [7] is shown to experience high error floor under the Doppler shift.

In Figure 3, the BER of the proposed multiple-symbol double-differential detection (MSDD) for TWR is compared with that of the ANC-DM [7] with different Doppler shifts. It can be observed that the proposed MSDD scheme nearly has the same performance under different Doppler shifts; it nearly has the same performance as the ANC-DM [7] without Doppler shift at high SNR. However, ANC-DM [7] is shown to experience high error floor under the doppler shift.

In Figure 4, the BER of the proposed DD scheme is compared with that of ANC-DM [7] under random Doppler shift. It can be observed that the performance improves significantly with the increasing of the observation length N, approaching a limit about 0.5 dB away from the performance of ANC-DM [7] under no Doppler shift with $N = 64$. However, ANC-DM [7] under random Doppler shift can not work.

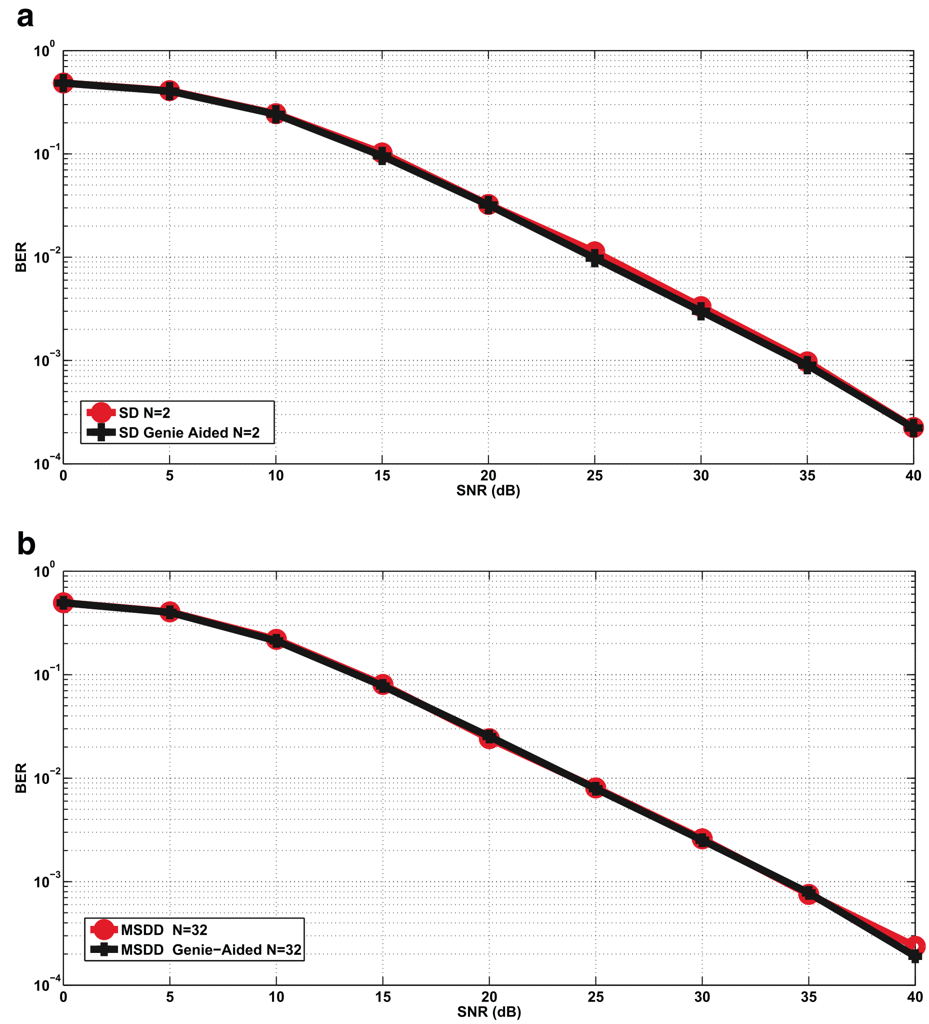

Figure 1 Simulated BER performance of the proposed (a) SD and (b) DD detection. μ is under estimation and perfectly known under random carrier offset.

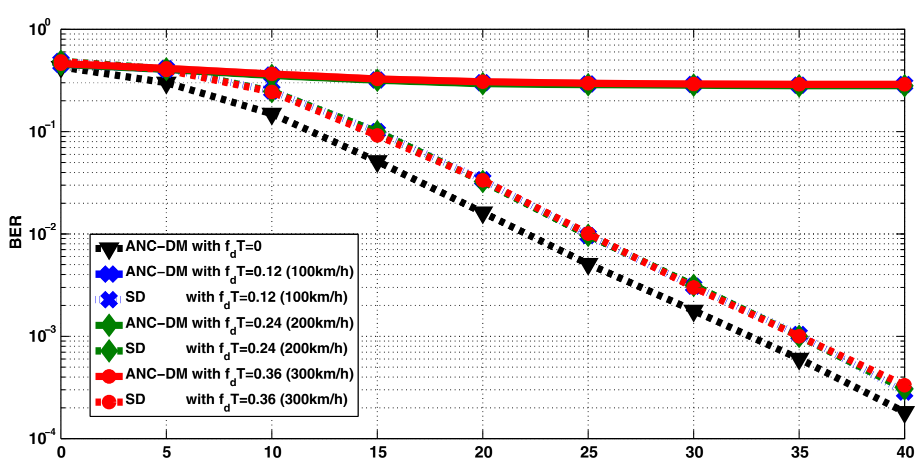

Figure 2 Simulated BER performance of the proposed single-differential detection with different Doppler shifts.

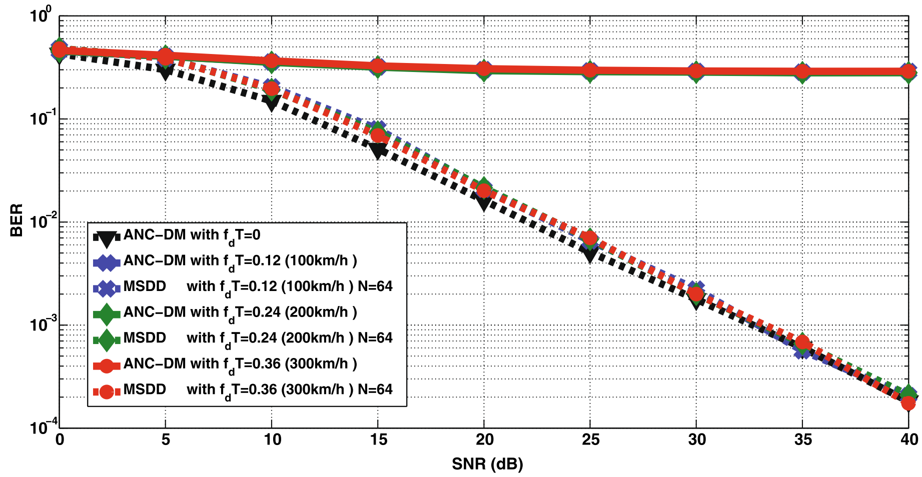

Figure 3 **Simulated BER performance of the proposed multiple-symbol double-differential detection with different Doppler shifts.**

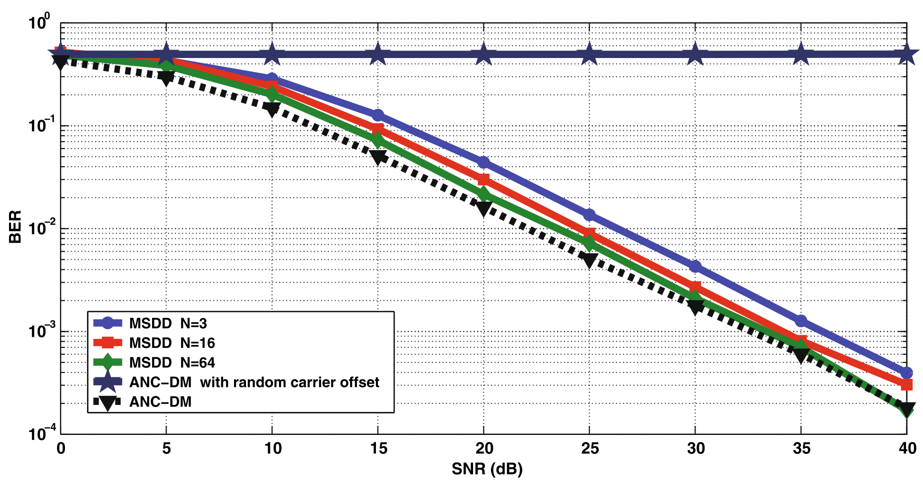

Figure 4 **Simulated BER performance of the proposed double-differential detection under random Doppler shift.**

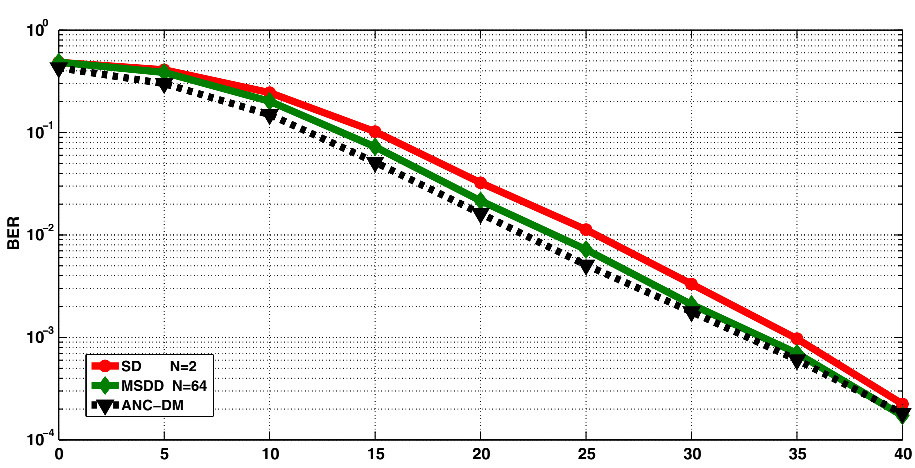

Figure 5 **Simulated BER performance comparison between single-differential and double-differential detection under random carrier offset.**

Figure 5 compares the BER performance between the proposed SD scheme and DD scheme. Liu et al. [13] shows the performance of multiple-symbol single-differential (MSSD) detection degrades with the increasing of the observation length N because of the inaccurate Doppler shift estimation caused by the short training symbols; therefore, for single-differential modulation in TWR, single-symbol detection is preferred. It is shown that the BER performance of DD with $N = 64$ is about 2 dB superior to that of SD with $P = 2$ with random carrier frequency offsets.

Next, the computational complexity of the two proposed methods is compared. The computational complexity of the proposed DD with multiple-symbol detection using fast algorithm is $O\left(N\log_2 N\right)$, which is independent of the constellation size M, while that of the SD is $O\left(M\right)$, which are both linear. It is also demonstrated that single-differential detector using the frequency offset estimation needs extra training symbols, which decreases the transmit rate, while double-differential detector has its insensitivity to unknown frequency offsets, allowing the hardware implementation to be easy, without the need of complicated frequency offset acquisition and tracking circuitry. Its inherent SNR loss can be greatly minimized by using the multiple-symbol detection.

Conclusion

In this paper, we have proposed two differential modulation schemes to effectively attenuate the degrading effects on performance due to the Doppler shifts in TWR using ANC, when neither the sources nor the relay has any knowledge of CSI. The simulation results indicate that the proposed algorithms can effectively remove the impact of Doppler shift in the presence of channel fading with low computational complexity in high-speed mobile environment.

Competing interests

The authors declare that they have no competing interests.

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