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# Power minimization of cooperative beamforming networks with spectrum sharing

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**Abstract**

Cooperative communication with spectrum sharing has been proved to be an efficient way to reduce energy consumption. In this work, an optimal power allocation algorithm is proposed for mobile cooperative beamforming networks, where several secondary users (SUs) collaboratively act as relay nodes to assist the transmission of a primary user (PU) in reward of transmitting their own information. The optimal power allocation scheme is obtained by solving a convex optimization problem aiming at minimizing network energy consumption with guaranteed quality of service (QoS) for both PU and SUs. Simulation results show that with the consideration of overheads, circuit power, and energy costs for message sharing, a cooperative beamforming network augmented with our proposed power allocation algorithm performs better in scenarios of more SUs in terms of higher energy efficiency and less power consumption per unit throughput. Additionally, the proposed scheme outperforms the conventional best-relay selection approach in the aspects of power consumption per unit throughput, energy efficiency, and network power consumption. Furthermore, simulations also demonstrate that the derived cooperative beamforming algorithm exhibits superior performance in energy saving than conventional methods in high-speed scenarios.

**Keywords:** Cooperative beamforming; Power minimization; Spectrum sharing; Mobile network

## 1 Introduction

Towards the fifth generation (5G) of wireless/mobile broadband communication, many advanced key technologies are going to be implemented, including spectrum sharing management, exploiting the Cloud-RAN (CRAN) and mobile clouds concepts as well as heterogeneous network coordination, in order to meet the demand for larger bandwidth, higher data rate up to Gbit/s and seamless connection access [1-3]. Furthermore, reduction of unnecessary energy consumption becomes one of the major concerns in 5G communications, which not only reduces environmental impact but also cuts overall network costs and helps make communication more practical and affordable. These studies belong to the research of 'green networking' which aims in increasing energy savings while maintaining satisfactory users' quality of service (QoS) [4-6]. Besides, cooperative techniques are promising in improving the reliability of the communications against channel impairments and achieved throughput through aggregating resources offered by different collaborative entities and realizing significant energy savings, targeting a so-called green and soft 5G system [7].

Apart from 'cell breathing' of base station coordination and clustered cooperative schemes [8,9], user cooperation, as a new form of spatial diversity, enables single-antenna mobiles in a multiuser environment to share their antennas and generate a virtual multiple-antenna transmitter, providing higher throughput, robustness to channel variations and ubiquitous mobile access for media-rich mobile devices. Recently, being extended with the concept of cognitive radio, user cooperation with spectrum sharing capability, i.e., secondary users (SUs) share the same spectrum with the primary user (PU), can further improve spectral efficiency. User-cooperative diversity protocols that combat fading induced by multipath propagation in wireless networks and the outage capacity characterization of various relaying protocols, i.e., fixed relaying, selective relaying, and incremental relaying, are characterized [10]. In literature, numerous criteria have been proposed for relay node selection based on channel conditions [11], the distances between nodes [12],

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and the transmission contention [13], etc. It has been demonstrated in [14,15] that when more relay nodes, also called as cooperators, are involved in transmission, less energy costs and higher network throughput can be achieved simultaneously, compared with the single-relay transmission. This motivated the research on methodologies of selecting multiple entities to act as cooperators, also called as 'cooperative beamforming'. However, several issues should be considered from a media access control (MAC) layer perspective when multiple cooperating entities are involved. The first issue is that more control signaling overhead should be considered for selecting and coordinating multiple cooperating entities [16]. The second issue is related to users' willingness to cooperate. Furthermore, incentive mechanisms are of necessity for promoting user cooperation in order to prevent unwillingness and selfishness of the SUs via reputation evaluation, cost reduction, or increase of transmission time [17-20].

As the energy expenditure becomes a major concern for the operators of communication networks, constructing cooperative networks with the energy consumption minimized has become an important research field [21]. Multiple solutions of resource allocation were derived based on convex optimization or game theory via lowering power consumption under certain constraints, such as minimizing the outage probability [22-24], maximizing energy efficiency [25-28], maximizing overall throughput [29,30], maintaining service quality, or maximizing the sum of SUs' capacities and signal-to-noise power ratio (SNR) [31]. Joint relay selection and power allocation schemes are proposed by factoring the cost of acquiring channel state information (CSI) to minimize the total energy consumption [32]. The allocation schemes obtained are applicable to adapting the transmission power, bandwidth, time, and accessibility for the nodes involved in the cooperative network. In addition, circuit power consumption model has been taken into account in calculating the energy efficiency of cooperative heterogeneous and beamforming networks in [25,33].

The aforementioned resource allocation methods share some common drawbacks which limit their applicabilities in cooperative spectrum sharing networks in cases where a PU and multiple SUs coexist. First, the energy consumption minimization strategies adopted in these methods usually consider a single relay node, which is typical merely for the best relay selection scheme. The adaptation of the strategies in the situation with multiple SUs selected simultaneously as relay nodes, as in the cooperative beamforming scenario, has not been investigated thoroughly so far. Furthermore, SUs may be reluctant to relay the traffic for the PU, if SUs do not forsee an immediate benefit or reward from their cooperation or they have some security and privacy concerns or encounter resource limitation, e.g., battery shortage [14]. This issue concerning the incentive mechanisms so as to prevent unwillingness and selfishness of the SUs has not been paid great attention in recent researches on cooperative beamforming networks. Moreover, the existing power allocation schemes are derived under an ideal channel assumption that channel coefficients for multiple links among the communication nodes are independent random variables following identical distributions. This leads to nonrealistic conclusions for performance evaluation. It is necessary to re-design the resource allocation methods by taking into account some new constraints posed by the existence of multiple relays, the energy costs of the communication coordination overheads, the influence of the rewards for the relay nodes, and realistic propagation scenarios.

In this paper, we propose an energy-saving strategy for cooperative beamforming by finding the optimal power allocation scheme. This method is particularly useful in cooperative beamforming for mobile spectrum sharing scenarios where a PU and multiple SUs co-exist. The proposed power allocation scheme is obtained by solving a convex optimization problem with the boundary conditions specifying the minimum QoS for the PU and SUs, similar to the approach described in [19]. A model of constant circuit power consumption originally introduced in [25] is considered for calculating the network power consumption. Simulations under the setting of realistic fast fading channels generated by using an infinite impulse response (IIR) filter demonstrate that the energy efficiency which is defined as the ratio of outage capacity over total energy consumed, i.e., a necessary extension from single-relay scenarios [22,34] to multi-relay cooperative beamforming cases, can be significantly improved by implementing the proposed optimization scheme. Furthermore, the proposed scheme exhibits superior performance compared with some conventional methods including the best relay selection approach.

The rest of the paper proceeds as follows. In Section 2, the considered cooperative beamforming scenario in a cognitive network is introduced. In Section 3, the optimization method of power allocation and the derivation of energy efficiency for cooperative beamforming are presented. Simulation results are elaborated in Section 4 for performance assessment of the proposed scheme. Eventually, conclusive remarks are given in Section 5.

## 2 System model

Let us consider a wireless network where totally $M$ SUs act simultaneously as relay nodes for one PU, as illustrated in Figure 1. A direct propagation path exists between the primary transmitter (PT) and primary receiver (PR). Each node is equipped with a single antenna for signal transmission and reception. A dual-hop decode-and-forward (DF) cooperative communication mode with

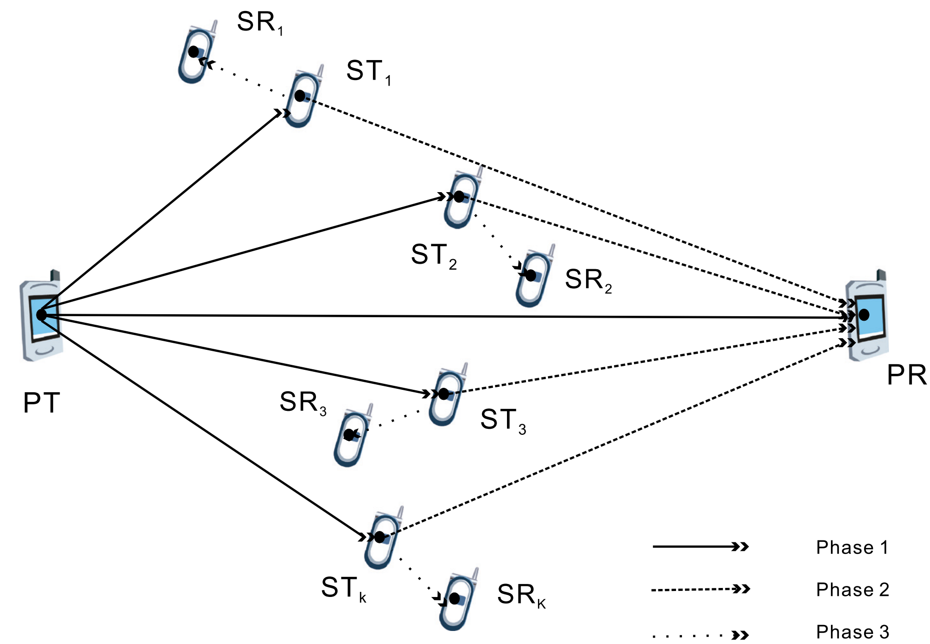


**Figure 1 Cooperative beamforming model with three phases.**

synchronization and diversity combining at the PR is considered. In return, the SU selected as a relay is rewarded with getting access to the channel for its own communication with a secondary receiver (SR) under the PU's authority.

Resource allocation can be performed with a frame or a slot structure according to the system design in a code division multiple access (CDMA) system, similar to Bluetooth, Wi-Fi, or sensor networks. The cooperative communication with all SUs and PU involved can be performed in three phases, as illustrated in Figure 2, where $T$ and $t_k$ denote the transmission time assigned to the PU and the SUs, respectively [35]. In the first phase, the PT broadcasts the data to the PR and all $M$ SUs that could act as relays. In the second phase, the SUs decode the received signals individually and forward to the PR, acting as secondary transmitters (STs). At the PR, the signals from the PT and all STs are soft-combined and decoded. An ST's own transmission to its SR is performed during a short period $t_k$ in the third phase. The overall time for completing these phases is then calculated as $T + t_k = \beta_k T$, where $\beta_k \geq 1$ can be determined optimally as a balance between

| Broadcast | Cooperative Beamforming | SUs' own transmission |
|---|---|---|
| $\frac{T}{2}$ | $\frac{T}{2}$ | $t$ |

**Figure 2 Time slot allocation for primary and secondary user.**

the ST's throughput and power consumption, as will be discussed later in this paper.

Noted that before the foregoing three phases, a short time period is usually allocated for the transmission of training signals and the estimation of CSI. The signal received for a link between any pair of nodes $i$ and $j$, can be written as:

$$y_j = h_{ij}\sqrt{P_{ij}}x_i + n_j, \tag{1}$$

where $h_{ij}$ is the complex channel coefficient from the node $i$ to the node $j$, $P_i$ denotes the transmitting power of the node $i$ to the node $j$ over the transmission bandwidth $W$, and $n_j$ is the thermal noise generated by the node $j$ and is modeled as an additive white Gaussian random variable with zero mean and spectral density height of $N_o$ (W/Hz). The complex channel coefficient $h_{ij}$ is assumed to be a complex circularly symmetric Gaussian random variable with magnitude $|h_{ij}|$ following a Rayleigh distribution. The mean of the Rayleigh distribution is dependent on the distance $d_{ij}$ between the $i$th node and the $j$th node according to $d_{ij}^{-\alpha}$, where $\alpha$ is an exponent constant describing the attenuation caused by the wave propagation. It is easy to show that $|h_{ij}|^2$ follows an exponential distribution with its rate parameter being $\lambda_{ij} = d_{ij}^{\alpha}$ [22]. From Equation 1, the SNR for the signals received at the node $j$ when the node $i$ transmits can be calculated as:

$$\mathrm{SNR}_{ij} = \frac{P_{ij}|h_{ij}|^2}{N_0}, \quad \gamma_{ij} \triangleq \frac{P_{ij}}{N_0}, \tag{2}$$

where $\gamma_{ij}$ is defined as the ratio between the transmission signal power over the noise variance.

In order to successfully decode the information received by $\mathrm{ST}_k$, the SNR of the link between PT and $\mathrm{ST}_k$, i.e., $\mathrm{SNR}_{ps_k}$ is usually required to be higher than a threshold $\theta_1$, i.e.,

$$\mathrm{SNR}_{ps_k} = \frac{P_{ps_k}\left|h_{ps_k}\right|^2}{N_0} = \frac{P_{ps_k}d_{ps_k}^{-\alpha}}{N_0} > \theta_1, \tag{3}$$

where $d_{ps_k}$ represents the distance from the PT to $\mathrm{ST}_k$, $P_{ps_k}$ denotes the transmission power of the PT, and $h_{ps_k}$ is the channel coefficient of the link between the PT and $\mathrm{ST}_k$. Applying the triangular principle for $d_{ps_k}$, the distance $d_p$ from the PT to the PR, and the distance $d_{s_kd}$ from $\mathrm{ST}_k$ to the PR, it is easy to show that $d_{ps_k}$ should be confined within the range described by the following inequalities:

$$\left|d_p - d_{s_kd}\right| < d_{ps_k} < \min\left\{\left(d_p + d_{s_kd}\right), \left(\frac{P_{\max}}{N_0\theta_1}\right)^{\frac{1}{\alpha}}\right\}, \tag{4}$$

where $P_{\max}$ represents the maximum transmission power of the network. Notice that Equation 4 can also be applied as a fundamental principle for relay selection when multiple STs are available in the network. In order to improve the understandings, explanations of the adopted symbols are demonstrated in Table 1.

In the following, an analytical expression is derived for the transmission rate of the cooperative communication system illustrated in Figure 1. In the first phase shown in Figure 2, the PT directly transmits to the PR without spectrum cooperation. The data rate per Hz $R_d$ for the link between the PT and the PR can be calculated as:

$$R_d = \log_2\left(1 + \gamma_d|h_d|^2\right), \tag{5}$$

where $\gamma_d$ denotes the ratio of the direct transmitting power $P_d$ of the PT to the noise variance and $h_d$ is the channel coefficient for the direct link from the PT to the PR.

In the second phase, cooperative beamforming is performed together with the direct transmission from the PT to the PR. The STs forward the signals to the PR after the decoding step, and the PR combines the signals originally transmitted by the PT and all STs using, e.g., the maximal ratio combining (MRC). The maximum data rate per second at the PR can be calculated by considering that all $M$ STs transmit during the time $T/2$ to be:

$$R_\mathrm{c} = \min_{k\in\kappa}\left\{R_{ps_k}, R_\mathrm{MRC}\right\}, \tag{6}$$

with $k$ being the indices of the STs, $\kappa \in \{1, 2, \ldots, M\}$, $R_{ps_k}$ representing the data rate between the PT and $\mathrm{ST}_k$, and $R_\mathrm{MRC}$ denoting the combination rate at the PR of the signals arriving from the PT and all STs. They can be calculated as respectively:

**Table 1 Explanation of adopted symbols**

| Symbol | Explanation |
|---|---|
| $y_j$ | Received signal at the $j$th node |
| $h_{ij}$ | Channel coefficient from $i$th to $j$th node |
| $P_{ij}$ | Transmitting power per bandwidth from the $i$th node to the $j$th node |
| $x_i$ | Transmitting signal at the $i$th node |
| W | Transmission bandwidth |
| $N_o$ | Spectral density height of thermal noise |
| $n_j$ | Thermal noise |
| $\lambda_{ij}$ | Rate parameter of exponential distribution |
| $d_{ij}$ | Distance between the $i$th and $j$th nodes |
| $R_d$ | Direct transmission data rate |
| $\gamma_d$ | Transmitting SINR of direct transmission |
| $h_d$ | Channel coefficient of direct transmission |
| $R_c$ | Data rate of PR |
| $R_{ps_k}$ | Data rate of the link of PT-$\mathrm{SR}_k$ |
| $R_\mathrm{MRC}$ | Data rate of MRC |
| $\gamma_p$ | Ratio between the transmission signal power over the noise variance for PU |
| $h_{ps_k}$ | Channel coefficient of the link PT-$\mathrm{SR}_k$ |
| $\gamma_{s_kd}$ | Ratio between the transmission signal power over the noise variance between $\mathrm{ST}_k$ and PR |
| $h_{s_kd}$ | Channel coefficient of the link $\mathrm{ST}_k$-PR |
| $R_{su_k}$ | Data rate of the $k$th SU |
| $\gamma_{su_k}$ | Ratio between the transmission signal power over the noise variance for the $k$th SU |
| $h_{su_k}$ | Channel coefficient of the link ST-SR |
| $R_N$ | Cooperative network throughput |
| T | Time for the 1st and 2nd phase transmission |
| $t_k$ | Time for the third phase transmission |
| $P_p$ | Transmitting power of PT |
| $P_d$ | Transmitting power of non-cooperative scheme |
| $P_{s_kd}$ | Transmitting power of the $\mathrm{ST}_k$ |
| $P_{su_k}$ | Transmitting power of the third phase |
| $w_k$ | Beamforming weight of the $\mathrm{ST}_k$ |
| $P_\mathrm{OV}$ | Circuit power |
| $P_\mathrm{SR}$ | Overheads of cooperative beamforming |
| $P_\mathrm{TR}$ | Overheads of message sharing |
| $P_c$ | Circuit power |
| M | Number of cooperative SUs |
| $b_\mathrm{RD}$ | Number of bits per symbol for cooperative beamforming |
| $b_\mathrm{SR}$ | Number of bits per symbol for message sharing |
| $\rho$ | Amplifier efficiency |
| $N_\mathrm{TR}$ | Number of symbols of training period |
| $N_\mathrm{RD}$ | Number of symbols of cooperative beamforming |
| $P_b$ | Power consumption per throughput |

**Table 1 Explanation of adopted symbols** *(Continued)*

| Symbol | Explanation |
|---|---|
| $\beta_k$ | Time allocation ratio of the *k*th SU |
| $Q_p$ | QoS of PU |
| $Q_s$ | QoS of SU |
| $P_d^{out}$ | Outage probability of direct transmission |
| $C_d^{out}$ | Outage capacity of direct transmission |
| $C_{su}^{out}$ | Outage capacity of SU |
| $\varepsilon$ | Outage probability threshold |
| $C_{cb}^{out}$ | Outage capacity of cooperative beamforming |
| $\eta$ | Energy efficiency |
| $\theta_1$ | SNR threshold for decoding |
| $\theta_2$ | SNR of the worst link between $ST_k$ and PR |
| $d_p$ | Distance between PT and PR |
| $d_{s_kd}$ | Distance between PT and PR |
| $d_{su}$ | Distance between ST and SR |
| $d_{ps_k}$ | Distance between PT and ST |
| $P_{max}$ | Maximum transmitting power |
| $\gamma_d$ | Ratio between the direct transmission signal power over the noise variance |
| $\zeta$ | Network energy consumption for unit bit of throughput |
| $L_s$ | Number of SUs |
| $v$ | Moving speed of users |

$$R_{ps_k} = \frac{1}{2}\log_2\left(1+\gamma_p|h_{ps_k}|^2\right) \tag{7}$$

$$R_{\text{MRC}} = \frac{1}{2}\log_2\left(1+\gamma_p|h_d|^2+\sum_{k=1}^{M}\gamma_{s_kd}|h_{s_kd}|^2\right), \tag{8}$$

where $\gamma_p$ denotes the ratio of the transmission power $P_p$ of the PT to the noise variance $N_0$, $\gamma_{s_kd}$ represents the ratio of the transmission power of the $\text{ST}_k$ and $N_0$, and $h_{s_kd}$ is the channel coefficient for the link between the $\text{ST}_k$ and the PR.

In the last phase after the cooperative beamforming is executed, each ST gets the permission to use the channel between $\text{ST}_k$ and $\text{SR}_k$. The data transmission rate can be calculated as:

$$R_{su_k} = \log_2\left(1+\gamma_{su_k}|h_{su_k}|^2\right), \tag{9}$$

where $\gamma_{su_k}$ and $h_{su_k}$ denote respectively the ratio of transmission power of the $\text{ST}_k$ and the noise variance and the channel coefficient of the link between the $\text{ST}_k$ and the $\text{SR}_k$.

The total network's throughput $R_\text{N}$ can then be calculated as:

$$R_N = \frac{TR_c + \sum_{k=1}^{M} t_k R_{su_k}}{\bar{t}+T}, \tag{10}$$

where $t_k$ is the time utilized in the third phase for the $k$th ST, and $\bar{t}$ represents the average of $t_k$, $k = 1, \ldots, M$. In the system considered here, every SU transmits its own information in its allocated channel, and the individual transmission time $t_k$ is determined using an optimization algorithm described in the later sections of the paper.

The network's power consumption $P_N$, as a holistic and system-wide metric, which includes all the energy consumption such as transmission power, circuit power, and signaling overhead in the entire network, can be calculated as:

$$P_N = \frac{\frac{T}{2}\left(P_p + \sum_{k=1}^{M} w_k P_{s_kd}\right) + \sum_{k=1}^{M} w_k t_k P_{su_k}}{\bar{t}+T} + P_{\text{OV}}, \tag{11}$$

where the optimal beamforming weight $w_k$ for the $k$th ST can be calculated as $|h_{s_kd}|/\|\boldsymbol{h}\|$ with $\|\cdot\|$ being the norm of given vector and $\boldsymbol{h} = \left[|h_{s_1d}|, |h_{s_2d}|, \ldots, |h_{s_Md}|\right]^T$ representing the vector consisting of the channel gains from the STs to the PR and $P_{\text{OV}}$ denotes the power overheads required in the source message sharing and channel estimation phases [25].

The overhead consumption $P_{\text{OV}}$ is obtained by using the constant circuit-power consumption model [25]:

$$P_{\text{OV}} = \left(\frac{P_{\text{SR}}}{\rho} + (1+M)P_c\right)\frac{b_{\text{RD}}}{b_{\text{SR}}} + \left(\frac{P_{\text{TR}}}{\rho} + (1+M)P_c\right)\frac{N_{\text{TR}}}{N_{\text{RD}}}, \tag{12}$$

where $P_{\text{TR}}$ denotes the power used for the training signals, $P_{\text{SR}}$ signifies the overhead power of message sharing period, $\rho$ represents the amplifier efficiency, $P_c$ is the circuit power, $b_{\text{RD}}$ and $b_{\text{SR}}$ denote respectively the number of bits per symbol during the cooperative beamforming period and during the message sharing period, $W$ is the available transmission bandwidth, and finally, $N_{\text{TR}}$ and $N_{\text{RD}}$ represent the number of symbols transmitted during the two periods, respectively. $P_{\text{TR}}$, is selected such that the SNR of the worst link between the $\text{ST}_k$ and PR is $\theta_2$. In addition, $P_{\text{SR}}$ can be calculated as:

$$\begin{aligned} P_{\text{SR}} &= \frac{\gamma_{\text{th}}(b_{\text{SR}})N_0W}{\min\left(|h_{ps_k}|^2\right)}, \\ \gamma_{\text{th}}(b_{\text{SR}}) &= -\frac{1}{c_2}\left(2^{b_{\text{SR}}}-1\right)\ln\left(\frac{p_{\text{th}}}{c_1}\right), \\ p_{\text{th}}(\text{b}) &= c_1\exp\left(-\frac{c_2 p_p}{\left(2^b-1\right)N_oW}\right), \end{aligned} \tag{13}$$

where $\gamma_{\text{th}}(b_{\text{SR}})$ is the SNR required to achieve the target M-ary quadrature amplitude modulation (M-QAM) bit error probability, $p_{\text{th}}$ and $c_1 = 0.2$, $c_2 = 1.5$.

To compute the average transmission rates for the entire time period ($T + t_k = \beta_k T$), we consider that the cooperative communication is performed within $1/\beta_k$ of the whole time period, and the secondary communication utilizes the spectrum in the rest of time, i.e., $(\beta_k-1)/\beta_k$ of the whole period. The average transmission rates of PU in the first and second phases and of SUs in the third phase can then be calculated as, respectively:

$$\bar{R}_p = \frac{R_c}{\bar{\beta}_k}, \bar{R}_{su} = \sum_{k=1}^{M} \frac{\beta_k-1}{\beta_k} R_{su_k} \text{ with } k = 1, \ldots, M. \tag{14}$$

## 3 An algorithm for energy consumption minimization and cooperative beamforming

### 3.1 Optimal power allocation for cooperative beamforming

We now derive an algorithm for determining the optimal power allocation with minimal energy consumption for the considered cooperative beamforming system. It is reasonable to assume that within the channel coherence time, CSI, correctly detected by corresponding UEs, has been sent to the PT. Based on the CSI, the transmission powers, i.e., $P_p$, $P_{s_kd}$, and $P_{su_k}$, shall be determined with the objective to minimize the average energy consumption of the network while guaranteeing users' QoS. To be specific, $P_{s_kd}$ denotes the transmission power of the $k$th ST to the PR, and $P_{su_k}$ represents the transmission power of the SU$_k$'s own transmission in the third phase. The predefined QoS of the PU and SU denoted with $Q_p$ and $Q_s$, respectively, are identified with the required transmission rate. The optimization problem can be formulated as:

$$\min_{P,\beta_k} T \cdot \left( \frac{1}{2\beta_k} \left(P_p + P_{s_kd}\right) + \frac{\beta_k - 1}{\beta_k} P_{su_k} \right), \tag{15}$$

$$\text{s.t.} \frac{1}{2\beta_k} \left( \log_2 \left( 1 + \frac{P_p |h_p|^2}{N_o} \right) + \log_2 \left( 1 + \frac{P_{s_kd} |h_{s_kd}|^2}{N_o} \right) \right) \geq Q_p, \tag{16}$$

$$\frac{\beta_k - 1}{\beta_k} \log_2 \left( 1 + \frac{P_{su_k} |h_{su_k}|^2}{N_o} \right) \geq Q_s, \tag{17}$$

$$\beta_k > 1. \tag{18}$$

The inequalities Equations 16 and 17 represent respectively the QoS requirements of the PU and of the SU. The left-hand sides of Equations 16 and 17 separately denote the transmission rate for PU during the cooperative communication (phases 1 and 2) and SU's own transmission (phase 3).

This convex optimization problem in Equations 15 to 18 satisfies the Karush-Kuhn-Tucker conditions, and it can be solved using the Lagrangian equation [36,37]. Besides, $T$ is normalized as 1. Detailed convexity analysis and the solution of the optimization problem are given in Appendix 1 and Appendix 2, respectively.

Despite the unawareness of $\beta_k$, the optimal powers can be computed as, respectively:

$$\begin{aligned} P_p &= N_o \left( \frac{2^{\beta_k Q_p}}{|h_p| \, |h_{s_kd}|} - \frac{1}{|h_p|^2} \right), \\ P_{s_kd} &= N_o \left( \frac{2^{\beta_k Q_P}}{|h_p| \, |h_{s_kd}|} - \frac{1}{|h_{s_kd}|^2} \right), \\ P_{su_k} &= \frac{N_o}{|h_{su_k}|^2} \left( 2^{\frac{\beta_k Q_s}{\beta_k - 1}} - 1 \right). \end{aligned} \tag{19}$$

Once these optimal transmission powers are known, the optimal $\beta_k$ can be calculated by solving the function:

$$\begin{aligned} &\frac{2^{\beta_k Q_p}}{|h_{s_kd}| \, |h_p|} \left( Q_p \beta_k - 1 \right) - \frac{\beta_k Q_s 2^{\frac{\beta_k Q_s}{\beta_k - 1}}}{|h_{su_k}|^2 (\beta_k - 1)} \\ &+ \frac{2^{\frac{\beta_k Q_s}{\beta_k - 1}} - 1}{|h_{su_k}|^2} + \frac{1}{2|h_p|^2} + \frac{1}{2|h_{s_kd}|^2} = 0. \end{aligned} \tag{20}$$

It can be readily shown in Appendix 2 that the left-hand side of Equation 20 has the derivative with respect to $\beta_k$ always larger than zero and, hence, increases monotonically along with $\beta_k$. Thus, the optimal $\beta_k$ can be determined by applying the Newton's iterative method as described in [38].

Notice that when the cooperative beamforming is not executed, the optimization problem for the power allocation via minimizing the energy consumption reduces to;

$$\min \{P_d\} \quad \text{s.t.} \log_2 \left( 1 + \frac{P_d |h_d|^2}{N_o} \right) \geq Q_p. \tag{21}$$

The optimal power for the direct transmission in this case can be calculated as:

$$P_d = \frac{1}{|h_d|^2} \left( e^{Q_p} - 1 \right). \tag{22}$$

This scheme is used as a reference for evaluating the performance of the proposed method in simulations.

### 3.2 Energy efficiency of the cooperative beamforming scheme

The expressions of the outage probability and outage capacity for the proposed cooperative beamforming scheme are derived in the following. It is easy to show that during the direct transmission from the PT to the PR, the outage probability between the PT and PR can be calculated as:

$$P_d^{\text{out}} = 1 - \exp \left( -d_d^{\alpha} \cdot \frac{2^R - 1}{\gamma_d} \right). \tag{23}$$

Hence, for a fixed outage probability $\varepsilon$, the outage capacity can be obtained as:

$$C_d^{\text{out}} = \log_2\left(1 + \frac{\gamma_d}{d_d^{\alpha}} \ln \frac{1}{(1-\varepsilon)}\right). \tag{24}$$

In the third phase, the outage capacity for an SU can be formulated similarly as:

$$C_{su}^{\text{out}} = \log_2\left(1 + \frac{\gamma_{\text{su}}}{d_{\text{su}}^{\alpha}} \ln \frac{1}{(1-\varepsilon)}\right), \tag{25}$$

where $d_{\text{su}}$ represents the distance between an ST and the corresponding SR.

From the data rate expressions in the second phase as shown in Equations 6 to 7, the outage probability $P_c^{\text{out}}$ of cooperative beamforming can be calculated as:

$$\begin{aligned} P_c^{\text{out}} &= \Pr\left(\frac{1}{2}\min\left(R_{ps_1}, R_{ps_2} \cdot\cdot R_{ps_k} \cdot\cdot, R_{ps_M}, R_{\text{MRC}}\right) < R\right) \\ &= 1 - \prod_{k=1}^{M}\left(1 - \Pr\left(R_{ps_k} < 2R\right)\right)\left(1 - \Pr(R_{\text{MRC}} < 2R)\right). \end{aligned} \tag{26}$$

The two cumulative density functions $\Pr\left(R_{ps_k} < 2R\right)$ and $\Pr(R_{\text{MRC}} < 2R)$ in the right-hand side of Equation 26 can be calculated as, respectively:

$$\Pr\left(R_{ps_k} < 2R\right) = 1 - \exp\left(-d_{ps_k}^{\alpha} \cdot \frac{2^R - 1}{\gamma_{ps_k}}\right), \tag{27}$$

$$\Pr(R_{\text{MRC}} < 2R) = F\left(2^{2R} - 1\right), \tag{28}$$

where $F(z)$ can be shown to be:

$$F(z) = \sum_{k=1}^{M+1} \frac{\left(1 - \exp(-\lambda_k z)\right) \prod_{i=1, i\neq k}^{i=M+1} \lambda_i}{\prod_{i=1, i\neq k}^{i=M+1} (\lambda_k - \lambda_i)} \tag{29}$$

with $z = \sum_{k=1}^{M} \gamma_{s_k d} \left|h_{s_k d}\right|^2 + \gamma_p \left|h_p\right|^2$, $\lambda_k = d_{s_k d}^{\alpha}$, and $\lambda_{M+1} = d_p^{\alpha}/\gamma_p$.

Substituting Equations 27 to 29 into 26, the outage probability $P_c^{\text{out}}$ is rewritten as:

$$P_c^{\text{out}} = 1 - \sum_{k=1}^{M} \left(g_{s_k d}\right)^{d_{s_k d}^{\alpha}} \left(1 - a\left(g_p\right)^{d_p^{\alpha}} + (1+a)\left(g_{s_k d}\right)^{d_{s_k d}^{\alpha}}\right), \tag{30}$$

with

$$\begin{aligned} a &= \left(\frac{P_{s_k d}}{P_p}\left(\frac{d_p}{d_{s_k d}}\right)^{\alpha} - 1\right)^{-1}, \\ g_{s_k d} &= \exp\left(-\frac{N_0\left(2^{2R} - 1\right)}{P_{s_k d}}\right), \\ g_p &= \exp\left(-\frac{N_0\left(2^{2R} - 1\right)}{P_p}\right). \end{aligned} \tag{31}$$

From Equations 26 to 27, it is obvious that $P_{\text{c}}^{\text{out}}$ increases along with $R$, while $g(x)$ is a decreasing function of $R$. Since $g_{s_k d} = g_p^{\frac{P_p}{P_{s_k d}}}$, by replacing $g_{s_k d}$ in Equation 30 with $g_p$, it can be shown that $P_c^{\text{out}}$ is a monotonically decreasing function of $g_p$. Thus, by equating Equation 30 with a threshold $\varepsilon$ of outage probability, we can solve for a specific $g_p$, denoted with $g_{p|\varepsilon}$ by using the Newton method.

$$C_{\text{cb}}^{\text{out}} = \log_2\left(1 + \gamma_{\text{p}} \ln \frac{1}{g_{p|\varepsilon}}\right). \tag{32}$$

Eventually, the energy efficiency of the cooperative beamforming scheme, which can be used as a metric evaluating the amount of useful information bit per unitary energy, is obtained as:

$$\eta = \frac{C_{\text{cb}}^{\text{out}}}{\text{P}_N}, \tag{33}$$

where $\text{P}_N$ can be obtained from Equation 11.

A widely adopted metric for evaluating the validity for saving network energy consumption is the value of 'bits-per-Joule' for the network, i.e., the network energy consumption for unit bit of throughput, denoted with $\zeta$ and calculated as $\zeta = P_{\text{N}}/R_{\text{N}}$. Energy efficiency, denoted with $\eta$, is defined as ratio of outage capacity over network power consumption.

## 4 Simulation results for performance evaluation

Monte Carlo simulations are conducted to evaluate the performance of the proposed minimum power allocation algorithm utilized in cooperative beamforming for a spectrum sharing system. The mobile wireless channel is generated using an IIR filter followed by linear interpolation based on the 'sum-of-sinusoids' principle proposed by Jakes in [39,40]. The distance $d_p$ between the PT and the PR varies within the range from 0 to 1,000 m. The distances between the STs and the PR are fixed to be 300 m, and the distances between the STs and their corresponding SRs are set to be 200 m. In the simulations, a ST is selected to be a relay if the distance between the PT and the ST satisfies Equation 4 and is upper-bounded by 800 m. The parameter setting of the simulations is reported in Table 2.

**Table 2 Parameter settings for simulation**

| Description | Value |
|---|---|
| Amplifier efficiency $\rho$ | 38% |
| Circuit power $p_C$ | 1 mw |
| Power for training signals $P_{TR}$ | 1 mw |
| Bits per symbol of MS $b_{RD}$ | 6 |
| Bits per symbol of CB $b_{SR}$ | 4 |
| Number of training symbols $N_{TR}$ | 1 |
| Number of symbols for CB $N_{RD}$ | 99 |
| Outage probability threshold $\varepsilon$ | 0.01 |
| Required QoS of PU/SU | 1/0.5 bps/Hz |
| Pathloss exponent $\alpha$ | 3.5 |
| PT-PR distance $d_p$ | $0 < d_p \leq 1{,}000$ m |
| PT-ST distance $d_{ps_k}$ | $d_{ps_k} \leq 800$ m |
| ST-PR distance $d_{s_kd}$ | $d_{s_kd} = 300$ m |
| ST-SR distance $d_{su}$ | $d_{su} = 200$ m |
| Maximum transmission power $P_{max}$ | 40 mw |
| Noise spectral density $N_o$ | 0.5 |
| Number of snapshots per meter | 20 |
| SNR threshold for decoding $\theta_1$ | 0.01 |
| SNR of the worst link between $ST_k$ and PR $\theta_2$ | 0.0001 |

### 4.1 Performance evaluation with the number $L_s$ of SUs as a parameter

Figure 3a,b depicts the average minimum network power consumption $P_N$ required for performing cooperative beamforming versus the PT-PR distance $d_d$ for the cases with and without considering the overhead cost $P_{OV}$, respectively. The number $L_s$ of SUs is considered as a parameter. Totally, 2,000 snapshots were simulated to calculate each point along the graph of average $P_N$ versus $d_d$. It can be observed by comparing Figure 3a,b that in the case without considering $P_{OV}$, the average minimum network power consumption $P_N$ decreases dramatically when $L_s$ increases, while in the situation where $P_{OV}$ is taken into account, the average $P_N$ graphs appear to be less deviated from each other when $L_s$ changes. This observation indicates that the energy costs for relay scheduling and information sharing can increase along with the number of the SUs involved. The advantage of cooperative beamforming is counteracted by the increasing of overheads to some extent.

Figure 4 illustrates the variation of $\zeta$, i.e., the network energy consumption per unit throughput versus the PT-PR distance $d_d$ with the overhead consumption $P_{OV}$ considered. The number $L_s$ of SUs is also taken as a parameter in the simulations. It can be observed that $\zeta$ decreases significantly with the growth of the SU number. This is due to the fact that the network throughput $R_N$ increases enormously along with $L_s$, and meanwhile, no evident

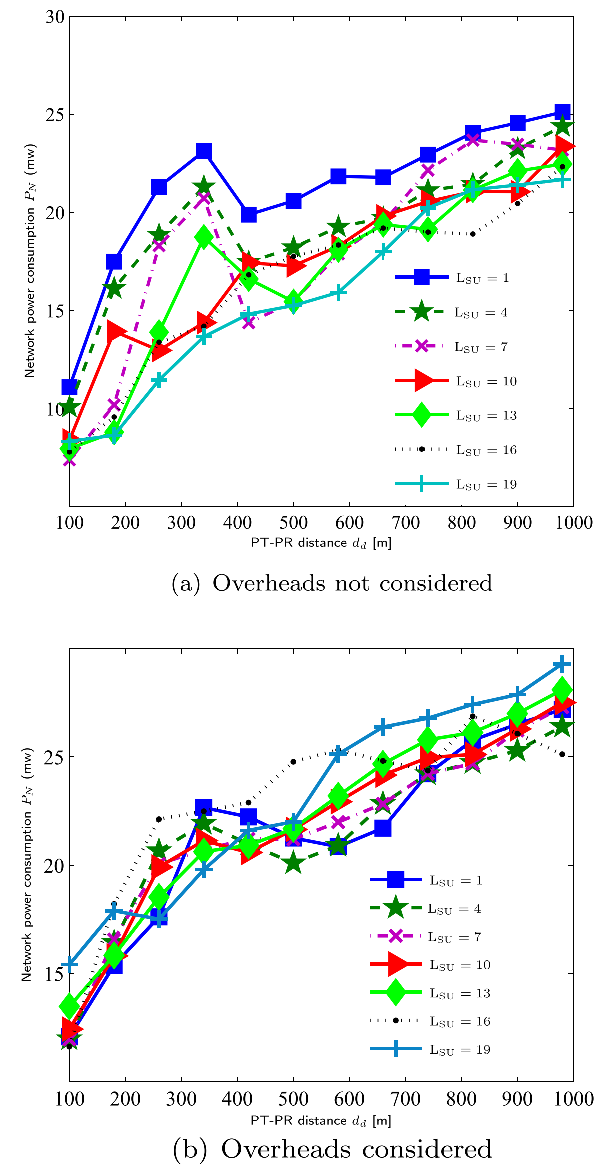


**Figure 3 Average minimum power consumption versus the PT-PR distance $d_d$ for different SU numbers with/without overhead considered. (a)** Overheads not considered. **(b)** Overheads considered.

increase of network power consumption is observed when $L_s$ grows when the overhead and scheduling consumption is considered.

The variation of the energy efficiency $\eta$ for the cooperative beamforming scheme when $L_s$ changes is illustrated in Figure 5. The overhead consumption has been considered in the simulations. Every scatter plot in Figure 5 is calculated by averaging the energy efficiency obtained with various $d_d$ from 2,000 snapshots. It is obvious from Figure 5 that the average $\eta$ grows linearly with the increasing number of SUs. A linear curve with a slope of 10

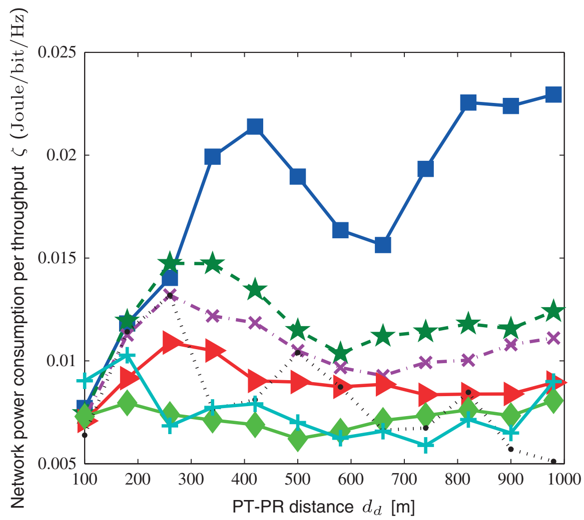


**Figure 4 Network power consumption per unit throughput $\zeta$ versus the PT-PR distance $d_d$ for different SU numbers with overhead considered.** The legend shown in Figure 3 also applies here.

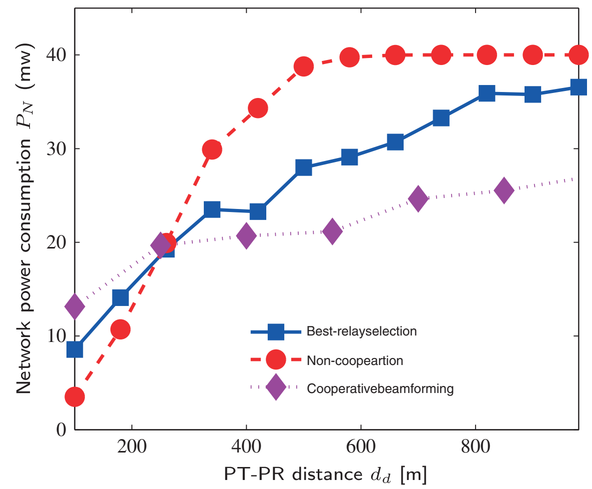


**Figure 6 Average minimum power consumption versus the PT-PR distance $d_d$ for three communication schemes with the number of SUs $L_s = 6$.**

bit/s/J/SU is found fitting well with the simulation results. The observation of more bits being transmitted when $L_s$ increases is due to the joint effect that the enhanced capacity of the cooperative beamforming system for larger $L_s$, and at the same time, the network power consumption in the cooperative beamforming phase is reduced with more SUs involved.

### 4.2 Comparison among three communication schemes

Figure 6 compares the network power consumption for three cases, i.e., best relay selection with the number of SUs $L_s = 6$, non-cooperation scheme with single relay, and the cooperative beamforming with $L_s = 6$.

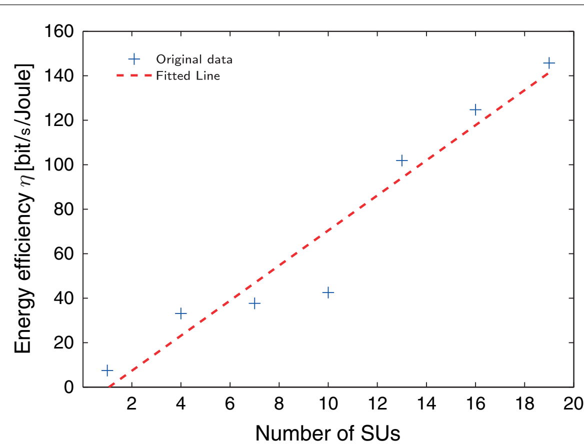


**Figure 5 Energy efficiency versus SU number with overhead considered.**

In the best relay selection algorithm, the optimal relay is selected with the criterium of maximum energy efficiency from $M$ candidate relays [35]. The threshold of QoS requirement of the PU and the SUs are set as respectively, 1 and 0.5 bps/Hz in the simulated communication schemes. It can be observed that the proposed cooperative beamforming scheme outperforms the other two, and the best-relay selection algorithm exhibits the intermediate performance. This is the rationale as the implementation of the relay selection process may deteriorate the performance in some cases when compared with the cooperative beamforming scheme. Furthermore, for a fixed number of SUs, without significant benefits from the energy minimization, the best relay selection scheme consumes the similar amount of energy for scheduling and of circuit power as the cooperative beamforming does.

Figure 7a,b illustrates the throughput $R_d$ of the direct transmission, the network throughput $R_N$, the average PU's throughput $R_p$ in the first and second phases, and the average SU's throughput $\bar{R}_S = \frac{1}{M}\sum_{k=1}^{M} R_{su_k}$, versus the PT-PR distance $d_d$ for the best-relay-selection scheme and the proposed cooperative beamforming scheme, respectively. The number $L_s$ of SUs equals 6 in the simulations. It can be found from Figure 7a,b that the direct transmission provides less $R_d$ for increasing $d_d$, and on the contrary, the PU's throughput $R_P$ and the network throughput $R_N$ with cooperative beamforming increase along with $d_d$. The throughput of SUs remains a constant level irrespective of $d_d$ as expected. These results reveal that the relay and the cooperative beamforming systems can provide higher throughput than the direct transmission. It can be furthermore observed by comparing Figure 7a,b that

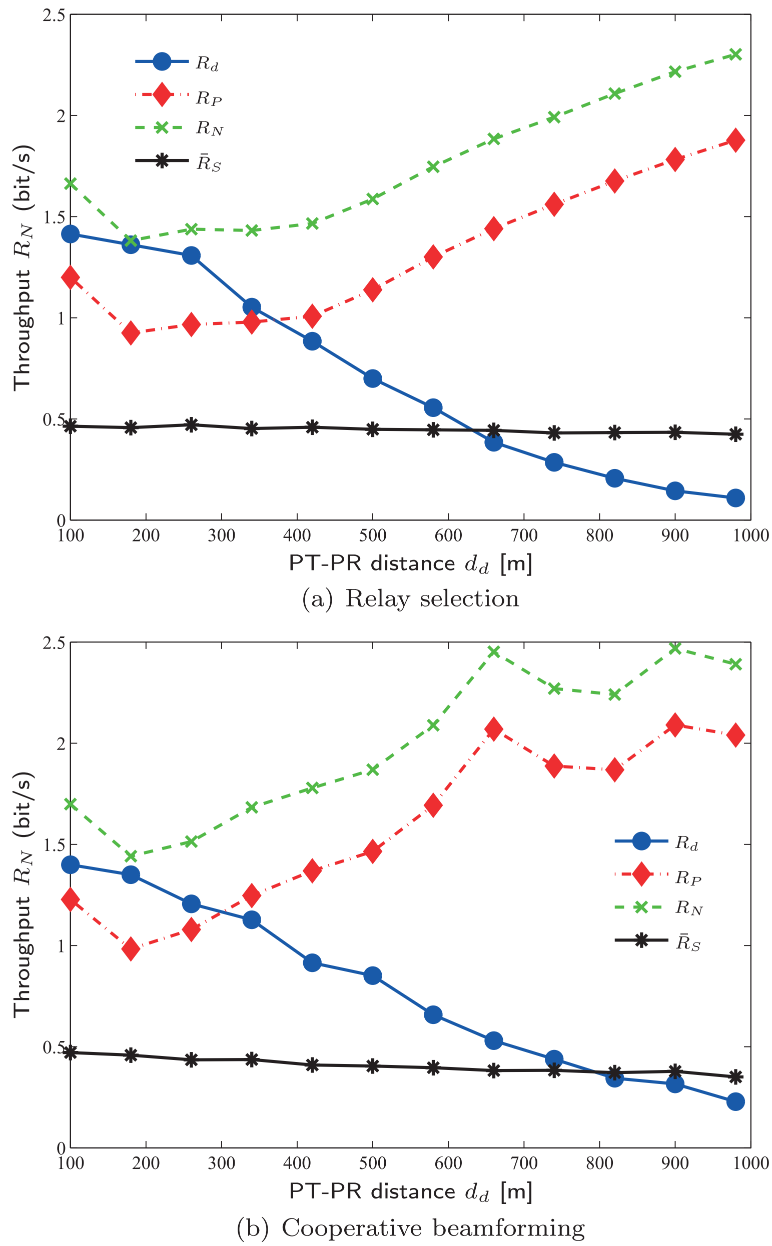


**Figure 7 Throughput versus the PT-PR distance $d_d$ for relay-selection and cooperative beamforming schemes with the number of SUs $L_s = 6$. (a)** relay selection. **(b)** Cooperative beamforming.

the proposed cooperative beamforming scheme exhibits throughput graphs similar to those obtained for the best relay scheme. However, since the network consumption is less for the cooperative beamforming than for the best relay scheme as illustrated in Figure 6, it is a natural conjecture that the energy efficiency, i.e., the network power consumption per unit throughput would be less in the case of cooperative beamforming than of the best relay. This is confirmed by the results shown in Figure 8 where the 'Joule-per-bit' for the cooperative beamforming is nearly a half of that observed for best relay selection with large $d_{\mathrm{d}}$.

Figure 9 compares the network energy efficiency $\eta$ versus the PT-PR distance $d_d$ for the best relay selection, non-cooperative and cooperative beamforming schemes with $L_s = 6$. It can be observed that the network energy efficiency of cooperative beamforming remains constant for different values of $d_d$ and is the highest among all three schemes considered. Furthermore, the graph of the energy efficiency of the best relay scheme exhibits

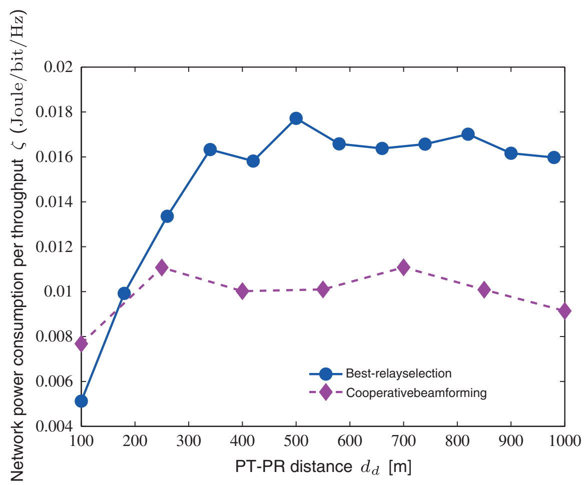


**Figure 8 Network power consumption per unit throughput $\zeta$ versus the PT-PR distance $d_d$ of two cases with $L_s = 6$.**

a concave shape with the maximum obtained with the distance equal to 500 m. This is consistent with the analysis shown in [41,42] that the network energy efficiency is concave with respect to the transmission power which monotonically increases along with the distance. Furthermore, in the case with short distances, the outage capacity is small, and a certain level of total power consumption still remains. Consequently, the energy efficiency becomes extremely low. In cases where the distance is sufficiently large, the outage capacity becomes saturated. In such case, since the power consumption keeps increasing, the energy efficiency decreases and converges to zero for large $d_d$.

### 4.3 Performance evaluation in vehicular scenarios

The performance of the cooperative beamforming scheme with proposed power allocation strategy is also evaluated by simulations in time-variant cases where the nodes in the network move in high speeds. In the simulations, the speeds of the PU and of the SUs are set to be identical and represented with $v$. The channel coefficients among the nodes are generated by using a tap-delay-line (TDL) model with six discrete paths. The number $L_s$ of SUs is fixed to be 5.

Figure 10 depicts the network power consumption $P_N$ versus the PT-PR distance $d_d$ for moving speed $v = 5$, 50, and 100 km/h. It can be observed from Figure 10 that the graph of $P_N$ versus $d_d$ observed for $v = 100$ km/h is only slightly above those obtained with less $v$. This indicates that the network power consumption for the cooperative beamforming is insensible to the speed of the nodes involved. Figures 11, 12, and 13 depict, respectively, the network throughput $R_N$, the energy consumption per unit throughput, and the energy efficiency $\eta$ versus the PT-PR distance $d_d$ for different $v$. From these figures, it can be observed that the performance of the cooperative beamforming scheme with proposed power allocation only slightly degrades when the speed of nodes increases. These results demonstrate the robustness of the proposed scheme against the time variability of the channels and the applicability of the scheme in vehicular scenarios.

## 5 Conclusions

In this paper, a novel power allocation scheme has been proposed for energy-efficient mobile cooperative communication with spectrum sharing. In the system, SUs act collaboratively as beamforming relays and help the PU in data transmission in reward of transmitting their own information. The optimal power allocation derived solves a convex optimization problem of minimizing network

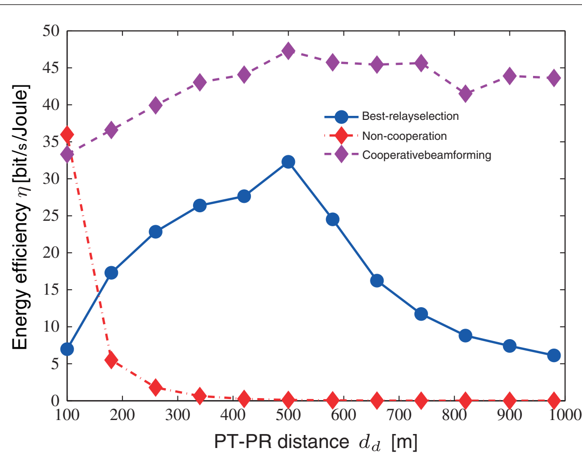


**Figure 9 Network energy efficiency versus the PT-PR distance $d_d$ of three cases with $L_s = 6$.**

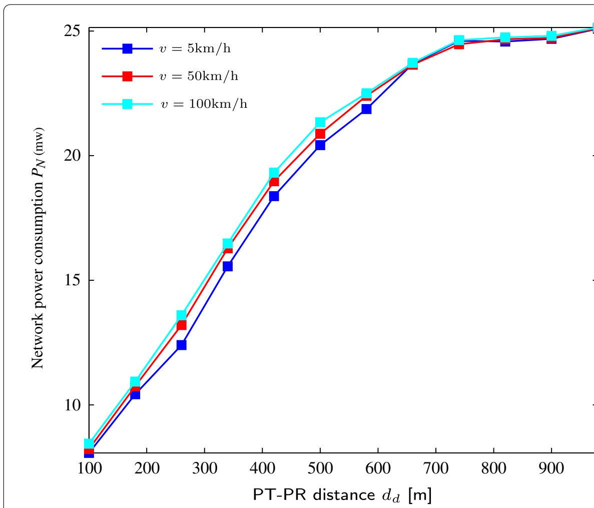


**Figure 10 Network power consumption $P_N$ versus the PT-PR distance $d_d$ with UE speed as a parameter.**

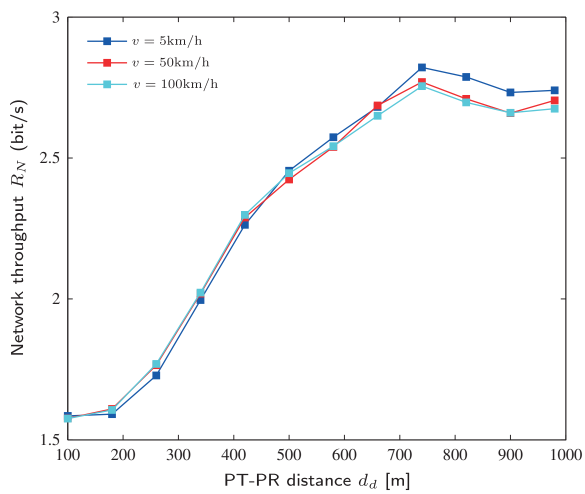


**Figure 11** Network throughput $R_N$ versus the PT-PR distance $d_d$ with UE speed as a parameter.

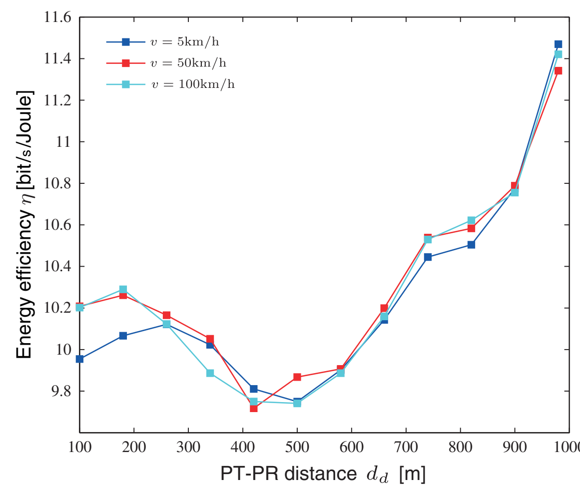


**Figure 13** Network energy efficiency $\eta$ versus the PT-PR distance $d_d$ with UE speed as a parameter.

energy consumption under the constraint of the guaranteed quality of service for both PU and SUs. Analytical expressions were derived based on the resultant optimal power allocation for outage capacity of the system. Numerical simulations have been conducted for evaluating the performance of the proposed scheme. The results demonstrated that the network power consumption exhibited different behaviors depending on whether the overhead power consumption is considered. By taking into account the overhead power consumption, circuit power, and message sharing energy costs, the cooperative beamforming cognitive network together with the proposed power allocation method performs better when more SUs are involved, in terms of enhanced energy efficiency and reduced power consumption per unit throughput. Furthermore, simulations demonstrated that the proposed cooperative beamforming scheme is superior to the conventional best relay selection scheme in the aspects of network power consumption, power consumption per unit throughput, and energy efficiency. In addition, the cooperative beamforming scheme with the optimal power allocation proposed here exhibited stable performance in the time-variant scenarios where both the PU and SUs move in high speeds.

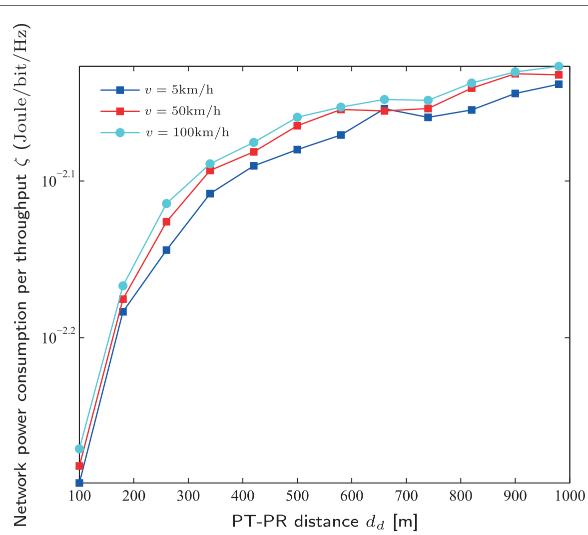


**Figure 12** Network power consumption per unit throughput $\zeta$ versus the PT-PR distance $d_d$ with UE speed as a parameter.

## Appendix 1

### Convexity analysis of the optimization problem in Equations 15 to 18

A convex optimization problem is one in which the objective and constraint functions are convex [37]. In general, a continuous and twice differentiable function is concave, if and only if its Hessian matrix is negative semi-definite. On the contrary, if the Hessian matrix is positively semi-definite, the function is convex. Now, we discuss the convexity of the two inequality constraints (16) and (17), i.e.,

$$\beta_k Q_p - \frac{1}{2}\log_2\left(1+\frac{P_p\left|h_p\right|^2}{N_o}\right) - \frac{1}{2}\log_2\left(1+\frac{P_{s_kd}\left|h_{s_kd}\right|^2}{N_o}\right) \leq 0 \quad \text{and}$$

$$Q_s - \frac{\beta_k - 1}{\beta_k}\log_2\left(1+\frac{P_{su_k}\left|h_{su_k}\right|^2}{N_0}\right) \leq 0.$$

Denoting $f\left(P_p, P_{s_kd}, \beta_k\right) = \beta_k Q_p - \frac{1}{2}\log_2\left(1+\frac{P_p\left|h_p\right|^2}{N_o}\right) - \frac{1}{2}\log_2\left(1+\frac{P_{s_kd}\left|h_{s_kd}\right|^2}{N_o}\right)$, we can compute the Hessian matrix of $f\left(P_p, P_{s_kd}, \beta_k\right)$ as:

$$H_f = \begin{pmatrix} \frac{\partial^2 f\left(P_p, P_{s_kd}, \beta_k\right)}{\partial {P_p}^2} & \frac{\partial^2 f\left(P_p, P_{s_kd}, \beta_k\right)}{\partial P_p \partial P_{s_kd}} & \frac{\partial^2 f\left(P_p, P_{s_kd}, \beta_k\right)}{\partial P_p \partial \beta_k} \\ \frac{\partial^2 f\left(P_p, P_{s_kd}, \beta_k\right)}{\partial P_{s_kd} \partial P_p} & \frac{\partial^2 f\left(P_p, P_{s_kd}, \beta_k\right)}{\partial {P_{s_kd}}^2} & \frac{\partial^2 f\left(P_p, P_{s_kd}, \beta_k\right)}{\partial P_{s_kd} \partial \beta_k} \\ \frac{\partial^2 f\left(P_p, P_{s_kd}, \beta_k\right)}{\partial \beta_k \partial P_p} & \frac{\partial^2 f\left(P_p, P_{s_kd}, \beta_k\right)}{\partial \beta_k \partial P_{s_kd}} & \frac{\partial^2 f\left(P_p, P_{s_kd}, \beta_k\right)}{\partial {\beta_k}^2} \end{pmatrix} = \begin{pmatrix} \frac{1}{2}\left(\frac{|h_p|^2 \ln 2}{N_0 + P_p |h_p|^2}\right)^2 & 0 & 0 \\ 0 & \frac{1}{2}\left(\frac{\left|h_{s_kd}\right|^2 \ln 2}{N_0 + P_{s_kd}\left|h_{s_kd}\right|^2}\right)^2 & 0 \\ 0 & 0 & 0 \end{pmatrix}.$$

Thus, the eigenvalues of $H_f$ are all above or equal to zero, which shows that $H_f$ is a positive semi-definite matrix, an implication that $f\left(P_p, P_{s_kd}, \beta_k\right)$ is a convex function and Equation 16 is a convex constraint.

Similarly, denoting $g\left(P_{su_k}, \beta_k\right) = Q_s - \frac{\beta_k - 1}{\beta_k} \log_2 \left(1 + \frac{P_{su_k}\left|h_{su_k}\right|^2}{N_0}\right)$, we can obtain the Hessian matrix of $g\left(P_{su_k}, \beta_k\right)$ as:

$$H_g = \begin{pmatrix} \frac{\beta_k - 1}{\beta_k} \frac{\left|h_{su_k}\right|^4 (\ln 2)^2}{\left[N_0 + P_{su_k}\left|h_{su_k}\right|^2\right]^2} & \frac{1}{{\beta_k}^2} \frac{\left|h_{su_k}\right|^2 \ln 2}{\left[N_0 + P_{su_k}\left|h_{su_k}\right|^2\right]} \\ \frac{1}{{\beta_k}^2} \frac{\left|h_{su_k}\right|^2 \ln 2}{\left[N_0 + P_{su_k}\left|h_{su_k}\right|^2\right]} & \frac{2}{{\beta_k}^3} \log_2\left(1 + \frac{P_{su_k}\left|h_{su_k}\right|^2}{N_0}\right) \end{pmatrix}.$$

Thus, the determinants of all leading principal minors of the $H_g$ are given by:

$$\left|H_g^1\right| = \frac{\beta_k - 1}{\beta_k} \frac{\left|h_{su_k}\right|^4 (\ln 2)^2}{\left[N_0 + P_{su_k}\left|h_{su_k}\right|^2\right]^2} > 0$$

$$\left|H_g^2\right| = \left[\frac{1}{{\beta_k}^4} \frac{\left|h_{su_k}\right|^4 (\ln 2)^2}{\left[N_0 + P_{su_k}\left|h_{su_k}\right|^2\right]^2}\right] [2(\beta_k - 1)\log_2\left(1 + \frac{P_{su_k}\left|h_{su_k}\right|^2}{N_0}\right) - 1].$$

From Equation 17, it can be shown that:

$$\left[2\left(\beta_k - 1\right)\log_2\left(1 + \frac{P_{su_k}\left|h_{su_k}\right|^2}{N_0}\right) - 1\right] \geq 2\beta_k Q_s - 1.$$

Thus, as long as the requirement of $2\beta_k Q_s - 1 \geq 0$, i.e., $Q_s \geq 0.5$ is satisfied, it is easy to show that Equation 17 is a convex constraint.

Furthermore, denoting the objective function in Equation 15 $z\left(P_p, P_{s_kd}, P_{su_k}, \beta_k\right) = \frac{1}{2\beta_k}\left(P_p + P_{s_kd}\right) + \frac{\beta_k - 1}{\beta_k} P_{su_k}$, the Hessian matrix of $z\left(P_p, P_{s_kd}, P_{su_k}, \beta_k\right)$ can be shown as:

$$H_z = \begin{pmatrix} 0 & 0 & 0 & -\frac{1}{2{\beta_k}^2} \\ 0 & 0 & 0 & -\frac{1}{2{\beta_k}^2} \\ 0 & 0 & 0 & -\frac{1}{{\beta_k}^2} \\ -\frac{1}{2{\beta_k}^2} & -\frac{1}{2{\beta_k}^2} & -\frac{1}{{\beta_k}^2} & \frac{1}{{\beta_k}^3}\left[P_p + P_{s_kd} - 2P_{su_k}\right] \end{pmatrix}.$$

It can be seen that all leading principal minors of $H_z$ are equal to zero matrixes. Thus, $z\left(P_p, P_{s_kd}, P_{su_k}, \beta_k\right)$ is a convex function in terms of $\left(P_p, P_{s_kd}, P_{su_k}, \beta_k\right)$ and the objective function in Equation 15 is convex.

In addition, the constraint (18) is a linear function concerning to $\beta_k$. Thus, the Hessian matrixes is zero matrix, indicating the convexity of the constraint (18).

Based on these analyses, we conclude that the formulated optimization problem in Equations 15 to 18 is convex, under the condition that QoS of SU is larger than or equal to 0.5.

## Appendix 2

### Solution of the optimization problem in Equations 15 to 18

As Equations 15 to 18 are a convex optimization problem with differentiable objective and constraint, the KKT conditions are necessary and sufficient for the optimality. Furthermore, $T$ is normalized to be 1 without loss of generality. By adding Lagrange multipliers, i.e., $\lambda$, $\mu$, $\nu$, according to the constraints of Equations 16 to 18, the Lagrange dual function can be formulated as:

$$\begin{aligned} L\left(P, \beta_k, \lambda, \mu, \nu\right) = & \frac{1}{2\beta_k}\left(P_p + P_{s_kd}\right) + \frac{\beta_k - 1}{\beta_k} P_{su_k} \\ & + \lambda\left(\beta_k Q_p - \frac{1}{2}\log_2\left(1 + \frac{P_p|h_p|^2}{N_o}\right) - \frac{1}{2}\log_2\left(1 + \frac{P_{s_kd}|h_{s_kd}|^2}{N_o}\right)\right) \\ & + \mu\left(Q_s - \frac{\beta_k - 1}{\beta_k}\log_2\left(1 + \frac{P_{su_k}|h_{su_k}|^2}{N_0}\right)\right) + \nu\left(1 - \beta_k\right). \end{aligned}$$

By solving the following equations simultaneously:

$$\begin{aligned} & \frac{\partial L\left(P, \beta_k, \lambda, \mu, \nu\right)}{\partial P_p} = 0, \\ & \frac{\partial L\left(P, \beta_k, \lambda, \mu, \nu\right)}{\partial P_{s_kd}} = 0, \\ & \frac{\partial L\left(P, \beta_k, \lambda, \mu, \nu\right)}{\partial P_{su_k}} = 0, \\ & \frac{\partial L\left(P, \beta_k, \lambda, \mu, \nu\right)}{\partial \beta_k} = 0, \\ & \lambda\left(\beta_k Q_p - \frac{1}{2}\log_2\left(1 + \frac{P_p|h_p|^2}{N_o}\right) - \frac{1}{2}\log_2\left(1 + \frac{P_{s_kd}|h_{s_kd}|^2}{N_o}\right)\right) = 0, \\ & \mu\left(Q_s - \frac{\beta_k - 1}{\beta_k}\log_2\left(1 + \frac{P_{su_k}\left|h_{su_k}\right|^2}{N_0}\right)\right) = 0, \\ & \nu\left(1 - \beta_k\right) = 0, \\ & \beta_k Q_p - \frac{1}{2}\log_2\left(1 + \frac{P_p|h_p|^2}{N_o}\right) - \frac{1}{2}\log_2\left(1 + \frac{P_{s_kd}|h_{s_kd}|^2}{N_o}\right) \geq 0, \\ & Q_s - \frac{\beta_k - 1}{\beta_k}\log_2\left(1 + \frac{P_{su_k}\left|h_{su_k}\right|^2}{N_0}\right) \geq 0, \\ & 1 - \beta_k \geq 0, \end{aligned}$$

we obtain the optimal point in terms of the powers:

$$P_p = N_o\left(\frac{2^{\beta_k Q_p}}{|h_p|\,|h_{s_k d}|} - \frac{1}{|h_p|^2}\right),$$
$$P_{s_k d} = N_o\left(\frac{2^{\beta_k Q_P}}{|h_p|\,|h_{s_k d}|} - \frac{1}{|h_{s_k d}|^2}\right),$$
$$P_{su_k} = \frac{N_o}{|h_{su_k}|^2}\left(2^{\frac{\beta_k Q_s}{\beta_k - 1}} - 1\right). \tag{34}$$

Once these optimal transmission powers are known, the optimal $\beta_k$ can be calculated by solving the function:

$$\frac{2^{\beta_k Q_p}}{|h_{s_k d}|\,|h_p|}(Q_p\beta_k - 1) - \frac{\beta_k Q_s 2^{\frac{\beta_k Q_s}{\beta_k - 1}}}{|h_{su_k}|^2(\beta_k - 1)} + \frac{2^{\frac{\beta_k Q_s}{\beta_k - 1}} - 1}{|h_{su_k}|^2} + \frac{1}{2|h_p|^2} + \frac{1}{2|h_{s_k d}|^2} = 0. \tag{35}$$

Denoting the left-hand side of Equation 35 as $f(\beta_k)$ and taking the derivative of $f(\beta_k)$ with respect to $\beta_k$ as follows:

$$f'(\beta_k) = \frac{{Q_p}^2 \beta_k 2^{\beta_k Q_p}}{|h_{s_k d}|\,|h_p|} + \frac{{Q_s}^2 \beta_k 2^{\frac{\beta_k Q_s}{\beta_k - 1}}}{|h_{su_k}|^2(\beta_k - 1)^3}, \tag{36}$$

it is clear that Equation 36 is always larger than zero, and hence, $f(\beta_k)$ increases monotonically along with $\beta_k$. Thus, the optimal $\beta_k$ can be determined by applying the Newton's iterative method as described in [38]. Given an initial $\beta_k$ as $\beta_k(0)$, the following process is repeated:

$$\beta_k(n+1) = \beta_k(n) - \frac{f(\beta_k(n))}{f'(\beta_k(n))} \tag{37}$$

until a sufficiently accurate value is reached.

**Competing interests**
The authors declare that they have no competing interests.

**Authors' contributions**
CL carried out the generic studies, performed the simulation platform, and drafted the manuscript. XY gave guidance in the implementation of the simulation software and modification of the contribution. SRB conceived of the study, participated in its design and coordination, and helped to draft the manuscript. MGL offered some help in the conceiving of the research. All authors read and approved the final manuscript.

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