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Spectrum sensing using low-complexity principal components for cognitive radios

Zeba Idrees[1]*, Farrukh A Bhatti[2] and Adnan Rashdi[1]

Abstract

Principal component (PC) algorithm has recently been shown as a very accurate blind detection technique in comparison with other covariance-based detection algorithms. However, it also has a higher complexity owing to the computation of the eigenvectors. We propose a low-complexity Lanczos principal component (LPC) algorithm that utilizes Lanczos iterative method to compute the eigenvectors. In comparison with the PC algorithm, the proposed LPC algorithm offers significant reduction in complexity while giving a similar detection performance. Low-complexity LPC algorithm allows for the use of larger sized covariance matrix that further improves the detection performance. Maximum-minimum eigenvalue (MME) algorithm is also included in the comparison and it gives an inferior performance as compared to both PC and LPC algorithm. All the algorithms were tested with experimental data while using universal software radio peripheral (USRP) testbed that was controlled by GNU radio software.

Keywords: Spectrum sensing; Cognitive radio; Covariance-based detection; Software defined radio

1 Introduction

Cognitive radio has the ability to communicate over the unused frequency spectrum intelligently and adaptively. Spectrum sensing in a cognitive radio (CR) is crucial in generating awareness about the radio environment [1]. Blind detection methods such as covariance-based detection (CBD) algorithms enable signal detection in low signal-to-noise ratio (SNR) conditions without relying on the prior knowledge of the primary user's (PU) signal. CBD techniques also overcome the issue of noise power uncertainty that exists in an energy detector [2, 3]. These methods use the covariance and variances of the received signal and do not require information about the noise variance. The performance of the CBD algorithms is associated with the number of samples involved in the detection. However, using a large number of samples also increases the sensing time and complexity [4–8]. Recently, principal component analysis (PCA) has been applied for spectrum sensing in cognitive radios [9–11]. The principal of dimension reduction has been used in [12] to devise PC algorithm that outperforms other CBD algorithms,

such as Maximum-minimum eigenvalue (MME), maximum eigenvalue detection (MED), and energy with maximum eigenvalue (EME). PCA reduces the dimensionality of the data and retains the most significant components that account for the greatest variation of the original data [13–16]. However, the PC algorithm also has the highest complexity in comparison with other CBD techniques.

As envisaged in internet of things (IoT), the number of things (or devices) connected to the network might exceed the number of human users. The same idea also derives the research on 5G networks where the network capacity will be enhanced by a 1000 fold. Opportunistic spectrum access may help in this scenario where multiple overlaid devices try to access the spectrum. Spectrum sensing can help eradicate collisions and excessive contention delay experienced by dense node deployment. Such devices/sensors have embedded computing nature, hence energy efficiency is their major concern. Therefore, low complexity, energy efficient spectrum sensing algorithms are vital for implementation in such devices.

In this paper, we firstly analyse the performance of the PC algorithm while considering the effect of dimension reduction on the detection performance and the related complexity. Next we describe the proposed Lanczos principal component (LPC) algorithm, which is an energy efficient version of the PC algorithm. The LPC can be used

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in low powered devices (e.g. sensors employed in IoT) for blind signal detection. The performance of LPC is compared with PC as well as MME algorithms. MME has been included in the comparison as it is the best-known CBD technique. All the algorithms were tested with actual wireless microphone signals using a universal software radio peripheral (USRP)2 testbed and GNU radio software. The specific contributions of this paper are as follows.

- The detection performance of the PC algorithm is analysed under a low SNR (< -15 dB) scenario while varying the number of principal components included in the decision test statistic. The effect of dimension reduction on the sensing performance is also considered along with the complexity involved in each case.
- A low-complexity LPC algorithm is proposed that employs an iterative approach to compute the principal components and achieves the similar detection performance as of the PC algorithm with a reduced complexity. This reduction in complexity saves the sensing time and improves the energy efficiency.
- The performance of the proposed LPC algorithm is compared with the PC and the MME algorithms while using the actual signals. In addition, computational complexity of all the three algorithms (MME, PC and LPC) is also computed and compared mathematically and graphically.
- All the algorithms has been evaluated under both single and multiple receive antenna system with actual wireless microphone signals. Experimental setup is established using USRP2 and GNU radio software.

2 System model, detection with MME and PCA
2.1 System model
We have considered both a single and a multiple receive antenna system. f_s is the signal's sampling rate and $f_s >> w$, here w is the received signal's bandwidth. The signal is over-sampled to get a high correlation between the samples. Let $T_s = 1/f_s$ be the sampling period. We define $y(n) \triangleq y(nT_s)$, $s(n) \triangleq s(nT_s)$ and $w(n) \triangleq w(nT_s)$, where $y(n)$ is the received signal after passing through the channel, $s(n)$ is the PU's signal and $w(n)$ is the additive white Gaussian noise (AWGN). $w(n)$ follows a normal distribution $w(n) \sim \mathcal{N}(0, 1)$. Complex baseband samples from a single receive antenna system can be represented as

$$\mathbf{y}(n) = [y_1(n), y_1(n-1), \cdots, y_1(n-N+1)]. \quad (1)$$

Where N is the total number of samples used in making a single decision about the signal's presence under both hypotheses. For a multiple receive antenna system with M

radio frequency (RF) front ends, the complex base band samples are represented as

$$\mathbf{y}(n) = [y_1(n), \cdots, y_M(n), y_1(n-1) \cdots, y_M(n-1),$$
$$y_1(n-N/M+1) \cdots, y_M(n-N/M+1)]. \quad (2)$$

Spectrum sensing can be expressed as

$$y(n) = \begin{cases} w(n): & H_0 \\ s(n) + w(n): & H_1, \end{cases} \quad (3)$$

Where H_0 is the null hypothesis that shows the absence of the PU's signal. H_1 is the alternative hypothesis which indicates the presence of the signal. Probability of detection (P_d) and probability of false alarm (P_{fa}) characterizes the sensing performance. Where $P_d = \text{Pr}(H_1|H_1)$ and $P_{fa} = \text{Pr}(H_1|H_0)$.

2.2 MME
MME algorithm calculates the eigenvalues (λ) and finds the ratio of maximum to minimum eigenvalue [7].

$$\lambda_{\max}/\lambda_{\min} > \psi \quad (4)$$

PU's signal exists if the ratio is greater than ψ, where ψ is the threshold set according to desired probability of false alarm. MME is a blind detection algorithm without noise uncertainty issue, but calculation of eigenvalues using conventional method is computationally intensive.

2.3 PC Algorithm
Considering L consecutive outputs where L is the smoothing factor and N samples are involved in making a detection decision. Data set consists of baseband samples that are complex, and follow a normal distribution. $k = 2M$ dimensional data set in the matrix form is

$$\mathbf{X} = [\text{X}(n), \text{X}(n-1), \ldots, \text{X}(n-N+1)]. \quad (5)$$

We have a finite number of samples which needs to define a sample covariance matrix instead of a statistical covariance matrix, expressing $\text{X}(n-i)$ as X_i covariance matrix will be

$$\mathbf{R}(N) = \frac{1}{N} \sum_{i=1}^{N} \text{X}_i \text{X}_i^T. \quad (6)$$

Let $\lambda_1 \geq \lambda_2 \geq \lambda_3 \geq \ldots \lambda_k$ be the eigenvalues of $\mathbf{R}$ such that

$$|\mathbf{R} - \lambda \mathbf{I}| = 0, \quad (7)$$

Here $\mathbf{I}$ is the identity matrix with dimension as of covariance matrix $\mathbf{R}$. $\gamma_1, \gamma_2, \gamma_3 \ldots \gamma_k$ are the characteristic vectors of $\mathbf{R}$. To form a feature matrix, first, we need to select G, that is, the number of most significant eigenvector corresponds to the highest eigenvalues where $1 \leq G \leq k$. Principal components associated with eigenvalues larger in magnitude than the average of the eigenvalues are

taken. The transformation of the original data set is as below

$$\mathbf{p}_i = \mathbf{F}^T \mathbf{x}_i, \tag{8}$$

$\mathbf{F}$ is the feature matrix containing G most significant eigenvectors $i = 1, 2, .., N$ complete PC can be expressed as $P_j = [p_{j1}, p_{j2}, \ldots, p_{jN}]$, $j = 1, 2, \ldots . G$. The set of PCs with no redundant data is $\mathbf{P} = [P_1, P_2, \ldots, P_G]^T$ with g row vectors each containing N entries.

$$\sum_{i=1}^{N} p_{ji}^2 = \lambda_j. \tag{9}$$

(9) follows that jth PC gives the distribution of the energy given by the jth eigenvalue [15]. Test statistic T to distinguish between signal and noise is as follows [12]

$$T = 1/N \sum_{j=1}^{g} p_{j1}^2 + p_{j2}^2 + p_{j3}^2 + \ldots + p_{jN}^2 > \psi. \tag{10}$$

Here p_{ji} is the ith element of jth PC and ψ is the detection threshold determined empirically at a desired probability of false alarm. PU exists if $T > \psi$.

2.4 Performance analysis of the PC algorithm
Wireless microphone signals were used to formulate the PU, simulations were done using N samples. The value of N is set to 20,000. Figure 1 represents the probability of detection versus SNR curves with varying number of PCs. It can be seen from the figure that the probability of detection is improved as we include more principal

components for detection; in Fig. 1, 'pc' represents the number of principal components included in test statistic. It is also clear that the first two PCs retain the most useful information as compared to the last ones. There is an improvement in detection performance while moving from one to two and two to three PCs, but no significant improvement is observed when we include all four PCs, that is the point where we encounter the redundant data that can be discarded to save the sensing time. This supports the statement that the variables with little variances could be discarded without significantly influencing the total variance, thereby reducing the number of variables. The use of smaller number of PCs not only reduces the sensing time but also loses some useful info that results in a low probability of detection in our case. As we increase the number of PCs, detection time also increases because it gets involved more number of operations to calculate the test statistic that increases the complexity of the algorithm.

3 Proposed LPC algorithm
From the performance analysis of the PC algorithm, it is observed that the detection performance can be improved by including more PCs while using larger sized covariance matrix. However, doing so increases the complexity. In this section, we propose the LPC algorithm that uses the iterative method to compute the eigenvectors, required to generate PCs; the use of this approach significantly reduces the complexity. The proposed method performs much faster than the existing method, as it only computes
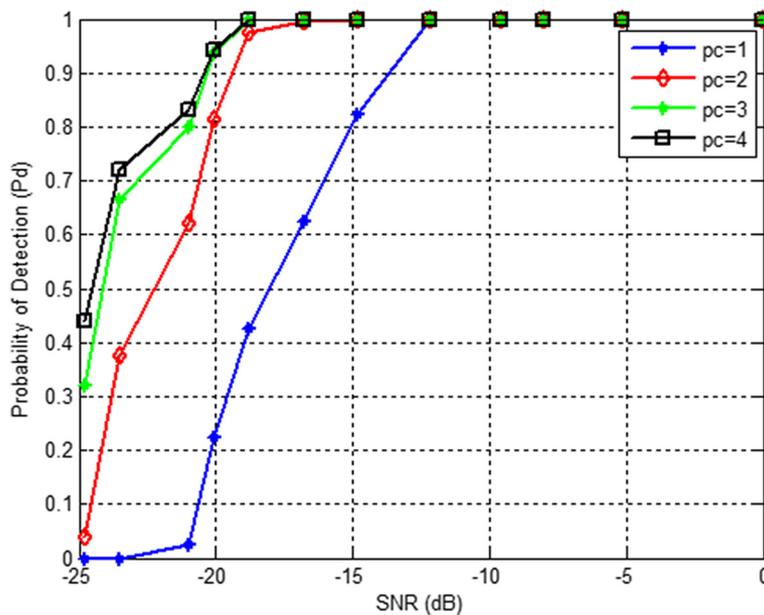
Fig. 1 Analysis of the PC algorithm. Improvement in probability of detection with increasing number of principal components at 10 % probability of false alarm and $N = 20,000$

the eigenvectors corresponding to the highest eigenvalues, whereas the direct method computes all the eigenvectors, thus wasting resources in computing in significant eignvectors. This approach is more efficient as it obviates the calculation of all the eignvectors and then sorting them at the end. Advance algorithms such as Lanczos and Arnoldi save this data and use the Gram-Schmidt process or Householder algorithm to reorthogonalize them into a basis spanning the Krylov subspace corresponding to the matrix. As the matrix size increases, the direct method becomes very slow, therefore not feasible practically, while the proposed method only calculates the desired eigenvectors via an iterative approach.

There are many iterative approaches like Arnoldi algorithm, Jacobi-Davidson algorithm and Lanczos algorithm [17]. We found Lanczos algorithm appropriate for our scenario as it has the least convergence time as compared to other approaches [17]. A disadvantage of this algorithm is that the number of iterations can be large. To cater for this issue, a variation of the Lanczos algorithm known as implicitly restarted Lanczos algorithm (IRLA) is used to compute the desired eigenvectors [18]. Implicitly restart (IR) extracts the useful information from a large Krylove subspace and resolves the storage issue and the difficulties associated with the standard approach. IR does this by compressing the useful data into a fixed size k dimensional subspace. IRLA is summarized in Algorithm 1 [18]. A is the symmetric matrix of interest v is the starting vector, $T_k \in \Re^{k \times k}$ is real, symmetric and tridiagonal with nonnegative subdiagonal elements. $V_k \in C^{n \times k}$ (the columns of V_k) are the Lanczos vectors.

Here k represents the desired number of eigenvectors to calculate and r is the residual. Selection of shift μ_j depends upon the user's required set of vectors. o steps of shifted QR iterations are applied to T_m using $\mu_1, \mu_2, ...\mu_o$ as shifts. $[\beta_k = T_m(k+1,k)] = 0$ if we use exact shifts. LPCA can be summarized as follows:

1. Calculate the covariance matrix as in (6).
2. Decompose the covariance matrix via implicitly restarted Lanczos algorithm as described in Algorithm 1.
3. Generate the principal components.
4. Calculate test statistic T as in (9).
5. Decide between H_1 and H_0 by comparing the T with a predetermined threshold (Empirically determined at the desired probability of false alarm).

3.1 Computational complexity comparison
As of the other covariance-based detection techniques, complexity of the PC algorithm also comprises of two major steps, one is the computation of the covariance matrix as in (6) and the other is the decomposition of the covariance matrix to calculate eigenvectors in our

case. As the covariance matrix is a block Toeplitz and Hermitian, due to these properties of covariance matrix, we only need to evaluate its first block. Calculation of the covariance matrix requires $(M^2 L N_s)$ multiplications and $O(M^2 L(N_s - 1))$ additions; here M is the number of receive antennas, L is the smoothing factor and N_s is the number of samples [4]. Eigen decomposition of the covariance matrix requires $O(M^3 L^3)$ multiplications and additions [5].

Therefore, the total complexity of eigenvalue-based detection algorithm (i.e. MME) is $O(M^2 L N_s) + O(M^3 L^3)$. Hence, after sorting, the total complexity of the PC algorithm becomes $O(M^2 L N_s) + O(M^3 L^3) + O(L^2)$, the complexity involved in the generation of principal components is negligible as compare to the calculation and decomposition of covariance matrix. The use of the iterative approach reduces the complexity up to $2L^2$ [17]. Complexity with purposed LPC algorithm reduced to $O(M^2 L N_s) + O(M^3 L)$. Figure 2 shows the lognormal plot of the complexity of all three algorithms (MME, PCA and LPCA).

Algorithm 1: Implicitly restarted Lanczos algorithm

Starting from $v_1 = v/\|v\|$
here v is the starting vector
Compute an m-step Lanczos factorization ($m = k + o$)
$AV_m = V_m T_m + r_m e_m^*$
Repeat until convergence ($T_k = D_k diagonal$)
Compute $\sigma(T_m)$ and select p shift $\mu_1 \mu_2, ..., \mu_o$
Initialize $Q = I_m$
For $j = 1, 2, ..., o$
QR-factorize $Q_j R_j = T_m - \mu_j I$
Update $T_m = Q_j^* T_m Q_j$, $Q = Q Q_j$
End For
$r_k = v_{(k+1)} \beta_k + r_m \sigma_k$, with
$\beta_k = T_m(k+1, k)$ and $\sigma_k = Q(m.k)$
$V_k = V_m Q(:, 1:k)$, $T_k = T_m(1:k, 1:k)$
Beginning with $k - steps$ Lanczos Factorization
$AV_K = V_k T_k + r_k e_k^*$
To get a new m-step Lanczos Factorization do apply on additional steps
$AV_m = V_m T_m + r_m e_m^*$
End repeat

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4 Experimental setup
The algorithms were tested using the USRP testbed. Two receiver systems with single ($M = 1$) and multiple antenna ($M = 2$) were set up with the help of USRP2 for receiving correlated wireless signals, as shown in Fig. 3. Transmission parameters were set as in [12]. USRP and USRP2 are generally used hardware platforms in the field
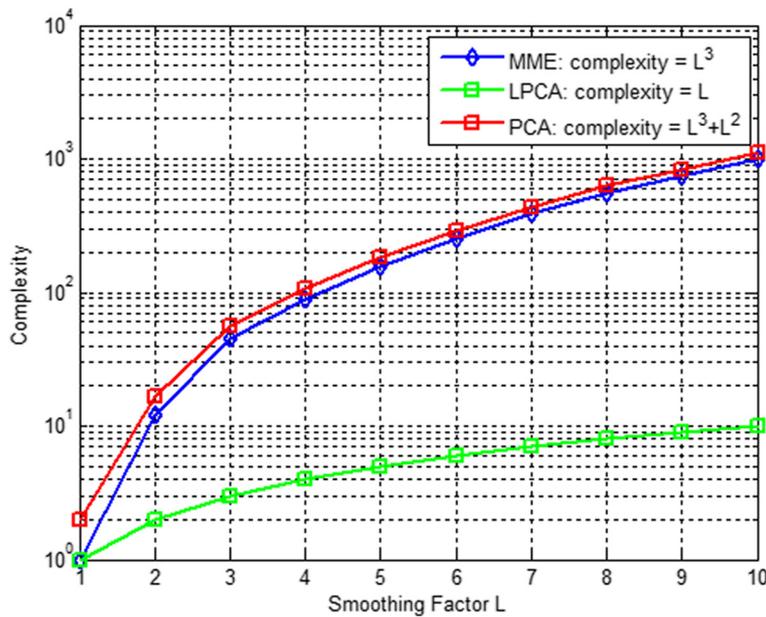
Fig. 2 Lognormal plot of complexity. Complexity plot of proposed LPCA, existing PC and MME method

of cognitive radios and SDR provided by Ettus Research [19]. It consists of a motherboard and a selectable RF daughter boards along with a gigabit ethernet port that can be attached to a host computer for further processing. WBX was connected to the motherboard. It has a single transmit and receive antenna and covers a wide frequency range of 50 MHz to 2.2 GHz [19]. One of the advantages of USRP2 is that it works with GNU radio, open source software along with an abundance of resources, which simplify its usage [20]. Signal reception

and down conversion was performed by the RF daughterboard. Afterwards a gigabit ethernet was used to pass the down-converted signal to host computer for further processing. A master/slave configuration was established to connect the two USRPs. The host computer was connected to the master USRP through an ethernet interface. The two USRP kits were interconnected via a MIMO cable that enables the synchronization between them. In addition, it also transfers down-converted signals from slave to master. Synchronization is achieved by feeding two
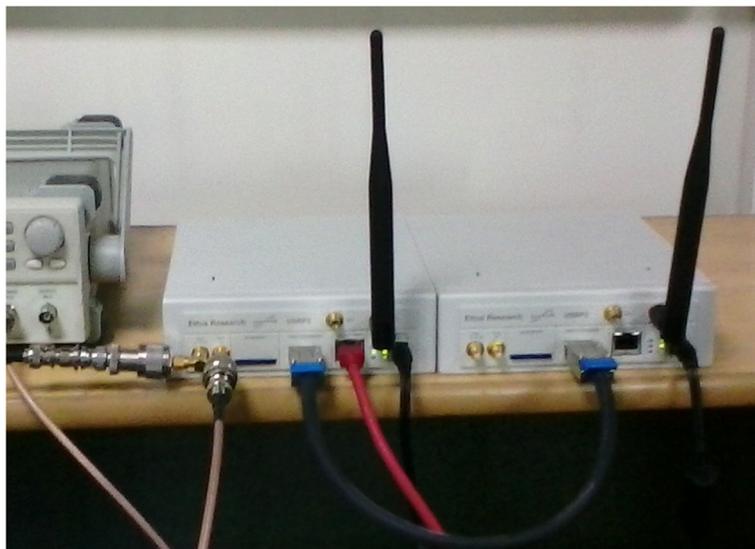
Fig. 3 Experimental setup. Experimental setup with two receive antennas

reference signals to the master USRP. A frequency reference is provided by a 10-MHz signal and sample time is synchronized by a one pulse per second (1 PPS) signal. In our experimental setup, the PU was emulated using an Rhode & Schwarz SMF100A microwave signal generator that transmits an FM signal at a frequency of 410 MHz with a bandwidth of 200 kHz [21]. The SNR was varied by changing the transmit power of the signal generator. The sampling rate of the received signal was 6.25 Mega samples per second. A high correlation between the samples is achieved by over-sampling. The received signals were fed to the host computer through an ethernet cable for subsequent performance analysis. The SNR was measured by tuning off the transmitter and recording the noise at each RF front end. The power of the signal at M_{th} front end is calculated by $P_M = 1/S \sum_{n=1}^{S} |X_M(n)|^2$. Where S is the total samples used for computing power, $P_{M,1}$ represents the power of signal and $P_{M,0}$ is the power of noise.

$$SNR_M = 10log_{10}[\,(P_{M,1} - P_{M,0})/P_{M,0}\,]. \tag{11}$$

5   Results and discussions

This section describes the performance comparison of the LPC, PC and MME algorithms. Figure 4 shows the performance analysis of the three algorithms in terms of probability of detection vs. SNR at 10 % probability of false alarm for single ($M = 1$) and multiple ($M = 2$) receive antenna cases. Probability of detection improves for the $M = 2$ case as the use of multiple antennas at the receiver overcomes the effect of channel fading and also

enhances the robustness against interference [22]. Antennas were configured for correlated reception (i.e antenna spacing $< \lambda_c/2$) which enhance the received signal's correlation between the samples. The results obtained from the PC and LPC algorithms were with $G = 2$ and the number of samples used in making a single detection decision N was kept constant for all the three algorithms. It was observed that the proposed method (LPCA) gives similar detection performance as of the PC method with significantly reduced complexity and it also outperforms the MME algorithm. There is no effect on the detection test statistic except the reduction in computational complexity, this allows the inclusion of more PCs that ultimately improve the detection performance. The proposed algorithm reduces the complexity by using an iterative approach and subspace method. Improvement in probability of detection is attained using LPCA, this improvement is achieved by using the covariance matrix of order $L = 6$ instead of $L = 4$ while in case of PC and MME algorithm $L = 4$. With this L the complexity of PC algorithm is $L^3 + L^2 = 80$ and MME has the complexity $L^3 = 64$ while that of the LPCA is $L = 6$, the proposed LPCA gives the better detection with low complexity even using large covariance matrix.

The direct method becomes slow as the matrix size increases; therefore, it is not feasible practically. With the direct method, while getting the required eigenvector, we also got a series of vectors, which were, finally, discarded, this can result in a large amount of disregarded information; LPCA overcomes this issue and makes the things simpler. Receiver operating curves at different SNR are

Fig. 4 Performance comparison. Comparison of performance in term of probability of detection for single and multiple receive antennas cases at 10 % probability of false alarm $G = 2, N = 60,000$
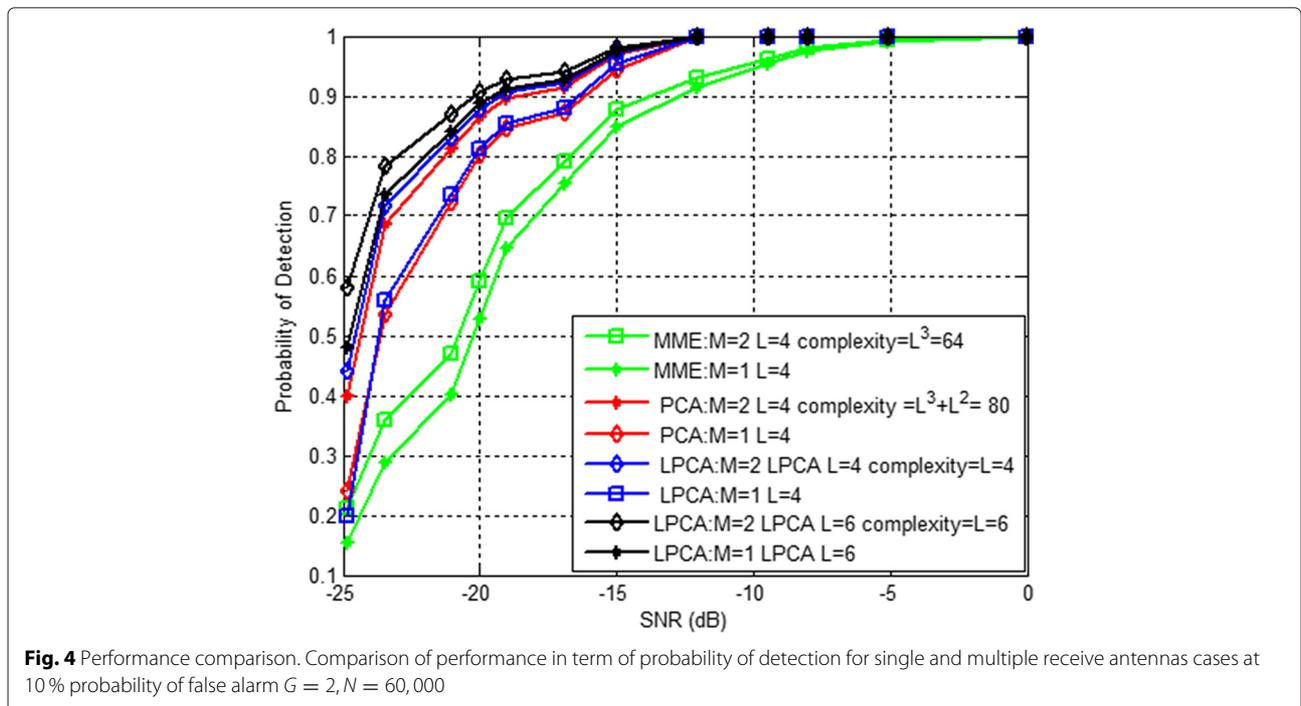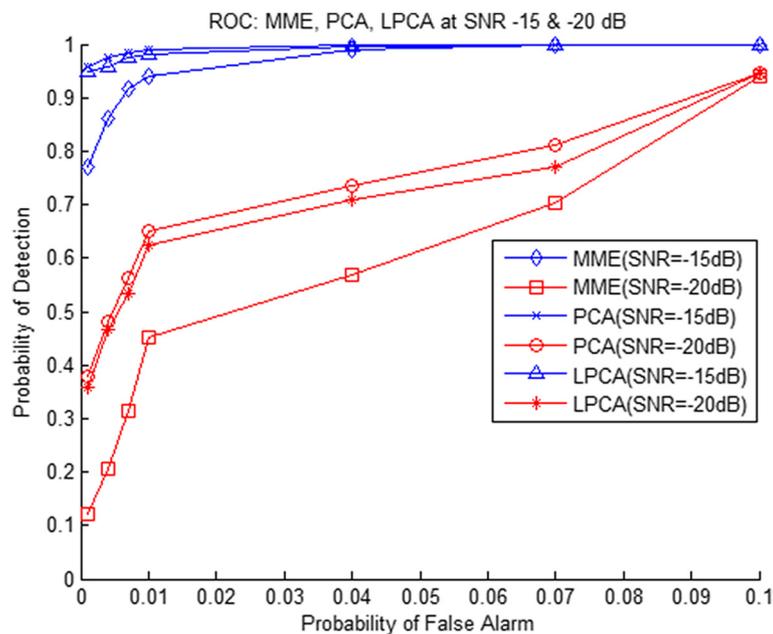
Fig. 5 ROC. Receiver operating curve probability of detection (P_d) vs. probability of false alarm (P_f) at SNR −15 and −20 dB

shown in Fig. 5 for MME, PC and LPCA at SNR −15 and −20 dB with $L = 4$ and $N = 60,000$. Detection threshold was empirically calculated by considering the normalized histogram of the test statistic under H_0 and corresponds to probability of false alarm of range 0 to 1. It can be seen from the figure that receiver operating characteristics (ROC) of both PC and LPCA are almost the same (Fig. 5). As expected, the proposed algorithm gives the same performance as that of the PC algorithm with a reduced complexity and significantly exceeds the sensing requirements defined by the FCC, that is, achieving 90 % P_d with 10 % $P_f a$ at SNR of −12 dB for wireless microphone signals.

6 Conclusion

Complexity is a major issue in blind signal detection algorithms that are based on a covariance matrix. The use of a large number of received samples increases the size of the covariance matrix and as a result the complexity. In this paper, we proposed a novel algorithm for blind signal detection, i.e. LPC algorithm that has an iterative nature which reduces the complexity and saves sensing time. LPC achieves the same detection performance as PC, yet its complexity is significantly less than the PC algorithm. Thus, LPC can be used in even low-powered devices for blind signal detection. The performance of LPC is compared with PC as well as MME algorithms. The proposed method gives the best sensing performance while reducing the complexity from $O(L^3 + L^2)$ to $O(L)$. All the algorithms are tested with actual wireless microphone signals while using a USRP2 testbed and GNU radio

software. In the future, these algorithms can be tested on a stand-alone platform for real-time performance evaluation.

Authors' information
Zeba Idrees received her B.E. degree in telecommunication engineering from the Govt. College University Faisalabad, Pakistan and a Masters degree in electrical engineering from the National University of Sciences and Technology (NUST), Islamabad, Pakistan in 2012 and 2014, respectively. She possesses professional and research experience of more than 2 years in academia as well as industry. During her research period, she conducted research on Cognitive radio networks. She has a research interest in the areas of Wireless communications and Signal Processing.

Farrukh A Bhatti received his B.E. and M.S. degrees in avionics engineering and electrical engineering from the National University of Sciences and Technology, Pakistan, in 2004 and 2008, respectively. He received his Ph.D. degree in electrical and electronic engineering from The University of Auckland, New Zealand, in 2014. During his Ph.D., he undertook research field trips to the Industrial Research Ltd Wellington and the Wireless @ Virginia Tech research group, Virginia Polytechnic Institute and State University, Blacksburg, VA, USA. Presently, he is an Assistant Professor in the department of electrical engineering at the Institute of Space Technology, Islamabad, Pakistan. His research interests include advanced cellular networks, multiple-antenna systems, software-defined radios and cognitive radio networks.

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