Tabakovic and Grgic *EURASIP Journal on Wireless Communications and Networking* (2016) 2016:45
DOI 10.1186/s13638-016-0536-1

EURASIP Journal on Wireless Communications and Networking
a SpringerOpen Journal

RESEARCH Open Access

Cognitive radio frequency assignment with interference weighting and categorization

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Abstract

Cognitive radio is one of the technologies promoting flexible and efficient use of radio frequency spectrum, thus solving spectrum scarcity problem. Frequency assignment is an integral part of the cognitive radio spectrum management and a critical point of success or failure of the cognitive radio concept. In this paper, cognitive radio frequency assignment with a novel interference weighting and categorization is proposed, as an extension of the solution to the graph coloring problem. In our approach, the edge weights quantifying the interference potential are appended to the conflict graph, co- and adjacent channel interference are treated, and dynamically changing local lists of blocked frequencies are included. We propose the improved cognitive radio saturation metric for the dynamic vertex ordering and to introduce interference categorization which will reduce the communication overhead. Using the proposed model, resource manager can quantify individual interference components, as well as aggregate interference from multiple users, resulting in more knowledgeable frequency decisions. Generalization of the proposed model is suggested. The suggested generalization consists of the selection of the central frequency and optimal bandwidth to be used, according to the user requirements. We have developed interference-sensitive algorithms for minimizing the interference and maximizing the throughput, both in centralized and distributed implementation. The results show significant reduction of the interference, improved spectrum efficiency, and increase in network throughput, comparing to the benchmark algorithms.

Keywords: Cognitive radio, Frequency assignment, Resource allocation, Graph coloring, Spectrum management, Dynamic spectrum access, Interference modeling

1 Introduction

The future market demand for the wireless broadband services, internet of things, machine to machine communications, and wireless data offload requires the deployment of the next generation wireless networks, which will need a rapid and a more flexible access to the radio frequency spectrum. Since radio frequencies are a limited natural resource, which cannot be saved for the future use or transferred from underused to overloaded areas, efficient usage of the frequencies has become one of the major concerns of the industry, governments, and scientific research. As a result of these research efforts, many different proposals for better and more efficient usage of the radio frequencies have emerged. One of the promising technologies, which could make a big paradigm shift towards the future wireless networks, is a cognitive radio (CR) technology. CR is introduced in [1, 2] and refers to intelligent radio communications system employing technology that allows the system to obtain knowledge of its operational environment, using technology to learn from the environment and dynamically adjust its operational parameters and protocols [3].

Problem addressed. CR frequency assignment represents the core of the cognitive radio process, since its unique features, along with reconfigurability and learning, make the cognitive radio what it is. The frequency assignment (FA) problem in wireless communications is well-studied, due to the extensive research of FA in mobile radio networks, wireless mesh networks, broadcasting networks, or in military applications [4–6]. Most of these studies do not consider primary-secondary spectrum sharing nor the dynamic availability of the spectrum due to the presence of primary users. Taking a deeper look

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into these research approaches, we can deduce that the cognitive radio FA has many unique and specific features, which makes CR unsuitable for transposition of the same FA models and algorithms from other types of wireless networks.

Task of the frequency assignment in the CR context is to select the best available radio frequency spectrum to satisfy user's communication needs, while avoiding causing harmful interference to the primary users (PUs) and minimizing influence to the other concurrent secondary users (SUs) sharing the same frequency band. Since the spectrum opportunities used for dynamic spectrum access in CR are limited in both time and space, CR has to be very fast and pragmatic in using the available spectrum bands. Suboptimal but fast converging and robust solutions are preferred in relation to the optimal one which usually requires extensive data sets and too much time to obtain results. Having this in mind, we can present basic characteristics of FA in CR:

- CR has to operate as a secondary service on non-interfering, non-protection basis, alongside other services using the same frequency band.
- CR has to work non-intrusively, has to protect PUs, and vacate the spectrum in case of a PU appearance.
- CR FA has to be fast, adaptive and easy to implement and does not necessarily have to provide an optimal FA.
- CR FA has to re-initiate and re-converge in cases of the quality degradation due to a dynamic changes in the radio channel or SU and PU activation.
- CR FA has to allocate the channels and also the spectrum fragments of a different bandwidth.
- CR FA has to work with the limited information in an environment with either cooperative or selfish SUs.
- CR FA has to be flexible, applicable in centralized and distributed manner.
- CR FA has to assign frequencies continuously and sequentially when a part of the PUs and SUs are already operating.
- CR FA has to be applicable in the heterogeneous wireless environment with the different classes of the dynamic spectrum access, different user requirements, and different protection requirements.

Our contributions. Our paper is focusing on the FA component of the dynamic spectrum management in the CR context. The problem of quantifying the interference using interference weighting and categorization in CR FA process, as well as CR FA algorithms with a numerical measure of interference levels between SUs as described in this paper, have not been previously addressed to the best of our knowledge. The FA in the CR networks presented in this paper is formulated as a variation of the graph coloring problem with the dynamic vertex ordering, where specific CR features are included. In this paper, the special attention is granted to the modeling and control of the interference. Numerical quantification of the interference is enabled with the inclusion of the conflict graph edge weights representing co- and adjacent channel interference between the SUs. We have developed the centralized and distributed algorithms with interference minimization and throughput maximization as objectives. Specifically, the contributions and novelty of this paper can be summarized in the following:

- The extension of the CR network conflict graph with:
  - Introduction of the continuous value edge weights quantifying the potential level of co- and adjacent channel interference between SUs
  - Incorporation of the influence of the adjacent channel interference by introducing an additional layer in the conflict graph
- Introduction of the SU interference categorization for interference susceptibility of the FA algorithms, while reducing the communication overhead
- The proposal of the spectrum fragments assignment with the identification of the central frequency and optimal bandwidth for the CR transmissions
- Introduction of a new saturation metric for a dynamic sorting of the vertices in the process of the sequential FA, taking into consideration the channel limitations due to the PU transmissions as hard constraints and a level of the interference from the adjacent assigned CRs as soft constraints
- The proposal and evaluation of the interference sensitive FA algorithms with the objective of:
  - Minimal total CR network interference
  - Maximal CR network throughput with consideration of the network interference as a comprehensive metric

In our paper, the condition of not causing harmful interference to the active PUs is a hard constraint, which has to be respected and cannot be violated by any FA plan. This is realized by the list of the blocked channels at each SU. Each SU has its own local list of the blocked channels, which is dynamically changed in time and space as the PUs frequency usage pattern constantly changes. The soft constraints are conditions which can be violated in the FA process if there is no better choice of the channel. The violations of the soft constraints are penalized by the FA algorithm, resulting in the deterioration of the objective function. The objective function takes into consideration the interference among SUs and the throughput achieved by the SU transmissions.

Our work differs from the previous works on the graph coloring-based CR networks FA in several aspects: the level of the interference between the SUs and between the SUs and PUs are numerically quantified using both the continuous and discrete weights; the influence of the co-channel and adjacent channel interference are considered; all individual interference contributions and cumulative interference values are monitored in the course of FA process. Proposed FA algorithms use this unique feature of the numerical quantification of the interference to precisely control and assign frequencies that have minimal interference influence in relation to other users. The advantages of the proposed interference sensitive FA strategy are numerous. It significantly reduces the interference in the CR networks, fully protects the PUs, reduces the frequency reuse distance between the CR users, improves spectrum usage efficiency, and increases the overall networks throughput with the available radio spectrum.

Paper organization. The rest of the paper is organized in the following way. Section 2 presents the overview of the previous research along with the comparison and main differences with respect to our paper. In Section 3, we formulate a CR network FA problem, present the system model and propose a novel framework for the interference sensitive FA in the CR. In Section 4, an interference estimation model used in the algorithm design is elaborated, characterization and categorization of co- and adjacent channel interference is introduced, and determination of weighting coefficients is elaborated. We describe the phases of the proposed FA algorithms, saturation metric, and coloring label with the objective of interference minimization and throughput maximization and the transmit power control strategy in Section 5. In Section 6, considerations of the centralized and distributed implementation of the FA algorithms are discussed. In Section 7, generalization to the frequency and bandwidth spectrum assignment are proposed, as well as implementation consideration in practical CR systems. In Section 8, we present an evaluation of the proposed algorithms' performance in the simulated environment of the PU and CR networks. Finally, we conclude the paper and outline the key results of this paper in Section 9.

2 Related work

Spectrum management is technical, procedural, and policy approach to the planning, coordination, and managing the use of the electromagnetic spectrum as a limited resource. The CR spectrum management functions have some specific capabilities, since the CR networks are focusing on the secondary access and spectrum sharing, while protecting the PUs and minimizing interference with the other SUs. An insightful overview of the spectrum management in the CR networks is presented in [7, 8]. The problem of the FA is commonly solved using different heuristic methods, the graph theory, game theory, or linear programming. In the FA modeled as a the graph coloring problem, the task is to color all the nodes of the graph with the minimum number of colors, in a way that no two adjacent nodes (nodes connected with an edge) have the same color [4–6]. A comprehensive analysis of the frequency assignment in the CR networks can be found in [9, 10].

The comparisons of the different target objectives, the approaches for the FA, and the used methods for the selected previous works presented in this section are summarized in Table 1. A deeper discussion on the insights of the related work and the main differences with respect to this paper are presented below. In [11], the authors propose a spectrum decision framework determining a set of spectrum bands taking into account the application requirements and dynamic nature of spectrum bands. For the real time applications, authors propose spectrum decision with the objective of total capacity variance minimization, while for the best effort applications objective is network capacity maximization. The authors propose a new metric for calculating the capacity of a CR user, namely, the expected normalized capacity of a user taking into account network switching delay. This spectrum switching delay includes the time intervals for the spectrum decision process in the base station, signaling for establishing new channels and the RF front-end

Table 1 Summary of FA problem solutions presented in related work

Ref.	Objective	Approach	Method
[11]	Throughput/ variance	Centralized single ch	Linear integer optimization
[12]	Power/total throughput	Distributed / centralized	Graph theory
[13]	Fairness/throughput	Centralized multi ch	Graph theory
[14]	Node connectivity/ interference	Local	Heuristic algorithm
[15]	Throughput surplus	Distributed / centralized	Nash bargaining / game theory
[16]	Fairness/QoS	Distributed list coloring	Graph theory
[17]	Fairness	Local multi ch	Graph theory / heuristic
[18]	Throughput	Macro BS femtocell	Graph coloring
[19]	Interference	Wireless mesh	Graph coloring
[20]	Interference/power	Distributed	Game theory

reconfiguration. The dynamic resource management for the spectrum decision is elaborated including admission and decision control. The authors propose centralized spectrum management using linear integer optimization method with a single channel assignment. Two phase CR network power control and channel allocation is proposed in [12]. In the first phase, distributed power updating process is employed in order to maximize the coverage of the CR network, while assuring that the signal-to-interference-plus-noise ratio (SINR) constraints at all PU receivers are met. In the second phase, a centralized channel allocation scheme based on weighted bipartite graph matching is used. The objective of the channel assignment is the maximization of the total network throughput while protecting the PUs. Both, uplink and downlink scenarios, are investigated with distributed and centralized implementation. The authors consider a cellular CR network with base and terminal stations, where the interference between the SUs are not considered. Contrary to this, in our approach, the SUs protect the PUs using the local list of blocked channels, the interference between the SUs is measured with conflict edge weights, and FA algorithms consider the interference quantification in the frequency decisions. In [13], the FA problem for the opportunistic spectrum access is investigated. The authors propose a novel FA approach using color-sensitive graph coloring. The authors present a solution of the graph coloring problem using the sequential coloring with a vertex labeling. The vertex labeling is based on the channel capacity divided by the vertex degree, giving priority to the most valuable vertices. Based on those premises, the authors propose coloring algorithms with an objective of maximal cumulative reward, maximal minimal reward, and maximal proportional fairness. Results of the simulations show improvement of the network performance in throughput and in interference reduction compared to the other algorithms. In [13], the authors use binary edge conflict graph and interference free channel assignment. In our proposal, the interference is continuous or have discrete characteristic values. The FA algorithms allow the interference with matching network penalty. In [14], localized efficient channel assignment in the CR networks is proposed with a goal to maximize node connectivity after the FA, which is important for packet delivery. Partition of the network into two-level trees, conflict resolution strategy and trimming of nonessential links are proposed. Two basic localized algorithms and an advanced localized algorithm for the FA are presented. In assigning, the priority is given to the nodes with a higher link degree. Node-link-based algorithm with Hungarian method of maximal matching has almost twice the broadcast reachable nodes and assigned link rate, compared to the basic algorithms. As opposed to our approach, in the proposed model, the CR throughput is not considered, all channels have the same performance, there is no transit power control, and the interference range is the same as the communication range. In [15], the authors consider cooperative cognitive radio networks and propose the resource allocation in an OFDMA-based CR network with the SUs relaying data of the PUs. A new flexible channel cooperation (FLEC) design of the PU and SU cooperation is proposed. In the first time slot, PUs transmit to SU. In the second time slot, the SUs transmit to the primary BS. A game theory Nash bargaining-based solution for relay and subchannel assignment, as well as strategy optimization and power control, is considered. Centralized and distributed algorithms are proposed and show throughput improvement comparing to the conventional channel cooperation model. Distributed bargaining algorithm converges within 20 iterations, while centralized algorithm proves to be impractical due to the slow convergence. Consequently, the authors propose a heuristic method for the centralized problem. In [16], the authors propose a graph coloring-based fair channel allocation for base stations self-coexistence in the CR networks. The proposed solution is different from our approach in the following: each base station has a list of the available channels with equal characteristics and throughput. Graph representation of the system employs a binary interference model between the base stations. The proposed model allows operation of the CR networks with the different traffic requirements and guarantees the certain quality of service (QoS). Depending on the access category and priorities for the different packets, quality metric and share of the resources is determined. Simulation results show fairness improvement in terms of bandwidth allocation ratio and faster convergence than traditional list multi-coloring algorithm. An opportunistic distributed channel graph coloring algorithms for the spectrum access in dynamic spectrum access networks is presented in [17]. Maximum spectrum packing algorithm uses the local topology information of one-hop neighbors assigning a channel to the node with the highest degree. The algorithm ensures that all the SUs obtain one color and some SU's multiple colors. The algorithm assigns the color to the node with the highest degree and also enables assigning multiple colors to some nodes, thus improving overall spectrum efficiency. The improved heuristic with probabilistic color selection using truncated geometric distribution based on node degree ranking reduces the algorithm termination time. In probabilistic heuristic, the SUs request different channels with the probability that depends on the relative degree of the SU and its one-hop neighbors, reducing the possibility of two users contend for the same frequency. Random backoff is used to resolve the contentions of the same degree users during channel access. The performance of the algorithm is tested on randomly generated Bernoulli graphs with specified graph density. Comparing to our approach, proposed model does not consider the protection of the PUs and only simple binary conflict graphs are considered without interference susceptibility in the FA. In [18], the authors consider downlink resource allocation for hybrid macro/femtocell LTE-advanced network. Both macro and femto base stations have the CR functionalities. The cross tier interference is reduced by using the cross-cognition scheme, where base stations have higher priority in utilizing its licensed spectrum and lower priority in opportunistically utilizing non-licensed spectrum. The largest degree first based graph coloring algorithm is applied for the frequency block assignment reducing intra-tier interference between the femtocell nodes. Simulation results show increased throughput of the CR femtocell network compared to non-spectrum shared macro-femto cell network and improved interference mitigation of the graph coloring resource allocation compared to the random resource allocation. The problem of the FA to the communication links in the multi-hop wireless mesh networks with the objective of minimizing overall network interference is addressed in [19]. In the proposed mesh network, router nodes have multiple radio interfaces. The authors propose a centralized algorithm based on the heuristic Tabu search technique, where in the first phase, random initial FA solution is improved using the iterative algorithm with Tabu list, and in second phase, the algorithm eliminates remaining interface constraints. Distributed algorithm is based on a greedy heuristic algorithm for Max K-cut problem of the constrained graph optimization, where algorithm chooses frequency which produces the largest decrease in the local interference. The authors also propose an extension of the model with consideration of the non-orthogonal channels and non-uniform node traffic useful for practical implementation. The empirical evaluation on randomly generated network graph shows that proposed algorithms perform close to lower bounds established by the semi-definite and integer linear programming. However, compared to the approach presented in this paper, the authors assume quasi-static assignment of the channels, all channels are not available for the FA to all users, binary interference model is assumed in simulations, and finally, there is no sharing between the primary and the secondary users. In [20], a game theoretic approach for the distributed channel selection and power allocation in the cognitive radio networks is investigated. Proposed algorithm enforces the cooperation between the SUs in order to reduce the energy consumption and to find Nash equilibrium. Game theory model fits well with the cognitive FA problem because the spectrum decision of one of the SU influences the performance of neighboring SUs. Potential game formulation and no-regret learning model have been presented as a solution to the channel allocation problem. The influence of the cooperation on observed SINR and power consumption is investigated. Results show that the proposed iterative algorithm converges to a pure strategy Nash equilibrium solution.

So far, all the work done on the CR networks graph coloring FA employed simplistic binary co-channel interference model in order to assign frequencies and optimize utility function [10, 12–14, 16–19]. As we know, the interference control and management are of utmost importance for the spectrum sharing and efficient spectrum management in the CR systems. The CR FA algorithms have to deal with the mutual interference from co-channel or adjacent channel transmissions, which may cause significant performance degradation. Nevertheless, in the existing work, it was not possible to assess, quantify, or compare interference influence of the CR frequency selection on the neighboring SUs, nor to share spectrum between users when the interference exists, but it is bearable. Also, the adjacent channel interference was mainly neglected. To alleviate this deficiency, we propose a novel conflict graph model and algorithms with interference weighting and categorization, which enable interference sensitive frequency decisions, better interference control, and quantitative estimation of the individual components, as well as the calculation of the cumulative interference at the neighboring users. As a result, the proposed FA in the CR is performed with the interference sensitivity, achieving the efficient spectrum utilization, while enabling reliable transmissions.

3 The proposed FA framework

The CR spectrum management task is to achieve efficient wireless communications without causing a harmful interference or service interruptions. The CR spectrum management functions have some specific capabilities since the CR networks are focusing on the secondary access and spectrum sharing, while protecting the PUs and minimizing interference to the other SUs. A functional overview of the CR spectrum management framework is shown in Fig. 1. It consists of four major steps:

- Spectrum sensing. A CR user can only utilize temporary unused parts of the spectrum. Therefore, CR should monitor the available spectrum bands, collect the information on the spectrum use, and identify possible spectrum holes and their characteristics.
- Spectrum decision. Based on the spectrum availability information acquired through the spectrum sensing, policy guidelines, user requirements, and registry of the spectrum use, CR characterizes radio frequency spectrum possible for various models of the dynamic spectrum access. In the spectrum decision, CR or the centralized entity determines the carrier frequency,

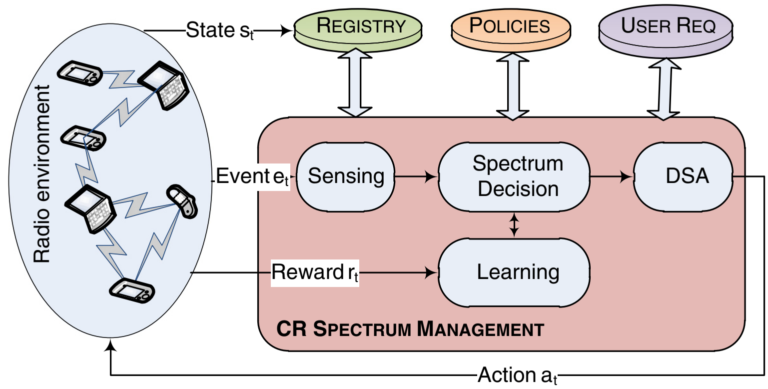

Fig. 1 CR spectrum management framework

channel bandwidth, transmission power, modulation, coding, communication technology, together with other operational, and technical parameters used for the CR operation.

- Dynamic spectrum access (DSA). The CR reconfigures its technical parameters in line with the selected operational technical parameters and operates in order to satisfy its primary goal of successful communication with a required QoS.
- Learning. Since CR is operating in heterogeneous radio environment with different user characteristics, different requirements and many parameters determining its environment and performance, CR has to adapt to the constantly changing environment, observe performance of its operation, and has to adapt its spectrum decision function using reinforcement learning.

The process of assigning frequencies to the CR users as part of the spectrum decision step is of crucial importance for the CR network functioning and represents a focal point of the efficient radio spectrum use, distinguishing the CR from other wireless communications systems.

3.1 Problem formulation and system model

In this paper, we are focusing on the FA component of the CR spectrum management. We will interchangeably use the following terms: radio links and vertices, frequencies and colors, interference and edges, cognitive radio and secondary user. The problem of the FA in the CR networks can be informally described as follows: given a set of the transceivers with the CR capabilities, a network of the PU transceivers with the administrative licenses and priority access rights to the radio frequency spectrum, and predefined constrained number of the frequencies, we wish to efficiently assign radio frequencies to the CR users. The CR FA has to satisfy the following requirements:

- CR self-goal: successfully transmit as much information as possible with required QoS
- CR network goal:
  - Minimizing the total network interference
  - Maximizing the network throughput
- Requirements towards other users: not to cause excessive interference with the operating PUs, to keep the CR mutual interference under the reasonable limit and to efficiently share the radio frequency spectrum

The FA problem in the wireless communications can be modeled as a graph-theoretical problem by defining an undirected conflict graph describing the interference relations and assigning frequencies by coloring vertices. The graph coloring problem is known to be NP-complete problem [21] and therefore computationally hard. Although any given solution of the NP-complete problem can be verified in the polynomial time, there is no known efficient way to find the optimal solution. Also, the time required to solve the problem increases very quickly as the size of the problem grows. For example, 3.16 million years are needed to find a solution to the FA problem using brute force search approach for 20 terminals and 10 frequencies with 1 μs per solution [6]. Considering the basic characteristics of the FA in the CR already mentioned, and the computational complexity of finding an optimal solution, we are directing towards the use of the heuristic method for solving this problem. Although the heuristic algorithm leads to a suboptimal solution, it provides a reasonably good solution in the limited time, which works well in the practical applications of the assigning frequencies to the CR SUs.

In this paper, the CR FA is modeled as a graph coloring problem using novel dynamic vertex ordering, interference weighting, and categorization. To illustrate the concept presented in this paper, an example of the dynamic spectrum access scenario is depicted in Fig. 2, with the corresponding communication and conflict graph. The communication graph represents the communication links between the radio transceivers, while conflict graph shows possible interference between the radio links. In the communication graph, the PU and SU transceivers are represented as the vertices, while communication links between radio transceivers are represented by communication graph edges. In the communication graph shown in Fig. 2, PU network is illustrated as two base stations serving a set of PUs, establishing the primary links, while CR network deployed in the same area is represented by a set of CR links $\{A, B, \ldots, I\}$. The circles illustrate the service zones of the PU networks. The corresponding conflict graph is shown in Fig. 2, where the vertices represent the CR communication links with a dynamically changing local list of blocked frequencies due to PU transmissions. In the same figure, the edges show possible interference occurrences where co- and adjacent channel interference levels are proportional to the edge weights coefficients.

Let us assume there are five available frequencies represented by numbers, which are opportunistically available to the CR operation. Since the PU and the CR networks share the same radio frequency band, the frequencies temporarily used by PUs cannot be utilized by the CR

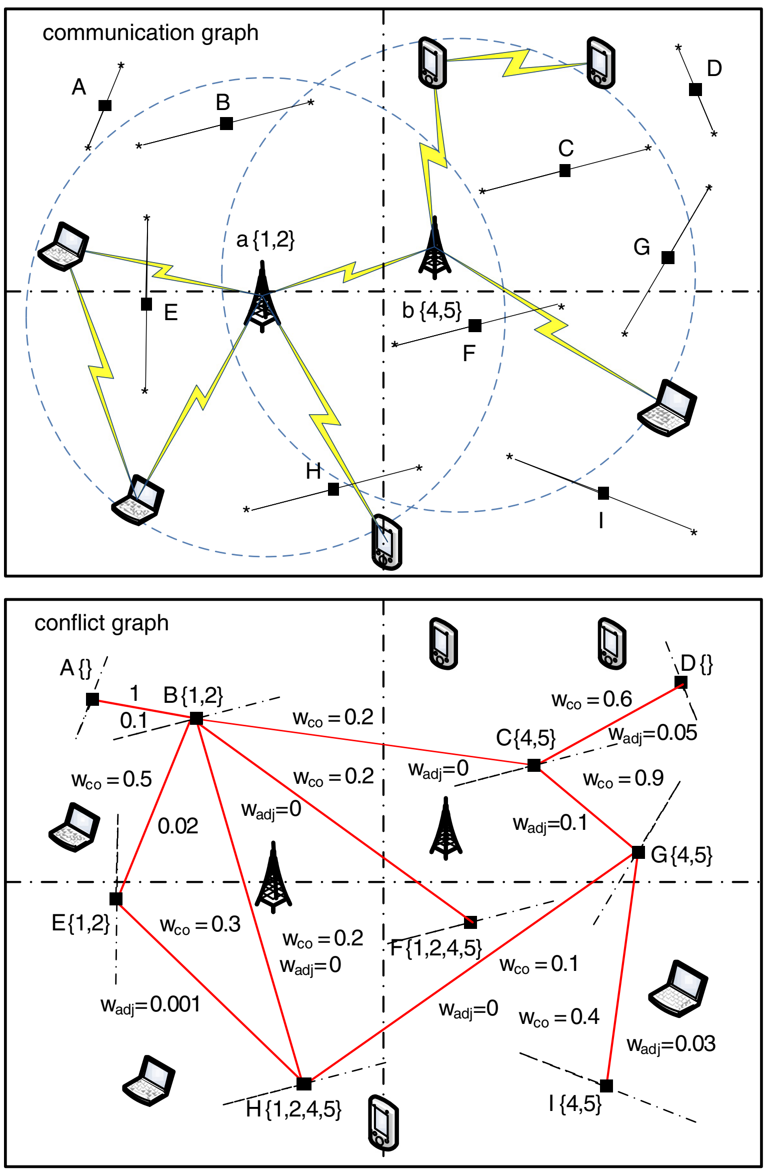

Fig. 2 Communication and conflict graph illustration. The figure shows example of dynamic spectrum access scenario with corresponding communication and conflict graph

users if they are in the interference range, and therefore, those channels are blocked for corresponding CR link indicated by the corresponding blocked channels list. For the CR link, B channels $\{1, 2\}$ cannot be used due to the possible interference with the service area of the PU base station a, for the CR link C channels $\{3, 4\}$ are blocked, for the CR link F channels $\{1, 2, 4, 5\}$ are blocked, etc. The conflict edges represent the potential interference between the CR links with two edge weights per edge indicating co-channel and adjacent channel interference potential. In Fig. 2, the edge between the CR link A and CR link B has weights of 1 for co-channel and 0.1 for the adjacent channel, indicating very high interference potential between those links, and therefore, those links cannot use the same channel. The edge between the CR link C and CR link D has a conflict weight of 0.6/0.05 indicating a medium interference potential and the edge between CR link B and CR link F has a weight of 0.2/0 indicating a low co-channel interference and no adjacent channel interference. This indicates the possibility for sharing the channel. The existence of the conflict graph edges and the corresponding weights depends on the propagation channel characteristics and the antenna discrimination between the CR link pairs analyzed. For example, between the CR link A and CR link E, there is no edge because the terrain obstacle prevents possible interference path and corresponding links can use the same channel without danger of the mutual harmful interference. Practical implementation of constructing conflict graph and determining its weights is elaborated in Section 7.2.

In our work, we abstract the CR network topology into the communication graph $G = (V,E)$ as a general undirected graph over the set of network transceivers represented by the vertices V and PU or CR links represented by the edges E in Fig. 2. The FA channel selection constraints and potential interference between the radio links are represented by a conflict graph $G_c = (V_c, E_{co}, E_{adj})$, where V_c is set of conflict graph vertices, and E_{co} and E_{adj} are co-channel and adjacent channel conflict edges with associated interference potential quantified by the edge weights w_{co} and w_{adj}. Using the described model, the FA problem can then be further formulated as a complete frequency assignment of the CR network represented by a conflict graph $G_c = (V_c, E_{co}, E_{adj})$. The spectrum of interest is divided in M frequencies represented by constrained set of the radio frequencies $F = \{1, .., M\}$, and the dynamically changing local set of blocked frequencies protecting the PU transmissions represented by B_v. Using the abovementioned notation, frequency assignment is a graph coloring function $f : (G_c, B_v, W_{co}, W_{adj}) \rightarrow F$, such that the objective function is maximized. To facilitate further discussion, we will use the notation and acronyms as defined in Table 2.

Table 2 Important notation and acronyms

Notation	Description
$G_c = (V_c, E_{co}, E_{adj})$	Conflict graph with vertices V_c and edges E_{co}, E_{adj}
$F = \{1, .., M\}$	Set of M available frequencies
B_v	List of blocked frequencies at CRv due to PU hard constraints
$W_{co} = \left\{w^{co}_{vi\,vj}\right\} \in [0, 1]$	Set of co-channel continuous interference weight coefficients
$W_{adj} = \left\{w^{adj}_{vi\,vj}\right\} \in [0, 1]$	Set of adjacent channel continuous interference weight coefficients
N_{PU}, N_{SU}	Number of primary and secondary users
$h^{PUi'}_{PUi}\ h^{SUk'}_{PUi}$	Channel coefficients between PU Tx i', SU Tx k', and PU Rx i
$h^{SUk'}_{SUl}\ h^{PUj'}_{SUl}$	Channel coefficients between SU Tx k', PU Tx j', and SU Rx l
$P_{Rx\_SUl\_D}, P_{Rx\_SUl\_I}$	Desired received signal strength and interfering received signal strength at the l*th* SU
$P_{Tx\_PUm'}, P_{Tx\_SUl'}$	Transmitting power at m'th PU and l'th SU
S_{vi}	Saturation metrics label
Col_{vi}	Vertex coloring argument
Acronyms	**Description**
CR	Cognitive radio
FA	Frequency assignment
PU, SU	Primary user, secondary user
Tx, Rx	Transmitter, receiver
QoS	Quality of service
CminSumInt	Centralized minimum cumulative network interference FA algorithm
CMaxSumCap	Centralized interference sensitive maximum throughput FA algorithm
DminInt	Distributed minimum interference FA algorithm
DMaxCap	Distributed interference sensitive maximum throughput FA algorithm

3.2 Framework overview

The CR FA task is to assign the frequencies to the SUs when and where they request access to the spectrum, to assign new frequencies to the CR users when co- or adjacent channel PU transmission occurs in the interference range, or when the quality of service reduces below the requested threshold. As a result of that, we are facing with the task of assigning frequencies in a partially colored graph. In that respect, the FA is a continuous process, as it is not possible to assign frequencies and optimize coloring to the entire CR network at the same moment. There is a need to constantly assign and re-assign frequencies to active SUs. If the PU reactivates, new SU appears, or signal level drops due to dynamic characteristics of the radio channel, the FA process has to be re-initiated and channel allocation has to re-converge for the SUs influenced by these changes.

Based on the system model described in Section 3.1, we have developed a novel framework for assigning the frequencies in the CR environment. In the preparatory step, input data and the CR technical parameters are determined, including available frequencies, compatibility criteria, CR coverage area, and the interference potential. In the second step, the CR adjacency, interference levels, and weights are estimated, followed by communication and the conflict graph construction determining whether the CRs can share radio frequencies. In the CR FA step, an iterative process is performed consisting of labeling and ordering of the CRs, selecting the next transmitter to assign the frequency, and selecting the frequency maximizing network objective.

In the previous research of the CR FA application of graph coloring, the interference model assumed the conflict edges to be binary variables representing the interference or no-interference occurrence. The adjacent channel interference was not taken into account since the channels were assumed to be orthogonal. In the proposed FA framework, the CR network conflict graph illustrated in Fig. 2 is extended with the novel features:

- The local list of the blocked frequencies B_{v} associated with each conflict graph vertex representing frequencies which cannot be used by CR
- Two layers of edges in conflict graph taking into consideration co-channel interference E_{co} and adjacent channel interference E_{adj}
- Continuous interference weight coefficients W_{co} and W_{adj} associated to the conflict graph co-channel and adjacent channel edges incorporating quantification of possible interference between adjacent vertices corresponding to the interference "strength"
- The categorization of the interference weights reducing communication overhead

Since the CR user has to protect and not cause the interference to the licensed PUs, each CR keeps its own list of blocked frequencies and the FA algorithm avoids selecting the channels causing harmful interference to the PUs which are temporarily operating in the close vicinity of the observed CR. As the PUs frequency usage is constantly changing, the list of the blocked channels at the CR is dynamically updated using the spectrum sensing, the geo-location database or some other method of the spectrum awareness. In the proposed approach, the FA is interference sensitive, as the edges have associated co-channel and adjacent channel weights determining the level of the interference between the vertices. In this paper, we are proposing the algorithms which can be categorized under the sequential graph coloring class of algorithms with a saturation degree dynamic vertex ordering. This class of algorithms was first introduced by Brélaz [22] as saturation largest first algorithm known under the name Degree of saturation algorithm (DSATUR). In DSATUR algorithm, the vertices are ordered by a decreasing order of saturation, where the saturation degree is determined as a number of different colors to which a vertex is adjacent. In [13], color-sensitive graph coloring is used where dynamic vertex ordering is determined using labeling based on channel capacity divided by vertex degree and selecting most valuable SU to assign frequencies first (CR link contributing the most to the network throughput).

In our approach, we are also using labeling and dynamic SUs ordering to determine the prioritization of assigning frequencies to the CR users, but with the different saturation metric and network objectives compared to the existing literature on graph coloring FA. We are selecting the most difficult SUs to assign frequencies first using a specially designed CR saturation metric. Proposed novel CR saturation metric is determined as a level of freedom in selecting the frequency for the SU, taking into consideration channel limitations due to local PUs transmissions and a level of interference from adjacent assigned SUs as a combination of hard and soft frequency assignment constraints. The saturation metric is calculated using the interference weight coefficients. In this paper, we are proposing two different FA problem solutions with objective functions determined as minimal total network interference and as maximal throughput with the interference susceptibility.

Taking into account the CR network topology, we are considering centralized FA where centralized resource manager optimizes frequency use on the network or cluster level, and distributed FA where the selection of the frequencies is performed on a level of CR and its neighboring nodes. More detailed algorithm description and the mathematical formulation of the saturation metrics and objective functions are provided in Sections 5 and 6.

4 Interference modeling and characterization

4.1 Interference model

Modeling and estimation of the mutual interference are of vital importance for exploiting the potential of the radio spectrum band to the full extent, while keeping the interference under control and below level causing the network performance deterioration. Since the interference is the primary limiting factor influencing the performance of the CR networks, we have built our FA model with the interference susceptibility. The determined level of interference is a basis for the interference weighting and categorization used in the proposed FA algorithms. Channel model for the co-existing PUs and cognitive SUs is illustrated in Fig. 3. The figure shows one PU transmitter-

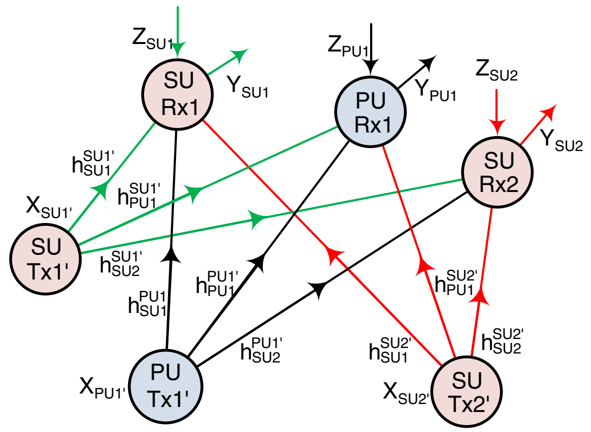

Fig. 3 Channel model for scenario of co-existing PUs and SUs

receiver pair (PU Tx1' - PU Rx1) and two SU transmitter-receiver pairs (SU Tx1' - SU Rx1, SU Tx2' - SU Rx2). The received signal at Rx is noted by Y, transmitted signal from Tx is noted by X and Z is the noise. Lines between Tx and Rx represent TX-Rx channel paths, where each Tx is connected to all receivers with corresponding channel coefficients h. We assume that the quasi-static fading is present, and the channel coefficients between users are considered to be independent random variables. Channel coefficients take into consideration the total loss of the radio link between the transmitter and the receiver, as well as the antenna gain and the propagation effects (e.g., large scale fading due to the shadowing and small scale fading due to the multipath).

The signal received at the PU or at the SU can be represented with a sum of: the desired received signal, interfering received signal, and the noise. Signal received at ith PU and signal received at lth SU is a superposition of various components and can be expressed as

$$Y_{\mathrm{PUi}} = h_{\mathrm{PU}i}^{\mathrm{PU}i'} X_{\mathrm{PU}i'} + \sum_{j'=1, j'\neq i}^{N_{\mathrm{PU}}} h_{\mathrm{PU}i}^{\mathrm{PU}j'} X_{\mathrm{PU}j'} + \sum_{k'=1}^{N_{\mathrm{SU}}} h_{\mathrm{PU}i}^{\mathrm{SU}k'} X_{\mathrm{SU}k'} + Z_{\mathrm{PU}i}, \quad (1)$$

$$Y_{\mathrm{SU}l} = h_{\mathrm{SU}l}^{\mathrm{SU}l'} X_{\mathrm{SU}l'} + \sum_{j'=1}^{N_{\mathrm{PU}}} h_{\mathrm{SU}l}^{\mathrm{PU}j'} X_{\mathrm{PU}j'} + \sum_{k'=1, k'\neq i}^{N_{\mathrm{SU}}} h_{\mathrm{SU}l}^{\mathrm{SU}k'} X_{\mathrm{SU}k'} + Z_{\mathrm{SU}l}, \quad (2)$$

where $X_{\mathrm{PU}i'}$ is the signal from PU transmitter i', $X_{\mathrm{SU}l'}$ is the signal from SU transmitter l', $h_{\mathrm{PU}i}^{\mathrm{PU}i'}$ denotes the channel coefficient between PU transmitter i' and PU receiver i, $h_{\mathrm{PU}i}^{\mathrm{SU}k'}$ is the channel coefficient between SU transmitter k' and PU receiver i, $h_{\mathrm{SU}l}^{\mathrm{SU}l'}$ is the channel coefficient between SU transmitter l' and SU receiver l, $h_{\mathrm{SU}l}^{\mathrm{PU}j'}$ is the channel coefficient between PU transmitter j' and SU receiver l, N_{PU} is the number of PUs, and N_{SU} is the number of SUs, as illustrated in the Fig. 3. $Z_{\mathrm{PU}i}$ and $Z_{\mathrm{SU}l}$ represent additive white Gaussian noise at the PU receiver i and SU receiver l with the variances $\sigma_{\mathrm{PU}i}^2$ and $\sigma_{\mathrm{SU}l}^2$.

The desired received signal strength represents the power of the useful signal received at the SU receiver from the wanted SU transmitter contributing to the information data flow. The interfering received signal strength is the superposition of all received interfering signals at the input of the same SU receiver. Accordingly, we can express the desired received signal strength $P_{\mathrm{Rx\_SU}l\_D}$, and interfering received signal strength $P_{\mathrm{Rx\_SU}l\_I}$ at SU receiver l as

$$P_{\mathrm{Rx\_SU}l\_D} = \left| h_{\mathrm{SU}l}^{\mathrm{SU}l'} \right|^2 P_{\mathrm{Tx\_SU}l'}, \quad (3)$$

$$P_{\mathrm{Rx\_SU}l\_I} = \sum_{k'=1, k'\neq l}^{N_{\mathrm{SU}}} \left| h_{\mathrm{SU}l}^{\mathrm{SU}k'} \right|^2 P_{\mathrm{Tx\_SU}k'} + \sigma_{\mathrm{SU}l}^2 + \sum_{m'=1}^{N_{\mathrm{PU}}} \left| h_{\mathrm{SU}l}^{\mathrm{PU}m'} \right|^2 P_{\mathrm{Tx\_PU}m'}, \quad (4)$$

where $P_{\mathrm{Tx\_PU}m'}$, $P_{\mathrm{Tx\_SU}l'}$, and $P_{\mathrm{Tx\_SU}k'}$ are PU and SU transmitting powers, and $\left| h_{\mathrm{SU}l}^{\mathrm{SU}l'} \right|^2$ and $\left| h_{\mathrm{SU}l}^{\mathrm{PU}m'} \right|^2$ are the wanted and interfering channel gain coefficients. Received interfering power $P_{\mathrm{Rx\_SU}l\_I}$ takes into account the interference received from PUs, interference from other SUs on co-channel, and adjacent channel operating in the interference range of the corresponding SU and the noise. All those interferences add up to the cumulative interference, which reduces quality and reliability of the SU link. Similarly, we can calculate the desired received signal, and interfering received signal at the PU, which is omitted for brevity.

The SU throughput is proportional to the normalized ergodic channel capacity, which is defined as the maximum achievable rate averaged over all fading blocks and it can be expressed as

$$C\,(\mathrm{SINR}) = \max E\left[log_2 \left(1 + F^{-1} \left(1 - \epsilon\right) \mathrm{SINR}\right)\right], \quad (5)$$

where $F(x) = P\left(\left| h_{\mathrm{SU}l}^{\mathrm{SU}k'} \right|^2 > x\right)$ is the complementary cumulative distribution function of SU channel gain $\left| h_{\mathrm{SU}l}^{\mathrm{SU}k'} \right|^2$, ϵ is an error probability of the secondary transmissions, and SINR is the desired signal-to-interference-plus-noise ratio at the SU receiver. If the SU channel gain is below minimal threshold level x, no useful data transmission can be obtained between the SU transmitter and receiver. In Eq. (5), the theoretical relation between SU throughput and an ergodic channel capacity for the fading channel is shown. In the numerical simulation in Section 8, we assume that SU system utilizes M-ary quadrature amplitude modulation (M-QAM) and we use the approximation (19) for SU throughput. Additionally, in the simulation, we have set up the upper limit of the achievable individual SU throughput in order to avoid unfair spectrum usage between the SUs.

4.2 Weighting coefficients characterization

Since all of the channel parameters and fading statistics between interfering nodes are generally unknown to the SUs, it would be demanding to implement full network interference model. On the other hand, reliable and precise interference estimation is important for constructing the CR conflict graph needed for the FA algorithm implementation. Therefore, to encompass the sensitivity to the potential levels of the interference in the process of assignment of the frequencies to the SUs, we propose the classification of the interference in four categories. This enables an efficient practical implementation of the FA protocol with control and quantification of the interference levels, while not adding too much of an additional burden to the network communication overhead. Considering the spectrum sharing between the SUs, for each interfering path between SUs we designate co-channel and adjacent channel conflict edge and introduce associated interference weights determining the level of the potential interference between the corresponding SUs as follows:

$$w_{\mathrm{co}} = p_{\mathrm{co}} \cdot w_i \quad , \quad w_{\mathrm{adj}} = p_{\mathrm{adj}} \cdot w_i, \tag{6}$$

where w_{co} and w_{adj} are co- and adjacent channel interference edge weights, p_{co} and p_{adj} are co- and adjacent channel penalties, and w_i is the nominal edge weight.

Co- and adjacent channel interference edge weights are the network penalties for violation of the soft FA constraints in the neighboring vertex, resulting in the deterioration of the objective function used for the optimization of the FA process. The channel penalties p_{co} and p_{adj} represent the portion of the CR transmitter power which is absorbed by the CR receiver on the same or adjacent channel. Generally, channel penalty is calculated as the reduction of the interference power caused by the filter shape of the transmitter spectrum density mask and the receiver selectivity mask or it can be measured. As we are analyzing network with a predefined set of frequencies and aligned channel bandwidths, in network simulation in Section 8, we assume that the channel penalties are predefined and have the constant values. The nominal edge weight w_i corresponds to the channel coefficient including propagation loss, antenna gain, and discrimination between the SU Tx and SU Rx under the consideration.

We propose to classify level of the potential interference in four main categories (*harmful*—high interference; *disturbing*—medium-high interference; *annoying*—medium-low interference; *permissible*—low interference) and assign the corresponding conflict edge weight coefficients $w_i \in \{0; 0.4; 0.7; 1\}$. To classify the potential interference in the appropriate category, a value of the potential interference between the SU Tx and Rx is compared with the cumulative probability density function of the average received interference power between the SUs. The cumulative probability density function (cdf) is empirically determined from the previous spectrum sensing measurements in the frequency band under consideration. On the basis of this comparison, it is identified to which category out of four this potential interference belongs and the characteristic value of the nominal edge weight factor is assigned for the identified interference category. Proposed values for the categorization of the nominal edge weights are calculated by evaluating the cdf of the average received interference power between the SUs. Interference cdf is divided into four non-overlapping areas and a median cumulative probability value for each area is calculated. Channel weights representing each category are calculated as the ceiling round up (rounded off to the higher value) for the edge weight coefficients $(0.4; 0.7; 1)$ and as floor round up for edge weight coefficient (0). Specific weights for four different categories of the interference are chosen in such way that the possible harmful interference between SUs is emphasized. We have tested proposed algorithms with the variation of the values of discrete interference weights, and the results show similar behavior as the algorithms with a small variation of the nominal network weight coefficients representing four categories.

This novel interference categorization allows the introduction of the susceptibility of the FA protocol to the level of the interference, taking into account the probabilistic nature of the received signal as described above, as well as possible interference estimation errors.

To illustrate the used model of the interference classification, Fig. 4 shows the criteria for the forming of a list of blocked channels B_{v}, depending on a measured level of the PU signal at the corresponding SU. I_{PU} is the level of the interference from PUs transmissions at the observed frequency measured on the SU terminal. If the value of I_{PU} is above the threshold of the receiver interference limit, the said frequency cannot be temporarily used for the SU transmissions, since it may cause harmful interference to the PUs in the vicinity of the SU terminal. The frequency is included in the dynamic list of the blocked channels at this SU. If I_{PU} is below the threshold, the interference is small and SU can share the frequency with the PUs with appropriate transmission power. Probability density function of the received interfering signal is shown in the middle of

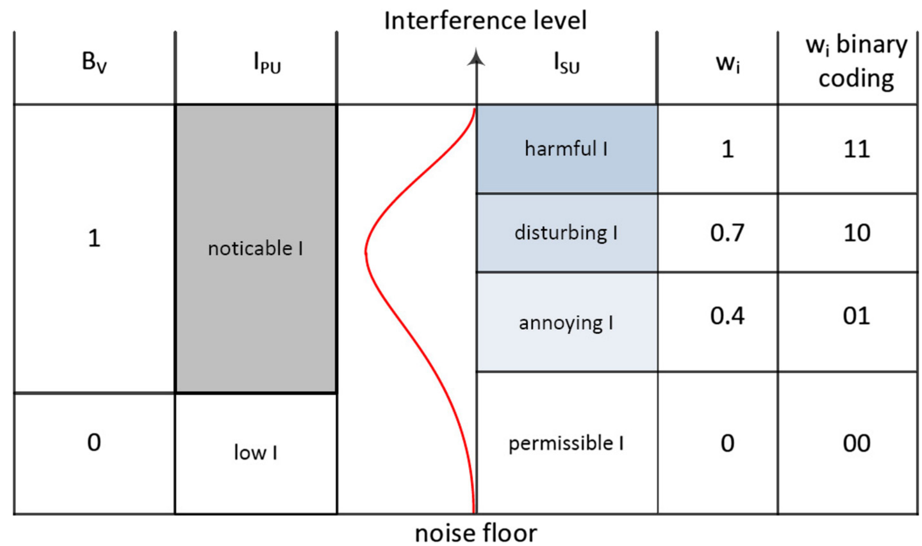

Fig. 4 Categorization of interference for FA in CR networks. The criteria for forming list of blocked channels is shown on the *left* side. Classification and quantification of SU interference is shown on the *right* side

Fig. 4. In the right side of Fig. 4, a proposed categorization of the SU interference corresponding to SU interference level, edge weights coefficients, and their binary coding is shown. I_{SU} is the level of the measured interference from the SU transmissions, used for determining the SU adjacency and the conflict edge weights. The SUs edge weights are used for quantitative estimation of the interference in the process of the assigning frequencies. In such a way, we can make more knowledgeable frequency decisions and control individual interference components as well as aggregate interference from multiple users during the FA process. In the proposed FA algorithms, we use the continuous edge weights in the interval $[0, 1]$, or the discrete weight values as shown in Fig. 4. The protocol for determining the SUs adjacency and the conflict graph edge weights is elaborated in Section 6.

5 Interference sensitive FA algorithms

The proposed FA in the CR networks is based on the efficient heuristic based algorithms that can run reasonably fast and provide a good quality solution. In this paper, non-uniform traffic on the different communication links and frequencies, weighted interference model, and non-orthogonal channels are considered based on the CR networks specifics.

5.1 Algorithm description

In a proposed model, we have a predefined set of frequencies $F = \{1, .., M\}$ and the task is to assign frequencies to all SUs requesting network access, taking into consideration the network interference. The SU is not allowed to use any of the frequencies which could cause noticeable interference to the PU network, determined by the spectrum awareness and expressed with a set of blocked channels B_V. If there are no available channels in the spectrum pool, SU's request is rejected or transferred to another spectrum band. In our approach, the SUs connected with the conflict graph edge (i.e., interfering each other) can share the same frequency or an adjacent frequency, but with a corresponding penalty, which is proportional to a level of the mutual interference calculated as a product of the nominal edge weights and co-/adjacent channel penalty. Having this quantitative metric, resource manager selects the frequencies with a minimal interference contribution towards the other users. Proposed FA algorithms consist of four main phases with specific tasks in each phase as follows:

- Phase 1. FA preparation:
  - Establish the list of the blocked channels at all SUs.
  - Determine the SU transmitting power for each available channel at all SUs.
  - Calculate the SU throughput for each available channel at all SUs.
  - Establish a conflict graph edges and determine the interference weights for co-channel and adjacent channel transmissions.
  - Interference categorization.
- Phase 2. Selecting the next SU to assign frequency:
  - Label the non-assigned SUs using specific CR FA saturation metric.

  - Dynamically order the SUs with decreasing saturation score.
  - Select the SU with the highest saturation metric to assign frequency as the next SU to process.

- Phase 3. Assigning frequency to the selected SU:
  - Calculate objective function for each available frequency at the selected SU.
  - Select and assign frequency maximizing objective function at the selected SU.
- Phase 4. CR FA performance evaluation and conflict graph update:
  - Calculate interference, throughput, and throughput variance for all SUs.
  - Determine the overall network performance and update conflict graph.

The initial SU transmitting power for each available channel at SUs is determined in a way that SU does not cause harmful interference to the closest PU using this channel and that the minimal required SINR at the SU user is achieved. In the following phases, the SU calculated transmitting power per channel is updated using power control ratio parameter. The details of the power control mechanism are described in Section 5.3, while establishing the list of blocked channels, calculation of the conflict graph edge weights, and its categorization are performed as described in Section 4.2.

We propose the centralized and distributed algorithms with the objective of minimizing the cumulative CR network interference and with the objective of maximizing the network throughput, as shown in Table 3.

5.2 Saturation metric labeling and coloring

The proposed algorithms process and assign frequencies to the SUs requiring spectrum access in a sequence where in each iteration step the most difficult SU is selected. Most difficult SU is the vertex with the smallest choice of channels determined by the saturation metric. The saturation metrics label S_{vi} is calculated as

$$S_{\mathrm{vi}} = \sum_{\mathrm{vi} \in V_c} B_{\mathrm{vi}} + \sum_{f \in F,\ \mathrm{vj} \in V_c} E_{\mathrm{co\,vi\,vj}} \cdot x_{\mathrm{vj}} \cdot w^{\mathrm{co}}_{\mathrm{vi\,vj}} + \sum_{f \in F,\ \mathrm{vk} \in V_c} E_{\mathrm{adj\,vi\,vk}} \cdot x_{\mathrm{vk}} \cdot w^{\mathrm{adj}}_{\mathrm{vi\,vk}}, \tag{7}$$

where B_{vi} are blocked frequencies, which cannot be used at the SU vi, $E_{\mathrm{co\,vi\,vj}}$, and $E_{\mathrm{adj\,vi\,vk}}$ are co- and adjacent channel edges indicating the interference between the SUs, $w^{\mathrm{co}}_{\mathrm{vi\,vj}}$ and $w^{\mathrm{adj}}_{\mathrm{vi\,vk}}$ are the conflict edge weights determining the interference potential of the adjacent SUs, x_{vj} and x_{vk} are variables indicating that the adjacent SUs have the frequency assigned. All the vertices are sorted in the decreasing order of the saturation label. The vertex with the largest label is selected for coloring. Saturation metrics score is changing in each iteration step due to the continuous process of the assigning frequencies to the SUs, and therefore, the order of the SUs also dynamically changes. In the assignment phase, the selected SU is colored with a color which contributes the most to the system utility function. Here, we investigate the different algorithms with the objective of the minimal interference and maximal throughput.

5.2.1 *Minimum cumulative network interference FA*

The proposed algorithm is minimizing the total network interference while taking into account co-channel and adjacent channel interference. Vertex coloring is performed according to

$$\mathrm{Col}_{\mathrm{vi}} = \mathrm{argmin}\left[\alpha + \beta\right], \tag{8}$$

$$\alpha = \sum_{vj \in E_{\mathrm{co\,vi\,vj}}} w^{co}_{\mathrm{vi\,vj}} \cdot x_{\mathrm{vj}}, \tag{9}$$

$$\beta = \sum_{\mathrm{vk} \in E_{\mathrm{adj\,vi\,vk}}} w^{\mathrm{adj}}_{\mathrm{vi\,vk}} \cdot x_{\mathrm{vk}}, \tag{10}$$

where α and β are co- and adjacent channel cumulative interference at the adjacent vertices. In coloring, the algorithm is selecting the frequency which contributes the least to the cumulative network interference. Having the interference represented by edge weights, the individual interference contributions can be quantified and the level of total network interference can be controlled.

5.2.2 *Interference sensitive maximum throughput FA*

The FA is performed using a comprehensive coloring argument that integrates the interference and throughput. The proposed algorithm is maximizing the total network throughput while taking into account co-channel and adjacent channel interference. The coloring argument is calculated as

$$\mathrm{Col}_{\mathrm{vi}} = \mathrm{argmax}\left[\frac{\sum C_{f\ \mathrm{vi}} \cdot x_{\mathrm{vi}}}{\sum_{\mathrm{vi} \in V_c} B_{\mathrm{vi}} + \alpha + \beta}\right], \tag{11}$$

Table 3 Summary of proposed FA algorithms

	Objective	
Implementation	Minimal interference	Maximal throughput
Centralized	CminSumInt	CMaxSumCap
Distributed	DminInt	DMaxCap

where $C_{f\,\mathrm{vi}}$ is the SU throughput at frequency f. The SU throughput is calculated using Eq. (5), and its value can be between 0 and the maximum available throughput per SU. This value is set to 16 Mbit/s in the numerical simulation in Section 8. The coloring is done with the preference given to the frequencies having the highest throughput, while having the minimal influence to the rest of the SU network.

5.3 Transmit power control

The proposed transmit power control strategy is based on the balancing of the SU transmit power level between minimal required to satisfy QoS requirements and maximal permissible level in order to avoid interference with the PUs. In order to minimize the interference to the other concurrent SUs, it is recommended to maintain the transmission power level at a minimum, while ensuring an adequate signal quality at the receiving end. Minimum required transmit power is determined by adjusting the power level until the targeted SINR at the SU receiver is satisfied. Maximum acceptable transmit power is established by considering the permissible interference at the PUs.

The initial SU power is calculated using the path loss estimation determined by spectrum sensing of the PUs as:

$$P_{\mathrm{Tx\_SU}} = P_{\mathrm{Tx\_C}} - P_{\mathrm{Rx\_C}} + S_{\mathrm{R}} - L_{\mathrm{A}} + P_{\mathrm{Rx\_SU\_I}}, \quad (12)$$

where $P_{\mathrm{Tx\_SU}}$ is the initial SU transmit power, $P_{\mathrm{Tx\_C}}$ is the predetermined transmitted power of the common pilot channel, $P_{\mathrm{Rx\_C}}$ is the measured power of the common pilot channel at the SU receiver, L_{A} is the additional gain, loss, and tolerances, S_{R} is the required SINR, and $P_{\mathrm{Rx\_SU\_I}}$ is the interfering power. The initial power level is used for the initial communication between SU terminals and as a starting power level for the process of adaptive adjusting of the transmitter power. Instant transmit power of the secondary user is determined as

$$P_{\mathrm{Tx\_SU}} = \left\{ \begin{array}{cc} P_{\mathrm{Tx\_SU\_R}} & \text{if } P_{\mathrm{Tx\_SU\_M}} \geq P_{\mathrm{Tx\_SU\_R}} \\ 0 & \text{if } P_{\mathrm{Tx\_SU\_M}} < P_{\mathrm{Tx\_SU\_R}} \end{array} \right\}, \quad (13)$$

where $P_{\mathrm{Tx\_SU\_M}}$ is the maximal acceptable transmit power and $P_{\mathrm{Tx\_SU\_R}}$ is the required transmit power. The maximal acceptable transmit power represents the maximal allowed power of the cognitive SU in order to satisfy the interference constraints, as illustrated in Fig. 4, and is determined by the spectrum sensing of the PU at the SU location. The required transmit power is the transmit power level just strong enough to satisfy SINR for obtainable throughput at the SU receiver. The comparison of the measured SINR at the SU receiver with the required SINR determines the SU transmit power control ratio for the transmit power adjustment. The new minimum required transmit power is obtained by multiplying present minimum required transmit power with the transmit power control ratio R_{TPC} as follows:

$$P_{\mathrm{Tx\_SU\_R}}\,(t+1) = P_{\mathrm{Tx\_SU\_R}}\,(t)\,R_{\mathrm{TPC}}. \quad (14)$$

A possible implementation of the proposed adaptive transmit power control strategy using fuzzy logic power controller is described in [23].

6 Centralized and distributed algorithm

6.1 Centralized FA algorithm

The centralized network resource manager has the advantage of collecting all the relevant information and using more complex processing engines. It can, therefore, provide an overall better solution. The centralized algorithm can keep track of the CR user mobility, achieving continuous connectivity of the established CR session. Pseudocode of the centralized FA algorithm is presented in Fig. 5.

In each iteration, the centralized algorithm calculates the saturation metrics of all unassigned vertices. In order to detect the most difficult vertex to process first, the vertices are sorted and vertex with the largest saturation metrics is selected. For the selected SU, coloring is performed with the frequency which maximizes the objective function. In CminSumInt algorithm, the frequency is selected on the basis of the minimal interference caused

Algorithm 1 :
Input:
Conflict graph $G_c(V_c,\ E_{co}, E_{adj})$, set of frequencies F, set of blocked frequencies B_v, set of co-channel and adjacent channel edge weights w_{co}, w_{adj}, penalties for co- and adjacent channel rule violation p^{co}, p^{adj}, set of transceiver user throughputs c_v
Output:
Frequency assignment function $f_{best}: V_c \to F$, cumulative network interference I_{sum}, total SU network throughput C_{CRN}
begin
initialize all
do while *not all vertices V_c have been colored*
    for *$\forall V_c$ calculate saturation metrics label S_{vi}*
    sort *vertices in decreasing order of S_{vi}*
    select *vertex with Max S_{vi}*
    color *vertex vi according to objective function*
    update all
repeat
$f_{best} = x_{vf}$
calculate *I_{sum}, C_{CRN}*
end

Fig. 5 Centralized FA algorithm. The figure shows pseudo code of proposed centralized algorithm to the adjacent vertices. In CMaxSumCap algorithm, the selected frequency maximizes the comprehensive metric that integrates both interference and throughput. The centralized solution requires an additional communication overhead to collect and distribute information between the vertices. In that respect, the common communication channel and protocol are necessary. Also, the centralized solution is sensitive to the potential failures of the central entity, which can result in the discontinuities of the FA in the CR networks. In the cluster topology, the CR network is subdivided in a number of geographically distinguished clusters. Each cluster uses a different resource manager and an independent algorithm determining the next SU to assign frequency using the saturation metric label and frequency assignment maximizing the network objective.

In order to determine the complexity of the algorithm, it is essential to determine the number of iterations needed for the algorithm to terminate. The number of iterations in the centralized algorithm is bounded by $O(N_{\mathrm{SU}})$, where N_{SU} is the number of the secondary users requesting access to the CR network. Since the single vertex coloring can be completed in $O(M)$ where M is the number of available frequencies, the overall complexity of the proposed centralized algorithm is $O(N_{\mathrm{SU}}M)$.

6.2 Distributed FA algorithm

The distributed algorithm finds a local objective, but it results in a more robust, flexible, faster and less communication demanding implementation. Proposed distributed algorithm works on a vertex level and each vertex only has the local information, which is communicated between the adjacent incident vertices. Pseudocode of the distributed FA algorithm is shown in Fig. 6.

Algorithm 2 :
Input:
Set of blocked frequencies, vertex co-channel and adjacent channel edge weights w_{co}, w_{adj}, penalties for co- and adjacent channel rule violation p^{co}, p^{adj}, vertex transceiver user throughputs c_v
Output:
Frequency assignment function $f_{best}: V_c \to F$ for vertices incident on $\boldsymbol{v_i}$, cumulative vertex interference I_{sum}, SU throughput C_{CRN}
begin
initialize all
***calculate** saturation metrics label S_{vi} for vertex $\boldsymbol{v_i}$*
***calculate** vertex contention window for backoff time*
***transmit** saturation metrics label S_{vi}, blocked frequencies vector B_{vi}*
***receive** S_{vi}, B_{vi} of all incident vertices*
***do while** not all V_c incident to vertex $\boldsymbol{v_i}$ are colored*
  ***sort** vertices in decreasing order of S_{vi}*
  ***select** vertex with Max S_{vi}*
  ***if** vertex $\boldsymbol{v_i}$ has Max S_{vi}*
    ***select frequency** with maximum objective*
    ***color** vertex vi according to objective function*
  endif
  update all
repeat
$f_{best} = x_{vf}$
***calculate** I_{sum}, C_{CRN} for $\boldsymbol{v_i}$*
end

Fig. 6 Distributed FA algorithm. The figure shows pseudo code of proposed distributed algorithm

The algorithm initializes and the SU requesting frequency access calculates its own saturation metrics label. Vertex saturation label and set of blocked frequencies are communicated to other adjacent non-assigned vertices requesting access to the same frequency band. In order to avoid transmission collision, contention window for the transmission backoff time is calculated as a random value in the interval $[0, \mathrm{window}]$, where the window is calculated as $1/S_{\mathrm{vi}}$. On the basis of the exchanged information on the local vertex saturation metric, ordering of the vertex coloring is determined. Vertex with a local maximum saturation score is selected first to color, and then it selects a frequency with the maximum contribution to the objective function. Selected frequency is then communicated to the adjacent vertices and the list of assigned frequencies is updated. Subsequently, new saturation labeling is performed and the next vertex to color is determined.

Each iteration of the single vertex coloring can be completed in $O(M)$ where M is the number of the available frequencies. The number of the iterations in the distributed algorithm is bounded by $O(\Delta)$, where Δ is a maximum degree of a vertex in the conflict graph. The overall complexity of the proposed distributed algorithm is $O(\Delta M)$. The complexity of the distributed algorithm is N_{SU}/Δ times smaller compared to the centralized algorithm, indicating that the distributed algorithm is simpler and converges faster.

7 Generalizations and implementation

7.1 Central frequency and bandwidth CR spectrum assignment

In the proposed FA, we assume fixed set of the channels with predetermined bandwidth and channel separation. In general, the problem of the FA in the CR networks should include not only the identification of the central frequency but also the optimal bandwidth to be used according to the service requirements of the SUs. Generally, spectrum holes can be of the various bandwidth and frequency separation, and CR devices by definition should be able to use these spectrum opportunities. The question of the assigning spectrum fragments of the different sizes is still an open area in research of the CR FA [10]. In this section, we present the generalization of the proposed model of the FA in order to relax input assumptions of the fixed channel division, to an implementation considering bandwidth and central frequency decision. This generalization is quite useful in the practical deployments of the CR systems.

The proposed solution of the central frequency and bandwidth assignment for the CR consists of the following steps: the admission control, the bandwidth reservation, and the spectrum assignment. In the admission control, the spectrum management system has to determine weather the new incoming CR communication request can be accepted or the request is queued. Fair spectrum bandwidth $B_{\mathrm{SU}i}$, which could be assigned to the SUi is calculated as

$$B_{\mathrm{SU}i} = \frac{C_{\mathrm{SU}r\_i} B_{\mathrm{free}}}{\sum_{j=1}^{N_{\mathrm{SU}}} C_{\mathrm{SU}r\_j}}, \tag{15}$$

where $C_{\mathrm{SUr\_i}}$ and $C_{\mathrm{SUr\_j}}$ are the requested throughputs of SU i and SU j and B_{free} is the available idle spectrum in the frequency band considered and not currently used by PUs.

The SUi request is accepted, if the achievable throughput in the bandwidth which can be assigned to the SU can support the minimal sustainable throughput needed for satisfying the service requirements for the SU as:

$$\mathrm{ack}_{\mathrm{SU}i} = \left\{ \begin{matrix} 1 \ \mathit{if} \ C_{\mathrm{SU}a} = B_{\mathrm{SU}i} \cdot c_{\mathrm{SINR}} \geq C_{\mathrm{SU}s\_i} \\ 0 \ \mathit{if} \ C_{\mathrm{SU}a} = B_{\mathrm{SU}i} \cdot c_{\mathrm{SINR}} < C_{\mathrm{SU}s\_i} \end{matrix} \right\}, \tag{16}$$

where $\mathrm{ack}_{\mathrm{SU}i}$ is the binary acceptance variable, $C_{\mathrm{SU}a}$ is the achievable SU throughput, c_{SINR} is the normalized channel throughput of the SU with observed SINR and $C_{\mathrm{SU}s\_i}$ is the minimal sustainable throughput. Achievable SU throughput of the SU is calculated using Eq. (5) or by approximation Eq. (19) in case of M-QAM.

For the bandwidth reservation, we propose to use MSA algorithm of the spectrum aggregation presented in [24]. The MSA algorithm is utilizing the worst spectrum band which can barely satisfy the bandwidth requirements B_{SUi} of the SU in consideration relating to all unassigned SUs. In MSA algorithm, the SUs are sorted by the bandwidth requirements and the idle spectrum is sorted by the available bandwidth of the spectrum regions. For the bandwidth reservation, the MSA algorithm firstly chooses the SU with the highest bandwidth requirements and try to assign the spectrum region with the least available bandwidth. The reservation continues in the decreasing order of SU's bandwidth requirements and ascending order of the available spectrum regions, until all users are assigned or there is no available spectrum.

The spectrum assignment phase can use proposed algorithms for the CR FA presented in this paper without the need for modification. In the preparatory phase, generalization of the interference weighting to include interference potential between the spectrum fragments is proposed. As we assign discontinuous spectrum regions of the different bandwidths to SUs, we have to extend the definition of the spectrum weights to be able to accommodate the evaluation of the level of the interference between the SUs. For each interfering path between the SUs, we designate the conflict edge and introduce the associated interference weight w_E determining the level of potential interference between corresponding SUs as follows:

$$w_E = p_E \cdot w_{N0}, \tag{17}$$

where p_E is the conflict edge channel penalty and w_{N0} is the nominal edge weight. The nominal edge weight w_{N0} corresponds to the channel coefficient including propagation loss, antenna gains, and discrimination between the SU Tx and SU Rx under the consideration. It is determined as described in Section 4. The conflict edge channel penalty p_E represents the portion of the SU transmitter power which is absorbed by the SU receiver. It is calculated as the reduction of the interference power caused by the filter shape of the transmitter spectrum density mask and the receiver selectivity mask as follows:

$$p_E = \int_{f_l}^{f_h} S_{\mathrm{SUTx}}\left(f\right) \cdot S_{\mathrm{SURx}}\left(f - \Delta f\right) \cdot df, \tag{18}$$

where S_{SUTx} is the Tx signal's power distribution across the frequency spectrum, S_{SURx} is the SU receiver filter frequency response, f is the frequency, f_l and f_h are lower and higher limits of the frequency band in consideration, and $\Delta f = \left|f_{\mathrm{Tx0}} - f_{\mathrm{Rx0}}\right|$ is the spectrum gap between the SU Tx and Rx center frequency. If $\Delta f = 0$, both Tx and Rx are operating at the same central frequency and we are coming to the special case of co-channel penalty.

7.2 Implementation considerations

The CR networks utilizing algorithms with the objective of the minimal total network interference (CminSumInt, DminInt) are more suitable for the user scenarios where there exists a large number of SUs and scarcity of the available frequencies, requiring reliable but not very data demanding communications. The real-world applications of CR networks with this objective function could be the wireless sensor networks, the radiometers in the smart grid networks, the medical body area network devices for monitoring, diagnosing or treating of the patients, the utility companies' telemetry networks, location services in the search and rescue applications, different machine to machine communications, backup system for the emergency networks, and the military communications in the hostile interference environment. The CR networks using algorithms with the objective of the maximal throughput with the interference susceptibility (CMaxSumCap, DMaxCap) are more appropriate for the capacity oriented applications encompassing different scenarios including internet connectivity, cellular networks, and TV white space applications. The real-world applications could be the cellular networks data offload, realizing femtocells in the cellular networks, the best effort wireless hotspots, the local internet connectivity, the internet of things applications, the public protection and disaster relief networks realizing communication between the emergency respondents and the public safety agencies, and the wireless cameras video transfer.

The FA in the CR is an event-based functionality and continuously re-iterated process. In a given moment in time, frequencies for only part of the active SUs have to be assigned. When the PU appears, all active SUs in the interference range have to move to the new spectrum bands [11]. This event triggers re-initiation of the FA process for multiple SUs. When new SU appears in the CR network, it needs to be assigned frequency for its transmissions. Similarly, when the quality of the SU transmissions deteriorates due to dynamic channel conditions, the SU wants to switch to a better frequency. In those cases, the process of a single frequency selection is initiated.

The network protocol for determining the conflict graph edges and corresponding weights can be as follows. Each of the active SUs interchangeably transmits SU ID and test message on the common communication channel, while all other SUs are temporarily monitoring the channel and receiving. The test message and default transmission parameters of the SU are known to all of the SUs in the same geographic area. By monitoring the received signal strength, each of the SUs can determine its adjacent SU transceivers and categorize the interference potential with the corresponding interference category (harmful, disturbing, annoying, permissible). Accordingly, each SU can construct a local conflict network graph with edges corresponding to all of the received SUs transmissions and associated weighting coefficient proportionate to the received signal strength. In the centralized implementation, these network weights are then communicated to the central spectrum management entity, which constructs the network graph and assigns frequencies. The implementation of the protocol determining network conflict graph and corresponding weights is easy to implement and it can be coded by only 2 bits. Since determining of the conflict graph edges and corresponding weights adds additional communication overhead to the CR communication, it is rational to construct the conflict graph and perform weight determining protocol only when needed. In the case of the stationary SU, the weight determining protocol should be performed at the time of the establishing the network or when some change in the network configuration or propagation parameters occurs, while in the case of moving SUs, the weight determining protocol should be performed more often. This could be determined by a network policy and triggered when the CR terminals determine the increased probability level of dropped or severely disturbed transmissions.

8 Numerical results and discussion

8.1 Simulation setup

In order to evaluate the performance of the proposed FA algorithms, we have implemented the stochastic simulation of the primary and the CR network. In the setup of the stochastic simulation, we assumed that the area under test is a square with dimensions of 30 km × 30 km. We considered the SU Tx−Rx pairs that co-exist with a cellular PU network in a configuration similar to the one illustrated in Fig. 2. PU network consists of a number of base stations covering a circular area around them. CR network consists of N_{SU} short range secondary links which access radio spectrum opportunistically in the same area. The frequency band considered is 2000 MHz, and available frequencies are consisting of the group of 15 to 25 channels with a channel bandwidth of 3.5 MHz. The number of the PUs active in the area under test is between 15 and 40; the number of the SU links requesting frequency access is between 10 and 50. The whole spectrum is considered to be open for an opportunistic spectrum access scenario, meaning the SUs can use any channel on the condition of not causing harmful interference to the PU network. We summarize the simulation parameters in Table 4.

PU network model. In the simulation, we considered the network of the PU base stations, each of them operating on one channel, which has to be protected by all SUs in the corresponding interference range. We have randomly placed a number of PUs in a given area. For simplicity, we assumed that the PUs have a constant Tx power of 43 dBm

Table 4 Simulation parameters

Parameter	Value
Area under test	30 km × 30 km
Number of PUs (N_{PU})	15–40
Number of SUs (N_{SU})	10–50
Frequency band	2000 MHz
Channel bandwidth	3.5 MHz
Number of frequencies (M)	15–25
PU transmission range (dPU)	10 km
SU transmission range (dSU)	1–4 km
PU interference range	20 km
SU interference range	2–8 km
Modulation M-QAM	M = [4, 16, 32, 64]
Maximal SU throughput (64 QAM)	16 Mbit/s

and omnidirectional antenna radiation pattern. Each one of the PUs is defined as an information source, which can be in two distinctive states: an active state in which PU constantly generates information packets and requests for data transmission or in a passive state in which PU is not active as an information source. The PU entity is modeled as a two state Markov birth-death process with birth rate α and death rate β. Since each user arrival is independent, transition follows the Poisson arrival process.

CR network model. Cognitive secondary system co-exists in the same area with a primary system using opportunistic radio spectrum access. The SU transmitter and receiver pairs are uniformly randomly generated in the analyzed area. The distance between SU Tx and SU Rx is set between 1 and 4 km. In order to test proposed algorithms in a difficult sharing environment with many interference cases and for simplicity reasons, we assume that SUs also have an omnidirectional antenna radiation pattern. The SU adjusts its transmitting power in order to avoid generating interference with the active PUs on the same frequency. The SU Tx power is limited to a maximal value of $P_{\mathrm{SUMAX}} = 27$ dBm, and it can have any continuous value in the interval $[0, P_{\mathrm{SUMAX}}]$. The SU Tx power is calculated for each available channel using the transmit power strategy as described in Section 5.3. For the channels which are included in the local list of blocked channels B_{v}, the SU TX power is set to 0 because they cannot temporarily be used. In the simulation, we assumed that the SUs use square M-QAM with Grey bit mapping, without losing the generality of the proposed algorithms. For the calculation of the SU throughput in simulation, we used the approximation in [25], tight within 1 dB for $M \geq 4$ and BER $\leq 10^{-3}$ as

$$C\,(\mathrm{SINR}) \approx \mathrm{BW}_{\mathrm{ch}} log_2 \left[1 + \frac{\mathrm{SINR}}{-ln\big(5\mathrm{BER}\big)/1.6}\right], \quad (19)$$

where $C\,(\mathrm{SINR})$ is the throughput, $\mathrm{BW}_{\mathrm{ch}}$ is the channel bandwidth, SINR is the wanted signal to interference plus noise ratio at the SU receiver and BER $= 10^{-3}$ is the target bit error rate. The SINR at each SU differs and depends on the channel used, obtainable SU Tx power, distance to SU Rx, and sum of all interference from other SUs. The maximal throughput of the individual SU is set to 16 Mbit/s, using 64-QAM modulation, as an upper limit of the achievable individual SU throughput. For each SU, the list of blocked channels is established, comparing calculated distance between the PU and SU with the PU interference range. If the distance is smaller than the interference range, this channel cannot be used at the SU. The corresponding channel is included in the SU list of blocked channels B_{v} in order to protect PU transmissions. This list for each SU is dynamically updated with each iteration of the FA algorithm. In the simulation, the SU edges and their corresponding co-channel and adjacent channel weights are calculated from the level of overlapping of interference ranges of the SUs as follows:

$$w_{\mathrm{co}} = p_{\mathrm{co}} \cdot w_i \ \ , \ \ w_{\mathrm{adj}} = p_{\mathrm{adj}} \cdot w_i, \quad (20)$$

$$w_i = \left\{ \begin{array}{cc} 1 & if\ d_{\mathrm{SU}j}^{\mathrm{SU}i} < 2d_{\mathrm{ST}} \\ \frac{2d_{\mathrm{ST}} - d_{\mathrm{SU}j}^{\mathrm{SU}i}}{2\left(d_{\mathrm{SI}} - d_{\mathrm{ST}}\right)} + 1 & if\ 2d_{\mathrm{ST}} \leq d_{\mathrm{SU}j}^{\mathrm{SU}i} \leq 2d_{\mathrm{SI}} \\ 0 & if\ 2d_{\mathrm{SI}} < d_{\mathrm{SU}j}^{\mathrm{SU}i} \end{array} \right\}, \quad (21)$$

where $d_{\mathrm{SU}j}^{\mathrm{SU}i}$ is the distance between SUi and SUj, d_{ST} is the SU transmission range, d_{SI} is the SU interference range, p_{co} is the co-channel penalty set to 1, p_{adj} is the adjacent channel penalty set to 0.1. Interference is classified in four categories as described in Section 4.2.

Methodology. The three parameters, the number of the available frequencies M, the number of the secondary users N_{SU}, and the number of the active primary users N_{PU} are tunable in the ranges shown in Table 4. Each time, we change one of the parameters in the tuning range and then compare the algorithms using the following metrics:

- Average throughput per SU (Mbit/s): the ratio of the cumulative CR network throughput over the number of active SUs
- Average interference per SU: the ratio of the cumulative interference of all interference sources over the number of active SUs
- SU throughput fairness: Jain's fairness index as a measure of the throughput distribution of all of the active SUs in the CR network

Instead of using the cumulative network values, we use the average throughput and average interference as a metric, to be able to compare the results of the simulations with different input parameters. The higher the average throughput per SU, the better the result. A smaller average interference indicates that the algorithm is better in respect of protecting the other users and more efficient in the spectrum usage. The fairness of the proposed algorithms is measured using Jain's fairness index (JFI) [26] calculated as

$$\mathrm{JFI}\,(x_1, x_2, \ldots, x_n) = \frac{\left(\sum_{i=1}^{n} x_i\right)^2}{n \sum_{i=1}^{n} x_i^2}, \quad (22)$$

where n is the number of the SU and x_i is the quantity of resources (i.e., throughput in our case) allocated to each user i. The value of JFI varies from $1/n$ representing poor fairness to 1 representing excellent fairness. Typically, systems with JFI larger than 0.5 are considered to provide a good fairness of the resource distribution.

On the basis of the simulated PU and CR network, we assign frequencies to each of the requesting SUs, using centralized and distributed FA algorithms under investigation. As a benchmark algorithm, we use the centralized and distributed version of Collaborative Max Sum Reward (CSUM) algorithm presented in [13], with a difference of assigning only one frequency per CR. As the performance of graph coloring depends on the conflict graph topology, we test algorithms on a large number of different networks for each setup of the input parameters, in order to have network neutral performance results. Each deployment of the PUs and SUs produces different network topology and a different corresponding conflict graph. All simulations are repeated 500 times with the selected set of input parameters. In each iteration, the new set of PUs and CR Tx-Rx pairs are generated, frequencies are assigned to the PUs, transmit power and throughput are calculated for all SUs, communication and conflict graph is constructed with corresponding interference weighting and categorization. After that, the process of assigning frequencies to all SUs is performed with the proposed algorithms and the experimental performance of the algorithms is compared to the benchmark algorithms. The results are averaged to reduce the influence of the network topology or simulations variance on the network performance and presented results.

8.2 Overall FA algorithms results

In this section, we analyze and present the performance of the FA algorithms in a simulated environment of the PU and CR network. Figure 7 shows the cumulative distribution function $F(x)$ of the average interference level and average throughput per individual SU for 2000 iterations of the FA process with the same input parameters.

The CR network using centralized interference minimizing algorithm (CminSumInt) for assigning frequencies results in the lowest interference with the other concurrent users but also results in a lower individual throughput reward comparing to other algorithms. Throughput maximization algorithm (CMaxSumCap), on the other hand, outperforms all other analyzed algorithms taking into account the achieved SU throughput but causes more interference with the other users compared to a minimum interference algorithm. Figure 7 also shows tradeoff of the average interference level and the average throughput for the proposed algorithms. Using cumulative CR network interference minimization as the objective for FA also results a downturn in the lower level of average throughput compared to the algorithm using throughput maximization as the objective. For example, for the 90 % of the SUs in the network utilizing CminSumInt algorithm, the average interference level is below 0.4, while the average throughput is higher than 8 Mbit/s. On the other side, for the 90 % of the SUs in the network using CMaxCap algorithm, the average interference is below 0.7, and the average throughput is higher than 10.2 Mbit/s. The

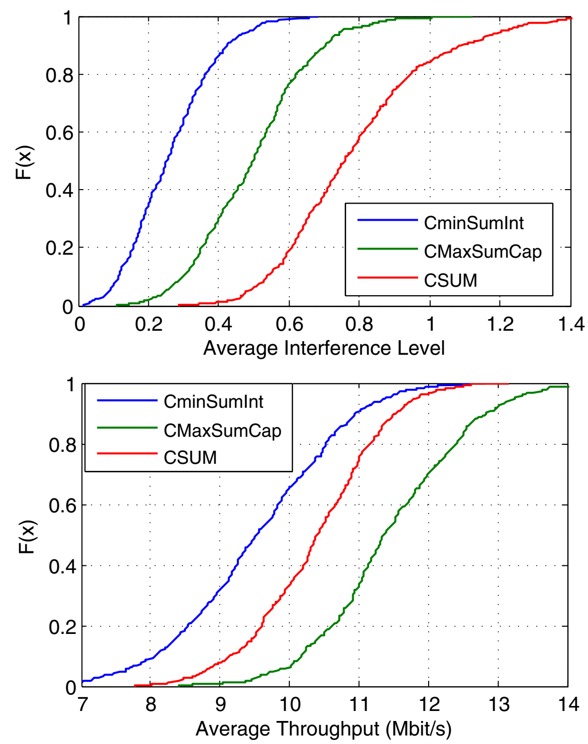

Fig. 7 Cumulative distribution functions of average interference level and throughput (NoPU = 25, NoSU = 40, $M = 15$)

cumulative distribution functions in Fig. 7 show that labeling and coloring rules have been appropriately selected to achieve the minimal interference and maximal throughput objective. In the analyzed case, the network using CSUM algorithm have 2.5–3.5 times larger average level of interference per user compared to the network using CminsumInt algorithm. Introducing the interference sensibility in the proposed FA algorithms with co-channel and adjacent channel interference modeling, edge weights coefficients, and interference categorization significantly contribute to the algorithm efficiency related to interference reduction compared to benchmark CSUM algorithm, which employs binary interference model.

Figure 8 shows the convergence time required by the centralized and distributed algorithms to perform the FA, depending on the number of the SUs, for a network with 25 active PUs and 15 frequencies. As the number of SUs accessing the FA system increases, the time required to assign the frequencies to all SUs also increases. The convergence time for the centralized algorithm grows much faster than for the distributed algorithm and also shows a non-linear increase related to the number of SUs. On the other side, the distributed algorithm converges much faster than the centralized algorithm and shows a linear dependence on the number of SUs. The centralized algorithm has the higher complexity, since more iterations are

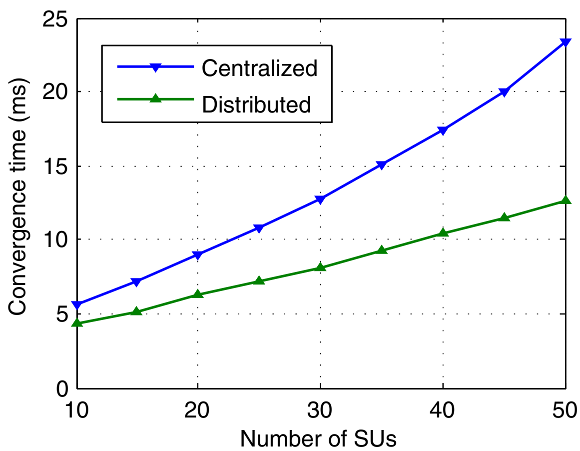

Fig. 8 Convergence time for centralized and distributed algorithm. The figure shows performance metrics of proposed centralized and distributed algorithm

needed compared to the distributed algorithm. The distributed algorithm performs local optimization involving adjacent vertices, and the FA process is done in parallel by the several SUs. As a result, differences in convergence times between algorithms become more significant when the number of the SUs increases. For the network with 50 SUs, the distributed algorithm is two times faster than its centralized counterpart. As we can see from Figs. 11 and 12, proposed centralized algorithms outperform distributed algorithms, but the time for calculating the FA scheme for all SUs is much longer. As a result, for bigger networks having more than hundred of SUs, usage of the centralized algorithms would become impractical. Due to the longer convergence time, the SUs would be in a situation of missing some of the spectrum opportunities (e.g., short temporal spectrum holes). We can conclude that, for the smaller CR networks, or for longer lasting spectrum opportunities (e.g., TV white spaces), the centralized algorithm is more suitable. On the other side, distributed algorithms would be a better choice for the bigger SU networks and spectrum band with more dynamic changes in the frequency availability.

8.3 Centralized FA algorithms performance

The performance of the centralized FA algorithms in the CR networks is presented in Figs. 9 and 10.

Figure 9 shows the average throughput per SU, the average interference per SU, and the throughput fairness depending on the number of the frequencies, for the network with 25 active PUs, 40 SUs, and other simulation parameters according to Table 4. It can be seen that the average throughput per SU steadily increases with the increase of a number of the available frequencies. CMaxSumCap algorithm reaches maximum throughput per SU, which is set at 16 Mbit/s with a frequency pool of 23 channels. In order to achieve an average throughput of 13 Mbit/s per SU, a network with CMaxSumCap algorithm uses 16 frequencies, with CSUM algorithm uses 18 frequencies and with CminSumInt uses 22 frequencies which is 35 % more than the best performing algorithm. The average interference per SU is decreasing with a larger number of the available frequencies, since the same number of SUs is distributed in a larger frequency pool, resulting in a lower number of SUs per frequency and therefore lower mutual interference. If the average mutual interference per SU is limited to 0.1, the CR network with 40 SUs and CminSumInt algorithm can operate with 16 frequencies. The network with CMaxSumCap algorithm needs more than 18 frequencies and network with CSUM algorithm needs at least 23 frequencies. If the number of frequencies is set to 20, it can be observed that the average interference is below 0.05 for CMaxSumCap and CminSumInt algorithms, while it is 0.22 for CSUM algorithm (i.e., 4.5 times larger). This clearly shows the benefits of the interference weighting and categorization on the network performance. A comparison of throughput fairness shows that the algorithms have Jain's fairness index larger than 0.8, meaning that algorithms provide a good fairness of network resources. As the number of available frequencies increases, fairness also increases. From Fig. 9, we conclude that the SU networks using CMaxSumCap and CSUM algorithms have excellent fairness. The network using CminsumInt algorithms has a higher discrepancy in the SU throughput, but nevertheless, it is still satisfactory.

Figure 10 shows the average throughput per SU, the average interference per SU and the throughput fairness depending on the number of SUs in the CR network with 25 active PUs, and the available pool of 15 frequencies. The average throughput decreases, and the interference increases with an increasing number of secondary users in the CR network. Intuitively, when we set the CR network area and available frequencies, while the number of SUs increases, there are more conflicts between SUs, since the distance between the interfering SUs is decreasing. Consequently, there is a smaller choice of the available frequencies, frequencies are more saturated, the SINR per SU is decreasing, and an average throughput per user decreases. If an average interference per SU is set to 0.2, the CR network with 15 frequencies using CSUM algorithm can accommodate 22 SUs, the network using CMaxSumCap algorithm can accommodate 33 SUs, and the network using CminSumInt algorithm can accommodate 42 SUs. Therefore, the results show that network using CMaxSumCap can accommodate almost two times more users than the network using CSUM algorithm. The CR network with 30 SUs using CminSumInt algorithm has a very low average interference of 0.03, comparing to 0.14 for the network using CMaxSumCap algorithm

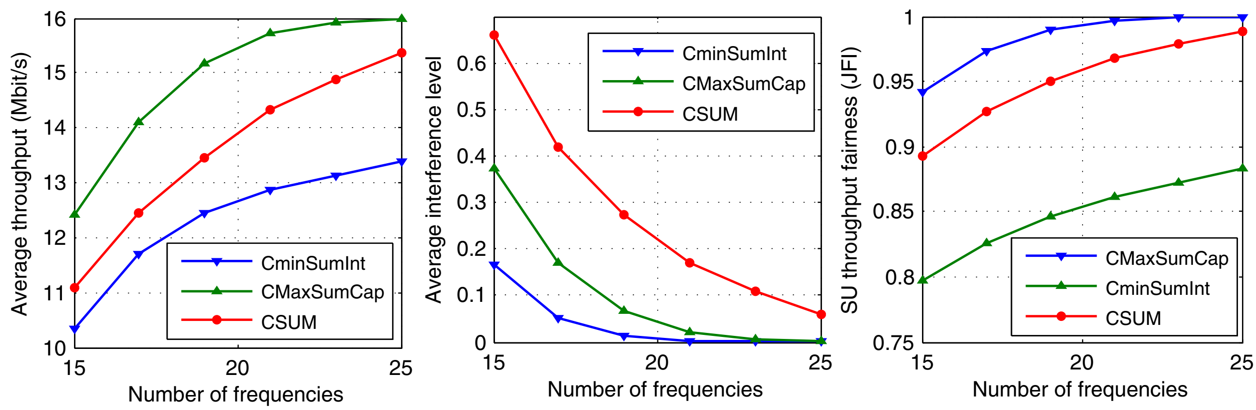

Fig. 9 Centralized FA algorithms' performance depending on the number of frequencies (NoPU = 25, NoSU = 40, M = 15–25). The figure shows average throughput per SU, average interference level per SU, and SU throughput fairness depending on number of frequencies

and 0.37 for the network using CSUM algorithm (i.e., 12 times larger). Due to its greedy nature, fairness of all three algorithms in Fig. 10 is decreasing with increasing number of the SUs accessing the network. CMaxSumCap results in Jain's fairness index above 0.9 in whole tuning range, while in networks using CminSumInt algorithm fairness is worse due to larger differences in SU's throughput. Generally, we can conclude that among the studied algorithms, the centralized minimum cumulative network interference algorithm CminSumInt causes the lowest interference levels in all investigated scenarios and most efficiently uses the radio frequency spectrum. The CR networks using CminSumInt algorithm have a low harmful interference footprint, and therefore, they cause low interference pollution of the radio spectrum environment. Having minimized the level of harmful interference to the SUs and limited interference to the primary network, the largest number of the SUs can be accommodated in the available radio spectrum space. Therefore, networks using CminSumInt algorithm provide the most efficient use of the radio spectrum among the studied algorithms. On the other hand, minimizing interference leads to the greater differences of the throughput between the SUs as the JFI score is lesser compared to the other studied algorithms. Centralized interference sensitive maximum throughput frequency assignment algorithm CMaxSumCap has the highest CR network throughput and excellent fairness among the studied algorithms. Considering that, the CR network using CMaxSumCap algorithm makes the best self-use of the available radio frequency spectrum without causing excessive interference to the PU network. Since the CMaxSumCap algorithm is selecting the frequency with high throughput, causing minimum interference in the selection phase, it is very efficient in frequency usage. CMaxSumCap algorithm causes more interference than CminSumInt algorithm, but it is a well-balanced algorithm, since it distributes throughput more fairly between the users. Both proposed algorithms show

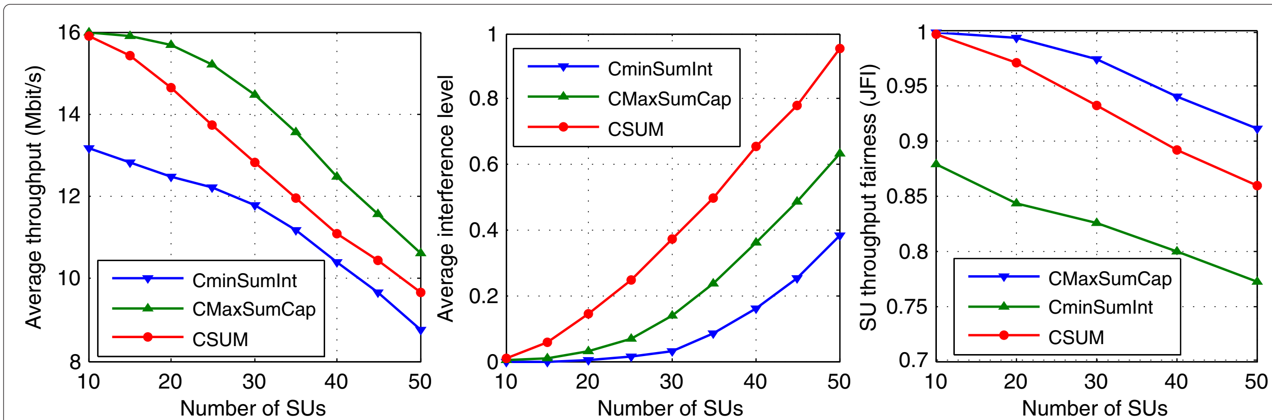

Fig. 10 Centralized FA algorithms' performance depending on the number of SUs (NoPU = 25, NoSU = 10–50, M = 15). The figure shows average throughput per SU, average interference level per SU, and SU throughput fairness depending on the number of SUs

significant reduction of the average interference compared to the benchmark binary interference algorithm CSUM. CMaxSumCap algorithm performs better than the benchmark algorithm in throughput value, average interference, and throughput fairness due to the introduction of a co-channel and an adjacent channel interference gradation with edge weights and new saturation metric for dynamic sorting of vertices in the process of frequency assignment.

8.4 Distributed FA algorithms performance

In this section, we evaluate the performance of the distributed FA algorithms in the CR networks, as presented in Figs. 11 and 12. Comparing the distributed algorithms to their centralized counterparts as a benchmark, distributed algorithms are inferior because they perform a local optimization only, with the cooperation among the neighboring SUs, while the centralized algorithms tend to objectivize the whole CR network under their control, resulting in an overall better results. However, distributed algorithms are much faster as shown in Fig. 8, more robust and simpler to implement.

Figure 11 shows an average throughput per SU depending on the number of SUs, the number of frequencies, and the number of PUs for centralized and distributed algorithms. In the distributed algorithms, the average throughput per SU decreases with the increase of the number of PUs or SUs, similarly to centralized algorithms. Also, an average throughput per SU increases with a larger number of available frequencies, due to a lower frequency congestion. An average SU throughput in the network of 30 SUs and with 15 frequencies is 14 Mbit/s for distributed CSUM algorithm (D-CSUM), 14.48 Mbit/s for DMaxCap algorithm, and 15 Mbit/s for CMaxSumCap algorithm. In the environment with 25 active PUs and CR network with 50 SUs, 18 frequencies (17.84) is needed for CMaxSumCap algorithm, 20 frequencies (19.31) for DMaxCap algorithm, and 21 frequencies (20.95) for D-CSUM algorithm, in case of setting 14 Mbit/s for the average throughput. In the CR network with 40 SUs and 19 frequencies assigned to 25 PUs, an average throughput for the CR network using D-CSUM algorithm is 13.1 Mbit/s, for the CR network using DMaxCap is 13.8 Mbit/s and for the CR network using CMaxSumCap is 14.5 Mbit/s. Comparing an average network throughput per SU, we can conclude that the distributed algorithm achieves 5 % lower average throughput per user compared to a centralized version of the algorithm with the advantage of a faster algorithm convergence and reduced network complexity and communications overhead. The throughput difference between centralized and distributed algorithms is not large, but if we sum over a large number of SUs, the total CR network throughput of the network with distributed frequency assignment is lower than the CR network with the centralized frequency assignment algorithm.

Figure 12 shows an average interference per SU depending on the number of SUs, the number of frequencies, and the number of PUs for centralized and distributed algorithms. Having primary network of 25 PUs using 15 frequencies, CR network utilizing D-CSUM algorithm can operate 23 SUs with an average interference 0.1, CR network utilizing DminInt algorithm can handle 40 SUs, and the CR network utilizing CminSumInt algorithm can operate 47 SUs. The CR network with 50 SUs using a distributed frequency assignment algorithm needs two to three frequencies more than the CR network using a centralized algorithm, which roughly corresponds to 15 % but significantly less than the SU network using D-CSUM algorithm. Increasing the number of PUs in the analyzed area deteriorates the performance of the CR network because the number of available frequencies is reduced

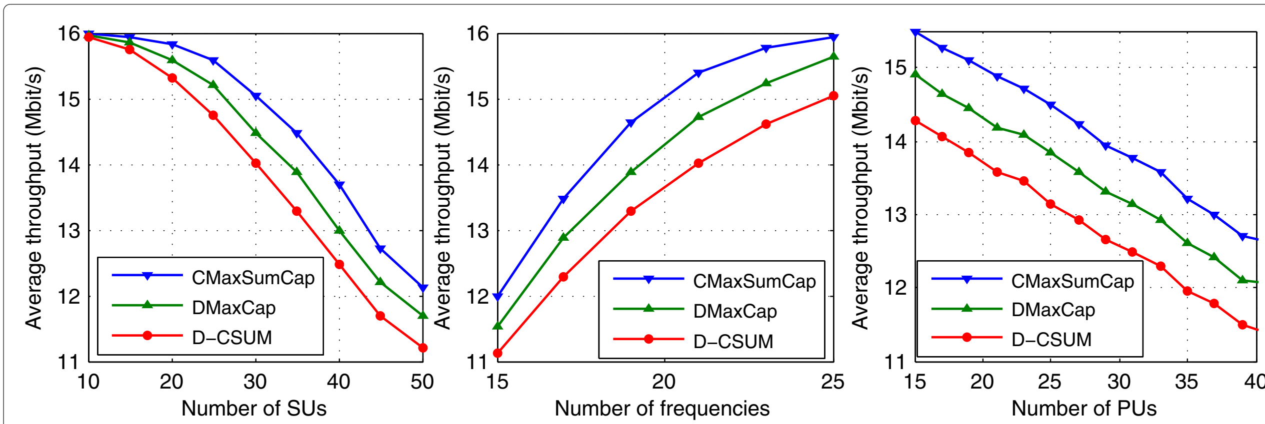

Fig. 11 Throughput performance of distributed and centralized FA algorithms (NoPU = 15–40, NoSU = 1–50, M = 15–25). The figure shows average throughput per SU depending on the number of SUs, number of frequencies, and number of PUs

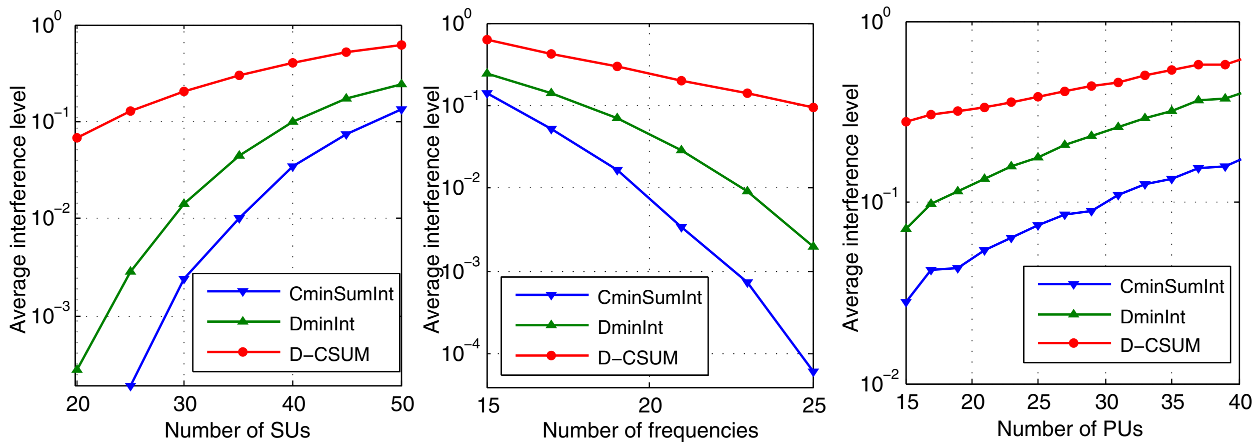

Fig. 12 Interference performance of distributed and centralized FA algorithms (NoPU = 15–40, NoSU = 10–50, M = 15–25). The figure shows average interference per SU depending on number of SUs, number of frequencies and number of PUs

due to the fact that SUs cannot operate on the same frequencies as PUs in the interfering area.

From the presented analysis, we can conclude that the distributed algorithms are less efficient in frequency assignment compared to their centralized counterparts. On the other hand, the distributed algorithms are simpler and easier to implement, since they do not require an extensive cooperation and communication between the SUs. Taking into account analyzed and presented distributed frequency assignment algorithms, we can conclude that the distributed minimum interference algorithm DminInt causes the lowest interference levels and most efficiently uses the radio frequency spectrum in all investigated distributed scenarios. Considering a throughput performance, distributed interference sensitive maximum throughput frequency assignment algorithm DMaxCap has the highest CR network throughput and the lowest SUs throughput imbalance among the studied distributed algorithms.

9 Conclusions

In this paper, we have formulated and addressed the FA problem in the CR networks. The FA ensures that the appropriate frequency is selected to satisfy the requirements of the SUs, while enabling an efficient usage of the radio frequency spectrum. The FA is a crucial function that limits the interference between the CR devices and PUs operating in a heterogeneous and dynamic radio environment. We have treated the FA problem as a graph coloring problem, while introducing the CR specificity. We have introduced the interference susceptibility with a two-layered conflict graph determining the interference potential using co- and adjacent channel edge weights. The protection of the PUs is realized with a local dynamically changing list of blocked channels in the PUs interference range. We have proposed the interference characterization with four categories as an extension to a continuous interference model. As a generalization of our model, we proposed frequency and bandwidth selection instead of channel selection in the FA decision process. We have presented centralized and distributed sequential algorithms that can assign channels to the SU communication links in the CR network with the objective of minimizing network interference (CminSumInt, DminInt) and maximizing network throughput (CMaxSumCap, DMaxCap). The algorithms use dynamic vertex ordering with CR specific novel saturation metrics, which takes into account the frequency limitations due to the PUs activities and interference potential of the already assigned neighboring SUs determining the degree of freedom in selecting the frequency. We showed the effectiveness of our algorithms in the reducing interference and improving the SU network throughput using the network simulation and performance evaluation.

CminSumInt algorithm minimizes the influence of the SU to the other PUs and SUs, resulting in 70 % average reduction of the interference compared to the benchmark CSUM algorithm. CMaxSumCap maximizes the SU network throughput resulting in 15 % increase of the average throughput per SU. CMaxSumCap is a well-balanced algorithm with a good performance in all three performance indicators: throughput, fairness, and interference. The CR networks using the FA algorithms with the interference weighting and categorization can accommodate almost twice as much CR users compared to the networks using algorithms with only binary interference model. Distributed algorithms DminInt and DMaxCap carrying out local optimization provide benefits comparable to a centralized approach. The distributed algorithms reduce the computation complexity and have the faster algorithm convergence and linear increase with the number of SUs. It is shown that the centralized algorithms are more suitable for smaller CR networks, or networks with longer lasting spectrum opportunities, while distributed algorithms are more suitable for the bigger SU networks and more dynamic spectrum environment.

We can conclude that the proposed interference weighting and categorization is beneficial to the CR network performance, since it is possible to quantify the individual interference components and aggregate interference. This approach results in a significantly reduced mutual influence between terminals (2.5–12 times less then with binary interference model) and more efficient spectrum usage. Based on the proposed interference metric, the resource manager can make more knowledgeable and more frequency efficient decisions. The downturn of the interference minimum algorithms is in lower fairness of the throughput. This problem is not present in the maximum capacity algorithms. In future work, the proposed framework can be extended to add a selection of appropriate frequency band and type of the CR spectrum access for multi-band CR operation in heterogeneous environment and to investigate implementation of the fuzzy logic membership function for the interference categorization.

Competing interests
The authors declare that they have no competing interests.

Acknowledgements
The authors would like to thank Dr. Hongjian Sun, the associate editor coordinating the review of this paper and approving it for publication. The authors would also like to express sincere appreciation to the anonymous reviewers and Dr. Ninoslav Majurec from NASA-JPL for their valuable comments which helped in significantly improving this manuscript.

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Received: 14 July 2015 Accepted: 21 January 2016
Published online: 09 February 2016

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