Duan *et al. EURASIP Journal on Wireless Communications and Networking* (2016) 2016:157
DOI 10.1186/s13638-016-0655-8

EURASIP Journal on Wireless Communications and Networking

RESEARCH Open Access

Sum-rate maximization and robust beamforming design for MIMO two-way relay networks with reciprocal and imperfect CSI

Wei Duan[1], Miaowen Wen[2], Xueqin Jiang[3*], Yier Yan[4] and Moon Ho Lee[1]

Abstract

In this paper, we investigate the robust relay beamforming design for a multi-input multi-output (MIMO) two-way relay networks (TWRN) by considering imperfect and reciprocal channel state informations (CSIs). In order to maximize the sum-rate (SR) subject to the individual relay power constraint, we first equivalently convert the objective problem into a sum of the inverse of the signal-to-residual-interference-plus-noise ratio (SI-SRINR) problem. The SI-SRINR problem can be reformulated as a biconvex semi-definite programming (SDP) which employs bounded channel uncertainties as the worst-case model. Then, we convert residual-interference-plus-noise (RIN) and relay power constraints into linear matrix inequalities (LMIs). By this way, the objective problem can be tackled by the proposed efficient iterative algorithm. The analysis demonstrates the procedures of the proposed SI-SRINR robust design.

Keywords: Two-way relay, Sum-rate, Imperfect CSI, SDP, LMI

1 Introduction

Recently, cooperative multi-input multi-output (MIMO) relaying system approach is popularized to increase the system capacity and improve the transmission reliability by leveraging spatial diversity. The MIMO relay network with perfect channel state information (CSI) has been studied in [1–3]. In [1], the authors developed a unified framework for optimizing two-way linear non-regenerative MIMO relay systems. In [2], the authors studied transceiver designs for a cognitive two-way relay network aiming at maximizing the achievable transmission rate of the secondary user. Based on iterative minimization of weighted mean-square error (MSE), a linear transceiver design algorithm for weighted sum-rate maximization has been investigated in the cellular network [3].

All the above works consider perfect CSI, which, however, is usually hard to obtain in practice, due to inaccurate channel estimation, feedback delay, and so on. To evaluate this imperfectness, by taking account into the channel uncertainties, the authors proposed a robust multi-branch Tomlinson-Harashima precoding (MB-THP) transceiver design in MIMO relay networks with imperfect CSI [4], where the Kronecker model is adopted for the covariance of the CSI mismatch. Moreover, the deterministic CSI uncertainty model has been widely used in the worst-case system [5–8]. In [5], the design of robust relay beamforming for two-way relay networks with channel feedback errors was studied, where each node is equipped with a single antenna. In [6], the authors investigated a robust beamforming scheme for the multi-antenna non-regenerative cognitive relay network with the bounded channel uncertainties which are modeled by the worst-case model. In [7], the authors investigated the multi-antenna non-regenerative relay network and addressed the joint source-relay-destination beamforming design problem under deterministic imperfect CSI model. In [8], based on the linear beamformer at the relay and QR successive interference cancelation (SIC) at the destination, the authors proposed a robust minimum MSE-regularized zero-forcing (MMSE-RZF) beamformer optimized in terms of rate. In this work, the authors assumed that the source can estimate the first-hop (form the source to the relay) CSI perfectly via the reciprocal channel when the high signal-to-noise (SNR) is training.

*Correspondence: xqjiang@dhu.edu.cn
[3]School of Information Science and Technology, Donghua University, 436 Shanghai, China
Full list of author information is available at the end of the article

In particular, in [9, 10], the authors considered the problem of robust minimum sum mean-square error (SMSE) relay precoder design for the two-way relay networking (TWRN).

For the one-way relay networks [4, 6–8, 10], the self-interference is not considered and the objective problem is easy to be converted into the convex version. For the single-antenna scenarios [5, 11], the equivalent channel can be expressed by employing the Hadamard product. For above references, the objective problem is actually only for the manipulation of a single node. By using ([12] Lemma 2), the robust minimum sum-MSE optimization problem [9, 10] can be easily converted into a convex problem. Therefore, the sum-rate (SR) maximization problem with multi-relay nodes for TWRN with the residual relay power constraint is more challenging and more general. In this paper, we propose a joint source and relay robust beamforming scheme for the MIMO TWRN where both the first- and second-hop CSIs are considered to be reciprocal and imperfectly known at each node. To get the accurate performance, the residual interference has been reserved. Since the considered sum-rate (SR) maximization problem is not only non-convex but also subject to the semi-infinite relay power constraints, we convert the objective problem into a sum of the inverse of the signal-to-residual-interference-plus-noise ratio (SI-SRINR) problem which is subject to the linear matrix inequality (LMIs) version of the constraints. In order to efficiently tackle the SI-SRINR problem, we first transform the problem into a biconvex semi-definite program (SDP) using the sign-definiteness lemma and then propose an alternating iterative algorithm with satisfactory convergence.

Notations: $\mathbf{A}^T$ and $\mathbf{A}^H$ denote the transpose and the Hermitian transpose of a matrix $\mathbf{A}$, respectively. $\mathbf{I}_N$ represents an $N \times N$ identity matrix. $\mathrm{E}(\cdot)$, $\otimes$, and $\|\cdot\|$ stand for the statistical expectation, the Kronecker product, and the Frobenius norm.

2 System model and objective problem

A TWRN consisting of two source nodes 1 and 2, S_1 and S_2, and L relay nodes $\{R_1, R_2, \ldots, R_L\}$ is considered. The source and relay nodes are equipped with M and N antennas, respectively. Each transmission involves two time slots. At the first time slot, after linearly processed by a transceiver beamforming matrix $\mathbf{B}_t \in \mathbb{C}^{M\times M_b}, \forall t \in \{1, 2\}$ and $M_b \leq M$, with the power constraint as $\|\mathbf{B}_t\|^2 \leq P_t$. Denote the data symbol vector transmitted from the source node S_t as $\mathbf{x}_t \in \mathbb{C}^{M_b\times 1}$ with $\mathrm{E}\left\{\mathbf{x}_t\mathbf{x}_t^H\right\} = \mathbf{I}_{M_b}$. The received signal at R_i can be expressed as

$$\mathbf{y}_{R_i} = \mathbf{F}_{i,1}\mathbf{B}_1\mathbf{x}_1 + \mathbf{F}_{i,2}\mathbf{B}_2\mathbf{x}_2 + \mathbf{n}_{R_i}, \tag{1}$$

where $\mathbf{F}_{i,t} \in \mathbb{C}^{N\times M}, \forall i \in \{1, ..., L\}$ represents the channel coefficient from the source node S_t to the relay node R_i and $\mathbf{n}_{R_i} \sim CN(0, \sigma^2_{R_i}\mathbf{I}_N)$ denotes the additive white Gaussian noise (AWGN) vector with zero mean and variance $\sigma^2_{R_i}\mathbf{I}_N$.

At the second time slot, the relay node R_i linearly amplifies $\mathbf{y}_{R_i}$ with an $N \times N$ matrix $\mathbf{W}_i$ and then broadcasts the amplified signal vector $\mathbf{x}_{R_i}$ to source nodes 1 and 2. The signal transmitted from relay node R_i can be expressed as

$$\mathbf{x}_{R_i} = \mathbf{W}_i\mathbf{y}_{R_i}. \tag{2}$$

From (2), the average transmit power consumed by the relay node R_i can be derived as

$$\mathrm{E}\left\{\left\|\mathbf{x}_{R_i}\right\|^2\right\} = \left\|\mathbf{W}_i\mathbf{F}_{i,1}\mathbf{B}_1\right\|^2 + \left\|\mathbf{W}_i\mathbf{F}_{i,2}\mathbf{B}_2\right\|^2 + \sigma^2_{R_i}\|\mathbf{W}_i\|^2. \tag{3}$$

The received signal at source node S_t for $t \in \{1, 2\}$ can be written as

$$\mathbf{y}_t = \sum_{i=1}^{L}\mathbf{G}_{t,i}\mathbf{W}_i\left(\mathbf{F}_{i,t}\mathbf{B}_t\mathbf{x}_t + \mathbf{F}_{i,\bar{t}}\mathbf{B}_{\bar{t}}\mathbf{x}_{\bar{t}}\right) + \sum_{i=1}^{L}\mathbf{G}_{t,i}\mathbf{W}_i\mathbf{n}_{R_i} + \mathbf{n}_t,$$

where $\mathbf{G}_{t,i}$ denotes the channel coefficient form the relay node R_i to the source node S_t of dimension $M \times N$ and $\mathbf{n}_t$ is the noise vector at the source node S_t with zero mean and variance $\sigma^2_{S_t}\mathbf{I}_M$. By taking into account the estimation error and delay, we further assume that the CSI is partially known at each node and the channels are reciprocal, i.e., $\mathbf{F}_{i,t} = \mathbf{G}^T_{t,i}$. To model this imperfect effect, we consider the following additive CSI uncertainties:

$$\mathbf{F}_{i,t} \triangleq \widetilde{\mathbf{F}}_{i,t} + \Delta_{\mathbf{F}_{i,t}}, \tag{4}$$

where $\widetilde{\mathbf{F}}_{i,t}$ and $\Delta_{\mathbf{F}_{i,t}}$ are the nominal values and channel uncertainty of the channel $\mathbf{F}_{i,t}$. For simplicity, we assumed that the channel uncertainties are norm-bounded errors (NBEs) in analogy with [13, 14], i.e.,

$$\left\|\Delta_{\mathbf{F}_{i,1}}\right\| = \left\|\Delta_{\mathbf{G}_{1,i}}\right\| = \alpha_i, \ \left\|\Delta_{\mathbf{F}_{i,2}}\right\| = \left\|\Delta_{\mathbf{G}_{2,i}}\right\| = \beta_i, \tag{5}$$

where the slack values satisfy $0 \leq \{\alpha_i, \beta_i\} \ll 1$, which is a reasonable assumption in a practical system.

In TWRN, since the signal transmitted by the transceiver nodes reappear as self-interference, by employing the successive interference cancelation (SIC), the self-interference can be completely eliminated with perfect CSI [15]. Nevertheless, considering the imperfect CSI in this paper, the self-interference at both source nodes cannot be completely canceled, and the approximate residual self-interference[1] at the source S_t is

$$\begin{aligned}\chi_t &= \sum_{i=1}^{L}\left(\mathbf{G}_{t,i}\mathbf{W}_i\mathbf{F}_{i,t}\mathbf{B}_t\mathbf{x}_t - \widetilde{\mathbf{G}}_{t,i}\mathbf{W}_i\widetilde{\mathbf{F}}_{i,t}\mathbf{B}_t\mathbf{x}_t\right)\\ &\approx \sum_{i=1}^{L}\left(\widetilde{\mathbf{G}}_{t,i}\mathbf{W}_i\Delta_{\mathbf{F}_{i,t}} + \Delta_{\mathbf{G}_{t,i}}\mathbf{W}_i\widetilde{\mathbf{F}}_{i,t}\right)\mathbf{B}_t\mathbf{x}_t,\end{aligned} \tag{6}$$

where the term $\Delta_{\mathbf{G}_{t,i}}\mathbf{W}_i\Delta_{\mathbf{F}_{i,t}}\mathbf{B}_t\mathbf{x}_t$ has been set to be 0 because if we retain this term in χ_t, it will result in some terms involving high order of channel uncertainties which is very close to 0 when calculating the covariance of the residual interference. Let $\widetilde{\chi}_t = \sum_{i=1}^{L}\left(\widetilde{\mathbf{G}}_{t,i}\mathbf{W}_i\Delta_{\mathbf{F}_{i,t}} + \Delta_{\mathbf{G}_{t,i}}\mathbf{W}_i\widetilde{\mathbf{F}}_{i,t}\right)$. The received SRINR at receivers can be thus expressed as

$$\mathrm{SRINR}_t = \frac{\left\| \sum_{i=1}^{L} \mathbf{G}_{t,i}\mathbf{W}_i\mathbf{F}_{i,\bar{t}}\mathbf{B}_{\bar{t}}\right\|^2}{\|\widetilde{\chi}_t\mathbf{B}_t\|^2 + \sigma_{R_i}^2\left\| \sum_{i=1}^{L}\mathbf{G}_{t,i}\mathbf{W}_i\right\|^2 + \sigma_{S_t}^2}, \quad (7)$$

where $\bar{t} = 3 - t$ for $t = \{1, 2\}$. The objective of this paper is to maximize the SR, which is subject to the individual relay transmit power constraint, shown as

$$T_1: \max_{\mathbf{B}_t,\mathbf{W}_i} \quad \sum_{t=1}^{2}\frac{1}{2}\log_2\left(1 + \mathrm{SRINR}_t\right)$$
$$s.t. \quad \left\|\mathbf{x}_{R_i}\right\|^2 \le P_{R_i}, \|\mathbf{B}_t\|^2 \le P_t,$$

where P_{R_i} is the maximum allocated power to the relay and the factor $\frac{1}{2}$ is due to the half-duplex relay. Obviously, the objective problem T_1 is non-convex since the variables $\mathbf{W}_i$ and $\mathbf{B}_t$ are in the numerator and the denominator of the SRINR_t. Moreover, since the semi-infinite expressions of the optimal $\mathbf{W}_i$, $\mathbf{B}_t$ are intractable, it is difficult to obtain the globally optimal solution. In the next section, we will propose a subpotimal solution to solve T_1.

3 Joint optimal beamformer design and proposed algorithm

3.1 Joint optimal beamformer design

Let $N_t = \sigma_{R_i}^2\left\|\sum_{i=1}^{L}\mathbf{G}_{t,i}\mathbf{W}_i\right\|^2 + \sigma_{S_t}^2$ and $\mathbf{H}_{\bar{t}} = \sum_{i=1}^{L}\mathbf{G}_{t,i}\mathbf{W}_i\mathbf{F}_{i,\bar{t}}$. The beamforming matrix $\mathbf{W}_i$ and $\mathbf{B}_t$ are optimized by solving the following problem:

$$T_2: \max_{\mathbf{B}_t,\mathbf{W}_i} \quad \sum_{t=1}^{2}\frac{1}{2}\log\left(1 + \frac{\left\|\mathbf{H}_{\bar{t}}\mathbf{B}_{\bar{t}}\right\|^2}{\left\|\widetilde{\chi}_t\mathbf{B}_t\right\|^2 + N_t}\right)$$
$$s.t. \quad \left\|\mathbf{x}_{R_i}\right\|^2 \le P_{R_i}, \ \|\mathbf{B}_t\|^2 \le P_t, \forall t = \{1,2\}.$$

Further denoting $a_t = \frac{\|\mathbf{H}_{\bar{t}}\mathbf{B}_{\bar{t}}\|^2}{\left\|\widetilde{\chi}_t\mathbf{B}_t\right\|^2 + N_t}$, for $t = 1, 2$, the objective function can be expressed as

$$\begin{aligned} &\max\left\{\frac{1}{2}\log\left(1 + a_1\right) + \frac{1}{2}\log\left(1 + a_2\right)\right\} \\ &= \max\frac{1}{2}\log\left\{(1 + a_1) \times (1 + a_2)\right\} \\ &\stackrel{a}{=} \max\left\{a_1 + a_2 + a_1 a\right\}, \end{aligned} \quad (8)$$

where (a) follows $\log(\cdot)$ is a monotonic function and 1 is a constant. Now, we have the equivalent optimization problem as

$$Q_1: \max_{\mathbf{B}_t,\mathbf{W}_i} \quad a_1 + a_2 + a_1 a_2$$
$$s.t. \quad \left\|\mathbf{x}_{R_i}\right\|^2 \le P_{R_i}, \|\mathbf{B}_t\|^2 \le P_t.$$

Since the CSIs are imperfectly known, Q_1 cannot be solved by using zero-gradient (ZG) algorithm [16]. In particular, due to a_1 and a_2 are both not only non-convex but also non-concave, the objective function $a_1 + a_2 + a_1 a_2$ is difficult to convert into the convex version. To efficiently solve T_2, we propose a biconvex SDP to obtain the suboptimal solution of the worst-case SR.

Proposition 1. *The problem T_2 is equivalent to T_3 with inequality constraints which is given as*

$$T_3: \min_{\mathbf{B}_t,\mathbf{W}_i,\gamma_t} \quad \gamma_1 + \gamma_2$$
$$s.t. \quad q_t \le \gamma_t, \left\|\mathbf{x}_{R_i}\right\|^2 \le P_{R_i}, \|\mathbf{B}_t\|^2 \le P_t,$$

where $q_t = \frac{\|\widetilde{\chi}_t\mathbf{B}_t\|^2 + N_t}{\|\mathbf{H}_{\bar{t}}\mathbf{B}_{\bar{t}}\|^2}$ and γ_t is an auxiliary optimization variable which serves as upper bound of q_t for $t = 1, 2$.

Proof. The objective problem of the SR can be formulated as follows:

$$\begin{aligned} &\tfrac{1}{2}\log\left(1 + \tfrac{\|\mathbf{H}_2\mathbf{B}_2\|^2}{\|\widetilde{\chi}_1\mathbf{B}_1\|^2 + N_1}\right) + \tfrac{1}{2}\log\left(1 + \tfrac{\|\mathbf{H}_1\mathbf{B}_1\|^2}{\|\widetilde{\chi}_2\mathbf{B}_2\|^2 + N_2}\right) \\ &= \tfrac{1}{2}\log\left\{1\left(1 + \tfrac{\|\mathbf{H}_2\mathbf{B}_2\|^2}{\|\widetilde{\chi}_1\mathbf{B}_1\|^2 + N_1}\right)\left(1 + \tfrac{\|\mathbf{H}_1\mathbf{B}_1\|^2}{\|\widetilde{\chi}_2\mathbf{B}_2\|^2 + N_2}\right)\right\}. \end{aligned} \quad (9)$$

Since the optimal solution of $\{\max\,(1 + A)(1 + B)\}$ is equivalent to the problem $\left\{\min\,\left(\frac{1}{A} + \frac{1}{B}\right)\right\}$ [17] and the fact that $\log(\cdot)$ is a monotonic function, letting $q_t = \frac{\|\widetilde{\chi}_t\mathbf{B}_t\|^2 + N_t}{\|\mathbf{H}_{\bar{t}}\mathbf{B}_{\bar{t}}\|^2}$, the SR maximization problem can be equivalently converted into $\left\{\min\,(q_1 + q_2)\right\}$. Similar to [9], by introducing the auxiliary optimization variable γ_t, the problem $\left\{\min\,(q_1 + q_2)\right\}$ can be recast in the epigraph form [18] as $\{\min\,(\gamma_1 + \gamma_2)\}$, $s.t.\ q_t \le {\gamma_t}^2$, and we have the objective problem T_3. □

The problem T_3 is still non-convex with respect to the constraint q_t. In order to solve this problem, $q_t \le \gamma_t$ can be converted into following three convex subproblems which are:

$$\begin{aligned} &(1): N_t \le \varsigma_t, \ (2): \|\widetilde{\chi}_t\mathbf{B}_t\|^2 \le \tau_t, \\ &(3): \left\|\mathbf{H}_{\bar{t}}\mathbf{B}_{\bar{t}}\right\|^2 \ge \tfrac{1}{\gamma_t}\left(\varsigma_t + \tau_t\right), \end{aligned} \quad (10)$$

where ς_t and τ_t are slack values which serve as upper bounds of N_t and $\|\widetilde{\chi}_t\mathbf{B}_t\|^2$, respectively. Now, the problem T_3 is equivalent to a convex SDP with inequality constraints as:

$$T_4: \min_{\mathbf{B}_t,\mathbf{W}_i,\gamma_t} \quad \gamma_1 + \gamma_2$$
$$s.t. \quad N_t \le \varsigma_t, \left\|\mathbf{x}_{R_i}\right\|^2 \le P_{R_i}, \|\widetilde{\chi}_t\mathbf{B}_t\|^2 \le \tau_t,$$
$$\left\|\mathbf{H}_{\bar{t}}\mathbf{B}_{\bar{t}}\right\|^2 \ge \frac{1}{\gamma_t}\left(\varsigma_t + \tau_t\right), \|\mathbf{B}_t\|^2 \le P_t.$$

Proposition 2. *The constraint $N_t \leq \varsigma_t$ can be equivalently converted into the linear matrix inequality (LMI) version as*

$$\begin{bmatrix} \Gamma_1 - \sum_{i=1}^{l} \nu_i \mathbf{q}_1^H \mathbf{q}_1 & -\varrho_{1,t}\Omega_1^H & \cdots & -\varrho_{l,t}\Omega_l^H \\ -\varrho_{1,t}\Omega_1 & \nu_1 \mathbf{I}_{MN} & \cdots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ -\varrho_{l,t}\Omega_l & \mathbf{0} & \cdots & \nu_l \mathbf{I}_{MN} \end{bmatrix} \succeq 0, \quad (11)$$

where $\varrho_{i,1} = \alpha_i$ and $\varrho_{i,2} = \beta_i$, $\nu_i \geq 0$ are slack variables, $\mathbf{q}_1 = [-1, \mathbf{0}_{1\times MN}]$, $\Omega_i = [\mathbf{0}_{MN\times 1}, \Psi_i^H]$, for $i = 1, ..., l$ and

$$\Gamma_1 = \begin{bmatrix} \varsigma_t^\star & \widetilde{\Upsilon}^H \\ \widetilde{\Upsilon} & \mathbf{I}_{MN} \end{bmatrix}.$$

Proof. See Appendix 1. □

Using $\|\mathbf{A}+\mathbf{B}\| \leq \|\mathbf{A}\| + \|\mathbf{B}\|$ and $\|\mathbf{AB}\| \leq \|\mathbf{A}\| \|\mathbf{B}\|$ for the residual interference covariance, we have

$$\begin{aligned} \|\widetilde{\chi}_t \mathbf{B}_t\|^2 &= \left\| \left(\sum_{i=1}^{L} \widetilde{\mathbf{G}}_{t,i} \mathbf{W}_i \Delta_{\mathbf{F}_{i,t}} + \sum_{i=1}^{L} \Delta_{\mathbf{G}_{t,i}} \mathbf{W}_i \widetilde{\mathbf{F}}_{i,t} \right) \mathbf{B}_{\bar{t}} \right\|^2 \\ &\leq \left\| \sum_{i=1}^{L} \widetilde{\mathbf{G}}_{t,i} \mathbf{W}_i \Delta_{\mathbf{F}_{i,t}} \mathbf{B}_{\bar{t}} \right\|^2 + \left\| \sum_{i=1}^{L} \Delta_{\mathbf{G}_{t,i}} \mathbf{W}_i \widetilde{\mathbf{F}}_{i,t} \mathbf{B}_{\bar{t}} \right\|^2 \\ &\quad + 2 \left\| \sum_{i=1}^{L} \widetilde{\mathbf{G}}_{t,i} \mathbf{W}_i \Delta_{\mathbf{F}_{i,t}} \overline{\mathbf{B}}_t \Delta_{\mathbf{G}_{t,i}} \mathbf{W}_i \widetilde{\mathbf{F}}_{i,t} \mathbf{B}_{\bar{t}} \right\| \\ &\leq 4\varrho_{i,t}^2 \left\| \sum_{i=1}^{L} \mathbf{B}_{\bar{t}} \widetilde{\mathbf{G}}_{t,i} \mathbf{W}_i \right\|^2. \end{aligned} \quad (12)$$

Letting $4\varrho_{i,t}^2 \left\| \sum_{i=1}^{L} \mathbf{B}_{\bar{t}} \widetilde{\mathbf{G}}_{t,i} \mathbf{W}_i \right\|^2 = \tau_t$, similar to Appendix 1, the SRINR constraint can be further rewritten as

$$\begin{aligned} \left\| \mathbf{H}_{\bar{t}} \mathbf{B}_{\bar{t}} \right\|^2 &= \left\| \varphi + \sum_{i=1}^{L} \mathbf{M}_{\mathbf{G}_{t,i}} \mathrm{vec}\left(\Delta_{\mathbf{G}_{t,i}}\right) + \sum_{i=1}^{L} \mathbf{M}_{\mathbf{F}_{i,\bar{t}}} \mathrm{vec}\left(\Delta_{\mathbf{F}_{i,\bar{t}}}\right) \right\|^2 \\ &\geq \tfrac{1}{\gamma_t} \left(\tau_t + \varsigma_t\right), \end{aligned} \quad (13)$$

where $\varphi = \sum_{i=1}^{L} \mathrm{vec}\left(\widetilde{\mathbf{G}}_{t,i} \mathbf{W}_i \widetilde{\mathbf{F}}_{i,\bar{t}} \mathbf{B}_{\bar{t}}\right)$, $\mathbf{M}_{\mathbf{F}_{i,\bar{t}}} = \sum_{i=1}^{L} \mathbf{B}_{\bar{t}}^T \otimes \left(\widetilde{\mathbf{G}}_{t,i} \mathbf{W}_i\right)$ and $\mathbf{M}_{\mathbf{G}_{t,i}} = \sum_{i=1}^{L} \left(\mathbf{W}_i \widetilde{\mathbf{F}}_{i,\bar{t}} \mathbf{B}_{\bar{t}}\right)^T \otimes \mathbf{I}_M$. Substituting quantities φ, $\mathbf{M}_{\mathbf{F}_{i,\bar{t}}}$, and $\mathbf{M}_{\mathbf{G}_{t,i}}$ into the LMI version of (13), we have

$$\begin{bmatrix} \Gamma_2 - \sum_{j=1}^{2l} \phi_j \mathbf{q}_2^H \mathbf{q}_2 & -\xi_{1,t}\Xi_1^H & \cdots & -\xi_{2l,t}\Xi_{2l}^H \\ -\xi_{1,t}\Xi_1 & \phi_1 \mathbf{I}_{MN} & \cdots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ -\xi_{2l,t}\Xi_{2l} & \mathbf{0} & \cdots & \phi_{2l} \mathbf{I}_{MN} \end{bmatrix} \succeq 0, \quad (14)$$

where $\mathbf{q}_2 = \left[-1, \mathbf{0}_{1\times MM_b}\right]$, $\phi_j \geq 0$ is the slack variables for $j = 1, ..., 2l$. $\Xi_i = \left[\mathbf{0}_{M_b M \times 1}, \mathbf{M}_{\mathbf{G}_{t,i}}^H\right]$, and $\xi_{i,t} = \left\|\Delta_{\mathbf{G}_{t,i}}\right\|$ for $i = 1, ..., l$, $\Xi_i = \left[\mathbf{0}_{M_b M\times 1}, \mathbf{M}_{\mathbf{F}_{i,\bar{t}}}^H\right]$, and $\xi_{i,t} = \left\|\Delta_{\mathbf{F}_{i,\bar{t}}}\right\|$ for $i = l, ..., 2l$, and

$$\Gamma_2 = \begin{bmatrix} \frac{1}{\gamma_t}\left(\tau_t + \varsigma_t\right) & \varphi^H \\ \varphi & \mathbf{I}_{MM_b} \end{bmatrix}.$$

After introducing $\widehat{\mathbf{B}}_t = \mathrm{vec}(\mathbf{B}_t)\mathrm{vec}(\mathbf{B}_t)^H$, $\mathbf{W}_i \mathbf{F}_{i,t} = \mathbf{Q}_t$, $\mathbf{W}_i \widetilde{\mathbf{F}}_{i,t} = \widetilde{\mathbf{Q}}_t$, $\mathbf{W}_i \Delta_{\mathbf{F}_{i,t}} = \Delta_{\mathbf{Q}_t}$, and $P_{R_i} - \sigma_{R_i}^2 \|\mathbf{W}_i\|^2 \leq \widehat{P}_{R_i}$, the individual relay power constraint can be then rewritten as:

$$\begin{aligned} \sum_{t=1}^{2} \Big\{ & \mathrm{vec}\left(\widetilde{\mathbf{Q}}_t\right) \widehat{\mathbf{B}}_t \mathrm{vec}\left(\widetilde{\mathbf{Q}}_t\right) + \mathrm{vec}\left(\Delta_{\mathbf{Q}_t}\right)^H \widehat{\mathbf{B}}_t \mathrm{vec}\left(\Delta_{\mathbf{Q}_t}\right)^H \\ & + 2\Re\left\{ \mathrm{vec}\left(\widetilde{\mathbf{Q}}_t\right) \widehat{\mathbf{B}}_t \mathrm{vec}\left(\Delta_{\mathbf{Q}_t}\right)^H \right\} \Big\} - \widehat{P}_{R_i} \leq 0. \end{aligned} \quad (15)$$

Proposition 3. *The individual relay power constraints can be converted to the following LMI:*

$$\begin{bmatrix} \widehat{\mathbf{B}}_1 + \lambda_1 \mathbf{I} & \mathrm{vec}\left(\widetilde{\mathbf{Q}}_1\right) \widehat{\mathbf{B}}_1 & \mathbf{0} \\ \widehat{\mathbf{B}}_1 \mathrm{vec}\left(\widetilde{\mathbf{Q}}_1\right)^H & \Theta_t & \widehat{\mathbf{B}}_2 \mathrm{vec}\left(\widetilde{\mathbf{Q}}_2\right)^H \\ \mathbf{0} & \mathrm{vec}\left(\widetilde{\mathbf{Q}}_2\right) \widehat{\mathbf{B}}_2 & \widehat{\mathbf{B}}_2 + \lambda_2 \mathbf{I} \end{bmatrix} \succeq 0, \quad (16)$$

where $\Theta_t = \sum_{t=1}^{2} vec\left(\widetilde{\mathbf{F}}_{i,t}\right) \widehat{\mathbf{B}}_{t vec} \left(\widetilde{\mathbf{F}}_{i,t}\right)^H - \lambda_1 \omega_i^2 \alpha_i^2 - \lambda_2 \omega_i^2 \beta_i^2 - \widehat{P}_{R_i}$ with $\omega_i = \|\mathbf{W}_i\|$.

Proof. See Appendix 2. □

By putting all these components together, the objective problem T_4 becomes

$$\begin{aligned} T_5 : \quad & \min_{\mathbf{B}_t, \mathbf{W}_i, \gamma_t} \quad \gamma_1 + \gamma_2 \\ & s.t. \quad (11), (12), (14), (27), \|\mathbf{B}_t\|^2 \leq P_t, \\ & \qquad \tau_t \geq 0, \varsigma_t \geq 0, \lambda_t \geq 0, \nu_i \geq 0, \phi_j \geq 0, \\ & \qquad \forall t = 1, 2, \forall i = 1, ..., l, \forall j = 1, ..., 2l. \end{aligned}$$

It is clear that the problem T_5 is a biconvex SDP with linear objective function, which can be efficiently solved by an iterative algorithm. Furthermore, with fixed $\mathbf{B}_t/\mathbf{W}_i$, T_5 is convex with regard to $\mathbf{W}_i/\mathbf{B}_t$ which can be solved by CVX [18].

3.2 Proposed algorithm and computational complexity analysis

Now, we summarize the proposed beamforming method in Algorithm 1.

The proposed Algorithm 1 will converge to a suboptimal solution as $\sum_{t=1}^{2} \gamma_t^{(n)} - \sum_{t=1}^{2} \gamma_t^{(n-1)} \leq \xi$. Therefore, ξ is initialized to be a small value, and $N_{\max}$ is set to limit the number of iterations.

The process of Algorithm 1 with details are as follows: let $J\left(\{\mathbf{W}_i\}, \{\mathbf{B}_t\}\right)$ represent the objective function

Algorithm 1 The proposed SI-SRINR method

1. Initialize: $\xi = 10^{-3}$, N_{max}, $\mathbf{B}_t^{(0)}$, set $n = 0$;
2. Repeat:

for $n = 0$ to N_{max} **do**
for fixed $\mathbf{W}_i^{(n-1)}$, $\mathbf{B}_{\bar{t}}^{(n-1)}$ update $\mathbf{B}_{\bar{t}}^{(n)}$ via solving T_5;
for fixed $\mathbf{W}_i^{(n-1)}$, $\mathbf{B}_{\bar{t}}^{(n)}$ update $\mathbf{B}_t^{(n)}$ via solving T_5;
for given $\mathbf{B}_t^{(n)}$ and $\mathbf{B}_{\bar{t}}^{(n)}$ update $\mathbf{W}_i^{(n)}$ and $\sum_{t=1}^{2} \gamma_t^{(n)}$ via solving T_5;
 if $\sum_{t=1}^{2} \gamma_t^{(n)} - \sum_{t=1}^{2} \gamma_t^{(n-1)} \le \xi$, **then**
 break;
 end if
end for

$\gamma_1 + \gamma_2$. At the $(n+1)$th iteration, the value of $\{\mathbf{W}_i\}$ which can be denoted by $\left\{\mathbf{W}_i^{(n+1)}\right\}$ is the solution to T_4 that maximizes the objective J under the constraints. Because T_4 is convex (with fixed $\mathbf{W}_i$), updating $\mathbf{B}_t$ will only increase or maintain the objective J. By this way, with computed $\left\{\mathbf{W}_i^{(n+1)}\right\}$, we obtain $\left\{\mathbf{B}_t^{(n+1)}\right\}$ which implies that $J\left(\left\{\mathbf{W}_i^{(n+1)}\right\}, \left\{\mathbf{B}_t^{(n+1)}\right\}\right) \ge J\left(\left\{\mathbf{W}_i^{(n)}\right\}, \left\{\mathbf{B}_t^{(n+1)}\right\}\right)$. From the previous inequalities, we observe that

$$J\left(\left\{\mathbf{W}_i^{(n+1)}\right\}, \left\{\mathbf{B}_t^{(n+1)}\right\}\right) \ge J\left(\left\{\mathbf{W}_i^{(n)}\right\}, \left\{\mathbf{B}_t^{(n)}\right\}\right),$$

i.e., the objective function increases monotonically with the number of iterations. This observation, coupled with the fact that $J\left(\{\mathbf{W}_i\}, \{\mathbf{B}_t\}\right)$ is upper-bounded, implies that the proposed algorithm converges to a limit as number $n \longrightarrow \infty$.

Discussion 1: For two-way relay networks, once the second transmission phase finishes, the signal transmitted by the transceiver nodes reappears as self-interference. Without eliminating the self-interference, with the condition of the imperfect CSI, the exactly optimal solution is difficult to obtain. In spite of this, the proposed suboptimal solution is very close to the exactly optimal solution when the CSI uncertainty is small enough and the number of the iterations $n \longrightarrow \infty$ in the proposed algorithm.

To better analyze the complexity of Algorithm 1, the standard real-valued SDP problem is given as $\min\ \mathbf{c}^t\mathbf{x},\ s.t.\ \mathbf{A}_0 + \sum_{i=1}^{n} x_i\mathbf{A}_i$, where $\mathbf{A}_i$ denotes the symmetric block-diagonal matrices with K diagonal blocks of size $a_k \times a_k$, for $k = 1, ..., K$. The number of elementary arithmetic operations for solving this problem is given by [19]

$$\mathcal{O}(1)\left(1 + \sum_{k=1}^{K} a_k\right)^{1/2} n\left(n^2 + n\sum_{k=1}^{K} a_k^2 + \sum_{k=1}^{K} a_k^3\right). \quad (17)$$

We measure the performance of the proposed Algorithm 1 for each iteration in terms of the computational complexity compared with non-SI-SRINR one by using the total number of floating point operations (FLOPs). A FLOP is defined as a real floating operation, i.e., a real addition, multiplication, division, and so on. The details of the computational complexity of the proposed robust beamforming method is summarized in Table 1. The unknown variables to be determined for $\mathbf{B}_t$ is of size $n = 2MM_b + 3L + 6$, and for $\mathbf{W}_i$ are of size $n = 2N^2 + 3L + 6$, where the first term corresponds to the real and image parts of $\mathbf{B}_t$ and $\mathbf{W}_i$ while the other terms represent the additional slack variables $(\gamma_t, \tau_t, \varsigma_t, \lambda_t, \nu_i, \phi_j)$. To compute the optimal $\sum_{t=1}^{2} \gamma_t$, for $t = 1, 2.$, the number of diagonal blocks K is equal to 3, which are related to the SRINR constraint, the individual relay power constraint, and the noise power constraint. By employing (17), and further denoting β_δ and α_δ as the block dimensions and the number of the variables for $\delta \in \left\{\widehat{P}_{R_i}, \varsigma_t, \gamma_t\right\}$, respectively, the total FLOPs can be obtained as

$$R_{\text{FLOPs}} = \sum_{\delta = \widehat{P}_{R_i}, \varsigma_t, \gamma_t} \mathcal{O}(1)\left(1 + \beta_\delta\right)^{1/2} \alpha_\delta \left(\alpha_\delta^2 + \alpha_\delta \beta_\delta^2 + \beta_\delta^3\right). \quad (18)$$

Similar to [9], by introducing the slack variables ϵ_1, ϵ_2, and $\widetilde{\omega}$, we can recast the non-SI-SRINR method Q_1 as

$$\begin{aligned} Q_2: \ \max_{\mathbf{B}_t, \mathbf{W}_i} \quad & \epsilon_1 + \epsilon_2 + \widetilde{\omega} \\ s.t. \quad & \epsilon_i \le a_i, \frac{\widetilde{\omega}}{a_1 a_2} \le 1, \epsilon_i > 0, \widetilde{\omega} > 0, \left\|\mathbf{x}_{R_i}\right\|^2 \le P_{R_i}, \\ & \|\mathbf{B}_t\|^2 \le P_t, \forall\, i \in \{1, 2\}. \end{aligned} \quad (19)$$

We introduce further auxiliary variables $\theta_1 \ge 0$ and $\theta_2 \ge 0$, and assume $\theta_1^2 \le \frac{\widetilde{\omega}}{a_1}$, $\theta_2^2 \le \frac{1}{a_2}$. By this way, the constraint $\frac{\widetilde{\omega}}{a_1 a_2} \le 1$ can be converted into $\theta_1^2\theta_2^2 \le 1$. By employing Schur-complement theorem [18], we have

$$\theta_1^2 \le \frac{\widetilde{\omega}}{a_1} \longrightarrow \begin{bmatrix} \widetilde{\omega} & \theta_1 \\ \theta_1 & \frac{1}{a_1} \end{bmatrix} \succeq 0, \quad (20)$$

$$\theta_2^2 \le \frac{1}{a_2} \longrightarrow \begin{bmatrix} 1 & \theta_2 \\ \theta_2 & \frac{1}{a_2} \end{bmatrix} \succeq 0. \quad (21)$$

Table 1 Computational complexity of the proposed SI-SRINR algorithm

Step	Operations	Block dimensions (a_k)	Number of variables (n)
1	$\widehat{P}_{R_i}$	$2MM_b + 1$	$2MM_b + 2N^2 + 6L + 12$
2	ς_t	$(l+1)MN + 1$	$2N^2 + 3L + 6$
3.1	τ_t	l	$(2N^2 + 3L + 6) \times (2MM_b + 3L + 6)$
3.2	γ_t	$2lMN + MM_b + 1$	$(2N^2 + 3L + 6) \times (2MM_b + 3L + 6)$

Finally, putting (20) and (21) together, the optimization problem Q_2 can be efficiently converted into the following biconvex problem

$$\begin{aligned} Q_3: \max_{\mathbf{B}_t, \mathbf{W}_i} \quad & \epsilon_1 + \epsilon_2 + \widetilde{\omega} \\ s.t. \quad & \begin{bmatrix} \widetilde{\omega} & \theta_1 \\ \theta_1 & \frac{1}{a_1} \end{bmatrix} \succeq 0, \begin{bmatrix} 1 & \theta_2 \\ \theta_2 & \frac{1}{a_2} \end{bmatrix} \succeq 0, L\left(\epsilon_i \leq a_i\right), \widetilde{\omega} > 0, \\ & L\left(\left\|\mathbf{x}_{R_i}\right\|^2 \leq P_{R_i}\right), \|\mathbf{B}_t\|^2 \leq P_t, \forall\, i \in \{1, 2\}, \end{aligned}$$

where $L(A)$ denotes the LMI version of A. Obviously, the constraints $L\left(\epsilon_i \leq a_i\right)$ and $L\left(\left\|\mathbf{x}_{R_i}\right\|^2 \leq P_{R_i}\right)$ have the same computational complexity to the proposed SI-SRINR method as shown in Table 1. In addition, since the constraints (20) and (21) in Q_3 not only request more FOLPs but also lead to lower convergence performance, our proposed SI-SRINR method outperforms non-SI-SRINR one.

4 Simulation results

In this section, we study the performance of the proposed SI-SRINR robust beamforming design for TWRN. The channel estimates $\widetilde{\mathbf{G}}_{t,i}$, $\widetilde{\mathbf{F}}_{i,t}$ are assumed to be reciprocal and identically distributed complex Gaussian random variables. The proposed scenario is considered with two source nodes and $L = 2$ relay nodes. The source and relay nodes are equipped with $M_b = M = N = 4$ antennas. We further assume that the noise variances $\sigma_{R_i}^2, \sigma_{S_t}^2$ for $i = 1, ..., L$ and $t = 1, 2$, are equally given as $\sigma^2 = 1$. All results are averaged over $N_{\max} = 1000$ channel realizations with $\xi = 10^{-4}$.

With suboptimal $\mathbf{B}_t$ and $\mathbf{W}_i$ which are obtained by using Algorithm 1, we compare the convergence performance of the average worst-case SR for SI-SRINR method with the non-SI-SRINR one with fixed $\alpha_i = \beta_i = 0.01$ and transmit SNR $= 30$ dB as shown in Fig. 1. For non-SI-SRINR method Q_1, the near optimal solution is obtained by using $(1 + a_1)(1 + a_2) \approx a_1 a_2$, where $a_t = \mathrm{SRINR}_t$. It is found that the SI-SRINR and the non-SI-SRINR methods can achieve same optimal worst-case SR solution with almost 400 and 700 iterations, respectively. This is reasonable because, for the non-SI-SRINR method, the SR is calculated by using multiplication of SRINR which increases the complexity as discussed in Section 3.2.

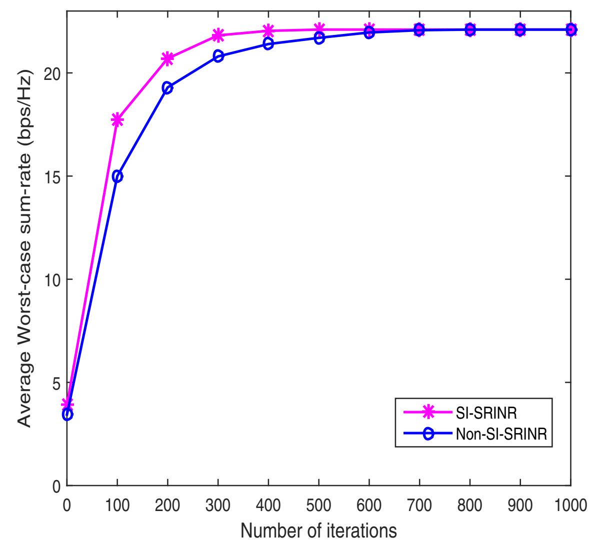

Fig. 1 The convergence performance of the output average worst-case SR versus number of iterations

In Fig. 2, we compare the proposed SI-SRINR method with the non-SI-SRINR one, non-robust one, and the perfect one with fixed CSI error as $\alpha_i = \beta_i = 0.03$ versus SNR. For the perfect CSI one, the channel coefficients are perfectly known at each node where the channel uncertainties $\Delta_{\mathbf{F}_{i,t}} = 0$, for $t = 1, 2$, which serves as the performance upper bound for our proposed robust beamforming design. For the robust one, the nominal values of the channels $\widetilde{\mathbf{F}}_{i,t}$ can be estimated and the channel uncertainties $\Delta_{\mathbf{F}_{i,t}}$ is NBEs as α_i, for $t = 1$, and β_i, for $t = 2$, respectively. For the non-robust one, the channel estimates are directly used as the actual channel responses without considering channel uncertainties. It is clear from Fig. 2 that, for different values of SNR, the solution of our proposed robust beamforming design shows better performance than the non-SI-SRINR one and the non-robust one.

In Fig. 3, we compare the average worst-case SR for SI-SRINR method with the non-SI-SRINR one for different CSI errors as 0.01, 0.05 versus relay power, where the

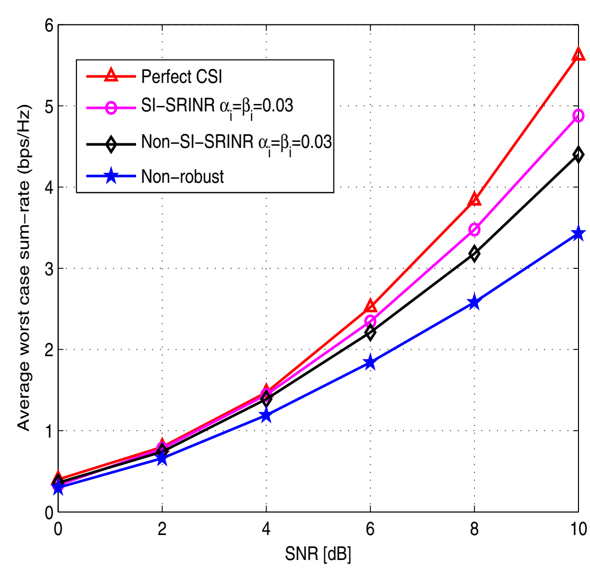

Fig. 2 The output average worst-case SR versus transmit SNR

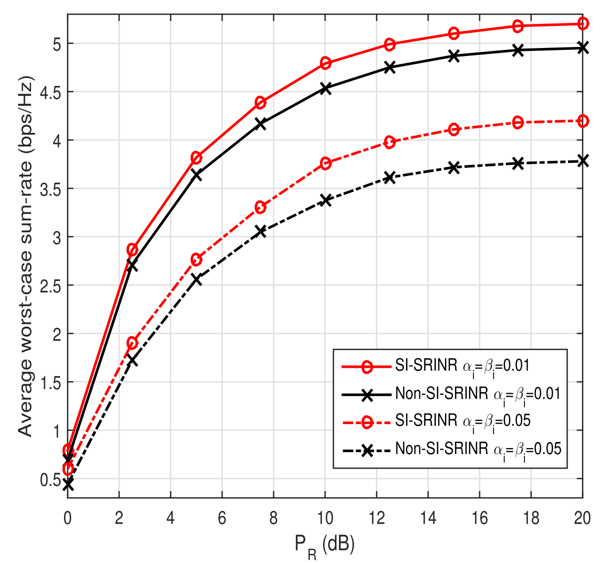

Fig. 3 The output average worst-case SR versus relay power

transmit SNR is given as SNR $= 10$ dB. It is clear from Fig. 1 that, the solution of our proposed SI-SRINR robust beamforming design shows better performance than the non-SI-SRINR one with increasing relay power. This is because the approximation $(1 + a_1)(1 + a_2) \approx a_1 a_2$ loses the performance gain at not extremely high SNR region.

Figure 4 depicts the performance of our proposed SI-SRINR method performance versus the number of the relays Z by comparing with the perfect case and the non-robust max-power beamforming solution [20] with fixed $\alpha_i = \beta_i = 0.01$. We consider a practical scenario with the relay power constraints as $P_R = 20$ dB. It is easy to see that the solution of our proposed SI-SRINR algorithm is close to the perfect one and outperforms the non-robust max-power beamforming one for different values of the transmit power P_1 and P_2.

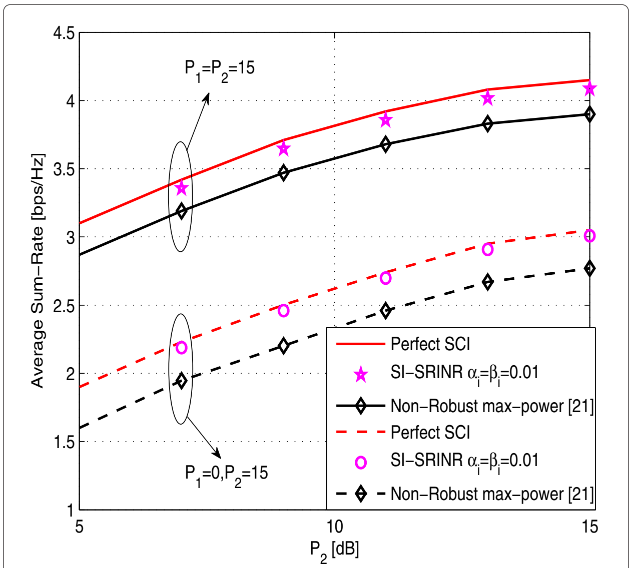

Fig. 4 The output average SR versus the number of the relay Z

5 Conclusions

In this paper, we considered MIMO TWRN with the robust relay beamforming design and proposed an efficient iterative algorithm to solve the SR maximization problem. The worst-case robust design problem was first converted into a SI-SRINR problem. After then, by utilizing the sign-definiteness lemma, the objective problems were represented as the tractable ones which are obtained through the SDP-based iterative optimization. Numerical results showed that the performance of the proposed SI-SRINR robust design is improved compared to the non-SI-SRINR one and non-robust one.

Appendix 1

Since the CSIs are imperfect, the upper bound of N_t cannot be straightforwardly obtained with existent channel uncertainties. Therefore, we employ the ([14] Lemma 1) to solve this problem. For the constraint

$$N_t = \sigma_{R_i}^2 \left\| \sum_{i=1}^{L} \mathbf{G}_{t,i} \mathbf{W}_i \right\|^2 + \sigma_{S_t}^2 \leq \varsigma_t, \tag{22}$$

assuming $\left(\varsigma_t - \sigma_{S_t}^2\right) / \sigma_{R_i}^2 = \varsigma_t^\star$, we have $\left\| \sum_{i=1}^{L} \mathbf{G}_{t,i} \mathbf{W}_i \right\|^2 \leq \varsigma_t^\star$. Using the identity $\|X\| = \|\mathrm{vec}[X]\|$ for any given matrix X, we have

$$\left\| \sum_{i=1}^{L} \mathbf{G}_{t,i} \mathbf{W}_i \right\|^2 = \left\| \sum_{i=1}^{L} \mathrm{vec}\left[\mathbf{G}_{t,i} \mathbf{W}_i\right] \right\|^2. \tag{23}$$

Using the identity $\mathrm{vec}[ABC] = \left(C^T \otimes A\right) \mathrm{vec}[B]$, where $(\cdot)^T$ and $\otimes$ denote the transpose and Kronecker product, we have

$$\sum_{i=1}^{L} \mathbf{G}_{t,i} \mathbf{W}_i = \sum_{i=1}^{L} \widetilde{\mathbf{G}}_{t,i} \mathbf{W}_i + \sum_{i=1}^{L} \left[\sum_{i=1}^{L} \mathbf{W}_i^T \otimes \mathbf{I}_M \right] \mathrm{vec}\left(\Delta_{\mathbf{G}_{t,i}}\right). \tag{24}$$

Further assuming $\sum_{i=1}^{L} \mathrm{vec}\left[\mathbf{G}_{t,i} \mathbf{W}_i\right] = \Upsilon$, the constraint $\left\| \sum_{i=1}^{L} \mathbf{G}_{t,i} \mathbf{W}_i \right\|^2 \leq \varsigma_t^\star$ can be represented in terms of the following LMI

$$\begin{bmatrix} \varsigma_t^\star & \Upsilon^H \\ \Upsilon & \mathbf{I}_{MN} \end{bmatrix} \succeq 0. \tag{25}$$

Let $\left\| \sum_{i=1}^{L} \widetilde{\mathbf{G}}_{t,i}\mathbf{W}_i \right\|^2 = \widetilde{\Upsilon}$, $\left[\sum_{i=1}^{L} \mathbf{W}_i^T \otimes \mathbf{I}_M\right] = \Psi_i$ and insert the structure of $\left\|\sum_{i=1}^{L} \mathbf{G}_{t,i}\mathbf{W}_i\right\|^2$ into (25), we have

$$\begin{bmatrix} \varsigma_t^\star & \widetilde{\Upsilon}^H \\ \widetilde{\Upsilon} & \mathbf{I}_{MN} \end{bmatrix} \succeq -\sum_{i=1}^{L} \begin{bmatrix} 0 & \left(\Psi_i \mathrm{vec}\left(\Delta_{\mathbf{G}_{t,i}}\right)\right)^H \\ \Psi_i \mathrm{vec}\left(\Delta_{\mathbf{G}_{t,i}}\right) & \mathbf{I}_{MN} \end{bmatrix}. \tag{26}$$

By employing S-Lemma, we can recast (26) as the following matrix inequality:

$$\begin{bmatrix} \varsigma_t^\star - \sum_{i=1}^{L} \nu_i & \widetilde{\Upsilon}^H & 0_{1\times MN} & \cdots & \cdots & \cdots & \cdots & 0_{1\times MN} \\ \widetilde{\Upsilon} & \mathbf{I}_{MN} & -\alpha_1\Psi_1^H & \cdots & -\alpha_l\Psi_l^H & -\beta_1\Psi_{l+1}^H & \cdots & -\beta_l\Psi_{2l}^H \\ 0_{MN\times 1} & -\alpha_1\Psi_1 & \nu_1\mathbf{I}_{MN} & \mathbf{0}_{MN} & \cdots & \cdots & \cdots & \mathbf{0}_{MN} \\ \vdots & \vdots & \mathbf{0}_{MN} & \ddots & \ddots & \ddots & \ddots & \vdots \\ \vdots & -\alpha_l\Psi_l & \vdots & \ddots & \nu_l\mathbf{I}_{MN} & \ddots & \ddots & \vdots \\ \vdots & -\beta_1\Psi_{l+1} & \vdots & \ddots & \ddots & \nu_{l+1}\mathbf{I}_{MN} & \ddots & \vdots \\ \vdots & \vdots & \vdots & \ddots & \ddots & \ddots & \ddots & \vdots \\ 0_{MN\times 1} & -\beta_l\Psi_{2l} & \mathbf{0}_{MN} & \cdots & \cdots & \cdots & \cdots & \nu_{2l}\mathbf{I}_{MN} \end{bmatrix} \succeq 0,$$

where $\alpha_i = \left\|\Delta_{\mathbf{F}_{i,1}}\right\| = \left\|\Delta_{\mathbf{G}_{1,i}}\right\|$, $\beta_i = \left\|\Delta_{\mathbf{F}_{i,2}}\right\| = \left\|\Delta_{\mathbf{G}_{2,i}}\right\|$ are the norm-bounded errors (NBEs) of channel uncertainties, and $\mathbf{0}_{MN}$ denotes $MN \times MN$ zero matrix. This completes the proof.

Appendix 2

(Lemma 1 [14]): Define the functions

$$f_j(\mathrm{x}) = \mathrm{x}^H \mathbf{A}_j \mathrm{x} + 2\mathrm{Re}\left\{\mathrm{b}_j^H \mathrm{x}\right\} + c_j, \;\; j = 1, 2 \tag{27}$$

where $\mathbf{A}_j$ is a square semi-definite matrix and c_j is a real constant. The implication of $f_j(\mathrm{x}) \leq 0$ holds true if and only if there exists $\lambda \geq 0$ such that

$$\lambda \begin{bmatrix} \mathbf{A}_1 & \mathrm{b}_1 \\ \mathrm{b}_1^H & c_1 \end{bmatrix} - \begin{bmatrix} \mathbf{A}_2 & \mathrm{b}_2 \\ \mathrm{b}_2^H & c_2 \end{bmatrix} \succeq 0.$$

By employing ([14] Lemma 1) and treating the terms involving $t = 2$ as constants, (15) can be rewritten as

$$\begin{bmatrix} \widehat{\mathbf{B}}_1 + \lambda_1 \mathbf{I} & \mathrm{vec}\left(\mathbf{Q}_1\right)\widehat{\mathbf{B}}_1 \\ \widehat{\mathbf{B}}_1 \mathrm{vec}\left(\mathbf{Q}_1\right)^H & \phi \end{bmatrix} \succeq 0,$$

where $\phi = \mathrm{vec}\left(\Delta_{\mathbf{Q}_1}\right)\widehat{\mathbf{B}}_1 \mathrm{vec}\left(\Delta_{\mathbf{Q}_1}\right)^H + \mathrm{vec}\left(\mathbf{Q}_2\right)\widehat{\mathbf{B}}_2 \mathrm{vec}\left(\mathbf{Q}_2\right)^H - \lambda_1\omega_i^2\alpha_i^2 - \widehat{P}_t$, with $\omega_i = \left\|\mathbf{W}_i^\circ\right\|$, with $\mathbf{W}_i^\circ$ denoting optimal solution of $\mathbf{W}_i$.

([21] Theorem 4.2): If $\mathbf{D} \succeq 0$, $i = 1, 2$, then the following QMI system

$$\begin{bmatrix} \mathbf{H}_1 & \mathbf{H}_2 + \mathbf{H}_3\mathbf{X} \\ \left(\mathbf{H}_2 + \mathbf{H}_3\mathbf{X}\right)^H & \mathbf{H}_4 + \mathbf{H}_5\mathbf{X} + \left(\mathbf{H}_5\mathbf{X}\right)^H + \mathbf{X}^H\mathbf{H}_6\mathbf{X} \end{bmatrix} \succeq 0, \;\; \forall \mathbf{X} : \mathrm{tr}(\mathbf{D}_i\mathbf{X}\mathbf{X}^H \leq 1). i = 1, 2,$$

is equivalent to the LMI system: $\exists\lambda \geq 0$, (28) is satisfied which is shown as

$$\begin{bmatrix} \mathbf{H}_1 & \mathbf{H}_2 & \mathbf{H}_3 \\ \mathbf{H}_2^H & \mathbf{H}_4 & \mathbf{H}_5 \\ \mathbf{H}_3^H & \mathbf{H}_5^H & \mathbf{H}_6 \end{bmatrix} - \sum_{i=1} \lambda_i \begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{I} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & -\mathbf{D}_i \end{bmatrix} \succeq 0. \tag{28}$$

Based on ([21] Theorem 4.2) and define $\mathbf{H}_1 = \widehat{\mathbf{B}}_1 + \lambda_1\mathbf{I}$, $\mathbf{H}_2 = \mathrm{vec}\left(\widetilde{\mathbf{F}}_{i,1}\right)\widehat{\mathbf{B}}_1$, $\mathbf{H}_3 = \mathbf{0}$, $\mathbf{H}_4 = \Theta_t$, $\mathbf{H}_5 = \widehat{\mathbf{B}}_2\mathrm{vec}\left(\widetilde{\mathbf{Q}}_2\right)^H$, $\mathbf{H}_6 = \widehat{\mathbf{B}}_2$, and $\mathbf{D} = \lambda_2\mathbf{I}$, the individual relay power constraints can be converted to the following LMI:

$$\begin{bmatrix} \widehat{\mathbf{B}}_1 + \lambda_1\mathbf{I} & \mathrm{vec}\left(\widetilde{\mathbf{Q}}_1\right)\widehat{\mathbf{B}}_1 & \mathbf{0} \\ \widehat{\mathbf{B}}_1\mathrm{vec}\left(\widetilde{\mathbf{Q}}_1\right)^H & \Theta_t & \widehat{\mathbf{B}}_2\mathrm{vec}\left(\widetilde{\mathbf{Q}}_2\right)^H \\ \mathbf{0} & \mathrm{vec}\left(\widetilde{\mathbf{Q}}_2\right)\widehat{\mathbf{B}}_2 & \widehat{\mathbf{B}}_2 + \lambda_2\mathbf{I} \end{bmatrix} \succeq 0, \tag{29}$$

where $\Theta_t = \sum_{t=1}^{2} \mathrm{vec}(\widetilde{\mathbf{F}}_{i,t})\widehat{\mathbf{B}}_t\mathrm{vec}\left(\widetilde{\mathbf{F}}_{i,t}\right)^H - \lambda_1\omega_i^2\alpha_i^2 - \lambda_2\omega_i^2\beta_i^2 - \widehat{P}_{R_i}$. This completes the LMI version of the individual relay power constraint.

Competing interests

The authors declare that they have no competing interests.

Acknowledgements

This work was supported by MEST 2015R1A2A1A05000977, NRF, South Korea, National Natural Science Foundation of China by Grants 61501190 and the Natural Science Foundation of Guangdong Province by Grant 2014A030310389, Shanghai Rising-Star Program (15QA1400100), Innovation Program of Shanghai Municipal Education Commission (15ZZ03), DHU Distinguished Young Professor Program (16D210402), and Key Lab of Information Processing & Transmission of Guangzhou 201605030014.

Author details

[1]Division of Electronic and Information Engineering, Chonbuk National University, Jeonju, South Korea. [2]School of Electronic and Information Engineering, South China University of Technology, Guangzhou, China. [3]School of Information Science and Technology, Donghua University, 436 Shanghai, China. [4]School of Mechanical and Electrical Engineering, Guangzhou University, Guangzhou, China.

Received: 14 January 2016 Accepted: 10 June 2016
Published online: 02 July 2016

References

1. Y Rong, Joint source and relay optimization for two-way linear nonregenerative MIMO relay communications. IEEE Trans. Signal Process. **60**(12), 6533–6546 (2012)
2. R Wang, M Tao, Y Liu, Optimal linear transceiver designs for cognitive two-way relay networks. IEEE Trans. Signal Process. **61**(4), 992–1005 (2013)
3. Q Shi, M Razaviyayn, Z Luo, C He, An iteratively weighted MMSE approach to distributed sum-utility maximization for a MIMO interfering broadcast channel. IEEE Trans. Signal Process. **59**(9), 4331–4340 (2011)
4. L Zhang, Y Cai, R Lamare, M Zhao, Robust multibranch Tomlinson-Harashima precoding design in amplify-and-forward MIMO relay systems. IEEE Trans. Commun. **62**(10), 3476–3490 (2014)
5. M Tao, R Wang, Robust relay beamforming for two-way relay networks. IEEE Commun. Lett. **16**(7), 1052–1055 (2012)
6. Q Li, Q Zhang, J Qin, Robust beamforming for cognitive multi-antenna relay networks with bounded channel uncertainties. IEEE Trans. Commun. **62**(2), 478–487 (2014)
7. H. Tang, W. Chen, J Li, Robust joint source-relay-destination design under per-antenna power constraints. IEEE Trans. Signal Process. **63**(10), 2639–2649 (2015)
8. Z Wang, W Chen, Relay beamforming design with SIC detection for MIMO multirelay networks with imperfect CSI. IEEE Trans. Veh. Technol. **62**(8), 3774–3785 (2013)

9. P Ubaidulla, S Aissa, Robust two-way cognitive relaying: precoder designs under interference constraints and imperfect CSI. IEEE Trans. Wireless Commun. **13**(5), 2478–2489 (2014)
10. J Wang, Y Wang, X Gui, P Zhang, Robust relay precoding design for two-way relay systems with delayed and limited feedback. IEEE Commun. Lett. **17**(4), 689–692 (2013)
11. S Ma, Z Dong, J Ma, D Sun, Robust relay beamforming in cognitive two-way relay networks. IEEE Commun. Lett. **17**(10), 2000–2003 (2013)
12. EA Jorswieck, EG Larsson, Complete characterization of Pareto boundary for the MISO interference channel. IEEE Trans. Signal Process. **56**(10), 5292–5296 (2008)
13. J Liu, F Gao, Z Qiu, Robust transceiver design for downlink multiuser MIMO AF relay systems. IEEE Trans. Wireless Commun. **14**(4), 2218–2231 (2015)
14. Y Huang, D Palomar, S Zhang, Lorentz-positive maps and quadratic matrix inequalities with applications to robust MISO transmit beamforming. IEEE Trans. Signal Process. **61**(5), 1121–1130 (2013)
15. A Gupta, A Singer, Successive interference cancellation using constellation structure. IEEE Trans. Signal Process. **55**(12), 5716–5730 (2007)
16. M Kobayashi, G Caire, Joint beamforming and scheduling for a multi-antenna downlink with imperfect transmitter channel knowledge. IEEE J. Sel. Areas Commun. **25**(7), 1468–1477 (2007)
17. K Xu, D Zhang, Y Xu, W Ma, The equivalence of two optimal power-allocation schemes for A-TWRC. IEEE Trans. Veh. Technol. **63**(4), 2478–2489 (2014)
18. S Boyd, L Vandenberghe, *Convex Optimization*. (Cambridge University Press, Cambridge, UK, 2004)
19. A Ben-Tal, A Nemirovski, *Lectures on Modern Convex Optimization: Analysis, Algorithms, Engineering Applications*. (SIAM, Philadelphia, PA, USA, 2001)
20. Y Yang, T Chang, M Lin, J Ge, Y Ji, Efficient solutions to distributed beamforming for two-way relay networks under individual relay power constraints. IEEE Trans. Signal Process. **64**(4), 1643–1649 (2012)
21. Q Zhang, X Huang, Q Li, J Qin, Cooperative jamming aided robust secure transmission for wireless information and power transfer in MISO channels. IEEE Trans. Commun. **63**(3), 906–915 (2015)