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# Power minimization for OFDM modulated two-way amplify-and-forward relay wireless sensor networks

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**Abstract**

This paper studies a power minimization problem of an orthogonal frequency-division multiplexing modulated two-way amplify-and-forward relay wireless sensor network (WSN). The objective is to develop effective strategies for controlling the transmit power levels of WSN nodes and assigning the bit rates among different subcarriers to minimize the overall transmit power of the network, while ensuring the end-to-end target rates of the two WSN sources. By using channel knowledge at transmitters, a two-step method is proposed to solve the power minimization problem, resulting in a joint power control and bit rate assignment (JPCBRA) algorithm. At the first step, a per-subcarrier optimization problem concerning the power minimization issue is solved, giving closed-form solutions for individual transmit powers at the WSN sources and the relay. At the second step, the end-to-end target rates of the two WSN sources are allocated to different subcarriers so that the total transmit power can be reduced further while satisfying the initial data-rate requirement of the two WSN sources. Computer simulation results confirmed the effectiveness of the proposed JPCBRA algorithm. It is shown that with the JPCBRA algorithm, the total transmit power of the network can be significantly reduced in any traffic and/or channel cases in contrast to the compared schemes.

**Keywords:** Two-way relay, Amplify-and-forward, Orthogonal frequency-division multiplexing, Wireless sensor network, Power minimization, Bit-rate assignment

## 1 Introduction

Wireless sensor networks (WSNs), sometimes referred to as wireless sensor and actor networks [1], are networks with densely deployed sensor nodes which are capable of monitoring physical or environmental conditions such as sound, pressure, humidity, temperature, wind direction, and speed and to cooperatively deliver their data through the network to a sink node. Normally, WSN nodes are geographically distributed in different places and are mostly battery powered. Thus, energy becomes the scarcest resource of WSN nodes, which determines the lifetime of the network. As such, the power minimization problem (i.e., saving as much energy as possible) has become a hot research topic for WSNs. On the other hand, cooperative relay scheme [2] as a novel technique for wireless communications promises enhancements in throughput and energy efficiency and thus has attracted much attention from the research community in recent years.

Since cooperative relay scheme could make the data delivery of WSNs more reliable and thus ensure the strict requirements of the network, some pioneer works [3–8] have been trying to apply the cooperative relay scheme to WSNs. In general, there are two transmission fashions for a two-hop cooperative relay scheme, namely, one- and two-way relaying. It should be noted that one-way relaying is generally more suitable for the case where data is delivered unidirectionally, namely, from a source to a destination only. In case that data from two nodes is forwarded in opposite directions, two-way relaying (TWR) could make the bi-directional transmission more efficient than one-way relaying [9]. Generally speaking, a TWR network, where two sources deliver data to each other through a relay node as shown in Fig. 1, can apply an amplify-and-forward (AF) or a decode-and-forward or

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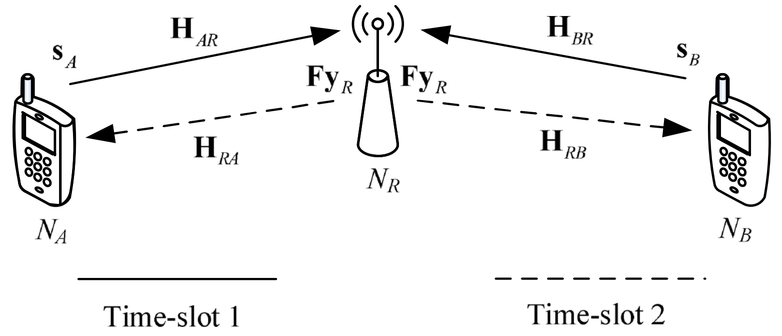

**Fig. 1** A TWR network, where two sources $N_A$ and $N_B$ deliver data to each via a relay node $N_R$

a denoise-and-forward [10], or a compress-and-forward [11] protocol while employing either the time division broadcast or the multiple-access broadcast (MABC) policy [12]. Due to the better flexibility of the AF protocol and the larger spectral efficiency of the MABC policy, more research effort has been focused on the AF protocol-based MABC policy in the research community [12].

## 2 Related work

In recent years, there are some publications focusing on the energy efficient design of TWR networks. By using the channel statistics, the authors of [12] proposed a transmit power allocation algorithm to minimize the overall transmit power of the TWR network with asymmetric traffic requirements. Considering the transmit and circuit power consumptions together, a hybrid one- and two-way relaying policy was developed in [13] to enhance the system energy efficiency. By using the instantaneous channel knowledge at transmitters, joint optimum power allocation and beamforming algorithms have been proposed for the two-phase TWR network in [14] and the three-phase TWR network in [15] to minimize the system transmit power, while guaranteeing that the received signal-to-noise ratios (SNRs) at the two sources be larger than the pre-defined thresholds. By exploiting the statistical channel knowledge at transmitters, the authors of [16] addressed an optimization problem minimizing the total transmit power of the TWR network with different outage probability constraints on the two sources. By exploiting the statistical channel knowledge at transmitters, a joint power allocation and relay location algorithm was proposed in [17] to reduce the power consumption of an asymmetric two-way AF relay network while ensuring the required network quality of service. In [18], the hybrid one- and two-way relaying policy proposed in [13] was extended to an orthogonal frequency-division multiplexing (OFDM) modulated TWR network. Moreover, the study of energy efficiency and spectrum efficiency trade-off for TWR networks can be found in [19] and references therein. In [20], a power allocation technique was proposed for maximizing the bits per unit of Joule of an OFDM modulated two-way AF relay network. The authors of [21] consider a two-way relay network with multiple pairs of source nodes and proposed a joint power and subchannel allocation and active subchannel selection technique maximizing the summation of the information bits per Joule of energy consumed at each terminal pair. The authors of [22] consider an AF two-way relay scenario with multiple relay nodes and CSI errors and proposed a joint implementation algorithm of subcarrier pairing and allocation, relay selection, and transmit power allocation. Moreover, [6–8] have expanded the idea of TWR into WSN settings. In [6], a novel policy incorporating single-threshold noncoherent energy detection and digital network coding at the relay side was developed for the WSN, where two sources would like to exchange low-rate data via a shared relay with complexity or latency limit. In order to maximize the secrecy sum rate, the authors of [7] proposed joint subcarrier assignment, subcarrier pairing, and power allocation algorithms for the orthogonal frequency division multiple access two-way relay WSN with and without cooperative jamming. The authors of [8] proposed a communication scheme employing TWR for three sensor nodes to support bi-directional communications of two applications. In [23], an intelligent power allocation algorithm for distributed beamforming is proposed to guarantee the required channel capacity and improve the network lifetime as well. The authors of [24] studied the outage performance of a two-way OFDM-based nonlinear AF relay network. In [25], an optimal power allocation algorithm is proposed to maximize the minimum signal-to-noise plus distortion ratio for a two-way AF OFDM-based relay network. It should be noted that most of the existing studies as mentioned earlier either focus on the single-carrier systems or the power minimization issue is not considered. Thus, it is natural to question how to extend the proposed energy efficient algorithms to multi-carrier settings? Do they need any modifications or should they be redesigned completely? So far, this issue has not been well studied in the literature.

In this paper, a two-phase OFDM modulated two-way relay WSN[1] is considered, where two WSN sources exchange messages via a half-duplex AF relay. Our objective is to develop effective strategies for controlling the transmit power levels of WSN nodes and assigning the bit rates among different subcarriers to minimize the overall transmit power of the network, while ensuring the end-to-end target rates of the two sources. First, a total power minimization problem is formulated by considering both power control of nodes and bit rate assignment. A two-step method is then proposed to solve this optimization problem. At the first step, a per-subcarrier optimization problem concerning the power minimization issue is solved, giving closed-form solutions for individual transmit powers at the sources and the relay. At the second step, the end-to-end target rates of the two sources are allocated to different subcarriers so that the total transmit power can be reduced further while satisfying the initial data-rate requirement of the two sources. Computer simulations are conducted, and the results validate the proposed algorithm, showing that the proposed algorithm can significantly reduce the total transmit power in any traffic and/or channel cases.

## 3 Network model

We consider a two-way relay WSN, where each WSN node is equipped with a single antenna and operates in an AF protocol-based half-duplex fashion. As depicted in Fig. 1, two sources $N_A$ and $N_B$ would like to deliver messages to each other via a relay node $N_R$. We denote by $\mathbf{s}_i \triangleq [s_i(1), s_i(2), \cdots, s_i(K)]^T$, $i \in \{A, B\}$, the OFDM symbol transmitted by node $N_i$, where $K$ is the number of subcarriers. Assume that $s_i(k)$, $k \in \{1, \cdots, K\}$ has a unit energy and no direct link exists between the two sources. We denote by $\mathbf{E}_i \triangleq \mathrm{diag}\{e_i(1), \cdots, e_i(K)\}$, $i \in \{A, B, R\}$, the transmit power at node $N_i$, where $e_i(k)$ represents the transmit power of the node for the $k$- th subcarrier. The channel gain from node $N_i$ to node $N_j$ is denoted by $\mathbf{H}_{ij} \triangleq \mathrm{diag}\{h_{ij}(1), \cdots, h_{ij}(K)\}$, $i, j \in \{A, B, R\}$ and $i \neq j$, where $h_{ij}(k)$ is the corresponding path gain for the $k-$th subcarrier. In the paper, $h_{ij}(k)$ is modeled as $h_{ij}(k) \triangleq g_{ij} \Big/ \sqrt{d_{ij}^{\alpha}}$, where $d_{ij}$ and $g_{ij}$ represent the distance and the large-scale behavior of the path gain between nodes $N_i$ and $N_j$, respectively, and $\alpha$ is the corresponding path-loss coefficient [12, 17]. Without loss of generality, we assume that $g_{ij}$ is a complex Gaussian random variable and obeys $\mathcal{CN}(0, 1)$, and the channels are reciprocal, i.e., $\mathbf{H}_{ij} = \mathbf{H}_{ji}$ and quasi-static. We denote by $\mathbf{w}_i$, $i \in \{A, B, R\}$, the additive white Gaussian noise (AWGN) at node $N_i$. It is assumed that all the elements of AWGNs obey i.i.d. $\mathcal{CN}(0, 1)$.

As shown in Fig. 1, two sources $N_A$ and $N_B$, respectively, send $\mathbf{s}_A$ and $\mathbf{s}_B$ to the relay $N_R$ at time slot 1. Then, the relay receives the following signal

$$\mathbf{y}_R = \mathbf{H}_{AR}\sqrt{\mathbf{E}_A}\mathbf{s}_A + \mathbf{H}_{BR}\sqrt{\mathbf{E}_B}\mathbf{s}_B + \mathbf{w}_R. \tag{1}$$

At time slot 2, the relay $N_R$ first scales each subcarrier of the received signal $\mathbf{y}_R$ by a factor $\mathbf{F} \triangleq \mathrm{diag}\{f(1), \cdots, f(K)\}$, where

$$f(k) = \frac{1}{\sqrt{e_A(k)\left|h_{AR}(k)\right|^2 + e_B(k)\left|h_{BR}(k)\right|^2 + 1}}. \tag{2}$$

Next, the relay broadcasts the resulting signal to the two sources. At the end of time slot 2, the node $N_i$ receives

$$\mathbf{y}_i = \mathbf{F}\left(\underbrace{\mathbf{H}_{iR}\mathbf{H}_{iR}\sqrt{\mathbf{E}_i\mathbf{E}_R}\mathbf{s}_i}_{\text{self}-\text{interference}} + \mathbf{H}_{iR}\mathbf{H}_{jR}\sqrt{\mathbf{E}_j\mathbf{E}_R}\mathbf{s}_j + \mathbf{H}_{iR}\sqrt{\mathbf{E}_R}\mathbf{w}_R\right) + \mathbf{w}_i, \tag{3}$$

where $i, j \in \{A, B\}$, and $i \neq j$. Assuming perfect channel state information (CSI) at receivers, the source $N_i$ can implement self-interference removal and then obtain

$$\mathbf{y}_i' = \mathbf{F}\left(\mathbf{H}_{iR}\mathbf{H}_{jR}\sqrt{\mathbf{E}_j\mathbf{E}_R}\mathbf{s}_j + \mathbf{H}_{iR}\sqrt{\mathbf{E}_R}\mathbf{w}_R\right) + \mathbf{w}_i. \tag{4}$$

As such, at the end of time slot 2, the mutual information achieved by the source $N_i$ can be written as [26]

$$I_i = \sum_{k=1}^{K} \frac{1}{2}\log_2\left[1 + \mathrm{SNR}_i(k)\right], \tag{5}$$

where the factor 1/2 arises because of the fact that the signal is transmitted over two time slots, $I_i(k)$ and $\mathrm{SNR}_i(k)$ are the mutual information and the SNR for the $k$- th subcarrier and can be given as the following

$$I_i(k) = \frac{1}{2}\log_2\left[1 + \mathrm{SNR}_i(k)\right], \tag{6}$$

$$\mathrm{SNR}_i(k) = \frac{e_j e_R \left|h_{AR}(k)\right|^2 \left|h_{BR}(k)\right|^2}{(e_i + e_R)\left|h_{iR}(k)\right|^2 + e_j\left|h_{jR}(k)\right|^2 + 1}. \tag{7}$$

## 4 Problem description and solution

The objective here is to develop a joint power control and bit rate assignment (JPCBRA) algorithm to minimize the total transmit power of an OFDM modulated TWR network. Thus, a general optimization problem minimizing the transmit power of the network can be written as

$$\left[e_A^*(k), e_B^*(k), e_R^*(k), r^*(k)\right] = \underset{e_A(k),e_B(k),e_R(k),r(k)}{\arg} \quad \min \sum_{k=1}^{K} \frac{e_A(k) + e_B(k) + e_R(k)}{2} \tag{8a}$$

$$\text{subject to} \quad \sum_{k=1}^{K} r(k) = r \tag{8b}$$

$$\min(I_A, I_B) \geq r \tag{8c}$$

where $r$ is the end-to-end target rate of the two sources, $r(k)$ is the assigned bit rate for the $k$-th subcarrier, and factor $1/2$ in (8a) indicates that each node in the network only uses one-half of the transmission time during one cycle message delivery.

It is observed that closed-form solutions for per-subcarrier optimization problems minimizing the network transmit power can be obtained. In addition, problem (8) can be decomposed into a two-step problem, (i) step 1: power control minimizing the total transmit power without bit rate assignment; (ii) step 2: bit rate assignment (BRA) for reducing the transmit power further. Thus, to solve the JPCAB problem given by (8), a per-subcarrier optimization problem minimizing the transmit power of the network without bit rate assignment is examined first. The per-subcarrier optimization problem is written as

$$\left[e_A^{\mathrm{P}}(k), e_B^{\mathrm{P}}(k), e_R^{\mathrm{P}}(k)\right] = \underset{e_A(k),e_B(k)e_R(k)}{\arg} \quad \min \frac{e_A(k) + e_B(k) + e_R(k)}{2} \tag{9a}$$

$$\text{subject to} \quad \min\left[I_A(k), I_B(k)\right] \geq r(k) \tag{9b}$$

where $r(k)$ is the assigned bit rate of the node $N_i$ for the $k$-th subcarrier.

The optimal solution to the per-subcarrier optimization problem (9) is presented in the following proposition.

**Proposition 1** *For an AF protocol-based TWR network, the optimal solution to the per-subcarrier optimization problem* (9) *can be given by*

$$e_A^{\mathrm{P}}(k) = \frac{z(k)\left[|h_{AR}(k)| + |h_{BR}(k)|\right]}{|h_{AR}(k)|^2 |h_{BR}(k)|}, \tag{10}$$

$$e_B^{\mathrm{P}}(k) = \frac{z(k)\left[|h_{AR}(k)| + |h_{BR}(k)|\right]}{|h_{AR}(k)| |h_{BR}(k)|^2}, \tag{11}$$

$$e_R^{\mathrm{P}}(k) = \frac{z(k)\left[|h_{AR}(k)| + |h_{BR}(k)|\right]^2 + |h_{AR}(k)| |h_{BR}(k)|}{|h_{AR}(k)|^2 |h_{BR}(k)|^2}, \tag{12}$$

*where* $z(k) = 2^{2r(k)} - 1$.

*Proof* See Appendix 1. ☐

Thus, the total transmit power of the network is given by

$$e_T^{\mathrm{P}}(k) = \frac{2z(k)\left[|h_{AR}(k)| + |h_{BR}(k)|\right]^2 + |h_{AR}(k)| |h_{BR}(k)|}{|h_{AR}(k)|^2 |h_{BR}(k)|^2}. \tag{13}$$

In the next step, we address the BRA problem allocating the end-to-end target rate of the two sources to different subcarriers so that the total transmit power can be reduced further while satisfying the end-to-end target rate of the two sources. The BRA problem is formulated as

$$r^*(k) = \underset{r(k)}{\arg\min} \sum_{k=1}^{K} e_T^{\mathrm{P}}(k) \tag{14a}$$

$$\text{subject to} \quad \sum_{k=1}^{K} r(k) = r \tag{14b}$$

$$z(k) = 2^{2r(k)} - 1 \tag{14c}$$

$$r(1), r(2), \cdots, r(K) \geq 0 \tag{14d}$$

where $e_T^{\mathrm{P}}(k)$ is given by (13).

Ignoring constraint (14d), (14) can be solved straightforwardly by dealing with a linear system and the solution is presented in the following proposition.

**Proposition 2** *For an AF protocol-based TWR network, the optimal solution to the BRA problem* (14) *without constraint* (14d) *can be classified as the following two cases.*

*(i) Case 1 :* $k = 1$ *or* $k = K$. *The solution can be given by*

$$r^*(1) = \frac{r}{K} + \sum_{n=1}^{K-1} \frac{(K-n)\,c(n)}{K} \text{ or } r^*(K) = \frac{r}{K} - \sum_{m=1}^{K-1} \frac{m c(m)}{K}. \tag{15}$$

*(ii) Case 2:* $k \neq 1, K$. *The solution is given by*

$$r^*(k) = \frac{r}{K} - \sum_{m=1}^{k-1} \frac{m c(m)}{K} + \sum_{n=k}^{K-1} \frac{(K-n)\,c(n)}{K}. \tag{16}$$

*Here,* $c(k)$ *is defined as*

$$c(k) \triangleq \log_2 \left\{ \frac{|h_{AR}(k)| |h_{BR}(k)| \left[|h_{AR}(k+1)| + |h_{BR}(k+1)|\right]}{|h_{AR}(k+1)| |h_{BR}(k+1)| \left[||h_{AR}(k)| + h_{BR}(k)|\right]} \right\}. \tag{17}$$

*Proof* See Appendix 2. ☐

*Discussion:* Since the allocated bit rate for each subcarrier must be larger than or equal to 0, namely, ensuring (14d), we need to check the value of $r^*(k)$ given in Proposition 2. In case that $r^*(k)$, $k \in \{1, 2, \cdots, K\}$, can satisfy (14d), they are considered as the final solution of problem (14). Otherwise, the subcarrier with the minimum bit rate will be discarded and the target rate will be reallocated again among the other subcarriers until all the assigned bit rates are larger than or equal to 0. Then, based on Proposition 2, a JPCBRA algorithm minimizing the total transmit power of the network is proposed as given in the following table.

**Algorithm 1** JPCBRA minimizing the network transmit power.

```
Step 1. Set $L = K$.
Step 2. Calculate the assigned bit rate according to
        Proposition 2.
Step 3. Check $r^*(l)$. If $r^*(l) \geq 0, l \in \{1, 2, \cdots, K\}$, go to
        step Otherwise, discard the subcarrier with the
        minimum assigned bit rate and set $L = L - 1$ and
        then go to step 2.
Step 4. Based on the assigned bit rate on each subcarrier,
        the individual transmit powers $e_A^{\mathrm{P}}(k)$, $e_B^{\mathrm{P}}(k)$ and
        $e_R^{\mathrm{P}}(k)$ are calculated according to (10), (11), and
        (12), respectively.
Step 5. End of the algorithm.
```

## 5 Simulation results

In this section, simulation results are provided to evaluate the performance of our proposed algorithm. Assume that the relay node is situated on the line between $N_A$ and $N_B$, and $d_{AB} = 1$. The parameter $\alpha$ is set to four to model radio propagation in urban areas [12]. It means that $0 < d_{AR} < 1$, $0 < d_{BR} < 1$, and $d_{AR} + d_{BR} = 1$. In our simulations, the transmit power levels are obtained by finding the mean of the node's transmit powers.

Figure 2 provides the detailed information with respect to the assigned bit rates on each subcarrier, where three different channel cases are involved. In Fig. 2, $d_{AR} = 0.5$ means the relay node is located at the middle of the two sources and is referred to as the symmetric channel case. $d_{AR} = 0.3$ and $d_{AR} = 0.1$ are the moderately and strongly asymmetric channel cases, respectively. Here, $K$ and $r$ are set to 16 and 10 bit/s/Hz, respectively. It can be observed that the bit rates are not equally assigned among all the subcarriers even in the symmetric channel case. Some of the subcarriers are assigned more bit rates than the others, and some of the subcarriers are not used at all, all of which depend upon the actual transmission conditions.

In Fig. 3, we plot the total transmit power as a function of $d_{AR}$, where $K$ and $r$ are set to 16 and 10 bit/s/Hz, respectively. Here, "equal power" means each node and each subcarrier have identical transmit powers, "without BRA" means bit rates are equally assigned to all the subcarriers, and "equal power of nodes" means each node has the same transmit power. It can be seen that the JPCBRA algorithm can significantly reduce the network transmit power in contrast to the compared schemes. When the relay is close to one of the sources, more transmit power can be saved with the proposed algorithm.

In Fig. 4, we plot the total transmit power as a function of $r$, where the three channel cases considered in Fig. 2 are involved, the parameter $K$ is maintained the same as before and $r$ is ranged from 1 to 10 bit/s/Hz. It can be observed that with the JPCBRA algorithm, the total transmit power can be significantly reduced and with increasing the level of asymmetry of the channel, the superiority of the proposed algorithm becomes more evident. In addition, the total transmit power almost increases linearly with the increasing of $r$. However, the total transmit power of the three compared schemes increase logarithmically when the sources' target rate increases.

## 6 Conclusions

In this paper, we have addressed an optimization problem concerning the total transmit power minimization of an OFDM modulated two-way AF relay WSN with end-to-end target rate constraint on the two sources. By exploiting the channel knowledge at the transmitters, a two-step method has been proposed to solve the power minimization problem, leading to a JPCBRA algorithm. Computer simulation experiments were conducted with comparison with the schemes of "equal power without BRA," "equal power of nodes without BRA," and "power control without BRA." Simulation results validated the correctness and the efficiency of our proposed algorithm. It was shown that with the proposed JPCBRA algorithm, the total transmit power of the network is significantly reduced. It was also shown that with increasing the level of asymmetry of the channel, the performance enhancement in terms of the reduced power turns into more evident.

## Endnotes

[1] Usually, WSNs only need low data rates. However, in some settings, e.g., wireless multimedia sensor networks, multimedia content such as video and audio streams and images should be perceived and then delivered. Thus, in order to enable the retrieval of multimedia streams, some advanced physical layer techniques, such as OFDM and ultra-wide band, are employed [27, 28].

## Appendix 1: proof of proposition 1

First, it can be observed that the global minimum of the per-subcarrier optimization problem (9) is achieved only when the inequality constraint (9b) holds with equality. In addition, it can be readily proved that for any set of $[e_A(k), e_B(k), e_R(k)]$ satisfying (9b) and $e_A(k) + e_B(k), +e_R(k) = e_A^{\mathrm{P}}(k) + e_B^{\mathrm{P}}(k) + e_R^{\mathrm{P}}(k)$, where

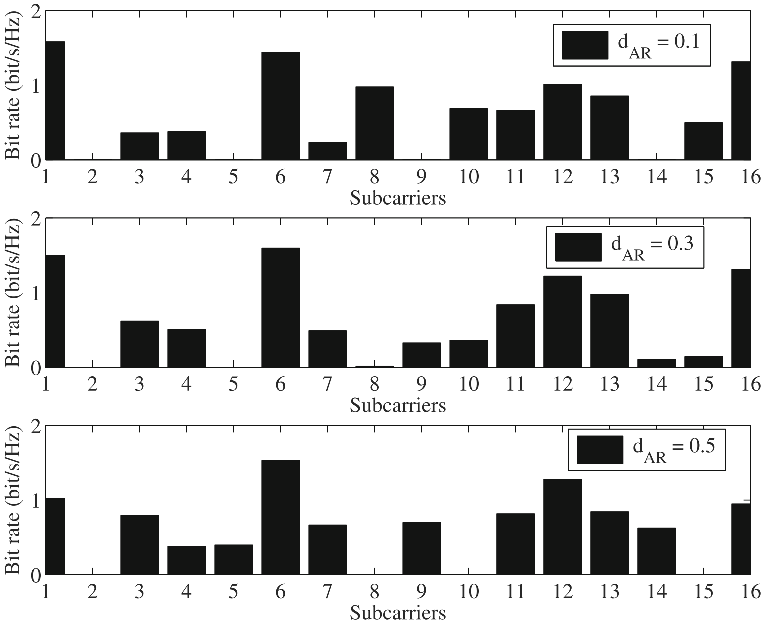

**Fig. 2** The detailed information with respect to the assigned bit rates on each subcarrier, where three different channel cases are involved

$\left[e_A^{\mathrm{P}}(k), e_B^{\mathrm{P}}(k), e_R^{\mathrm{P}}(k)\right]$ is the optimal solution of (9), $\min\left\{I_A\left[e_A^{\mathrm{P}}(k), e_B^{\mathrm{P}}(k), e_R^{\mathrm{P}}(k)\right], I_B\left[e_A^{\mathrm{P}}(k), e_B^{\mathrm{P}}(k), e_R^{\mathrm{P}}(k)\right]\right\} \geq \min\left\{I_A\left[e_A(k), e_B(k), e_R(k)\right], I_B\left[e_A(k), e_B(k), e_R(k)\right]\right\}$ holds. Based on the above results, the per-subcarrier optimization problem (9) can be solved by transforming it to a two-phase problem, phase 1: maximizing the mutual information of the sources subject to a total power constraint; phase 2: minimizing the total transmit power with the target-rate requirement based on the achieved results at phase 1. The optimization problem at phase 1 can be recognized as a special case of that in [29]. It is worth-mentioning that in [29], exact SNR expressions of the two sources are used to solve the optimization problem. For the sake of simplicity, here, the approximate SNR expression as given below is used instead.

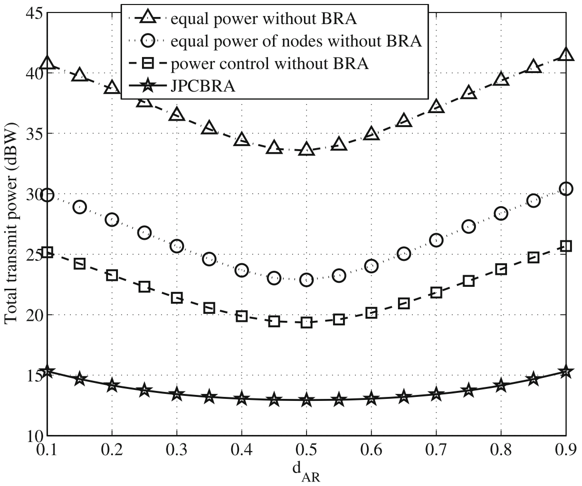

**Fig. 3** The total transmit power as a function of $d_{AR}$, where $K$ and $r$ are set to 16 and 10 bit/s/Hz, respectively

$$\mathrm{SNR}_i\left(k\right) \approx \frac{e_j e_R \left|h_{AR}\left(k\right)\right|^2 \left|h_{BR}\left(k\right)\right|^2}{\left(e_i + e_R\right)\left|h_{iR}\left(k\right)\right|^2 + e_j\left|h_{jR}\left(k\right)\right|^2}$$

Then using the method similar to that in [29], the optimization problem at phase 1 can be solved and the solution is given by

$$\begin{cases} e_A\left(k\right) = \frac{|h_{BR}(k)|e_T(k)}{2(|h_{AR}(k)|+|k_{BR}(k)|)} \\ e_B\left(k\right) = \frac{|h_{AR}(k)|e_T(k)}{2[|h_{AR}(k)|+|h_{BR}(k)|]} \\ e_R\left(k\right) = \frac{(|h_{AR}(k)|+|h_{BR}(k)|)e_T(k)}{2(|h_{AR}(k)|+|h_{BR}(k)|)} \end{cases}, \tag{18}$$

where $e_T(k)$ is the imposed total power constraint at phase 1.

Next, based on (18) and using simple algebraic manipulations, one can obtain the solution of the phase 2 problem being given by

$$\begin{cases} e_A^*\left(k\right) = \frac{(|h_{AR}(k)|+|h_{BR}(k)|)\left[2^{2r(k)}-1\right]}{|h_{AR}(k)|^2|h_{BR}(k)|} \\ e_B^*\left(k\right) = \frac{(|h_{AR}(k)|+|h_{BR}(k)|)\left[2^{2r(k)}-1\right]}{|h_{AR}(k)||h_{BR}(k)|^2} \\ e_R^*\left(k\right) = \frac{(|h_{AR}(k)|+|h_{BR}(k)|)^2\left[2^{2r(k)}-1\right]}{|h_{AR}(k)|^2|h_{BR}(k)|^2} \end{cases}. \tag{19}$$

Of note is that the approximate SNR expressions rather than the exact ones are used in the development of the solution to the optimization problem. Thus, the end-to-end target rates of the two sources cannot be stringently

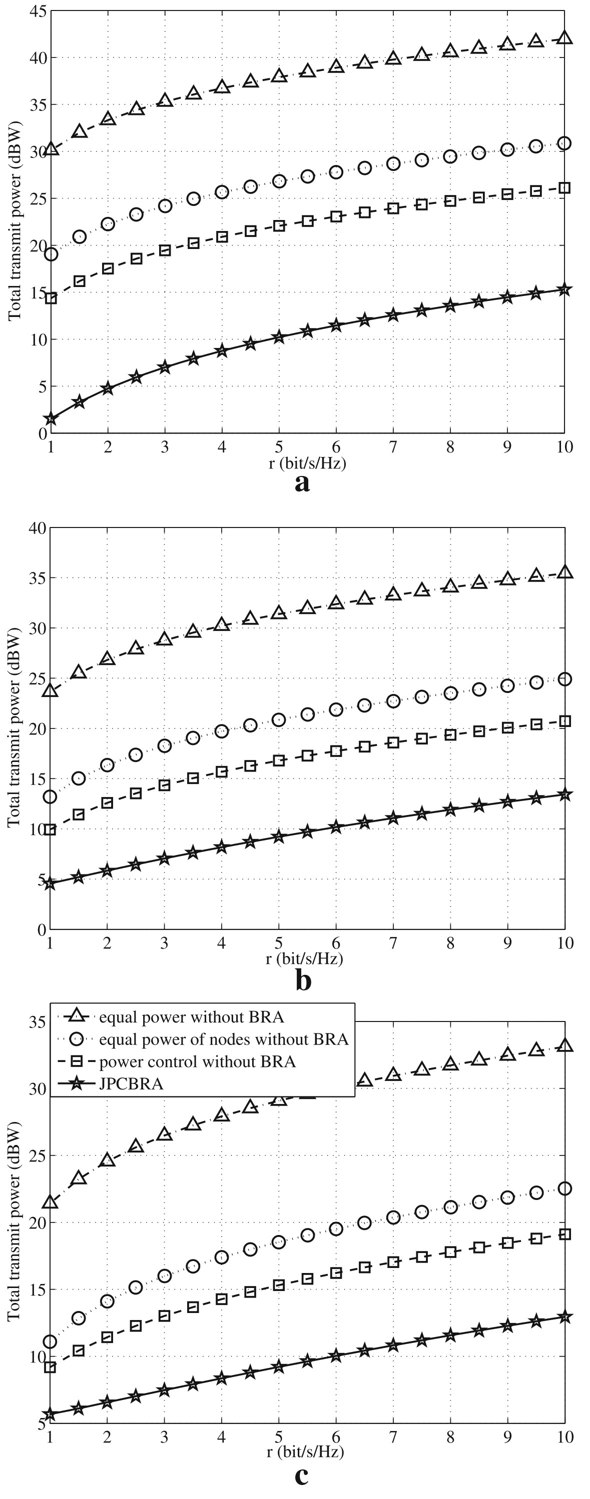

**Fig. 4** The total transmit power as a function of *r*, where the three channel cases considered in Fig. 2 are involved, the parameter *K* is maintained the same as before and *r* is ranged from 1 to 10 bit/s/Hz

satisfied. To that end, we make a modification to the solution achieved earlier. According to (7), one can find that the received SNRs of the two sources are both monotonically increasing functions of $e_R(k)$. Thus, we modify $e_R(k)$ in (18). The idea is to increase $e_R(k)$ such that the resulting SNRs of the two sources can satisfy the initial requirements. Then, by performing some simple manipulations, we can obtain the final solution of (9) as presented in Proposition 1.

## Appendix 2: proof of proposition 2

The Lagrangian function of (14) without constraint (14d) can be given by

$$L = \sum_{k=1}^{K} \frac{2\left[2^{2r(k)} - 1\right]\left[|h_{AR}(k)| + |h_{BR}(k)|\right]^2 + |h_{AR}(k)|\,|h_{BR}(k)|}{|h_{AR}(k)|^2\,|h_{BR}(k)|^2} + \nu\left[\sum_{k=1}^{K} r(k) - r\right], \tag{20}$$

where $\nu$ is a Lagrangian multiplier. Thus, the Karush-Khun-Tucker conditions can be formulated as

$$\left\{\begin{array}{l} \frac{(\ln 2)2^{2[r(k)+1]}[|h_{AR}(k)|+|h_{BR}(k)|]^2}{|h_{AR}(k)|^2|h_{BR}(k)|^2} + \nu = 0 \\ \sum\limits_{k=1}^{K} r(k) - r = 0 \end{array}\right. . \tag{21}$$

Based on (21), it is ready to deduce that the optimal solution should satisfy the following equation:

$$\left\{\begin{array}{l} \frac{(\ln 2)\,2^{2[r(k+1)+1]}\left[|h_{AR}(k+1)| + |h_{BR}(k+1)|\right]^2}{|h_{AR}(k+1)|^2\,|h_{BR}(k+1)|^2} \\ \quad = \frac{(\ln 2)\,2^{2[r(k)+1]}\left[|h_{AR}(k)| + |h_{BR}(k)|\right]^2}{|h_{AR}(k)|^2\,|h_{BR}(k)|^2} \\ \sum\limits_{k=1}^{K} r(k) - r = 0 \end{array}\right. \tag{22}$$

Further, (22) can be rewritten as

$$\begin{pmatrix} 1 & -1 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 1 & -1 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 1 & -1 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ldots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & -1 & 0 \\ 0 & 0 & 0 & 0 & \cdots & 1 & -1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix} \begin{bmatrix} r(1) \\ r(2) \\ r(3) \\ \vdots \\ r(K) \end{bmatrix} = \begin{bmatrix} c(1) \\ c(2) \\ \vdots \\ c(K-1) \\ r \end{bmatrix}, \tag{23}$$

where $c(k)$, $k \in \{1, 2, \cdots, K-1\}$ is defined in (17). It is clear that linear system (23) can be solved by calculating the inverse matrix of $\mathbf{X}$. The inverse matrix of $\mathbf{X}$ can be calculated as

$$\mathbf{X}^{-1}=\begin{pmatrix} \frac{K-1}{K} & \frac{K-2}{K} & \frac{K-3}{K} & \frac{K-4}{K} & \cdots & \frac{K-(K-1)}{K} & \frac{1}{K} \\ -\frac{1}{K} & \frac{K-2}{K} & \frac{K-3}{K} & \frac{K-4}{K} & \cdots & \frac{K-(K-1)}{K} & \frac{1}{K} \\ -\frac{1}{K} & -\frac{2}{K} & \frac{K-3}{K} & \frac{K-4}{K} & \cdots & \frac{K-(K-1)}{K} & \frac{1}{K} \\ -\frac{1}{K} & -\frac{2}{K} & -\frac{3}{K} & \frac{K-4}{K} & \cdots & \frac{K-(K-1)}{K} & \frac{1}{K} \\ \vdots & \vdots & \vdots & \vdots & \ddots & & \vdots \\ -\frac{1}{K} & -\frac{2}{K} & -\frac{3}{K} & -\frac{4}{K} & \cdots & \frac{K-(K-1)}{K} & \frac{1}{K} \\ -\frac{1}{K} & -\frac{2}{K} & -\frac{3}{K} & -\frac{4}{K} & \cdots & -\frac{(K-1)}{K} & \frac{1}{K} \end{pmatrix}. \tag{24}$$

Then, according to $\mathbf{y} = \mathbf{X}^{-1}\mathbf{c}$, one can straightforwardly obtain the final solution as presented in Proposition 2.

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**Competing interests**

The authors declare that they have no competing interests.

## Publisher's Note

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**References**

1. F Akyildiz, IH Kasimoglu, Wireless sensor and actor networks: research challenges. Ad Hoc Net. **2**(4), 351–367 (2004)
2. JN Laneman, DNC Tse, GW Wornell, Cooperative diversity in wireless networks: efficient protocols and outage behavior. IEEE Trans. Inform. Theory. **50**(12), 3062–3080 (2004)
3. F Engel, T Abrao, L Hanzo, in *Proc. WCNC2013 Con*. Relay selection methods for maximizing the lifetime of wireless sensor networks, (Shanghai, 2013), pp. 2339–2344
4. W Fang, Q Zhou, Z Wang, Q Liu, An adaptive transmission scheme for wireless sensor networks. Int. J. Future Generation Commun. Netw. **6**(1), 35–44 (2013)
5. R Liu, IJ Wassell, K Soga, in *Proc. WiMob2010 Con*. Relay node placement for wireless sensor networks deployed in tunnels, (Niagara Falls, 2010), pp. 144–150
6. T Krikidis, Z Ding, CD Charalambous, Noncoherent energy detection with orthogonal signaling for an uncoded two-way relay channel. IEEE Trans. Veh. Technol. **61**(1), 404–409 (2012)
7. H Zhang, H Xing, J Cheng, A Nallanathan, VCM Leung, Secure resource allocation for OFDMA two-way relay wireless sensor networks without and with cooperative jamming. IEEE Trans. Ind. Inform. **12**(5), 1714–1725 (2016)
8. Z Chen, A Kuehne, A Klein, A Loch, M Hollick, J Widmer, in *Proc. WSA2012 Con*. Two-way relaying for multiple applications in wireless sensor networks, (Dresden, 2012), pp. 318–324
9. X Ji, On the study of cognitive bidirectional relaying with asymmetric traffic demands. J. RF-Eng. Telecommun. **69**(5–6), 185–303 (2015)
10. Z Zhao, M Peng, Z Ding, W Wang, H-H Chen, Denoise-and-forward network coding for two-way relay MIMO systems. IEEE Trans. Veh. Technol. **63**(2), 775–788 (2014)
11. R Vaze, RW Heath J, On the ccapacity and diversity-multiplexing tradeoff of the two-way relay channel. IEEE Trans. Inform. Theory. **57**(7), 4219–4234 (2011)
12. X Ji, W-P Zhu, D Massicotte, Transmit power allocation for asymmetric bi-directional relay networks using channel statistics. IET Commun. **9**(13), 1649–1660 (2015)
13. C Sun, C Yang, Energy efficient hybrid one- and two-way relay transmission. IEEE Trans. Veh. Technol. **62**(8), 3737–3751 (2013)
14. V Havary-Nassab, S Shahbazpanahi, A Grami, Optimal distributed beamforming for two-way relay networks. IEEE Trans. Signal Process. **58**(3), 1238–1250 (2010)
15. M Zaeri-Amirani, S Shahbazpanahi, T Mirfakhraie, K Ozdemir, Performance tradeoffs in amplify-and-forward bidirectional network beamforming. IEEE Trans. Signal Process. **60**(8), 4196–4209 (2012)
16. Y Li, X Zhang, M Peng, W Wang, Power provisioning and relay positioning for two-way relay channel with analog network coding. IEEE Signal Process. Lett. **18**(9), 517–520 (2011)
17. X Ji, W-P Zhu, D Massicotte, M Ahmed-Ouameur, in *Proc. GlobalSIP2015 Con*. Energy efficient power allocation and relay location for asymmetric bi-directional relaying, (Orlando, 2015), pp. 1–5
18. C Sun, Y Cen, C Yang, Energy efficient OFDM relay systems. IEEE Trans. Commun. **61**(5), 1797–1809 (2013)
19. C Zhang, J Ge, Y Cai, J Li, Y Ji, MA Farah, Energy efficiency and spectral efficiency tradeoff for asymmetric two-way AF relaying with statistical CSI. IEEE Trans. Veh. Technol. **65**(4), 2833–2839 (2016)
20. S Chung, I Joe, Intelligent transmission power allocation for distributed beamforming in wireless sensor networks. Int. J. Distrib. Sens. Net. **2015**, 1–10 (2015)
21. S Kim, YH Lee, Energy-efficient power allocation for OFDM signaling over a two-way AF relay. IEEE Trans. Veh. Technol. **64**(10), 4856–4863 (2015)
22. Z Chang, Q Zhang, X Guo, T Ristaniemi, Energy-efficient resource allocation for OFDMA two-way relay networks with imperfect CSI. EURASIP J. Wirel.Comm. **2015**(225), 1–11 (2015)
23. C Xiong, L Lu, GY Li, Energy-efficient OFDMA-based two-way relay. IEEE Trans. Comm. **63**(9), 3157–3169 (2015)
24. DE Simmons, JP Coon, Two-way OFDM-based nonlinear amplify-and-forward relay systems. IEEE Trans. Veh. Technol. **65**(5), 3808–3812 (2016)
25. DE Simmons, JP Coon, in *Proc. EuCNC2015 Con*. Two-way OFDM-based nonlinear amplify-and-forward relaying: power allocation, (Paris, 2015), pp. 280–284
26. Y-U Jang, E-R Jeong, YH Lee, A two-step approach to power allocation for OFDM signals over two-way amplify-and-forward relay. IEEE Trans. Signal Process. **58**(4), 2426–2430 (2010)
27. IA Akyildiz, T Melodia, KR Chowdhury, A survey on wireless multimedia sensor networks. Comput. Net. **51**(4), 921–960 (2007)
28. R Han, W Yang, K You, MB-OFDM-UWB based wireless multimedia sensor networks for underground coalmine: a survey. Sensors. **16**(12), 1–20 (2016)
29. X Ji, W-P Zhu, D Massicotte, Adaptive power control for asymmetric two-way amplify-and-forward relaying with individual power constraints. IEEE Trans. Veh. Technol. **63**(9), 4315–4333 (2014)