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Energy efficient power allocation strategy for 5G carrier aggregation scenario

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Abstract

Carrier aggregation (CA) is considered to be a potential technology in next generation wireless communications. While boosting system throughput, CA has also put forward challenges to the resource allocation problems. In this paper, we firstly construct the energy efficiency optimization problem and prove that the function is strictly quasi concave. Then we propose a binary search-based power allocation algorithm to solve the strictly quasi concave optimization problem. Simulation results show that the proposed algorithm can greatly improve the system energy efficiency while keeping low computation complexity.

Keywords: Carrier aggregation, Power allocation, Binary search, Strictly quasi concave, Optimization

1 Introduction

The next generation wireless communication system should meet the characteristic of 1G+ bits per second data rate to meet the requirements of various high-speed multimedia applications. To achieve this goal, the use of a larger bandwidth for transmission becomes the most direct way to boost transmission rate. But a large segment of continuous spectrum is not available easily for most of the wireless network operators due to the practical constraints, which makes the effective use of a plurality of non-continuous frequency spectrum a viable alternative option. The international standardization organization, for example 3GPP, has carried out research on spectrum expansion technologies, which is called carrier aggregation (CA) [1]. With CA, multiple spectrum fragments, whether continuous or not, can be aggregated together to be used by single user, which can substantially improve single user's peak data rate. Thanks to its high spectral efficiency, except for cognitive radios [2, 3], millimeter-wave communication [4], it is likely that carrier aggregation technology will also play an important role in future 5G wireless networks.

Though CA technology has significantly improved the system throughput, it has also increased the complexity of resource scheduling for the network. In addition, due to that multiple component carriers are used by the UE simultaneously, more transmission power will be consumed by the eNodeB as well as by the UE [5]. Therefore, energy-saving problem cannot be ignored for carrier aggregation and it is necessary to reduce the extra energy consumed by CA operation, thus reduce the carbon emission and contribute to green communication. Up to now, there are few researches on energy efficiency optimization algorithm focusing on power allocation in carrier aggregation systems. With respect to energy efficiency optimization for CA systems, there are mainly two kinds of algorithms. For the first one, it is assumed that the collection of resource blocks occupied by each user is known [6] and then the power allocation is performed. For the second one, the users' target data rate is constraint, and then the power and resource block are jointly allocated [7]. Because of these non-practical assumptions, these two kinds of optimization methods cannot be applied to the practical systems.

In this paper, we have proposed a binary search-based power allocation scheme for CA systems, which can greatly improves the energy efficiency of carrier aggregation system while keep low computation complexity at the base station.

2 System model and problem formulation

2.1 System model

Consider a single cell network model with a total of K users and N CCs. Each CC is composed of M PRBs each with bandwidth B. Define $\mathcal{K}=\{1,2,\ldots,K\}$, $\mathcal{N}=\{1,2,\ldots,N\}$, and $\mathcal{M}=\{1,2,\ldots,M\}$ as the user set, CC set, and PRB set,

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respectively. Indicator variable $c_{k,n,m} \in \{0, 1\}$ is used to indicate the relationships among the CCs, the users, and the PRBs, where value "1" indicates that PRB # m of CC # n is allocated to user # k and value "0" indicates the opposite. There is a rule that each PRB can only be assigned to single user at any moment.

According to the Shannon formula, the throughput that user # k can obtain from PRB # m of CC # n can be written as:

$$r_{k,n,m} = B\log_2\left(1 + \frac{p_{k,n,m} \times g_{k,n,m}}{\mathrm{BN}_0}\right) \quad (1)$$

where $p_{k,n,m} \geq 0$ is the transmit power of the base station, $g_{k,n,m}$ is the channel gain, and N_0 is additive white Gaussian noise(AWGN) power spectral density.

Based on Eq. 1 and the definition of $c_{k,n,m}$, we can obtain the total system throughput by summing up the transmission rate of all the users as following:

$$R = \sum_{k=1}^{K}\sum_{n=1}^{N}\sum_{m=1}^{M} r_{k,n,m} \times c_{k,n,m} \quad (2)$$

The power consumption of the base station is the sum of the power consumption of each user on each resource block of each carrier, which can be expressed as follows:

$$P_{\mathrm{BS}} = \sum_{k=1}^{K}\sum_{n=1}^{N}\sum_{m=1}^{M} p_{k,n,m} \times c_{k,n,m} \quad (3)$$

where $p_{k,n,m}$ is the base station's transmit power on PRB # m of CC # n for user k.

In addition to transmit power, there is also other extra power consumption at the base station, such as the energy consumption of air conditioning and cooling facilities. Considering that this kind of power consumption is irrelevant to the radio transmission and is steady during a long period of time, we model it as a constant P_C. Without losing generality, the energy efficiency is defined as the number of bits that are transmitted per joule:

$$\eta_{\mathrm{EE}} = \frac{R}{P_{\mathrm{BS}} + P_C} \quad (4)$$

According to Eqs. 2, 3, and 4, we can obtain the energy efficiency optimization problem for carrier aggregation system, which is modeled as problem P:

$$\begin{aligned}
&P: \max \frac{\sum_{k=1}^{K}\sum_{n=1}^{N}\sum_{m=1}^{M} r_{k,n,m} \times c_{k,n,m}}{P_{\mathrm{BS}}+P_C} \\
&\text{st.} \\
&P_{\mathrm{BS}} \leq P_{\max} && (C1) \\
&\sum_{k=1}^{K} c_{k,n,m} \leq 1, \forall n, m && (C2) \\
&\sum_{n=1}^{N} \tau\left(\sum_{m=1}^{M} c_{k,n,m}\right) \leq T, \forall k && (C3) \\
&c_{k,n,m} \in \{0, 1\}, p_{k,n,m} \geq 0, \forall k, n, m && (C4)
\end{aligned} \quad (5)$$

where $P_{\max}$ is the maximum transmit power of the base station. Function $\tau(x)$ is defined as following:

$$\tau(x) = \begin{cases} 1, & x > 0 \\ 0, & x = 0 \end{cases} \quad (6)$$

The constraint condition C2 limits the maximum number of CCs that each user can aggregate simultaneously. It is specified that by 3GPP that the largest number of CCs each user can aggregate should be no more than 5 [1]. Therefore, throughout this paper, the value of T is set to less than or equal to 5. The constraint condition C4 guarantees the effectiveness of the indicator variable as well as the transmit power.

2.2 Problem analysis

Because of the existence of 0–1 indicating variables, the scope of the constrained optimization problem presented in 5 is not convex. At the same time, since the base station's transmit power, P_{BS}, appears both in the numerator and in the denominator of the optimization objective function, the optimization objective function is not convex. According to the above observations, the optimization problem as given in Eq. 5 is not a convex optimization problem, and it is not solved by convex optimization theory.

To solve this kind of non-convex optimization problem, reference [8] adopted a fixed value of base station transmission power and expanded the range of indicator variable $c_{k,n,m}$ from discrete value of 0 or 1 to continuous real numbers that range in [0, 1], and then convex optimization theory was used to solve the problem [9]. Since too many approximate operations are used in the above schemes, the accuracy of the solution is not fine enough. Furthermore, the total number of resource blocks is much larger for 5G carrier aggregation systems than that of the LTE systems, so the computing complexity of the scheme is too high and it cannot be effectively applied in the realistic 5G communication systems.

Assume that the total transmit power of the base station is a fixed value, which is denoted by P_{BS}, then the optimization problem presented by Eq. 5 is reduced to a resource allocation optimization problem that under fixed given transmit power value.

$$R_{\max}(P_{\mathrm{BS}}) = \max\sum_{k=1}^{K}\sum_{n=1}^{N}\sum_{m=1}^{M} r_{k,n,m} \times c_{k,n,m}$$

st.

$$\sum_{k=1}^{K}\sum_{n=1}^{N}\sum_{m=1}^{M} p_{k,n,m} \times c_{k,n,m} = P_{\mathrm{BS}} \qquad (\mathrm{C1})$$

$$\sum_{k=1}^{K} c_{k,n,m} \le 1, \forall n, m \qquad (C2)$$

$$\sum_{n=1}^{N} \tau\left(\sum_{m=1}^{M} c_{k,n,m}\right) \le T, \forall k \qquad (C3)$$

$$c_{k,n,m} \in \{0,1\}, p_{k,n,m} \ge 0, \forall k, n, m \qquad (C4) \tag{7}$$

where $R_{\max}(P_{\mathrm{BS}})$ represents the maximum data rate of the carrier aggregation system under determined P_{BS} value. The definition of $c_{k,n,m}$, $r_{k,n,m}$, and τ function are the same with that of optimization problem (Eq. 5).

On the basis of Eq. 7, we define the optimal energy efficiency of the carrier aggregation system under determined base station transmit power P_{BS}, which is:

$$\eta_{\mathrm{EE}}^{\max}(P_{\mathrm{BS}}) \triangleq \frac{R_{\max}(P_{\mathrm{BS}})}{P_{\mathrm{BS}} + P_C} \tag{8}$$

Based on the above analysis, we can see that the optimization problem (Eq. 5) is equivalent to solving the following constraint optimization problem:

$$\eta_{\mathrm{EE}}^{*} = \max_{0 \le P_{\mathrm{BS}} \le P_{\max}} \eta_{\mathrm{EE}}^{\max}(P_{\mathrm{BS}}) \tag{9}$$

where the constraint conditions are the same with that in the optimization problem (Eq. 7). For the newly constructed optimization problem (Eq. 9), the most direct method is traversing all of the possible base station transmit power values to obtain the optimal power allocation scheme that has the highest energy efficiency using water-filling theorem. At last, find the power allocation scheme that has the highest energy efficiency from all possible power allocation schemes, then the optimization problem is solved.

3 Power allocation scheme

Considering that the base station's transmission power, P_{BS}, has real value, if we aim at traversing all P_{BS} values at equal intervals in $[0, P_{\max}]$, the step must be set small enough to ensure sufficient accuracy. That is to say, we should solve the optimization problem for a large number of transmit power values. Obviously, the complexity of the algorithm is too high, so we need to optimize it with low complexity method.

Through in-depth analysis of the optimization problem (Eq. 9), we find that $\eta_{\mathrm{EE}}^{\max}(P_{\mathrm{BS}})$ is a strictly quasi concave function on P_{BS} (please see Appendix for the proof). It means that $\eta_{\mathrm{EE}}^{\max}(P_{\mathrm{BS}})$ has only one local optimal solution, and the local optimal solution is also global optimal. For this kind of optimization problems, the literature [8] proposed an iterative algorithm based on extreme point idea, which adopted a large number of approximate operations that greatly increased the algorithm's time and space overhead. Considering this, we in this paper propose a binary search-based power allocation algorithm (BSPAA), which can provide high precision without too much iterations.

Assume that the number of users in the system is K and the user set is $\mathcal{K} = \{1, 2, \ldots, K\}$. Since the resource allocation scheme has already been determined, it is not necessary to distinguish which component carrier each resource block belongs to, i.e., we can consider all of the resource blocks as a set, which is denoted as $\mathcal{N} = \{1, 2, \ldots, N\}$. Defined $p_{k,n}$ as the transmission power allocated to user #k on resource block #n and $c_{k,n}$ as the indication whether the resource block #n is assigned to user #k, where value 1 represents "yes" and value "0" represents "no".

According to the Shannon formula, the transmission rate of the user #k on the resource block #n can be represented by the following formula:

$$r_{k,n} = B\log_2\left(1 + \frac{p_{k,n} \times g_{k,n}}{BN_0}\right) \tag{10}$$

where $g_{k,n}$ is the channel gain of user #k on resource block n. B is the bandwidth of a single resource block and N_0 is the Gauss white noise.

According to the Eq. 10, we can get the total downlink transmission rate of the carrier aggregation system:

$$R = \sum_{k=1}^{K}\sum_{n=1}^{N} r_{k,n} \times c_{k,n} \tag{11}$$

According to above analysis, the energy efficiency optimization problem under fixed base station's transmit power and determined resource block allocation scheme is equivalent to the optimization of the downlink transmission rate of the system, therefore we can get the following mathematical model:

$$\max\sum_{k=1}^{K}\sum_{n=1}^{N}r_{k,n}\times c_{k,n}$$
$$\text{st.}$$
$$\sum_{k=1}^{K}c_{k,n}\leq 1,\forall n \quad (\text{C1})$$
$$\sum_{k=1}^{K}\sum_{n=1}^{N}p_{k,n}=P \quad (\text{C2}) \tag{12}$$

where $r_{k,n}$ is given by Eq. 10. Constraint C1 shows that one resource block can only be assigned to at most one user. Constraint C2 shows that the total transmit power of the base station is equal to the sum of the transmit power on all of the resource blocks.

Considering the above optimization problem, because the resource allocation scheme has been determined, the constraint C1 can be satisfied at any case and the above optimization problem can be reduced to the following form:

$$\max\sum_{k=1}^{K}\sum_{n=1}^{N}r_{k,n}\times c_{k,n}$$
$$\text{st.}$$
$$\sum_{k=1}^{K}\sum_{n=1}^{N}p_{k,n}=P \quad (\text{C1}) \tag{13}$$

It is clear that above optimization problem is a typical convex optimization problem, and the constraint condition is equality, so we can solve the problem by water -filling theorem [10]. Defined λ_k as the Lagrange multiplier of user #k, then we can get the transmission power allocated to user #k on resource block #n:

$$p_{k.n}^{*}=\left(\frac{1}{\lambda_k}-\frac{N_0}{g_{k,n}}\right)^{+} \tag{14}$$

In Eq. 14, x^+ equals to x when x is larger than 0, and x^+ equals to 0 when x is less than or equal to 0. The Lagrange multiplier, λ_k, must satisfy the following inequality:

$$\sum_{k=1}^{K}\left(\frac{1}{\lambda_k}-\frac{N_0}{g_{k,n}}\right)^{+}=P \tag{15}$$

It can be seen from Eqs. 14 and 15 that the optimal power allocation scheme can be solved as long as the value of Lagrange multiplier is determined. The solution of the Lagrange multiplier is usually achieved by binary search method, and the specific steps can be found in [10], which is not addressed in detail here.

Binary search algorithm can determine the total transmission power of the base station while water-filling can solve the problem of power allocation under fixed transmission power and determined resource allocation scheme. Combining binary search algorithm and water-filling method, we can adopt BSPAA algorithm to achieve the optimal resource allocation results, which is as follows:

step1. Use GCSRAA algorithm to get the optimal resource allocation scheme $\mathcal{A}$.
step2. Initialize all the parameters, including the lower limit of transmission power $P_{\mathrm{lo}} = 0$, the upper limit of transmission power $P_{\mathrm{hi}} = 0$, and the transmit power adjustment step Δ.
step3. Set the initial transmit power of the base station to $P_{\mathrm{cur}} = (P_{\mathrm{hi}} + P_{\mathrm{lo}})/2$.
step4. Using water-filling based power allocation algorithm to calculate $\eta_{\mathrm{EE}}(P_{\mathrm{cur}} - \Delta)$, $\eta_{\mathrm{EE}}(P_{\mathrm{cur}})$, and $\eta_{\mathrm{EE}}(P_{\mathrm{cur}} + \Delta)$, i.e., the energy efficiency under the condition that the base station transmit power is $P_{\mathrm{cur}} - \Delta$, P_{cur}, and $P_{\mathrm{cur}} + \Delta$, respectively.
step5. If the energy efficiency values obtained in step3 satisfy the following inequality:

$$\begin{aligned}&\eta_{\mathrm{EE}}(P_{\mathrm{cur}}-\Delta)\leq\eta_{\mathrm{EE}}(P_{\mathrm{cur}})\\&\eta_{\mathrm{EE}}(P_{\mathrm{cur}}+\Delta)\leq\eta_{\mathrm{EE}}(P_{\mathrm{cur}})\end{aligned} \tag{16}$$

it means that the energy efficiency under transmit power P_{cur} is higher than that under transmit power $P_{\mathrm{cur}} - \Delta$ and $P_{\mathrm{cur}} + \Delta$, so it can be declared that P_{cur} is the global optimal solution if the adjustment step Δ is fine enough, then jump to step6.

Else if the energy efficiency values obtained in step3 satisfy the following inequality:

$$\begin{aligned}&\eta_{\mathrm{EE}}(P_{\mathrm{cur}}-\Delta)\leq\eta_{\mathrm{EE}}(P_{\mathrm{cur}})\\&\eta_{\mathrm{EE}}(P_{\mathrm{cur}}+\Delta)\geq\eta_{\mathrm{EE}}(P_{\mathrm{cur}})\end{aligned} \tag{17}$$

it means that the energy efficiency value is increasing under transmit power P_{cur}, i.e., the local optimal solution is larger than P_{cur}, so we can update the lower limit of the transmission power to P_{cur} and then jump to step3.

Else if the energy efficiency values obtained in step3 satisfy the following inequality:

$$\begin{aligned}&\eta_{\mathrm{EE}}(P_{\mathrm{cur}}-\Delta)\geq\eta_{\mathrm{EE}}(P_{\mathrm{cur}})\\&\eta_{\mathrm{EE}}(P_{\mathrm{cur}}+\Delta)\leq\eta_{\mathrm{EE}}(P_{\mathrm{cur}})\end{aligned} \tag{18}$$

it means that the energy efficiency value is decreasing under transmit power P_{cur}, i.e., the local optimal solution is smaller than P_{cur}, so we can update the upper limit of the transmission power to P_{cur} and then jump to step3.

step6. The algorithm ends. The current base station transmit power is the optimal value, and the corresponding power allocation scheme and resource allocation scheme are also the optimal strategy.

The algorithm is expressed in a flow chart as depicted in Fig. 1.

4 Numerical results

This chapter focuses on the performance and complexity analysis of the binary search-based power allocation scheme for carrier aggregation systems, to prove that the BSPAA algorithm can obtain fairly good results with lower complexity. The basic parameters used in the simulation are shown in Table 1. In this paper, two kinds of classical resource allocation algorithms for carrier aggregation system are evaluated and compared: nonlinear programming resource allocation algorithm (NPRAA) [11] and Markov-based carrier selected and resource allocation algorithm (MCSRAA). In order to simplify the simulation results, the normalized system energy efficiency is adopted in the following simulations.

Figure 2 shows the relationship between the energy efficiency of the system and the value of the base station transmission power under different constant extra power consumption values. In the simulation, the transmission power of the base station is exhaustively searched with 0.1 W search interval. As can be seen from the diagram, the energy efficiency of the system is a strictly quasi concave function of the base station transmission power. When the base station transmission power is about 10 W, the optimal system energy efficiency can be obtained. For the reason that the value of the additional power consumption is constant and independent to other system parameters, it cannot be optimized, so the

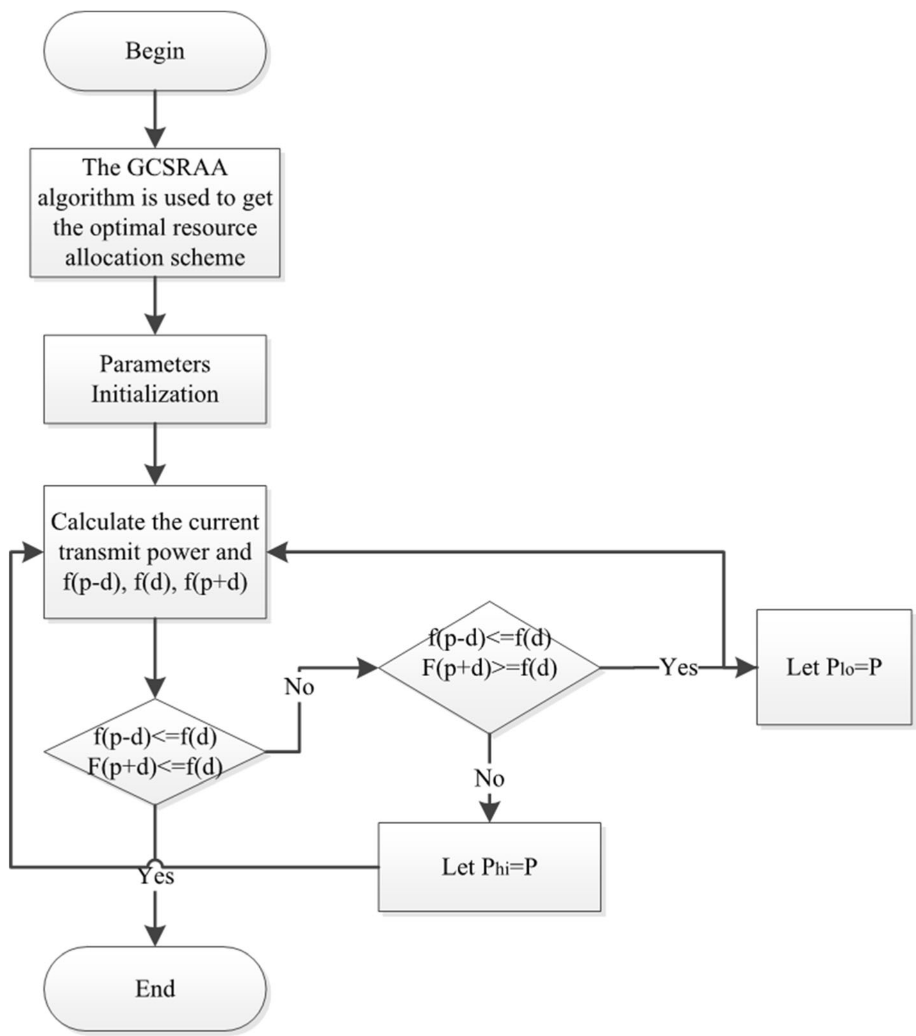

Fig. 1 Energy efficiency optimization algorithm for carrier aggregation systems

Table 1 Simulation parameters

Parameters	Value
Carrier frequency	3.5 GHz
Cell layout	1 layer, 3 sectors
Channel model	SCME model
Scenario	Urban macro
Carrier bandwidth	20 MHz
Maximal transmit power	49 dBm
Minimum BS-UE distance	35 m
Cell range expansion parameter	0 dB
Service type	Full buffer
TTI length	1 ms
TTI number	5000

bigger its value is, the smaller the energy efficiency can be obtained by the proposed algorithm.

Figure 3 shows the relationship between the system energy efficiency and the number of component carriers that can be occupied by a single user in a carrier aggregation system with total of 5 users. For users working in carrier aggregation mode, the more component carriers a user can occupy, the more resource blocks it can be assigned, and the higher transmission rate can be obtained, which can significantly improve the energy efficiency of the whole system. Therefore, all the three curves show an upward trend for the three cases that different maximal CCs a user can occupy simultaneously. In addition, the more carrier frequencies are used in the system, the more available resource blocks there will be, so the user is more likely to be assigned to more resources and the energy efficiency value increases faster.

Figure 4 shows the relationship between the system energy efficiency and the transmission power of the base station, where the number of users is 5 and the maximum number of component carriers per user can occupy is also 5. As can be seen from the figure, the optimal performance can be achieved when the base station transmission power is about 10 W for 5, 10, and 15 carrier frequency cases, which is consistent with the observations obtained from Fig. 2. When the base station transmit power is small, increasing the transmission power of the base station can increase the power on the resource blocks assigned to the user and the transmission rate can be increased accordingly, which can improve the energy efficiency of the whole system. When the base station transmission power exceeds a certain value, the number of available resources in the system will be limited. Therefore, the energy efficiency of the system tends to be stable when increasing the transmission power of the base station. Based on the analysis of Fig. 4, it can be concluded that the higher the number of available carriers in the system, the higher the energy efficiency of the system will be.

Figure 5 shows the relationship between the system energy efficiency and the number of component carriers that single user can occupy simultaneously, where there are 15 available carrier frequencies and the base station transmission power is 10 W. For carrier aggregation users, the more component carriers a user can occupy,

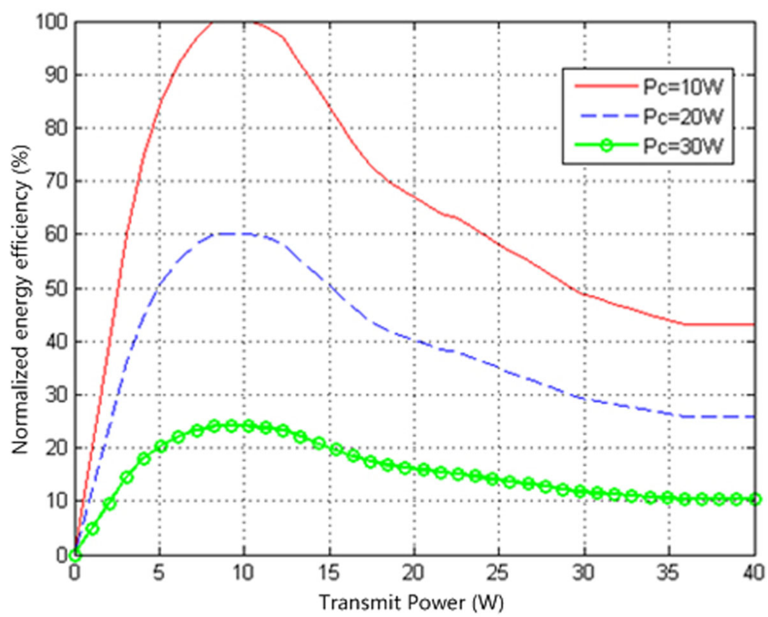

Fig. 2 System energy efficiency v.s. transmit power

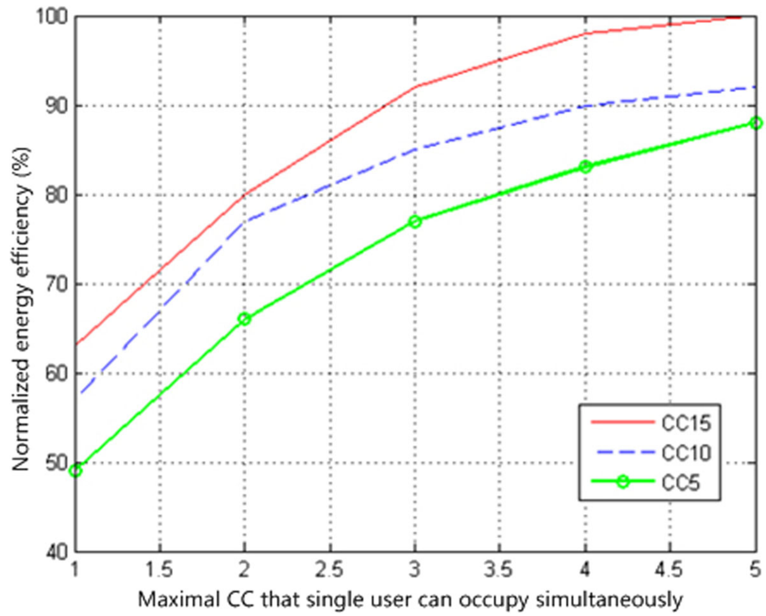

Fig. 3 System energy efficiency v.s. maximal CC that single user can occupy simultaneously

the more resource blocks can be assigned, and the higher transmission rate can be obtained, thereby the energy efficiency of the system can be significantly improved. Therefore, all the three curves show an upward trend. In addition, it can be seen from the figure that the energy efficiency optimization algorithm BSPAA proposed in this paper has achieved an average performance improvement of 11% compared with the NLPRAA algorithm, and an average performance improvement of 25% is obtained with respect to the MCSRAA algorithm. It can be concluded that the energy efficiency optimization algorithm proposed can get better results than the existing energy efficiency optimization algorithm, and has higher energy efficiency.

Figure 6 shows the computational complexity comparison between the energy efficiency optimization

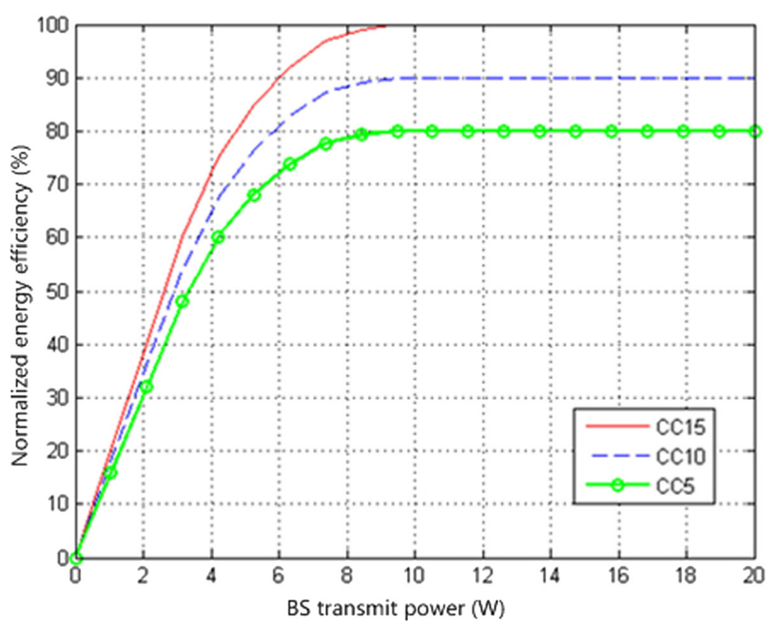

Fig. 4 System energy efficiency v.s.BS transmit power with different number of available carrier frequencies

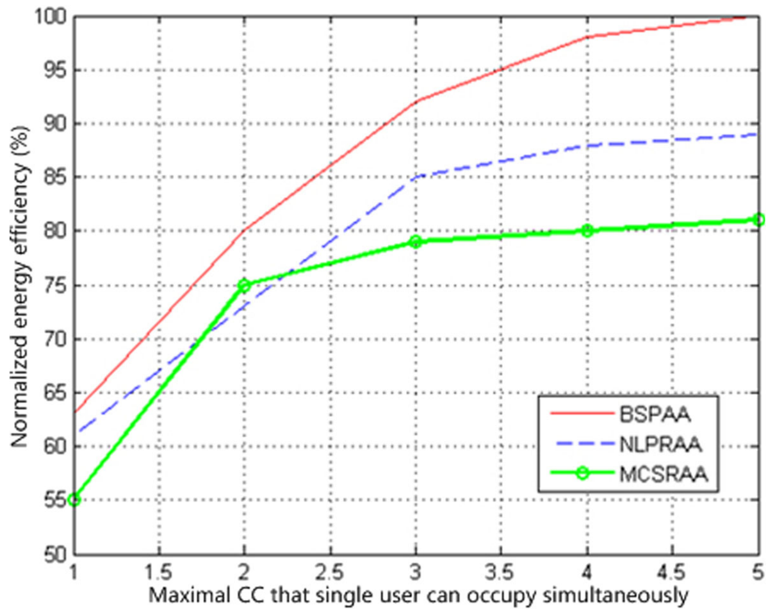

Fig. 5 System energy efficiency v.s. maximal CC that a single user can occupy simultaneously

algorithm and the other two baseline algorithms. For the optimization algorithm proposed in this paper, because the resource allocation algorithm and the power allocation algorithm are executed independently, the total iteration number of the algorithm is the sum of that for the two sub algorithms. As can be seen from the figure, the algorithm proposed in this paper can reach 90% of the optimal performance after about 300 iterations, and the optimal performance can be obtained after about 400 iterations. However, the other two baseline algorithms can only achieve the optimal solution after 600 iterations, and the energy efficiency is far worse than the algorithm proposed in this paper. Therefore, it can be proved that the proposed algorithm can achieve better optimization results while ensuring low computation complexity.

5 Conclusions

This paper presents the problem of power allocation in carrier aggregation systems. Energy efficiency is adopted as the evaluation metric, and it is found that the

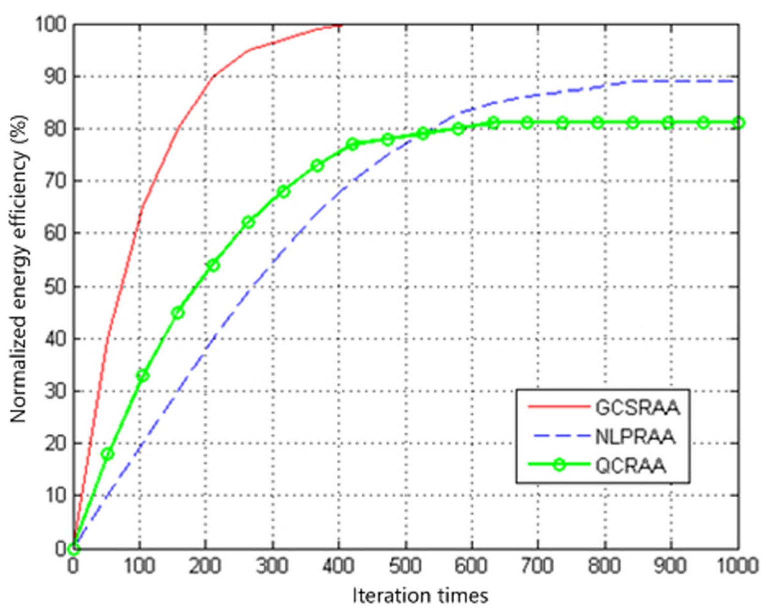

Fig. 6 System energy efficiency v.s. iteration times

function of energy efficiency optimization is strictly quasi concave, which cannot be solved easily with traditional optimization method. We propose a binary search-based power allocation scheme, which can significantly improve the system energy efficiency while keeping low computation complexity. Future work will consider jointly optimization of component carrier selection, radio resource allocation, and power allocation to further improve the system performance.

6 Appendix

Firstly, we give the definition of strictly quasi concave function: for function $F(x)$and independent variable x, define the super set of x as $\mathcal{S}_\alpha \triangleq \{x|F(x)\geq\alpha, x\geq 0\}$, where α is a real number greater than 0. For any $x_1\in\mathcal{S}_\alpha$, $x_2\in\mathcal{S}_\alpha$, and any real number in the interval (0, 1), if $\beta x_1 + (1-\beta)x_2\in\mathcal{S}_\alpha$ holds, then the function $F(x)$ is strictly concave on x.

We will prove that $\eta_{\mathrm{EE}}^{\max}(P_{\mathrm{BS}})$ is a strictly quasi concave function on P_{BS} according to the definition of strictly quasi concave function. The certification process is as following:

Let the super set of independent variable P_{BS} for function $\eta_{\mathrm{EE}}^{\max}(P_{\mathrm{BS}})$ be $\mathcal{S}_\alpha \triangleq \left\{P_{\mathrm{BS}}|\eta_{\mathrm{EE}}^{\max}(P_{\mathrm{BS}})\geq\alpha, P_{\mathrm{BS}}\geq 0\right\}$, where α is a real number greater than 0 and the definition of P_{BS} is the same as above. According to the Eqs. 4, 5, 6, and 7, we substitute the expression $\eta_{\mathrm{EE}}^{\max}(P_{\mathrm{BS}})$ into $\mathcal{S}_\alpha$ and perform a series of simplification and transposition, we can get $\mathcal{S}_\alpha = \{P_{\mathrm{BS}}|R_{\max}(P_{\mathrm{BS}})-\alpha P_{\mathrm{BS}}-\alpha P_C\geq 0, P_{\mathrm{BS}}\geq 0\}$. Let $P_{\mathrm{BS}_1}, P_{\mathrm{BS}_2}\in\mathcal{S}_\alpha$, and β be any real number in the interval (0, 1), next we will prove that $\beta P_{\mathrm{BS}_1} + (1-\beta)P_{\mathrm{BS}_2}$ also belongs to $\mathcal{S}_\alpha$.

For $\eta_{\mathrm{EE}}^{\max}(P_{\mathrm{BS}})$, we have:

$$\begin{aligned}\eta_{\mathrm{EE}}^{\max}(\beta P_{\mathrm{BS}_1} + (1-\beta)P_{\mathrm{BS}_2}) = R_{\max}(\beta P_{\mathrm{BS}_1} + (1-\beta)P_{\mathrm{BS}_2})\\ -\alpha(\beta P_{\mathrm{BS}_1} + (1-\beta)P_{\mathrm{BS}_2})-\alpha P_C\end{aligned} \tag{19}$$

We can see from Eqs. 4–1) that $R_{\max}(P_{\mathrm{BS}})$ and P_{BS} are logarithmic, so $R_{\max}(P_{\mathrm{BS}})$ is a strictly convex function on P_{BS}, therefore we can get the following inequality:

$$R_{\max}(\beta P_{\mathrm{BS}_1} + (1-\beta)P_{\mathrm{BS}_2})\geq R_{\max}(\beta P_{\mathrm{BS}_1}) + R_{\max}((1-\beta)P_{\mathrm{BS}_2}) \tag{20}$$

Substitute Eq. 20 into the Eq. 19, we can have the following inequality:

$$\begin{aligned}&R_{\max}(\beta P_{\mathrm{BS}_1} + (1-\beta)P_{\mathrm{BS}_2})-\alpha(\beta P_{\mathrm{BS}_1} + (1-\beta)P_{\mathrm{BS}_2})-\alpha P_C\geq\\ &R_{\max}(\beta P_{\mathrm{BS}_1}) + R_{\max}((1-\beta)P_{\mathrm{BS}_2})-\alpha(\beta P_{\mathrm{BS}_1} + (1-\beta)P_{\mathrm{BS}_2})-\alpha P_C\end{aligned} \tag{21}$$

Based on the inequality (Eq. 21) and inequality (Eq. 19), the following inequalities are obtained:

$$\begin{aligned}\eta_{EE}^{\max}(\beta P_{\mathrm{BS}_1} + (1-\beta)P_{\mathrm{BS}_2})\geq R_{\max}(\beta P_{\mathrm{BS}_1}) + R_{\max}((1-\beta)P_{\mathrm{BS}_2})\\ -\alpha(\beta P_{\mathrm{BS}_1} + (1-\beta)P_{\mathrm{BS}_2})-\alpha P_C\end{aligned} \tag{22}$$

By splitting, transposing, and merging operations to the inequality (Eq. 22), we can get the following inequalities:

$$\begin{aligned}&R_{\max}(\beta P_{\mathrm{BS}_1} + (1-\beta)P_{\mathrm{BS}_2})-\alpha(\beta P_{\mathrm{BS}_1} + (1-\beta)P_{\mathrm{BS}_2})-\alpha P_C\geq\\ &(R_{\max}(\beta P_{\mathrm{BS}_1})-\alpha\beta P_{\mathrm{BS}_1}-\alpha\beta P_C) +\\ &(R_{\max}((1-\beta)P_{\mathrm{BS}_2})-\alpha(1-\beta)P_{\mathrm{BS}_2}-\alpha(1-\beta)P_C)\end{aligned} \tag{23}$$

Because $R_{\max}(P_{\mathrm{BS}})$ and P_{BS} are logarithmic, for any real number $0 < \beta < 1$, we have:

$$R_{\max}(\beta P_{\mathrm{BS}})\geq\beta R_{\max}(P_{\mathrm{BS}}) \tag{24}$$

Substitute Eq. 24 into the inequality Eq. 23, we can get the following inequality:

$$\begin{aligned}&R_{\max}(\beta P_{\mathrm{BS}_1} + (1-\beta)P_{\mathrm{BS}_2})-\alpha(\beta P_{\mathrm{BS}_1} + (1-\beta)P_{\mathrm{BS}_2})-\alpha P_C\geq\\ &\beta(R_{\max}(P_{\mathrm{BS}_1})-\alpha P_{\mathrm{BS}_1}-\alpha P_C) + (1-\beta)(R_{\max}(P_{\mathrm{BS}_2})-\alpha P_{\mathrm{BS}_2}-\alpha P_C)\end{aligned} \tag{25}$$

Because $P_{\mathrm{BS}_1}, P_{\mathrm{BS}_2}\in\mathcal{S}_\alpha$, i.e., both of P_{BS_1} and P_{BS_2} belong to the super set $\mathcal{S}_\alpha$ defined above, the following inequality holds:

$$\begin{aligned}&\beta(R_{\max}(P_{\mathrm{BS}_1})-\alpha P_{\mathrm{BS}_1}-\alpha P_C)\geq 0\\ &(1-\beta)(R_{\max}(P_{\mathrm{BS}_2})-\alpha P_{\mathrm{BS}_2}-\alpha P_C)\geq 0\end{aligned} \tag{26}$$

Substitute Eq. 26 into the formula Eq. 25,we can get the following inequality:

$$R_{\max}(\beta P_{\mathrm{BS}_1} + (1-\beta)P_{\mathrm{BS}_2})-\alpha(\beta P_{\mathrm{BS}_1} + (1-\beta)P_{\mathrm{BS}_2})-\alpha P_C\geq 0 \tag{27}$$

So it is proved that $\beta P_{\mathrm{BS}_1} + (1-\beta)P_{\mathrm{BS}_2}\in\mathcal{S}_\alpha$, that is to say, $\beta P_{\mathrm{BS}_1} + (1-\beta)P_{\mathrm{BS}_2}$ also belongs to super set $\mathcal{S}_\alpha$, which proves the strictly quasi concave characteristic of $\eta_{\mathrm{EE}}^{\max}(P_{\mathrm{BS}})$ on P_{BS}. Therefore, $\eta_{\mathrm{EE}}^{\max}(P_{\mathrm{BS}})$ is a strictly quasi concave function on P_{BS}.

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Authors' contributions
WG conceived and designed the study. LM performed the experiments. GC reviewed and edited the manuscript. All authors read and agreed the manuscript.

Competing interests
The authors have declared that no competing interests exist.

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