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Dynamic power allocation for a multiuser transmitter with hybrid energy sources

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Abstract

In this paper, we investigate the problem of dynamic power allocation for a multiuser transmitter supplied by hybrid energy sources in details. Specifically, we focus on the hybrid energy sources which include both the traditional power grid and various renewable sources whereby there are a few issues in considerations: (1) The energy harvested jointly from various renewable sources is time-varying and possibly unpredictable and is stored in a limited capacity buffer with battery leakage. (2) At the meantime, the data arrives randomly to the transmitter and queues according to the individual receivers to wait to be transmitted. (3) In addition, the wireless channels fluctuate randomly due to fading. Taking into account the time variant and dynamic features of this system, we develop a dynamic power allocation algorithm for the transmitter with the aim of minimizing the average amount of energy consumption from the power grid over an infinite horizon, subject to all data in queues cannot exceed a given deadline of receivers. The research question is formulated as a stochastic optimization problem, then we utilize Lyapunov optimization to exploit an online algorithm with low complexity, and it does not require prior statistical knowledge of the stochastic processes. Performance analysis of the proposed algorithm is carried out in theory, which shows that the proposed algorithm performs arbitrarily close to the optimal objective value; meanwhile, the algorithm ensures that the maximum delay of all data queues cannot exceed a given value. Finally, performance comparison shows that our proposed algorithm provides not only better performance but also less time delay than other two algorithms.

Keywords: Energy harvesting, Power allocation, Hybrid energy sources, Lyapunov optimization, Wireless communication

1 Introduction

As the vast energy consumption of the devices in wireless communication systems has recently raised considerable environmental concerns, eco-friendly green communication, aiming at maximizing energy efficiency (bit-per-Joule), have drawn considerable research interests [1–3]. A large number of green technologies/methods for different wireless communication systems have been reported in the literatures [4–6]. Most of these works assume that the communication systems are powered by a constant energy source (such as traditional power grid, and diesel generator) or a rechargeable battery, such that the energy can be continuously used for system operations whenever needed.

On the other hand, as an economical and environmental-friendly supply of energy for communication nodes compared to traditional sources of energy, energy harvesting (EH) has recently attracted a large amount of attention of researchers [7–9]. EH nodes can harvest energy from natural resources, such as solar, wind, vibration, electromagnetic, and thermoelectric, thereby the harvested energy is substantially free of cost and can be unlimitedly available. As such, wireless networks composed of EH nodes can be energy self-sustained and reduce the use of conventional energy and accompanying carbon footprint. In addition, EH devices do not require conventional recharging; it enables untethered mobility and therefore can be deployed in hard-to-reach places such as remote rural areas, even within the human body [7, 10, 11]. However, the energy that can be harvested from the environment is unstable and varies over time, e.g., energy fluctuation caused by time-dependent solar and wind patterns. Therefore, EH brings new problems of intermittency and randomness of

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available energy. As a result, all wireless nodes powered by renewable energy are subject to the EH constraints over time, i.e., the total energy consumption up to any time must be less than the energy harvested by that time. Within the past few years, a large body of research works on power management has been done [12–19] in EH wireless communication systems including single-user setting, broadcast channels, relay channels, interference channels, and multiple access channels.

However, the above works with EH capability [12–19] assume that EH is the only source of energy for the transmitter, the proposed schemes just apply to the communication system with low traffic demands. As a matter of fact, the productions of renewable energy, strongly influenced by weather conditions, are intermittent and cannot be forecasted accurately [6]. Therefore, a sole EH source may not be able to maintain stable operation or guarantee a certain quality of service (QoS) of the system. To achieve both reliable and green communication, the concept of hybrid energy sources, i.e., using different energy sources in a complementary manner, has also drawn interests from both industry and academia [20–23]. For instance, Huawei Pty Ltd. has already developed base stations which draw energy from both solar panels and a wind energy harvester [20], and power grid as a supplement, as shown in Fig. 1.

With the hybrid energy sources [23–29], most of the researches focus on two categories: (1) deterministic EH model and (2) statistical EH model. The first category refers to the model that the energy arrival times and the amount of harvested energy are to be known a priori at the transmitter. And the second model is referred to that the prior knowledge of the statistical distribution of the EH process is known. Paper [24], based on the deterministic EH model, developed an energy efficient resource allocation scheme for timesharing multiuser systems by Lagrange dual decomposition method. And [25] focused on the joint energy-bandwidth allocation problems in multiuser channels based on the first EH

model and proposed an iterative algorithm using the Proximal Jacobian ADMM. For the deterministic EH model, [26] proposed the optimal offline transmission scheme for the point-to-point transmission to reduce the grid energy consumption. Considering random energy and data arrivals, Gong et al. [27] explored the structure of the optimal power allocation policy based on the statistical EH model. At the same time, paper [28] studied the energy-efficient resource allocation problem in an interference-free network and proposed the optimal offline and online algorithms based on the two EH, respectively. Similarly, the authors in [29] exploited an optimal resource allocation scheme to meet outage probability constraint by using dynamic programming (DP) approach.

All these works [24–29] based on both models provided many important references for our research. In practice, it is difficult to know the energy profile a priori at the transmitter [6]. Especially, it is more difficult to obtain the statistical knowledge of the energy generated jointly by both solar and wind energy sources, even more renewable sources. Besides, both the time-varying channel conditions and the dynamic mobile traffic have the common features of randomness and unpredictability, resulting in that their statistical properties are uncertain or hard to obtain in a longtime. However, all works in [24–26, 28] only consider full buffer networks without taking into account the dynamics of the data queues. Although Han and Ansari [27] took into account this factor, just focused on the single-link scenario and their proposed algorithms are not suitable for networks with multiple users. As such, the algorithms proposed in [24–29] are hardly implemented in practice because they require prior knowledge of the EH process, data arrival process and the channel state process.

In this paper, we develop a dynamic power allocation algorithm for the multiuser transmitter with hybrid energy sources, which are independent of the prior knowledge of any stochastic events, with the goal of minimizing the time average energy drawn from the power grid over an infinite horizon under certain delay requirement. We consider hybrid energy sources including both the traditional power grid and various renewable sources. The energy harvested from various renewable sources is stored in a buffer (battery) with limited capacity, and the harvested energy is time-varying and possibly unpredictable. Moreover, the battery is not perfect, such as storage loss and energy leakage, which degrade the efficiency of the renewable energy. The data arrives randomly to the transmitter and queues according to the individual receivers, and the wireless channel fluctuates randomly due to fading.

Taking into account the time variant and dynamic features of this system, we formulate the problem as a

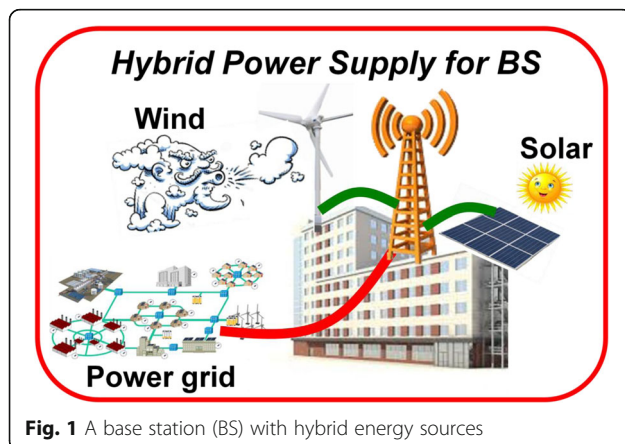


Fig. 1 A base station (BS) with hybrid energy sources

stochastic network optimization problem and solved by Lyapunov optimization approach initially developed in [30, 31]. Researchers show in [30, 31] that the optimization technique is well-suited for the queuing model in the scheduling problem for renewable energy supply and present a simple algorithm that does not require prior statistical information and is provably close to optimal. The authors in [32, 33] applied the technique in smart grid to solve the problems of power management and energy trading respectively. Paper [34] studied the issue of electric vehicles charging with renewable energy based on Lyapunov optimization.

The work in this paper is an extension of our earlier work in [35] that only considered a single user in point-to-point communication system. Note that our problem formulation is different from that in [30–35]; we apply the approach to the multiuser transmitter with hybrid energy sources in fading channels, which have multiple data queues to individual receivers. What is more, different receivers have individual tolerable delay times. As such, multiple queues will compete the limited resources with each other in the case of limited energy harvested by the transmitter, while at the same time satisfying the maximal transmission power constraint and the rate-power relationship constraint. The work in this paper is not just only incremental with respect to our earlier work in [35]. The research problem is now more complicated and practical; a simple algorithm for power allocation cannot resolve the problem anymore.

Our major contributions for this research are three-fold: (1) No need to know the statistical information of the EH process, data arrivals, and channel states; we develop a dynamic power allocation algorithm for a multiuser transmitter with the aim of minimizing the energy consumption from the power grid, taking into account the battery imperfection. (2) The proposed dynamic algorithm can be easily implemented in practice, just according to the current queue backlogs, channel states, and EH condition. In addition, we reveal the tradeoff between performance and delay by theoretical analysis. (3) The solution of the optimization problem considered in this paper provides a universal power allocation policy for multiuser transmitter with hybrid energy sources over an infinite horizon and facilitates the design of reliable green communication.

The remainder of this paper is organized as follows. In Section 2, the model of a multiuser communication system where the transmitter is powered by hybrid energy sources is described. In Section 3, minimization problem of the time average energy consumption from the power grid is formulated and the dynamic power allocation policy is elaborated. Simulation results are presented in Section 4. In Section 5, some concluding remarks are given.

2 System model

We consider a multiuser EH transmitter supplied by hybrid energy sources (composed of both power grid and multiple renewable energy sources) in fading channels, as shown in Fig. 2. The energy harvested from various renewable sources is first stored in a limited capacity buffer with imperfection before it can be used by the transmitter. Without loss of generality, we assume that the energy harvested is used only for transmission and the energy consumed by circuit or for signal processing is supplied by the power grid. The system operates in slotted time $t \in \{0, 1, 2, \dots\}$ with fixed time slots, the interval Δt is given at 1 s.

The transmitter has N receiver users; in every slot, the new data randomly arrives at the transmitter and queues according to individual receivers to await transmission through individual wireless channels. Let $\mathbf{a}(t) = [a_1(t), a_2(t), \dots, a_N(t)]$ be the vector of new data arrivals on slot t ; here, $a_n(t)$, $n \in \{1, 2, \dots, N\}$, is the rate of data incoming to the n -th data queue on slot t . We assume that $0 \leq a_n(t) \leq a_{\max}$, $\forall n, t$, a_{\max} is the maximum arrival rate for every data queue. Let $\boldsymbol{\mu}(t) = [\mu_1(t), \mu_2(t), \dots, \mu_N(t)]$ denotes the vector of departure rate from data queues; here, $\mu_n(t)$, $n \in \{1, 2, \dots, N\}$, in practice, is the transmission rate over corresponding wireless channel. Thus, the data queue is updated by

$$Q_n(t+1) = \max[Q_n(t) - \mu_n(t), 0] + a_n(t), \quad \forall n, t \quad (1)$$

where $Q_n(t)$ expresses the backlog of n -th data queue. We assume $Q_n(0) = 0$ for all n , that is, each data queue is empty before transmission.

For every slot, the transmission rate $\boldsymbol{\mu}(t)$ depends on transmission power allocated by the transmitter and current channels condition. We assume that the wireless channels fluctuate randomly due to fading and all channels are orthogonal. Let $\mathbf{h}(t) = [h_1(t), h_2(t), \dots, h_N(t)]$ be the vector of channels condition between the transmitter and receivers, and $h_n(t)$ represents the attenuation value and/or noise level of the n -th channel state on slot t . Suppose that the channel state information (CSI) at the beginning of every timeslot is known at the transmitter via channel monitoring and feedback link, and the overhead incurred by channel monitoring is neglected for simplicity [15, 29]. The channel conditions remain constant for the duration of each slot but may change at slot boundaries. For any n and t , the value of $h_n(t)$ is deterministically bounded by constants, $h_{\min} \leq h_n(t) \leq h_{\max}$.

The transmission rate μ_{ab} over the wireless link (a, b) depends on the channel state h_{ab} and transmission power P_{ab} ; the rate-power curve is shown in Fig. 3.

Further, the relationship between the channel state, transmission power, and rate on slot t can be described

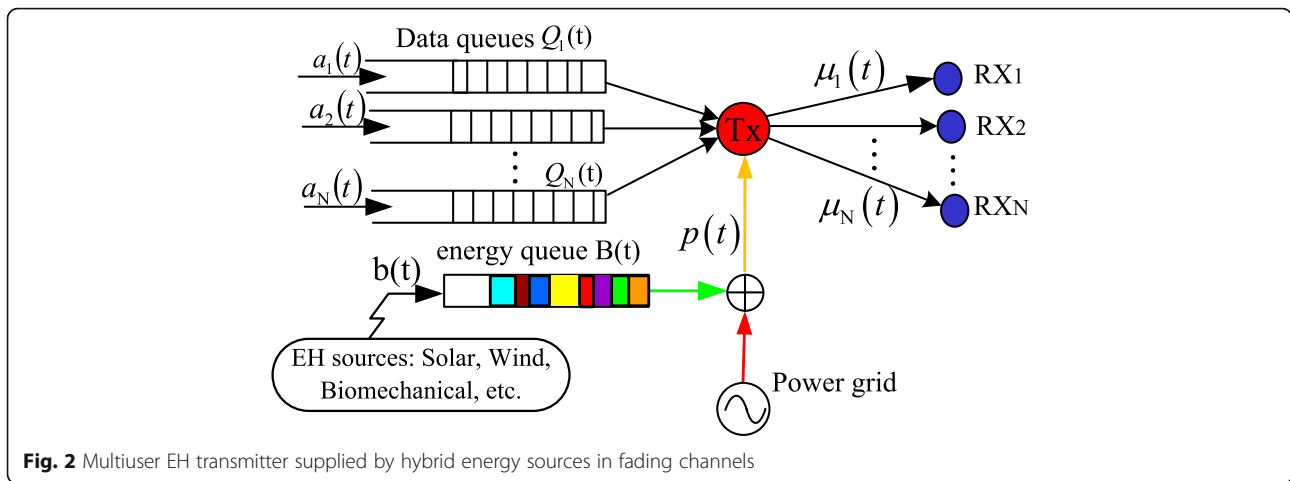


Fig. 2 Multiuser EH transmitter supplied by hybrid energy sources in fading channels

by the function $g(p_n(t), h_n(t))$ given by Shannon's capacity formula [28, 36].

$$\mu_n(t) = g(p_n(t), h_n(t)) = \frac{1}{2} \log_2(1 + p_n(t) \cdot h_n(t)), \forall n, t \tag{2}$$

where $n \in \{1, 2, \dots, N\}$, the rate-power function $g(\cdot)$ is assumed to be monotonically non-decreasing, determining the number of bits in data queue that can be transferred over the wireless link. However, the data in queues may be packets; we allow arbitrary fragmentation of packets during transmission.

From the above discussion, in order to finite backlog of all data queues (i.e., the data queues are all stable) [30], the transmitter must make a decision of transmission power on each slot according both the backlog of each data queue and current channels condition. Assume that the transmission power vector on slot t is denoted as $\mathbf{p}(t) = [p_1(t), p_2(t), \dots, p_N(t)]$. The total

transmission power on slot t is $\sum_{n=1}^N p_n(t)$, which is supplied by both the EH sources and the power grid.

$$\sum_{n=1}^N p_n(t) = P_p(t) + P_h(t), \quad \forall t \tag{3}$$

where $P_p(t)$ and $P_h(t)$ are supplied by the power grid and the energy queue buffered the energy harvested from EH sources, respectively. Furthermore, the total power consumed from the two types of energy sources is given by $\rho \sum_{n=1}^N p_n(t)$. Here, $\rho \geq 1$ is a constant, which accounts for the inefficiency of the non-ideal transmitter [26].

We assume that $b(t)$ Joules of energy is collected jointly from various renewable sources at the end of the t -th interval, the harvested energy is buffered in the battery before it can be used in the next time slot, $b(t) \leq B_{\max}$, where B_{\max} represents the maximum capacity of the buffer, i.e., the rechargeable battery can store at most B_{\max} Joules of energy. Due to the battery defects, such as energy leakage, supposing that a factor of $1 - \beta$ of the stored harvested energy is leaked per time interval due to the inefficiency of the battery [37], where $0 < \beta < 1$ represents the efficiency of the battery per time slot. Let $B(t)$ be the amount of the available energy in the rechargeable battery (energy queue), thus we have the following update equation of energy queue:

$$B(t + 1) = \min \left(\max [\beta(B(t) - \rho P_h(t) \Delta t), 0] + b(t), B_{\max} \right) \tag{4}$$

We assume $B(0) = 0$, which denotes the available energy before transmission.

The optimization goal is to minimize the time average energy consumption from the power grid over a long time, subject to the constraints of the stability of all data queues. Due to the finite storage capacity and the possible leakage of the battery, it is beneficial to draw the energy as quickly as possible from the battery so that

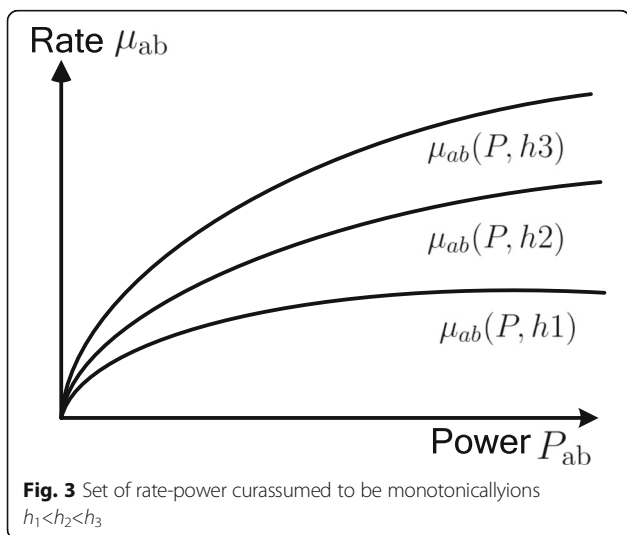


Fig. 3 Set of rate-power curves assumed to be monotonically increasing $h_1 < h_2 < h_3$

more harvested energy can be stored in the future, and thus the amount of possibly wasted harvested energy is minimized. Based the above discussion and the objective, we expect to find a dynamic power allocation scheme in the next section, which provides insight into how to efficiently utilize the energy supplied by the EH source, saving the traditional energy.

3 Problem formulation and solution

Our objective is to minimize the energy consumption from the power grid by making a decision of transmission power $\mathbf{p}(t)$ in every slot; the problem can be formulated as a stochastic optimization problem as follows:

$$\min : \lim_{t \rightarrow \infty} \frac{1}{t} \sum_{\tau=0}^{t-1} E \left\{ \max \left[\rho \sum_{n=1}^N p_n(\tau) - B(\tau), 0 \right] \right\} \quad (5)$$

$$s.t : \bar{Q}_n < \infty, \forall n \quad (6)$$

$$\text{Tolerable delay for the } n\text{-th user } \leq D_n^{\max}, \forall n \quad (7)$$

$$\mu_n(t) = \frac{1}{2} \log_2(1 + p_n(t) \cdot h_n(t)), \forall n, t \quad (8)$$

$$0 \leq p_n(t) \leq p_n^{\max} \quad (9)$$

where the optimization goal (5) shows that the time average expected energy consumed by the transmitter from the power grid is minimized over an infinite horizon, therein $\max[\rho \sum_{n=1}^N p_n(\tau) - B(\tau), 0]$ represents the energy consumption from the power grid on slot τ , $E\{\cdot\}$ denotes statistical expectation. Constraint Eq. (6) guarantees that all the queues are stable which defined as: $\bar{Q}_n \triangleq \limsup_{t \rightarrow \infty} \frac{1}{t} \sum_{\tau=0}^{t-1} E\{Q_n(\tau)\} < \infty$. Constraint Eq. (7) shows that the data queues have individual delay requirements, namely, the data in each queue waits for be transmitted to corresponding user, and the waiting time cannot exceed a deadline of corresponding user. In constraint Eq. (9), p_n^{\max} is the maximum transmission power allocated by the transmitter to the n -th user. To ensure that the problem Eqs. (5)–(9) are always feasible, we assume that the set of data arrivals vector is in the feasible region of the problem. The authors in [38, 39] defined the set of data arrivals vector that can be transmitted reliably under some power-allocated algorithm. In addition, the sum of the maximum allocation power for each user is assumed no more than the

maximum transmission power of the transmitter P_{\max} , i.e., $\sum_{n=1}^N p_n^{\max} \leq P_{\max}$.

3.1 The delay-aware virtual queue

To ensure that the optimization objective satisfies the delay constraints (7), we utilize virtual queues accounting for the constraints, which initially introduced in [30]. Let $Z_n(t)$, $n \in \{1, 2, \dots, N\}$ be the virtual queues. Fix any parameter $\sigma_n > 0$, define $Z_n(0) = 0$ for all n , and the virtual queues update according to the following:

$$Z_n(t+1) = \max[Z_n(t) + \sigma_n \cdot 1_{\{Q_n(t) > 0\}} - \mu_n(t), 0] \quad (10)$$

where $1_{\{Q_n(t) > 0\}}$ is an indicator function that is 1 if $Q_n(t) > 0$ and 0 else. We can see from Eq. (9), the virtual queue $Z_n(t)$ has an arrival process that add σ_n whenever the backlog of the actual queue $Q_n(t)$ is non-empty. This ensures that $Z_n(t)$ grows when there is unserved data in the actual data queue $Q_n(t)$. The constant σ_n can adjust the growth rate of the virtual queue $Z_n(t)$. If we can control the transmitter to guarantee that the queues $Q_n(t)$ and $Z_n(t)$ have finite upper bounds, then we can ensure that all bits in the n -th data queue are served within maximum delay of D_n^{\max} slots, which is given in the following lemma.

Lemma 1 Suppose the system is controlled so that the queue $Q_n(t)$ and $Z_n(t)$ have finite upper bounds, e.g., $Z_n(t) \leq Z_n^{\max}$ and $Q_n(t) \leq Q_n^{\max}$ for all t , then all bits in data queue n are served with a maximum delay of D_n^{\max} slots, which is defined as:

$$D_n^{\max} = \frac{Q_n^{\max} + Z_n^{\max}}{\sigma_n} \quad (11)$$

The proof of Lemma 1 follows the approach of Lyapunov optimization in [30].

3.2 Lyapunov optimization

Define $\Theta(t) \triangleq (\mathbf{Q}(t), \mathbf{Z}(t))$ as the concatenated vector of the real and virtual queues, here $\mathbf{Q}(t) = [Q_1(t), Q_2(t), \dots, Q_N(t)]$, $\mathbf{Z}(t) = [Z_1(t), Z_2(t), \dots, Z_N(t)]$. As a scalar measure of the congestion in all queues, we define the following Lyapunov function:

$L(\Theta(t)) \triangleq \frac{1}{2} \sum_{n=1}^N [Q_n(t)^2 + Z_n(t)^2]$. Define the conditional 1-slot Lyapunov drift as follows:

$$\Delta L(\Theta(t)) \triangleq E\{L(\Theta(t+1)) - L(\Theta(t)) | \Theta(t)\} \quad (12)$$

Making a decision $\mathbf{p}(t)$ to minimize $\Delta L(\Theta(t))$ alone would push all queues towards lower backlog (i.e., delay) [30] but which incur more energy consumption from the power grid. Considering both the energy consumption from the power grid (5) and queues backlog growth (1) and (10), our objective is then to minimize the following function in each timeslot t :

$$\min: \Delta L(\Theta(t)) + V \cdot E \left\{ \max \left[\rho \sum_{n=1}^N p_n(t) - B(t), 0 \right] \middle| \Theta(t) \right\} \quad (13)$$

Note that the left part of Eq. (13) is the growth of the queues and the right part of Eq. (13) is the expected energy consumption from the power grid (be called penalty), and Eq. (13) is called drift-plus-penalty expression [30]. The parameter $V > 0$ is used to tune performance-delay tradeoff between performance and queue backlog (i.e., delay). So our approach minimizes a weighted sum of drift and penalty, which can be proven bounded.

Lemma 2 The drift-plus-penalty expression for all slots t satisfied:

$$\begin{aligned} & \Delta L(\Theta(t)) + V \cdot E \left\{ \max \left[\rho \sum_{n=1}^N p_n(t) - B(t), 0 \right] \middle| \Theta(t) \right\} \\ & \leq C + V \cdot E \left\{ \max \left[\rho \sum_{n=1}^N p_n(t) - B(t), 0 \right] \middle| \Theta(t) \right\} \\ & \quad + \sum_{n=1}^N Q_n(t) E \{ a_n(t) - \mu_n(t) \mid \Theta(t) \} \\ & \quad + \sum_{n=1}^N Z_n(t) E \{ \sigma_n - \mu_n(t) \mid \Theta(t) \} \end{aligned} \quad (14)$$

where the constant C is defined as:

$$\begin{aligned} C = & \frac{\sum_{n=1}^N [a_{\max}^2 + \mu_{\max}^2]}{2} \\ & + \frac{\sum_{n=1}^N \max[\sigma_n^2, \mu_{\max}^2]}{2} \end{aligned} \quad (15)$$

The proof of Lemma 2 follows the approach of drift-plus-penalty in [30] using the following inequality:

$$[\max(b-c, 0) + a]^2 \leq b^2 + c^2 + a^2 + 2b(a-c) \quad (16)$$

which holds for $a \geq 0$, $b \geq 0$, and $c \geq 0$, then we can yield Eq. (14).

4 Real-time power allocation algorithm

By referring to Lyapunov optimization approach, we transform the problem Eqs. (5)–(9) to minimize the drift-plus-penalty expression in each slot, thus it is equivalent to minimizing the right-hand-side of the drift-plus-penalty bound Eq. (14) in each slot t ,

$$\begin{aligned} \min: & V \max \left[\rho \sum_{n=1}^N p_n(t) - B(t), 0 \right] \\ & + \sum_{n=1}^N Q_n(t) [a_n(t) - \mu_n(t)] + \sum_{n=1}^N Z_n(t) [\sigma_n - \mu_n(t)] \end{aligned} \quad (17)$$

Simplifying Eq. (17) and removing the parts which have nothing with our decision variable vector $\mathbf{p}(t)$, then we obtain:

$$\min: \rho V \sum_{n=1}^N p_n(t) - \sum_{n=1}^N [Z_n(t) + Q_n(t)] \mu(p_n(t), h_n(t)) \quad (18)$$

4.1 Real-time optimization algorithm

Our online optimization algorithm is described as follows:

Step 1. Every slot t , observe $\mathbf{Z}(t)$, $\mathbf{Q}(t)$, $\mathbf{h}(t)$, $\mathbf{a}(t)$ and $b(t)$, then choose $\mathbf{p}(t) = [p_1(t), p_2(t), \dots, p_N(t)]$ to minimize Eq. (18), subjecting to the constraints Eqs. (7)–(9);

Step 2. Update the real queues, virtual queues, and energy queue according to Eq. (1), Eq. (10), and Eq. (4), respectively.

The optimization solution of Eq. (18) can be solved by examining each vertex formed by the solution space. We denote the optimal power allocated to n -th data queue in timeslot t as $p_n^*(t)$,

$$\begin{aligned} p_n^*(t) = & \arg \min [\rho V p_n(t) \\ & - \frac{1}{2} [Z_n(t) + Q_n(t)] \log_2(1 + p_n(t) h_n(t))] \quad \forall n \end{aligned}$$

To solve $p_n^*(t)$, substituting the rate-power function Eq. (2) into Eq. (18), then differentiating with respect to the transmit power $p_n(t)$ (decision variable), we will obtain:

$$p_n^*(t) = \frac{Q_n(t) + Z_n(t)}{2 \ln 2 \cdot \rho V} - \frac{1}{h_n(t)} \quad \forall n, t \quad (19)$$

However, subjected to the constraints $0 \leq p_n(t) \leq p_n^{\max}$ for any n, t , the actual transmit power allocation $p_n(t)$ to n -th user in slot t can be obtained according to:

$$p_n(t) = \min(p_n^{\max}, \max(0, p_n^*(t))) \quad (20)$$

If $p \sum_{n=1}^N p_n(t) \leq B(t)$ holds, the transmitter does not need to draw additional energy from the power grid in slot t ; otherwise, the transmitter needs to draw the amount of $p \sum_{n=1}^N p_n(t) - B(t)$ additional energy in slot t .

4.2 Performance analysis

Theorem 1 Suppose $g(p_n^{\max}, h_{\min}) \geq a_n^{\max}, \forall n \in \{1, 2, \dots, N\}$. If $Q_n(0) = Z_n(0) = 0$, then for any fixed parameter

σ_n , $0 \leq \sigma_n \leq a_n^{\max}$ and $V > 0$ for all t , the proposed algorithm has the following properties for each queue n :

1. In all slots, for all queues, $Q_n(t)$ and $Z_n(t)$ are upper bounded by Q_n^{\max} and Z_n^{\max} respectively, where:

$$Q_n^{\max} = 2\ln 2 \cdot \rho V \left(\frac{1}{h_{\min}} + p_n^{\max} \right) + a_{\max} \quad (21)$$

$$Z_n^{\max} = 2\ln 2 \cdot \rho V \left(\frac{1}{h_{\min}} + p_n^{\max} \right) + \sigma_n \quad (22)$$

2. The maximum delay of the data queue n can be calculated according to (11) given by:

$$D_n^{\max} = \frac{4\ln 2 \cdot V \rho \left(\frac{1}{h_{\min}} + p_n^{\max} \right) + a_{\max} + \sigma_n}{\sigma_n} \quad (23)$$

3. Given that $\sigma_n \leq E\{a_n\}$, the time average expected additional energy drawn from the power grid using the proposed algorithm is upper bounded with C/V of the optimal value T_{opt} , i.e.,

$$\lim_{t \rightarrow \infty} \frac{1}{t} \sum_{\tau=0}^{t-1} E \left\{ \max \left[\rho \sum_{n=1}^N p_n(\tau) - B(\tau), 0 \right] \right\} \leq T_{\text{opt}} + \frac{C}{V} \quad (24)$$

where T_{opt} is the optimal value of minimizing average energy drawn from the constant source, and C is given in Eq. (15).

The proof further conducts the performance analysis of the Lyapunov optimization as described in [30], and the proof of Theorem 1 is given in the Appendix of this paper.

The performance analysis shows that the congestions of the queues grow linearly with V , while our goal decreases with increased V value, which is a tuning parameter to balance performance and delay. The performance can be pushed arbitrarily close to the optimum by tuning V , but the queues backlog may be longer. Thus, we should choose appropriate V value. To reduce D_n^{\max} value, we should use σ_n as large as possible while still meet $\sigma_n \leq E\{a_n\}$. We can choose $\sigma_n = E\{a_n\}$ if this expectation is given.

4.3 Comparison between the proposed algorithm and DP

The optimization problems considered in related works [26, 28] are based on dynamic programming (DP); the obtained algorithms can achieve the optimal objective value. While the power allocation algorithm based on Lyapunov optimization in this paper performs asymptotically close to the optimal objective value by tuning the value of V , as shown in Eq. (23). However, DP requires more stringent system modeling assumptions, i.e., requiring the prior knowledge of the probabilistic characteristics of the EH process, data arrivals, and channel states. In contrast, the Lyapunov optimization technique does not need the prior knowledge of these stochastic events. If the prior knowledge of energy harvesting, data arrivals and channel state values ($a(t)$, $b(t)$, $h(t)$) were known in advance, one could in principle make $\mathbf{p}(t)$ decisions that minimize average energy consumption from the power grid. One of the contributions of this paper is to provide an efficient algorithm without knowing the prior knowledge of any stochastic events. So our proposed algorithm is suitable for broader applications.

Besides, our proposed algorithm just needs the observations of the current system states firstly, i.e., $\mathbf{Z}(t); \mathbf{Q}(t); \mathbf{h}(t); \mathbf{a}(t)$, and $b(t)$, then make $\mathbf{p}(t)$ decisions according to Eq. (18). So the proposed algorithm is simple to implement, the complexity is linear with the number of queues. In contrast, the algorithms in [26–28] based on DP showed that the complexity increase exponentially with the number of time intervals. DP approach involves computation of value function that can be difficult when the state space of the system is large and suffers from a curse of dimensionality when being applied to large-dimensional systems (such as systems with many queues)

[31]. Therefore, as aforementioned, our proposed algorithm has better scalability and easy to use.

5 Simulation results

To evaluate the performance of the proposed dynamic power allocation algorithm, we assume that there are three users and the energy is harvested from both solar and wind energy, the energy output characteristic follows an i.i.d. Poisson process. We evaluate the performance of the proposed algorithm on daily data set, i.e., in 3600 timeslots (the time interval is fixed as 1 s). Note that we adopt the distribution just for exposition purpose; the analysis in the previous section does not depend on the distributions. The related simulation settings are summarized in Table 1.

To better evaluate the performance of our proposed algorithm, three strategies are considered in the simulations. The first strategy uses Lyapunov optimization algorithm. The latter two strategies (second and third strategies) use simple greedy algorithm. The second strategy deploys “absorb-upon-arrival” policy, which describes such

Table 1 Simulation setting

Parameters	Value
Number of users	3
Timeslot length	1 s
EH sources	Solar and wind energy
Harvest process	i.i.d. Poisson process
Data arrival process	Uniform distribution
Inefficiency factor ρ	1.2
Energy leaked factor β	0.9
Bandwidth B	1 M/Hz
Channel Fading	Gaussian
Average SNR	10 dB
Max transmit power P_{\max}	6 W
p_n^{\max} For every user	2 W
$\sigma_n, n = 1, 2, 3$	$\frac{3}{4}E\{a_1\}, \frac{4}{5}E\{a_2\}, \frac{3}{4}E\{a_3\}$
$\sigma_n^{\max} n = 1,2,3$	4.4, 2.9, 3.9 bit/slot

scenario: when the energy in rechargeable battery cannot meet the need of the transmitter, the transmitter immediately draws energy from the power grid for sending data, which results in the least data delay time, but possibly more energy consumption from the power grid. The third strategy deploys the policy “absorb-at-deadline,” which means that the transmitter uses only renewable energy before deadline, and does not draw energy from the power

grid even if the EH source cannot meet the need of the transmitter until any data delay exceed the maximum, where the deadline is set to 25 slots.

The performance comparison of three strategies is shown in Fig. 4. Among the figures, Fig. 4a shows the energy drawn by the transmitter from power grid in every timeslot, and Fig. 4b shows the accumulated amount of energy drawn from the power grid over a day (i.e., 3600 timeslots). From Fig. 4, we can see that Lyapunov optimization algorithm (our proposed algorithm) achieves the best performance (the minimum amount of energy consumption from the power grid) among the three strategies, that is, more traditional energy is saved using our proposed algorithm. Under this condition of parameter setting, about 2337 J of traditional energy can be saved using the proposed algorithm in comparison with using the strategy of absorb-upon-arrival only over a day (3600 timeslots), and about 1225 J of traditional energy can be saved in comparison with using the strategy of absorb-at-deadline over a day. Here, V is set to 80 by trail and error.

Figure 5 shows the accumulated amount of the energy consumption from power grid in different cases of average amount of energy harvested b_{av} ($b_{av1} \leq b_{av2} \leq b_{av3}$). From Fig. 5, we can see that no matter under which case, Lyapunov optimization algorithm can achieve the best performance among the three strategies, and absorb-upon-arrival policy provides the worst performance. The reason is that Lyapunov optimization

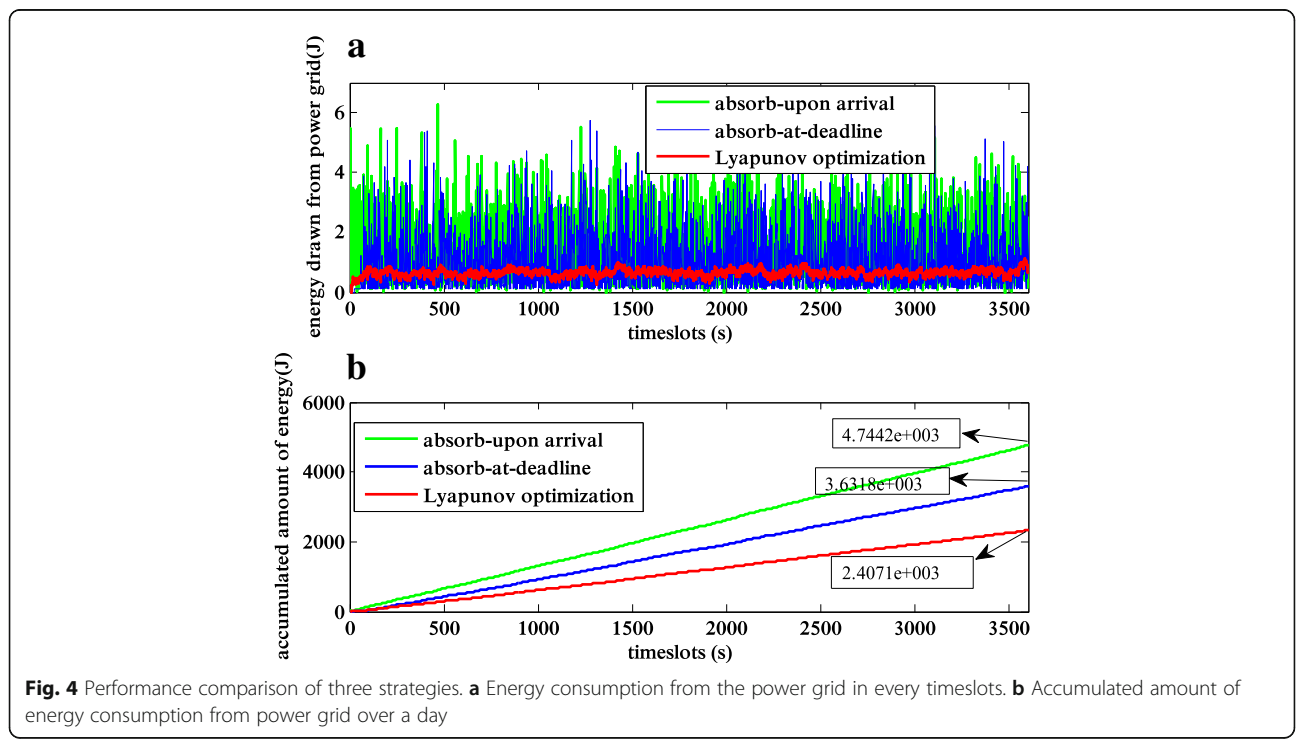


Fig. 4 Performance comparison of three strategies. **a** Energy consumption from the power grid in every timeslots. **b** Accumulated amount of energy consumption from power grid over a day

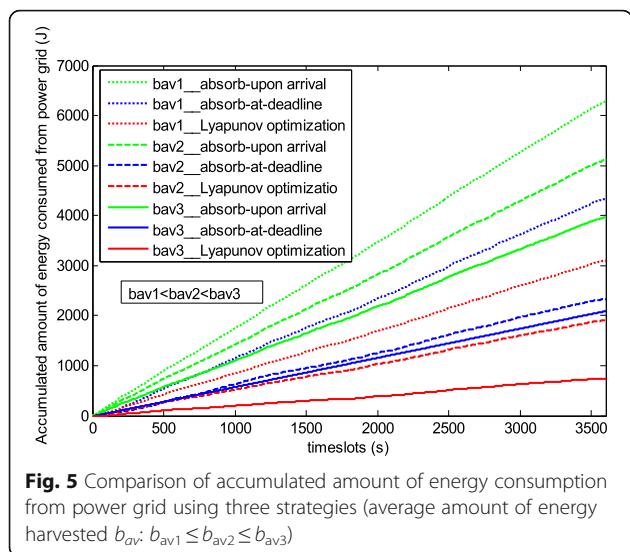


Fig. 5 Comparison of accumulated amount of energy consumption from power grid using three strategies (average amount of energy harvested $b_{av1} < b_{av2} < b_{av3}$)

algorithm enables the transmitter to send more data when channel state is better, while absorb-upon-arrival policy can provide the least data delay time, which resulting in the worst performance. However, no matter which strategy is adopted among three strategies, the smaller the amount of energy harvested by the transmitter from the renewable energy sources is, the more energy from the power grid is supplied for the transmitter. Here, V is set to 80.

To have a better insight of the delay time reduction, two strategies (Lyapunov optimization algorithm and absorb-at-deadline policy) has been compared. Simulation results on the fraction of waiting data of three queues are shown in Fig. 6. Seen from Fig. 6, using

Lyapunov optimization algorithm results in much smaller delay than the deadline. Most of the arrival data wait about 5 slots used our proposed algorithm, while the strategy absorb-at-deadline wait mostly 24 slots. Based on Lyapunov optimization algorithm, the maximum delay D_n^{max} (time-slots) computed by formula (11) and the actual average delay D_n^{actu} of 3 data queues by simulate, and the average delay D_n^{dead} based on absorb-at-deadline strategy of 3 data queues are shown in Table 2.

In order to study the impact of parameter V on the additional energy cost from the power grid and average delay of the data in the data queues, we have plotted Fig. 7 showing the relationship between the energy cost and the value of V and the relationship between the average delay time and the value V . We can see that as we expected, the average delay increases non-linearly with the value of V while the energy cost decreases with V . The energy cost and average delay reach saturation when V is larger than a certain value ($V = 80$, seen from Fig. 7), which illustrates that when V is large enough, the average delay will reach its maximum and the energy cost is close the optimal value (T_{opt}).

6 Conclusions

In this paper, we develop a dynamic power allocation algorithm for a multiuser transmitter powered by hybrid energy sources (including the traditional power grid and EH sources). The proposed algorithm provides insight into how to efficiently utilize the energy supplied by the EH sources, namely how to minimize the time average energy consumption from the power grid at the same time ensure the QoS of communication. Firstly, we

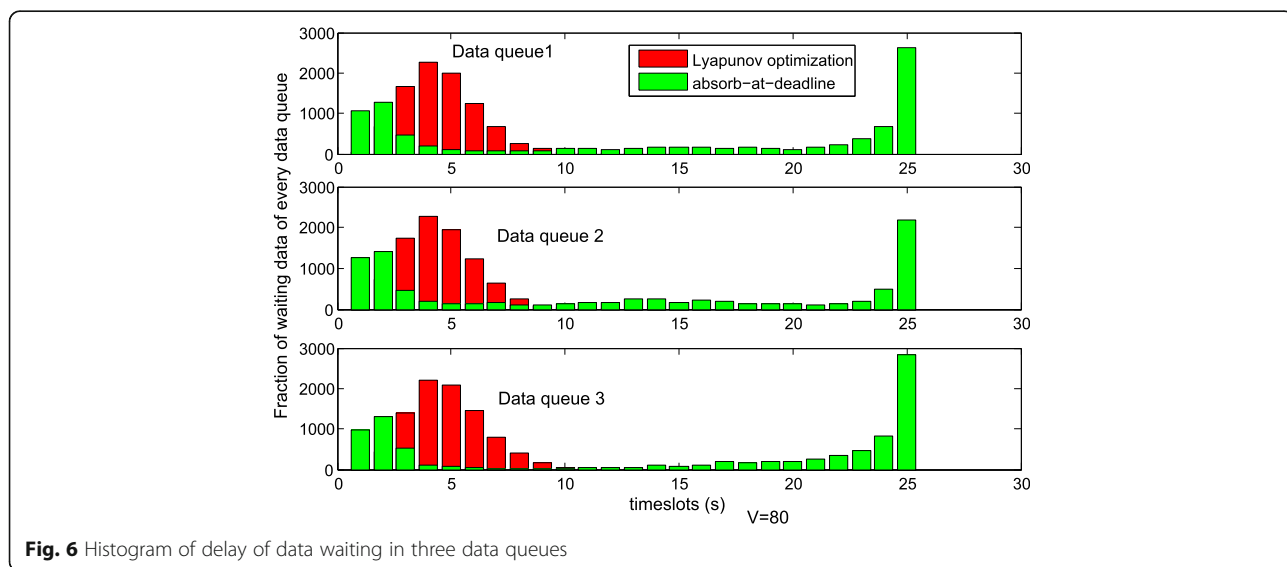


Fig. 6 Histogram of delay of data waiting in three data queues

Table 2 Simulation results of two algorithms

Algorithms	Queue 1	Queue 2	Queue 3
Lyapunov D_n^{\max}	31.7638	20.7181	23.8009
Lyapunov D_n^{actu}	4.6008	4.5276	4.8873
absorb-at-deadline D_n^{dead}	14.5031	12.7796	15.6809

model this kind transmitter in fading channels. The data arrivals and energy harvested from surrounding both randomly arrived at the transmitter, in addition the wireless channels fluctuate randomly, without knowing their statistical probabilities. Secondly the issue is formulated as a stochastic optimization problem, and a real-time power allocation algorithm is exploited with low complexity. The theoretical performance analysis shows that the proposed algorithm outperforms the state-of-the-art algorithms in terms of achieving a near optimal value by tuning the parameter V , while ensure the time delay of data queues would not exceed the maximum delay D_n^{\max} . A further comparison of the proposed algorithm with other two greedy algorithms demonstrates the proposed algorithm can consume much less energy from the power grid. Moreover, the algorithm of the optimization problem in this paper does not require the knowledge of statistical probabilities of the random processes; thereby, it provides a universal power allocation policy for multiuser transmitter with hybrid energy sources and facilitates the design of reliable green communication paradigm.

7 Appendix

7.1 Proof of Theorem 1

1. We use induction method to show that:

$$Q_n^{\max} = 2\ln 2 \cdot \rho V \left(\frac{1}{h_{\min}} + p_n^{\max} \right) + a_{\max}, \quad \forall n, t$$

It holds clearly for $t = 0$ (because $Q_n(0) = 0$). Next we assume

$$Q_n(t) \leq 2\ln 2 \cdot \rho V \left(\frac{1}{h_{\min}} + p_n^{\max} \right) + a_{\max}, \quad \forall n, t$$

what we can do is to prove it also true for slot $t + 1$. If

$$Q_n(t) \leq 2\ln 2 \cdot \rho V \left(\frac{1}{h_{\min}} + p_n^{\max} \right)$$

the maximum queue backlog growth is a_{\max} , then

$$Q_n(t) \leq 2\ln 2 \cdot \rho V \left(\frac{1}{h_{\min}} + p_n^{\max} \right) + a_{\max}$$

If $Q_n(t) \geq 2\ln 2 \cdot \rho V \left(\frac{1}{h_{\min}} + p_n^{\max} \right)$, since $Z_n(t) \geq 0$, we have:

$$\begin{aligned} Q_n(t) + Z_n(t) &\geq 2\ln 2 \cdot \rho V \left(\frac{1}{h_{\min} + p_n^{\max}} \right) \\ &\geq 2\ln 2 \cdot \rho V \left(\frac{1}{h_n(t) + p_n^{\max}} \right) \end{aligned}$$

In this case, according to the algorithm proposed above we will have $p_n^*(t) > p_n^{\max}$ by formula (19). Then we will choose $p_n(t) = p_n^{\max}$ on slot t according to (20), thus the data queue is served by at least a_{\max} , because

$$g(p_n^{\max}, h_{\min}) \geq \max [a_{\max}, \sigma_n]$$

Hence the data queue backlog cannot grow on the next slot, i.e.,

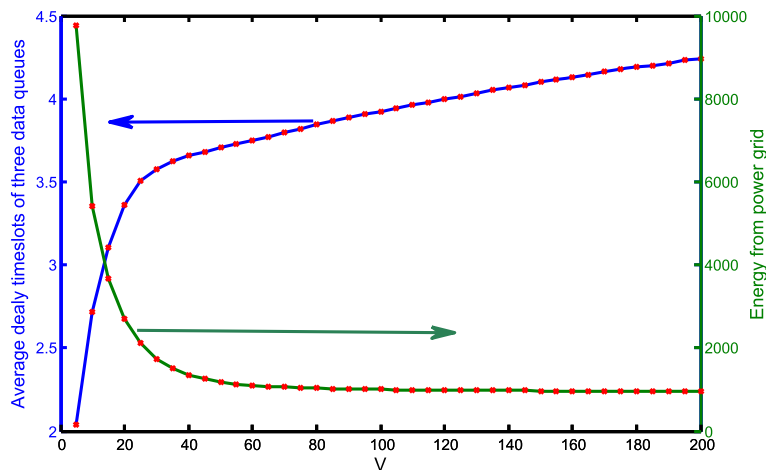


Fig. 7 Energy from power grid and average delay for different V values using our proposed algorithm

$$Q_n(t + 1) \leq Q_n(t) \leq 2\ln 2 \cdot \rho V \left(\frac{1}{h_{\min}} + p_n^{\max} \right) + a_{\max}$$

Therefore, we have

$$Q_n(t) \leq 2\ln 2 \cdot \rho V \left(\frac{1}{h_{\min}} + p_n^{\max} \right) + a_{\max}$$

for all slot t .

The proof that $Z_n(t) \leq 2\ln 2 \cdot \rho V \left(\frac{1}{h_{\min}} + p_n^{\max} \right) + \sigma_n$ is similar above.

2. It is very easy to prove according to Lemma 1 and the conclusion of Theorem 1.
3. Since the proposed algorithm will always try to minimize the right-hand-side part of the inequality (14) among all feasible solutions, even the optimal solution, assume the solution given by the proposed algorithm and optimal solution are $p_{n,\text{pro}}(t)$ and $p_{n,\text{opt}}(t)$ respectively, and the optimal result for minimizing average energy drawn from the constant source is T_{opt} , then by plugging the solution into the inequality (14), we can have the following:

$$\begin{aligned} & \Delta L(\Theta(t)) + VE \left\{ \max \left[\rho \sum_{n=1}^N p_{n,\text{pro}}(t) - B(t), 0 \right] \middle| \Theta(t) \right\} \\ & \leq C + VE \left\{ \max \left[\rho \sum_{n=1}^N p_{n,\text{opt}}(t) - B(t), 0 \right] \middle| \Theta(t) \right\} \\ & \quad + \sum_{n=1}^N Q_n(t) E \{ a_n(t) - \mu_n(p_{n,\text{opt}}(t), h_n(t)) \mid \Theta(t) \} \\ & \quad + \sum_{n=1}^N Z_n(t) E \{ \sigma_n - \mu_n(p_{n,\text{opt}}(t), h_n(t)) \mid \Theta(t) \} \\ & \leq C + VT_{\text{opt}} \end{aligned} \tag{25}$$

The result of (25) is based on the facts that

$$\begin{aligned} \lim_{T \rightarrow \infty} \frac{1}{T} \sum_{t=0}^{T-1} E \{ a_n(t) - \mu_n(p_{n,\text{opt}}(t), h_n(t)) \mid \Theta(t) \} & \leq 0 \\ \lim_{T \rightarrow \infty} \frac{1}{T} \sum_{t=0}^{T-1} E \{ \sigma_n - \mu_n(p_{n,\text{opt}}(t), h_n(t)) \mid \Theta(t) \} & \leq 0 \end{aligned}$$

Summing inequality (25) over slots $t \in \{0, \dots, T\}$, we can have:

$$\begin{aligned} & L(\Theta(T)) - L(\Theta(0)) \\ & + VE \left\{ \max \left[\rho \sum_{n=1}^N p_{n,\text{pro}}(t) - B(t), 0 \right] \right\} \\ & \leq CT + VT \cdot T_{\text{opt}} \end{aligned} \tag{26}$$

Using the fact that $L(\Theta(T)) \geq 0$ and $L(\Theta(0)) = 0$, dividing both sides of (26) by VT and letting $T \rightarrow \infty$ results in:

$$\lim_{T \rightarrow \infty} \frac{1}{T} \sum_{t=0}^{T-1} E \left\{ \max \left[\rho \sum_{n=1}^N p_{n,\text{pro}}(t) - B(t), 0 \right] \right\} \leq T_{\text{opt}} + \frac{C}{V}$$

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Authors' contributions

DL, JL, and JW conceived and designed the study. DL and JW performed the experiments. DL provided the performance analysis of the proposed algorithm in theory. DL and JL wrote the paper. FJ reviewed and edited the manuscript. All authors read and approved the manuscript.

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Competing interests

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