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On performance analysis for optimum combining of DF relaying with fast-fading multiple correlated CCIs, correlated source-relay, and thermal noise

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Abstract

This paper analyzes the outage probability (OP) and the average symbol error rate (SER) of decode-and-forward (DF) relaying. The paper derives closed-form expressions for the OP and the average SER with optimum combining (OC) considering fast-fading multiple correlated CCIs, the correlated source-relay, and thermal noise. It is shown that the performance of the large distance between the source and the relay is better than that of the small distance, regardless of interference fading speed at the destination. We also show that given the source-relay distance, the performance of slow-fading interference is basically better than that of fast fading, except in the low signal-to-noise-ratio (SNR) regime for the distance being small. In result, the source-relay distance is generally a more dominating factor for the performance than fading CCIs.

Keywords: Decode-and-forward relaying, Fast-fading correlated multiple CCIs, Correlated source-relay, Optimum combining, Rayleigh fading, Thermal noise, Outage probability, Symbol error rate

1 Introduction

Cooperative communications have been prominent because of diversity gain [1]. In cooperative networks, there are mainly two methods, such as the amplify-and-forward (AF) relay network and the decode-and-forward (DF) relay network [2]. Then with multiple copies, the destination can achieve cooperative diversity. In order to do so, we can use maximal-ratio combining (MRC) [3] (p., 316) or optimum combining (OC) [4]. MRC maximizes the signal-to-noise-ratio (SNR), while OC maximizes the signal-to-interference-plus-noise ratio (SINR). When co-channel interferers (CCIs) are present at the destination, OC reduces CCIs' power and increases diversity [5]. Since the analytical expressions for the outage probability (OP) and the average symbol error rate (SER) are complex for derivation, some simplified models have been used [6, 7]. Usually, thermal noise is ignored for the tractability of analytical expressions, assuming CCIs being dense, and the system models are simplified. In this case, when the effect of thermal noise is greater than that of CCIs, the analysis of the simplified model might be incorrect [8]. Therefore thermal noise is considered, but fading is still assumed to be slow so that fading in phase 1 and 2 is unchanged and constant. In this case, the analysis is limited for slow fading with MRC [8] or with OC [9]. In addition, it has been assumed that the source and the relay are always far enough to be uncorrelated, which is not always true. Sometimes they become so close that the correlation between them occurs, with which the performance degrades to some extent.

Recently, there have been many research advances in the DF relay network: the opportunistic relaying (OR) in the presence of CCIs is investigated in [10]; a new transmission scheme for selective DF relaying networks is presented, considering the employment of different modulation levels at the transmitting nodes [11]; and a joint scheme (JS) has been proposed for a multiple-relay multiple-input multiple-output (MIMO) network with a DF relaying strategy [12]. In [13], a novel distributed space-time coding (DSTC) transmission scheme for a two-path successive DF relay network is proposed. The average SER is analyzed for

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a wireless-powered three-node DF relaying system in Nakagami-m fading environment [14].

In this paper, a DF protocol is considered. It is assumed that at the relay symbol-by-symbol decoding is executed, and at the destination, full decoding is carried out [2]. We also assume that multiple correlated CCIs are fast faded, the source and relay are correlated, and thermal noise is present. To the best of our knowledge, the performance analysis for this system has not been reported. First, we derive closed-form expressions for the OP and the average SER with OC considering fast-fading multiple correlated CCIs, the correlated source-relay, and thermal noise. Second, we investigate the effects of the source-relay distance and fast/slow-fading CCIs on the performance.

The paper is organized as follows: Section 2 defines the system and channel model. In Section 3, the exact analytical expressions are derived for the OP and the average SER. Section 4 presents the analytical and simulation results, which we discuss in detail. The paper is concluded in Section 5.

2 System and channel model

We define the full cooperative case as relaying with no symbol errors and the non-cooperative case as relaying with the symbol error probability being one. Let the probability of symbol errors at the relay be $P_{\mathrm{e}}^{(R)}$. For the full cooperative case, $P_{\mathrm{e}}^{(R)}=0$, and for the non-cooperative case, $P_{\mathrm{e}}^{(R)}=1$. For $0<P_{\mathrm{e}}^{(R)}<1$, we say simply the cooperative case. We assume that the destination knows whether or not the relay sends the symbol with the probability one. We suppose a time division duplex (TDD) mode [1, 8]. The DF protocol is composed of two time slots, t_1 and t_2. One time interval t_1 is for phase 1 and the other t_2 is for phase 2. Therefore, a single transmission duration (STD) t_{STD} becomes t_1+t_2. Note that under fast fading assumption, channel states change and are not constant over t_{STD}. The relay system consists of a source (S), a relay (R), a destination (D), interferers ($I_R^{(j)}$, $j=1,2,\cdots,N_{I_R}$) at the relay, and interferers ($I_D^{(i)}$, $i=1,2,\cdots,N_{I_D}$) at the destination. We model thermal noise as circularly symmetric additive white Gaussian noise (AWGN). Each channel is affected by AWGN. The system and channel model is depicted in Fig. 1. (The source-relay channel correlation coefficient r_{SR} and the destination interferer channel correlation coefficient r_{I_D} are defined in the following sections.)

Under the above assumptions, for the first time slot t_1, i.e., in phase 1, the source transmits its data symbols. The received signal at the destination is expressed by:

$$y^{(S,D,t_1)}=\sqrt{E_S^{t_1}}g_0^{(S,D,t_1)}b_0+\sum_{i=1}^{N_{I_D}}\sqrt{E_{I_D}^{t_1}}g_i^{(I_D,D,t_1)}b_i+n^{(S,D,t_1)} \quad (1)$$

where $E_S^{t_1}$, $E_{I_D}^{t_1}$, b_0, and b_i are the power transmitted by the source over the time slot t_1, the power transmitted by each interferer over the time slot t_1, the source data symbol with unit average power, and each interferer data symbol with unit average power for $i=1,\cdots,N_{I_D}$, respectively, and N_{I_D} is the number of interferers. Furthermore, the channel propagation parameters $g_0^{(S,D,t_1)}$ and $g_i^{(I_D,D,t_1)}$, $i=1,2,\cdots,N_{I_D}$, $\sim\mathcal{CN}(0,1^2)$ are Rayleigh faded, and thermal noise $n^{(S,D)}\sim\mathcal{CN}(0,N_0)$ is complex AWGN, where the notation $\mathcal{CN}(\mu,\Sigma)$ denotes the complex circularly symmetric normal distribution with

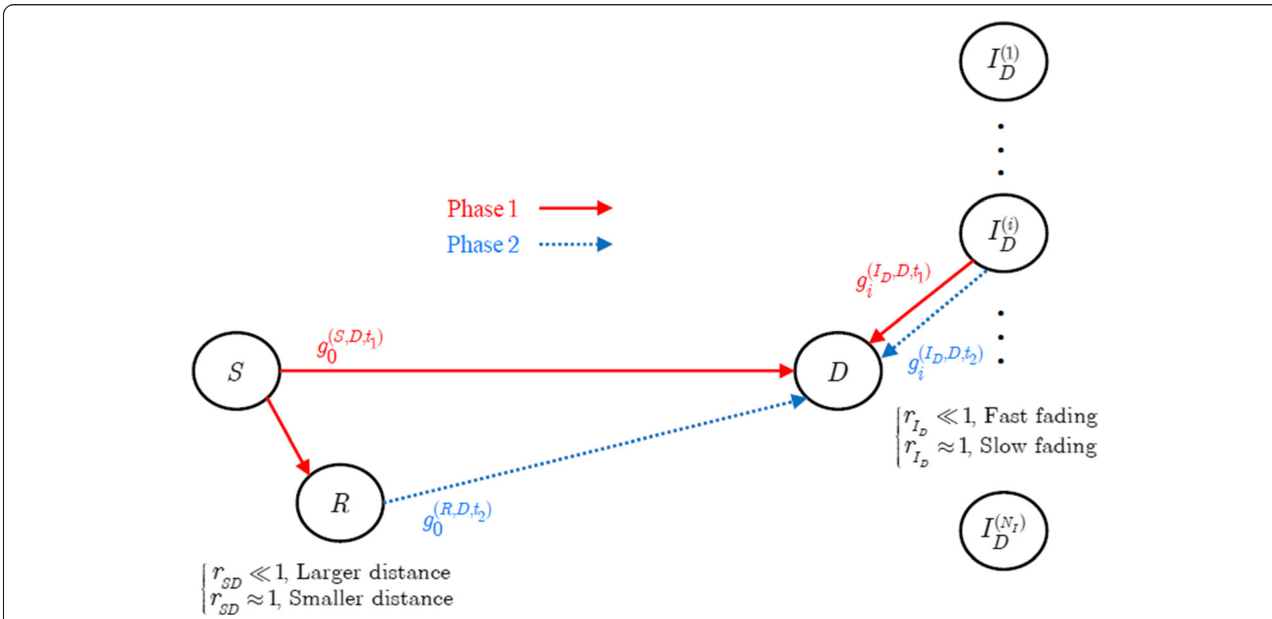

Fig. 1 System and channel model

mean μ and variance Σ. The received signal at the relay is expressed by:

$$y^{(S,R,t_1)} = \sqrt{E_S^{t_1}} g_0^{(S,R,t_1)} b_0 + \sum_{j=1}^{N_{I_R}} \sqrt{E_{I_R}^{t_1}} g_i^{(I_R,R,t_1)} r_i + n^{(S,R,t_1)} \quad (2)$$

where $E_{I_R}^{t_1}$ is each interferer power over the time slot t_1 and r_i, and $j = 1, \cdots, N_{I_R}$ are the interferer data symbols each with unit average power. The channel parameters $g_i^{(I_R,R,t_1)}$, $i = 1, 2, \cdots, N_{I_R}$, $\sim\mathcal{CN}(0, 1^2)$ are Rayleigh faded, and $n^{(S,R,t_1)} \sim \mathcal{CN}(0, N_0)$ is complex AWGN.

For the second time slot t_2, i.e., in phase 2, if the relay correctly decodes the symbol, then it forwards the symbol to the destination. In this case, the signal at the destination is expressed by:

$$y^{(R,D,t_2)} = \sqrt{E_R^{t_2}} g_0^{(R,D,t_2)} b_0 + \sum_{i=1}^{N_{I_D}} \sqrt{E_{I_D}^{t_2}} g_i^{(I_D,D,t_2)} b_i + n^{(R,D,t_2)} \quad (3)$$

where $E_R^{t_2}$ is the transmitter power and $E_{I_D}^{t_2}$ is each interferer power. The channel parameters $g_0^{(R,D,t_2)}$ and $g_i^{(I,D,t_2)}$, $i = 1, 2 \cdots, N_{I_D,}$, $\sim\mathcal{CN}(0, 1^2)$ are Rayleigh faded, and $n^{(R,D,t_2)} \sim \mathcal{CN}(0, N_0)$ is complex AWGN.

Thus, assuming $E_S = E_S^{t_1} = E_R^{t_2}$ and $E_{I_D} = E_{I_D}^{t_1} = E_{I_D}^{t_2}$, for $P_e^{(R)} = 0$, the received signal at the destination is expressed by:

$$\mathbf{y}_{\text{full-co}} = \begin{bmatrix} y^{(S,D,t_1)} \\ y^{(R,D,t_2)} \end{bmatrix} = \sqrt{E_S} \begin{bmatrix} g_0^{(S,D,t_1)} \\ g_0^{(R,D,t_2)} \end{bmatrix} b_0 + \sum_{i=1}^{N_1} \sqrt{E_{I_D}} \begin{bmatrix} g_i^{(I_D,D,t_1)} \\ g_i^{(I_D,D,t_2)} \end{bmatrix} b_i + \begin{bmatrix} n^{(S,D,t_1)} \\ n^{(R,D,t_2)} \end{bmatrix} = \sqrt{E_S} \boldsymbol{g}_0 b_0 + \sum_{i=1}^{N_1} \sqrt{E_{I_D}} \boldsymbol{g}_i b_i + \mathbf{n} \quad (4)$$

where $\mathbf{y}_{\text{full-co}} \triangleq \begin{bmatrix} y^{(S,D,t_1)} \\ y^{(R,D,t_2)} \end{bmatrix}$, $\boldsymbol{g}_0 \triangleq \begin{bmatrix} g_0^{(S,D,t_1)} \\ g_0^{(R,D,t_2)} \end{bmatrix}$, $\boldsymbol{g}_i \triangleq \begin{bmatrix} g_i^{(I_D,D,t_1)} \\ g_i^{(I_D,D,t_2)} \end{bmatrix}$, $i = 1, 2, \cdots, N_{I_D}$, and $\mathrm{n} \triangleq \begin{bmatrix} n^{(S,D,t_1)} \\ n^{(R,D,t_2)} \end{bmatrix}$. Here, we assume $\boldsymbol{g}_0$, $\boldsymbol{g}_i$, $i = 1, 2 \cdots, N_{I_D}$, and n are (2×1) zero-mean complex symmetric Gaussian random vectors. For $P_e^{(R)} = 1$, the received signal at the destination is expressed by:

$$\mathbf{y}_{\text{non-co}} = \begin{bmatrix} y^{(S,D,t_1)} \\ 0 \end{bmatrix} = \sqrt{E_S} \begin{bmatrix} g_0^{(S,D,t_1)} \\ 0 \end{bmatrix} b_0 + \sum_{i=1}^{N_1} \sqrt{E_{I_D}} \begin{bmatrix} g_i^{(I_D,D,t_1)} \\ 0 \end{bmatrix} b_i + \begin{bmatrix} n^{(S,D,t_1)} \\ 0 \end{bmatrix} \quad (5)$$

3 OP and SER derivation

We first derive the OP, $P_{\text{out}}^{(\text{full-co})}(\gamma_{Th}^{(\text{full-co})})$, and the average SER, $\text{SER}_{\text{full}-\infty}$, for $P_e^{(R)} = 0$ and later derive the OP, $P_{\text{out}}^{(\text{non-co})}(\gamma_{Th}^{(\text{non-co})})$, and the average SER, $\text{SER}_{\text{non-}\infty}$, for $P_e^{(R)} = 1$. In order to obtain the decision $x_{\text{full-co}} = \mathrm{w}_{\text{full-co}}^{\dagger} \mathrm{y}_{\text{full-co}}$, the weight vector $\mathrm{w}_{\text{full-co}}$ is expressed by $\mathrm{w}_{\text{full-co}} = \mathbf{R}^{-1}\boldsymbol{g}_0$ with the interference-plus-noise correlation matrix $\mathbf{R} = \mathrm{N}_0\mathbf{I}_2 + E_I\mathbf{G}\mathbf{G}^{\dagger}$ and $\mathrm{G} = [\boldsymbol{g}_1 \boldsymbol{g}_2 \cdots \boldsymbol{g}_{N_{I_D}}]$ [4]. The notation $\mathbf{I}_2$ is the (2×2) identity matrix, and the notation $(\bullet)^{\dagger}$ is the conjugation and transposition. The instantaneous maximum output SINR at the destination is expressed as

$$\gamma_{\text{full-co}} = E_S \boldsymbol{g}_0^{\dagger} \mathbf{R}^{-1} \boldsymbol{g}_0 \quad (6)$$

The moment-generating function (MGF) of $\gamma_{\text{full-co}}$ is given by:

$$\begin{aligned} M_{\gamma_{\text{full-co}}}(s) &= \mathbb{E}_{\gamma_{\text{full-co}}}[e^{\gamma_{\text{full-co}} s}] = \mathbb{E}_{\gamma_{\text{full-co}}}\left[e^{E_S \boldsymbol{g}_0^{\dagger} \mathbf{R}^{-1} \boldsymbol{g}_0 s}\right] \\ &= \mathbb{E}_{\gamma_{\text{full-co}}}\left[e^{E_S \boldsymbol{g}_0^{\dagger}(N_0\mathbf{I}_2 + E_I\mathbf{G}\mathbf{G}^{\dagger})^{-1}\boldsymbol{g}_0 s}\right] \\ &= \mathbb{E}_{\mathbf{G}\mathbf{G}^{\dagger}}\left[\mathbb{E}_{\gamma_{\text{full-co}}|\mathbf{G}\mathbf{G}^{\dagger}}\left[e^{E_S \boldsymbol{g}_0^{\dagger}(N_0\mathbf{I}_2 + E_I\mathbf{G}\mathbf{G}^{\dagger})^{-1}\boldsymbol{g}_0 s}\right]\right] \\ &= \mathbb{E}_{\mathbf{G}\mathbf{G}^{\dagger}}\left[\frac{1}{\left|\mathbf{I}_2 - E_S \mathbb{E}[\boldsymbol{g}_0 \boldsymbol{g}_0^{\dagger}](N_0\mathbf{I}_2 + E_I\mathbf{G}\mathbf{G}^{\dagger})^{-1} s\right|}\right] \\ &= \mathbb{E}_{\mathbf{G}\mathbf{G}^{\dagger}}\left[\frac{1}{\left|\mathbf{I}_2 - \Gamma_0 \mathbf{\Sigma}_{SR}(\mathbf{I}_2 + \Gamma_1\mathbf{G}\mathbf{G}^{\dagger})^{-1} s\right|}\right] \end{aligned} \quad (7)$$

where on the fifth line in the above equation, we use the general central quadratic form [15] of the MGF, the notation $|\mathcal{A}|$ is the determinant of a matrix $\mathcal{A}$, the source-relay channel parameter (2×2) correlation matrix $\mathbb{E}[\boldsymbol{g}_0 \boldsymbol{g}_0^{\dagger}]$ is denoted as $\mathbf{\Sigma}_{SR}$, the power ratio $\Gamma_0 \triangleq E_S/N_0$ is the SNR over each time slot, i.e., t_1 or t_2, and the power ratio $\Gamma_1 \triangleq E_{I_D}/N_0$ is the interference-to-noise ratio (INR) over each time slot, i.e., t_1 or t_2. We express the (2×2) Hermitian matrix $\mathbf{G}\mathbf{G}^{\dagger}$ as the eigenvalue decomposition [16].

$$\mathbf{G}\mathbf{G}^{\dagger} = \mathbf{U}\mathbf{B}\mathbf{U}^{\dagger}, \qquad \mathbf{B} = \begin{bmatrix} \beta_1 & 0 \\ 0 & \beta_2 \end{bmatrix} \quad (8)$$

where β_1 and β_2 with $\beta_1 \geq \beta_2$ are the non-zero ordered real eigenvalues of $\mathbf{G}\mathbf{G}^{\dagger}$ and $\mathbf{U}$ is the (2×2) unitary matrix. The MGF $M_{\gamma_{\text{full-co}}}(s)$ is given by:

$$\begin{aligned} M_{\gamma_{\text{full-co}}}(s) &= \mathbb{E}_{\mathbf{G}\mathbf{G}^{\dagger}}\left[\frac{1}{\left|\mathbf{I}_2 - \Gamma_0 \mathbf{\Sigma}_{SR}(\mathbf{I}_2 + \Gamma_1\mathbf{G}\mathbf{G}^{\dagger})^{-1} s\right|}\right] = \mathbb{E}_{\mathbf{B},\mathbf{U}}\left[\frac{1}{\left|\mathbf{I}_2 - \Gamma_0 \mathbf{\Sigma}_{SR}(\mathbf{I}_2 + \Gamma_1\mathbf{G}\mathbf{G}^{\dagger})^{-1} s\right|}\right] \\ &= \mathbb{E}_{\mathbf{B}}\left[\mathbb{E}_{\mathbf{U}|\mathbf{B}}\left[\frac{1}{\left|\mathbf{I}_2 - \Gamma_0 \mathbf{\Sigma}_{SR}(\mathbf{I}_2 + \Gamma_1\mathbf{G}\mathbf{G}^{\dagger})^{-1} s\right|}\right]\right] = \mathbb{E}_{\mathbf{B}}\left[\mathbb{E}_{\mathbf{U}|\mathbf{B}}\left[\frac{(1+\Gamma_1\beta_1)(1+\Gamma_1\beta_2)}{(1+\Gamma_1\beta_1 - \Gamma_0\sigma_1 s)(1+\Gamma_1\beta_2 - \Gamma_0\sigma_2 s) + (\Gamma_0\sigma_1 s - \Gamma_0\sigma_2 s)(\Gamma_1\beta_2 - \Gamma_1\beta_1)\sin^2\theta}\right]\right] \end{aligned} \quad (9)$$

where on the fourth line in the above equation, we use the fact that the random variable (RV) θ is uniformly distributed in the interval $[-\pi, \pi)$ [17]. The MGF $M_{\gamma_{\text{co}}}(s)$ is simplified by the integration over the RV θ as:

$$\begin{aligned} M_{\gamma_{\text{full-co}}}(s) &= \mathbb{E}_{\mathbf{B}}\left[\mathbb{E}_{\mathbf{U}|\mathbf{B}}\left[\frac{(1+\Gamma_1\beta_1)(1+\Gamma_1\beta_2)}{(1+\Gamma_1\beta_1-\Gamma_0\sigma_1 s)(1+\Gamma_1\beta_2-\Gamma_0\sigma_2 s)+(\Gamma_0\sigma_1 s-\Gamma_0\sigma_2 s)(\Gamma_1\beta_2-\Gamma_1\beta_1)\sin^2\theta}\right]\right] \\ &= \mathbb{E}_{\mathbf{B}}\left[\frac{1}{2\pi}\int_{-\pi}^{\pi}\frac{(1+\Gamma_1\beta_1)(1+\Gamma_1\beta_2)}{(1+\Gamma_1\beta_1-\Gamma_0\sigma_1 s)(1+\Gamma_1\beta_2-\Gamma_0\sigma_2 s)+(\Gamma_0\sigma_1 s-\Gamma_0\sigma_2 s)(\Gamma_1\beta_2-\Gamma_1\beta_1)\sin^2\theta}\,d\theta\right] \\ &= \mathbb{E}_{\mathbf{B}}\left[\frac{|(1+\Gamma_1\beta_1)(1+\Gamma_1\beta_2)|}{\sqrt{(1+\Gamma_1\beta_1-\Gamma_0\sigma_1 s)(1+\Gamma_1\beta_1-\Gamma_0\sigma_2 s)(1+\Gamma_1\beta_2-\Gamma_0\sigma_2 s)(1+\Gamma_1\beta_2-\Gamma_0\sigma_1 s)}}\right]. \end{aligned} \tag{10}$$

where the notation $|a|$ is the absolute value of a scalar a. The expectation over $\mathbf{B}$ is obtained using the probability density function (PDF) $f_{\mathbf{B}}(\mathbf{B})$ [18] as:

$$M_{\gamma_{\text{full-co}}}(s) = K\left|\left[\int_0^\infty \frac{x^{N_{I_D}+j-3}e^{-x/\alpha_i}|1+\Gamma_1 x|}{\sqrt{(1+\Gamma_1 x-s\Gamma_0\sigma_1)(1+\Gamma_1 x-s\Gamma_0\sigma_2)}}dx\right]_{i,j=1}^2\right| \tag{11}$$

where $[a_{i,j}]_{i,j=1}^2$ is a (2×2) matrix with elements $a_{i,j}$, i, $j = 1, 2$, and the constant K is given by:

$$K = \frac{1}{\Gamma(N_{I_D})\Gamma(N_{I_D}-1)|\mathbf{\Sigma}_D|^{N_{I_D}}\left|\left[\left(-\alpha_j^{-1}\right)^{i-1}\right]_{i,j=1}^2\right|}. \tag{12}$$

The function $\Gamma(\cdot)$ is the gamma function. The values α_1 and α_2 with $\alpha_1 \geq \alpha_2$ are the eigenvalues of the destination interferer channel parameter (2×2) correlation matrix $\mathbf{\Sigma}_{I_D} \triangleq \mathbb{E}[\boldsymbol{g}_i\boldsymbol{g}_i^\dagger]$, $i = 1, 2, \cdots, N_{I_D}$. Using the (2×2) determinant formula, the MGF $M_{\gamma_{\text{full-co}}}(s)$ is given by:

$$M_{\gamma_{\text{full-co}}}(s) = K\left(\begin{aligned}&\int_0^\infty \frac{x^{N_{I_D}-2}e^{-x/\alpha_1}|1+\Gamma_1 x|}{\sqrt{(1+\Gamma_1 x-s\Gamma_0\sigma_1)(1+\Gamma_1 x-s\Gamma_0\sigma_2)}}dx\int_0^\infty \frac{x^{N_{I_D}-1}e^{-x/\alpha_2}|1+\Gamma_1 x|}{\sqrt{(1+\Gamma_1 x-s\Gamma_0\sigma_1)(1+\Gamma_1 x-s\Gamma_0\sigma_2)}}dx \\ &-\int_0^\infty \frac{x^{N_{I_D}-1}e^{-x/\alpha_1}|1+\Gamma_1 x|}{\sqrt{(1+\Gamma_1 x-s\Gamma_0\sigma_1)(1+\Gamma_1 x-s\Gamma_0\sigma_2)}}dx\int_0^\infty \frac{x^{N_{I_D}-2}e^{-x/\alpha_2}|1+\Gamma_1 x|}{\sqrt{(1+\Gamma_1 x-s\Gamma_0\sigma_1)(1+\Gamma_1 x-s\Gamma_0\sigma_2)}}dx\end{aligned}\right) \tag{13}$$

With some algebraic manipulations, the MGF $M_{\gamma_{\text{full-co}}}(s)$ is expressed as:

$$M_{\gamma_{\text{full-co}}}(s) = K\left(\begin{aligned}&\int_0^\infty \frac{x^{N_{I_D}-2}e^{-x/\alpha_1}|1+\Gamma_1 x|}{\sqrt{(1+\Gamma_1 x-s\Gamma_0\sigma_1)(1+\Gamma_1 x-s\Gamma_0\sigma_2)}}dx\int_0^\infty x\frac{x^{N_{I_D}-2}e^{-x/\alpha_2}|1+\Gamma_1 x|}{\sqrt{(1+\Gamma_1 x-s\Gamma_0\sigma_1)(1+\Gamma_1 x-s\Gamma_0\sigma_2)}}dx \\ &-\int_0^\infty \frac{x^{N_{I_D}-2}e^{-x/\alpha_2}|1+\Gamma_1 x|}{\sqrt{(1+\Gamma_1 x-s\Gamma_0\sigma_1)(1+\Gamma_1 x-s\Gamma_0\sigma_2)}}dx\int_0^\infty x\frac{x^{N_{I_D}-2}e^{-x/\alpha_1}|1+\Gamma_1 x|}{\sqrt{(1+\Gamma_1 x-s\Gamma_0\sigma_1)(1+\Gamma_1 x-s\Gamma_0\sigma_2)}}dx\end{aligned}\right) \tag{14}$$

The MGF $M_{\gamma_{\text{full-co}}}(s)$ is further simplified as:

$$\begin{aligned} M_{\gamma_{\text{full-co}}}(s) = K\Big(&\int_0^\infty g(x;\alpha_1)dx\int_0^\infty xg(x;2-\alpha_1)dx \\ &-\int_0^\infty xg(x;\alpha_1)dx\int_0^\infty g(x;2-\alpha_1)dx\Big) \end{aligned} \tag{15}$$

where

$$g(x;\alpha) = \frac{x^{N_{I_D}-2}e^{-x/\alpha}|1+\Gamma_1 x|}{\sqrt{(1+\Gamma_1 x-s\Gamma_0\sigma_1)(1+\Gamma_1 x-s\Gamma_0\sigma_2)}} \tag{16}$$

and we use the fact that $\mathbf{\Sigma}_{I_D} = \mathbb{E}[\boldsymbol{g}_i\boldsymbol{g}_i^\dagger]$, $i = 1, 2, \cdots, N_{I_D}$, has unit diagonal elements because the channel parameters $g_i^{(I_D,D,t_1)}$ and $g_i^{(I_D,D,t_2)}$, $i = 1, 2, \cdots, N_{I_D}$, are distributed according to $\mathcal{CN}(0, 1^2)$, so that the trace of $\mathbf{\Sigma}_{I_D}$ is $\text{tr}(\mathbf{\Sigma}_{I_D}) = 2 = \alpha_1 + \alpha_2$. The result in Eq. (15) is valid for the multiple correlated CCIs with $N_{I_D} \geq 2$. For the single interferer case with $N_{I_D} = 1$, we obtain the simpler MGF $M_{\gamma_{\text{full-co}}}(s)$. Let the conditional MGF $M_{\gamma_{\text{full-co}}|\lambda_1}(s)$ be the MGF $M_{\gamma_{\text{full-co}}}(s)$ conditioned on λ_1 [19] and the RV λ_1 be the random eigenvalue of $\mathbf{R}$ (the other eigenvalue of $\mathbf{R}$ is the constant N_0). (Note that λ_1 and N_0 are the eigenvalues of $\mathbf{R} = N_0\mathbf{I}_2 + E_I\mathbf{G}\mathbf{G}^\dagger$, and β_1 and β_2 are the eigenvalues of $\mathbf{G}\mathbf{G}^\dagger$.) Then the MGF $M_{\gamma_{\text{full-co}}}(s)$ of $\gamma_{\text{full-co}}$ is derived as:

$$\begin{aligned} M_{\gamma_{\text{full-co}}}(s) &= \int_{N_0}^\infty M_{\gamma_{\text{co}}|\lambda_1}(s)f_{\lambda_1}(\lambda_1)\,d\lambda_1 \\ &= \int_{N_0}^\infty \frac{1}{\left(1-\frac{E_S}{N_0}s\right)\left(1-\frac{E_S}{\lambda_1}s\right)}f_{\lambda_1}(\lambda_1)\,d\lambda_1 \\ &= \int_{N_0}^\infty \frac{1}{\left(1-\frac{E_S}{N_0}s\right)\left(1-\frac{E_S}{\lambda_1}s\right)}\frac{1}{E_{I_D}^2}(\lambda_1-N_0)e^{-(\lambda_1-N_0)/E_{I_D}}\,d\lambda_1 \\ &= \int_{N_0}^\infty \frac{(\lambda_1-N_0)e^{-(\lambda_1-N_0)/E_{I_D}}}{E_{I_D}^2(1-\Gamma_0 s)\left(1-\frac{E_S}{\lambda_1}s\right)}d\lambda_1 \end{aligned} \tag{17}$$

In the derivation of Eq. (17), we use $\lambda_1 \sim \chi_1^2$, where the notation χ_1^2 denotes a complex chi-squared distribution with one complex degree of freedom; $f_{\lambda_1}(\lambda_1) = (\lambda_1 - N_0) e^{-(\lambda_1 - N_0)/E_{I_D}}/E_{I_D}^2$, $\lambda_1 \geq N_0$. Now we have derived $M_{\gamma_{\text{full-co}}}(s)$ for all $N_{I_D} \geq 1$. From $M_{\gamma_{\text{full-co}}}(s)$, we obtain the characteristic function (CF) $\phi_{\gamma_{\text{full-co}}}(t) = M_{\gamma_{\text{full-co}}}(\sqrt{-1}\,t)$. The PDF $f_{\gamma_{\text{full-co}}}(\gamma_{\text{full-co}})$ of $\gamma_{\text{full-co}}$ is obtained from $\phi_{\gamma_{\text{full-co}}}(t)$ by the Fourier transform, which is easily calculated using the fast Fourier transform (FFT). Then, $\text{SER}_{\text{full-co}}$ with the coherent binary phase shift keying (BPSK) is calculated as [19]:

$$\text{SER}_{\text{full-co}} = \int_0^\infty \text{Q}\left(\sqrt{2\gamma_{\text{full-co}}}\right) f_{\text{full-co}}(\gamma_{\text{full-co}}) d\gamma_{\text{full-co}} \tag{18}$$

where $Q(x) \triangleq 1/\sqrt{2\pi}\int_x^\infty e^{-y^2/2}dy$. For $N_{I_D} = 1$, $\text{SER}_{\text{full-co}}$ can be alternatively calculated by Eq. (10.20) in [19]. For a given threshold $\gamma_{Th}^{(\text{full-co})}$, the OP $P_{\text{out}}^{(\text{full-co})}(\gamma_{Th}^{(\text{full-co})})$ is defined and is calculated as:

$$\begin{aligned} P_{\text{out}}^{(\text{full-co})}\left(\gamma_{Th}^{(\text{full-co})}\right) &\triangleq p\left(\gamma_{\text{full-co}} \leq \gamma_{Th}^{(\text{full-co})}\right) \\ &= \int_0^{\gamma_{Th}^{(\text{full-co})}} f_{\text{full-co}}(\gamma_{\text{full-co}}) d\gamma_{\text{full-co}} \end{aligned} \tag{19}$$

Next, we derive $P_{\text{out}}^{(\text{non-co})}(\gamma_{Th}^{(\text{non-co})})$ and $\text{SER}_{\text{non-co}}$ for $P_e^{(R)} = 1$. To obtain the decision $x_{\text{non-co}} = \mathrm{w}_{\text{non-co}}^\dagger y^{(S,D,t_1)}$, the weight $\mathrm{w}_{\text{non-co}}$ is expressed by $\mathrm{w}_{\text{non-co}} = \mathrm{R}_{\text{non-co}}^{-1} g_0^{(S,D,t_1)}$ with $\mathrm{R}_{\text{non-co}} = N_0 + E_I \langle g_i^{(I_D,D,t_1)} \rangle_{i=1}^{N_{I_D}} (\langle g_i^{(I_D,D,t_1)} \rangle_{i=1}^{N_{I_D}})^\dagger$. The notation $\langle a_i \rangle_{i=1}^N$ is a $(1 \times N)$ matrix with elements a_i, $i = 1, 2, \cdots, N_{I_D}$. The maximum instantaneous output SINR at the destination can be expressed as:

$$\begin{aligned} \gamma_{\text{non-co}} &= E_S g_0^{(S,D,t_1)\dagger} \mathrm{R}_{\text{non-co}}^{-1} g_0^{(S,D,t_1)} \\ &= E_S \left| g_0^{(S,D,t_1)} \right|^2 \mathrm{R}_{\text{non-co}}^{-1} \\ &= \frac{E_S \left| g_0^{(S,D,t_1)} \right|^2}{N_0 + E_{I_D} \left\langle g_i^{(I_D,D,t_1)} \right\rangle_{i=1}^{N_{I_D}} \left(\left\langle g_i^{(I_D,D,t_1)} \right\rangle_{i=1}^{N_{I_D}} \right)^\dagger} \\ &= \frac{X}{1/\Gamma_0 + \Gamma_1/\Gamma_0 W}. \end{aligned} \tag{20}$$

The RV $X \triangleq |g_0^{(S,D,t_1)}|^2$ is exponentially distributed with the PDF $f_X(x) \triangleq e^{-x}$, $x \geq 0$. The chi-squared-distributed RV $W \triangleq \langle g_i^{(I_D,D,t_1)} \rangle_{i=1}^{N_{I_D}} (\langle g_i^{(I_D,D,t_1)} \rangle_{i=1}^{N_{I_D}})^\dagger \sim \chi_{N_{I_D}}^2$ has the PDF $f_W(w) \triangleq 1/(N_{I_D}-1)! \cdot w^{N_{I_D}-1} e^{-w}$, $w \geq 0$, with N_{I_D} complex degree of freedom. The RV $Y \triangleq 1/\Gamma_0 + \Gamma_1/\Gamma_0 W$ has the PDF

$$f_Y(y) \triangleq \frac{\Gamma_0}{\Gamma_1} ((\Gamma_0 y - 1)/\Gamma_1)^{N_{I_D}-1} e^{-((\Gamma_0 y - 1)/\Gamma_1)} / (N_{I_D}-1)! \tag{21}$$

with $y \geq 1/\Gamma_0$. Then, the RV $\gamma_{\text{non-co}} = X/Y$ is ratio distributed, and the $f_{\gamma_{\text{non-co}}}(\gamma_{\text{non-co}})$ is derived as:

$$\begin{aligned} f_{\gamma_{\text{non-co}}}(\gamma_{\text{non-co}}) &= \int_{1/\Gamma_0}^\infty y f_X(\gamma_{\text{non-co}} y) f_Y(y) dy \\ &= \int_{1/\Gamma_0}^\infty y e^{-\gamma_{\text{non-co}} y} \frac{\Gamma_0}{\Gamma_1} \frac{1}{(N_{I_D}-1)!} \left((\Gamma_0 y - 1)/\Gamma_1\right)^{N_{I_D}-1} e^{-((\Gamma_0 y-1)/\Gamma_1)} dy. \end{aligned} \tag{22}$$

Similarly as in the $P_e^{(R)} = 0$ case, with $f_{\gamma_{\text{non-co}}}(\gamma_{\text{non-co}})$, we calculate $P_{\text{out}}^{(\text{non-co})}(\gamma_{Th}^{(\text{non-co})})$ and $\text{SER}_{\text{non-co}}$ for $P_e^{(R)} = 1$.

Now based on the total probability theorem, finally, we obtain a closed-form expression for the OP $P_{\text{out}}(\gamma_{Th})$ at the destination as:

$$\begin{aligned} P_{\text{out}}(\gamma_{Th}) = P_{\text{out}}^{(\text{full-co})}(\gamma_{Th})\left(1 - P_e^{(R)}\right) \\ + P_{\text{out}}^{(\text{non-co})}(\gamma_{Th}) P_e^{(R)} \end{aligned} \tag{23}$$

and the average SER at the destination is derived as:

$$\begin{aligned} \text{SER} &= \int_0^\infty \text{Q}\left(\sqrt{2\gamma}\right) f_\gamma(\gamma) d\gamma \\ &= \text{SER}_{\text{full-co}}\left(1 - P_e^{(R)}\right) + \text{SER}_{\text{non-co}} P_e^{(R)}. \end{aligned} \tag{24}$$

With these exact analytical expressions, we can investigate the effects of the distance between the source and the relay, i.e., $\mathbf{\Sigma}_{SR}$ and fast/slow-fading interference at the destination, i.e., $\mathbf{\Sigma}_{I_D}$.

4 Results and discussion

We assume that the signals have the exponential correlation [20]. Thus, with the source-relay channel correlation coefficient $r_{SR} \in [0, 1)$:

$$\mathbf{\Sigma}_{SR} = \begin{bmatrix} 1 & r_{SR} \\ r_{SR} & 1 \end{bmatrix} \tag{25}$$

and with the destination interferer channel correlation coefficient $r_{I_D} \in [0, 1)$:

$$\mathbf{\Sigma}_{I_D} = \begin{bmatrix} 1 & r_{I_D} \\ r_{I_D} & 1 \end{bmatrix}. \tag{26}$$

We define the total SNR as $\Gamma_0^{\text{total}} \triangleq E_S^{\text{total}}/N_0$ and the total INR $\Gamma_1^{\text{total}} \triangleq E_{I_D}^{\text{total}}/N_0$, where $E_S^{\text{total}} = E_S^{t_1} + E_R^{t_2} = 2E_S$ and $E_{I_D}^{total} = (2N_{I_D})E_{I_D}$, where the factor 2 represents two time slots. The correlation coefficients are explained as follows: the smaller the r_{SR} is, the larger the distance between the source node and the relay node is. On the other hand, the smaller the r_{I_D} is, the more independent, i.e., the less correlated, the two channel coefficients

$g_i^{(I_D,D,t_1)}$ and $g_i^{(I_D,D,t_2)}$ are, for a given i among $i = 1, 2, \cdots, N_{I_D}$. This means that the maximum Doppler spread is larger so that the coherence time is smaller, i.e., fast fading [21]. Thinking in the opposite direction, i.e., slow fading, is also true.

First, we investigate the effect of the probability of symbol errors $P_e^{(R)}$ at the relay on the OP $P_{out}(\gamma_{Th})$ at the destination. We assume that with $N_{I_D} = 2$, $\Gamma_1^{total} = (2N_{I_D})\Gamma_1 = 3\ \text{dB} + 3\ \text{dB} + 4\ \text{dB} = 10\ \text{dB}$ is fixed. We also assume that there are almost uncorrelated users ($r_{SR} = 0.01$) and slow-fading multiple correlated CCIs ($r_{I_D} = 0.99$), which are the assumptions of the previous researches, i.e., independent users and flat-fading interference over t_{STD}. In Fig. 2, the OP performance is shown for various $P_e^{(R)}$ values. We observe in Fig. 2 that the OP performance with $P_e^{(R)} \leq 0.001$ reaches that with the full cooperative case $P_e^{(R)} = 0$. Since this paper focuses on the performance analysis for the source-relay distance and fast/slow fading CCIs, from now on, we set $P_e^{(R)} = 0.001$. In Fig. 2, we also show the analytical and simulation results, which are in good agreement, so that the following analyses are based on the analytical expressions.

Next, we analyze the OP $P_{out}(\gamma_{Th})$ at the destination for various r_{SR} and r_{I_D} values. In Fig. 3, for the fixed $r_{I_D} = 0.99$, i.e., slow fading CCIs (which is the previous research assumption), the OP $P_{out}(\gamma_{Th})$ at the destination is shown for various r_{SR}. We observe in Fig. 3 that as the correlation between the source and the relay becomes larger, the OP performance degrades severely and cooperative diversity decreases. It is shown in Fig. 4 that for the fixed $r_{I_D} = 0.01$, i.e., fast fading CCIs (which is considered in this paper), the OP $P_{out}(\gamma_{Th})$ at the destination is shown for various r_{SR}. The results in Fig. 4 are similar with those in Fig. 3, but the patterns of the OP performance degradation are different. In order to investigate the difference, we plot the combination of Fig. 3 and Fig. 4 in Fig. 5. It is investigated in Fig. 5 that the performance of the large distance between the source and the relay is better than that of the small distance, regardless of interference fading speed at the destination. We define the impact of fast-fading CCIs on the performance as the SNR Γ_0 loss in decibel compared with slow-fading CCIs. We observe in Fig. 5 that given the distance between the source and the relay ($r_{SR} = 0.01$ or 0.99), the performance of fast-fading interference at the destination is basically worse than that of slow fading, except in the low SNR regime for the distance being small ($r_{SR} = 0.99$). The exception is explained as follows: since the small distance results in lost diversity, in the

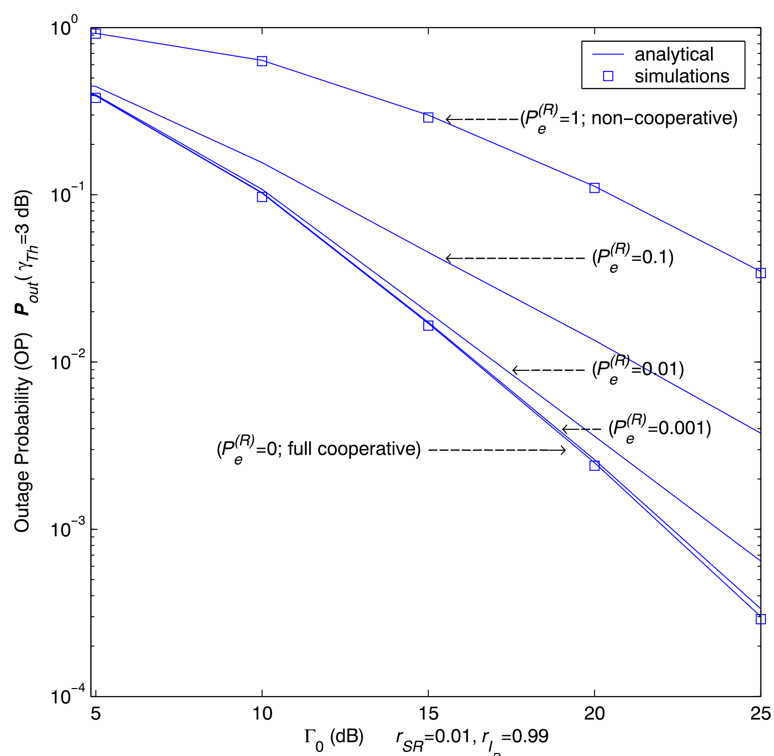

Fig. 2 The outage probability, $P_{out}(\gamma_{Th})$, for various $P_e^{(R)}$ values in the presence of almost uncorrelated users ($r_{SR} = 0.01$), and slow-fading multiple correlated CCIs ($r_{I_D} = 0.99$), and thermal noise, with $N_{I_D} = 2$, $\Gamma_1^{total} = 10\ \text{dB}$, and $\gamma_{Th} = 3\ \text{dB}$

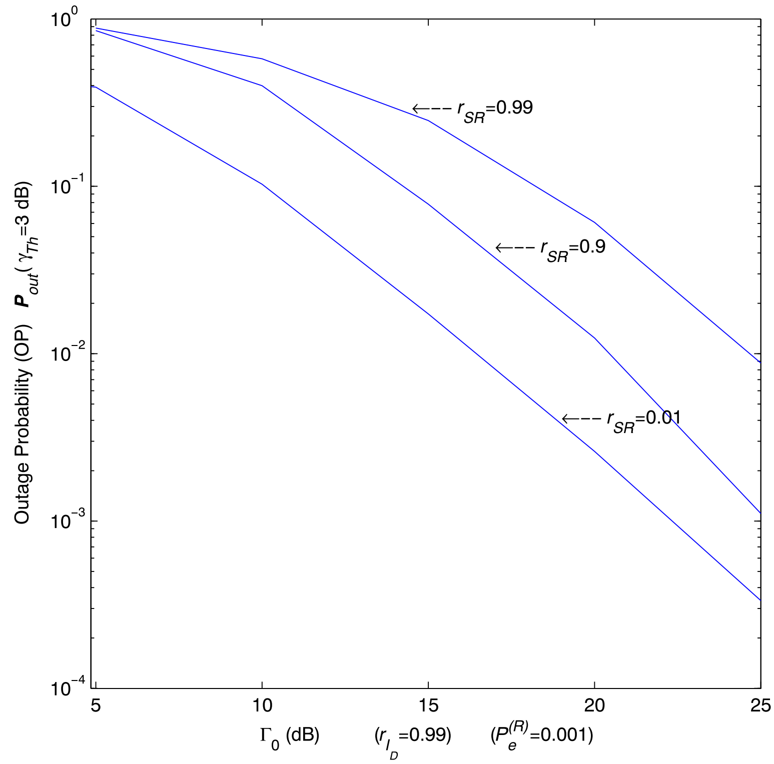

Fig. 3 The outage probability, $P_{\text{out}}(\gamma_{Th})$, for various r_{SR} values in the presence of slow-fading multiple correlated CCIs ($r_{I_D} = 0.99$), and thermal noise, with $N_{I_D} = 2$, $\Gamma_1^{\text{total}} = 10$ dB, $\gamma_{Th} = 3$ dB, and $P_e^{(R)} = 0.001$

low SNR regime, the weak power correlated signals transmitted by the source and the relay are more vulnerable to highly correlated CCIs ($r_{I_D} = 0.99$) than almost uncorrelated CCIs ($r_{I_D} = 0.01$). (Note that if we ignored thermal noise, we could not observe the exception in the low SNR regimes.) In other words, besides the exception, DF-relaying OC more easily cancels out almost flat-fading CCIs ($r_{I_D} = 0.99$) than fast-fading CCIs ($r_{I_D} = 0.01$). Slow-fading CCIs represent highly correlated CCIs, and fast-fading CCIs represent weakly correlated CCIs. In result, the source-relay distance (r_{SR}) is generally a more dominating factor than the fading CCIs (r_{I_D}) at the destination, when the performance of OC for these systems is analyzed. It is also shown in Fig. 5 that the previous research assumption ($r_{I_D} = 0.01$, $r_{I_D} = 0.99$) is the most optimistic. In order to further investigate the effects of various r_{SR} and r_{I_D} on the average SER for the DF-relaying OC system, Fig. 6 shows the SER of the BPSK modulation versus the SNR Γ_0. It is clearly shown in Fig. 6 that there is the gap between the most optimistic case ($r_{I_D} = 0.01$, $r_{I_D} = 0.99$) and the most conservative case ($r_{SR} = 0.99$, $r_{I_D} = 0.01$). The gap is about 8 dB in the SNR Γ_0 at the SER of 10^{-4}. We also observe that the results in Fig. 6 are consistent with those in Fig. 5.

Now, we discuss the difference between OC and non-OC. In order to achieve cooperative diversity, OC maximizes the SINR, reduces CCIs' power, and increases diversity. On the other hand, non-OC, such as MRC, maximizes only the SNR so that a smaller output SINR is produced and the performance is degraded severely in the presence of CCIs.

5 Conclusion

In this paper, we investigated the effects of the source-relay distance and fast/slow-fading CCIs on the performance of the DF-relaying OC system. Conditioned on the probability of symbol errors at the relay, we first developed the MGF of the instantaneous maximum output SINR. Using the total probability theorem, we then derived closed-form expressions for the OP and the average SER at the destination. With these analytical expressions, it was shown that the performance of the large distance between the source and the relay is better than that of the small distance, regardless of interference fading speed at the destination. Furthermore, we also showed that given the distance, the performance of slow-fading interference is basically better than that of fast fading, except in the low SNR regime for the

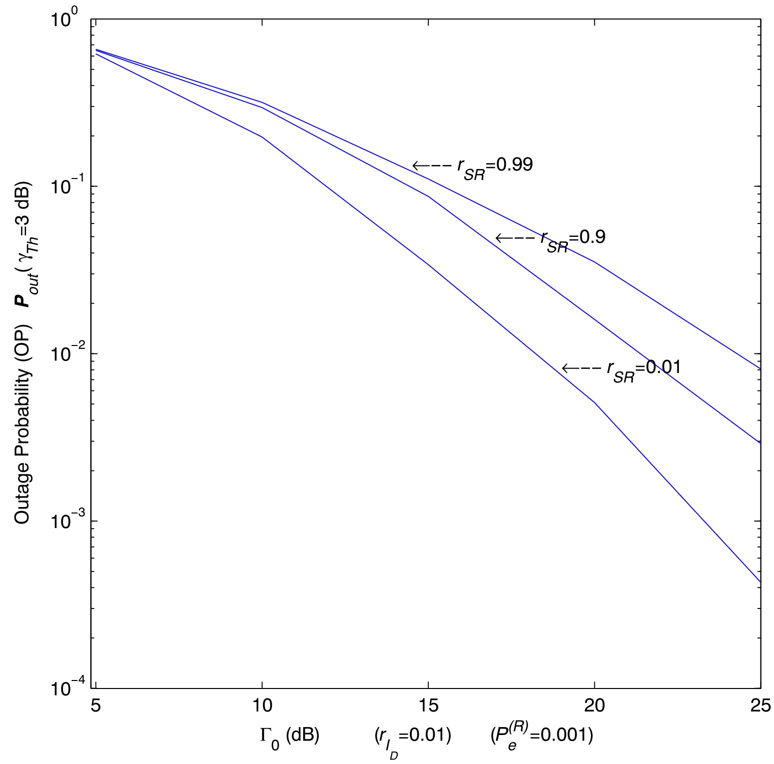

Fig. 4 The outage probability, $P_{\text{out}}(\gamma_{Th})$, for various r_{SR} values in the presence of fast-fading multiple correlated CCIs ($r_{I_D} = 0.01$), and thermal noise, with $N_{I_D} = 2$, $\Gamma_1^{\text{total}} = 10$ dB, $\gamma_{Th} = 3$ dB, and $P_e^{(R)} = 0.001$

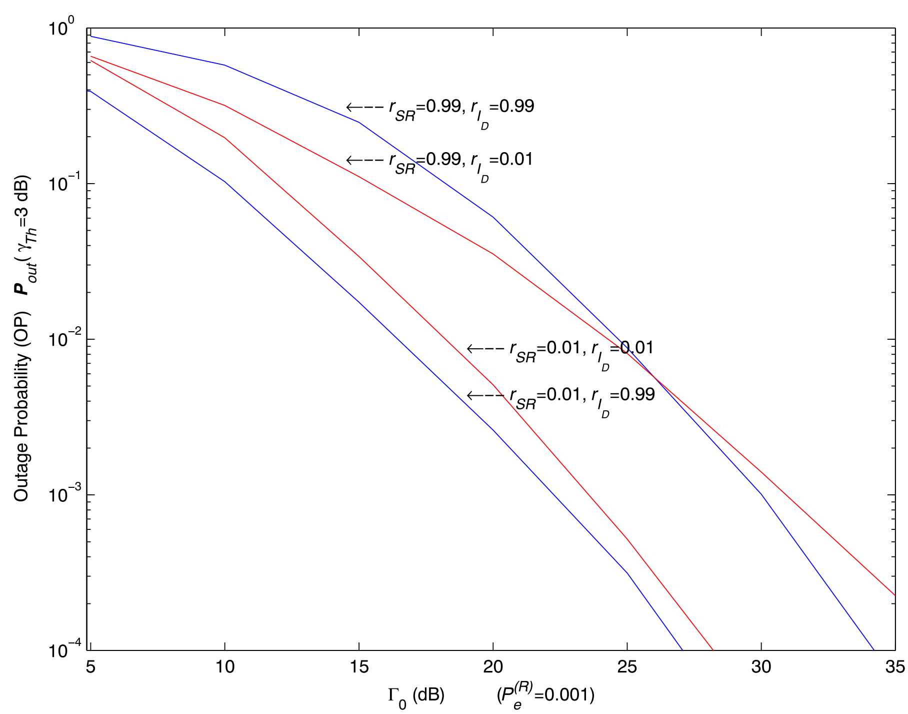

Fig. 5 The outage probability, $P_{out}(\gamma_{Th})$, for various r_{SR} and r_{I_D} values in the presence of fading multiple correlated CCIs, and thermal noise, with $N_{I_D} = 2$, $\Gamma_1^{total} = 10$ dB, $\gamma_{Th} = 3$ dB, and $P_e^{(R)} = 0.001$

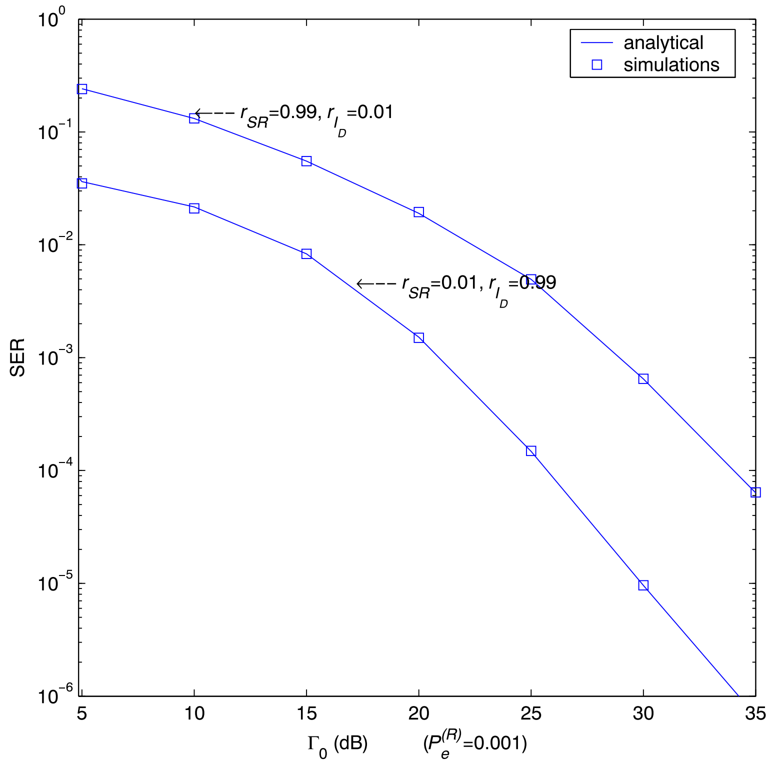

Fig. 6 SER for various r_{SR} and r_{I_D} values in the presence of fading multiple correlated CCIs, and thermal noise, with $N_{I_D} = 2$, $\Gamma_1^{total} = 10$ dB, $\gamma_{Th} = 3$ dB, and $P_e^{(R)} = 0.001$

distance being small. In result, the source-relay distance is generally a more dominating factor than the fading CCIs. Finally, we presented the average SER performance, which showed the gap between the most optimistic case and the most conservative case.

Abbreviations
AF: Amplify-and-forward; AWGN: Additive white Gaussian noise; BPSK: Binary phase shift keying; CCIs: Co-channel interferers; DF: Decode-and-forward; DSTC: Distributed space-time coding; FFT: Fast Fourier transform; MGF: Moment-generating function; MIMO: Multiple-input multiple-output; MRC: Maximal-ratio combining; OC: Optimum combining; OP: Outage probability; OR: Opportunistic relaying; PDF: Probability density function; SER: Symbol error rate; SINR: Signal-to-interference-plus-noise ratio; SNR: Signal-to-noise-ratio; STD: Single transmission duration; TDD: Time division duplex

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Authors' contributions
KC analyzed the outage probability (OP) and the average symbol error rate (SER) of decode-and-forward (DF) relaying. KC derived closed-form expressions for the OP and the average SER with optimum combining (OC) considering fast fading multiple correlated CCIs, the correlated source-relay, and thermal noise. KC read and approved the final manuscript.

Competing interests
The authors declare that they have no competing interests.

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