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Performance analysis of NOMA-based mobile edge computing with imperfect CSI

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Abstract

In this paper, a non-orthogonal multiple access (NOMA)-based mobile edge computing (MEC) system is proposed, where paired users (the mth user and the nth user) offload tasks to the MEC server under imperfect channel state information (ipCSI) condition. To evaluate the effect on performance under ipCSI condition in NOMA-MEC system, this study derives new exact and asymptotic expressions of the offloading outage probability for two users under ipCSI and perfect channel state information (pCSI) conditions. On the basis of the theoretical derivation, the diversity orders of the nth user under ipCSI and pCSI conditions are *zero* and n, respectively, while those for the distant user are all *zeros*. In addition, we also investigate the system throughput and energy efficiency of NOMA-MEC in delay-limited transmission. Numerical results show that (1) the offloading outage behaviors of NOMA-MEC are better than those of time division multiple address (TDMA), (2) the offloading outage performance of paired users under ipCSI is worse than that under pCSI, and (3) the NOMA-MEC system achieves higher throughput and energy efficiency than does the TDMA-MEC system.

Keywords: Mobile edge computing, Non-orthogonal multiple access, Imperfect channel state information, Offloading outage probability

1 Introduction

Future wireless communication will face the challenges of network requirements, such as high-density users, high speed, and ultra-reliable and low latency communications (URLLC). The latency of URLLC is as low as 1 ms, and the reliability is as high as 99.999%, which can ensure mission-critical communications. In the future 6th generation networks, URLLC will provide powerful guarantee for emerging applications that have strict requirements on latency and reliability [1]. Traditional orthogonal multiple access (OMA) divides physical resources into orthogonal resource blocks in time/frequency/code domains, which limits the number of accessing users. To meet the high requirements of communication networks, highly efficient multiple access technologies are required to improve the system capacity and spectrum utilization under limited spectrum resources. Non-orthogonal multiple access (NOMA)[1] has become a popular research topic in

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[1]The superposed signals for multiple users can be mapped to single physical resource element or multiple elements. Based on these, NOMA can be divided into single carrier NOMA (SC-NOMA) and multi-carrier NOMA [2]. In this paper, we focus on single carrier NOMA and use NOMA to represent SC-NOMA.

communication systems [3, 4] and has been discussed in the 6th generation mobile communication system for large-scale access. NOMA adopts superposition coding to realize the transmission of multiple signals in the same time/frequency/code domains; at the receiver, the information required is extracted by the successive interference cancelation (SIC) scheme to improve the utilization of spectrum resources.

The uplink and downlink communications in NOMA system have been studied by many treatises. For downlink communications, the authors in [5] conducted a detailed study on the performance of NOMA with the perfect channel state information (pCSI) condition by analyzing the outage probability. Furthermore, the ergodic rate of NOMA with pCSI was optimized in [6], through which the authors obtained a closed-form globally optimal solution. Considering the low latency in combined with high reliability is a challenging task, the performance of NOMA for URLLC was analyzed in [7], and the results showed that NOMA has reduced the physical layer latency and improved the reliability of supporting the time-critical applications obviously. To draw more conclusions, the authors in [2] proposed a unified framework of NOMA and derived the exact expressions of outage probability as well as the system throughput in delay-limited mode. To study paired users, the authors in [8] developed the optimal and suboptimal schemes. For uplink communications, the performance with pCSI of the NOMA system was characterized by outage probability and achievable sum rate, which is concerned with the power back-off step [9]. To further analyze the performance of uplink NOMA, the authors provide the expressions of effective capacity in [10] and studied the performance of two users under quality of service (QoS) delay constraints. In [11], the authors discussed external and internal eavesdropping scenarios and derived expressions of secrecy outage probability and throughput with pCSI. The impact of pCSI on NOMA systems was investigated carefully in the above researches, while the imperfect channel state information (ipCSI) should be discussed, which is suitable to the practical NOMA scenarios. Based on these, the authors derived a closed-form approximation of the outage probability for users and the high signal-to-noise ratio (SNR) expressions in downlink NOMA with ipCSI in [12]. To prevent the interception of information between base station and receiver, the authors of [13] studied the secrecy outage probability and average secrecy capacity under ipCSI condition. The influence of ipCSI on the security performance of system could be seen when their is the existence of multiple eavesdropping channels.

URLLC is an important application scenario of the 5th generation networks, with new features of high reliability, low latency, and high availability. In [14], the authors proposed a cross-layer optimization framework to ensure that the wireless access network has ultra-high reliability and ultra-low delay. In order to focus on local communication scenarios, the authors discussed the delay and packet loss of URLLC, as well as the network availability that supports the QoS of users [15]. As cloud capabilities tend toward the network edge, mobile edge computing (MEC) has become a new trend in mitigating the terminal computing abilities. MEC is considered as an effective solution for URLLC, which is capable of reducing the delay of computationally intensive tasks by invoking great computing cells within a short distance [16]. Given that its computing ability is near that of mobile devices and ultra-low latency is its greatest advantage, MEC is widely regarded as the key technology of the next Internet generation [17]. Intelligent applications and networks are deeply integrated based on MEC and Internet of Things to achieve the corresponding requirements of users. The MEC server can be distributed at the edge of networks

and perform computation-intensive and delay-critical tasks from mobile devices [18, 19]. In view of the ultra-low latency characteristic of MEC, the authors evaluated its actual delay and throughput performance in cellular networks and found that MEC reduces the delay of downlink communication [20]. To improve the efficiency of offloading tasks, the authors of [21] proposed an energy-saving offloading strategy that the computational offloading problem of MEC is transformed into a system cost minimization considering the completion time and energy.

Combining MEC and NOMA is an effective method to utilize computing capacities and improve energy efficiency. The authors in [22] discussed the effect of NOMA on delay and energy efficiency of offloading tasks in MEC, where both uplink and downlink NOMA are taken into consideration. In [23], the authors proposed a NOMA-based computational offloading scheme to reduce the task execution time for users. A distributed algorithm was also proposed to optimize users' transmission time in [24]. The results showed that NOMA-based MEC has more advantages in delay than traditional frequency division multiple access-assisted MEC. In order to reduce the energy consumption of offloading tasks, the authors developed an optimization framework based on NOMA to optimize communication resource allocation and transmission power [25]. In [26], the authors studied energy consumption, where NOMA-based MEC offload scheduling can reduce the system energy consumption compared with OMA. Moreover, the authors in [27] minimized the offloading delay and analyzed the convergence speed.

This treatise focuses on the combination of MEC and NOMA technology under the ipCSI condition in actual scenarios. Based on [28], the NOMA-assisted MEC network is considered where all users transmit the tasks to the MEC server through uplink transmission. The researches in [29, 30] illustrate the advantages of combining NOMA with MEC. However, NOMA-MEC is still in its infancy under ipCSI condition. In addition, NOMA has better outage performance than OMA. There are many OMA schemes this paper focuses on comparing NOMA and time division multiple address (TDMA). These are the motivations of this paper. The contributions of this work are summarized as follows:

1) We study the outage performance under two conditions in NOMA-MEC. The NOMA framework studied is applied to the MEC scenario, and the closed-form expressions of the offloading outage probability for paired users (the mth user and the nth user) under pCSI and ipCSI conditions through setting the target transmission rate v_n and v_m are derived. In order to get more conclusions, we also obtain the expressions of the asymptotic offloading outage probability at high SNRs and provide the diversity orders of user. Additionally, we obtain the diversity orders of the both two users are *zeros* under ipCSI condition. The diversity order of the nth user under pCSI is n, while that of the mth user is *zero*.

2) We evaluate theoretical results of system performance by simulation, which shows that the offloading outage probability under ipCSI is larger than that under pCSI. We further analyze the impact of changing channel estimation errors on system performance. With the increasing of channel estimation errors, the offloading outage behaviors for users are becoming more worse. In addition, when the offloading tasks are reduced or the offloading time is increased, the offloading outage probability for the users will be decreased.

3)   We study the throughput and energy efficiency for two users in delay-limited transmission mode of NOMA-MEC system and derive the corresponding expressions. We find that NOMA-MEC has higher system throughput and energy efficiency than TDMA-MEC. In addition, the throughput and energy efficiency under ipCSI are lower than those under pCSI, while the system throughput and energy efficiency will decrease as the channel estimation errors increase.

2   System model

Considering NOMA-based MEC communication scenario, M users offload tasks to a single MEC server illustrated in Fig. 1. Assume that each node is a single antenna device and operates in half duplex mode. All communication links in network are subject to Rayleigh fading and disturbed by additive white Gaussian noise (AWGN). $\widehat{h}_i \sim \mathcal{CN}\left(0, \widehat{\Omega}_i\right)$ denote the channel coefficients of links between the user and the MEC server, where $i \in \{1, 2, \cdots, M\}$. $\widehat{\Omega}_i = d_i^{-\alpha}$, where d_i represents the distance between the user and the server, and α is the path loss exponent. Due to channel estimation errors, it is difficult to obtain the pCSI of channels for NOMA-MEC system in practical communication scenarios. To evaluate the influence of ipCSI in NOMA-MEC system, the channel coefficient is modeled as $\widehat{h}_i = h_i + \varpi e_i$, where $\varpi \in (0, 1)$, h_i represents the channel gain under the pCSI condition. $\varpi = 0$ denotes that the system has ability to obtain the pCSI, and $\varpi = 1$ denotes that the system cannot obtain the pCSI and will suffer from the channel estimation error $e_i \sim \mathcal{CN}\left(0, \sigma_{e_i}^2\right)$. Assuming that h_i is statistically independent of e_i, $\gamma_i = \sigma_{e_i}^2 \Big/ \widehat{\Omega}_i$ represents the relative channel estimation error and has $\sigma_{e_i}^2 = \gamma_i d_i^{-\alpha}$.

In this paper, two users are selected from M users, i.e., the nth and mth users for non-orthogonal transmission, where a pair of users simultaneously offload tasks to the MEC server. The channel gains between users and the MEC server are sorted as $|\widehat{h}_m|^2 \le |\widehat{h}_n|^2$, where the nth and mth users have similar channel estimation errors (i.e., $|h_m|^2 \le |h_n|^2$). On the basis of the principle of NOMA, the received expression of offloading tasks at MEC server is given by:

Fig. 1 NOMA-MEC system model
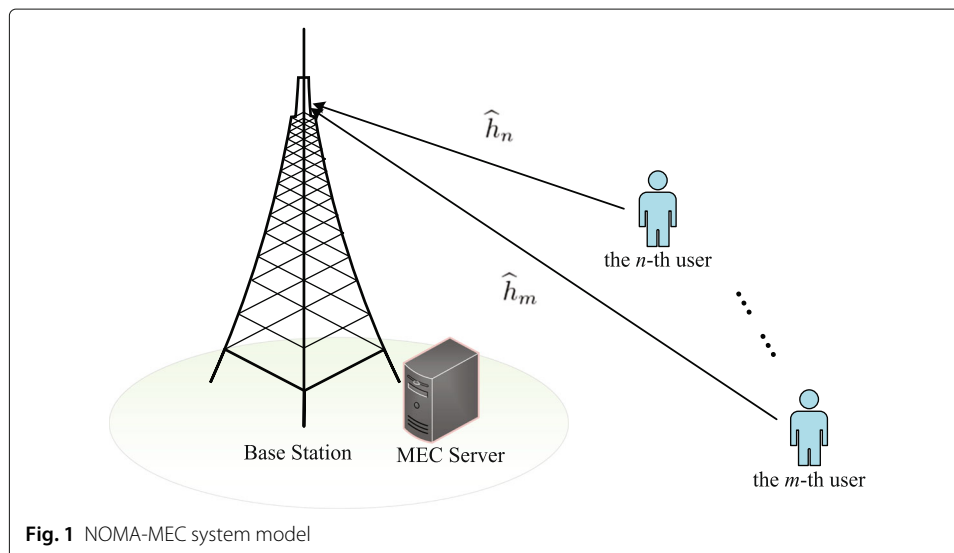
$$y_{\text{MEC}} = (h_m + \varpi e_m)\sqrt{P_m}x_m + (h_n + \varpi e_n)\sqrt{P_n}x_n + n_{\text{MEC}}, \tag{1}$$

where x_j denotes the offloading task of the jth user, $j \in \{m, n\}$. $n_{\text{MEC}} \sim \mathcal{CN}\left(0, \sigma_{\text{MEC}}^2\right)$ represents the AWGN at the MEC server. The transmission power of the jth user is denoted as P_j, i.e., $P_j = a_j P$ and P is the total power of the two users. To guarantee better fairness between the users, assume that $a_m > a_n$ with $a_m + a_n = 1$. Note that optimal power allocation coefficients [9] can further improve the performance in this system; however, it is beyond the scope of this paper. The mth user has an exclusive time slot in TDMA when offloading the tasks, while the nth user will also enter the slot to complete its offloading tasks in NOMA-MEC. The nth user does not need additional time slot, which is an advantage of NOMA-MEC compared with TDMA-MEC, thus reducing the offloading delay of the system.

According to the principle of uplink NOMA, the MEC server first decodes the task x_m with large power allocation coefficient by treating the task x_n with small power allocation coefficient as noise and then subtracts this component. After carrying out SIC procedure, the task x_n with small power coefficient can be detected. Hence, the signal-to-interference-plus-noise ratios (SINRs) for the MEC server to decode x_m and x_n are given by:

$$\Gamma_m = \frac{a_m \rho |h_m|^2}{a_n \rho |h_n|^2 + \varpi\left(\theta_n + \theta_m\right) + 1}, \tag{2}$$

and

$$\Gamma_n = \frac{a_n \rho |h_n|^2}{\varpi \theta_n + 1}, \tag{3}$$

respectively, where $\rho \stackrel{\Delta}{=} \frac{P}{\sigma_{\text{MEC}}^2}$ is the transmit SNR, $\theta_j = \sigma_{e_j}^2 a_j \rho$.

Assuming that the ith user has N_i-bits tasks and offloads these to the MEC server, where the time required to execute the tasks is $T_{\text{MEC}} = \frac{2NC}{f_{\text{MEC}}}$, N represents the total tasks; C is the number of central processing unit (CPU) cycles demanding for computing one input bit, and f_{MEC} is the CPU frequency at the MEC server.

3 Performance evaluation

In this section, the offloading outage performance for the paired users under ipCSI/pCSI conditions in the uplink NOMA-MEC system is analyzed. Firstly, we derive the exact closed-form expressions of the offloading outage probability and the asymptotic offloading outage probability in the high SNR region for the users. Then, so as to further study the outage performance in NOMA-MEC, we obtain the diversity orders and evaluate the performance indicators of users such as system throughput and energy efficiency.

3.1 Outage probability

Considering that target rates for two users are determined by their QoS, the offloading outage probability becomes a prime indicator to evaluate the system performance. The offloading outage means that the user cannot complete offloading to the MEC server within the specified time. Hence, in uplink NOMA-MEC scenario, the offloading outage performance for the users under ipCSI/pCSI conditions is analyzed in detail.

When the nth user completes N_n-bits offloading tasks within T_1, the target transmission rate v_n of the nth user is denoted by $v_n = \frac{N_n}{T_1}$. Once the actual transmission rate R_n is less

than v_n, the nth user has an outage behavior, and then, the offloading outage probability of the nth user under ipCSI is given by:

$$
\begin{aligned}
\mathrm{P}_{\mathrm{ipCSI}}^n &= \mathrm{Pr}\,(R_n < v_n) \\
&= \mathrm{Pr}\left[\log\left(1 + \Gamma_n\right) < v_n\right].
\end{aligned}
\tag{4}
$$

Theorem 1 *The exact closed-form expression for offloading outage probability of the nth user under ipCSI condition in NOMA-MEC system is given by:*

$$
\mathrm{P}_{\mathrm{ipCSI}}^n = \frac{M!}{(M-n)!\,(n-1)!} \sum_{i=0}^{M-n} \binom{M-n}{i} \frac{(-1)^i}{n+i}\left(1 - e^{-\frac{\kappa(\varpi\theta_n + 1)}{a_n\rho\Omega_n}}\right)^{n+i},
\tag{5}
$$

where $\kappa = 2^{v_n} - 1$, $\varpi = 1$.

Proof The SINR of the nth user Γ_n can be obtained by (3), and (4) is rewritten as:

$$
\mathrm{P}_{\mathrm{ipCSI}}^n = \mathrm{Pr}\left[\log\left(1 + \frac{a_n\rho|h_n|^2}{\varpi\theta_n + 1}\right) < v_n\right].
\tag{6}
$$

Additionally, the offloading outage probability of the nth user is given by:

$$
\mathrm{P}_{\mathrm{ipCSI}}^n = \mathrm{Pr}\left(|h_n|^2 < \frac{\kappa\left(\varpi\theta_n + 1\right)}{a_n\rho}\right).
\tag{7}
$$

$|h_m|^2$ and $|h_n|^2$ are independent random variables that obey variances Ω_m and Ω_n, respectively, and with the aid of order statistics [31] and binomial theorem, the PDF of the nth user's sorted channel gain $|h_n|^2$ can be expressed as:

$$
f_{|h_n|^2}(y) = \frac{M!}{(M-n)!\,(n-1)!} \frac{1}{\Omega_n} \sum_{k=0}^{n-1} \binom{n-1}{k}(-1)^k e^{-\frac{y(M-n+k+1)}{\Omega_n}}.
\tag{8}
$$

Substituting (8) into (7) and performing some simple operations, we can attain (5), which completes the proof. $\square$

Corollary 1 *Substituting $\varpi = 0$ into (5), the exact closed-form expression for offloading outage probability of the nth user under pCSI condition is given by:*

$$
\mathrm{P}_{\mathrm{pCSI}}^n = \frac{M!}{(M-n)!\,(n-1)!} \sum_{i=0}^{M-n} \binom{M-n}{i} \frac{(-1)^i}{n+i}\left(1 - e^{-\frac{\kappa}{a_n\rho\Omega_n}}\right)^{n+i}.
\tag{9}
$$

The offloading outage event of the mth user can be expressed that the MEC server first decodes the task x_m by treating the task x_n. At this moment, an offloading outage event occurs when the actual transmission rate $R_m = \log\left(1 + \Gamma_m\right)$ is lower than the transmission rate v_m $\left(v_m = \frac{N_m}{T_1}\right)$. Hence, the offloading outage probability of the mth user with ipCSI can be expressed as:

$$
\mathrm{P}_{\mathrm{ipCSI}}^m = \mathrm{Pr}\,(R_m < v_m).
\tag{10}
$$

The offloading outage probability of the mth user in the NOMA-MEC system will be given below.

Theorem 2 *The exact closed-form expression for offloading outage probability of the mth user under ipCSI condition in NOMA-MEC system is given by:*

$$
\begin{aligned}
\mathrm{P}^m_{\mathrm{ipCSI}} = {} & \frac{M!}{(M-m)!\,(m-1)!} \frac{M!}{(M-n)!\,(n-1)!} \sum_{i=0}^{M-m} \sum_{s=0}^{m+i} \sum_{r=0}^{M-n} \sum_{k=0}^{n+r-1} \\
& \times \binom{M-m}{i} \binom{m+i}{s} \binom{M-n}{r} \binom{n+r-1}{k} \frac{(-1)^{i+s+r+k}}{m+i} \\
& \times \frac{a_m \Omega_m}{(1+k)\, a_m \Omega_m + s\chi a_n \Omega_n} e^{-\frac{s\chi[\varpi(\theta_n+\theta_m)+1]}{a_m \rho \Omega_m}},
\end{aligned}
\tag{11}
$$

where $\chi = 2^{v_m} - 1$, $\varpi = 1$.

Proof See the Appendix. □

Corollary 2 *By substituting* $\varpi = 0$ *into* (11), *the exact closed-form expression for offloading outage probability of the mth user under pCSI condition is given by:*

$$
\begin{aligned}
\mathrm{P}^m_{\mathrm{pCSI}} = {} & \frac{M!}{(M-m)!\,(m-1)!} \frac{M!}{(M-n)!\,(n-1)!} \sum_{i=0}^{M-m} \sum_{s=0}^{m+i} \sum_{r=0}^{M-n} \sum_{k=0}^{n+r-1} \\
& \times \binom{M-m}{i} \binom{m+i}{s} \binom{M-n}{r} \binom{n+r-1}{k} \frac{(-1)^{i+s+r+k}}{m+i} \\
& \times \frac{a_m \Omega_m}{(1+k)\, a_m \Omega_m + s\chi a_n \Omega_n} e^{-\frac{s\chi}{a_m \rho \Omega_m}}.
\end{aligned}
\tag{12}
$$

3.2 Diversity order

In this subsection, we obtain the diversity orders of users under the different channel state conditions, which is defined as follows:

$$
\mu = -\lim_{\rho \to \infty} \frac{\log\left[\mathrm{P}^\infty(\rho)\right]}{\log \rho},
\tag{13}
$$

where $\mathrm{P}^\infty(\rho)$ represents the offloading outage probability at high SNR of the users.

Corollary 3 *When* $\rho \to \infty$ *is substituted into* (5), *with* $\rho \to \infty$ $(x \to 0)$, $1 - e^{-x} \sim x$, *and the nth user's asymptotic offloading outage probability in the high SNR region under the ipCSI condition is given by:*

$$
\mathrm{P}^{n,\infty}_{\mathrm{ipCSI}} = \frac{M!}{(M-n)!\,n!} \left(\frac{\kappa(\theta_n+1)}{a_n \rho \Omega_n}\right)^n.
\tag{14}
$$

Remark 1 *Substituting* (14) *into* (13), *the diversity order for the nth user under ipCSI condition* $\mu^n_{\mathrm{ipCSI}} = 0$ *can be obtained .*

Corollary 4 *Substituting* $\rho \to \infty$ *into* (9), *the nth user's asymptotic offloading outage probability in the high SNR region under the pCSI condition is given by:*

$$
\mathrm{P}^{n,\infty}_{\mathrm{pCSI}} = \frac{M!}{(M-n)!\,n!} \left(\frac{\kappa}{a_n \rho \Omega_n}\right)^n.
\tag{15}
$$

Remark 2 *Substituting* (15) *into* (13), *the diversity order for the nth user under pCSI condition* $\mu^n_{\mathrm{pCSI}} = n$ *can be obtained.*

Corollary 5 *Substituting $\rho \to \infty$ into (11), the mth user's asymptotic offloading outage probability in the high SNR region under the ipCSI condition is given by:*

$$
\begin{aligned}
\mathrm{P}^{m,\infty}_{\mathrm{ipCSI}} =& \frac{M!}{(M-m)!\,(m-1)!} \frac{M!}{(M-n)!\,(n-1)!} \sum_{i=0}^{M-m}\sum_{s=0}^{m+i}\sum_{r=0}^{M-n}\sum_{k=0}^{n+r-1} \\
& \times \binom{M-m}{i}\binom{m+i}{s}\binom{M-n}{r}\binom{n+r-1}{k}\frac{(-1)^{i+s+r+k}}{m+i} \\
& \times \frac{a_m\Omega_m}{(1+k)\,a_m\Omega_m + s\chi a_n\Omega_n}\left(1 - \frac{s\chi\,(\theta_n+\theta_m+1)}{a_m\rho\Omega_m}\right).
\end{aligned}
\tag{16}
$$

Proof When $\rho \to \infty$ ($x \to 0$), we have $e^{-\frac{s\chi(\theta_n+\theta_m+1)}{a_m\rho\Omega_m}} \sim 1 - \frac{s\chi(\theta_n+\theta_m+1)}{a_m\rho\Omega_m}$. By substituting it into (11), (16) can be determined. The proof is completed. □

Remark 3 *Substituting (16) into (13), the diversity order for the mth user under ipCSI condition $\mu^m_{\mathrm{ipCSI}} = 0$ can be obtained.*

Corollary 6 *Substituting $\rho \to \infty$ into (12), the mth user's asymptotic offloading outage probability in the high SNR region under the pCSI condition is given by:*

$$
\begin{aligned}
\mathrm{P}^{m,\infty}_{\mathrm{pCSI}} =& \frac{M!}{(M-m)!\,(m-1)!} \frac{M!}{(M-n)!\,(n-1)!} \sum_{i=0}^{M-m}\sum_{s=0}^{m+i}\sum_{r=0}^{M-n}\sum_{k=0}^{n+r-1} \\
& \times \binom{M-m}{i}\binom{m+i}{s}\binom{M-n}{r}\binom{n+r-1}{k}\frac{(-1)^{i+s+r+k}}{m+i} \\
& \times \frac{a_m\Omega_m}{(1+k)\,a_m\Omega_m + s\chi a_n\Omega_n}\left(1 - \frac{s\chi}{a_m\rho\Omega_m}\right).
\end{aligned}
\tag{17}
$$

Remark 4 *Substituting (17) into (13), the diversity order for the mth user under pCSI condition $\mu^m_{\mathrm{pCSI}} = 0$ can be obtained.*

3.3 Throughput analysis

In this subsection, the system throughput of NOMA-MEC in the delay-limited transmission mode is discussed. The paired users offload tasks to the MEC server at constant rates of v_m and v_n, respectively.

Under the condition of channel estimation error, the throughput for the users in NOMA-MEC system can be expressed as:

$$
R_{\mathrm{ipCSI}} = \left(1 - \mathrm{P}^n_{\mathrm{ipCSI}}\right)v_n + \left(1 - \mathrm{P}^m_{\mathrm{ipCSI}}\right)v_m,
\tag{18}
$$

where $\mathrm{P}^n_{\mathrm{ipCSI}}$ and $\mathrm{P}^m_{\mathrm{ipCSI}}$ have been derived in (5) and (11), respectively.

In the absence of channel estimation error, the throughput for the users in NOMA-MEC system can be expressed as:

$$
R_{\mathrm{pCSI}} = \left(1 - \mathrm{P}^n_{\mathrm{pCSI}}\right)v_n + \left(1 - \mathrm{P}^m_{\mathrm{pCSI}}\right)v_m,
\tag{19}
$$

where $\mathrm{P}^n_{\mathrm{pCSI}}$ and $\mathrm{P}^m_{\mathrm{pCSI}}$ have been derived in (9) and (12), respectively.

3.4 Energy efficiency

In this subsection, the energy efficiency in NOMA-MEC system is analyzed based on the system throughput analysis above. Energy efficiency [32] is defined as:

$$\eta = \frac{\text{Total data rate}}{\text{Total energy consumption}}. \tag{20}$$

In this system, the total data rate is expressed as the corresponding system throughput, and the total energy consumption can be expressed as the sum of two users' transmitted power. According to the results derived above, the system energy efficiency under the ipCSI and pCSI conditions is expressed as follows:

$$\eta_{\text{ipCSI}} = \frac{R_{\text{ipCSI}}}{TP} \tag{21}$$

and

$$\eta_{\text{pCSI}} = \frac{R_{\text{pCSI}}}{TP} \tag{22}$$

respectively, where T represents the transmission time of the entire offloading process and η_{ipCSI} and η_{pCSI} are the energy efficiency of the system with or without channel estimation errors respectively in the delay-limited transmission mode.
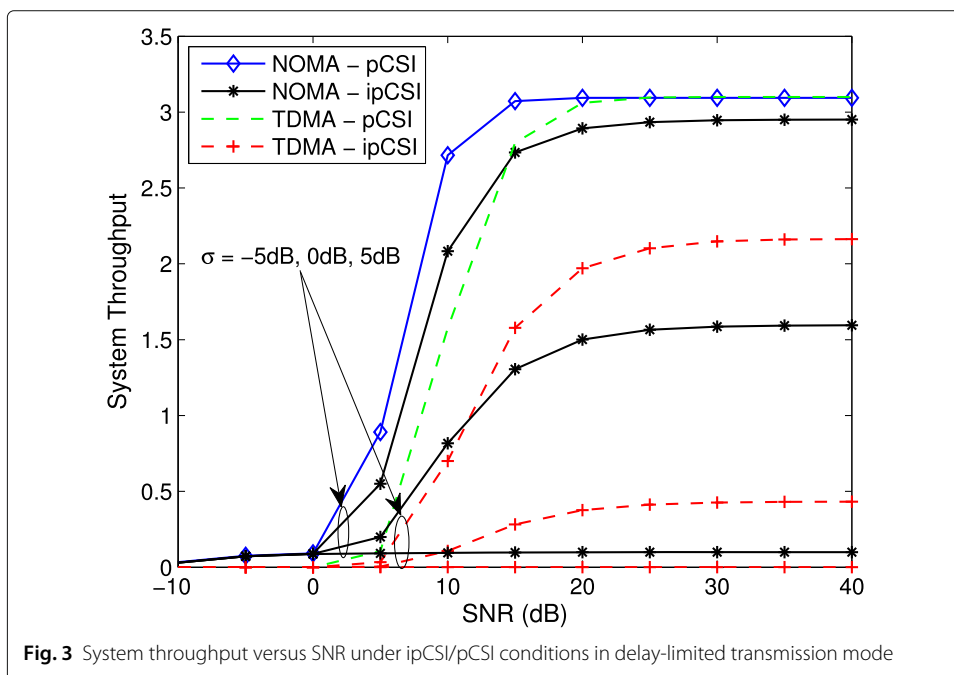
4 Results and discussion

In this section, the numerical results are given to verify the above theoretical expressions derived. The performance under the ipCSI and pCSI conditions in NOMA-MEC system is further evaluated. Assume that the distance from the MEC server to the nth user is d_n=0.3 m, while the distance from the mth user is d_m=0.7 m. The path loss exponent is set to α=2, and the power allocation factors are a_n=0.2 and a_m=0.8. We assume that the target transmission rates of this system are set to v_n=3 bit/s and v_m=0.1 bit/s, respectively. Compared with the performance of the traditional OMA, the entire communication process of TDMA is completed in two time slots. In other words, the mth and nth users occupy one time slot each in the system.
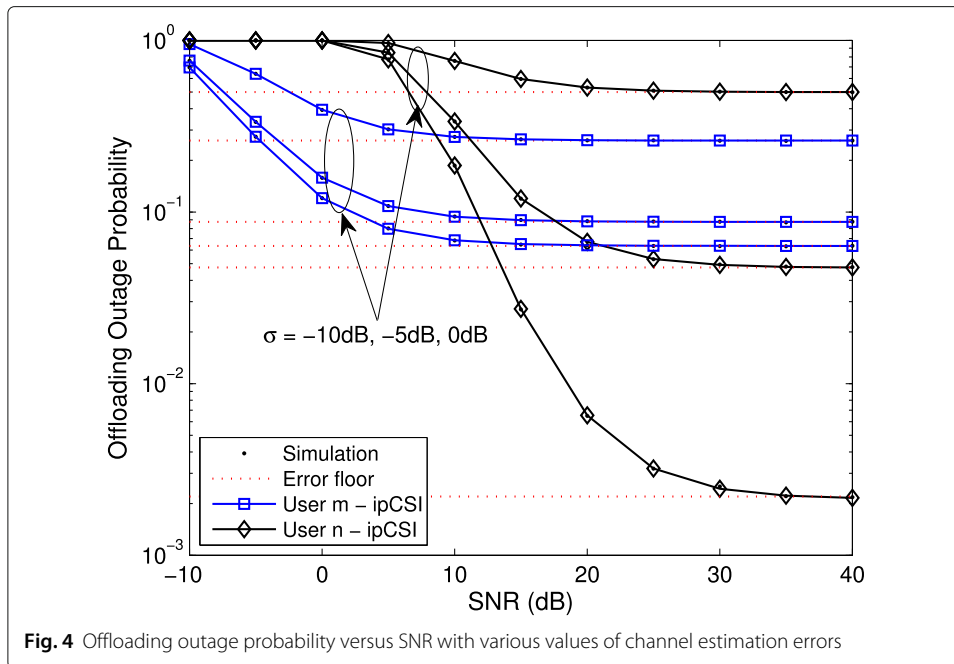
Figure 2 depicts the offloading outage probability for the two users versus the transmit SNR while the channel estimation errors are $\sigma_{e_n}^2 = -10$dB and $\sigma_{e_m}^2 = 0$ dB. The exact theoretical curves for the offloading outage probability of the two users under the ipCSI/pCSI conditions are plotted according to (5), (9) and (11), (12), respectively. It is clear that the exact curves clearly match the simulation curves. The offloading outage probability of the mth user is lower than the nth user's probability at low SNR, and the opposite is true at the high SNR. Error floors exit with the users under the ipCSI condition because of the interference of channel estimation errors during transmission. Meanwhile, the offloading performance of the nth user is higher than that in TDMA-MEC under the same conditions. Hence, the existence of channel estimation errors must be considered in the actual NOMA-MEC scenarios.

As shown in Fig. 3, we present the system throughput versus the SNR under ipCSI/pCSI conditions in delay-limited transmission mode, and the channel estimation errors are $\sigma_{e_n}^2 = 0$ dB and $\sigma_{e_m}^2 = -10$ dB. The solid curves are the throughput in the NOMA-MEC system with or without channel estimation error, in which obtained according to (18) and (19). The dashed curve represents the throughput in the TDMA-MEC system with or without channel estimation error. It is observed that with increasing the $\sigma_{e_i}^2$, the system throughput of TDMA-MEC with ipCSI is becoming much smaller. This is due to the fact

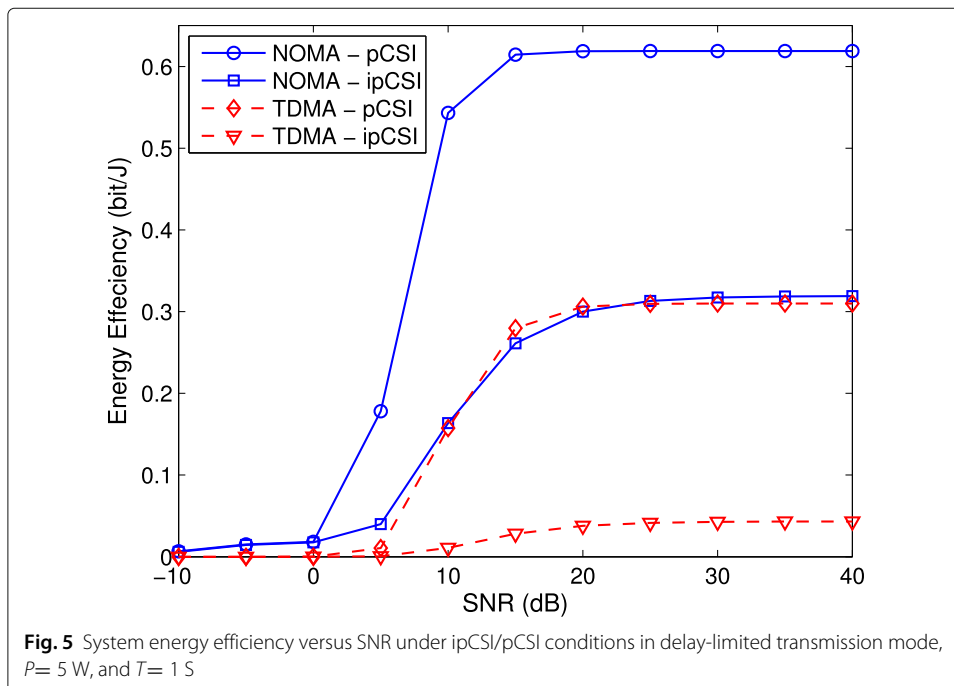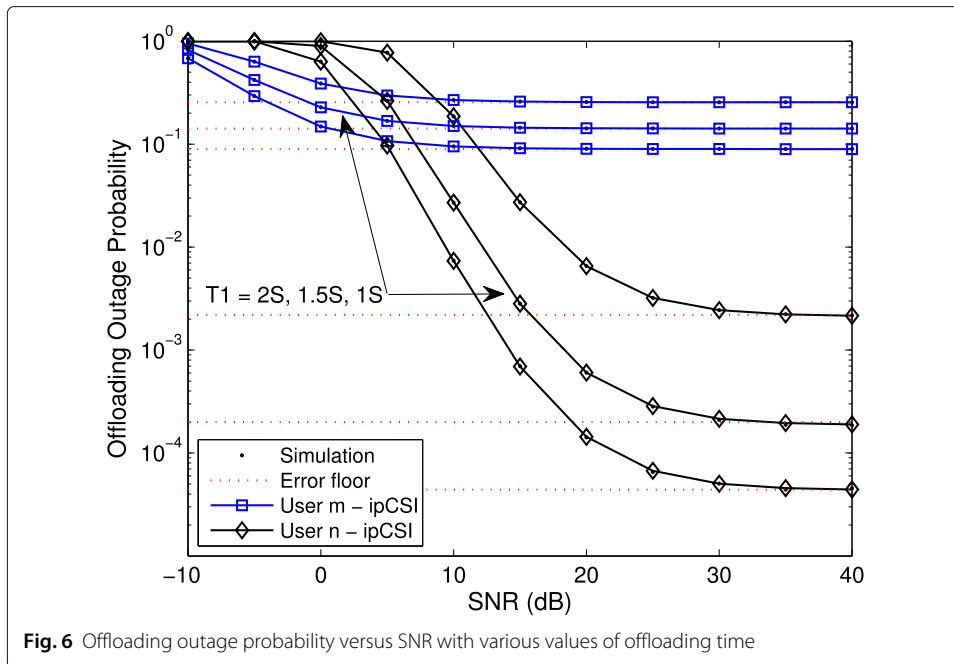**Fig. 2** Offloading outage probability for two users versus SNR
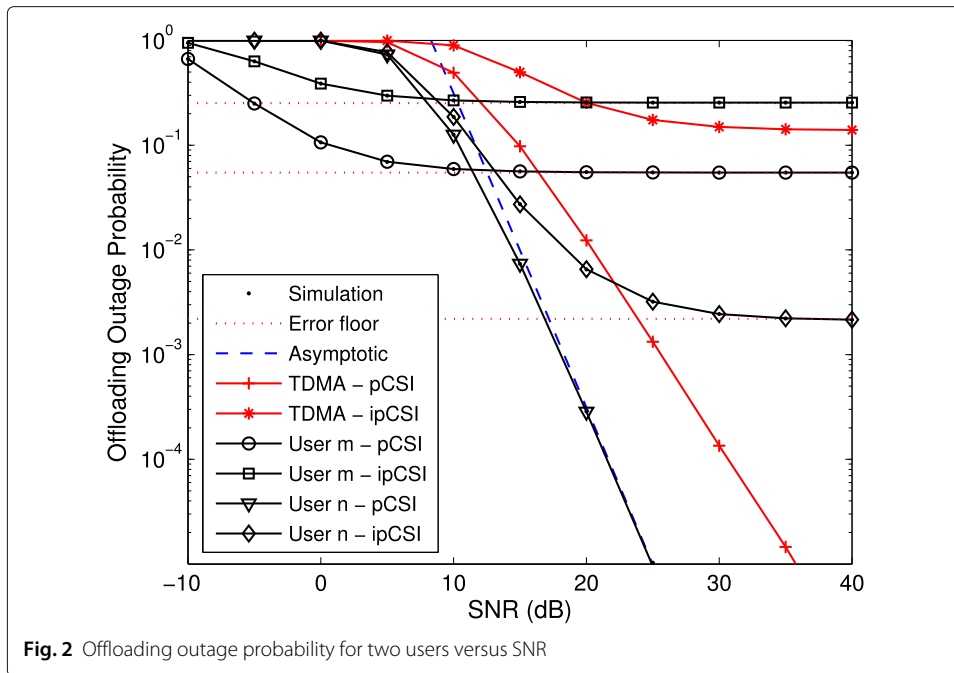
that the channel estimate error $\sigma_{e_i}^2$ leads to the worse offloading outage probability. Additionally, we can observe that channel estimation errors affect the performance index of this system, because the offloading outage probability for the users under pCSI is lower than that under ipCSI. The results show that with the $\sigma_{e_n}^2$ value increases, the offloading outage probability of the users increases, but the system throughput at the high SNR region decreases.

Fig. 3 System throughput versus SNR under ipCSI/pCSI conditions in delay-limited transmission mode

Fig. 4 Offloading outage probability versus SNR with various values of channel estimation errors
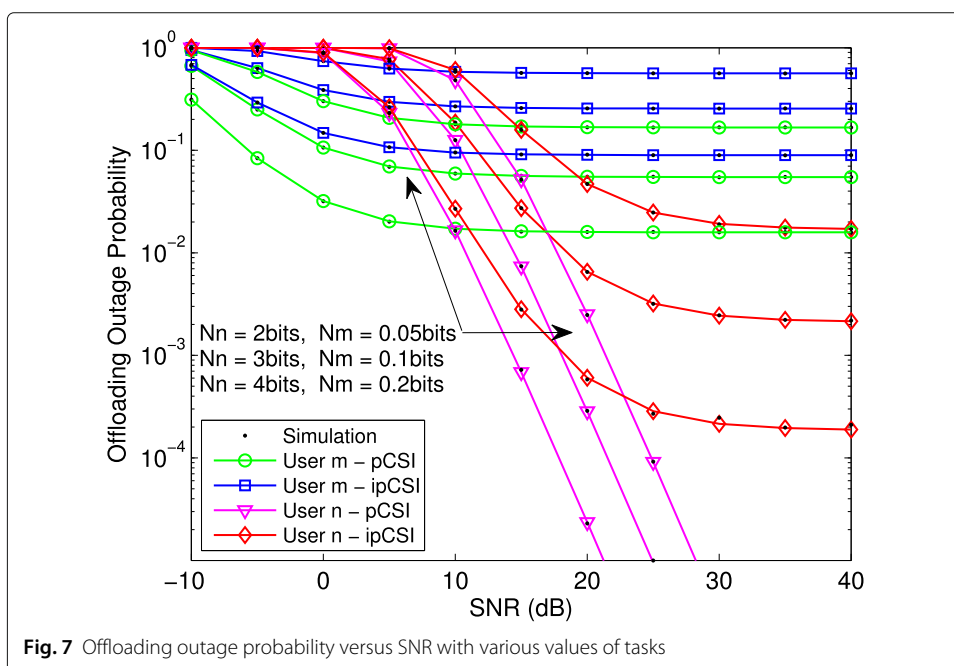
In Fig. 4, the offloading outage probability for the two users with channel estimation errors from $\sigma_{e_n}^2 = \sigma_{e_m}^2 = 0$ dB to $\sigma_{e_n}^2 = \sigma_{e_m}^2 = -10$ dB is shown. We can observe that error floors exist under the ipCSI condition, which verify the conclusions in Remark 1 and Remark 3. The offloading outage probability gradually increases with the increase of the channel estimation error values. We can also see that the impact on the nth user is more obvious than the mth user because of the interference of the nth user and the channel

Fig. 5 System energy efficiency versus SNR under ipCSI/pCSI conditions in delay-limited transmission mode, $P = 5$ W, and $T = 1$ S

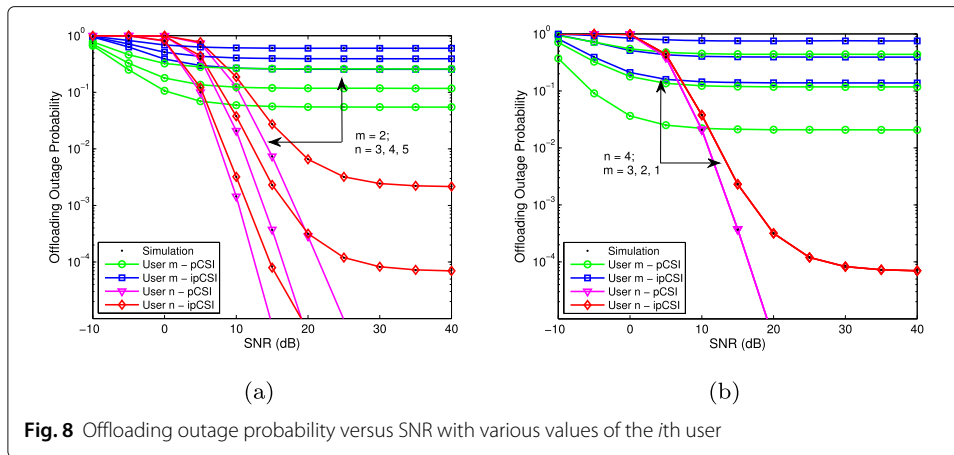**Fig. 6** Offloading outage probability versus SNR with various values of offloading time

estimation error. By contrast, the nth user is only interfered by the channel estimation error.

In Fig. 5, the system energy efficiency versus the SNR for the two users under ipCSI/pCSI conditions in delay-limited transmission mode is shown. The solid curves represent the energy efficiency for the NOMA-MEC system which are obtained from (18), (21) and (19), (22) with the throughput. The energy efficiency for the NOMA-MEC system is much higher than that of TDMA-MEC. At high SNR, the energy efficiency of

Fig. 7 Offloading outage probability versus SNR with various values of tasks

Fig. 8 Offloading outage probability versus SNR with various values of the *i*th user

the NOMA-MEC system with channel estimation error is higher than that of the TDMA-MEC system without channel estimation error, because NOMA-ipCSI can achieve greater throughput than TDMA-pCSI in such transmission mode.

In Fig. 6, the offloading outage probability for the two users versus offloading times from 1 S to 2 S is shown. It is clear that when users are allowed less time to offload, the offloading outage probabilities will be increased. This is due to the smaller the offloading time is, the higher the target transmission rate of the users, and the greater the offloading outage probability is. Therefore, the offloading time must be considered in actual NOMA-MEC systems.

In Fig. 7, we present the offloading outage probability for the two users versus various values of tasks. At $N_n = 4$ bits, $N_m = 0.2$ bits; $N_n = 3$ bits, $N_m = 0.1$ bits; and $N_n = 2$ bits, $N_m = 0.05$ bits, we can observe that with offloading tasks of both the *n*th and *m*th users increase simultaneously, the offloading outage probability also increases gradually. This is because that with the amount of tasks increases, the requirements for system performance are becoming higher. Hence, it is also necessary to consider the offloading tasks in NOMA-MEC.

Figure 8 plots the offloading outage probabilities for the two users versus various values of the *i*th user. In Fig. 8a, $m = 2$ and the values of n are 3, 4, and 5, while in Fig. 8b, $n = 4$ and the values of m are 1, 2, and 3. We can observe that when the user is closer to the MEC server, the outage probability is becoming smaller. This is consistent with the fact that MEC is closer to the mobile devices.

5 Conclusions

We have investigated the offloading performance of uplink NOMA-based MEC with ipCSI/pCSI. The exact and asymptotic expressions of offloading outage probability for the paired users were derived in detail. The analytical results have shown that the offloading probability of NOMA-MEC with pCSI is superior to TDMA-MEC. As a result of channel estimation errors, the offloading behaviors of NOMA-MEC with ipCSI are worse than that of pCSI. When the channel estimation errors increase, the offloading outage probability of NOMA-MEC is becoming larger. Finally, the throughput and energy efficiency of NOMA-MEC have been investigated with ipCSI/pCSI. In addition, the impact

on the outage behaviors for users when the offloading time or tasks change has also been discussed.

Appendix: Proof of Theorem 2

By substituting (2) into (10), the offloading outage probability P_{ipCSI}^m can be given by:

$$P_{ipCSI}^m = \Pr \left[\log \left(1 + \frac{a_m \rho |h_m|^2}{a_n \rho |h_n|^2 + \varpi (\theta_n + \theta_m) + 1} \right) < v_m \right]$$

(23)

Furthermore, the above equation can be calculated as:

$$P_{ipCSI}^m = \Pr \left(|h_m|^2 < \frac{\chi \left[a_n \rho |h_n|^2 + \varpi (\theta_n + \theta_m) + 1 \right]}{a_m \rho} \right)$$

$$= \int_0^\infty f_{|h_n|^2} (y) dy \int_0^{\frac{\chi \left[a_n \rho |h_n|^2 + \varpi (\theta_n + \theta_m) + 1 \right]}{a_m \rho}} f_{|h_m|^2} (x) dx.$$

(24)

With some arithmetic operations, the above expression can be given by:

$$P_{ipCSI}^m = \int_0^\infty f_{|h_n|^2} (y) \left[F_{|h_m|^2} \left(\frac{\chi \left[a_n \rho y + \varpi (\theta_n + \theta_m) + 1 \right]}{a_m \rho} \right) \right] dy.$$

(25)

$|h_m|^2$ and $|h_n|^2$ are independent random variables that obey variances Ω_m and Ω_n, respectively, and with the aid of order statistics and binomial theorem, the CDF of the mth user's sorted channel gain $|h_m|^2$ can be expressed as:

$$F_{|h_m|^2} (x) = \frac{M!}{(M-m)! (m-1)!} \sum_{i=0}^{M-m} \binom{M-m}{i} \frac{(-1)^i}{m+i} \left(1 - e^{-\frac{x}{\Omega_m}} \right)^{m+i}.$$

(26)

Substituting (26) into (25), the offloading outage probability of the mth user is given by:

$$P_{ipCSI}^m = \frac{M!}{(M-m)! (m-1)!} \sum_{i=0}^{M-m} \sum_{s=0}^{m+i} \binom{M-m}{i} \binom{m+i}{s}$$

$$\times \frac{(-1)^{i+s}}{m+i} \int_0^\infty f_{|h_n|^2} (y) e^{-\frac{s \chi [a_n \rho y + \varpi (\theta_n + \theta_m) + 1]}{a_m \rho \Omega_m}} dy.$$

(27)

The PDF of the nth user's sorted channel gain $|h_n|^2$ is known, and substituting it into the above expression can obtain the offloading outage probability as follows:

$$P_{ipCSI}^m = \frac{M!}{(M-m)! (m-1)!} \frac{M!}{(M-n)! (n-1)! \Omega_n} \sum_{i=0}^{M-m} \sum_{s=0}^{m+i} \sum_{r=0}^{M-n}$$

$$\times \sum_{k=0}^{n+r-1} \binom{M-m}{i} \binom{m+i}{s} \binom{M-n}{r} \binom{n+r-1}{k}$$

$$\times \frac{(-1)^{i+s+r+k}}{m+i} \int_0^\infty e^{-\frac{(1+k)y}{\Omega_n}} e^{-\frac{s \chi [a_n \rho y + \varpi (\theta_n + \theta_m) + 1]}{a_m \rho \Omega_m}} dy.$$

(28)

By sorting the above expression further, (11) can be attained easily. The proof is completed.

Abbreviations
URLLC: Ultra-reliable and low latency communications; OMA: Orthogonal multiple access; NOMA: Non-orthogonal multiple access; SIC: Successive interference cancelation; pCSI: Channel state information; QoS: Quality of service; ipCSI: Imperfect channel state information; MEC: Mobile edge computing; SNR: Signal-to-noise ratio; AWGN: Additive white Gaussian noise; SINR: Signal-to-interference-plus-noise ratio; CPU: Central processing unit; TDMA: Time division multiple address

Authors' contributions
All authors contributed to the design and analysis of the research, to the simulation results, and to the writing of the manuscript. All authors read and approved the final manuscript.

Competing interests
The authors declare that they have no competing interests.

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